Computer algebra independent integration tests

6-Hyperbolic-functions/6.5-Hyperbolic-secant/6.5.7-d-hyper-^m-a+b-c-sech-^n-^p

Nasser M. Abbasi

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [220]. This is test number [180].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (220)	% 0.00 (0)
Mathematica	% 100.00 (220)	% 0.00 (0)
Maple	% 81.82 (180)	% 18.18 (40)
Maxima	% 66.82 (147)	% 33.18 (73)
Fricas	% 95.91 (211)	% 4.09 (9)
Sympy	% 4.55 (10)	% 95.45 (210)
Giac	% 64.09 (141)	% 35.91 (79)
Mupad	% 55.00 (121)	% 45.00 (99)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

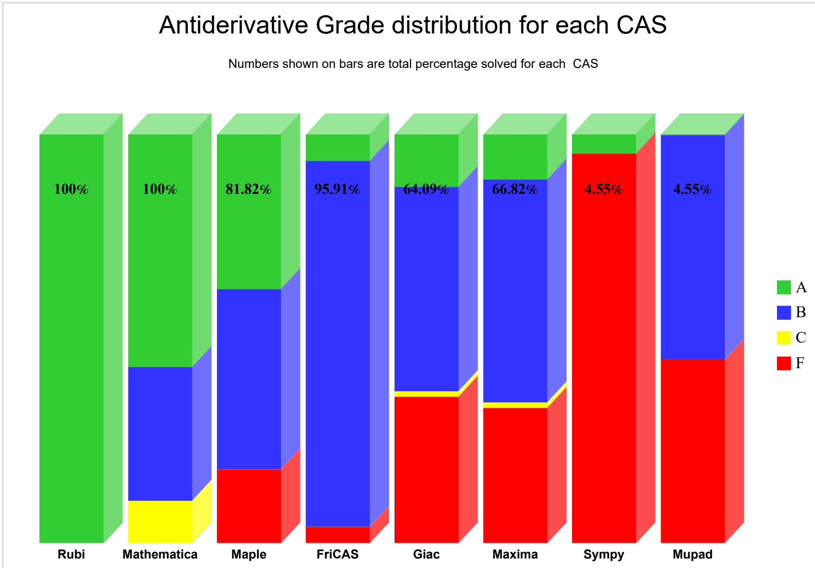
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

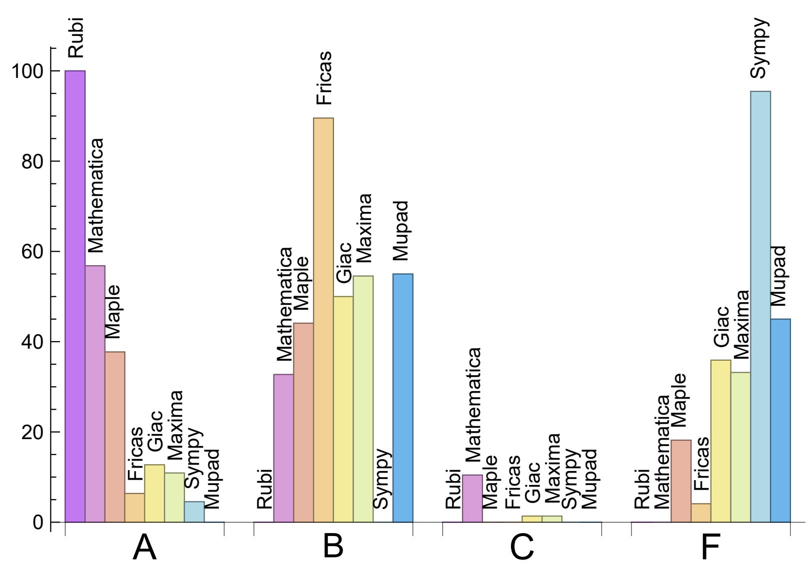
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	56.82	32.73	10.45	0.00
Maple	37.73	44.09	0.00	18.18
Maxima	10.91	54.55	1.36	33.18
Fricas	6.36	89.55	0.00	4.09
Sympy	4.55	0.00	0.00	95.45
Giac	12.73	50.00	1.36	35.91
Mupad	0.00	55.00	0.00	45.00

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	40	100.00 %	0.00 %	0.00 %
Maxima	73	98.63 %	0.00 %	1.37 %
Fricas	9	0.00 %	100.00 %	0.00 %
Sympy	210	79.52 %	20.48 %	0.00 %
Giac	79	0.00 %	0.00 %	100.00 %
Mupad	99	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

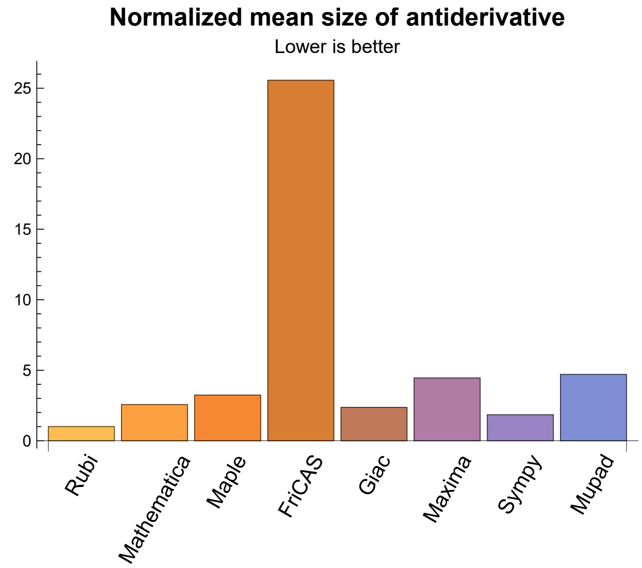
1.3 Performance

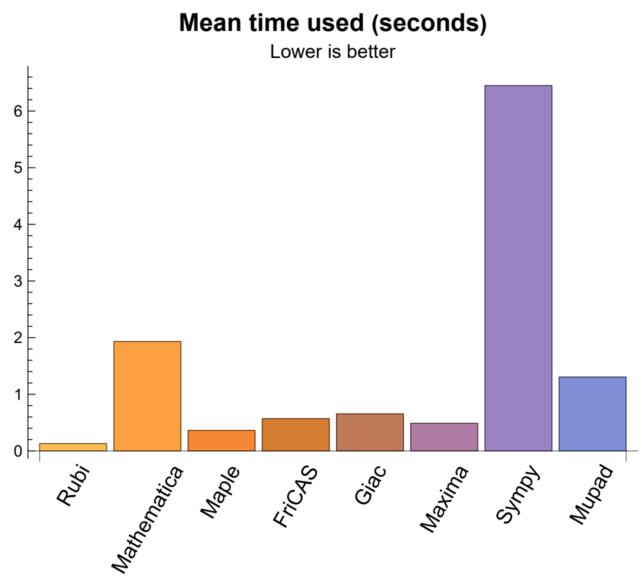
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.13	84.68	1.00	76.00	1.00
Mathematica	1.93	262.51	2.56	130.50	1.77
Maple	0.36	341.99	3.24	146.00	2.07
Maxima	0.49	450.10	4.45	244.00	3.00
Fricas	0.57	2489.24	25.57	1569.00	21.42
Sympy	6.45	87.20	1.84	72.50	1.54
Giac	0.65	200.89	2.36	167.00	2.19
Mupad	1.30	347.68	4.70	238.00	3.79

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {33, 34, 35, 41, 42, 43, 44, 46, 48, 66, 68, 69, 151, 155, 157, 158, 160, 162, 164, 166, 168, 169}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at https://

ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

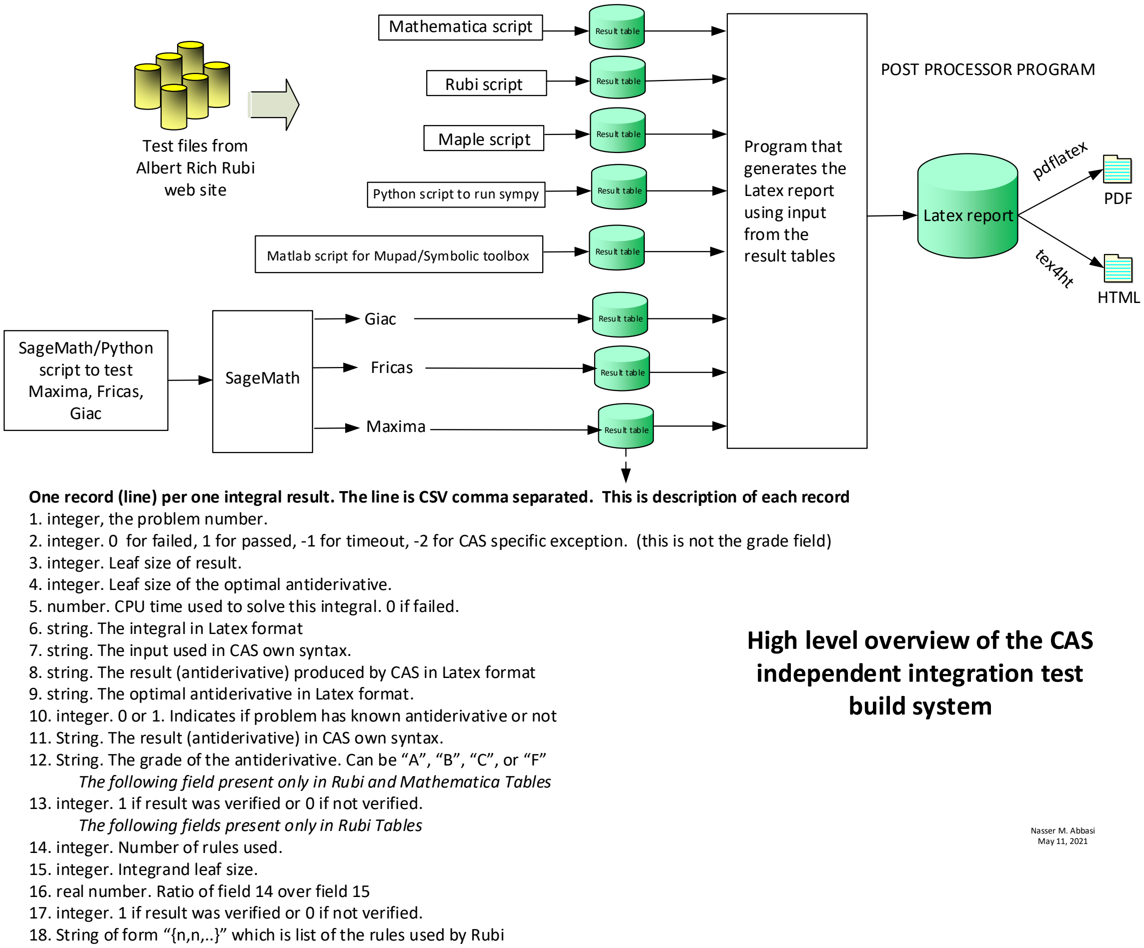
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 6, 8, 9, 10, 11, 12, 15, 18, 20, 21, 23, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 67, 71, 73, 74, 75, 76, 77, 78, 83, 84, 86, 88, 89, 93, 94, 95, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 109, 111, 113, 115, 117, 119, 121, 123, 125, 127, 129, 131, 133, 135, 138, 140, 142, 144, 146, 148, 150, 152, 154, 156, 159, 161, 163, 165, 167, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 182, 183, 184, 185, 186, 187, 188, 190, 192, 193, 194, 195, 197, 201, 202, 203, 204, 208, 209, 210, 211, 212, 214, 216, 217, 219, 220 }

B grade: { 5, 7, 13, 14, 16, 17, 19, 22, 24, 25, 27, 30, 32, 35, 38, 40, 41, 43, 48, 70, 72, 79, 80, 81, 82, 85, 87, 90, 91, 92, 96, 112, 114, 116, 118, 120, 122, 124, 126, 128, 130, 132, 134, 136, 137, 139, 141, 143, 145, 147, 149, 151, 153, 155, 157, 158, 160, 162, 164, 180, 181, 191, 196, 198, 199, 200, 205, 206, 207, 213, 215, 218 }

C grade: { 26, 28, 29, 31, 33, 34, 36, 37, 39, 42, 44, 45, 46, 47, 66, 68, 69, 108, 110, 166, 168, 169, 189 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 23, 28, 29, 31, 36, 44, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 103, 104, 105, 106, 107, 108, 109, 111, 113, 115, 116, 117, 118, 119, 123, 125, 127, 128, 129, 130, 131, 133, 136, 137, 142, 152, 163, 180, 189, 198, 207, 216 }

B grade: { 22, 24, 25, 26, 27, 30, 32, 33, 34, 35, 37, 38, 39, 40, 41, 42, 43, 45, 46, 47, 48, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 110, 112, 114, 120, 121, 122, 124, 126, 132, 134, 135, 138, 139, 140, 141, 143, 144, 145, 146, 147, 148, 149, 150, 151, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175 }

C grade: { }

F grade: { 176, 177, 178, 179, 181, 182, 183, 184, 185, 186, 187, 188, 190, 191, 192, 193, 194, 195, 196, 197, 199, 200, 201, 202, 203, 204, 205, 206, 208, 209, 210, 211, 212, 213, 214, 215, 217, 218, 219, 220 }

2.1.4 Maxima

A grade: { 3, 4, 6, 12, 20, 49, 51, 52, 57, 59, 60, 65, 103, 104, 105, 106, 115, 118, 127, 138, 140, 170, 171, 172 }

B grade: { 1, 2, 5, 7, 8, 9, 10, 11, 13, 14, 15, 16, 17, 18, 19, 21, 22, 23, 24, 25, 27, 30, 32, 33, 35, 38, 40, 41, 43, 46, 48, 50, 53, 54, 55, 56, 58, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 75, 78, 80, 82, 84, 87, 89, 91, 93, 96, 98, 100, 102, 107, 108, 109, 110, 111, 112, 113, 114, 116, 117, 119, 120, 121, 122, 123, 124, 125, 126, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 139, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169 }

C grade: { 173, 174, 175 }

F grade: { 26, 28, 29, 31, 34, 36, 37, 39, 42, 44, 45, 47, 74, 76, 77, 79, 81, 83, 85, 86, 88, 90, 92, 94, 95, 97, 99, 101, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220 }

2.1.5 FriCAS

A grade: { 1, 3, 4, 49, 50, 51, 57, 59, 65, 171, 172, 173, 174, 175 }

B grade: { 2, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 52, 53, 54, 55, 56, 58, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 170, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 187, 188, 189, 190, 191, 192, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 212, 213, 214, 215, 216, 217, 219 }

C grade: { }

F grade: { 47, 48, 168, 169, 186, 193, 211, 218, 220 }

2.1.6 Sympy

A grade: { 51, 103, 105, 113, 115, 125, 127, 142, 207, 216 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 104, 106, 107, 108, 109, 110, 111, 112, 114, 116, 117, 118, 119, 120, 121, 122, 123, 124, 126, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 208, 209, 210, 211, 212, 213, 214, 215, 217, 218, 219, 220 }

2.1.7 Giac

A grade: { 4, 6, 8, 12, 16, 17, 20, 30, 32, 33, 35, 52, 56, 57, 58, 75, 78, 80, 82, 87, 89, 106, 107, 108, 118, 143, 151, 153 }

B grade: { 1, 2, 3, 5, 7, 9, 10, 11, 13, 14, 15, 18, 19, 21, 22, 23, 24, 25, 27, 38, 40, 41, 43, 46, 48, 49, 50, 51, 53, 54, 55, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 84, 91, 93, 96, 98, 100, 102, 103, 104, 105, 109, 110, 111, 112, 113, 114, 115, 116, 117, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 144, 148, 149, 150, 152, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 191, 192 }

C grade: { 173, 174, 175 }

F grade: { 26, 28, 29, 31, 34, 36, 37, 39, 42, 44, 45, 47, 74, 76, 77, 79, 81, 83, 85, 86, 88, 90, 92, 94, 95, 97, 99, 101, 138, 139, 140, 141, 142, 145, 146, 147, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 36, 44, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 152, 163, 180, 189, 198, 207, 216 }

C grade: { }

F grade: { 33, 34, 35, 37, 38, 39, 40, 41, 42, 43, 45, 46, 47, 48, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 148, 149, 150, 151, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 181, 182, 183, 184, 185, 186, 187, 188, 190, 191, 192, 193, 194, 195, 196, 197, 199, 200, 201, 202, 203, 204, 205, 206, 208, 209, 210, 211, 212, 213, 214, 215, 217, 218, 219, 220 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	54	78	129	114	0	130	73
normalized size	1	1.00	0.77	1.11	1.84	1.63	0.00	1.86	1.04
time (sec)	N/A	0.081	0.333	0.327	0.313	0.400	0.000	1.103	1.609

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	53	56	111	85	0	85	44
normalized size	1	1.00	1.20	1.27	2.52	1.93	0.00	1.93	1.00
time (sec)	N/A	0.056	0.052	0.312	0.338	0.441	0.000	0.133	0.177

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	57	45	62	67	0	92	55
normalized size	1	1.00	1.33	1.05	1.44	1.56	0.00	2.14	1.28
time (sec)	N/A	0.056	0.194	0.176	0.327	0.405	0.000	0.164	1.428

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	35	26	36	38	0	45	26
normalized size	1	1.00	1.46	1.08	1.50	1.58	0.00	1.88	1.08
time (sec)	N/A	0.033	0.022	0.090	0.321	0.428	0.000	0.125	0.083

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	67	36	80	180	0	72	79
normalized size	1	1.00	2.48	1.33	2.96	6.67	0.00	2.67	2.93
time (sec)	N/A	0.045	0.048	0.196	0.325	0.418	0.000	0.126	1.412

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	37	44	39	91	0	34	34
normalized size	1	1.00	1.37	1.63	1.44	3.37	0.00	1.26	1.26
time (sec)	N/A	0.046	0.073	0.381	0.333	0.400	0.000	0.143	1.376

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	131	70	198	924	0	142	160
normalized size	1	1.00	2.43	1.30	3.67	17.11	0.00	2.63	2.96
time (sec)	N/A	0.071	0.049	0.382	0.342	0.431	0.000	0.154	0.166

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	84	73	187	246	0	80	172
normalized size	1	1.00	1.87	1.62	4.16	5.47	0.00	1.78	3.82
time (sec)	N/A	0.058	0.054	0.437	0.339	0.384	0.000	0.135	1.401

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	153	109	211	342	0	231	269
normalized size	1	1.00	1.34	0.96	1.85	3.00	0.00	2.03	2.36
time (sec)	N/A	0.138	1.663	0.343	0.338	0.417	0.000	0.177	0.270

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	83	93	266	212	0	140	201
normalized size	1	1.00	1.15	1.29	3.69	2.94	0.00	1.94	2.79
time (sec)	N/A	0.087	0.526	0.329	0.340	0.394	0.000	0.189	1.496

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	126	90	160	252	0	144	236
normalized size	1	1.00	1.73	1.23	2.19	3.45	0.00	1.97	3.23
time (sec)	N/A	0.107	0.952	0.336	0.332	0.409	0.000	0.177	0.159

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	59	43	65	133	0	75	45
normalized size	1	1.00	1.31	0.96	1.44	2.96	0.00	1.67	1.00
time (sec)	N/A	0.046	0.173	0.096	0.321	0.403	0.000	0.142	1.466

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	108	72	197	1148	0	139	232
normalized size	1	1.00	2.08	1.38	3.79	22.08	0.00	2.67	4.46
time (sec)	N/A	0.077	0.577	0.215	0.337	0.425	0.000	0.140	1.503

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	109	91	140	284	0	111	215
normalized size	1	1.00	2.18	1.82	2.80	5.68	0.00	2.22	4.30
time (sec)	N/A	0.062	1.767	0.490	0.337	0.416	0.000	0.151	1.487

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	144	126	354	2930	0	228	316
normalized size	1	1.00	1.38	1.21	3.40	28.17	0.00	2.19	3.04
time (sec)	N/A	0.136	1.645	0.425	0.339	0.453	0.000	0.157	1.655

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	151	138	285	408	0	115	115
normalized size	1	1.00	2.01	1.84	3.80	5.44	0.00	1.53	1.53
time (sec)	N/A	0.083	1.320	0.549	0.343	0.400	0.000	0.175	0.213

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	651	182	422	727	0	339	686
normalized size	1	1.00	3.58	1.00	2.32	3.99	0.00	1.86	3.77
time (sec)	N/A	0.232	2.515	0.440	0.345	0.453	0.000	0.218	0.367

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	119	130	489	403	0	193	348
normalized size	1	1.00	1.20	1.31	4.94	4.07	0.00	1.95	3.52
time (sec)	N/A	0.111	1.253	0.371	0.342	0.415	0.000	0.209	0.330

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	143	480	145	443	595	0	278	592
normalized size	1	1.28	4.29	1.29	3.96	5.31	0.00	2.48	5.29
time (sec)	N/A	0.193	1.916	0.418	0.339	0.429	0.000	0.202	0.230

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	93	58	94	276	0	101	288
normalized size	1	1.00	1.45	0.91	1.47	4.31	0.00	1.58	4.50
time (sec)	N/A	0.059	0.257	0.130	0.322	0.394	0.000	0.191	1.497

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	134	118	358	3443	0	228	434
normalized size	1	1.00	1.61	1.42	4.31	41.48	0.00	2.75	5.23
time (sec)	N/A	0.100	1.281	0.245	0.329	0.445	0.000	0.168	1.532

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	380	148	358	622	0	249	644
normalized size	1	1.00	5.43	2.11	5.11	8.89	0.00	3.56	9.20
time (sec)	N/A	0.073	2.793	0.591	0.335	0.470	0.000	0.181	1.479

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	224	192	556	6717	0	341	536
normalized size	1	1.00	1.56	1.33	3.86	46.65	0.00	2.37	3.72
time (sec)	N/A	0.178	3.887	0.443	0.338	0.596	0.000	0.197	1.644

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	213	213	664	955	0	355	745
normalized size	1	1.00	2.05	2.05	6.38	9.18	0.00	3.41	7.16
time (sec)	N/A	0.105	2.466	0.659	0.344	0.587	0.000	0.189	1.602

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	294	708	526	1681	0	220	328
normalized size	1	1.00	2.51	6.05	4.50	14.37	0.00	1.88	2.80
time (sec)	N/A	0.196	2.483	0.411	0.462	0.593	0.000	2.883	2.426

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	372	261	0	1246	0	0	473
normalized size	1	1.00	5.24	3.68	0.00	17.55	0.00	0.00	6.66
time (sec)	N/A	0.105	2.226	0.336	0.000	0.492	0.000	0.000	1.841

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	236	383	352	805	0	132	276
normalized size	1	1.00	3.15	5.11	4.69	10.73	0.00	1.76	3.68
time (sec)	N/A	0.110	0.997	0.319	0.424	0.483	0.000	1.603	2.029

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	328	44	0	595	0	0	42
normalized size	1	1.00	6.98	0.94	0.00	12.66	0.00	0.00	0.89
time (sec)	N/A	0.048	1.015	0.079	0.000	0.512	0.000	0.000	0.145

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	232	67	0	533	0	0	616
normalized size	1	1.00	4.22	1.22	0.00	9.69	0.00	0.00	11.20
time (sec)	N/A	0.082	0.928	0.250	0.000	0.507	0.000	0.000	2.181

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	179	147	100	588	0	75	847
normalized size	1	1.00	3.38	2.77	1.89	11.09	0.00	1.42	15.98
time (sec)	N/A	0.071	0.752	0.300	0.426	0.557	0.000	0.633	2.467

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	338	134	0	1881	0	0	1586
normalized size	1	1.00	3.89	1.54	0.00	21.62	0.00	0.00	18.23
time (sec)	N/A	0.118	2.050	0.325	0.000	0.464	0.000	0.000	3.070

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	216	258	195	1753	0	123	248
normalized size	1	1.00	2.88	3.44	2.60	23.37	0.00	1.64	3.31
time (sec)	N/A	0.099	2.082	0.357	0.456	0.524	0.000	0.669	2.170

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F(-1)	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	1330	1034	1299	5169	0	323	-1
normalized size	1	1.00	6.86	5.33	6.70	26.64	0.00	1.66	-0.01
time (sec)	N/A	0.273	14.252	0.395	0.514	0.562	0.000	3.728	0.000

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	861	561	0	3804	0	0	-1
normalized size	1	1.00	7.55	4.92	0.00	33.37	0.00	0.00	-0.01
time (sec)	N/A	0.144	4.819	0.368	0.000	0.486	0.000	0.000	0.000

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	791	543	696	2925	0	234	-1
normalized size	1	1.00	6.04	4.15	5.31	22.33	0.00	1.79	-0.01
time (sec)	N/A	0.196	11.332	0.371	0.459	0.502	0.000	2.051	0.000

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	479	74	0	1780	0	0	71
normalized size	1	1.00	5.70	0.88	0.00	21.19	0.00	0.00	0.85
time (sec)	N/A	0.065	2.763	0.172	0.000	0.544	0.000	0.000	1.645

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	377	431	0	2376	0	0	-1
normalized size	1	1.00	3.81	4.35	0.00	24.00	0.00	0.00	-0.01
time (sec)	N/A	0.130	1.251	0.336	0.000	0.580	0.000	0.000	0.000

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	220	313	262	2407	0	239	-1
normalized size	1	1.00	2.39	3.40	2.85	26.16	0.00	2.60	-0.01
time (sec)	N/A	0.084	2.625	0.394	0.461	0.623	0.000	0.732	0.000

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	462	496	0	6878	0	0	-1
normalized size	1	1.00	3.14	3.37	0.00	46.79	0.00	0.00	-0.01
time (sec)	N/A	0.206	2.227	0.423	0.000	0.733	0.000	0.000	0.000

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	295	577	430	6143	0	253	-1
normalized size	1	1.00	2.40	4.69	3.50	49.94	0.00	2.06	-0.01
time (sec)	N/A	0.195	6.078	0.504	0.520	0.525	0.000	0.772	0.000

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	3080	1668	2468	12353	0	518	-1
normalized size	1	1.00	12.73	6.89	10.20	51.05	0.00	2.14	-0.00
time (sec)	N/A	0.399	27.316	0.426	0.603	0.637	0.000	6.128	0.000

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	B	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	1217	1296	0	8667	0	0	-1
normalized size	1	1.00	7.90	8.42	0.00	56.28	0.00	0.00	-0.01
time (sec)	N/A	0.218	10.382	0.381	0.000	0.564	0.000	0.000	0.000

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	2544	1329	1373	9730	0	370	-1
normalized size	1	1.00	13.60	7.11	7.34	52.03	0.00	1.98	-0.01
time (sec)	N/A	0.292	17.845	0.389	0.533	0.596	0.000	3.702	0.000

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	453	107	0	4829	0	0	103
normalized size	1	1.00	3.91	0.92	0.00	41.63	0.00	0.00	0.89
time (sec)	N/A	0.084	9.604	0.168	0.000	0.514	0.000	0.000	1.607

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	440	1476	0	8742	0	0	-1
normalized size	1	1.00	2.86	9.58	0.00	56.77	0.00	0.00	-0.01
time (sec)	N/A	0.223	2.498	0.359	0.000	0.637	0.000	0.000	0.000

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	981	816	533	7275	0	347	-1
normalized size	1	1.00	7.79	6.48	4.23	57.74	0.00	2.75	-0.01
time (sec)	N/A	0.100	6.920	0.406	0.528	0.549	0.000	1.761	0.000

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	524	1555	0	0	0	0	-1
normalized size	1	1.00	2.46	7.30	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.342	4.117	0.447	0.000	0.000	0.000	0.000	0.000

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	F(-1)	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	985	1443	782	0	0	406	-1
normalized size	1	1.00	5.97	8.75	4.74	0.00	0.00	2.46	-0.01
time (sec)	N/A	0.273	5.530	0.483	0.595	0.000	0.000	1.664	0.000

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	45	66	97	61	0	116	50
normalized size	1	1.00	0.74	1.08	1.59	1.00	0.00	1.90	0.82
time (sec)	N/A	0.046	0.093	0.457	0.308	0.397	0.000	0.137	0.129

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	50	34	85	41	0	72	34
normalized size	1	1.00	1.67	1.13	2.83	1.37	0.00	2.40	1.13
time (sec)	N/A	0.050	0.018	0.426	0.309	0.394	0.000	0.157	0.085

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	33	37	38	28	60	66	23
normalized size	1	1.00	1.06	1.19	1.23	0.90	1.94	2.13	0.74
time (sec)	N/A	0.031	0.030	0.306	0.310	0.405	11.245	0.146	0.077

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	35	26	28	93	0	36	62
normalized size	1	1.00	1.46	1.08	1.17	3.88	0.00	1.50	2.58
time (sec)	N/A	0.030	0.018	0.286	0.412	0.450	0.000	0.146	1.436

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	48	45	81	321	0	84	124
normalized size	1	1.00	1.20	1.12	2.02	8.02	0.00	2.10	3.10
time (sec)	N/A	0.027	0.024	0.300	0.403	0.409	0.000	0.124	0.159

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	43	39	34	112	158	0	61	61
normalized size	1	1.43	1.30	1.13	3.73	5.27	0.00	2.03	2.03
time (sec)	N/A	0.040	0.012	0.336	0.317	0.383	0.000	0.124	1.382

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	60	83	184	1112	0	156	283
normalized size	1	1.00	0.86	1.19	2.63	15.89	0.00	2.23	4.04
time (sec)	N/A	0.050	0.105	0.417	0.410	0.422	0.000	0.158	1.383

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	65	71	56	300	343	0	85	292
normalized size	1	1.30	1.42	1.12	6.00	6.86	0.00	1.70	5.84
time (sec)	N/A	0.046	0.014	0.366	0.321	0.384	0.000	0.145	1.435

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	58	79	105	78	0	151	66
normalized size	1	1.00	0.71	0.96	1.28	0.95	0.00	1.84	0.80
time (sec)	N/A	0.093	0.131	0.397	0.306	0.399	0.000	0.155	1.371

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	72	66	105	414	0	94	114
normalized size	1	1.00	1.47	1.35	2.14	8.45	0.00	1.92	2.33
time (sec)	N/A	0.059	0.024	0.364	0.412	0.409	0.000	0.151	0.166

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	52	51	63	80	0	128	65
normalized size	1	1.00	1.11	1.09	1.34	1.70	0.00	2.72	1.38
time (sec)	N/A	0.078	0.149	0.355	0.322	0.392	0.000	0.173	0.164

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	80	63	101	653	0	112	172
normalized size	1	1.00	1.43	1.12	1.80	11.66	0.00	2.00	3.07
time (sec)	N/A	0.068	0.041	0.365	0.420	0.418	0.000	0.139	1.439

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	71	106	201	1372	0	170	303
normalized size	1	1.00	0.79	1.18	2.23	15.24	0.00	1.89	3.37
time (sec)	N/A	0.084	0.133	0.385	0.423	0.429	0.000	0.141	1.518

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	93	70	324	404	0	156	452
normalized size	1	1.00	1.75	1.32	6.11	7.62	0.00	2.94	8.53
time (sec)	N/A	0.066	0.031	0.418	0.335	0.406	0.000	0.173	1.465

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	104	169	348	2946	0	293	569
normalized size	1	1.00	0.81	1.32	2.72	23.02	0.00	2.29	4.45
time (sec)	N/A	0.149	0.244	0.460	0.420	0.432	0.000	0.160	1.570

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	144	102	671	677	0	197	692
normalized size	1	1.00	1.80	1.28	8.39	8.46	0.00	2.46	8.65
time (sec)	N/A	0.076	0.024	0.445	0.329	0.390	0.000	0.159	1.416

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	70	93	130	153	0	177	117
normalized size	1	1.00	0.83	1.11	1.55	1.82	0.00	2.11	1.39
time (sec)	N/A	0.111	0.427	0.479	0.324	0.395	0.000	0.214	1.529

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F(-1)	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	483	103	179	1409	0	163	218
normalized size	1	1.00	5.96	1.27	2.21	17.40	0.00	2.01	2.69
time (sec)	N/A	0.093	6.882	0.454	0.415	0.423	0.000	0.213	0.223

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	64	77	160	270	0	152	221
normalized size	1	1.00	0.89	1.07	2.22	3.75	0.00	2.11	3.07
time (sec)	N/A	0.092	0.508	0.461	0.319	0.427	0.000	0.195	0.167

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	575	125	221	1992	0	199	344
normalized size	1	1.00	6.18	1.34	2.38	21.42	0.00	2.14	3.70
time (sec)	N/A	0.103	7.875	0.507	0.412	0.466	0.000	0.191	0.191

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	1430	193	365	3465	0	310	535
normalized size	1	1.00	9.73	1.31	2.48	23.57	0.00	2.11	3.64
time (sec)	N/A	0.143	9.836	0.460	0.414	0.473	0.000	0.189	1.472

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	319	116	695	816	0	302	978
normalized size	1	1.00	4.31	1.57	9.39	11.03	0.00	4.08	13.22
time (sec)	N/A	0.080	1.486	0.571	0.329	0.437	0.000	0.191	1.495

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	297	280	556	6114	0	485	931
normalized size	1	1.00	1.52	1.43	2.84	31.19	0.00	2.47	4.75
time (sec)	N/A	0.232	9.396	0.472	0.421	0.505	0.000	0.177	1.599

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	348	158	1245	1190	0	360	1333
normalized size	1	1.00	3.22	1.46	11.53	11.02	0.00	3.33	12.34
time (sec)	N/A	0.094	1.756	0.520	0.343	0.417	0.000	0.182	1.473

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	95	493	526	1713	0	208	260
normalized size	1	1.00	0.81	4.21	4.50	14.64	0.00	1.78	2.22
time (sec)	N/A	0.189	0.495	0.541	0.457	0.458	0.000	2.623	1.998

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	79	256	0	1616	0	0	332
normalized size	1	1.00	1.04	3.37	0.00	21.26	0.00	0.00	4.37
time (sec)	N/A	0.090	0.274	0.489	0.000	0.465	0.000	0.000	2.213

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	67	278	352	829	0	125	206
normalized size	1	1.00	0.89	3.71	4.69	11.05	0.00	1.67	2.75
time (sec)	N/A	0.116	0.235	0.464	0.424	0.457	0.000	1.574	1.968

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	128	0	718	0	0	277
normalized size	1	1.00	1.00	2.46	0.00	13.81	0.00	0.00	5.33
time (sec)	N/A	0.061	0.124	0.402	0.000	0.459	0.000	0.000	1.739

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	82	0	487	0	0	108
normalized size	1	1.00	1.00	2.28	0.00	13.53	0.00	0.00	3.00
time (sec)	N/A	0.042	0.066	0.278	0.000	0.423	0.000	0.000	1.896

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	104	66	411	0	47	125
normalized size	1	1.00	1.00	2.89	1.83	11.42	0.00	1.31	3.47
time (sec)	N/A	0.061	0.080	0.252	0.417	0.429	0.000	0.639	0.570

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	194	108	0	526	0	0	307
normalized size	1	1.00	3.53	1.96	0.00	9.56	0.00	0.00	5.58
time (sec)	N/A	0.069	0.712	0.252	0.000	0.437	0.000	0.000	1.815

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	182	141	91	645	0	72	166
normalized size	1	1.00	3.50	2.71	1.75	12.40	0.00	1.38	3.19
time (sec)	N/A	0.070	0.632	0.217	0.426	0.426	0.000	0.622	0.475

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	213	189	0	1518	0	0	946
normalized size	1	1.00	2.48	2.20	0.00	17.65	0.00	0.00	11.00
time (sec)	N/A	0.103	1.836	0.281	0.000	0.489	0.000	0.000	3.500

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	214	316	160	1905	0	118	334
normalized size	1	1.00	2.78	4.10	2.08	24.74	0.00	1.53	4.34
time (sec)	N/A	0.089	2.178	0.282	0.461	0.488	0.000	0.639	1.990

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	113	517	0	5842	0	0	-1
normalized size	1	1.00	0.90	4.14	0.00	46.74	0.00	0.00	-0.01
time (sec)	N/A	0.154	0.919	0.533	0.000	0.539	0.000	0.000	0.000

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	103	557	696	3739	0	323	-1
normalized size	1	1.00	0.72	3.87	4.83	25.97	0.00	2.24	-0.01
time (sec)	N/A	0.235	1.379	0.513	0.466	0.492	0.000	2.018	0.000

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	234	385	0	3154	0	0	-1
normalized size	1	1.00	2.34	3.85	0.00	31.54	0.00	0.00	-0.01
time (sec)	N/A	0.132	1.812	0.514	0.000	0.487	0.000	0.000	0.000

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	124	331	0	1856	0	0	-1
normalized size	1	1.00	1.51	4.04	0.00	22.63	0.00	0.00	-0.01
time (sec)	N/A	0.066	0.314	0.345	0.000	0.453	0.000	0.000	0.000

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	187	263	150	1489	0	130	-1
normalized size	1	1.00	2.53	3.55	2.03	20.12	0.00	1.76	-0.01
time (sec)	N/A	0.074	1.015	0.253	0.456	0.444	0.000	0.744	0.000

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	108	241	0	1570	0	0	-1
normalized size	1	1.00	1.48	3.30	0.00	21.51	0.00	0.00	-0.01
time (sec)	N/A	0.071	0.228	0.270	0.000	0.475	0.000	0.000	0.000

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	88	374	165	1569	0	139	-1
normalized size	1	1.00	1.06	4.51	1.99	18.90	0.00	1.67	-0.01
time (sec)	N/A	0.090	0.216	0.294	0.478	0.451	0.000	0.769	0.000

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	282	361	0	2069	0	0	-1
normalized size	1	1.00	2.79	3.57	0.00	20.49	0.00	0.00	-0.01
time (sec)	N/A	0.117	2.529	0.272	0.000	0.498	0.000	0.000	0.000

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	229	419	244	2958	0	225	-1
normalized size	1	1.00	2.27	4.15	2.42	29.29	0.00	2.23	-0.01
time (sec)	N/A	0.142	3.813	0.276	0.554	0.480	0.000	0.789	0.000

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	489	448	0	6499	0	0	-1
normalized size	1	1.00	3.20	2.93	0.00	42.48	0.00	0.00	-0.01
time (sec)	N/A	0.200	5.193	0.288	0.000	0.581	0.000	0.000	0.000

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	156	1435	1373	11740	0	395	-1
normalized size	1	1.00	0.76	7.03	6.73	57.55	0.00	1.94	-0.00
time (sec)	N/A	0.377	4.229	0.591	0.546	0.655	0.000	3.671	0.000

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	292	1238	0	9856	0	0	-1
normalized size	1	1.00	1.90	8.04	0.00	64.00	0.00	0.00	-0.01
time (sec)	N/A	0.185	3.653	0.559	0.000	0.639	0.000	0.000	0.000

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	214	1172	0	6806	0	0	-1
normalized size	1	1.00	1.51	8.25	0.00	47.93	0.00	0.00	-0.01
time (sec)	N/A	0.137	2.936	0.398	0.000	0.541	0.000	0.000	0.000

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	258	615	353	5109	0	282	-1
normalized size	1	1.00	2.39	5.69	3.27	47.31	0.00	2.61	-0.01
time (sec)	N/A	0.090	2.169	0.344	0.514	0.508	0.000	1.754	0.000

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	159	1038	0	6037	0	0	-1
normalized size	1	1.00	1.29	8.44	0.00	49.08	0.00	0.00	-0.01
time (sec)	N/A	0.108	0.789	0.342	0.000	0.526	0.000	0.000	0.000

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	250	1084	369	5447	0	274	-1
normalized size	1	1.00	2.00	8.67	2.95	43.58	0.00	2.19	-0.01
time (sec)	N/A	0.107	3.386	0.284	0.535	0.519	0.000	1.734	0.000

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	125	592	0	5006	0	0	-1
normalized size	1	1.00	1.18	5.58	0.00	47.23	0.00	0.00	-0.01
time (sec)	N/A	0.092	0.287	0.320	0.000	0.512	0.000	0.000	0.000

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	125	1245	395	5887	0	302	-1
normalized size	1	1.00	0.87	8.65	2.74	40.88	0.00	2.10	-0.01
time (sec)	N/A	0.142	1.012	0.345	1.367	0.528	0.000	1.751	0.000

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	247	1201	0	7993	0	0	-1
normalized size	1	1.00	1.61	7.85	0.00	52.24	0.00	0.00	-0.01
time (sec)	N/A	0.205	4.721	0.299	0.000	0.624	0.000	0.000	0.000

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	57	98	92	327	0	105	433
normalized size	1	1.00	1.19	2.04	1.92	6.81	0.00	2.19	9.02
time (sec)	N/A	0.064	0.030	0.329	0.615	0.421	0.000	0.202	1.545

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	45	64	78	1072	80	116	173
normalized size	1	1.00	0.92	1.31	1.59	21.88	1.63	2.37	3.53
time (sec)	N/A	0.056	0.022	0.201	0.747	0.430	1.662	0.179	0.125

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	41	60	42	155	0	69	163
normalized size	1	1.00	1.28	1.88	1.31	4.84	0.00	2.16	5.09
time (sec)	N/A	0.057	0.017	0.328	0.405	0.398	0.000	0.161	1.503

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	29	27	359	42	80	72
normalized size	1	1.00	1.00	1.00	0.93	12.38	1.45	2.76	2.48
time (sec)	N/A	0.028	0.021	0.138	0.331	0.419	0.487	0.134	1.470

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	23	36	0	23	23
normalized size	1	1.00	1.00	1.07	1.53	2.40	0.00	1.53	1.53
time (sec)	N/A	0.013	0.003	0.269	0.370	0.403	0.000	0.121	1.395

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	44	26	65	69	0	56	167
normalized size	1	1.00	1.57	0.93	2.32	2.46	0.00	2.00	5.96
time (sec)	N/A	0.051	0.041	0.256	0.534	0.420	0.000	0.153	0.172

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	41	30	47	39	0	27	25
normalized size	1	1.00	2.28	1.67	2.61	2.17	0.00	1.50	1.39
time (sec)	N/A	0.059	0.031	0.333	0.336	0.414	0.000	0.146	0.111

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	52	42	108	378	0	81	76
normalized size	1	1.00	1.68	1.35	3.48	12.19	0.00	2.61	2.45
time (sec)	N/A	0.057	0.179	0.329	0.333	0.419	0.000	0.208	1.414

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	49	70	170	140	0	67	161
normalized size	1	1.00	1.44	2.06	5.00	4.12	0.00	1.97	4.74
time (sec)	N/A	0.063	0.023	0.389	0.344	0.414	0.000	0.214	1.492

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	62	78	251	1099	0	117	179
normalized size	1	1.00	1.22	1.53	4.92	21.55	0.00	2.29	3.51
time (sec)	N/A	0.078	0.263	0.313	0.512	0.426	0.000	0.232	0.110

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	395	181	649	721	0	275	1022
normalized size	1	1.00	5.13	2.35	8.43	9.36	0.00	3.57	13.27
time (sec)	N/A	0.105	1.185	0.437	0.548	0.417	0.000	0.249	0.183

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	107	110	333	2591	129	241	349
normalized size	1	1.00	1.39	1.43	4.32	33.65	1.68	3.13	4.53
time (sec)	N/A	0.095	0.290	0.242	0.439	0.450	5.089	0.202	1.522

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	281	115	325	435	0	193	513
normalized size	1	1.00	4.76	1.95	5.51	7.37	0.00	3.27	8.69
time (sec)	N/A	0.098	0.864	0.405	0.524	0.423	0.000	0.171	0.142

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	81	48	55	1180	63	159	182
normalized size	1	1.00	1.69	1.00	1.15	24.58	1.31	3.31	3.79
time (sec)	N/A	0.050	0.145	0.158	0.482	0.418	1.665	0.142	1.689

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	106	47	120	176	0	79	163
normalized size	1	1.00	2.65	1.18	3.00	4.40	0.00	1.98	4.08
time (sec)	N/A	0.034	0.377	0.365	0.457	0.396	0.000	0.144	1.458

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	84	60	161	665	0	171	308
normalized size	1	1.00	1.58	1.13	3.04	12.55	0.00	3.23	5.81
time (sec)	N/A	0.080	0.246	0.262	0.698	0.458	0.000	0.171	0.296

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	82	64	71	106	0	65	60
normalized size	1	1.00	2.28	1.78	1.97	2.94	0.00	1.81	1.67
time (sec)	N/A	0.085	0.741	0.388	0.545	0.438	0.000	0.201	1.422

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	82	78	206	637	0	161	240
normalized size	1	1.00	1.49	1.42	3.75	11.58	0.00	2.93	4.36
time (sec)	N/A	0.089	0.200	0.311	0.480	0.449	0.000	0.252	0.220

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	160	96	268	201	0	97	183
normalized size	1	1.00	3.48	2.09	5.83	4.37	0.00	2.11	3.98
time (sec)	N/A	0.092	0.810	0.360	0.427	0.427	0.000	0.272	1.424

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	77	102	282	1252	0	147	207
normalized size	1	1.00	1.48	1.96	5.42	24.08	0.00	2.83	3.98
time (sec)	N/A	0.094	0.232	0.321	0.449	0.444	0.000	0.337	1.466

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	256	163	613	425	0	167	511
normalized size	1	1.00	4.00	2.55	9.58	6.64	0.00	2.61	7.98
time (sec)	N/A	0.097	1.101	0.445	0.542	0.422	0.000	0.374	1.439

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	107	164	696	2548	0	216	377
normalized size	1	1.00	1.24	1.91	8.09	29.63	0.00	2.51	4.38
time (sec)	N/A	0.125	0.516	0.337	0.408	0.478	0.000	0.459	1.530

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	301	274	1453	1323	0	472	1834
normalized size	1	1.00	2.74	2.49	13.21	12.03	0.00	4.29	16.67
time (sec)	N/A	0.118	5.936	0.554	0.385	0.435	0.000	0.305	1.623

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	128	156	652	4658	178	384	573
normalized size	1	1.00	1.24	1.51	6.33	45.22	1.73	3.73	5.56
time (sec)	N/A	0.098	0.754	0.298	0.462	0.513	13.795	0.273	1.638

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	479	180	788	881	0	356	1133
normalized size	1	1.00	5.21	1.96	8.57	9.58	0.00	3.87	12.32
time (sec)	N/A	0.110	1.790	0.515	0.442	0.417	0.000	0.197	0.213

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	100	67	85	2519	87	268	347
normalized size	1	1.00	1.41	0.94	1.20	35.48	1.23	3.77	4.89
time (sec)	N/A	0.058	0.301	0.161	0.431	0.441	4.911	0.186	0.215

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	268	83	332	470	0	182	502
normalized size	1	1.00	3.67	1.14	4.55	6.44	0.00	2.49	6.88
time (sec)	N/A	0.047	0.928	0.420	0.411	0.425	0.000	0.154	1.442

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	114	111	300	2376	0	325	360
normalized size	1	1.00	1.36	1.32	3.57	28.29	0.00	3.87	4.29
time (sec)	N/A	0.107	0.663	0.308	0.469	0.457	0.000	0.183	1.638

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	126	111	172	359	0	132	234
normalized size	1	1.00	2.07	1.82	2.82	5.89	0.00	2.16	3.84
time (sec)	N/A	0.103	1.799	0.573	0.413	0.413	0.000	0.211	0.142

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	110	137	314	1701	0	292	324
normalized size	1	1.00	1.36	1.69	3.88	21.00	0.00	3.60	4.00
time (sec)	N/A	0.116	1.242	0.359	0.470	0.443	0.000	0.289	1.723

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	343	149	366	354	0	155	260
normalized size	1	1.00	5.72	2.48	6.10	5.90	0.00	2.58	4.33
time (sec)	N/A	0.101	1.726	0.483	0.521	0.401	0.000	0.343	1.479

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	101	153	422	1830	0	248	384
normalized size	1	1.00	1.25	1.89	5.21	22.59	0.00	3.06	4.74
time (sec)	N/A	0.121	0.907	0.318	0.497	0.449	0.000	0.410	1.636

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	303	199	826	521	0	210	547
normalized size	1	1.00	4.39	2.88	11.97	7.55	0.00	3.04	7.93
time (sec)	N/A	0.109	1.087	0.425	0.361	0.408	0.000	0.427	1.603

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	98	189	727	2632	0	239	411
normalized size	1	1.00	1.27	2.45	9.44	34.18	0.00	3.10	5.34
time (sec)	N/A	0.121	0.797	0.363	0.420	0.464	0.000	0.586	1.614

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	455	129	703	941	0	334	1083
normalized size	1	1.00	4.10	1.16	6.33	8.48	0.00	3.01	9.76
time (sec)	N/A	0.070	1.666	0.525	0.567	0.424	0.000	0.151	0.196

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	724	185	1277	1652	0	537	1952
normalized size	1	1.00	4.44	1.13	7.83	10.13	0.00	3.29	11.98
time (sec)	N/A	0.095	6.567	0.606	0.349	0.427	0.000	0.167	0.317

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	98	331	131	736	0	0	421
normalized size	1	1.00	1.40	4.73	1.87	10.51	0.00	0.00	6.01
time (sec)	N/A	0.112	0.307	0.316	0.459	0.485	0.000	0.000	1.812

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	196	386	637	683	0	0	183
normalized size	1	1.00	3.32	6.54	10.80	11.58	0.00	0.00	3.10
time (sec)	N/A	0.178	1.086	0.349	0.636	0.449	0.000	0.000	1.873

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	41	196	77	112	0	0	238
normalized size	1	1.00	0.91	4.36	1.71	2.49	0.00	0.00	5.29
time (sec)	N/A	0.088	0.117	0.333	0.474	0.460	0.000	0.000	1.650

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	174	251	291	419	0	0	105
normalized size	1	1.00	3.78	5.46	6.33	9.11	0.00	0.00	2.28
time (sec)	N/A	0.140	0.304	0.358	0.497	0.455	0.000	0.000	0.426

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	26	38	51	76	124	0	51
normalized size	1	1.00	1.13	1.65	2.22	3.30	5.39	0.00	2.22
time (sec)	N/A	0.035	0.183	0.146	0.378	0.421	7.035	0.000	0.345

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	172	149	83	436	0	64	470
normalized size	1	1.00	3.74	3.24	1.80	9.48	0.00	1.39	10.22
time (sec)	N/A	0.046	0.250	0.277	0.467	0.441	0.000	0.390	2.161

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	42	133	100	115	0	97	228
normalized size	1	1.00	0.91	2.89	2.17	2.50	0.00	2.11	4.96
time (sec)	N/A	0.087	0.092	0.375	0.421	0.461	0.000	0.334	1.817

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	193	189	429	749	0	0	977
normalized size	1	1.00	3.11	3.05	6.92	12.08	0.00	0.00	15.76
time (sec)	N/A	0.180	1.161	0.457	0.477	0.457	0.000	0.000	3.389

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	100	199	187	862	0	0	523
normalized size	1	1.00	1.37	2.73	2.56	11.81	0.00	0.00	7.16
time (sec)	N/A	0.122	0.236	0.476	0.435	0.524	0.000	0.000	2.069

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	380	298	1435	2705	0	0	779
normalized size	1	1.00	4.37	3.43	16.49	31.09	0.00	0.00	8.95
time (sec)	N/A	0.286	3.325	0.493	0.627	0.465	0.000	0.000	4.146

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	109	351	154	853	0	208	-1
normalized size	1	1.00	1.43	4.62	2.03	11.22	0.00	2.74	-0.01
time (sec)	N/A	0.117	0.454	0.372	0.446	0.495	0.000	1.376	0.000

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	228	676	1053	1479	0	187	-1
normalized size	1	1.00	2.51	7.43	11.57	16.25	0.00	2.05	-0.01
time (sec)	N/A	0.194	2.133	0.402	0.837	0.450	0.000	1.162	0.000

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	81	186	108	485	0	121	-1
normalized size	1	1.00	1.59	3.65	2.12	9.51	0.00	2.37	-0.02
time (sec)	N/A	0.090	0.760	0.351	0.375	0.416	0.000	0.889	0.000

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	326	411	597	1846	0	147	-1
normalized size	1	1.00	3.84	4.84	7.02	21.72	0.00	1.73	-0.01
time (sec)	N/A	0.169	4.629	0.351	0.547	0.473	0.000	0.718	0.000

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	79	60	106	476	0	121	53
normalized size	1	1.00	1.61	1.22	2.16	9.71	0.00	2.47	1.08
time (sec)	N/A	0.062	0.593	0.185	0.423	0.418	0.000	0.423	1.730

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	221	423	187	1690	0	163	-1
normalized size	1	1.00	2.38	4.55	2.01	18.17	0.00	1.75	-0.01
time (sec)	N/A	0.096	2.042	0.382	0.694	0.461	0.000	0.435	0.000

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	115	292	209	1031	0	246	-1
normalized size	1	1.00	1.39	3.52	2.52	12.42	0.00	2.96	-0.01
time (sec)	N/A	0.129	0.295	0.423	0.351	0.551	0.000	0.556	0.000

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	268	481	1070	3624	0	282	-1
normalized size	1	1.00	2.21	3.98	8.84	29.95	0.00	2.33	-0.01
time (sec)	N/A	0.284	2.824	0.457	0.917	0.508	0.000	0.955	0.000

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	130	367	384	3624	0	383	-1
normalized size	1	1.00	1.18	3.34	3.49	32.95	0.00	3.48	-0.01
time (sec)	N/A	0.175	1.276	0.530	0.473	0.716	0.000	1.318	0.000

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	350	634	2961	9849	0	302	-1
normalized size	1	1.00	2.17	3.94	18.39	61.17	0.00	1.88	-0.01
time (sec)	N/A	0.411	5.093	0.520	1.414	0.612	0.000	1.604	0.000

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	515	1713	3239	5463	0	371	-1
normalized size	1	1.00	3.48	11.57	21.89	36.91	0.00	2.51	-0.01
time (sec)	N/A	0.320	5.782	0.446	1.993	0.524	0.000	2.251	0.000

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	136	579	206	1741	0	187	-1
normalized size	1	1.00	1.77	7.52	2.68	22.61	0.00	2.43	-0.01
time (sec)	N/A	0.120	2.168	0.417	0.420	0.459	0.000	1.883	0.000

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	1457	1306	2201	6464	0	295	-1
normalized size	1	1.00	10.48	9.40	15.83	46.50	0.00	2.12	-0.01
time (sec)	N/A	0.294	13.087	0.481	1.020	0.573	0.000	1.614	0.000

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	131	672	209	1753	0	175	-1
normalized size	1	1.00	1.62	8.30	2.58	21.64	0.00	2.16	-0.01
time (sec)	N/A	0.126	1.568	0.405	0.336	0.447	0.000	1.221	0.000

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	1457	1173	1255	7158	0	309	-1
normalized size	1	1.00	10.48	8.44	9.03	51.50	0.00	2.22	-0.01
time (sec)	N/A	0.246	12.056	0.430	0.927	0.568	0.000	0.966	0.000

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	129	82	193	1666	0	187	94
normalized size	1	1.00	1.77	1.12	2.64	22.82	0.00	2.56	1.29
time (sec)	N/A	0.082	2.010	0.207	0.420	0.454	0.000	0.526	1.604

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	301	1283	402	6538	0	327	-1
normalized size	1	1.00	2.06	8.79	2.75	44.78	0.00	2.24	-0.01
time (sec)	N/A	0.181	6.209	0.411	0.528	0.570	0.000	0.856	0.000

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	155	1046	419	4132	0	475	-1
normalized size	1	1.00	1.19	8.05	3.22	31.78	0.00	3.65	-0.01
time (sec)	N/A	0.186	1.056	0.473	0.721	0.847	0.000	0.716	0.000

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	2083	1433	1971	11606	0	402	-1
normalized size	1	1.00	11.45	7.87	10.83	63.77	0.00	2.21	-0.01
time (sec)	N/A	0.403	7.228	0.507	0.941	0.630	0.000	1.272	0.000

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	172	1128	692	10255	0	766	-1
normalized size	1	1.00	1.13	7.42	4.55	67.47	0.00	5.04	-0.01
time (sec)	N/A	0.243	1.922	0.570	0.538	1.341	0.000	2.475	0.000

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	F(-1)	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	3334	1610	4920	0	0	482	-1
normalized size	1	1.00	14.37	6.94	21.21	0.00	0.00	2.08	-0.00
time (sec)	N/A	0.514	7.738	0.567	1.838	0.000	0.000	2.587	0.000

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	F(-1)	F(-1)	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	1405	2880	718	0	0	594	-1
normalized size	1	1.00	6.79	13.91	3.47	0.00	0.00	2.87	-0.00
time (sec)	N/A	0.346	6.926	0.446	0.840	0.000	0.000	0.853	0.000

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	25	120	33	183	0	72	-1
normalized size	1	1.00	0.86	4.14	1.14	6.31	0.00	2.48	-0.03
time (sec)	N/A	0.031	0.023	0.284	0.460	0.671	0.000	0.126	0.000

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	79	13	18	0	26	-1
normalized size	1	1.00	1.00	5.64	0.93	1.29	0.00	1.86	-0.07
time (sec)	N/A	0.022	0.006	0.299	0.606	0.773	0.000	0.117	0.000

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	79	22	18	0	31	-1
normalized size	1	1.00	1.00	5.64	1.57	1.29	0.00	2.21	-0.07
time (sec)	N/A	0.024	0.010	0.258	0.670	0.534	0.000	0.134	0.000

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	27	123	33	1	0	83	-1
normalized size	1	1.00	0.79	3.62	0.97	0.03	0.00	2.44	-0.03
time (sec)	N/A	0.029	0.014	0.316	0.489	0.679	0.000	0.142	0.000

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	81	13	1	0	31	-1
normalized size	1	1.00	1.00	5.06	0.81	0.06	0.00	1.94	-0.06
time (sec)	N/A	0.020	0.006	0.321	0.545	0.513	0.000	0.125	0.000

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	81	22	1	0	37	-1
normalized size	1	1.00	1.00	5.06	1.38	0.06	0.00	2.31	-0.06
time (sec)	N/A	0.022	0.010	0.299	0.439	0.492	0.000	0.141	0.000

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	114	0	0	4594	0	0	-1
normalized size	1	1.00	1.37	0.00	0.00	55.35	0.00	0.00	-0.01
time (sec)	N/A	0.147	0.749	0.453	0.000	1.326	0.000	0.000	0.000

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	192	0	0	8852	0	0	-1
normalized size	1	1.00	1.54	0.00	0.00	70.82	0.00	0.00	-0.01
time (sec)	N/A	0.313	0.470	0.397	0.000	1.218	0.000	0.000	0.000

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	90	0	0	2394	0	0	-1
normalized size	1	1.00	1.53	0.00	0.00	40.58	0.00	0.00	-0.02
time (sec)	N/A	0.107	0.397	0.319	0.000	0.730	0.000	0.000	0.000

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	150	0	0	4316	0	0	-1
normalized size	1	1.00	1.72	0.00	0.00	49.61	0.00	0.00	-0.01
time (sec)	N/A	0.218	0.287	0.273	0.000	0.889	0.000	0.000	0.000

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	90	43	0	1608	0	0	32
normalized size	1	1.00	2.25	1.08	0.00	40.20	0.00	0.00	0.80
time (sec)	N/A	0.063	0.274	0.067	0.000	0.486	0.000	0.000	1.710

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	134	0	0	2949	0	0	-1
normalized size	1	1.00	2.27	0.00	0.00	49.98	0.00	0.00	-0.02
time (sec)	N/A	0.045	0.741	0.308	0.000	0.584	0.000	0.000	0.000

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	111	0	0	3597	0	0	-1
normalized size	1	1.00	1.98	0.00	0.00	64.23	0.00	0.00	-0.02
time (sec)	N/A	0.108	0.130	0.381	0.000	0.616	0.000	0.000	0.000

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	75	0	0	1303	0	0	-1
normalized size	1	1.00	1.56	0.00	0.00	27.15	0.00	0.00	-0.02
time (sec)	N/A	0.178	0.521	0.388	0.000	0.523	0.000	0.000	0.000

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	156	0	0	5247	0	0	-1
normalized size	1	1.00	1.88	0.00	0.00	63.22	0.00	0.00	-0.01
time (sec)	N/A	0.143	0.594	0.411	0.000	0.634	0.000	0.000	0.000

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	149	0	0	2341	0	0	-1
normalized size	1	1.00	1.77	0.00	0.00	27.87	0.00	0.00	-0.01
time (sec)	N/A	0.248	0.625	0.412	0.000	0.622	0.000	0.000	0.000

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	191	0	0	0	0	0	-1
normalized size	1	1.00	1.53	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.221	1.022	0.419	0.000	0.000	0.000	0.000	0.000

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	129	0	0	4226	0	0	-1
normalized size	1	1.00	1.70	0.00	0.00	55.61	0.00	0.00	-0.01
time (sec)	N/A	0.128	0.971	0.302	0.000	2.146	0.000	0.000	0.000

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	197	0	0	8582	0	0	-1
normalized size	1	1.00	1.58	0.00	0.00	68.66	0.00	0.00	-0.01
time (sec)	N/A	0.361	0.853	0.293	0.000	1.153	0.000	0.000	0.000

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	65	56	0	2312	0	0	45
normalized size	1	1.00	1.14	0.98	0.00	40.56	0.00	0.00	0.79
time (sec)	N/A	0.084	0.116	0.049	0.000	0.842	0.000	0.000	3.359

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	152	0	0	4140	0	0	-1
normalized size	1	1.00	1.73	0.00	0.00	47.05	0.00	0.00	-0.01
time (sec)	N/A	0.091	0.275	0.283	0.000	1.851	0.000	0.000	0.000

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	159	0	0	4123	0	134	-1
normalized size	1	1.00	2.27	0.00	0.00	58.90	0.00	1.91	-0.01
time (sec)	N/A	0.132	0.517	0.365	0.000	1.307	0.000	0.473	0.000

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	144	0	0	3349	0	222	-1
normalized size	1	1.00	1.78	0.00	0.00	41.35	0.00	2.74	-0.01
time (sec)	N/A	0.232	0.350	0.375	0.000	4.041	0.000	0.523	0.000

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	280	0	0	0	0	0	-1
normalized size	1	1.00	1.65	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.194	9.281	0.620	0.000	0.000	0.000	0.000	0.000

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	109	0	0	2678	0	0	-1
normalized size	1	1.00	1.65	0.00	0.00	40.58	0.00	0.00	-0.02
time (sec)	N/A	0.131	0.520	0.414	0.000	0.613	0.000	0.000	0.000

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	169	0	0	4569	0	0	-1
normalized size	1	1.00	1.88	0.00	0.00	50.77	0.00	0.00	-0.01
time (sec)	N/A	0.228	0.530	0.401	0.000	0.658	0.000	0.000	0.000

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	105	0	0	1650	0	0	-1
normalized size	1	1.00	2.50	0.00	0.00	39.29	0.00	0.00	-0.02
time (sec)	N/A	0.093	0.225	0.358	0.000	0.524	0.000	0.000	0.000

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	107	0	0	2856	0	0	-1
normalized size	1	1.00	1.78	0.00	0.00	47.60	0.00	0.00	-0.02
time (sec)	N/A	0.193	0.181	0.352	0.000	0.562	0.000	0.000	0.000

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	70	30	0	1430	0	0	19
normalized size	1	1.00	2.80	1.20	0.00	57.20	0.00	0.00	0.76
time (sec)	N/A	0.053	0.083	0.118	0.000	0.465	0.000	0.000	1.667

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	62	0	0	1059	0	0	-1
normalized size	1	1.00	2.14	0.00	0.00	36.52	0.00	0.00	-0.03
time (sec)	N/A	0.025	0.041	0.369	0.000	0.453	0.000	0.000	0.000

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	124	0	0	3663	0	0	-1
normalized size	1	1.00	2.21	0.00	0.00	65.41	0.00	0.00	-0.02
time (sec)	N/A	0.103	0.261	0.402	0.000	0.574	0.000	0.000	0.000

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	94	0	0	1365	0	0	-1
normalized size	1	1.00	1.77	0.00	0.00	25.75	0.00	0.00	-0.02
time (sec)	N/A	0.188	0.112	0.470	0.000	0.510	0.000	0.000	0.000

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	159	0	0	6475	0	0	-1
normalized size	1	1.00	1.77	0.00	0.00	71.94	0.00	0.00	-0.01
time (sec)	N/A	0.161	0.848	0.411	0.000	0.734	0.000	0.000	0.000

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	129	0	0	3360	0	0	-1
normalized size	1	1.00	1.90	0.00	0.00	49.41	0.00	0.00	-0.01
time (sec)	N/A	0.147	0.531	0.382	0.000	0.676	0.000	0.000	0.000

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	169	0	0	5170	0	0	-1
normalized size	1	1.00	1.97	0.00	0.00	60.12	0.00	0.00	-0.01
time (sec)	N/A	0.252	0.409	0.378	0.000	0.700	0.000	0.000	0.000

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	103	0	0	2194	0	0	-1
normalized size	1	1.00	2.10	0.00	0.00	44.78	0.00	0.00	-0.02
time (sec)	N/A	0.115	0.234	0.335	0.000	0.537	0.000	0.000	0.000

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	128	0	0	1733	0	0	-1
normalized size	1	1.00	2.51	0.00	0.00	33.98	0.00	0.00	-0.02
time (sec)	N/A	0.218	0.755	0.334	0.000	0.542	0.000	0.000	0.000

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	98	46	0	2034	44	0	35
normalized size	1	1.00	2.28	1.07	0.00	47.30	1.02	0.00	0.81
time (sec)	N/A	0.073	0.396	0.099	0.000	0.519	5.925	0.000	1.741

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	107	0	0	2095	0	0	-1
normalized size	1	1.00	1.88	0.00	0.00	36.75	0.00	0.00	-0.02
time (sec)	N/A	0.045	0.841	0.280	0.000	0.550	0.000	0.000	0.000

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	155	0	0	6939	0	0	-1
normalized size	1	1.00	1.96	0.00	0.00	87.84	0.00	0.00	-0.01
time (sec)	N/A	0.140	0.686	0.393	0.000	0.738	0.000	0.000	0.000

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	120	0	0	3941	0	0	-1
normalized size	1	1.00	1.36	0.00	0.00	44.78	0.00	0.00	-0.01
time (sec)	N/A	0.274	0.458	0.399	0.000	0.670	0.000	0.000	0.000

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	178	0	0	0	0	0	-1
normalized size	1	1.00	1.51	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.339	0.747	0.395	0.000	0.000	0.000	0.000	0.000

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	126	0	0	5184	0	0	-1
normalized size	1	1.00	1.66	0.00	0.00	68.21	0.00	0.00	-0.01
time (sec)	N/A	0.158	0.564	0.377	0.000	0.764	0.000	0.000	0.000

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	290	0	0	3559	0	0	-1
normalized size	1	1.00	3.22	0.00	0.00	39.54	0.00	0.00	-0.01
time (sec)	N/A	0.262	2.168	0.382	0.000	0.696	0.000	0.000	0.000

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	124	0	0	4644	0	0	-1
normalized size	1	1.00	1.82	0.00	0.00	68.29	0.00	0.00	-0.01
time (sec)	N/A	0.124	0.999	0.385	0.000	0.726	0.000	0.000	0.000

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	290	0	0	4989	0	0	-1
normalized size	1	1.00	3.30	0.00	0.00	56.69	0.00	0.00	-0.01
time (sec)	N/A	0.248	1.199	0.334	0.000	0.770	0.000	0.000	0.000

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	112	61	0	3994	65	0	50
normalized size	1	1.00	1.81	0.98	0.00	64.42	1.05	0.00	0.81
time (sec)	N/A	0.087	0.718	0.106	0.000	0.703	12.680	0.000	3.069

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	130	0	0	6299	0	0	-1
normalized size	1	1.00	1.37	0.00	0.00	66.31	0.00	0.00	-0.01
time (sec)	N/A	0.090	0.451	0.285	0.000	0.801	0.000	0.000	0.000

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	242	0	0	0	0	0	-1
normalized size	1	1.00	2.22	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.200	1.114	0.381	0.000	0.000	0.000	0.000	0.000

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	155	0	0	11205	0	0	-1
normalized size	1	1.00	1.17	0.00	0.00	84.25	0.00	0.00	-0.01
time (sec)	N/A	0.376	0.829	0.401	0.000	4.182	0.000	0.000	0.000

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	330	0	0	0	0	0	-1
normalized size	1	1.00	1.80	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.191	7.985	0.604	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [190] had the largest ratio of [.5833]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	5	1.00	21	0.238
2	A	3	2	1.00	21	0.095
3	A	4	4	1.00	21	0.190

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
4	A	3	2	1.00	19	0.105
5	A	3	3	1.00	19	0.158
6	A	3	2	1.00	21	0.095
7	A	4	4	1.00	21	0.190
8	A	3	2	1.00	21	0.095
9	A	6	5	1.00	23	0.217
10	A	3	2	1.00	23	0.087
11	A	5	5	1.00	23	0.217
12	A	3	2	1.00	21	0.095
13	A	4	3	1.00	21	0.143
14	A	3	2	1.00	23	0.087
15	A	5	5	1.00	23	0.217
16	A	3	2	1.00	23	0.087
17	A	6	5	1.00	23	0.217
18	A	3	2	1.00	23	0.087
19	A	6	5	1.28	23	0.217
20	A	3	2	1.00	21	0.095
21	A	4	3	1.00	21	0.143
22	A	3	2	1.00	23	0.087
23	A	5	4	1.00	23	0.174
24	A	3	2	1.00	23	0.087
25	A	6	6	1.00	23	0.261
26	A	4	4	1.00	23	0.174
27	A	5	5	1.00	23	0.217
28	A	3	3	1.00	21	0.143
29	A	4	4	1.00	21	0.190

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
30	A	3	3	1.00	23	0.130
31	A	5	5	1.00	23	0.217
32	A	4	4	1.00	23	0.174
33	A	7	6	1.00	23	0.261
34	A	5	4	1.00	23	0.174
35	A	6	6	1.00	23	0.261
36	A	4	4	1.00	21	0.190
37	A	5	5	1.00	21	0.238
38	A	4	4	1.00	23	0.174
39	A	6	6	1.00	23	0.261
40	A	5	4	1.00	23	0.174
41	A	8	6	1.00	23	0.261
42	A	6	5	1.00	23	0.217
43	A	7	6	1.00	23	0.261
44	A	5	4	1.00	21	0.190
45	A	6	6	1.00	21	0.286
46	A	5	4	1.00	23	0.174
47	A	7	7	1.00	23	0.304
48	A	6	5	1.00	23	0.217
49	A	3	3	1.00	21	0.143
50	A	3	2	1.00	21	0.095
51	A	2	2	1.00	21	0.095
52	A	2	2	1.00	19	0.105
53	A	2	2	1.00	19	0.105
54	A	3	3	1.43	21	0.143
55	A	3	3	1.00	21	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
56	A	3	2	1.30	21	0.095
57	A	4	4	1.00	23	0.174
58	A	4	3	1.00	23	0.130
59	A	5	4	1.00	23	0.174
60	A	5	4	1.00	21	0.190
61	A	4	4	1.00	21	0.190
62	A	3	2	1.00	23	0.087
63	A	5	5	1.00	23	0.217
64	A	3	2	1.00	23	0.087
65	A	6	5	1.00	23	0.217
66	A	5	4	1.00	23	0.174
67	A	5	4	1.00	23	0.174
68	A	6	5	1.00	21	0.238
69	A	5	5	1.00	21	0.238
70	A	3	2	1.00	23	0.087
71	A	6	6	1.00	23	0.261
72	A	3	2	1.00	23	0.087
73	A	6	6	1.00	23	0.261
74	A	4	3	1.00	23	0.130
75	A	5	5	1.00	23	0.217
76	A	3	3	1.00	21	0.143
77	A	2	2	1.00	21	0.095
78	A	2	2	1.00	23	0.087
79	A	4	4	1.00	23	0.174
80	A	3	3	1.00	23	0.130
81	A	5	5	1.00	23	0.217

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
82	A	4	3	1.00	23	0.130
83	A	5	4	1.00	23	0.174
84	A	6	6	1.00	23	0.261
85	A	5	4	1.00	21	0.190
86	A	3	3	1.00	21	0.143
87	A	3	3	1.00	23	0.130
88	A	3	3	1.00	23	0.130
89	A	3	3	1.00	23	0.130
90	A	5	5	1.00	23	0.217
91	A	5	4	1.00	23	0.174
92	A	6	6	1.00	23	0.261
93	A	7	6	1.00	23	0.261
94	A	6	5	1.00	21	0.238
95	A	4	4	1.00	21	0.190
96	A	4	3	1.00	23	0.130
97	A	4	4	1.00	23	0.174
98	A	4	4	1.00	23	0.174
99	A	4	3	1.00	23	0.130
100	A	4	4	1.00	23	0.174
101	A	6	6	1.00	23	0.261
102	A	4	3	1.00	21	0.143
103	A	4	3	1.00	21	0.143
104	A	4	3	1.00	21	0.143
105	A	3	2	1.00	19	0.105
106	A	3	2	1.00	12	0.167
107	A	4	3	1.00	19	0.158

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
108	A	4	3	1.00	21	0.143
109	A	4	3	1.00	21	0.143
110	A	4	3	1.00	21	0.143
111	A	4	3	1.00	21	0.143
112	A	4	3	1.00	23	0.130
113	A	4	3	1.00	23	0.130
114	A	4	3	1.00	23	0.130
115	A	4	3	1.00	21	0.143
116	A	4	3	1.00	14	0.214
117	A	4	3	1.00	21	0.143
118	A	4	3	1.00	23	0.130
119	A	4	3	1.00	23	0.130
120	A	4	3	1.00	23	0.130
121	A	4	3	1.00	23	0.130
122	A	4	3	1.00	23	0.130
123	A	5	4	1.00	23	0.174
124	A	4	3	1.00	23	0.130
125	A	5	4	1.00	23	0.174
126	A	4	3	1.00	23	0.130
127	A	4	3	1.00	21	0.143
128	A	4	3	1.00	14	0.214
129	A	4	3	1.00	21	0.143
130	A	4	3	1.00	23	0.130
131	A	4	3	1.00	23	0.130
132	A	4	3	1.00	23	0.130
133	A	4	3	1.00	23	0.130

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
134	A	4	3	1.00	23	0.130
135	A	4	3	1.00	23	0.130
136	A	4	3	1.00	14	0.214
137	A	4	3	1.00	14	0.214
138	A	4	3	1.00	23	0.130
139	A	6	6	1.00	23	0.261
140	A	4	3	1.00	23	0.130
141	A	5	5	1.00	23	0.217
142	A	2	2	1.00	21	0.095
143	A	3	3	1.00	14	0.214
144	A	4	3	1.00	21	0.143
145	A	6	6	1.00	23	0.261
146	A	4	3	1.00	23	0.130
147	A	7	7	1.00	23	0.304
148	A	4	3	1.00	23	0.130
149	A	6	6	1.00	23	0.261
150	A	4	3	1.00	23	0.130
151	A	6	6	1.00	23	0.261
152	A	4	3	1.00	21	0.143
153	A	5	5	1.00	14	0.357
154	A	4	3	1.00	21	0.143
155	A	7	7	1.00	23	0.304
156	A	4	3	1.00	23	0.130
157	A	8	7	1.00	23	0.304
158	A	7	7	1.00	23	0.304
159	A	4	3	1.00	23	0.130

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
160	A	7	7	1.00	23	0.304
161	A	4	3	1.00	23	0.130
162	A	7	7	1.00	23	0.304
163	A	4	3	1.00	21	0.143
164	A	6	6	1.00	14	0.429
165	A	4	3	1.00	21	0.143
166	A	8	8	1.00	23	0.348
167	A	4	3	1.00	23	0.130
168	A	9	8	1.00	23	0.348
169	A	7	6	1.00	14	0.429
170	A	4	4	1.00	12	0.333
171	A	3	3	1.00	12	0.250
172	A	3	3	1.00	12	0.250
173	A	4	4	1.00	10	0.400
174	A	3	3	1.00	10	0.300
175	A	3	3	1.00	10	0.300
176	A	7	6	1.00	17	0.353
177	A	9	9	1.00	17	0.529
178	A	6	6	1.00	17	0.353
179	A	8	8	1.00	17	0.471
180	A	5	5	1.00	15	0.333
181	A	6	6	1.00	12	0.500
182	A	7	5	1.00	15	0.333
183	A	6	6	1.00	17	0.353
184	A	8	6	1.00	17	0.353
185	A	7	7	1.00	17	0.412

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
186	A	9	7	1.00	17	0.412
187	A	7	6	1.00	17	0.353
188	A	9	9	1.00	17	0.529
189	A	6	5	1.00	15	0.333
190	A	7	7	1.00	12	0.583
191	A	8	6	1.00	15	0.400
192	A	8	8	1.00	17	0.471
193	A	8	8	1.00	16	0.500
194	A	6	5	1.00	17	0.294
195	A	8	8	1.00	17	0.471
196	A	5	5	1.00	17	0.294
197	A	7	7	1.00	17	0.412
198	A	4	4	1.00	15	0.267
199	A	3	3	1.00	12	0.250
200	A	7	5	1.00	15	0.333
201	A	6	6	1.00	17	0.353
202	A	8	6	1.00	17	0.353
203	A	6	5	1.00	17	0.294
204	A	8	8	1.00	17	0.471
205	A	5	5	1.00	17	0.294
206	A	5	5	1.00	17	0.294
207	A	5	5	1.00	15	0.333
208	A	4	4	1.00	12	0.333
209	A	8	6	1.00	15	0.400
210	A	7	7	1.00	17	0.412
211	A	9	9	1.00	17	0.529

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
212	A	6	5	1.00	17	0.294
213	A	7	7	1.00	17	0.412
214	A	6	6	1.00	17	0.353
215	A	7	7	1.00	17	0.412
216	A	6	5	1.00	15	0.333
217	A	6	6	1.00	12	0.500
218	A	9	7	1.00	15	0.467
219	A	8	8	1.00	17	0.471
220	A	7	6	1.00	16	0.375

Chapter 3

Listing of integrals

3.1 $\int \left(a+b\,\mathbf{sech}^2(c+dx)\right)\sinh^4(c+dx)\,dx$

Optimal. Leaf size=70

$$-\frac{(5a-4b)\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{3}{8}x(a-4b)+\frac{a\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{b\tanh(c+dx)}{d}$$

```
[Out] 3/8*(a-4*b)*x-1/8*(5*a-4*b)*cosh(d*x+c)*sinh(d*x+c)/d+1/4*a*cosh(d*x+c)^3*s
inh(d*x+c)/d+b*tanh(d*x+c)/d
```

Rubi [A] time = 0.08, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4132, 455, 1157, 388, 206}

$$-\frac{(5a-4b)\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{3}{8}x(a-4b)+\frac{a\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{b\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)*Sinh[c + d*x]^4,x]
```

```
[Out] (3*(a - 4*b)*x)/8 - ((5*a - 4*b)*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) + (a*Co
sh[c + d*x]^3*Sinh[c + d*x])/(4*d) + (b*Tanh[c + d*x])/d
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \left(a+b\text{sech}^2(c+dx)\right)\sinh^4(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x^4\left(a+b-bx^2\right)}{\left(1-x^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{a\cosh^3(c+dx)\sinh(c+dx)}{4d} + \frac{\text{Subst}\left(\int \frac{-a-4ax^2+4bx^4}{\left(1-x^2\right)^2}\,dx, x, \tanh(c\right.}{4d}$$

$$= -\frac{(5a-4b)\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{a\cosh^3(c+dx)\sinh(c+d}{4d}$$

$$= -\frac{(5a-4b)\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{a\cosh^3(c+dx)\sinh(c+d}{4d}$$

$$= \frac{3}{8}(a-4b)x - \frac{(5a-4b)\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{a\cosh^3(c+dx}{4d}$$

Mathematica [A] time = 0.33, size = 54, normalized size = 0.77

$$\frac{12(a-4b)(c+dx)-8(a-b)\sinh(2(c+dx))+a\sinh(4(c+dx))+32b\tanh(c+dx)}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)*Sinh[c + d*x]^4,x]
```

```
[Out] (12*(a - 4*b)*(c + d*x) - 8*(a - b)*Sinh[2*(c + d*x)] + a*Sinh[4*(c + d*x)]
 + 32*b*Tanh[c + d*x])/(32*d)
```

fricas [A] time = 0.40, size = 114, normalized size = 1.63

$$\frac{a\sinh(dx+c)^5 + \left(10\,a\cosh(dx+c)^2 - 7\,a + 8\,b\right)\sinh(dx+c)^3 + 8\,(3\,(a-4\,b)dx - 8\,b)\cosh(dx+c) + \left(5\,a\,\mathrm{c}\right.}{64\,d\cosh(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)*sinh(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/64*(a*sinh(d*x + c)^5 + (10*a*cosh(d*x + c)^2 - 7*a + 8*b)*sinh(d*x + c)^
3 + 8*(3*(a - 4*b)*d*x - 8*b)*cosh(d*x + c) + (5*a*cosh(d*x + c)^4 - 3*(7*a
 - 8*b)*cosh(d*x + c)^2 - 8*a + 72*b)*sinh(d*x + c))/(d*cosh(d*x + c))
```

giac [B] time = 1.10, size = 130, normalized size = 1.86

$$\frac{24\,(dx+c)(a-4\,b) + ae^{(4\,dx+4\,c)} - 8\,ae^{(2\,dx+2\,c)} + 8\,be^{(2\,dx+2\,c)} - \left(18\,ae^{(4\,dx+4\,c)} - 72\,be^{(4\,dx+4\,c)} - 8\,ae^{(2\,dx+2\,c)} + 8\,be\right.}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)*sinh(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/64*(24*(d*x + c)*(a - 4*b) + a*e^(4*d*x + 4*c) - 8*a*e^(2*d*x + 2*c) + 8*
b*e^(2*d*x + 2*c) - (18*a*e^(4*d*x + 4*c) - 72*b*e^(4*d*x + 4*c) - 8*a*e^(2
*d*x + 2*c) + 8*b*e^(2*d*x + 2*c) + a)*e^(-4*d*x - 4*c) - 128*b/(e^(2*d*x +
 2*c) + 1))/d
```

maple [A] time = 0.33, size = 78, normalized size = 1.11

$$\frac{a\left(\left(\frac{\left(\sinh^3(dx+c)\right)}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh\left(dx+c\right)+\frac{3dx}{8}+\frac{3c}{8}\right)+b\left(\frac{\sinh^3(dx+c)}{2\cosh(dx+c)}-\frac{3dx}{2}-\frac{3c}{2}+\frac{3\tanh(dx+c)}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)*sinh(d*x+c)^4,x)
```

```
[Out] 1/d*(a*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)+b*(1
/2*sinh(d*x+c)^3/cosh(d*x+c)-3/2*d*x-3/2*c+3/2*tanh(d*x+c)))
```

maxima [B] time = 0.31, size = 129, normalized size = 1.84

$$\frac{1}{64}\,a\left(24\,x+\frac{e^{(4\,dx+4\,c)}}{d}-\frac{8\,e^{(2\,dx+2\,c)}}{d}+\frac{8\,e^{(-2\,dx-2\,c)}}{d}-\frac{e^{(-4\,dx-4\,c)}}{d}\right)-\frac{1}{8}\,b\left(\frac{12\,(dx+c)}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}-\frac{17\,e^{(-2\,dx-2\,c)}}{d\left(e^{(-2\,dx-2\,c)}+e^{(-}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)*sinh(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/64*a*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*c)
/d - e^(-4*d*x - 4*c)/d) - 1/8*b*(12*(d*x + c)/d + e^(-2*d*x - 2*c)/d - (17
*e^(-2*d*x - 2*c) + 1)/(d*(e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c))))
```

mupad [B] time = 1.61, size = 73, normalized size = 1.04

$$\frac{3\,a\,x}{8}-\frac{3\,b\,x}{2}-\frac{a\sinh\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{a\sinh\left(4\,c+4\,d\,x\right)}{32\,d}+\frac{b\sinh\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{b\sinh\left(c+d\,x\right)}{d\cosh\left(c+d\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^4*(a + b/cosh(c + d*x)^2),x)
```

```
[Out] (3*a*x)/8 - (3*b*x)/2 - (a*sinh(2*c + 2*d*x))/(4*d) + (a*sinh(4*c + 4*d*x))
/(32*d) + (b*sinh(2*c + 2*d*x))/(4*d) + (b*sinh(c + d*x))/(d*cosh(c + d*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\,\mathrm{sech}^2\,(c + dx)\right)\sinh^4\,(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)*sinh(d*x+c)**4,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)*sinh(c + d*x)**4, x)
```

3.2 $\int \left(a+b\,\mathbf{sech}^2(c+dx)\right)\sinh^3(c+dx)\,dx$

Optimal. Leaf size=44

$$-\frac{(a-b)\cosh(c+dx)}{d}+\frac{a\cosh^3(c+dx)}{3d}+\frac{b\,\mathrm{sech}(c+dx)}{d}$$

[Out] -(a-b)*cosh(d*x+c)/d+1/3*a*cosh(d*x+c)^3/d+b*sech(d*x+c)/d

Rubi [A] time = 0.06, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4133, 448}

$$-\frac{(a-b)\cosh(c+dx)}{d}+\frac{a\cosh^3(c+dx)}{3d}+\frac{b\,\mathrm{sech}(c+dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sech[c + d*x]^2)*Sinh[c + d*x]^3,x]

[Out] -(((a - b)*Cosh[c + d*x])/d) + (a*Cosh[c + d*x]^3)/(3*d) + (b*Sech[c + d*x]
)/d

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \left(a+b\,\mathrm{sech}^2(c+dx)\right)\sinh^3(c+dx)\,dx = -\frac{\mathrm{Subst}\left(\int \frac{(1-x^2)(b+ax^2)}{x^2}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= -\frac{\mathrm{Subst}\left(\int \left(a\left(1-\frac{b}{a}\right)+\frac{b}{x^2}-ax^2\right)dx, x, \cosh(c+dx)\right)}{d}$$
$$= -\frac{(a-b)\cosh(c+dx)}{d}+\frac{a\cosh^3(c+dx)}{3d}+\frac{b\,\mathrm{sech}(c+dx)}{d}$$

Mathematica [A] time = 0.05, size = 53, normalized size = 1.20

$$-\frac{3a\cosh(c+dx)}{4d}+\frac{a\cosh(3(c+dx))}{12d}+\frac{b\cosh(c+dx)}{d}+\frac{b\,\mathrm{sech}(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)*Sinh[c + d*x]^3,x]
```

```
[Out] (-3*a*Cosh[c + d*x])/(4*d) + (b*Cosh[c + d*x])/d + (a*Cosh[3*(c + d*x)])/(1
2*d) + (b*Sech[c + d*x])/d
```

fricas [B] time = 0.44, size = 85, normalized size = 1.93

$$\frac{a\cosh(dx+c)^4+a\sinh(dx+c)^4-4(2a-3b)\cosh(dx+c)^2+2\left(3a\cosh(dx+c)^2-4a+6b\right)\sinh(dx+c}{24\,d\cosh(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)*sinh(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(a*cosh(d*x + c)^4 + a*sinh(d*x + c)^4 - 4*(2*a - 3*b)*cosh(d*x + c)^2
 + 2*(3*a*cosh(d*x + c)^2 - 4*a + 6*b)*sinh(d*x + c)^2 - 9*a + 36*b)/(d*cos
h(d*x + c))
```

giac [B] time = 0.13, size = 85, normalized size = 1.93

$$\frac{a\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3-12\,a\left(e^{(dx+c)}+e^{(-dx-c)}\right)+12\,b\left(e^{(dx+c)}+e^{(-dx-c)}\right)+\frac{48\,b}{e^{(dx+c)}+e^{(-dx-c)}}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)*sinh(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/24*(a*(e^(d*x + c) + e^(-d*x - c))^3 - 12*a*(e^(d*x + c) + e^(-d*x - c))
+ 12*b*(e^(d*x + c) + e^(-d*x - c)) + 48*b/(e^(d*x + c) + e^(-d*x - c)))/d
```

maple [A] time = 0.31, size = 56, normalized size = 1.27

$$\frac{a\left(-\frac{2}{3}+\frac{\left(\sinh^{2}(dx+c)\right)}{3}\right)\cosh\left(dx+c\right)+b\left(\frac{\sinh^{2}(dx+c)}{\cosh(dx+c)}+\frac{2}{\cosh(dx+c)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)*sinh(d*x+c)^3,x)
```

```
[Out] 1/d*(a*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)+b*(sinh(d*x+c)^2/cosh(d*x+c)+2/
cosh(d*x+c)))
```

maxima [B] time = 0.34, size = 111, normalized size = 2.52

$$\frac{1}{24}a\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)+\frac{1}{2}b\left(\frac{e^{(-dx-c)}}{d}+\frac{5\,e^{(-2\,dx-2\,c)}+1}{d\left(e^{(-dx-c)}+e^{(-3\,dx-3\,c)}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)*sinh(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/24*a*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x
- 3*c)/d) + 1/2*b*(e^(-d*x - c)/d + (5*e^(-2*d*x - 2*c) + 1)/(d*(e^(-d*x -
c) + e^(-3*d*x - 3*c))))
```

mupad [B] time = 0.18, size = 44, normalized size = 1.00

$$\frac{a\cosh\left(c+d\,x\right)^3}{3\,d}-\frac{\cosh\left(c+d\,x\right)\,\left(a-b\right)}{d}+\frac{b}{d\cosh\left(c+d\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^3*(a + b/cosh(c + d*x)^2),x)
```

```
[Out] (a*cosh(c + d*x)^3)/(3*d) - (cosh(c + d*x)*(a - b))/d + b/(d*cosh(c + d*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\,\mathrm{sech}^{2}\left(c+dx\right)\right)\sinh^{3}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)*sinh(d*x+c)**3,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)*sinh(c + d*x)**3, x)
```

3.3 $\int \left(a+b\,\mathbf{sech}^2(c+dx)\right)\sinh^2(c+dx)\,dx$

Optimal. Leaf size=43

$$-\frac{1}{2}x(a-2b)+\frac{a\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{b\tanh(c+dx)}{d}$$

```
[Out] -1/2*(a-2*b)*x+1/2*a*cosh(d*x+c)*sinh(d*x+c)/d-b*tanh(d*x+c)/d
```

Rubi [A] time = 0.06, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {4132, 455, 388, 206}

$$-\frac{1}{2}x(a-2b)+\frac{a\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{b\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)*Sinh[c + d*x]^2,x]
```

```
[Out] -((a - 2*b)*x)/2 + (a*Cosh[c + d*x]*Sinh[c + d*x])/(2*d) - (b*Tanh[c + d*x]
)/d
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \left(a+b\text{sech}^2(c+dx)\right)\sinh^2(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x^2\left(a+b-bx^2\right)}{\left(1-x^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{a\cosh(c+dx)\sinh(c+dx)}{2d} - \frac{\text{Subst}\left(\int \frac{a-2bx^2}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{2d}$$

$$= \frac{a\cosh(c+dx)\sinh(c+dx)}{2d} - \frac{b\tanh(c+dx)}{d} - \frac{(a-2b)\,\text{Subst}\left(\int \frac{1}{1-\ldots}\right.}{2\ldots}$$

$$= -\frac{1}{2}(a-2b)x + \frac{a\cosh(c+dx)\sinh(c+dx)}{2d} - \frac{b\tanh(c+dx)}{d}$$

Mathematica [A] time = 0.19, size = 57, normalized size = 1.33

$$\frac{a(-c-dx)}{2d} + \frac{a\sinh(2(c+dx))}{4d} + \frac{b\tanh^{-1}(\tanh(c+dx))}{d} - \frac{b\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)*Sinh[c + d*x]^2,x]
```

```
[Out] (a*(-c - d*x))/(2*d) + (b*ArcTanh[Tanh[c + d*x]])/d + (a*Sinh[2*(c + d*x)])
/(4*d) - (b*Tanh[c + d*x])/d
```

fricas [A] time = 0.41, size = 67, normalized size = 1.56

$$\frac{a\sinh(dx+c)^3 - 4\left((a-2b)dx - 2b\right)\cosh(dx+c) + \left(3a\cosh(dx+c)^2 + a - 8b\right)\sinh(dx+c)}{8d\cosh(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)*sinh(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(a*sinh(d*x + c)^3 - 4*((a - 2*b)*d*x - 2*b)*cosh(d*x + c) + (3*a*cosh(
d*x + c)^2 + a - 8*b)*sinh(d*x + c))/(d*cosh(d*x + c))
```

giac [B] time = 0.16, size = 92, normalized size = 2.14

$$-\frac{4\,(dx+c)(a-2\,b)-ae^{(2\,dx+2\,c)}-\frac{ae^{(4\,dx+4\,c)}-2\,be^{(4\,dx+4\,c)}+14\,be^{(2\,dx+2\,c)}-a}{e^{(4\,dx+4\,c)}+e^{(2\,dx+2\,c)}}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)*sinh(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/8*(4*(d*x + c)*(a - 2*b) - a*e^(2*d*x + 2*c) - (a*e^(4*d*x + 4*c) - 2*b*
e^(4*d*x + 4*c) + 14*b*e^(2*d*x + 2*c) - a)/(e^(4*d*x + 4*c) + e^(2*d*x + 2
*c)))/d
```

maple [A] time = 0.18, size = 45, normalized size = 1.05

$$\frac{a\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+b\,(dx+c-\tanh\,(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)*sinh(d*x+c)^2,x)
```

```
[Out] 1/d*(a*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c)+b*(d*x+c-tanh(d*x+c)))
```

maxima [A] time = 0.33, size = 62, normalized size = 1.44

$$-\frac{1}{8}\,a\left(4\,x-\frac{e^{(2\,dx+2\,c)}}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}\right)+b\left(x+\frac{c}{d}-\frac{2}{d\left(e^{(-2\,dx-2\,c)}+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)*sinh(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/8*a*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) + b*(x + c/d - 2/(d*(
e^(-2*d*x - 2*c) + 1)))
```

mupad [B] time = 1.43, size = 55, normalized size = 1.28

$$\frac{a\cosh\,(c+d\,x)\,\sinh\,(c+d\,x)}{2\,d}-\frac{\frac{a\,d\,x}{2}-b\,d\,x}{d}-\frac{b\sinh\,(c+d\,x)}{d\cosh\,(c+d\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^2*(a + b/cosh(c + d*x)^2),x)
```

```
[Out] (a*cosh(c + d*x)*sinh(c + d*x))/(2*d) - ((a*d*x)/2 - b*d*x)/d - (b*sinh(c +
 d*x))/(d*cosh(c + d*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\operatorname{sech}^2\left(c + dx\right)\right)\sinh^2\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)*sinh(d*x+c)**2,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)*sinh(c + d*x)**2, x)
```

3.4 $\int \left(a+b\,\mathbf{sech}^2(c+dx)\right)\sinh(c+dx)\,dx$

Optimal. Leaf size=24

$$\frac{a\cosh(c+dx)}{d}-\frac{b\,\text{sech}(c+dx)}{d}$$

```
[Out] a*cosh(d*x+c)/d-b*sech(d*x+c)/d
```

Rubi [A] time = 0.03, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {4133, 14}

$$\frac{a\cosh(c+dx)}{d}-\frac{b\,\text{sech}(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)*Sinh[c + d*x],x]
```

```
[Out] (a*Cosh[c + d*x])/d - (b*Sech[c + d*x])/d
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \left(a+b\,\text{sech}^2(c+dx)\right)\sinh(c+dx)\,dx &= \frac{\text{Subst}\left(\int \frac{b+ax^2}{x^2}\,dx,x,\cosh(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(a+\frac{b}{x^2}\right)dx,x,\cosh(c+dx)\right)}{d}\\
&= \frac{a\cosh(c+dx)}{d}-\frac{b\,\text{sech}(c+dx)}{d}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 35, normalized size = 1.46

$$\frac{a\sinh(c)\sinh(dx)}{d}+\frac{a\cosh(c)\cosh(dx)}{d}-\frac{b\operatorname{sech}(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)*Sinh[c + d*x],x]
[Out] (a*Cosh[c]*Cosh[d*x])/d - (b*Sech[c + d*x])/d + (a*Sinh[c]*Sinh[d*x])/d
```

fricas [A] time = 0.43, size = 38, normalized size = 1.58

$$\frac{a\cosh\left(dx+c\right)^{2}+a\sinh\left(dx+c\right)^{2}+a-2\,b}{2\,d\cosh\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)*sinh(d*x+c),x, algorithm="fricas")
[Out] 1/2*(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + a - 2*b)/(d*cosh(d*x + c))
```

giac [A] time = 0.12, size = 45, normalized size = 1.88

$$\frac{a\left(e^{(dx+c)}+e^{(-dx-c)}\right)-\frac{4\,b}{e^{(dx+c)}+e^{(-dx-c)}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)*sinh(d*x+c),x, algorithm="giac")
[Out] 1/2*(a*(e^(d*x + c) + e^(-d*x - c)) - 4*b/(e^(d*x + c) + e^(-d*x - c)))/d
```

maple [A] time = 0.09, size = 26, normalized size = 1.08

$$-\frac{b\operatorname{sech}\left(dx+c\right)-\frac{a}{\operatorname{sech}(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)*sinh(d*x+c),x)
[Out] -1/d*(b*sech(d*x+c)-a/sech(d*x+c))
```

maxima [A] time = 0.32, size = 36, normalized size = 1.50

$$\frac{a\cosh\left(dx+c\right)}{d}-\frac{2\,b}{d\left(e^{(dx+c)}+e^{(-dx-c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)*sinh(d*x+c),x, algorithm="maxima")
```

```
[Out] a*cosh(d*x + c)/d - 2*b/(d*(e^(d*x + c) + e^(-d*x - c)))
```

mupad [B] time = 0.08, size = 26, normalized size = 1.08

$$-\frac{b-a\cosh\left(c+d\,x\right)^2}{d\cosh\left(c+d\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)*(a + b/cosh(c + d*x)^2),x)
```

```
[Out] -(b - a*cosh(c + d*x)^2)/(d*cosh(c + d*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\operatorname{sech}^2\left(c+dx\right)\right)\sinh\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)*sinh(d*x+c),x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)*sinh(c + d*x), x)
```

3.5 $\int \mathbf{csch}(c+dx)\left(a+b\,\mathbf{sech}^2(c+dx)\right)\,dx$

Optimal. Leaf size=27

$$\frac{b\operatorname{sech}(c+dx)}{d}-\frac{(a+b)\tanh^{-1}(\cosh(c+dx))}{d}$$

```
[Out] -(a+b)*arctanh(cosh(d*x+c))/d+b*sech(d*x+c)/d
```

Rubi [A] time = 0.04, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {4133, 453, 206}

$$\frac{b\operatorname{sech}(c+dx)}{d}-\frac{(a+b)\tanh^{-1}(\cosh(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -(((a + b)*ArcTanh[Cosh[c + d*x]])/d) + (b*Sech[c + d*x])/d
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \operatorname{csch}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx = -\frac{\operatorname{Subst}\left(\int \frac{b+ax^2}{x^2(1-x^2)}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= \frac{b\operatorname{sech}(c+dx)}{d} - \frac{(a+b)\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= -\frac{(a+b)\tanh^{-1}(\cosh(c+dx))}{d} + \frac{b\operatorname{sech}(c+dx)}{d}$$

Mathematica [B] time = 0.05, size = 67, normalized size = 2.48

$$\frac{a\log\left(\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} - \frac{a\log\left(\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} + \frac{b\operatorname{sech}(c+dx)}{d} + \frac{b\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -((a*Log[Cosh[c/2 + (d*x)/2]])/d) + (a*Log[Sinh[c/2 + (d*x)/2]])/d + (b*Log
[Tanh[(c + d*x)/2]])/d + (b*Sech[c + d*x])/d
```

fricas [B] time = 0.42, size = 180, normalized size = 6.67

$$\frac{2\,b\cosh\left(dx+c\right) - \left((a+b)\cosh\left(dx+c\right)^2 + 2\,(a+b)\cosh\left(dx+c\right)\sinh\left(dx+c\right) + (a+b)\sinh\left(dx+c\right)^2 + a + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] (2*b*cosh(d*x + c) - ((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sin
h(d*x + c) + (a + b)*sinh(d*x + c)^2 + a + b)*log(cosh(d*x + c) + sinh(d*x
+ c) + 1) + ((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c
) + (a + b)*sinh(d*x + c)^2 + a + b)*log(cosh(d*x + c) + sinh(d*x + c) - 1)
 + 2*b*sinh(d*x + c))/(d*cosh(d*x + c)^2 + 2*d*cosh(d*x + c)*sinh(d*x + c)
+ d*sinh(d*x + c)^2 + d)
```

giac [B] time = 0.13, size = 72, normalized size = 2.67

$$-\frac{(a+b)\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right) - (a+b)\log\left(e^{(dx+c)}+e^{(-dx-c)}-2\right) - \frac{4\,b}{e^{(dx+c)}+e^{(-dx-c)}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/2*((a + b)*log(e^(d*x + c) + e^(-d*x - c) + 2) - (a + b)*log(e^(d*x + c)
 + e^(-d*x - c) - 2) - 4*b/(e^(d*x + c) + e^(-d*x - c)))/d
```

maple [A] time = 0.20, size = 36, normalized size = 1.33

$$\frac{-2a\operatorname{arctanh}\left(e^{dx+c}\right)+b\left(\frac{1}{\cosh(dx+c)}-2\operatorname{arctanh}\left(e^{dx+c}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)*(a+b*sech(d*x+c)^2),x)
```

```
[Out] 1/d*(-2*a*arctanh(exp(d*x+c))+b*(1/cosh(d*x+c)-2*arctanh(exp(d*x+c))))
```

maxima [B] time = 0.32, size = 80, normalized size = 2.96

$$-b\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}-\frac{2\,e^{(-dx-c)}}{d\left(e^{(-2\,dx-2\,c)}+1\right)}\right)+\frac{a\log\left(\tanh\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -b*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d - 2*e^(-d*x - c)/(d*(
e^(-2*d*x - 2*c) + 1))) + a*log(tanh(1/2*d*x + 1/2*c))/d
```

mupad [B] time = 1.41, size = 79, normalized size = 2.93

$$\frac{b}{d\cosh\left(c+d\,x\right)}-\frac{2\,\mathrm{atan}\left(\frac{e^{d\,x}\,e^{c}\,\left(a\,\sqrt{-d^2}+b\,\sqrt{-d^2}\right)}{d\,\sqrt{a^2+2\,a\,b+b^2}}\right)\,\sqrt{a^2+2\,a\,b+b^2}}{\sqrt{-d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cosh(c + d*x)^2)/sinh(c + d*x),x)
```

```
[Out] b/(d*cosh(c + d*x)) - (2*atan((exp(d*x)*exp(c)*(a*(-d^2)^(1/2) + b*(-d^2)^(
1/2)))/(d*(2*a*b + a^2 + b^2)^(1/2)))*(2*a*b + a^2 + b^2)^(1/2))/(-d^2)^(1/
2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\operatorname{sech}^{2}\left(c+dx\right)\right)\operatorname{csch}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)*csch(c + d*x), x)
```

3.6 $\int \operatorname{csch}^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx$

Optimal. Leaf size=27

$$-\frac{(a+b)\coth(c+dx)}{d}-\frac{b\tanh(c+dx)}{d}$$

```
[Out] -(a+b)*coth(d*x+c)/d-b*tanh(d*x+c)/d
```

Rubi [A] time = 0.05, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4132, 14}

$$-\frac{(a+b)\coth(c+dx)}{d}-\frac{b\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -(((a + b)*Coth[c + d*x])/d) - (b*Tanh[c + d*x])/d
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \operatorname{csch}^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)dx = \frac{\operatorname{Subst}\left(\int \frac{a+b-bx^2}{x^2}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\operatorname{Subst}\left(\int \left(-b+\frac{a+b}{x^2}\right)dx, x, \tanh(c+dx)\right)}{d}$$
$$= -\frac{(a+b)\coth(c+dx)}{d} - \frac{b\tanh(c+dx)}{d}$$

Mathematica [A] time = 0.07, size = 37, normalized size = 1.37

$$-\frac{a\coth(c+dx)}{d} - \frac{b\tanh(c+dx)}{d} - \frac{b\coth(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -((a*Coth[c + d*x])/d) - (b*Coth[c + d*x])/d - (b*Tanh[c + d*x])/d
```

fricas [B] time = 0.40, size = 91, normalized size = 3.37

$$-\frac{4\left((a+b)\cosh\left(dx+c\right)-b\sinh\left(dx+c\right)\right)}{d\cosh\left(dx+c\right)^3+3\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^2+d\sinh\left(dx+c\right)^3-d\cosh\left(dx+c\right)+\left(3\,d\cosh\left(dx+c\right)^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -4*((a + b)*cosh(d*x + c) - b*sinh(d*x + c))/(d*cosh(d*x + c)^3 + 3*d*cosh(
d*x + c)*sinh(d*x + c)^2 + d*sinh(d*x + c)^3 - d*cosh(d*x + c) + (3*d*cosh(
d*x + c)^2 + d)*sinh(d*x + c))
```

giac [A] time = 0.14, size = 34, normalized size = 1.26

$$-\frac{2\left(ae^{(2\,dx+2\,c)}+a+2\,b\right)}{d\left(e^{(4\,dx+4\,c)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -2*(a*e^(2*d*x + 2*c) + a + 2*b)/(d*(e^(4*d*x + 4*c) - 1))
```

maple [A] time = 0.38, size = 44, normalized size = 1.63

$$\frac{-\coth(dx+c)\,a+b\left(-\frac{1}{\sinh(dx+c)\cosh(dx+c)}-2\tanh(dx+c)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2*(a+b*sech(d*x+c)^2),x)
```

```
[Out] 1/d*(-coth(d*x+c)*a+b*(-1/sinh(d*x+c)/cosh(d*x+c)-2*tanh(d*x+c)))
```

maxima [A] time = 0.33, size = 39, normalized size = 1.44

$$\frac{2\,a}{d\left(e^{(-2\,dx-2\,c)}-1\right)}+\frac{4\,b}{d\left(e^{(-4\,dx-4\,c)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 2*a/(d*(e^(-2*d*x - 2*c) - 1)) + 4*b/(d*(e^(-4*d*x - 4*c) - 1))
```

mupad [B] time = 1.38, size = 34, normalized size = 1.26

$$-\frac{2\,\left(a+2\,b+a\,\mathrm{e}^{2\,c+2\,d\,x}\right)}{d\,\left(\mathrm{e}^{4\,c+4\,d\,x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cosh(c + d*x)^2)/sinh(c + d*x)^2,x)
```

```
[Out] -(2*(a + 2*b + a*exp(2*c + 2*d*x)))/(d*(exp(4*c + 4*d*x) - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\operatorname{sech}^{2}(c+dx)\right)\operatorname{csch}^{2}(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2*(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)*csch(c + d*x)**2, x)
```

3.7 $\int \operatorname{csch}^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx$

Optimal. Leaf size=54

$$\frac{(a+3b)\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{(a+b)\coth(c+dx)\operatorname{csch}(c+dx)}{2d}-\frac{b\operatorname{sech}(c+dx)}{d}$$

```
[Out] 1/2*(a+3*b)*arctanh(cosh(d*x+c))/d-1/2*(a+b)*coth(d*x+c)*csch(d*x+c)/d-b*se
ch(d*x+c)/d
```

Rubi [A] time = 0.07, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {4133, 456, 453, 206}

$$\frac{(a+3b)\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{(a+b)\coth(c+dx)\operatorname{csch}(c+dx)}{2d}-\frac{b\operatorname{sech}(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((a + 3*b)*ArcTanh[Cosh[c + d*x]])/(2*d) - ((a + b)*Coth[c + d*x]*Csch[c +
d*x])/(2*d) - (b*Sech[c + d*x])/d
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
```

```
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \operatorname{csch}^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)dx = \frac{\operatorname{Subst}\left(\int \frac{b+ax^2}{x^2(1-x^2)^2}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{(a+b)\coth(c+dx)\operatorname{csch}(c+dx)}{2d} - \frac{\operatorname{Subst}\left(\int \frac{-2b-(a+b)x^2}{x^2(1-x^2)}\,dx, x, \cosh\right.}{2d}$$

$$= -\frac{(a+b)\coth(c+dx)\operatorname{csch}(c+dx)}{2d} - \frac{b\operatorname{sech}(c+dx)}{d} + \frac{(a+3b)\operatorname{Subs}}{}$$

$$= \frac{(a+3b)\tanh^{-1}(\cosh(c+dx))}{2d} - \frac{(a+b)\coth(c+dx)\operatorname{csch}(c+dx)}{2d} -$$

Mathematica [B] time = 0.05, size = 131, normalized size = 2.43

$$-\frac{a\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8d} - \frac{a\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8d} - \frac{a\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{2d} - \frac{b\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8d} - \frac{b\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8d} - \frac{b}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -1/8*(a*Csch[(c + d*x)/2]^2)/d - (b*Csch[(c + d*x)/2]^2)/(8*d) - (a*Log[Tan
h[(c + d*x)/2]])/(2*d) - (3*b*Log[Tanh[(c + d*x)/2]])/(2*d) - (a*Sech[(c +
d*x)/2]^2)/(8*d) - (b*Sech[(c + d*x)/2]^2)/(8*d) - (b*Sech[c + d*x])/d
```

fricas [B] time = 0.43, size = 924, normalized size = 17.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a + 3*b)*cosh(d*x + c)^5 + 10*(a + 3*b)*cosh(d*x + c)*sinh(d*x + c
)^4 + 2*(a + 3*b)*sinh(d*x + c)^5 + 4*(a - b)*cosh(d*x + c)^3 + 4*(5*(a + 3
*b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^3 + 4*(5*(a + 3*b)*cosh(d*x + c)
^3 + 3*(a - b)*cosh(d*x + c))*sinh(d*x + c)^2 + 2*(a + 3*b)*cosh(d*x + c) -
 ((a + 3*b)*cosh(d*x + c)^6 + 6*(a + 3*b)*cosh(d*x + c)*sinh(d*x + c)^5 + (
a + 3*b)*sinh(d*x + c)^6 - (a + 3*b)*cosh(d*x + c)^4 + (15*(a + 3*b)*cosh(d
*x + c)^2 - a - 3*b)*sinh(d*x + c)^4 + 4*(5*(a + 3*b)*cosh(d*x + c)^3 - (a
+ 3*b)*cosh(d*x + c))*sinh(d*x + c)^3 - (a + 3*b)*cosh(d*x + c)^2 + (15*(a
+ 3*b)*cosh(d*x + c)^4 - 6*(a + 3*b)*cosh(d*x + c)^2 - a - 3*b)*sinh(d*x +
c)^2 + 2*(3*(a + 3*b)*cosh(d*x + c)^5 - 2*(a + 3*b)*cosh(d*x + c)^3 - (a +
3*b)*cosh(d*x + c))*sinh(d*x + c) + a + 3*b)*log(cosh(d*x + c) + sinh(d*x +
 c) + 1) + ((a + 3*b)*cosh(d*x + c)^6 + 6*(a + 3*b)*cosh(d*x + c)*sinh(d*x
+ c)^5 + (a + 3*b)*sinh(d*x + c)^6 - (a + 3*b)*cosh(d*x + c)^4 + (15*(a + 3
*b)*cosh(d*x + c)^2 - a - 3*b)*sinh(d*x + c)^4 + 4*(5*(a + 3*b)*cosh(d*x +
c)^3 - (a + 3*b)*cosh(d*x + c))*sinh(d*x + c)^3 - (a + 3*b)*cosh(d*x + c)^2
 + (15*(a + 3*b)*cosh(d*x + c)^4 - 6*(a + 3*b)*cosh(d*x + c)^2 - a - 3*b)*s
inh(d*x + c)^2 + 2*(3*(a + 3*b)*cosh(d*x + c)^5 - 2*(a + 3*b)*cosh(d*x + c)
^3 - (a + 3*b)*cosh(d*x + c))*sinh(d*x + c) + a + 3*b)*log(cosh(d*x + c) +
sinh(d*x + c) - 1) + 2*(5*(a + 3*b)*cosh(d*x + c)^4 + 6*(a - b)*cosh(d*x +
c)^2 + a + 3*b)*sinh(d*x + c))/(d*cosh(d*x + c)^6 + 6*d*cosh(d*x + c)*sinh(
d*x + c)^5 + d*sinh(d*x + c)^6 - d*cosh(d*x + c)^4 + (15*d*cosh(d*x + c)^2
- d)*sinh(d*x + c)^4 + 4*(5*d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x +
 c)^3 - d*cosh(d*x + c)^2 + (15*d*cosh(d*x + c)^4 - 6*d*cosh(d*x + c)^2 - d
)*sinh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^5 - 2*d*cosh(d*x + c)^3 - d*cosh(d
*x + c))*sinh(d*x + c) + d)
```

giac [B] time = 0.15, size = 142, normalized size = 2.63

$$\frac{(a+3\,b)\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)-(a+3\,b)\log\left(e^{(dx+c)}+e^{(-dx-c)}-2\right)-\frac{4\,(a(e^{(dx+c)}+e^{(-dx-c)})^2+3\,b(e^{(dx+c)}+e^{(-dx-c)})^2-8}{(e^{(dx+c)}+e^{(-dx-c)})^3-4\,e^{(dx+c)}-4\,e^{(-dx-c)}}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/4*((a + 3*b)*log(e^(d*x + c) + e^(-d*x - c) + 2) - (a + 3*b)*log(e^(d*x +
 c) + e^(-d*x - c) - 2) - 4*(a*(e^(d*x + c) + e^(-d*x - c))^2 + 3*b*(e^(d*x
 + c) + e^(-d*x - c))^2 - 8*b)/((e^(d*x + c) + e^(-d*x - c))^3 - 4*e^(d*x +
 c) - 4*e^(-d*x - c)))/d
```

maple [A] time = 0.38, size = 70, normalized size = 1.30

$$\frac{a\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(\mathrm{e}^{dx+c}\right)\right)+b\left(-\frac{1}{2\sinh(dx+c)^{2}\cosh(dx+c)}-\frac{3}{2\cosh(dx+c)}+3\operatorname{arctanh}\left(\mathrm{e}^{dx+c}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3*(a+b*sech(d*x+c)^2),x)
```

```
[Out] 1/d*(a*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))+b*(-1/2/sinh(d*x+
c)^2/cosh(d*x+c)-3/2/cosh(d*x+c)+3*arctanh(exp(d*x+c))))
```

maxima [B] time = 0.34, size = 198, normalized size = 3.67

$$\frac{1}{2}\,b\left(\frac{3\,\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{3\,\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(3\,e^{(-dx-c)}-2\,e^{(-3\,dx-3\,c)}+3\,e^{(-5\,dx-5\,c)}\right)}{d\left(e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}-e^{(-6\,dx-6\,c)}-1\right)}\right)+\frac{1}{2}\,a\left(\frac{\log\left(e^{(-dx-c)}+1\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*b*(3*log(e^(-d*x - c) + 1)/d - 3*log(e^(-d*x - c) - 1)/d + 2*(3*e^(-d*x
 - c) - 2*e^(-3*d*x - 3*c) + 3*e^(-5*d*x - 5*c))/(d*(e^(-2*d*x - 2*c) + e^(
-4*d*x - 4*c) - e^(-6*d*x - 6*c) - 1))) + 1/2*a*(log(e^(-d*x - c) + 1)/d -
log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d
*x - 2*c) - e^(-4*d*x - 4*c) - 1)))
```

mupad [B] time = 0.17, size = 160, normalized size = 2.96

$$\frac{\operatorname{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\left(a\,\sqrt{-d^2}+3\,b\,\sqrt{-d^2}\right)}{d\,\sqrt{a^2+6\,a\,b+9\,b^2}}\right)\sqrt{a^2+6\,a\,b+9\,b^2}}{\sqrt{-d^2}}-\frac{\mathrm{e}^{c+d\,x}\,(a+b)}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}-\frac{2\,\mathrm{e}^{c+d\,x}\,(a+b)}{d\,\left(\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1\right)}-\frac{2\,b\,\mathrm{e}^{c+d\,x}}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cosh(c + d*x)^2)/sinh(c + d*x)^3,x)
```

```
[Out] (atan((exp(d*x)*exp(c)*(a*(-d^2)^(1/2) + 3*b*(-d^2)^(1/2)))/(d*(6*a*b + a^2
 + 9*b^2)^(1/2)))*(6*a*b + a^2 + 9*b^2)^(1/2))/(-d^2)^(1/2) - (exp(c + d*x)
*(a + b))/(d*(exp(2*c + 2*d*x) - 1)) - (2*exp(c + d*x)*(a + b))/(d*(exp(4*c
 + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) - (2*b*exp(c + d*x))/(d*(exp(2*c + 2*d
*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\operatorname{sech}^{2}\left(c+dx\right)\right)\operatorname{csch}^{3}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3*(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)*csch(c + d*x)**3, x)
```

3.8 $\int \operatorname{csch}^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx$

Optimal. Leaf size=45

$$-\frac{(a+b)\coth^3(c+dx)}{3d}+\frac{(a+2b)\coth(c+dx)}{d}+\frac{b\tanh(c+dx)}{d}$$

```
[Out] (a+2*b)*coth(d*x+c)/d-1/3*(a+b)*coth(d*x+c)^3/d+b*tanh(d*x+c)/d
```

Rubi [A] time = 0.06, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4132, 448}

$$-\frac{(a+b)\coth^3(c+dx)}{3d}+\frac{(a+2b)\coth(c+dx)}{d}+\frac{b\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((a + 2*b)*Coth[c + d*x])/d - ((a + b)*Coth[c + d*x]^3)/(3*d) + (b*Tanh[c +
 d*x])/d
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \operatorname{csch}^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx = \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)(a+b-bx^2)}{x^4}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\operatorname{Subst}\left(\int \left(b+\frac{a+b}{x^4}+\frac{-a-2b}{x^2}\right)dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{(a+2b)\coth(c+dx)}{d} - \frac{(a+b)\coth^3(c+dx)}{3d} + \frac{b\tanh(c+dx)}{d}$$

Mathematica [A] time = 0.05, size = 84, normalized size = 1.87

$$\frac{2a\coth(c+dx)}{3d} - \frac{a\coth(c+dx)\operatorname{csch}^2(c+dx)}{3d} + \frac{b\tanh(c+dx)}{d} + \frac{5b\coth(c+dx)}{3d} - \frac{b\coth(c+dx)\operatorname{csch}^2(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] (2*a*Coth[c + d*x])/(3*d) + (5*b*Coth[c + d*x])/(3*d) - (a*Coth[c + d*x]*Cs
ch[c + d*x]^2)/(3*d) - (b*Coth[c + d*x]*Csch[c + d*x]^2)/(3*d) + (b*Tanh[c
+ d*x])/d
```

fricas [B] time = 0.38, size = 246, normalized size = 5.47

$$-\frac{}{3\left(d\cosh(dx+c)^6 + 6\,d\cosh(dx+c)\sinh(dx+c)^5 + d\sinh(dx+c)^6 - 2\,d\cosh(dx+c)^4 + \left(15\,d\cosh(dx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -8/3*((a - 2*b)*cosh(d*x + c)^2 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (
a - 2*b)*sinh(d*x + c)^2 + a + 4*b)/(d*cosh(d*x + c)^6 + 6*d*cosh(d*x + c)*
sinh(d*x + c)^5 + d*sinh(d*x + c)^6 - 2*d*cosh(d*x + c)^4 + (15*d*cosh(d*x
+ c)^2 - 2*d)*sinh(d*x + c)^4 + 4*(5*d*cosh(d*x + c)^3 - 2*d*cosh(d*x + c))
*sinh(d*x + c)^3 - d*cosh(d*x + c)^2 + (15*d*cosh(d*x + c)^4 - 12*d*cosh(d*
x + c)^2 - d)*sinh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^5 - 4*d*cosh(d*x + c)^
3 + d*cosh(d*x + c))*sinh(d*x + c) + 2*d)
```

giac [A] time = 0.13, size = 80, normalized size = 1.78

$$-\frac{2\left(\frac{3\,b}{e^{(2\,dx+2\,c)}+1} - \frac{3\,be^{(4\,dx+4\,c)} - 6\,ae^{(2\,dx+2\,c)} - 12\,be^{(2\,dx+2\,c)} + 2\,a + 5\,b}{\left(e^{(2\,dx+2\,c)}-1\right)^3}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -2/3*(3*b/(e^(2*d*x + 2*c) + 1) - (3*b*e^(4*d*x + 4*c) - 6*a*e^(2*d*x + 2*c
) - 12*b*e^(2*d*x + 2*c) + 2*a + 5*b)/(e^(2*d*x + 2*c) - 1)^3)/d
```

maple [A] time = 0.44, size = 73, normalized size = 1.62

$$\frac{a\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^{2}}{3}\right)\coth\left(dx+c\right)+b\left(-\frac{1}{3\sinh(dx+c)^{3}\cosh(dx+c)}+\frac{4}{3\sinh(dx+c)\cosh(dx+c)}+\frac{8\tanh(dx+c)}{3}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4*(a+b*sech(d*x+c)^2),x)
```

```
[Out] 1/d*(a*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)+b*(-1/3/sinh(d*x+c)^3/cosh(d*x+c
)+4/3/sinh(d*x+c)/cosh(d*x+c)+8/3*tanh(d*x+c)))
```

maxima [B] time = 0.34, size = 187, normalized size = 4.16

$$\frac{4}{3}a\left(\frac{3e^{(-2dx-2c)}}{d\left(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1\right)}-\frac{1}{d\left(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1\right)}\right)+\frac{16}{3}b\Bigg(\frac{}{d\big(2e^{(-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 4/3*a*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(
-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6
*d*x - 6*c) - 1))) + 16/3*b*(2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - 2*
e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) - 1)) - 1/(d*(2*e^(-2*d*x - 2*c) - 2*e^
(-6*d*x - 6*c) + e^(-8*d*x - 8*c) - 1)))
```

mupad [B] time = 1.40, size = 172, normalized size = 3.82

$$\frac{\frac{2b}{3d}+\frac{2b\,\mathrm{e}^{4c+4dx}}{3d}-\frac{4\,\mathrm{e}^{2c+2dx}\,(2a+3b)}{3d}}{3\,\mathrm{e}^{2c+2dx}-3\,\mathrm{e}^{4c+4dx}+\mathrm{e}^{6c+6dx}-1}-\frac{\frac{2(2a+3b)}{3d}-\frac{2b\,\mathrm{e}^{2c+2dx}}{3d}}{\mathrm{e}^{4c+4dx}-2\,\mathrm{e}^{2c+2dx}+1}+\frac{2b}{3d\,\left(\mathrm{e}^{2c+2dx}-1\right)}-\frac{2b}{d\,\left(\mathrm{e}^{2c+2dx}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cosh(c + d*x)^2)/sinh(c + d*x)^4,x)
```

```
[Out] ((2*b)/(3*d) + (2*b*exp(4*c + 4*d*x))/(3*d) - (4*exp(2*c + 2*d*x)*(2*a + 3*
b))/(3*d))/(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1)
```

```
 - ((2*(2*a + 3*b))/(3*d) - (2*b*exp(2*c + 2*d*x))/(3*d))/(exp(4*c + 4*d*x)
 - 2*exp(2*c + 2*d*x) + 1) + (2*b)/(3*d*(exp(2*c + 2*d*x) - 1)) - (2*b)/(d*
(exp(2*c + 2*d*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\,\mathrm{sech}^2\left(c + dx\right)\right)\mathrm{csch}^4\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4*(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)*csch(c + d*x)**4, x)
```

3.9 $\int \left(a + b\textbf{sech}^2(c + dx)\right)^2 \sinh^4(c + dx)\, dx$

Optimal. Leaf size=114

$$-\frac{\left(a^2 - 8ab + 4b^2\right)\tanh(c + dx)}{4d} + \frac{1}{8}x\left(3a^2 - 24ab + 8b^2\right) + \frac{a^2 \sinh^4(c + dx)\tanh(c + dx)}{4d} - \frac{a(a - 8b)\sinh(c + dx)\,c}{8d}$$

```
[Out] 1/8*(3*a^2-24*a*b+8*b^2)*x-1/8*a*(a-8*b)*cosh(d*x+c)*sinh(d*x+c)/d-1/4*(a^2
-8*a*b+4*b^2)*tanh(d*x+c)/d+1/4*a^2*sinh(d*x+c)^4*tanh(d*x+c)/d-1/3*b^2*tan
h(d*x+c)^3/d
```

Rubi [A] time = 0.14, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4132, 463, 455, 1153, 206}

$$-\frac{\left(a^2 - 8ab + 4b^2\right)\tanh(c + dx)}{4d} + \frac{1}{8}x\left(3a^2 - 24ab + 8b^2\right) + \frac{a^2 \sinh^4(c + dx)\tanh(c + dx)}{4d} - \frac{a(a - 8b)\sinh(c + dx)\,c}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^2*Sinh[c + d*x]^4,x]
```

```
[Out] ((3*a^2 - 24*a*b + 8*b^2)*x)/8 - (a*(a - 8*b)*Cosh[c + d*x]*Sinh[c + d*x])/
(8*d) - ((a^2 - 8*a*b + 4*b^2)*Tanh[c + d*x])/(4*d) + (a^2*Sinh[c + d*x]^4*
Tanh[c + d*x])/(4*d) - (b^2*Tanh[c + d*x]^3)/(3*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
```

```
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \left(a+b\operatorname{sech}^2(c+dx)\right)^2 \sinh^4(c+dx)\,dx = \frac{\operatorname{Subst}\left(\int \frac{x^4\left(a+b-bx^2\right)^2}{\left(1-x^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{a^2\sinh^4(c+dx)\tanh(c+dx)}{4d} - \frac{\operatorname{Subst}\left(\int \frac{x^4\left(5a^2-4(a+b)^2+4b^2x^2\right)}{\left(1-x^2\right)^2}\,dx, x\right.}{4d}$$

$$= -\frac{a(a-8b)\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{a^2\sinh^4(c+dx)\tanh(c+}{4d}$$

$$= -\frac{a(a-8b)\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{a^2\sinh^4(c+dx)\tanh(c+}{4d}$$

$$= -\frac{a(a-8b)\cosh(c+dx)\sinh(c+dx)}{8d} - \frac{\left(a^2-8ab+4b^2\right)\tanh(c+}{4d}$$

$$= \frac{1}{8}\left(3a^2-24ab+8b^2\right)x - \frac{a(a-8b)\cosh(c+dx)\sinh(c+dx)}{8d} - \frac{(a}{}$$

Mathematica [A] time = 1.66, size = 153, normalized size = 1.34

$$\frac{\operatorname{sech}^3(c+dx)\left(a\cosh^2(c+dx)+b\right)^2\left(3\cosh^3(c+dx)\left(4dx\left(3a^2-24ab+8b^2\right)+a^2\sinh(4(c+dx))-8a(a-2b)\right.\right.}{24d(a\cosh(2(}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^2*Sinh[c + d*x]^4,x]
```

```
[Out] ((b + a*Cosh[c + d*x]^2)^2*Sech[c + d*x]^3*(32*b^2*Sech[c]*Sinh[d*x] + 64*(
3*a - 2*b)*b*Cosh[c + d*x]^2*Sech[c]*Sinh[d*x] + 3*Cosh[c + d*x]^3*(4*(3*a^
2 - 24*a*b + 8*b^2)*d*x - 8*a*(a - 2*b)*Sinh[2*(c + d*x)] + a^2*Sinh[4*(c +
 d*x)]) + 32*b^2*Cosh[c + d*x]*Tanh[c]))/(24*d*(a + 2*b + a*Cosh[2*(c + d*x
)])^2)
```

fricas [B] time = 0.42, size = 342, normalized size = 3.00

$$3\,a^2\sinh\left(dx+c\right)^7+3\left(21\,a^2\cosh\left(dx+c\right)^2-5\,a^2+16\,ab\right)\sinh\left(dx+c\right)^5+8\left(3\left(3\,a^2-24\,ab+8\,b^2\right)dx-48\,ab+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2*sinh(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/192*(3*a^2*sinh(d*x + c)^7 + 3*(21*a^2*cosh(d*x + c)^2 - 5*a^2 + 16*a*b)*
sinh(d*x + c)^5 + 8*(3*(3*a^2 - 24*a*b + 8*b^2)*d*x - 48*a*b + 32*b^2)*cosh
(d*x + c)^3 + 24*(3*(3*a^2 - 24*a*b + 8*b^2)*d*x - 48*a*b + 32*b^2)*cosh(d*
x + c)*sinh(d*x + c)^2 + (105*a^2*cosh(d*x + c)^4 - 30*(5*a^2 - 16*a*b)*cos
h(d*x + c)^2 - 63*a^2 + 528*a*b - 256*b^2)*sinh(d*x + c)^3 + 24*(3*(3*a^2 -
 24*a*b + 8*b^2)*d*x - 48*a*b + 32*b^2)*cosh(d*x + c) + 3*(7*a^2*cosh(d*x +
 c)^6 - 5*(5*a^2 - 16*a*b)*cosh(d*x + c)^4 - (63*a^2 - 528*a*b + 256*b^2)*c
osh(d*x + c)^2 - 15*a^2 + 160*a*b)*sinh(d*x + c))/(d*cosh(d*x + c)^3 + 3*d*
cosh(d*x + c)*sinh(d*x + c)^2 + 3*d*cosh(d*x + c))
```

giac [B] time = 0.18, size = 231, normalized size = 2.03

$$3\,a^2e^{(4\,dx+4\,c)}-24\,a^2e^{(2\,dx+2\,c)}+48\,abe^{(2\,dx+2\,c)}+24\left(3\,a^2-24\,ab+8\,b^2\right)(dx+c)-3\left(18\,a^2e^{(4\,dx+4\,c)}-144\,abe^{(4\,dx+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2*sinh(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/192*(3*a^2*e^(4*d*x + 4*c) - 24*a^2*e^(2*d*x + 2*c) + 48*a*b*e^(2*d*x + 2
*c) + 24*(3*a^2 - 24*a*b + 8*b^2)*(d*x + c) - 3*(18*a^2*e^(4*d*x + 4*c) - 1
```

```
44*a*b*e^(4*d*x + 4*c) + 48*b^2*e^(4*d*x + 4*c) - 8*a^2*e^(2*d*x + 2*c) + 1
6*a*b*e^(2*d*x + 2*c) + a^2)*e^(-4*d*x - 4*c) - 256*(3*a*b*e^(4*d*x + 4*c)
- 3*b^2*e^(4*d*x + 4*c) + 6*a*b*e^(2*d*x + 2*c) - 3*b^2*e^(2*d*x + 2*c) + 3
*a*b - 2*b^2)/(e^(2*d*x + 2*c) + 1)^3)/d
```

maple [A] time = 0.34, size = 109, normalized size = 0.96

$$\frac{a^2\left(\left(\frac{\left(\sinh^3(dx+c)\right)}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+2ab\left(\frac{\sinh^3(dx+c)}{2\cosh(dx+c)}-\frac{3dx}{2}-\frac{3c}{2}+\frac{3\tanh(dx+c)}{2}\right)+b^2\left(dx+c\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)^2*sinh(d*x+c)^4,x)
```

```
[Out] 1/d*(a^2*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)+2*
a*b*(1/2*sinh(d*x+c)^3/cosh(d*x+c)-3/2*d*x-3/2*c+3/2*tanh(d*x+c))+b^2*(d*x+
c-tanh(d*x+c)-1/3*tanh(d*x+c)^3))
```

maxima [B] time = 0.34, size = 211, normalized size = 1.85

$$\frac{1}{64}a^2\left(24x+\frac{e^{(4dx+4c)}}{d}-\frac{8e^{(2dx+2c)}}{d}+\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right)+\frac{1}{3}b^2\left(3x+\frac{3c}{d}-\frac{4\left(3e^{(-2dx-2c)}+3e^{(-4\ldots}\right.}{d\left(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}\ldots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2*sinh(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/64*a^2*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*
c)/d - e^(-4*d*x - 4*c)/d) + 1/3*b^2*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) +
 3*e^(-4*d*x - 4*c) + 2)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-
6*d*x - 6*c) + 1))) - 1/4*a*b*(12*(d*x + c)/d + e^(-2*d*x - 2*c)/d - (17*e^
(-2*d*x - 2*c) + 1)/(d*(e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c))))
```

mupad [B] time = 0.27, size = 269, normalized size = 2.36

$$x\left(\frac{3a^2}{8}-3ab+b^2\right)-\frac{\frac{4\left(ab-b^2\right)}{3d}+\frac{4e^{4c+4dx}\left(ab-b^2\right)}{3d}+\frac{8abe^{2c+2dx}}{3d}}{3e^{2c+2dx}+3e^{4c+4dx}+e^{6c+6dx}+1}-\frac{\frac{4e^{2c+2dx}\left(ab-b^2\right)}{3d}+\frac{4ab}{3d}}{2e^{2c+2dx}+e^{4c+4dx}+1}-\frac{4\left(ab-b^2\right)}{3d\left(e^{2c+2dx}+1\right)}+\frac{e^{2c\ldots}}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^4*(a + b/cosh(c + d*x)^2)^2,x)
```

```
[Out] x*((3*a^2)/8 - 3*a*b + b^2) - ((4*(a*b - b^2))/(3*d) + (4*exp(4*c + 4*d*x)*
(a*b - b^2))/(3*d) + (8*a*b*exp(2*c + 2*d*x))/(3*d))/(3*exp(2*c + 2*d*x) +
```

```
3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) - ((4*exp(2*c + 2*d*x)*(a*b - b^
2))/(3*d) + (4*a*b)/(3*d))/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) - (4
*(a*b - b^2))/(3*d*(exp(2*c + 2*d*x) + 1)) + (exp(2*c + 2*d*x)*(2*a*b - a^2
))/(8*d) - (a^2*exp(- 4*c - 4*d*x))/(64*d) + (a^2*exp(4*c + 4*d*x))/(64*d)
+ (a*exp(- 2*c - 2*d*x)*(a - 2*b))/(8*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)**2*sinh(d*x+c)**4,x)
```

```
[Out] Timed out
```

3.10 $\int \left(a+b\,\mathbf{sech}^2(c+dx)\right)^2 \sinh^3(c+dx)\,dx$

Optimal. Leaf size=72

$$\frac{a^2\cosh^3(c+dx)}{3d}-\frac{a(a-2b)\cosh(c+dx)}{d}+\frac{b(2a-b)\text{sech}(c+dx)}{d}+\frac{b^2\text{sech}^3(c+dx)}{3d}$$

```
[Out] -a*(a-2*b)*cosh(d*x+c)/d+1/3*a^2*cosh(d*x+c)^3/d+(2*a-b)*b*sech(d*x+c)/d+1/
3*b^2*sech(d*x+c)^3/d
```

Rubi [A] time = 0.09, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4133, 448}

$$\frac{a^2\cosh^3(c+dx)}{3d}-\frac{a(a-2b)\cosh(c+dx)}{d}+\frac{b(2a-b)\text{sech}(c+dx)}{d}+\frac{b^2\text{sech}^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^2*Sinh[c + d*x]^3,x]
```

```
[Out] -((a*(a - 2*b)*Cosh[c + d*x])/d) + (a^2*Cosh[c + d*x]^3)/(3*d) + ((2*a - b)
*b*Sech[c + d*x])/d + (b^2*Sech[c + d*x]^3)/(3*d)
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \left(a+b\text{sech}^2(c+dx)\right)^2 \sinh^3(c+dx)\,dx = -\frac{\text{Subst}\left(\int \frac{(1-x^2)(b+ax^2)^2}{x^4}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \left(a(a-2b)+\frac{b^2}{x^4}+\frac{(2a-b)b}{x^2}-a^2x^2\right)dx, x, \cosh(c+dx)\right)}{d}$$
$$= -\frac{a(a-2b)\cosh(c+dx)}{d}+\frac{a^2\cosh^3(c+dx)}{3d}+\frac{(2a-b)b\text{sech}(c+dx}{d}$$

Mathematica [A] time = 0.53, size = 83, normalized size = 1.15

$$\frac{\text{sech}^3(c+dx)\left(-3\left(11a^2-64ab+16b^2\right)\cosh(2(c+dx))+a^2\cosh(6(c+dx))-26a^2-6a(a-4b)\cosh(4(c+dx)\right.}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^2*Sinh[c + d*x]^3,x]
```

```
[Out] ((-26*a^2 + 168*a*b - 16*b^2 - 3*(11*a^2 - 64*a*b + 16*b^2)*Cosh[2*(c + d*x
)] - 6*a*(a - 4*b)*Cosh[4*(c + d*x)] + a^2*Cosh[6*(c + d*x)])*Sech[c + d*x]
^3)/(96*d)
```

fricas [B] time = 0.39, size = 212, normalized size = 2.94

$$\frac{a^2\cosh(dx+c)^6+a^2\sinh(dx+c)^6-6\left(a^2-4ab\right)\cosh(dx+c)^4+3\left(5a^2\cosh(dx+c)^2-2a^2+8ab\right)\sinh(d}{24\left(d\cosh\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2*sinh(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(a^2*cosh(d*x + c)^6 + a^2*sinh(d*x + c)^6 - 6*(a^2 - 4*a*b)*cosh(d*x
+ c)^4 + 3*(5*a^2*cosh(d*x + c)^2 - 2*a^2 + 8*a*b)*sinh(d*x + c)^4 - 3*(11*
a^2 - 64*a*b + 16*b^2)*cosh(d*x + c)^2 + 3*(5*a^2*cosh(d*x + c)^4 - 12*(a^2
 - 4*a*b)*cosh(d*x + c)^2 - 11*a^2 + 64*a*b - 16*b^2)*sinh(d*x + c)^2 - 26*
a^2 + 168*a*b - 16*b^2)/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)*sinh(d*x + c
)^2 + 3*d*cosh(d*x + c))
```

giac [B] time = 0.19, size = 140, normalized size = 1.94

$$\frac{a^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3-12a^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)+24ab\left(e^{(dx+c)}+e^{(-dx-c)}\right)+\frac{16\left(6ab\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2-3b^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)\right.}{\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2*sinh(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/24*(a^2*(e^(d*x + c) + e^(-d*x - c))^3 - 12*a^2*(e^(d*x + c) + e^(-d*x -
c)) + 24*a*b*(e^(d*x + c) + e^(-d*x - c)) + 16*(6*a*b*(e^(d*x + c) + e^(-d*
x - c))^2 - 3*b^2*(e^(d*x + c) + e^(-d*x - c))^2 + 4*b^2)/(e^(d*x + c) + e^
(-d*x - c))^3)/d
```

maple [A] time = 0.33, size = 93, normalized size = 1.29

$$\frac{a^{2}\left(-\frac{2}{3}+\frac{\left(\sinh^{2}(dx+c)\right)}{3}\right)\cosh\left(dx+c\right)+2ab\left(\frac{\sinh^{2}(dx+c)}{\cosh(dx+c)}+\frac{2}{\cosh(dx+c)}\right)+b^{2}\left(-\frac{\sinh^{2}(dx+c)}{\cosh(dx+c)^{3}}-\frac{2}{3\cosh(dx+c)^{3}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)^2*sinh(d*x+c)^3,x)
```

```
[Out] 1/d*(a^2*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)+2*a*b*(sinh(d*x+c)^2/cosh(d*x
+c)+2/cosh(d*x+c))+b^2*(-sinh(d*x+c)^2/cosh(d*x+c)^3-2/3/cosh(d*x+c)^3))
```

maxima [B] time = 0.34, size = 266, normalized size = 3.69

$$\frac{1}{24}a^{2}\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)+ab\left(\frac{e^{(-dx-c)}}{d}+\frac{5\,e^{(-2\,dx-2\,c)}+1}{d\left(e^{(-dx-c)}+e^{(-3\,dx-3\,c)}\right)}\right)-\frac{2}{3}b^{2}\left(\frac{}{d\left(3\,e^{(-2\,dx-2\,c}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2*sinh(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/24*a^2*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*
x - 3*c)/d) + a*b*(e^(-d*x - c)/d + (5*e^(-2*d*x - 2*c) + 1)/(d*(e^(-d*x -
c) + e^(-3*d*x - 3*c)))) - 2/3*b^2*(3*e^(-d*x - c)/(d*(3*e^(-2*d*x - 2*c) +
 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 2*e^(-3*d*x - 3*c)/(d*(3*e^(
-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 3*e^(-5*d*x -
 5*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)))
```

mupad [B] time = 1.50, size = 201, normalized size = 2.79

$$\frac{e^{c+d\,x}\,\left(8\,a\,b-3\,a^{2}\right)}{8\,d}+\frac{e^{-c-d\,x}\,\left(8\,a\,b-3\,a^{2}\right)}{8\,d}+\frac{a^{2}\,e^{-3\,c-3\,d\,x}}{24\,d}+\frac{a^{2}\,e^{3\,c+3\,d\,x}}{24\,d}-\frac{8\,b^{2}\,e^{c+d\,x}}{3\,d\,\left(3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^3*(a + b/cosh(c + d*x)^2)^2,x)
```

```
[Out] (exp(c + d*x)*(8*a*b - 3*a^2))/(8*d) + (exp(- c - d*x)*(8*a*b - 3*a^2))/(8*
d) + (a^2*exp(- 3*c - 3*d*x))/(24*d) + (a^2*exp(3*c + 3*d*x))/(24*d) - (8*b
^2*exp(c + d*x))/(3*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c +
6*d*x) + 1)) + (2*exp(c + d*x)*(2*a*b - b^2))/(d*(exp(2*c + 2*d*x) + 1)) +
(8*b^2*exp(c + d*x))/(3*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)**2*sinh(d*x+c)**3,x)
```

```
[Out] Timed out
```

3.11 $\int \left(a+b\,\mathbf{sech}^2(c+dx)\right)^2 \sinh^2(c+dx)\,dx$

Optimal. Leaf size=73

$$\frac{a^2\sinh^2(c+dx)\tanh(c+dx)}{2d}+\frac{a(a-4b)\tanh(c+dx)}{2d}-\frac{1}{2}ax(a-4b)+\frac{b^2\tanh^3(c+dx)}{3d}$$

```
[Out] -1/2*a*(a-4*b)*x+1/2*a*(a-4*b)*tanh(d*x+c)/d+1/2*a^2*sinh(d*x+c)^2*tanh(d*x
+c)/d+1/3*b^2*tanh(d*x+c)^3/d
```

Rubi [A] time = 0.11, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4132, 463, 459, 321, 206}

$$\frac{a^2\sinh^2(c+dx)\tanh(c+dx)}{2d}+\frac{a(a-4b)\tanh(c+dx)}{2d}-\frac{1}{2}ax(a-4b)+\frac{b^2\tanh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^2*Sinh[c + d*x]^2,x]
```

```
[Out] -(a*(a - 4*b)*x)/2 + (a*(a - 4*b)*Tanh[c + d*x])/(2*d) + (a^2*Sinh[c + d*x]
^2*Tanh[c + d*x])/(2*d) + (b^2*Tanh[c + d*x]^3)/(3*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \left(a+b\text{sech}^2(c+dx)\right)^2\sinh^2(c+dx)\,dx=\frac{\text{Subst}\left(\int \frac{x^2\left(a+b-bx^2\right)^2}{\left(1-x^2\right)^2}\,dx,x,\tanh(c+dx)\right)}{d}$$

$$=\frac{a^2\sinh^2(c+dx)\tanh(c+dx)}{2d}-\frac{\text{Subst}\left(\int \frac{x^2\left(3a^2-2(a+b)^2+2b^2x^2\right)}{1-x^2}\,dx,x,\text{t}\right.}{2d}$$

$$=\frac{a^2\sinh^2(c+dx)\tanh(c+dx)}{2d}+\frac{b^2\tanh^3(c+dx)}{3d}-\frac{(a(a-4b))\,\text{Sub}}{}$$

$$=\frac{a(a-4b)\tanh(c+dx)}{2d}+\frac{a^2\sinh^2(c+dx)\tanh(c+dx)}{2d}+\frac{b^2\tanh^3(}{3d}$$

$$=-\frac{1}{2}a(a-4b)x+\frac{a(a-4b)\tanh(c+dx)}{2d}+\frac{a^2\sinh^2(c+dx)\tanh(c+}{2d}$$

Mathematica [A] time = 0.95, size = 126, normalized size = 1.73

$$\frac{\text{sech}^3(c+dx)\left(a\cosh^2(c+dx)+b\right)^2\left(3a\cosh^3(c+dx)(a\sinh(2(c+dx))-2dx(a-4b))-4b(6a-b)\text{sech}(c)\sinh\right.}{3d(a\cosh(2(c+dx))+a+2b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^2*Sinh[c + d*x]^2,x]
```

```
[Out] ((b + a*Cosh[c + d*x]^2)^2*Sech[c + d*x]^3*(-4*b^2*Sech[c]*Sinh[d*x] - 4*(6
*a - b)*b*Cosh[c + d*x]^2*Sech[c]*Sinh[d*x] + 3*a*Cosh[c + d*x]^3*(-2*(a -
4*b)*d*x + a*Sinh[2*(c + d*x)]) - 4*b^2*Cosh[c + d*x]*Tanh[c]))/(3*d*(a + 2
*b + a*Cosh[2*(c + d*x)])^2)
```

fricas [B] time = 0.41, size = 252, normalized size = 3.45

$$\frac{3\,a^2\sinh\left(dx+c\right)^5-4\left(3\left(a^2-4\,ab\right)dx-12\,ab+2\,b^2\right)\cosh\left(dx+c\right)^3-12\left(3\left(a^2-4\,ab\right)dx-12\,ab+2\,b^2\right)\cosh}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2*sinh(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(3*a^2*sinh(d*x + c)^5 - 4*(3*(a^2 - 4*a*b)*d*x - 12*a*b + 2*b^2)*cosh
(d*x + c)^3 - 12*(3*(a^2 - 4*a*b)*d*x - 12*a*b + 2*b^2)*cosh(d*x + c)*sinh(
d*x + c)^2 + (30*a^2*cosh(d*x + c)^2 + 9*a^2 - 48*a*b + 8*b^2)*sinh(d*x + c
)^3 - 12*(3*(a^2 - 4*a*b)*d*x - 12*a*b + 2*b^2)*cosh(d*x + c) + 3*(5*a^2*co
sh(d*x + c)^4 + (9*a^2 - 48*a*b + 8*b^2)*cosh(d*x + c)^2 + 2*a^2 - 16*a*b -
 8*b^2)*sinh(d*x + c))/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)*sinh(d*x + c)
^2 + 3*d*cosh(d*x + c))
```

giac [B] time = 0.18, size = 144, normalized size = 1.97

$$\frac{3\,a^2e^{(2\,dx+2\,c)}-12\left(a^2-4\,ab\right)(dx+c)+3\left(2\,a^2e^{(2\,dx+2\,c)}-8\,abe^{(2\,dx+2\,c)}-a^2\right)e^{(-2\,dx-2\,c)}+\frac{16\left(6\,abe^{(4\,dx+4\,c)}-3\,b^2e^{(4\,dx+4\,c)}\right.}{\left(e^{(2\,dx+2\,c)}\right.}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2*sinh(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/24*(3*a^2*e^(2*d*x + 2*c) - 12*(a^2 - 4*a*b)*(d*x + c) + 3*(2*a^2*e^(2*d*
x + 2*c) - 8*a*b*e^(2*d*x + 2*c) - a^2)*e^(-2*d*x - 2*c) + 16*(6*a*b*e^(4*d
*x + 4*c) - 3*b^2*e^(4*d*x + 4*c) + 12*a*b*e^(2*d*x + 2*c) + 6*a*b - b^2)/(
e^(2*d*x + 2*c) + 1)^3)/d
```

maple [A] time = 0.34, size = 90, normalized size = 1.23

$$\frac{a^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+2ab\left(dx+c-\tanh\left(dx+c\right)\right)+b^2\left(-\frac{\sinh(dx+c)}{2\cosh(dx+c)^3}+\frac{\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)^2*sinh(d*x+c)^2,x)
```

```
[Out] 1/d*(a^2*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c)+2*a*b*(d*x+c-tanh(d*x+
c))+b^2*(-1/2*sinh(d*x+c)/cosh(d*x+c)^3+1/2*(2/3+1/3*sech(d*x+c)^2)*tanh(d*
x+c)))
```

maxima [B] time = 0.33, size = 160, normalized size = 2.19

$$-\frac{1}{8}a^2\left(4x - \frac{e^{(2dx+2c)}}{d} + \frac{e^{(-2dx-2c)}}{d}\right) + 2ab\left(x + \frac{c}{d} - \frac{2}{d\left(e^{(-2dx-2c)}+1\right)}\right) + \frac{2}{3}b^2\left(\frac{3e^{(-4dx-4c)}}{d\left(3e^{(-2dx-2c)} + 3e^{(-4dx-4c)} + e^{(-6dx-}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2*sinh(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/8*a^2*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) + 2*a*b*(x + c/d -
2/(d*(e^(-2*d*x - 2*c) + 1))) + 2/3*b^2*(3*e^(-4*d*x - 4*c)/(d*(3*e^(-2*d*x
 - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x -
 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)))
```

mupad [B] time = 0.16, size = 236, normalized size = 3.23

$$\frac{\frac{2\left(b^2+2ab\right)}{3d} + \frac{2e^{2c+2dx}\left(2ab-b^2\right)}{3d}}{2e^{2c+2dx} + e^{4c+4dx} + 1} + x\left(2ab - \frac{a^2}{2}\right) + \frac{\frac{2\left(2ab-b^2\right)}{3d} + \frac{4e^{2c+2dx}\left(b^2+2ab\right)}{3d} + \frac{2e^{4c+4dx}\left(2ab-b^2\right)}{3d}}{3e^{2c+2dx} + 3e^{4c+4dx} + e^{6c+6dx} + 1} + \frac{2\left(2ab-b^2\right)}{3d\left(e^{2c+2dx}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^2*(a + b/cosh(c + d*x)^2)^2,x)
```

```
[Out] ((2*(2*a*b + b^2))/(3*d) + (2*exp(2*c + 2*d*x)*(2*a*b - b^2))/(3*d))/(2*exp
(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) + x*(2*a*b - a^2/2) + ((2*(2*a*b - b^
2))/(3*d) + (4*exp(2*c + 2*d*x)*(2*a*b + b^2))/(3*d) + (2*exp(4*c + 4*d*x)*
(2*a*b - b^2))/(3*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c +
6*d*x) + 1) + (2*(2*a*b - b^2))/(3*d*(exp(2*c + 2*d*x) + 1)) - (a^2*exp(- 2
*c - 2*d*x))/(8*d) + (a^2*exp(2*c + 2*d*x))/(8*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\operatorname{sech}^2(c + dx)\right)^2 \sinh^2(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)**2*sinh(d*x+c)**2,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**2*sinh(c + d*x)**2, x)
```

3.12 $\int \left(a+b\,\mathrm{sech}^2(c+dx)\right)^2 \sinh(c+dx)\,dx$

Optimal. Leaf size=45

$$\frac{a^2\cosh(c+dx)}{d}-\frac{2ab\,\mathrm{sech}(c+dx)}{d}-\frac{b^2\mathrm{sech}^3(c+dx)}{3d}$$

```
[Out] a^2*cosh(d*x+c)/d-2*a*b*sech(d*x+c)/d-1/3*b^2*sech(d*x+c)^3/d
```

Rubi [A] time = 0.05, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4133, 270}

$$\frac{a^2\cosh(c+dx)}{d}-\frac{2ab\,\mathrm{sech}(c+dx)}{d}-\frac{b^2\mathrm{sech}^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^2*Sinh[c + d*x],x]
```

```
[Out] (a^2*Cosh[c + d*x])/d - (2*a*b*Sech[c + d*x])/d - (b^2*Sech[c + d*x]^3)/(3*
d)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \left(a+b\operatorname{sech}^2(c+dx)\right)^2\sinh(c+dx)\,dx=\frac{\operatorname{Subst}\left(\int\frac{\left(b+ax^2\right)^2}{x^4}\,dx,x,\cosh(c+dx)\right)}{d}$$
$$=\frac{\operatorname{Subst}\left(\int\left(a^2+\frac{b^2}{x^4}+\frac{2ab}{x^2}\right)dx,x,\cosh(c+dx)\right)}{d}$$
$$=\frac{a^2\cosh(c+dx)}{d}-\frac{2ab\operatorname{sech}(c+dx)}{d}-\frac{b^2\operatorname{sech}^3(c+dx)}{3d}$$

Mathematica [**A**] time = 0.17, size = 59, normalized size = 1.31

$$\frac{\operatorname{sech}^3(c+dx)\left(3a^2\cosh(4(c+dx))+9a^2+12a(a-2b)\cosh(2(c+dx))-24ab-8b^2\right)}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^2*Sinh[c + d*x],x]
```

```
[Out] ((9*a^2 - 24*a*b - 8*b^2 + 12*a*(a - 2*b)*Cosh[2*(c + d*x)] + 3*a^2*Cosh[4*
(c + d*x)])*Sech[c + d*x]^3)/(24*d)
```

fricas [**B**] time = 0.40, size = 133, normalized size = 2.96

$$\frac{3\,a^2\cosh\left(dx+c\right)^4+3\,a^2\sinh\left(dx+c\right)^4+12\left(a^2-2\,ab\right)\cosh\left(dx+c\right)^2+6\left(3\,a^2\cosh\left(dx+c\right)^2+2\,a^2-4\,ab\right)\sin}{6\left(d\cosh\left(dx+c\right)^3+3\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^2+3\,d\cosh\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2*sinh(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/6*(3*a^2*cosh(d*x + c)^4 + 3*a^2*sinh(d*x + c)^4 + 12*(a^2 - 2*a*b)*cosh(
d*x + c)^2 + 6*(3*a^2*cosh(d*x + c)^2 + 2*a^2 - 4*a*b)*sinh(d*x + c)^2 + 9*
a^2 - 24*a*b - 8*b^2)/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)*sinh(d*x + c)^
2 + 3*d*cosh(d*x + c))
```

giac [**A**] time = 0.14, size = 75, normalized size = 1.67

$$\frac{3\,a^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)-\frac{8\left(3\,ab\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2+2\,b^2\right)}{\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2*sinh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/6*(3*a^2*(e^(d*x + c) + e^(-d*x - c)) - 8*(3*a*b*(e^(d*x + c) + e^(-d*x -
 c))^2 + 2*b^2)/(e^(d*x + c) + e^(-d*x - c))^3)/d
```

maple [A] time = 0.10, size = 43, normalized size = 0.96

$$-\frac{\frac{b^2\operatorname{sech}(dx+c)^3}{3} + 2ab\operatorname{sech}(dx+c) - \frac{a^2}{\operatorname{sech}(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)^2*sinh(d*x+c),x)
```

```
[Out] -1/d*(1/3*b^2*sech(d*x+c)^3+2*a*b*sech(d*x+c)-a^2/sech(d*x+c))
```

maxima [A] time = 0.32, size = 65, normalized size = 1.44

$$\frac{a^2\cosh(dx+c)}{d} - \frac{4\,ab}{d\left(e^{(dx+c)} + e^{(-dx-c)}\right)} - \frac{8\,b^2}{3\,d\left(e^{(dx+c)} + e^{(-dx-c)}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2*sinh(d*x+c),x, algorithm="maxima")
```

```
[Out] a^2*cosh(d*x + c)/d - 4*a*b/(d*(e^(d*x + c) + e^(-d*x - c))) - 8/3*b^2/(d*(
e^(d*x + c) + e^(-d*x - c))^3)
```

mupad [B] time = 1.47, size = 45, normalized size = 1.00

$$\frac{a^2\cosh(c+d\,x)}{d} - \frac{\frac{b^2}{3} + 2\,a\,b\cosh(c+d\,x)^2}{d\cosh(c+d\,x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)*(a + b/cosh(c + d*x)^2)^2,x)
```

```
[Out] (a^2*cosh(c + d*x))/d - (b^2/3 + 2*a*b*cosh(c + d*x)^2)/(d*cosh(c + d*x)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\operatorname{sech}^2(c+dx)\right)^2 \sinh(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)**2*sinh(d*x+c),x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**2*sinh(c + d*x), x)
```

3.13 $\int \operatorname{csch}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=52

$$\frac{b(2a+b)\operatorname{sech}(c+dx)}{d}-\frac{(a+b)^2\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b^2\operatorname{sech}^3(c+dx)}{3d}$$

```
[Out] -(a+b)^2*arctanh(cosh(d*x+c))/d+b*(2*a+b)*sech(d*x+c)/d+1/3*b^2*sech(d*x+c)
^3/d
```

Rubi [A] time = 0.08, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4133, 461, 207}

$$\frac{b(2a+b)\operatorname{sech}(c+dx)}{d}-\frac{(a+b)^2\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b^2\operatorname{sech}^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] -(((a + b)^2*ArcTanh[Cosh[c + d*x]])/d) + (b*(2*a + b)*Sech[c + d*x])/d + (
b^2*Sech[c + d*x]^3)/(3*d)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \operatorname{csch}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx = -\frac{\text{Subst}\left(\int \frac{\left(b+ax^2\right)^2}{x^4\left(1-x^2\right)}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{b^2}{x^4}+\frac{b(2a+b)}{x^2}-\frac{(a+b)^2}{-1+x^2}\right)dx, x, \cosh(c+dx)\right)}{d}$$
$$= \frac{b(2a+b)\operatorname{sech}(c+dx)}{d}+\frac{b^2\operatorname{sech}^3(c+dx)}{3d}+\frac{(a+b)^2\,\text{Subst}\left(\int \frac{1}{-1+x^2}$$
$$= -\frac{(a+b)^2\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b(2a+b)\operatorname{sech}(c+dx)}{d}+\frac{b^2\operatorname{sech}$$

Mathematica [B] time = 0.58, size = 108, normalized size = 2.08

$$-\frac{4\operatorname{sech}^3(c+dx)\left(a\cosh^2(c+dx)+b\right)^2\left(-3b(2a+b)\cosh^2(c+dx)+3(a+b)^2\cosh^3(c+dx)\left(\log\left(\cosh\left(\frac{1}{2}(c+\right.\right.\right.\right.}{3d(a\cosh(2(c+dx))+a+2b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (-4*(b + a*Cosh[c + d*x]^2)^2*(-b^2 - 3*b*(2*a + b)*Cosh[c + d*x]^2 + 3*(a
+ b)^2*Cosh[c + d*x]^3*(Log[Cosh[(c + d*x)/2]] - Log[Sinh[(c + d*x)/2]]))*S
ech[c + d*x]^3)/(3*d*(a + 2*b + a*Cosh[2*(c + d*x)])^2)
```

fricas [B] time = 0.42, size = 1148, normalized size = 22.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(6*(2*a*b + b^2)*cosh(d*x + c)^5 + 30*(2*a*b + b^2)*cosh(d*x + c)*sinh(
d*x + c)^4 + 6*(2*a*b + b^2)*sinh(d*x + c)^5 + 4*(6*a*b + 5*b^2)*cosh(d*x +
 c)^3 + 4*(15*(2*a*b + b^2)*cosh(d*x + c)^2 + 6*a*b + 5*b^2)*sinh(d*x + c)^
3 + 12*(5*(2*a*b + b^2)*cosh(d*x + c)^3 + (6*a*b + 5*b^2)*cosh(d*x + c))*si
nh(d*x + c)^2 + 6*(2*a*b + b^2)*cosh(d*x + c) - 3*((a^2 + 2*a*b + b^2)*cosh
(d*x + c)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^5 + (a^2 +
2*a*b + b^2)*sinh(d*x + c)^6 + 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 3*(5
```

```
*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 +
 4*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(d*x
+ c))*sinh(d*x + c)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 3*(5*(a^2 +
 2*a*b + b^2)*cosh(d*x + c)^4 + 6*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2
 + 2*a*b + b^2)*sinh(d*x + c)^2 + a^2 + 2*a*b + b^2 + 6*((a^2 + 2*a*b + b^2
)*cosh(d*x + c)^5 + 2*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 + 2*a*b +
b^2)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) +
 3*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(d*x +
c)*sinh(d*x + c)^5 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^6 + 3*(a^2 + 2*a*b +
 b^2)*cosh(d*x + c)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 + 2*
a*b + b^2)*sinh(d*x + c)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 3*(
a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(a^2 + 2*a*b + b^2)*c
osh(d*x + c)^2 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 6*(a^2 + 2*a*b
+ b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b + b^2)*sinh(d*x + c)^2 + a^2 + 2*a*b +
 b^2 + 6*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 + 2*(a^2 + 2*a*b + b^2)*cosh(
d*x + c)^3 + (a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x
 + c) + sinh(d*x + c) - 1) + 6*(5*(2*a*b + b^2)*cosh(d*x + c)^4 + 2*(6*a*b
+ 5*b^2)*cosh(d*x + c)^2 + 2*a*b + b^2)*sinh(d*x + c))/(d*cosh(d*x + c)^6 +
 6*d*cosh(d*x + c)*sinh(d*x + c)^5 + d*sinh(d*x + c)^6 + 3*d*cosh(d*x + c)^
4 + 3*(5*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^4 + 4*(5*d*cosh(d*x + c)^3 +
3*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*d*cosh(d*x + c)^2 + 3*(5*d*cosh(d*x
+ c)^4 + 6*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 6*(d*cosh(d*x + c)^5 +
2*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

giac [B] time = 0.14, size = 139, normalized size = 2.67

$$-\frac{3\left(a^2+2\,ab+b^2\right)\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)-3\left(a^2+2\,ab+b^2\right)\log\left(e^{(dx+c)}+e^{(-dx-c)}-2\right)-\frac{4\left(6\,ab\left(e^{(dx+c)}+e^{(-dx-c)}\right)\cdots\right.}{\left(e^{(dx+\cdots}\right.}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/6*(3*(a^2 + 2*a*b + b^2)*log(e^(d*x + c) + e^(-d*x - c) + 2) - 3*(a^2 +
2*a*b + b^2)*log(e^(d*x + c) + e^(-d*x - c) - 2) - 4*(6*a*b*(e^(d*x + c) +
e^(-d*x - c))^2 + 3*b^2*(e^(d*x + c) + e^(-d*x - c))^2 + 4*b^2)/(e^(d*x + c
) + e^(-d*x - c))^3)/d
```

maple [A] time = 0.22, size = 72, normalized size = 1.38

$$\frac{-2a^2\operatorname{arctanh}\left(e^{dx+c}\right)+2ab\left(\frac{1}{\cosh(dx+c)}-2\operatorname{arctanh}\left(e^{dx+c}\right)\right)+b^2\left(\frac{1}{3\cosh(dx+c)^3}+\frac{1}{\cosh(dx+c)}-2\operatorname{arctanh}\left(e^{dx+c}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)*(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(-2*a^2*arctanh(exp(d*x+c))+2*a*b*(1/cosh(d*x+c)-2*arctanh(exp(d*x+c)))
+b^2*(1/3/cosh(d*x+c)^3+1/cosh(d*x+c)-2*arctanh(exp(d*x+c))))
```

maxima [B] time = 0.34, size = 197, normalized size = 3.79

$$-\frac{1}{3}\,b^2\left(\frac{3\,\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{3\,\log\left(e^{(-dx-c)}-1\right)}{d}-\frac{2\left(3\,e^{(-dx-c)}+10\,e^{(-3\,dx-3\,c)}+3\,e^{(-5\,dx-5\,c)}\right)}{d\left(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1\right)}\right)-2\,ab\left(\frac{\log\left(e^{(-\cdots}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/3*b^2*(3*log(e^(-d*x - c) + 1)/d - 3*log(e^(-d*x - c) - 1)/d - 2*(3*e^(-
d*x - c) + 10*e^(-3*d*x - 3*c) + 3*e^(-5*d*x - 5*c))/(d*(3*e^(-2*d*x - 2*c)
 + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) - 2*a*b*(log(e^(-d*x - c) +
 1)/d - log(e^(-d*x - c) - 1)/d - 2*e^(-d*x - c)/(d*(e^(-2*d*x - 2*c) + 1))
) + a^2*log(tanh(1/2*d*x + 1/2*c))/d
```

mupad [B] time = 1.50, size = 232, normalized size = 4.46

$$\frac{2\,{\mathrm{e}}^{c+d\,x}\,\left(b^2+2\,a\,b\right)}{d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}-\frac{8\,b^2\,{\mathrm{e}}^{c+d\,x}}{3\,d\,\left(3\,{\mathrm{e}}^{2\,c+2\,d\,x}+3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}+1\right)}-\frac{2\,\mathrm{atan}\left(\frac{{\mathrm{e}}^{d\,x}\,{\mathrm{e}}^c\,\left(a^2\,\sqrt{-d^2}+b^2\,\sqrt{-d^2}+2\,a\,b\,\sqrt{-d^2}\right)}{d\,\sqrt{a^4+4\,a^3\,b+6\,a^2\,b^2+4\,a\,b^3+b^4}}\right)\,\sqrt{a^4\cdots}}{\sqrt{-d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cosh(c + d*x)^2)^2/sinh(c + d*x),x)
```

```
[Out] (2*exp(c + d*x)*(2*a*b + b^2))/(d*(exp(2*c + 2*d*x) + 1)) - (8*b^2*exp(c +
d*x))/(3*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)
) - (2*atan((exp(d*x)*exp(c)*(a^2*(-d^2)^(1/2) + b^2*(-d^2)^(1/2) + 2*a*b*(
-d^2)^(1/2)))/(d*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)^(1/2)))*(4*a*b
^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)^(1/2))/(-d^2)^(1/2) + (8*b^2*exp(c +
d*x))/(3*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\,\mathrm{sech}^2\left(c+dx\right)\right)^2\mathrm{csch}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**2*csch(c + d*x), x)
```

3.14 $\int \operatorname{csch}^2(c+dx) \left(a+b\operatorname{sech}^2(c+dx)\right)^2 \, dx$

Optimal. Leaf size=50

$$-\frac{2b(a+b)\tanh(c+dx)}{d}-\frac{(a+b)^2\coth(c+dx)}{d}+\frac{b^2\tanh^3(c+dx)}{3d}$$

```
[Out] -(a+b)^2*coth(d*x+c)/d-2*b*(a+b)*tanh(d*x+c)/d+1/3*b^2*tanh(d*x+c)^3/d
```

Rubi [A] time = 0.06, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4132, 270}

$$-\frac{2b(a+b)\tanh(c+dx)}{d}-\frac{(a+b)^2\coth(c+dx)}{d}+\frac{b^2\tanh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] -(((a + b)^2*Coth[c + d*x])/d) - (2*b*(a + b)*Tanh[c + d*x])/d + (b^2*Tanh[
c + d*x]^3)/(3*d)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \operatorname{csch}^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx = \frac{\operatorname{Subst}\left(\int \frac{\left(a+b-bx^2\right)^2}{x^2}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\operatorname{Subst}\left(\int \left(-2b(a+b)+\frac{(a+b)^2}{x^2}+b^2x^2\right)dx, x, \tanh(c+dx)\right)}{d}$$
$$= -\frac{(a+b)^2\coth(c+dx)}{d} - \frac{2b(a+b)\tanh(c+dx)}{d} + \frac{b^2\tanh^3(c+d}{3d}$$

Mathematica [B] time = 1.77, size = 109, normalized size = 2.18

$$-\frac{4\operatorname{sech}^3(c+dx)\left(a\cosh^2(c+dx)+b\right)^2\left(\sinh(dx)\cosh^2(c+dx)\left(b(6a+5b)\operatorname{sech}(c)-3(a+b)^2\operatorname{csch}(c)\coth(c+\right.\right.}{3d(a\cosh(2(c+dx))+a+2b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (-4*(b + a*Cosh[c + d*x]^2)^2*Sech[c + d*x]^3*(b^2*Sech[c]*Sinh[d*x] + Cosh
[c + d*x]^2*(-3*(a + b)^2*Coth[c + d*x]*Csch[c] + b*(6*a + 5*b)*Sech[c])*Si
nh[d*x] + b^2*Cosh[c + d*x]*Tanh[c]))/(3*d*(a + 2*b + a*Cosh[2*(c + d*x)])^
2)
```

fricas [B] time = 0.42, size = 284, normalized size = 5.68

$$-\frac{4\left(\left(3a^2+6ab+4b^2\right)\cosh(dx+c)^3+3\left(3a^2+6ab+4b^2\right)\cosh(dx+c)\sinh(dx\right.}{3\left(d\cosh(dx+c)^5+5d\cosh(dx+c)\sinh(dx+c)^4+d\sinh(dx+c)^5+d\cosh(dx+c)^3+\left(10d\cosh(dx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -4/3*((3*a^2 + 6*a*b + 4*b^2)*cosh(d*x + c)^3 + 3*(3*a^2 + 6*a*b + 4*b^2)*c
osh(d*x + c)*sinh(d*x + c)^2 - 2*(3*a*b + 2*b^2)*sinh(d*x + c)^3 + (9*a^2 +
 18*a*b + 8*b^2)*cosh(d*x + c) - 2*(3*(3*a*b + 2*b^2)*cosh(d*x + c)^2 + 3*a
*b + 4*b^2)*sinh(d*x + c))/(d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)*sinh(d*x
+ c)^4 + d*sinh(d*x + c)^5 + d*cosh(d*x + c)^3 + (10*d*cosh(d*x + c)^2 + 3*
d)*sinh(d*x + c)^3 + (10*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x +
c)^2 - 2*d*cosh(d*x + c) + (5*d*cosh(d*x + c)^4 + 9*d*cosh(d*x + c)^2 + 2*d
)*sinh(d*x + c))
```

giac [B] time = 0.15, size = 111, normalized size = 2.22

$$-\frac{2\left(\frac{3\left(a^{2}+2\,ab+b^{2}\right)}{e^{(2\,dx+2\,c)}-1}-\frac{6\,abe^{(4\,dx+4\,c)}+3\,b^{2}e^{(4\,dx+4\,c)}+12\,abe^{(2\,dx+2\,c)}+12\,b^{2}e^{(2\,dx+2\,c)}+6\,ab+5\,b^{2}}{\left(e^{(2\,dx+2\,c)}+1\right)^{3}}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -2/3*(3*(a^2 + 2*a*b + b^2)/(e^(2*d*x + 2*c) - 1) - (6*a*b*e^(4*d*x + 4*c)
+ 3*b^2*e^(4*d*x + 4*c) + 12*a*b*e^(2*d*x + 2*c) + 12*b^2*e^(2*d*x + 2*c) +
 6*a*b + 5*b^2)/(e^(2*d*x + 2*c) + 1)^3)/d
```

maple [A] time = 0.49, size = 91, normalized size = 1.82

$$\frac{-a^{2}\coth\left(dx+c\right)+2ab\left(-\frac{1}{\sinh(dx+c)\cosh(dx+c)}-2\tanh\left(dx+c\right)\right)+b^{2}\left(-\frac{1}{\sinh(dx+c)\cosh(dx+c)^{3}}-4\left(\frac{2}{3}+\frac{\mathrm{sech}(dx+c)^{2}}{3}\right)\mathrm{ta}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(-a^2*coth(d*x+c)+2*a*b*(-1/sinh(d*x+c)/cosh(d*x+c)-2*tanh(d*x+c))+b^2*
(-1/sinh(d*x+c)/cosh(d*x+c)^3-4*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c)))
```

maxima [B] time = 0.34, size = 140, normalized size = 2.80

$$-\frac{16}{3}\,b^{2}\left(\frac{2\,e^{(-2\,dx-2\,c)}}{d\left(2\,e^{(-2\,dx-2\,c)}-2\,e^{(-6\,dx-6\,c)}-e^{(-8\,dx-8\,c)}+1\right)}+\frac{1}{d\left(2\,e^{(-2\,dx-2\,c)}-2\,e^{(-6\,dx-6\,c)}-e^{(-8\,dx-8\,c)}+1\right)}\right)+\frac{2\,a^{2}}{d\left(e^{(-2\,dx-2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -16/3*b^2*(2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - 2*e^(-6*d*x - 6*c) -
 e^(-8*d*x - 8*c) + 1)) + 1/(d*(2*e^(-2*d*x - 2*c) - 2*e^(-6*d*x - 6*c) - e
^(-8*d*x - 8*c) + 1))) + 2*a^2/(d*(e^(-2*d*x - 2*c) - 1)) + 8*a*b/(d*(e^(-4
*d*x - 4*c) - 1))
```

mupad [B] time = 1.49, size = 215, normalized size = 4.30

$$\frac{\frac{2\left(3\,b^{2}+2\,a\,b\right)}{3\,d}+\frac{2\,\mathrm{e}^{2\,c+2\,d\,x}\left(b^{2}+2\,a\,b\right)}{3\,d}}{2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1}+\frac{\frac{2\left(b^{2}+2\,a\,b\right)}{3\,d}+\frac{2\,\mathrm{e}^{4\,c+4\,d\,x}\left(b^{2}+2\,a\,b\right)}{3\,d}+\frac{4\,\mathrm{e}^{2\,c+2\,d\,x}\left(3\,b^{2}+2\,a\,b\right)}{3\,d}}{3\,\mathrm{e}^{2\,c+2\,d\,x}+3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}+1}-\frac{2\,\left(a^{2}+2\,a\,b+b^{2}\right)}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}+\frac{2\,\left(b^{2}+2\right.}{3\,d\,\left(\mathrm{e}^{2\,c+2\,c}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cosh(c + d*x)^2)^2/sinh(c + d*x)^2,x)
```

```
[Out] ((2*(2*a*b + 3*b^2))/(3*d) + (2*exp(2*c + 2*d*x)*(2*a*b + b^2))/(3*d))/(2*e
xp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) + ((2*(2*a*b + b^2))/(3*d) + (2*exp
(4*c + 4*d*x)*(2*a*b + b^2))/(3*d) + (4*exp(2*c + 2*d*x)*(2*a*b + 3*b^2))/(
3*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) - (2
*(2*a*b + a^2 + b^2))/(d*(exp(2*c + 2*d*x) - 1)) + (2*(2*a*b + b^2))/(3*d*(
exp(2*c + 2*d*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\,\mathrm{sech}^2\left(c + dx\right)\right)^2 \mathrm{csch}^2\left(c + dx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2*(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**2*csch(c + d*x)**2, x)
```

3.15 $\int \mathbf{csch}^3(c+dx)\left(a+b\mathbf{sech}^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=104

$$-\frac{\left(3a^2+6ab+5b^2\right)\coth(c+dx)\text{csch}(c+dx)}{6d}-\frac{b(6a+5b)\text{sech}(c+dx)}{3d}+\frac{(a+b)(a+5b)\tanh^{-1}(\cosh(c+dx))}{2d}+\cdots$$

```
[Out] 1/2*(a+b)*(a+5*b)*arctanh(cosh(d*x+c))/d-1/6*(3*a^2+6*a*b+5*b^2)*coth(d*x+c
)*csch(d*x+c)/d-1/3*b*(6*a+5*b)*sech(d*x+c)/d+1/3*b^2*csch(d*x+c)^2*sech(d*
x+c)^3/d
```

Rubi [A] time = 0.14, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4133, 462, 456, 453, 206}

$$-\frac{\left(3a^2+6ab+5b^2\right)\coth(c+dx)\text{csch}(c+dx)}{6d}-\frac{b(6a+5b)\text{sech}(c+dx)}{3d}+\frac{(a+b)(a+5b)\tanh^{-1}(\cosh(c+dx))}{2d}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + b)*(a + 5*b)*ArcTanh[Cosh[c + d*x]])/(2*d) - ((3*a^2 + 6*a*b + 5*b^2)
*Coth[c + d*x]*Csch[c + d*x])/(6*d) - (b*(6*a + 5*b)*Sech[c + d*x])/(3*d) +
 (b^2*Csch[c + d*x]^2*Sech[c + d*x]^3)/(3*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
```

```
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \operatorname{csch}^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2 dx = \frac{\text{Subst}\left(\int \frac{\left(b+ax^2\right)^2}{x^4\left(1-x^2\right)^2}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= \frac{b^2\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{3d} + \frac{\text{Subst}\left(\int \frac{b(6a+5b)+3a^2x^2}{x^2\left(1-x^2\right)^2}\,dx, x, \text{cosh}\right.}{3d}$$

$$= -\frac{\left(3a^2+6ab+5b^2\right)\coth(c+dx)\operatorname{csch}(c+dx)}{6d} + \frac{b^2\operatorname{csch}^2(c+dx)\text{s}}{3d}$$

$$= -\frac{\left(3a^2+6ab+5b^2\right)\coth(c+dx)\operatorname{csch}(c+dx)}{6d} - \frac{b(6a+5b)\operatorname{sech}(c}{3d}$$

$$= \frac{(a+b)(a+5b)\tanh^{-1}(\cosh(c+dx))}{2d} - \frac{\left(3a^2+6ab+5b^2\right)\coth(c}{6d}$$

Mathematica [A] time = 1.64, size = 144, normalized size = 1.38

$$-\frac{\operatorname{sech}^3(c+dx)\left(a\cosh^2(c+dx)+b\right)^2\left(48b(a+b)\cosh^2(c+dx)+3(a+b)\cosh^3(c+dx)\left((a+b)\operatorname{csch}^2\left(\frac{1}{2}(c+d\right.\right.\right.}{6d(a\cosh(2(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] -1/6*((b + a*Cosh[c + d*x]^2)^2*(8*b^2 + 48*b*(a + b)*Cosh[c + d*x]^2 + 3*(
a + b)*Cosh[c + d*x]^3*((a + b)*Csch[(c + d*x)/2]^2 - 4*(a + 5*b)*(Log[Cosh
[(c + d*x)/2]] - Log[Sinh[(c + d*x)/2]]) + (a + b)*Sech[(c + d*x)/2]^2))*Se
ch[c + d*x]^3)/(d*(a + 2*b + a*Cosh[2*(c + d*x)])^2)
```

fricas [B] time = 0.45, size = 2930, normalized size = 28.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(6*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^9 + 54*(a^2 + 6*a*b + 5*b^2)*co
sh(d*x + c)*sinh(d*x + c)^8 + 6*(a^2 + 6*a*b + 5*b^2)*sinh(d*x + c)^9 + 8*(
3*a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^7 + 8*(27*(a^2 + 6*a*b + 5*b^2)*cosh(d
*x + c)^2 + 3*a^2 + 6*a*b + 5*b^2)*sinh(d*x + c)^7 + 56*(9*(a^2 + 6*a*b + 5
*b^2)*cosh(d*x + c)^3 + (3*a^2 + 6*a*b + 5*b^2)*cosh(d*x + c))*sinh(d*x + c
)^6 + 4*(9*a^2 + 6*a*b - 11*b^2)*cosh(d*x + c)^5 + 4*(189*(a^2 + 6*a*b + 5*
b^2)*cosh(d*x + c)^4 + 42*(3*a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^2 + 9*a^2 +
 6*a*b - 11*b^2)*sinh(d*x + c)^5 + 4*(189*(a^2 + 6*a*b + 5*b^2)*cosh(d*x +
c)^5 + 70*(3*a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^3 + 5*(9*a^2 + 6*a*b - 11*b
^2)*cosh(d*x + c))*sinh(d*x + c)^4 + 8*(3*a^2 + 6*a*b + 5*b^2)*cosh(d*x + c
)^3 + 8*(63*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^6 + 35*(3*a^2 + 6*a*b + 5*b
^2)*cosh(d*x + c)^4 + 5*(9*a^2 + 6*a*b - 11*b^2)*cosh(d*x + c)^2 + 3*a^2 +
6*a*b + 5*b^2)*sinh(d*x + c)^3 + 8*(27*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^
7 + 21*(3*a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^5 + 5*(9*a^2 + 6*a*b - 11*b^2)
*cosh(d*x + c)^3 + 3*(3*a^2 + 6*a*b + 5*b^2)*cosh(d*x + c))*sinh(d*x + c)^2
 + 6*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c) - 3*((a^2 + 6*a*b + 5*b^2)*cosh(d*
x + c)^10 + 10*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)*sinh(d*x + c)^9 + (a^2 +
 6*a*b + 5*b^2)*sinh(d*x + c)^10 + (a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^8 +
(45*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^2 + a^2 + 6*a*b + 5*b^2)*sinh(d*x +
 c)^8 + 8*(15*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^3 + (a^2 + 6*a*b + 5*b^2)
*cosh(d*x + c))*sinh(d*x + c)^7 - 2*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^6 +
 2*(105*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^4 + 14*(a^2 + 6*a*b + 5*b^2)*co
sh(d*x + c)^2 - a^2 - 6*a*b - 5*b^2)*sinh(d*x + c)^6 + 4*(63*(a^2 + 6*a*b +
```

5*b^2)*cosh(d*x + c)^5 + 14*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^3 - 3*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^4 + 2*(105*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^6 + 35*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^4 - 15*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^2 - a^2 - 6*a*b - 5*b^2)*sinh(d*x + c)^4 + 8*(15*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^7 + 7*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^5 - 5*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^3 - (a^2 + 6*a*b + 5*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + (a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^2 + (45*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^8 + 28*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^6 - 30*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^4 - 12*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^2 + a^2 + 6*a*b + 5*b^2)*sinh(d*x + c)^2 + a^2 + 6*a*b + 5*b^2 + 2*(5*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^9 + 4*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^7 - 6*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^5 - 4*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^3 + (a^2 + 6*a*b + 5*b^2)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) + 3*((a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^10 + 10*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)*sinh(d*x + c)^9 + (a^2 + 6*a*b + 5*b^2)*sinh(d*x + c)^10 + (a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^8 + (45*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^2 + a^2 + 6*a*b + 5*b^2)*sinh(d*x + c)^8 + 8*(15*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^3 + (a^2 + 6*a*b + 5*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 - 2*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^6 + 2*(105*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^4 + 14*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^2 - a^2 - 6*a*b - 5*b^2)*sinh(d*x + c)^6 + 4*(63*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^5 + 14*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^3 - 3*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^4 + 2*(105*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^6 + 35*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^4 - 15*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^2 - a^2 - 6*a*b - 5*b^2)*sinh(d*x + c)^4 + 8*(15*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^7 + 7*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^5 - 5*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^3 - (a^2 + 6*a*b + 5*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + (a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^2 + (45*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^8 + 28*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^6 - 30*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^4 - 12*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^2 + a^2 + 6*a*b + 5*b^2)*sinh(d*x + c)^2 + a^2 + 6*a*b + 5*b^2 + 2*(5*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^9 + 4*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^7 - 6*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^5 - 4*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^3 + (a^2 + 6*a*b + 5*b^2)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 2*(27*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^8 + 28*(3*a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^6 + 10*(9*a^2 + 6*a*b - 11*b^2)*cosh(d*x + c)^4 + 12*(3*a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^2 + 3*a^2 + 18*a*b + 15*b^2)*sinh(d*x + c))/(d*cosh(d*x + c)^10 + 10*d*cosh(d*x + c)*sinh(d*x + c)^9 + d*sinh(d*x + c)^10 + d*cosh(d*x + c)^8 + (45*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^8 + 8*(15*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^7 - 2*d*cosh(d*x + c)^6 + 2*(105*d*cosh(d*x + c)^4 + 14*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^6 + 4*(63*d*cosh(d*x + c)^5 + 14*d*cosh(d*x + c)^3 - 3*d*cosh(d*x + c))*sinh(d*x + c)^5 - 2*d*cosh(d*x + c)^4 + 2*(105*d*cosh(d*x + c)^6 + 35*d*cosh(d*x + c)^4 - 15*d*cosh(d*x

```
 + c)^2 - d)*sinh(d*x + c)^4 + 8*(15*d*cosh(d*x + c)^7 + 7*d*cosh(d*x + c)^
5 - 5*d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c)^3 + d*cosh(d*x + c
)^2 + (45*d*cosh(d*x + c)^8 + 28*d*cosh(d*x + c)^6 - 30*d*cosh(d*x + c)^4 -
 12*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 2*(5*d*cosh(d*x + c)^9 + 4*d*c
osh(d*x + c)^7 - 6*d*cosh(d*x + c)^5 - 4*d*cosh(d*x + c)^3 + d*cosh(d*x + c
))*sinh(d*x + c) + d)
```

giac [B] time = 0.16, size = 228, normalized size = 2.19

$$\frac{3\left(a^2+6\,ab+5\,b^2\right)\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)-3\left(a^2+6\,ab+5\,b^2\right)\log\left(e^{(dx+c)}+e^{(-dx-c)}-2\right)-\frac{12\left(a^2\left(e^{(dx+c)}+e^{(-dx-c}\right.\right.}{}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/12*(3*(a^2 + 6*a*b + 5*b^2)*log(e^(d*x + c) + e^(-d*x - c) + 2) - 3*(a^2
+ 6*a*b + 5*b^2)*log(e^(d*x + c) + e^(-d*x - c) - 2) - 12*(a^2*(e^(d*x + c)
 + e^(-d*x - c)) + 2*a*b*(e^(d*x + c) + e^(-d*x - c)) + b^2*(e^(d*x + c) +
e^(-d*x - c)))/((e^(d*x + c) + e^(-d*x - c))^2 - 4) - 16*(3*a*b*(e^(d*x + c
) + e^(-d*x - c))^2 + 3*b^2*(e^(d*x + c) + e^(-d*x - c))^2 + 2*b^2)/(e^(d*x
 + c) + e^(-d*x - c))^3)/d
```

maple [A] time = 0.42, size = 126, normalized size = 1.21

$$\frac{a^2\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(e^{dx+c}\right)\right)+2ab\left(-\frac{1}{2\sinh(dx+c)^2\cosh(dx+c)}-\frac{3}{2\cosh(dx+c)}+3\operatorname{arctanh}\left(e^{dx+c}\right)\right)+b^2\left(\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(a^2*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))+2*a*b*(-1/2/sin
h(d*x+c)^2/cosh(d*x+c)-3/2/cosh(d*x+c)+3*arctanh(exp(d*x+c)))+b^2*(-1/2/sin
h(d*x+c)^2/cosh(d*x+c)^3-5/6/cosh(d*x+c)^3-5/2/cosh(d*x+c)+5*arctanh(exp(d*
x+c))))
```

maxima [B] time = 0.34, size = 354, normalized size = 3.40

$$\frac{1}{6}\,b^2\left(\frac{15\,\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{15\,\log\left(e^{(-dx-c)}-1\right)}{d}-\frac{2\left(15\,e^{(-dx-c)}+20\,e^{(-3\,dx-3\,c)}-22\,e^{(-5\,dx-5\,c)}+20\,e^{(-7\,dx-7\,c)}+1\right.}{d\left(e^{(-2\,dx-2\,c)}-2\,e^{(-4\,dx-4\,c)}-2\,e^{(-6\,dx-6\,c)}+e^{(-8\,dx-8\,c)}+e^{(-10\,c}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/6*b^2*(15*log(e^(-d*x - c) + 1)/d - 15*log(e^(-d*x - c) - 1)/d - 2*(15*e^
(-d*x - c) + 20*e^(-3*d*x - 3*c) - 22*e^(-5*d*x - 5*c) + 20*e^(-7*d*x - 7*c
) + 15*e^(-9*d*x - 9*c))/(d*(e^(-2*d*x - 2*c) - 2*e^(-4*d*x - 4*c) - 2*e^(-
6*d*x - 6*c) + e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) + a*b*(3*log(e^
(-d*x - c) + 1)/d - 3*log(e^(-d*x - c) - 1)/d + 2*(3*e^(-d*x - c) - 2*e^(-3
*d*x - 3*c) + 3*e^(-5*d*x - 5*c))/(d*(e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) -
 e^(-6*d*x - 6*c) - 1))) + 1/2*a^2*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x -
 c) - 1)/d + 2*(e^(-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e
^(-4*d*x - 4*c) - 1)))
```

mupad [B] time = 1.65, size = 316, normalized size = 3.04

$$\frac{\operatorname{atan}\left(\frac{e^{d\,x}\,e^{c}\,\left(a^2\,\sqrt{-d^2}+5\,b^2\,\sqrt{-d^2}+6\,a\,b\,\sqrt{-d^2}\right)}{d\,\sqrt{a^4+12\,a^3\,b+46\,a^2\,b^2+60\,a\,b^3+25\,b^4}}\right)\,\sqrt{a^4+12\,a^3\,b+46\,a^2\,b^2+60\,a\,b^3+25\,b^4}}{\sqrt{-d^2}}-\frac{e^{c+d\,x}\,\left(a^2+2\,a\,b+b^2\right)}{d\,\left(e^{2\,c+2\,d\,x}-1\right)}+\frac{}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cosh(c + d*x)^2)^2/sinh(c + d*x)^3,x)
```

```
[Out] (atan((exp(d*x)*exp(c)*(a^2*(-d^2)^(1/2) + 5*b^2*(-d^2)^(1/2) + 6*a*b*(-d^2
)^(1/2)))/(d*(60*a*b^3 + 12*a^3*b + a^4 + 25*b^4 + 46*a^2*b^2)^(1/2)))*(60*
a*b^3 + 12*a^3*b + a^4 + 25*b^4 + 46*a^2*b^2)^(1/2))/(-d^2)^(1/2) - (exp(c
+ d*x)*(2*a*b + a^2 + b^2))/(d*(exp(2*c + 2*d*x) - 1)) + (8*b^2*exp(c + d*x
))/(3*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) -
 (2*exp(c + d*x)*(2*a*b + a^2 + b^2))/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*
d*x) + 1)) - (4*exp(c + d*x)*(a*b + b^2))/(d*(exp(2*c + 2*d*x) + 1)) - (8*b
^2*exp(c + d*x))/(3*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\operatorname{sech}^2(c + dx)\right)^2 \operatorname{csch}^3(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3*(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**2*csch(c + d*x)**3, x)
```

3.16 $\int \operatorname{csch}^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=75

$$\frac{b(2a+3b)\tanh(c+dx)}{d}-\frac{(a+b)^2\coth^3(c+dx)}{3d}+\frac{(a+b)(a+3b)\coth(c+dx)}{d}-\frac{b^2\tanh^3(c+dx)}{3d}$$

```
[Out] (a+b)*(a+3*b)*coth(d*x+c)/d-1/3*(a+b)^2*coth(d*x+c)^3/d+b*(2*a+3*b)*tanh(d*
x+c)/d-1/3*b^2*tanh(d*x+c)^3/d
```

Rubi [A] time = 0.08, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4132, 448}

$$\frac{b(2a+3b)\tanh(c+dx)}{d}-\frac{(a+b)^2\coth^3(c+dx)}{3d}+\frac{(a+b)(a+3b)\coth(c+dx)}{d}-\frac{b^2\tanh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + b)*(a + 3*b)*Coth[c + d*x])/d - ((a + b)^2*Coth[c + d*x]^3)/(3*d) + (
b*(2*a + 3*b)*Tanh[c + d*x])/d - (b^2*Tanh[c + d*x]^3)/(3*d)
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \operatorname{csch}^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2 dx = \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)(a+b-bx^2)^2}{x^4} dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\operatorname{Subst}\left(\int \left(b(2a+3b)+\frac{(a+b)^2}{x^4}+\frac{(-a-3b)(a+b)}{x^2}-b^2x^2\right) dx, x, \tanh(c+\right.}{d}$$
$$= \frac{(a+b)(a+3b)\coth(c+dx)}{d} - \frac{(a+b)^2\coth^3(c+dx)}{3d} + \frac{b(2a+3b}{}$$

Mathematica [B] time = 1.32, size = 151, normalized size = 2.01

$$-\frac{\operatorname{csch}(2c)\operatorname{csch}^3(2(c+dx))\left(-3a^2\sinh(2(c+dx))+a^2\sinh(6(c+dx))+3a^2\sinh(4c+2dx)+a^2\sinh(4c+6dx\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] -1/6*(Csch[2*c]*Csch[2*(c + d*x)]^3*(8*a*(a + 2*b)*Sinh[2*c] - 6*(a + 2*b)^
2*Sinh[2*d*x] - 3*a^2*Sinh[2*(c + d*x)] - 6*a*b*Sinh[2*(c + d*x)] + a^2*Sin
h[6*(c + d*x)] + 2*a*b*Sinh[6*(c + d*x)] + 3*a^2*Sinh[4*c + 2*d*x] + a^2*Si
nh[4*c + 6*d*x] + 8*a*b*Sinh[4*c + 6*d*x] + 8*b^2*Sinh[4*c + 6*d*x]))/d
```

fricas [B] time = 0.40, size = 408, normalized size = 5.44

$$-\frac{8\left(\left(a^2-4\,ab-4\,b^2\right)\cosh(dx+c)^4+8\left(a^2+2\,ab+2\,b^2\right)\cosh(dx+c)\sinh(\right.}{3\left(d\cosh(dx+c)^8+56\,d\cosh(dx+c)^3\sinh(dx+c)^5+28\,d\cosh(dx+c)^2\sinh(dx+c)^6+8\,d\cosh(dx+c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -8/3*((a^2 - 4*a*b - 4*b^2)*cosh(d*x + c)^4 + 8*(a^2 + 2*a*b + 2*b^2)*cosh(
d*x + c)*sinh(d*x + c)^3 + (a^2 - 4*a*b - 4*b^2)*sinh(d*x + c)^4 + 4*(a^2 +
 2*a*b)*cosh(d*x + c)^2 + 2*(3*(a^2 - 4*a*b - 4*b^2)*cosh(d*x + c)^2 + 2*a^
2 + 4*a*b)*sinh(d*x + c)^2 + 3*a^2 + 12*a*b + 12*b^2 + 8*((a^2 + 2*a*b + 2*
b^2)*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(
d*x + c)^8 + 56*d*cosh(d*x + c)^3*sinh(d*x + c)^5 + 28*d*cosh(d*x + c)^2*si
nh(d*x + c)^6 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 - 4*d
*cosh(d*x + c)^4 + 2*(35*d*cosh(d*x + c)^4 - 2*d)*sinh(d*x + c)^4 + 8*(7*d*
cosh(d*x + c)^5 - d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*d*cosh(d*x + c)^6
```

```
 - 6*d*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*(d*cosh(d*x + c)^7 - d*cosh(d*x
 + c)^3)*sinh(d*x + c) + 3*d)
```

giac [A] time = 0.18, size = 115, normalized size = 1.53

$$-\frac{4\left(3\,a^2e^{(8\,dx+8\,c)}+8\,a^2e^{(6\,dx+6\,c)}+16\,abe^{(6\,dx+6\,c)}+6\,a^2e^{(4\,dx+4\,c)}+24\,abe^{(4\,dx+4\,c)}+24\,b^2e^{(4\,dx+4\,c)}-a^2-8\,ab-8\,b\right.}{3\,d\left(e^{(4\,dx+4\,c)}-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -4/3*(3*a^2*e^(8*d*x + 8*c) + 8*a^2*e^(6*d*x + 6*c) + 16*a*b*e^(6*d*x + 6*c
) + 6*a^2*e^(4*d*x + 4*c) + 24*a*b*e^(4*d*x + 4*c) + 24*b^2*e^(4*d*x + 4*c)
 - a^2 - 8*a*b - 8*b^2)/(d*(e^(4*d*x + 4*c) - 1)^3)
```

maple [A] time = 0.55, size = 138, normalized size = 1.84

$$\frac{a^2\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^2}{3}\right)\coth\left(dx+c\right)+2ab\left(-\frac{1}{3\sinh(dx+c)^3\cosh(dx+c)}+\frac{4}{3\sinh(dx+c)\cosh(dx+c)}+\frac{8\tanh(dx+c)}{3}\right)+b^2\left(-\frac{}{3\sinh(dx\right.}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(a^2*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)+2*a*b*(-1/3/sinh(d*x+c)^3/cosh
(d*x+c)+4/3/sinh(d*x+c)/cosh(d*x+c)+8/3*tanh(d*x+c))+b^2*(-1/3/sinh(d*x+c)^
3/cosh(d*x+c)^3+2/sinh(d*x+c)/cosh(d*x+c)^3+8*(2/3+1/3*sech(d*x+c)^2)*tanh(
d*x+c)))
```

maxima [B] time = 0.34, size = 285, normalized size = 3.80

$$\frac{4}{3}\,a^2\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d\left(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1\right)}-\frac{1}{d\left(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1\right)}\right)+\frac{32}{3}\,ab\left(\frac{}{d\left(2\,e^{(\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 4/3*a^2*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e
^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(
-6*d*x - 6*c) - 1))) + 32/3*a*b*(2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c)
- 2*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) - 1)) - 1/(d*(2*e^(-2*d*x - 2*c) -
2*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) - 1))) + 32/3*b^2*(3*e^(-4*d*x - 4*c)
```

```
/(d*(3*e^(-4*d*x - 4*c) - 3*e^(-8*d*x - 8*c) + e^(-12*d*x - 12*c) - 1)) - 1
/(d*(3*e^(-4*d*x - 4*c) - 3*e^(-8*d*x - 8*c) + e^(-12*d*x - 12*c) - 1)))
```

mupad [B] time = 0.21, size = 115, normalized size = 1.53

$$-\frac{4\,\left(6\,a^2\,\mathrm{e}^{4\,c+4\,d\,x}-a^2-8\,b^2-8\,a\,b+8\,a^2\,\mathrm{e}^{6\,c+6\,d\,x}+3\,a^2\,\mathrm{e}^{8\,c+8\,d\,x}+24\,b^2\,\mathrm{e}^{4\,c+4\,d\,x}+24\,a\,b\,\mathrm{e}^{4\,c+4\,d\,x}+16\,a\,b\,\mathrm{e}^{6\,c+6}\right.}{3\,d\,{\left(\mathrm{e}^{4\,c+4\,d\,x}-1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cosh(c + d*x)^2)^2/sinh(c + d*x)^4,x)
```

```
[Out] -(4*(6*a^2*exp(4*c + 4*d*x) - a^2 - 8*b^2 - 8*a*b + 8*a^2*exp(6*c + 6*d*x)
+ 3*a^2*exp(8*c + 8*d*x) + 24*b^2*exp(4*c + 4*d*x) + 24*a*b*exp(4*c + 4*d*x
) + 16*a*b*exp(6*c + 6*d*x)))/(3*d*(exp(4*c + 4*d*x) - 1)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \operatorname{sech}^{2}{\left(c + d x \right)}\right)^{2} \operatorname{csch}^{4}{\left(c + d x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4*(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**2*csch(c + d*x)**4, x)
```

3.17 $\int \left(a+b\,\mathbf{sech}^2(c+dx)\right)^3 \sinh^4(c+dx)\,dx$

Optimal. Leaf size=182

$$\frac{b\left(6a^2-23ab-8b^2\right)\tanh^3(c+dx)}{8d}-\frac{3a\left(a^2-12ab+8b^2\right)\tanh(c+dx)}{8d}+\frac{3}{8}ax\left(a^2-12ab+8b^2\right)-\frac{3b^2(5a-16b)\,\text{ta}}{40d}$$

```
[Out] 3/8*a*(a^2-12*a*b+8*b^2)*x-3/8*a*(a^2-12*a*b+8*b^2)*tanh(d*x+c)/d+1/8*b*(6*
a^2-23*a*b-8*b^2)*tanh(d*x+c)^3/d-3/40*(5*a-16*b)*b^2*tanh(d*x+c)^5/d-3/8*(
a-2*b)*sinh(d*x+c)^2*tanh(d*x+c)*(a+b-b*tanh(d*x+c)^2)^2/d+1/4*cosh(d*x+c)*
sinh(d*x+c)^3*(a+b-b*tanh(d*x+c)^2)^3/d
```

Rubi [A] time = 0.23, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4132, 467, 577, 570, 206}

$$\frac{b\left(6a^2-23ab-8b^2\right)\tanh^3(c+dx)}{8d}-\frac{3a\left(a^2-12ab+8b^2\right)\tanh(c+dx)}{8d}+\frac{3}{8}ax\left(a^2-12ab+8b^2\right)-\frac{3b^2(5a-16b)\,\text{ta}}{40d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^3*Sinh[c + d*x]^4,x]
```

```
[Out] (3*a*(a^2 - 12*a*b + 8*b^2)*x)/8 - (3*a*(a^2 - 12*a*b + 8*b^2)*Tanh[c + d*x
])/(8*d) + (b*(6*a^2 - 23*a*b - 8*b^2)*Tanh[c + d*x]^3)/(8*d) - (3*(5*a - 1
6*b)*b^2*Tanh[c + d*x]^5)/(40*d) - (3*(a - 2*b)*Sinh[c + d*x]^2*Tanh[c + d*
x]*(a + b - b*Tanh[c + d*x]^2)^2)/(8*d) + (Cosh[c + d*x]*Sinh[c + d*x]^3*(a
 + b - b*Tanh[c + d*x]^2)^3)/(4*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 570

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rule 577

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*b*g*n*(p + 1)), x] + Dist[1/(a*
b*n*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b
*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m + n
*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && IGtQ[n, 0
] && LtQ[p, -1] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a
*f])
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \left(a+b\text{sech}^2(c+dx)\right)^3 \sinh^4(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x^4\left(a+b-bx^2\right)^3}{\left(1-x^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\cosh(c+dx)\sinh^3(c+dx)\left(a+b-b\tanh^2(c+dx)\right)^3}{4d} - \frac{\text{Subst}\left(\int \frac{x^2}{-}\right.}{}$$

$$= -\frac{3(a-2b)\sinh^2(c+dx)\tanh(c+dx)\left(a+b-b\tanh^2(c+dx)\right)^2}{8d} +$$

$$= -\frac{3(a-2b)\sinh^2(c+dx)\tanh(c+dx)\left(a+b-b\tanh^2(c+dx)\right)^2}{8d} +$$

$$= -\frac{3a\left(a^2-12ab+8b^2\right)\tanh(c+dx)}{8d} + \frac{b\left(6a^2-23ab-8b^2\right)\tanh^3(c}{8d}$$

$$= \frac{3}{8}a\left(a^2-12ab+8b^2\right)x - \frac{3a\left(a^2-12ab+8b^2\right)\tanh(c+dx)}{8d} + \frac{b\left(6a\right.}{}$$

Mathematica [B] time = 2.51, size = 651, normalized size = 3.58

$$\frac{\text{sech}(c)\text{sech}^5(c+dx)\left(a\cosh^2(c+dx)+b\right)^3\left(-180a^3\sinh(2c+dx)-310a^3\sinh(2c+3dx)-310a^3\sinh(4c+3a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^3*Sinh[c + d*x]^4,x]
```

```
[Out] ((b + a*Cosh[c + d*x]^2)^3*Sech[c]*Sech[c + d*x]^5*(1200*a*(a^2 - 12*a*b +
8*b^2)*d*x*Cosh[d*x] + 1200*a*(a^2 - 12*a*b + 8*b^2)*d*x*Cosh[2*c + d*x] +
600*a^3*d*x*Cosh[2*c + 3*d*x] - 7200*a^2*b*d*x*Cosh[2*c + 3*d*x] + 4800*a*b
^2*d*x*Cosh[2*c + 3*d*x] + 600*a^3*d*x*Cosh[4*c + 3*d*x] - 7200*a^2*b*d*x*C
osh[4*c + 3*d*x] + 4800*a*b^2*d*x*Cosh[4*c + 3*d*x] + 120*a^3*d*x*Cosh[4*c
+ 5*d*x] - 1440*a^2*b*d*x*Cosh[4*c + 5*d*x] + 960*a*b^2*d*x*Cosh[4*c + 5*d*
x] + 120*a^3*d*x*Cosh[6*c + 5*d*x] - 1440*a^2*b*d*x*Cosh[6*c + 5*d*x] + 960
*a*b^2*d*x*Cosh[6*c + 5*d*x] - 180*a^3*Sinh[d*x] + 12120*a^2*b*Sinh[d*x] -
14080*a*b^2*Sinh[d*x] + 1280*b^3*Sinh[d*x] - 180*a^3*Sinh[2*c + d*x] - 7080
*a^2*b*Sinh[2*c + d*x] + 11520*a*b^2*Sinh[2*c + d*x] - 310*a^3*Sinh[2*c + 3
*d*x] + 8760*a^2*b*Sinh[2*c + 3*d*x] - 8960*a*b^2*Sinh[2*c + 3*d*x] - 310*a
^3*Sinh[4*c + 3*d*x] - 840*a^2*b*Sinh[4*c + 3*d*x] + 3840*a*b^2*Sinh[4*c +
```

```
3*d*x] - 640*b^3*Sinh[4*c + 3*d*x] - 150*a^3*Sinh[4*c + 5*d*x] + 2520*a^2*b
*Sinh[4*c + 5*d*x] - 2560*a*b^2*Sinh[4*c + 5*d*x] + 128*b^3*Sinh[4*c + 5*d*
x] - 150*a^3*Sinh[6*c + 5*d*x] + 600*a^2*b*Sinh[6*c + 5*d*x] - 15*a^3*Sinh[
6*c + 7*d*x] + 120*a^2*b*Sinh[6*c + 7*d*x] - 15*a^3*Sinh[8*c + 7*d*x] + 120
*a^2*b*Sinh[8*c + 7*d*x] + 5*a^3*Sinh[8*c + 9*d*x] + 5*a^3*Sinh[10*c + 9*d*
x]))/(1280*d*(a + 2*b + a*Cosh[2*(c + d*x)])^3)
```

fricas [B] time = 0.45, size = 727, normalized size = 3.99

$$\frac{5\,a^3\sinh\left(dx+c\right)^9+15\left(12\,a^3\cosh\left(dx+c\right)^2-a^3+8\,a^2b\right)\sinh\left(dx+c\right)^7-8\left(120\,a^2b-160\,ab^2+8\,b^3-15\left(a^3\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3*sinh(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/320*(5*a^3*sinh(d*x + c)^9 + 15*(12*a^3*cosh(d*x + c)^2 - a^3 + 8*a^2*b)*
sinh(d*x + c)^7 - 8*(120*a^2*b - 160*a*b^2 + 8*b^3 - 15*(a^3 - 12*a^2*b + 8
*a*b^2)*d*x)*cosh(d*x + c)^5 - 40*(120*a^2*b - 160*a*b^2 + 8*b^3 - 15*(a^3
- 12*a^2*b + 8*a*b^2)*d*x)*cosh(d*x + c)*sinh(d*x + c)^4 + (630*a^3*cosh(d*
x + c)^4 - 150*a^3 + 1560*a^2*b - 1280*a*b^2 + 64*b^3 - 315*(a^3 - 8*a^2*b)
*cosh(d*x + c)^2)*sinh(d*x + c)^5 - 40*(120*a^2*b - 160*a*b^2 + 8*b^3 - 15*
(a^3 - 12*a^2*b + 8*a*b^2)*d*x)*cosh(d*x + c)^3 + 5*(84*a^3*cosh(d*x + c)^6
 - 105*(a^3 - 8*a^2*b)*cosh(d*x + c)^4 - 62*a^3 + 792*a^2*b - 512*a*b^2 - 6
4*b^3 - 4*(75*a^3 - 780*a^2*b + 640*a*b^2 - 32*b^3)*cosh(d*x + c)^2)*sinh(d
*x + c)^3 - 40*(2*(120*a^2*b - 160*a*b^2 + 8*b^3 - 15*(a^3 - 12*a^2*b + 8*a
*b^2)*d*x)*cosh(d*x + c)^3 + 3*(120*a^2*b - 160*a*b^2 + 8*b^3 - 15*(a^3 - 1
2*a^2*b + 8*a*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^2 - 80*(120*a^2*b - 16
0*a*b^2 + 8*b^3 - 15*(a^3 - 12*a^2*b + 8*a*b^2)*d*x)*cosh(d*x + c) + 5*(9*a
^3*cosh(d*x + c)^8 - 21*(a^3 - 8*a^2*b)*cosh(d*x + c)^6 - 2*(75*a^3 - 780*a
^2*b + 640*a*b^2 - 32*b^3)*cosh(d*x + c)^4 - 36*a^3 + 504*a^2*b - 256*a*b^2
 + 128*b^3 - 6*(31*a^3 - 396*a^2*b + 256*a*b^2 + 32*b^3)*cosh(d*x + c)^2)*s
inh(d*x + c))/(d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)*sinh(d*x + c)^4 + 5*d*
cosh(d*x + c)^3 + 5*(2*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)
^2 + 10*d*cosh(d*x + c))
```

giac [A] time = 0.22, size = 339, normalized size = 1.86

$$\frac{5\,a^3e^{(4\,dx+4\,c)}-40\,a^3e^{(2\,dx+2\,c)}+120\,a^2be^{(2\,dx+2\,c)}+120\left(a^3-12\,a^2b+8\,ab^2\right)(dx+c)-5\left(18\,a^3e^{(4\,dx+4\,c)}-216\,a^2b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3*sinh(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/320*(5*a^3*e^(4*d*x + 4*c) - 40*a^3*e^(2*d*x + 2*c) + 120*a^2*b*e^(2*d*x
+ 2*c) + 120*(a^3 - 12*a^2*b + 8*a*b^2)*(d*x + c) - 5*(18*a^3*e^(4*d*x + 4*
c) - 216*a^2*b*e^(4*d*x + 4*c) + 144*a*b^2*e^(4*d*x + 4*c) - 8*a^3*e^(2*d*x
 + 2*c) + 24*a^2*b*e^(2*d*x + 2*c) + a^3)*e^(-4*d*x - 4*c) - 128*(15*a^2*b*
e^(8*d*x + 8*c) - 30*a*b^2*e^(8*d*x + 8*c) + 5*b^3*e^(8*d*x + 8*c) + 60*a^2
*b*e^(6*d*x + 6*c) - 90*a*b^2*e^(6*d*x + 6*c) + 90*a^2*b*e^(4*d*x + 4*c) -
110*a*b^2*e^(4*d*x + 4*c) + 10*b^3*e^(4*d*x + 4*c) + 60*a^2*b*e^(2*d*x + 2*
c) - 70*a*b^2*e^(2*d*x + 2*c) + 15*a^2*b - 20*a*b^2 + b^3)/(e^(2*d*x + 2*c)
 + 1)^5)/d
```

maple [A] time = 0.44, size = 182, normalized size = 1.00

$$\frac{a^3\left(\left(\frac{\left(\sinh^3(dx+c)\right)}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+3a^2b\left(\frac{\sinh^3(dx+c)}{2\cosh(dx+c)}-\frac{3dx}{2}-\frac{3c}{2}+\frac{3\tanh(dx+c)}{2}\right)+3a\,b^2\left(dx\cdots\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)^3*sinh(d*x+c)^4,x)
```

```
[Out] 1/d*(a^3*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)+3*
a^2*b*(1/2*sinh(d*x+c)^3/cosh(d*x+c)-3/2*d*x-3/2*c+3/2*tanh(d*x+c))+3*a*b^2
*(d*x+c-tanh(d*x+c)-1/3*tanh(d*x+c)^3)+b^3*(-1/2*sinh(d*x+c)^3/cosh(d*x+c)^
5-3/8*sinh(d*x+c)/cosh(d*x+c)^5+3/8*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c
)^2)*tanh(d*x+c)))
```

maxima [B] time = 0.34, size = 422, normalized size = 2.32

$$\frac{1}{64}a^3\left(24x+\frac{e^{(4dx+4c)}}{d}-\frac{8e^{(2dx+2c)}}{d}+\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right)+ab^2\left(3x+\frac{3c}{d}-\frac{4\left(3e^{(-2dx-2c)}+3e^{(-4dx-\cdots}\right.}{d\left(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3*sinh(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/64*a^3*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*
c)/d - e^(-4*d*x - 4*c)/d) + a*b^2*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3
*e^(-4*d*x - 4*c) + 2)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*
d*x - 6*c) + 1))) - 3/8*a^2*b*(12*(d*x + c)/d + e^(-2*d*x - 2*c)/d - (17*e^
(-2*d*x - 2*c) + 1)/(d*(e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c)))) + 2/5*b^3*(1
0*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*
d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 5*e^(-8*d*x -
8*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5
*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*d*x - 2*c) + 1
```

```
0*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x
- 10*c) + 1)))
```

mupad [B] time = 0.37, size = 686, normalized size = 3.77

$$\frac{\frac{2\left(-3a^2b+3ab^2+b^3\right)}{5d}-\frac{6e^{2c+2dx}\left(3a^2b-2ab^2+b^3\right)}{5d}+\frac{6e^{4c+4dx}\left(-3a^2b+3ab^2+b^3\right)}{5d}-\frac{2e^{6c+6dx}\left(3a^2b-6ab^2+b^3\right)}{5d}}{4e^{2c+2dx}+6e^{4c+4dx}+4e^{6c+6dx}+e^{8c+8dx}+1}-\frac{\frac{2\left(3a^2b-6ab^2+b^3\right)}{5d}-\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^4*(a + b/cosh(c + d*x)^2)^3,x)
```

```
[Out] ((2*(3*a*b^2 - 3*a^2*b + b^3))/(5*d) - (6*exp(2*c + 2*d*x)*(3*a^2*b - 2*a*b
^2 + b^3))/(5*d) + (6*exp(4*c + 4*d*x)*(3*a*b^2 - 3*a^2*b + b^3))/(5*d) - (
2*exp(6*c + 6*d*x)*(3*a^2*b - 6*a*b^2 + b^3))/(5*d))/(4*exp(2*c + 2*d*x) +
6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) - ((2*(3*a^
2*b - 6*a*b^2 + b^3))/(5*d) - (8*exp(2*c + 2*d*x)*(3*a*b^2 - 3*a^2*b + b^3)
)/(5*d) + (12*exp(4*c + 4*d*x)*(3*a^2*b - 2*a*b^2 + b^3))/(5*d) - (8*exp(6*
c + 6*d*x)*(3*a*b^2 - 3*a^2*b + b^3))/(5*d) + (2*exp(8*c + 8*d*x)*(3*a^2*b
- 6*a*b^2 + b^3))/(5*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp
(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1) + ((2*(3*a*b^2
 - 3*a^2*b + b^3))/(5*d) - (2*exp(2*c + 2*d*x)*(3*a^2*b - 6*a*b^2 + b^3))/(
5*d))/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) - ((2*(3*a^2*b - 2*a*b^2
+ b^3))/(5*d) - (4*exp(2*c + 2*d*x)*(3*a*b^2 - 3*a^2*b + b^3))/(5*d) + (2*e
xp(4*c + 4*d*x)*(3*a^2*b - 6*a*b^2 + b^3))/(5*d))/(3*exp(2*c + 2*d*x) + 3*e
xp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) + (3*a*x*(a^2 - 12*a*b + 8*b^2))/8
- (a^3*exp(- 4*c - 4*d*x))/(64*d) + (a^3*exp(4*c + 4*d*x))/(64*d) - (2*(3*a
^2*b - 6*a*b^2 + b^3))/(5*d*(exp(2*c + 2*d*x) + 1)) + (a^2*exp(- 2*c - 2*d*
x)*(a - 3*b))/(8*d) - (a^2*exp(2*c + 2*d*x)*(a - 3*b))/(8*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)**3*sinh(d*x+c)**4,x)
```

```
[Out] Timed out
```

3.18 $\int \left(a+b\,\mathrm{sech}^2(c+dx)\right)^3 \sinh^3(c+dx)\,dx$

Optimal. Leaf size=99

$$\frac{a^3\cosh^3(c+dx)}{3d}-\frac{a^2(a-3b)\cosh(c+dx)}{d}+\frac{b^2(3a-b)\mathrm{sech}^3(c+dx)}{3d}+\frac{3ab(a-b)\mathrm{sech}(c+dx)}{d}+\frac{b^3\mathrm{sech}^5(c+dx)}{5d}$$

```
[Out] -a^2*(a-3*b)*cosh(d*x+c)/d+1/3*a^3*cosh(d*x+c)^3/d+3*a*(a-b)*b*sech(d*x+c)/
d+1/3*(3*a-b)*b^2*sech(d*x+c)^3/d+1/5*b^3*sech(d*x+c)^5/d
```

Rubi [A] time = 0.11, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4133, 448}

$$-\frac{a^2(a-3b)\cosh(c+dx)}{d}+\frac{a^3\cosh^3(c+dx)}{3d}+\frac{b^2(3a-b)\mathrm{sech}^3(c+dx)}{3d}+\frac{3ab(a-b)\mathrm{sech}(c+dx)}{d}+\frac{b^3\mathrm{sech}^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^3*Sinh[c + d*x]^3,x]
```

```
[Out] -((a^2*(a - 3*b)*Cosh[c + d*x])/d) + (a^3*Cosh[c + d*x]^3)/(3*d) + (3*a*(a
- b)*b*Sech[c + d*x])/d + ((3*a - b)*b^2*Sech[c + d*x]^3)/(3*d) + (b^3*Sech
[c + d*x]^5)/(5*d)
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \left(a+b\text{sech}^2(c+dx)\right)^3 \sinh^3(c+dx)\,dx = -\frac{\text{Subst}\left(\int \frac{(1-x^2)(b+ax^2)^3}{x^6}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(a^2(a-3b)+\frac{b^3}{x^6}+\frac{(3a-b)b^2}{x^4}+\frac{3a(a-b)b}{x^2}-a^3x^2\right)dx, x, \text{cosh}$$

$$= -\frac{a^2(a-3b)\cosh(c+dx)}{d}+\frac{a^3\cosh^3(c+dx)}{3d}+\frac{3a(a-b)b\text{sech}(c}{d}$$

Mathematica [A] time = 1.25, size = 119, normalized size = 1.20

$$\frac{4\text{sech}^5(c+dx)\left(a\cosh^2(c+dx)+b\right)^3\left(5a^2\cosh^6(c+dx)(a\cosh(2(c+dx))-5a+18b)+10b^2(3a-b)\cosh^2(c}{15d(a\cosh(2(c+dx))+a+2b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^3*Sinh[c + d*x]^3,x]
```

```
[Out] (4*(b + a*Cosh[c + d*x]^2)^3*(6*b^3 + 10*(3*a - b)*b^2*Cosh[c + d*x]^2 + 90
*a*(a - b)*b*Cosh[c + d*x]^4 + 5*a^2*Cosh[c + d*x]^6*(-5*a + 18*b + a*Cosh[
2*(c + d*x)]))*Sech[c + d*x]^5)/(15*d*(a + 2*b + a*Cosh[2*(c + d*x)])^3)
```

fricas [B] time = 0.41, size = 403, normalized size = 4.07

$$\frac{5\,a^3\cosh\left(dx+c\right)^8+5\,a^3\sinh\left(dx+c\right)^8-20\left(a^3-9\,a^2b\right)\cosh\left(dx+c\right)^6+20\left(7\,a^3\cosh\left(dx+c\right)^2-a^3+9\,a^2b\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3*sinh(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/120*(5*a^3*cosh(d*x + c)^8 + 5*a^3*sinh(d*x + c)^8 - 20*(a^3 - 9*a^2*b)*c
osh(d*x + c)^6 + 20*(7*a^3*cosh(d*x + c)^2 - a^3 + 9*a^2*b)*sinh(d*x + c)^6
 - 20*(11*a^3 - 90*a^2*b + 36*a*b^2)*cosh(d*x + c)^4 + 10*(35*a^3*cosh(d*x
+ c)^4 - 22*a^3 + 180*a^2*b - 72*a*b^2 - 30*(a^3 - 9*a^2*b)*cosh(d*x + c)^2
)*sinh(d*x + c)^4 - 425*a^3 + 3960*a^2*b - 1200*a*b^2 + 64*b^3 - 20*(31*a^3
 - 279*a^2*b + 96*a*b^2 + 16*b^3)*cosh(d*x + c)^2 + 20*(7*a^3*cosh(d*x + c)
^6 - 15*(a^3 - 9*a^2*b)*cosh(d*x + c)^4 - 31*a^3 + 279*a^2*b - 96*a*b^2 - 1
6*b^3 - 6*(11*a^3 - 90*a^2*b + 36*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2)/
(d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)*sinh(d*x + c)^4 + 5*d*cosh(d*x + c)^
3 + 5*(2*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 + 10*d*cosh
(d*x + c))
```

giac [B] time = 0.21, size = 193, normalized size = 1.95

$$\frac{5\,a^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3-60\,a^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)+180\,a^2b\left(e^{(dx+c)}+e^{(-dx-c)}\right)+\frac{16\left(45\,a^2b\left(e^{(dx+c)}+e^{(-dx-c)}\right)^4-45\,ab^2\left(e^{(dx+c)}\right.\right.}{}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3*sinh(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/120*(5*a^3*(e^(d*x + c) + e^(-d*x - c))^3 - 60*a^3*(e^(d*x + c) + e^(-d*x
 - c)) + 180*a^2*b*(e^(d*x + c) + e^(-d*x - c)) + 16*(45*a^2*b*(e^(d*x + c)
 + e^(-d*x - c))^4 - 45*a*b^2*(e^(d*x + c) + e^(-d*x - c))^4 + 60*a*b^2*(e^
(d*x + c) + e^(-d*x - c))^2 - 20*b^3*(e^(d*x + c) + e^(-d*x - c))^2 + 48*b^
3)/(e^(d*x + c) + e^(-d*x - c))^5)/d
```

maple [A] time = 0.37, size = 130, normalized size = 1.31

$$\frac{a^3\left(-\frac{2}{3}+\frac{\left(\sinh^2(dx+c)\right)}{3}\right)\cosh(dx+c)+3a^2b\left(\frac{\sinh^2(dx+c)}{\cosh(dx+c)}+\frac{2}{\cosh(dx+c)}\right)+3a\,b^2\left(-\frac{\sinh^2(dx+c)}{\cosh(dx+c)^3}-\frac{2}{3\cosh(dx+c)^3}\right)+b^3\left(-\frac{\sinh^2(dx+c)}{3\cosh(dx+c)^5}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)^3*sinh(d*x+c)^3,x)
```

```
[Out] 1/d*(a^3*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)+3*a^2*b*(sinh(d*x+c)^2/cosh(d
*x+c)+2/cosh(d*x+c))+3*a*b^2*(-sinh(d*x+c)^2/cosh(d*x+c)^3-2/3/cosh(d*x+c)^
3)+b^3*(-1/3*sinh(d*x+c)^2/cosh(d*x+c)^5-2/15/cosh(d*x+c)^5))
```

maxima [B] time = 0.34, size = 489, normalized size = 4.94

$$\frac{1}{24}\,a^3\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)+\frac{3}{2}\,a^2b\left(\frac{e^{(-dx-c)}}{d}+\frac{5\,e^{(-2\,dx-2\,c)}+1}{d\left(e^{(-dx-c)}+e^{(-3\,dx-3\,c)}\right)}\right)-2\,ab^2\left(\frac{}{d\left(3\,e^{(-2\,dx-}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3*sinh(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/24*a^3*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*
x - 3*c)/d) + 3/2*a^2*b*(e^(-d*x - c)/d + (5*e^(-2*d*x - 2*c) + 1)/(d*(e^(-
d*x - c) + e^(-3*d*x - 3*c)))) - 2*a*b^2*(3*e^(-d*x - c)/(d*(3*e^(-2*d*x -
2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 2*e^(-3*d*x - 3*c)/(d*
(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 3*e^(-5
*d*x - 5*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c)
```

```
+ 1))) - 8/15*b^3*(5*e^(-3*d*x - 3*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x
 - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1
)) - 2*e^(-5*d*x - 5*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e
^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 5*e^(-7*d
*x - 7*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c
) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)))
```

mupad [B] time = 0.33, size = 348, normalized size = 3.52

$$\frac{a^3\,\mathrm{e}^{-3\,c-3\,d\,x}}{24\,d}+\frac{a^3\,\mathrm{e}^{3\,c+3\,d\,x}}{24\,d}-\frac{3\,a^2\,\mathrm{e}^{-c-d\,x}\,\left(a-4\,b\right)}{8\,d}+\frac{8\,\mathrm{e}^{c+d\,x}\,\left(3\,a\,b^2-b^3\right)}{3\,d\,\left(2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1\right)}-\frac{64\,b^3\,\mathrm{e}^{c+d}}{5\,d\,\left(4\,\mathrm{e}^{2\,c+2\,d\,x}+6\,\mathrm{e}^{4\,c+4\,d\,x}+4\,\mathrm{e}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^3*(a + b/cosh(c + d*x)^2)^3,x)
```

```
[Out] (a^3*exp(- 3*c - 3*d*x))/(24*d) + (a^3*exp(3*c + 3*d*x))/(24*d) - (3*a^2*ex
p(- c - d*x)*(a - 4*b))/(8*d) + (8*exp(c + d*x)*(3*a*b^2 - b^3))/(3*d*(2*ex
p(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (64*b^3*exp(c + d*x))/(5*d*(4*exp
(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x)
+ 1)) - (8*exp(c + d*x)*(15*a*b^2 - 17*b^3))/(15*d*(3*exp(2*c + 2*d*x) + 3*
exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) + (32*b^3*exp(c + d*x))/(5*d*(5*e
xp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8
*d*x) + exp(10*c + 10*d*x) + 1)) - (6*exp(c + d*x)*(a*b^2 - a^2*b))/(d*(exp
(2*c + 2*d*x) + 1)) - (3*a^2*exp(c + d*x)*(a - 4*b))/(8*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)**3*sinh(d*x+c)**3,x)
```

```
[Out] Timed out
```

3.19 $\int \left(a+b\,\mathbf{sech}^2(c+dx)\right)^3 \sinh^2(c+dx)\,dx$

Optimal. Leaf size=112

$$\frac{a^3}{4d(1-\tanh(c+dx))}-\frac{a^3}{4d(\tanh(c+dx)+1)}-\frac{3a^2b\tanh(c+dx)}{d}-\frac{1}{2}a^2x(a-6b)+\frac{b^2(3a+b)\tanh^3(c+dx)}{3d}-\frac{b^3\tanh$$

```
[Out] -1/2*a^2*(a-6*b)*x+1/4*a^3/d/(1-tanh(d*x+c))-3*a^2*b*tanh(d*x+c)/d+1/3*b^2*
(3*a+b)*tanh(d*x+c)^3/d-1/5*b^3*tanh(d*x+c)^5/d-1/4*a^3/d/(1+tanh(d*x+c))
```

Rubi [A] time = 0.19, antiderivative size = 143, normalized size of antiderivative = 1.28, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4132, 467, 528, 388, 206}

$$-\frac{b\left(81a^2-28ab-4b^2\right)\tanh(c+dx)}{30d}-\frac{1}{2}a^2x(a-6b)-\frac{7b\tanh(c+dx)\left(a-b\tanh^2(c+dx)+b\right)^2}{10d}-\frac{b(33a-2b)\tanh$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^3*Sinh[c + d*x]^2,x]
```

```
[Out] -(a^2*(a - 6*b)*x)/2 - (b*(81*a^2 - 28*a*b - 4*b^2)*Tanh[c + d*x])/(30*d) -
 ((33*a - 2*b)*b*Tanh[c + d*x]*(a + b - b*Tanh[c + d*x]^2))/(30*d) - (7*b*T
anh[c + d*x]*(a + b - b*Tanh[c + d*x]^2)^2)/(10*d) + (Cosh[c + d*x]*Sinh[c
+ d*x]*(a + b - b*Tanh[c + d*x]^2)^3)/(2*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
```

```
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \left(a+b\operatorname{sech}^2(c+dx)\right)^3 \sinh^2(c+dx)\,dx = \frac{\operatorname{Subst}\left(\int \frac{x^2\left(a+b-bx^2\right)^3}{\left(1-x^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)\left(a+b-b\tanh^2(c+dx)\right)^3}{2d} - \frac{\operatorname{Subst}\left(\int \ldots\right.}{\ldots}$$

$$= -\frac{7b\tanh(c+dx)\left(a+b-b\tanh^2(c+dx)\right)^2}{10d} + \frac{\cosh(c+dx)\sinh(\ldots}{\ldots}$$

$$= -\frac{(33a-2b)b\tanh(c+dx)\left(a+b-b\tanh^2(c+dx)\right)}{30d} - \frac{7b\tanh(c+\ldots}{\ldots}$$

$$= -\frac{b\left(81a^2-28ab-4b^2\right)\tanh(c+dx)}{30d} - \frac{(33a-2b)b\tanh(c+dx)\left(\ldots\right.}{30\ldots}$$

$$= -\frac{1}{2}a^2(a-6b)x - \frac{b\left(81a^2-28ab-4b^2\right)\tanh(c+dx)}{30d} - \frac{(33a-2b)b\ldots}{\ldots}$$

Mathematica [B] time = 1.92, size = 480, normalized size = 4.29

$$\frac{\operatorname{sech}(c)\operatorname{sech}^5(c+dx)\left(75a^3\sinh(2c+dx)+135a^3\sinh(2c+3dx)+135a^3\sinh(4c+3dx)+75a^3\sinh(4c+5dx)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^3*Sinh[c + d*x]^2,x]
```

```
[Out] (Sech[c]*Sech[c + d*x]^5*(-600*a^2*(a - 6*b)*d*x*Cosh[d*x] - 600*a^2*(a - 6
*b)*d*x*Cosh[2*c + d*x] - 300*a^3*d*x*Cosh[2*c + 3*d*x] + 1800*a^2*b*d*x*Co
sh[2*c + 3*d*x] - 300*a^3*d*x*Cosh[4*c + 3*d*x] + 1800*a^2*b*d*x*Cosh[4*c +
 3*d*x] - 60*a^3*d*x*Cosh[4*c + 5*d*x] + 360*a^2*b*d*x*Cosh[4*c + 5*d*x] -
60*a^3*d*x*Cosh[6*c + 5*d*x] + 360*a^2*b*d*x*Cosh[6*c + 5*d*x] + 75*a^3*Sin
h[d*x] - 4320*a^2*b*Sinh[d*x] + 960*a*b^2*Sinh[d*x] - 160*b^3*Sinh[d*x] + 7
5*a^3*Sinh[2*c + d*x] + 2880*a^2*b*Sinh[2*c + d*x] - 1440*a*b^2*Sinh[2*c +
d*x] - 480*b^3*Sinh[2*c + d*x] + 135*a^3*Sinh[2*c + 3*d*x] - 2880*a^2*b*Sin
h[2*c + 3*d*x] + 480*a*b^2*Sinh[2*c + 3*d*x] + 160*b^3*Sinh[2*c + 3*d*x] +
135*a^3*Sinh[4*c + 3*d*x] + 720*a^2*b*Sinh[4*c + 3*d*x] - 720*a*b^2*Sinh[4*
c + 3*d*x] + 75*a^3*Sinh[4*c + 5*d*x] - 720*a^2*b*Sinh[4*c + 5*d*x] + 240*a
*b^2*Sinh[4*c + 5*d*x] + 32*b^3*Sinh[4*c + 5*d*x] + 75*a^3*Sinh[6*c + 5*d*x
] + 15*a^3*Sinh[6*c + 7*d*x] + 15*a^3*Sinh[8*c + 7*d*x]))/(3840*d)
```

fricas [B] time = 0.43, size = 595, normalized size = 5.31

$$\frac{15\,a^3\sinh\left(dx+c\right)^7+4\left(90\,a^2b-30\,ab^2-4\,b^3-15\left(a^3-6\,a^2b\right)dx\right)\cosh\left(dx+c\right)^5+20\left(90\,a^2b-30\,ab^2-4\,b^3-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3*sinh(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/120*(15*a^3*sinh(d*x + c)^7 + 4*(90*a^2*b - 30*a*b^2 - 4*b^3 - 15*(a^3 -
6*a^2*b)*d*x)*cosh(d*x + c)^5 + 20*(90*a^2*b - 30*a*b^2 - 4*b^3 - 15*(a^3 -
 6*a^2*b)*d*x)*cosh(d*x + c)*sinh(d*x + c)^4 + (315*a^3*cosh(d*x + c)^2 + 7
5*a^3 - 360*a^2*b + 120*a*b^2 + 16*b^3)*sinh(d*x + c)^5 + 20*(90*a^2*b - 30
*a*b^2 - 4*b^3 - 15*(a^3 - 6*a^2*b)*d*x)*cosh(d*x + c)^3 + 5*(105*a^3*cosh(
d*x + c)^4 + 27*a^3 - 216*a^2*b - 24*a*b^2 + 16*b^3 + 2*(75*a^3 - 360*a^2*b
 + 120*a*b^2 + 16*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 20*(2*(90*a^2*b -
 30*a*b^2 - 4*b^3 - 15*(a^3 - 6*a^2*b)*d*x)*cosh(d*x + c)^3 + 3*(90*a^2*b -
 30*a*b^2 - 4*b^3 - 15*(a^3 - 6*a^2*b)*d*x)*cosh(d*x + c))*sinh(d*x + c)^2
+ 40*(90*a^2*b - 30*a*b^2 - 4*b^3 - 15*(a^3 - 6*a^2*b)*d*x)*cosh(d*x + c) +
 5*(21*a^3*cosh(d*x + c)^6 + (75*a^3 - 360*a^2*b + 120*a*b^2 + 16*b^3)*cosh
(d*x + c)^4 + 15*a^3 - 144*a^2*b - 48*a*b^2 - 64*b^3 + 3*(27*a^3 - 216*a^2*
b - 24*a*b^2 + 16*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^5 +
```

```
 5*d*cosh(d*x + c)*sinh(d*x + c)^4 + 5*d*cosh(d*x + c)^3 + 5*(2*d*cosh(d*x
+ c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 + 10*d*cosh(d*x + c))
```

giac [B] time = 0.20, size = 278, normalized size = 2.48

$$\frac{15\,a^3e^{(2\,dx+2\,c)} - 60\left(a^3 - 6\,a^2b\right)(dx+c) + 15\left(2\,a^3e^{(2\,dx+2\,c)} - 12\,a^2be^{(2\,dx+2\,c)} - a^3\right)e^{(-2\,dx-2\,c)} + \frac{16\left(45\,a^2be^{(8\,dx+8\,c)} - 45\,\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3*sinh(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/120*(15*a^3*e^(2*d*x + 2*c) - 60*(a^3 - 6*a^2*b)*(d*x + c) + 15*(2*a^3*e^
(2*d*x + 2*c) - 12*a^2*b*e^(2*d*x + 2*c) - a^3)*e^(-2*d*x - 2*c) + 16*(45*a
^2*b*e^(8*d*x + 8*c) - 45*a*b^2*e^(8*d*x + 8*c) + 180*a^2*b*e^(6*d*x + 6*c)
 - 90*a*b^2*e^(6*d*x + 6*c) - 30*b^3*e^(6*d*x + 6*c) + 270*a^2*b*e^(4*d*x +
 4*c) - 60*a*b^2*e^(4*d*x + 4*c) + 10*b^3*e^(4*d*x + 4*c) + 180*a^2*b*e^(2*
d*x + 2*c) - 30*a*b^2*e^(2*d*x + 2*c) - 10*b^3*e^(2*d*x + 2*c) + 45*a^2*b -
 15*a*b^2 - 2*b^3)/(e^(2*d*x + 2*c) + 1)^5)/d
```

maple [A] time = 0.42, size = 145, normalized size = 1.29

$$\frac{a^3\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2} - \frac{dx}{2} - \frac{c}{2}\right) + 3a^2b\left(dx+c-\tanh\left(dx+c\right)\right) + 3a\,b^2\left(-\frac{\sinh(dx+c)}{2\cosh(dx+c)^3} + \frac{\left(\frac{2}{3}+\frac{\mathrm{sech}(dx+c)^2}{3}\right)\tanh(dx+c)}{2}\right)\cdots}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)^3*sinh(d*x+c)^2,x)
```

```
[Out] 1/d*(a^3*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c)+3*a^2*b*(d*x+c-tanh(d*
x+c))+3*a*b^2*(-1/2*sinh(d*x+c)/cosh(d*x+c)^3+1/2*(2/3+1/3*sech(d*x+c)^2)*t
anh(d*x+c))+b^3*(-1/4*sinh(d*x+c)/cosh(d*x+c)^5+1/4*(8/15+1/5*sech(d*x+c)^4
+4/15*sech(d*x+c)^2)*tanh(d*x+c)))
```

maxima [B] time = 0.34, size = 443, normalized size = 3.96

$$-\frac{1}{8}\,a^3\left(4\,x - \frac{e^{(2\,dx+2\,c)}}{d} + \frac{e^{(-2\,dx-2\,c)}}{d}\right) + 3\,a^2b\left(x + \frac{c}{d} - \frac{2}{d\left(e^{(-2\,dx-2\,c)}+1\right)}\right) + \frac{4}{15}\,b^3\left(\frac{5\cdots}{d\left(5\,e^{(-2\,dx-2\,c)} + 10\,e^{(-4\,dx-4\,c)} + 10\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3*sinh(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/8*a^3*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) + 3*a^2*b*(x + c/d
- 2/(d*(e^(-2*d*x - 2*c) + 1))) + 4/15*b^3*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*
d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c)
 + e^(-10*d*x - 10*c) + 1)) - 5*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 1
0*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x
- 10*c) + 1)) + 15*e^(-6*d*x - 6*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x -
 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))
 + 1/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5
*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) + 2*a*b^2*(3*e^(-4*d*x - 4*c)
/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 1/(
d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)))
```

mupad [B] time = 0.23, size = 592, normalized size = 5.29

$$\frac{\frac{2\left(9a^2b+3ab^2+4b^3\right)}{15d}-\frac{6e^{4c+4dx}\left(ab^2-a^2b\right)}{5d}+\frac{4e^{2c+2dx}\left(3a^2b-b^3\right)}{5d}}{3e^{2c+2dx}+3e^{4c+4dx}+e^{6c+6dx}+1}+\frac{\frac{2\left(3a^2b-b^3\right)}{5d}-\frac{6e^{6c+6dx}\left(ab^2-a^2b\right)}{5d}+\frac{2e^{2c+2dx}\left(9a^2b+3ab^2+4b^3\right)}{5d}}{4e^{2c+2dx}+6e^{4c+4dx}+4e^{6c+6dx}+e^{8c+8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^2*(a + b/cosh(c + d*x)^2)^3,x)
```

```
[Out] ((2*(3*a*b^2 + 9*a^2*b + 4*b^3))/(15*d) - (6*exp(4*c + 4*d*x)*(a*b^2 - a^2*
b))/(5*d) + (4*exp(2*c + 2*d*x)*(3*a^2*b - b^3))/(5*d))/(3*exp(2*c + 2*d*x)
 + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) + ((2*(3*a^2*b - b^3))/(5*d)
- (6*exp(6*c + 6*d*x)*(a*b^2 - a^2*b))/(5*d) + (2*exp(2*c + 2*d*x)*(3*a*b^2
 + 9*a^2*b + 4*b^3))/(5*d) + (6*exp(4*c + 4*d*x)*(3*a^2*b - b^3))/(5*d))/(4
*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d
*x) + 1) + ((2*(3*a^2*b - b^3))/(5*d) - (6*exp(2*c + 2*d*x)*(a*b^2 - a^2*b)
)/(5*d))/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) + ((4*exp(4*c + 4*d*x)
*(3*a*b^2 + 9*a^2*b + 4*b^3))/(5*d) - (6*exp(8*c + 8*d*x)*(a*b^2 - a^2*b))/
(5*d) - (6*(a*b^2 - a^2*b))/(5*d) + (8*exp(2*c + 2*d*x)*(3*a^2*b - b^3))/(5
*d) + (8*exp(6*c + 6*d*x)*(3*a^2*b - b^3))/(5*d))/(5*exp(2*c + 2*d*x) + 10*
exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10
*d*x) + 1) - (6*(a*b^2 - a^2*b))/(5*d*(exp(2*c + 2*d*x) + 1)) - (a^2*x*(a -
 6*b))/2 - (a^3*exp(- 2*c - 2*d*x))/(8*d) + (a^3*exp(2*c + 2*d*x))/(8*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)**3*sinh(d*x+c)**2,x)
```

```
[Out] Timed out
```

3.20 $\int \left(a+b\,\mathbf{sech}^2(c+dx)\right)^3 \sinh(c+dx)\,dx$

Optimal. Leaf size=64

$$\frac{a^3\cosh(c+dx)}{d}-\frac{3a^2b\,\mathrm{sech}(c+dx)}{d}-\frac{ab^2\mathrm{sech}^3(c+dx)}{d}-\frac{b^3\mathrm{sech}^5(c+dx)}{5d}$$

```
[Out] a^3*cosh(d*x+c)/d-3*a^2*b*sech(d*x+c)/d-a*b^2*sech(d*x+c)^3/d-1/5*b^3*sech(
d*x+c)^5/d
```

Rubi **[A]** time = 0.06, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4133, 270}

$$-\frac{3a^2b\,\mathrm{sech}(c+dx)}{d}+\frac{a^3\cosh(c+dx)}{d}-\frac{ab^2\mathrm{sech}^3(c+dx)}{d}-\frac{b^3\mathrm{sech}^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^3*Sinh[c + d*x],x]
```

```
[Out] (a^3*Cosh[c + d*x])/d - (3*a^2*b*Sech[c + d*x])/d - (a*b^2*Sech[c + d*x]^3)
/d - (b^3*Sech[c + d*x]^5)/(5*d)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \left(a+b\operatorname{sech}^2(c+dx)\right)^3\sinh(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{\left(b+ax^2\right)^3}{x^6}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(a^3+\frac{b^3}{x^6}+\frac{3ab^2}{x^4}+\frac{3a^2b}{x^2}\right)dx, x, \cosh(c+dx)\right)}{d}$$
$$= \frac{a^3\cosh(c+dx)}{d} - \frac{3a^2b\operatorname{sech}(c+dx)}{d} - \frac{ab^2\operatorname{sech}^3(c+dx)}{d} - \frac{b^3\operatorname{sech}^5(c}{5d}$$

Mathematica [A] time = 0.26, size = 93, normalized size = 1.45

$$\frac{8\operatorname{sech}^5(c+dx)\left(a\cosh^2(c+dx)+b\right)^3\left(5a^3\cosh^6(c+dx)-15a^2b\cosh^4(c+dx)-5ab^2\cosh^2(c+dx)-b^3\right)}{5d(a\cosh(2(c+dx))+a+2b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^3*Sinh[c + d*x],x]
```

```
[Out] (8*(b + a*Cosh[c + d*x]^2)^3*(-b^3 - 5*a*b^2*Cosh[c + d*x]^2 - 15*a^2*b*Cos
h[c + d*x]^4 + 5*a^3*Cosh[c + d*x]^6)*Sech[c + d*x]^5)/(5*d*(a + 2*b + a*Co
sh[2*(c + d*x)])^3)
```

fricas [B] time = 0.39, size = 276, normalized size = 4.31

$$\frac{5\,a^3\cosh\left(dx+c\right)^6+5\,a^3\sinh\left(dx+c\right)^6+30\left(a^3-2\,a^2b\right)\cosh\left(dx+c\right)^4+15\left(5\,a^3\cosh\left(dx+c\right)^2+2\,a^3-4\,a^2b\right)}{10\left(d\cosh\left(dx+c\right)^5+5\,d\cosh\left(dx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3*sinh(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/10*(5*a^3*cosh(d*x + c)^6 + 5*a^3*sinh(d*x + c)^6 + 30*(a^3 - 2*a^2*b)*co
sh(d*x + c)^4 + 15*(5*a^3*cosh(d*x + c)^2 + 2*a^3 - 4*a^2*b)*sinh(d*x + c)^
4 + 50*a^3 - 180*a^2*b - 80*a*b^2 - 32*b^3 + 5*(15*a^3 - 48*a^2*b - 16*a*b^
2)*cosh(d*x + c)^2 + 5*(15*a^3*cosh(d*x + c)^4 + 15*a^3 - 48*a^2*b - 16*a*b
^2 + 36*(a^3 - 2*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^2)/(d*cosh(d*x + c)^
5 + 5*d*cosh(d*x + c)*sinh(d*x + c)^4 + 5*d*cosh(d*x + c)^3 + 5*(2*d*cosh(d
*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 + 10*d*cosh(d*x + c))
```

giac [A] time = 0.19, size = 101, normalized size = 1.58

$$\frac{5\,a^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)-\frac{4\left(15\,a^2b\left(e^{(dx+c)}+e^{(-dx-c)}\right)^4+20\,ab^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2+16\,b^3\right)}{\left(e^{(dx+c)}+e^{(-dx-c)}\right)^5}}{10\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3*sinh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/10*(5*a^3*(e^(d*x + c) + e^(-d*x - c)) - 4*(15*a^2*b*(e^(d*x + c) + e^(-d
*x - c))^4 + 20*a*b^2*(e^(d*x + c) + e^(-d*x - c))^2 + 16*b^3)/(e^(d*x + c)
 + e^(-d*x - c))^5)/d
```

maple [A] time = 0.13, size = 58, normalized size = 0.91

$$-\frac{\frac{b^3\mathrm{sech}(dx+c)^5}{5}+a\,b^2\mathrm{sech}\left(dx+c\right)^3+3a^2b\,\mathrm{sech}\left(dx+c\right)-\frac{a^3}{\mathrm{sech}(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)^3*sinh(d*x+c),x)
```

```
[Out] -1/d*(1/5*b^3*sech(d*x+c)^5+a*b^2*sech(d*x+c)^3+3*a^2*b*sech(d*x+c)-a^3/sec
h(d*x+c))
```

maxima [A] time = 0.32, size = 94, normalized size = 1.47

$$\frac{a^3\cosh\left(dx+c\right)}{d}-\frac{6\,a^2b}{d\left(e^{(dx+c)}+e^{(-dx-c)}\right)}-\frac{8\,ab^2}{d\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3}-\frac{32\,b^3}{5\,d\left(e^{(dx+c)}+e^{(-dx-c)}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3*sinh(d*x+c),x, algorithm="maxima")
```

```
[Out] a^3*cosh(d*x + c)/d - 6*a^2*b/(d*(e^(d*x + c) + e^(-d*x - c))) - 8*a*b^2/(d
*(e^(d*x + c) + e^(-d*x - c))^3) - 32/5*b^3/(d*(e^(d*x + c) + e^(-d*x - c))
^5)
```

mupad [B] time = 1.50, size = 288, normalized size = 4.50

$$\frac{a^3\,\mathrm{e}^{c+d\,x}}{2\,d}+\frac{a^3\,\mathrm{e}^{-c-d\,x}}{2\,d}+\frac{64\,b^3\,\mathrm{e}^{c+d\,x}}{5\,d\,\left(4\,\mathrm{e}^{2\,c+2\,d\,x}+6\,\mathrm{e}^{4\,c+4\,d\,x}+4\,\mathrm{e}^{6\,c+6\,d\,x}+\mathrm{e}^{8\,c+8\,d\,x}+1\right)}+\frac{8\,\mathrm{e}^{c+d\,x}\,\left(5\,a\,b^2-4\,b^3\right)}{5\,d\,\left(3\,\mathrm{e}^{2\,c+2\,d\,x}+3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)*(a + b/cosh(c + d*x)^2)^3,x)
```

```
[Out] (a^3*exp(c + d*x))/(2*d) + (a^3*exp(- c - d*x))/(2*d) + (64*b^3*exp(c + d*x
))/(5*d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp
(8*c + 8*d*x) + 1)) + (8*exp(c + d*x)*(5*a*b^2 - 4*b^3))/(5*d*(3*exp(2*c +
2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) - (32*b^3*exp(c + d*x)
```

```
)/(5*d*(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*
exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1)) - (6*a^2*b*exp(c + d*x))/(d*(ex
p(2*c + 2*d*x) + 1)) - (8*a*b^2*exp(c + d*x))/(d*(2*exp(2*c + 2*d*x) + exp(
4*c + 4*d*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)**3*sinh(d*x+c),x)
```

```
[Out] Timed out
```

3.21 $\int \operatorname{csch}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=83

$$\frac{b\left(3a^2+3ab+b^2\right)\operatorname{sech}(c+dx)}{d}+\frac{b^2(3a+b)\operatorname{sech}^3(c+dx)}{3d}-\frac{(a+b)^3\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b^3\operatorname{sech}^5(c+dx)}{5d}$$

```
[Out] -(a+b)^3*arctanh(cosh(d*x+c))/d+b*(3*a^2+3*a*b+b^2)*sech(d*x+c)/d+1/3*b^2*(
3*a+b)*sech(d*x+c)^3/d+1/5*b^3*sech(d*x+c)^5/d
```

Rubi [A] time = 0.10, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4133, 461, 207}

$$\frac{b\left(3a^2+3ab+b^2\right)\operatorname{sech}(c+dx)}{d}+\frac{b^2(3a+b)\operatorname{sech}^3(c+dx)}{3d}-\frac{(a+b)^3\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b^3\operatorname{sech}^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] -(((a + b)^3*ArcTanh[Cosh[c + d*x]])/d) + (b*(3*a^2 + 3*a*b + b^2)*Sech[c +
 d*x])/d + (b^2*(3*a + b)*Sech[c + d*x]^3)/(3*d) + (b^3*Sech[c + d*x]^5)/(5
*d)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
```

```
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \operatorname{csch}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx = -\frac{\operatorname{Subst}\left(\int \frac{\left(b+ax^2\right)^3}{x^6\left(1-x^2\right)}\,dx,x,\cosh(c+dx)\right)}{d}$$

$$= -\frac{\operatorname{Subst}\left(\int \left(\frac{b^3}{x^6}+\frac{b^2(3a+b)}{x^4}+\frac{b\left(3a^2+3ab+b^2\right)}{x^2}-\frac{(a+b)^3}{-1+x^2}\right)dx,x,\cosh(c+dx)\right.}{d}$$

$$= \frac{b\left(3a^2+3ab+b^2\right)\operatorname{sech}(c+dx)}{d}+\frac{b^2(3a+b)\operatorname{sech}^3(c+dx)}{3d}+\frac{b^3\operatorname{sech}}{}$$

$$= -\frac{(a+b)^3\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b\left(3a^2+3ab+b^2\right)\operatorname{sech}(c+dx)}{d}+$$

Mathematica [A] time = 1.28, size = 134, normalized size = 1.61

$$-\frac{8\operatorname{sech}^5(c+dx)\left(a\cosh^2(c+dx)+b\right)^3\left(-15b\left(3a^2+3ab+b^2\right)\cosh^4(c+dx)-5b^2(3a+b)\cosh^2(c+dx)+15(a\right.}{15d(a\cosh(2(c+dx))+a+2b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (-8*(b + a*Cosh[c + d*x]^2)^3*(-3*b^3 - 5*b^2*(3*a + b)*Cosh[c + d*x]^2 - 1
5*b*(3*a^2 + 3*a*b + b^2)*Cosh[c + d*x]^4 + 15*(a + b)^3*Cosh[c + d*x]^5*(L
og[Cosh[(c + d*x)/2]] - Log[Sinh[(c + d*x)/2]]))*Sech[c + d*x]^5)/(15*d*(a
+ 2*b + a*Cosh[2*(c + d*x)])^3)
```

fricas [B] time = 0.44, size = 3443, normalized size = 41.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/15*(30*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^9 + 270*(3*a^2*b + 3*a*b^2
 + b^3)*cosh(d*x + c)*sinh(d*x + c)^8 + 30*(3*a^2*b + 3*a*b^2 + b^3)*sinh(d
*x + c)^9 + 40*(9*a^2*b + 12*a*b^2 + 4*b^3)*cosh(d*x + c)^7 + 40*(9*a^2*b +
```

12*a*b^2 + 4*b^3 + 27*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^7 + 280*(9*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + (9*a^2*b + 12*a*b^2 + 4*b^3)*cosh(d*x + c))*sinh(d*x + c)^6 + 4*(135*a^2*b + 195*a*b^2 + 89*b^3)*cosh(d*x + c)^5 + 4*(945*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 135*a^2*b + 195*a*b^2 + 89*b^3 + 210*(9*a^2*b + 12*a*b^2 + 4*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 20*(189*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^5 + 70*(9*a^2*b + 12*a*b^2 + 4*b^3)*cosh(d*x + c)^3 + (135*a^2*b + 195*a*b^2 + 89*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 40*(9*a^2*b + 12*a*b^2 + 4*b^3)*cosh(d*x + c)^3 + 40*(63*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^6 + 35*(9*a^2*b + 12*a*b^2 + 4*b^3)*cosh(d*x + c)^4 + 9*a^2*b + 12*a*b^2 + 4*b^3 + (135*a^2*b + 195*a*b^2 + 89*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 40*(27*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^7 + 21*(9*a^2*b + 12*a*b^2 + 4*b^3)*cosh(d*x + c)^5 + (135*a^2*b + 195*a*b^2 + 89*b^3)*cosh(d*x + c)^3 + 3*(9*a^2*b + 12*a*b^2 + 4*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 30*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c) - 15*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^10 + 10*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^9 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sinh(d*x + c)^10 + 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^8 + 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 9*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 40*(3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + 10*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^6 + 10*(21*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 14*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(63*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^5 + 70*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 10*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 10*(21*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^6 + 35*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 40*(3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^7 + 7*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^5 + 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2 + 5*(9*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^8 + 28*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^6 + 30*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 12*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 10*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^9 + 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^7 + 6*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^5 + 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) + 15*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^10 + 10*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^9 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sinh(d*x + c)^10 + 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^8 + 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 9*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)

```
^2)*sinh(d*x + c)^8 + 40*(3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3
 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + 10*(a^3
 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^6 + 10*(21*(a^3 + 3*a^2*b + 3*a*b
^2 + b^3)*cosh(d*x + c)^4 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 14*(a^3 + 3*a^2
*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(63*(a^3 + 3*a^2*b
 + 3*a*b^2 + b^3)*cosh(d*x + c)^5 + 70*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh
(d*x + c)^3 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x +
c)^5 + 10*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 10*(21*(a^3 + 3
*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^6 + 35*(a^3 + 3*a^2*b + 3*a*b^2 + b^3
)*cosh(d*x + c)^4 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 15*(a^3 + 3*a^2*b + 3*a
*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 40*(3*(a^3 + 3*a^2*b + 3*a*b
^2 + b^3)*cosh(d*x + c)^7 + 7*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)
^5 + 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + (a^3 + 3*a^2*b + 3
*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^
3 + 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2 + 5*(9*(a^3 + 3*a^2*b
 + 3*a*b^2 + b^3)*cosh(d*x + c)^8 + 28*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh
(d*x + c)^6 + 30*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + a^3 + 3*
a^2*b + 3*a*b^2 + b^3 + 12*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)
*sinh(d*x + c)^2 + 10*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^9 + 4*
(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^7 + 6*(a^3 + 3*a^2*b + 3*a*b^
2 + b^3)*cosh(d*x + c)^5 + 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^
3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(
d*x + c) + sinh(d*x + c) - 1) + 10*(27*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x +
 c)^8 + 28*(9*a^2*b + 12*a*b^2 + 4*b^3)*cosh(d*x + c)^6 + 2*(135*a^2*b + 19
5*a*b^2 + 89*b^3)*cosh(d*x + c)^4 + 9*a^2*b + 9*a*b^2 + 3*b^3 + 12*(9*a^2*b
 + 12*a*b^2 + 4*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^10 +
10*d*cosh(d*x + c)*sinh(d*x + c)^9 + d*sinh(d*x + c)^10 + 5*d*cosh(d*x + c)
^8 + 5*(9*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^8 + 40*(3*d*cosh(d*x + c)^3
+ d*cosh(d*x + c))*sinh(d*x + c)^7 + 10*d*cosh(d*x + c)^6 + 10*(21*d*cosh(d
*x + c)^4 + 14*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^6 + 4*(63*d*cosh(d*x +
c)^5 + 70*d*cosh(d*x + c)^3 + 15*d*cosh(d*x + c))*sinh(d*x + c)^5 + 10*d*co
sh(d*x + c)^4 + 10*(21*d*cosh(d*x + c)^6 + 35*d*cosh(d*x + c)^4 + 15*d*cosh
(d*x + c)^2 + d)*sinh(d*x + c)^4 + 40*(3*d*cosh(d*x + c)^7 + 7*d*cosh(d*x +
 c)^5 + 5*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^3 + 5*d*cosh(d
*x + c)^2 + 5*(9*d*cosh(d*x + c)^8 + 28*d*cosh(d*x + c)^6 + 30*d*cosh(d*x +
 c)^4 + 12*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 10*(d*cosh(d*x + c)^9 +
 4*d*cosh(d*x + c)^7 + 6*d*cosh(d*x + c)^5 + 4*d*cosh(d*x + c)^3 + d*cosh(d
*x + c))*sinh(d*x + c) + d)
```

giac [B] time = 0.17, size = 228, normalized size = 2.75

$$-\frac{15\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)-15\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\log\left(e^{(dx+c)}+e^{(-dx-c)}-2\right)}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/30*(15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*log(e^(d*x + c) + e^(-d*x - c) +
2) - 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*log(e^(d*x + c) + e^(-d*x - c) - 2)
 - 4*(45*a^2*b*(e^(d*x + c) + e^(-d*x - c))^4 + 45*a*b^2*(e^(d*x + c) + e^(
-d*x - c))^4 + 15*b^3*(e^(d*x + c) + e^(-d*x - c))^4 + 60*a*b^2*(e^(d*x + c
) + e^(-d*x - c))^2 + 20*b^3*(e^(d*x + c) + e^(-d*x - c))^2 + 48*b^3)/(e^(d
*x + c) + e^(-d*x - c))^5)/d
```

maple [A] time = 0.24, size = 118, normalized size = 1.42

$$\frac{-2a^3\operatorname{arctanh}\left(e^{dx+c}\right)+3a^2b\left(\frac{1}{\cosh(dx+c)}-2\operatorname{arctanh}\left(e^{dx+c}\right)\right)+3a\,b^2\left(\frac{1}{3\cosh(dx+c)^3}+\frac{1}{\cosh(dx+c)}-2\operatorname{arctanh}\left(e^{dx+c}\right.\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)*(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(-2*a^3*arctanh(exp(d*x+c))+3*a^2*b*(1/cosh(d*x+c)-2*arctanh(exp(d*x+c)
))+3*a*b^2*(1/3/cosh(d*x+c)^3+1/cosh(d*x+c)-2*arctanh(exp(d*x+c)))+b^3*(1/5
/cosh(d*x+c)^5+1/3/cosh(d*x+c)^3+1/cosh(d*x+c)-2*arctanh(exp(d*x+c))))
```

maxima [B] time = 0.33, size = 358, normalized size = 4.31

$$-\frac{1}{15}b^3\left(\frac{15\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{15\log\left(e^{(-dx-c)}-1\right)}{d}-\frac{2\left(15e^{(-dx-c)}+80e^{(-3dx-3c)}+178e^{(-5dx-5c)}+80e^{(-7dx-}\right.}{d\left(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/15*b^3*(15*log(e^(-d*x - c) + 1)/d - 15*log(e^(-d*x - c) - 1)/d - 2*(15*
e^(-d*x - c) + 80*e^(-3*d*x - 3*c) + 178*e^(-5*d*x - 5*c) + 80*e^(-7*d*x -
7*c) + 15*e^(-9*d*x - 9*c))/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) +
10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) - a*b^
2*(3*log(e^(-d*x - c) + 1)/d - 3*log(e^(-d*x - c) - 1)/d - 2*(3*e^(-d*x - c
) + 10*e^(-3*d*x - 3*c) + 3*e^(-5*d*x - 5*c))/(d*(3*e^(-2*d*x - 2*c) + 3*e^
(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) - 3*a^2*b*(log(e^(-d*x - c) + 1)/d
 - log(e^(-d*x - c) - 1)/d - 2*e^(-d*x - c)/(d*(e^(-2*d*x - 2*c) + 1))) + a
^3*log(tanh(1/2*d*x + 1/2*c))/d
```

mupad [B] time = 1.53, size = 434, normalized size = 5.23

$$\frac{2\,\mathrm{e}^{c+d\,x}\,\left(3\,a^2\,b+3\,a\,b^2+b^3\right)}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)}-\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(a^3\,\sqrt{-d^2}+b^3\,\sqrt{-d^2}+3\,a\,b^2\,\sqrt{-d^2}+3\,a^2\,b\,\sqrt{-d^2}\right)}{d\,\sqrt{a^6+6\,a^5\,b+15\,a^4\,b^2+20\,a^3\,b^3+15\,a^2\,b^4+6\,a\,b^5+b^6}}\right)\,\sqrt{a^6+6\,a^5\,b+15\,a^4\,b^2+20\,a^3}}{\sqrt{-d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cosh(c + d*x)^2)^3/sinh(c + d*x),x)
```

```
[Out] (2*exp(c + d*x)*(3*a*b^2 + 3*a^2*b + b^3))/(d*(exp(2*c + 2*d*x) + 1)) - (2*
atan((exp(d*x)*exp(c)*(a^3*(-d^2)^(1/2) + b^3*(-d^2)^(1/2) + 3*a*b^2*(-d^2)
^(1/2) + 3*a^2*b*(-d^2)^(1/2)))/(d*(6*a*b^5 + 6*a^5*b + a^6 + b^6 + 15*a^2*
b^4 + 20*a^3*b^3 + 15*a^4*b^2)^(1/2)))*(6*a*b^5 + 6*a^5*b + a^6 + b^6 + 15*
a^2*b^4 + 20*a^3*b^3 + 15*a^4*b^2)^(1/2))/(-d^2)^(1/2) - (64*b^3*exp(c + d*
x))/(5*d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + ex
p(8*c + 8*d*x) + 1)) - (8*exp(c + d*x)*(15*a*b^2 - 7*b^3))/(15*d*(3*exp(2*c
 + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) + (32*b^3*exp(c + d
*x))/(5*d*(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) +
 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1)) + (8*exp(c + d*x)*(3*a*b^2 +
 b^3))/(3*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\,\mathrm{sech}^2\left(c + dx\right)\right)^3 \mathrm{csch}\left(c + dx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**3*csch(c + d*x), x)
```

3.22 $\int \operatorname{csch}^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=70

$$\frac{b^2(a+b)\tanh^3(c+dx)}{d}-\frac{3b(a+b)^2\tanh(c+dx)}{d}-\frac{(a+b)^3\coth(c+dx)}{d}-\frac{b^3\tanh^5(c+dx)}{5d}$$

```
[Out] -(a+b)^3*coth(d*x+c)/d-3*b*(a+b)^2*tanh(d*x+c)/d+b^2*(a+b)*tanh(d*x+c)^3/d-
1/5*b^3*tanh(d*x+c)^5/d
```

Rubi [A] time = 0.07, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4132, 270}

$$\frac{b^2(a+b)\tanh^3(c+dx)}{d}-\frac{3b(a+b)^2\tanh(c+dx)}{d}-\frac{(a+b)^3\coth(c+dx)}{d}-\frac{b^3\tanh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] -(((a + b)^3*Coth[c + d*x])/d) - (3*b*(a + b)^2*Tanh[c + d*x])/d + (b^2*(a
+ b)*Tanh[c + d*x]^3)/d - (b^3*Tanh[c + d*x]^5)/(5*d)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \operatorname{csch}^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3 dx = \frac{\operatorname{Subst}\left(\int \frac{\left(a+b-bx^2\right)^3}{x^2} dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\operatorname{Subst}\left(\int \left(-3b(a+b)^2 + \frac{(a+b)^3}{x^2} + 3b^2(a+b)x^2 - b^3x^4\right) dx, x, \tanh(c\right.}{d}$$
$$= -\frac{(a+b)^3\coth(c+dx)}{d} - \frac{3b(a+b)^2\tanh(c+dx)}{d} + \frac{b^2(a+b)\tanh^3}{d}$$

Mathematica [B] time = 2.79, size = 380, normalized size = 5.43

$$-\frac{\operatorname{csch}(c)\operatorname{sech}(c)\coth(c+dx)\left(-25a^3\sinh(2(c+dx)) - 20a^3\sinh(4(c+dx)) - 5a^3\sinh(6(c+dx)) - 25a^3\sinh(2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] -1/40*(Coth[c + d*x]*Csch[c]*Sech[c]*(a + b*Sech[c + d*x]^2)^3*(10*a*(5*a^2
 + 12*a*b + 8*b^2)*Sinh[2*c] - 10*(5*a^3 + 18*a^2*b + 20*a*b^2 + 8*b^3)*Sin
h[2*d*x] - 25*a^3*Sinh[2*(c + d*x)] + 50*a*b^2*Sinh[2*(c + d*x)] + 30*b^3*S
inh[2*(c + d*x)] - 20*a^3*Sinh[4*(c + d*x)] + 40*a*b^2*Sinh[4*(c + d*x)] +
24*b^3*Sinh[4*(c + d*x)] - 5*a^3*Sinh[6*(c + d*x)] + 10*a*b^2*Sinh[6*(c + d
*x)] + 6*b^3*Sinh[6*(c + d*x)] - 25*a^3*Sinh[2*(c + 2*d*x)] - 120*a^2*b*Sin
h[2*(c + 2*d*x)] - 160*a*b^2*Sinh[2*(c + 2*d*x)] - 64*b^3*Sinh[2*(c + 2*d*x
)] + 25*a^3*Sinh[4*c + 2*d*x] + 30*a^2*b*Sinh[4*c + 2*d*x] + 5*a^3*Sinh[6*c
 + 4*d*x] - 5*a^3*Sinh[4*c + 6*d*x] - 30*a^2*b*Sinh[4*c + 6*d*x] - 40*a*b^2
*Sinh[4*c + 6*d*x] - 16*b^3*Sinh[4*c + 6*d*x]))/(d*(a + 2*b + a*Cosh[2*(c +
 d*x)])^3)
```

fricas [B] time = 0.47, size = 622, normalized size = 8.89

$$-\frac{4\left(\left(5a^3+15a^2b+20ab^2+8b^3\right)\cosh(dx+c)^5 + 5\left(5a^3+15a^2b+20ab^2+8b^3\right)\cosh(dx+c)\sinh(dx+c)^4\right.}{5\left(d\cosh(dx+c)^7 + 7d\cosh(dx+c)\sinh(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -4/5*((5*a^3 + 15*a^2*b + 20*a*b^2 + 8*b^3)*cosh(d*x + c)^5 + 5*(5*a^3 + 15
*a^2*b + 20*a*b^2 + 8*b^3)*cosh(d*x + c)*sinh(d*x + c)^4 - (15*a^2*b + 20*a
```

```
*b^2 + 8*b^3)*sinh(d*x + c)^5 + (25*a^3 + 75*a^2*b + 80*a*b^2 + 32*b^3)*cos
h(d*x + c)^3 - (45*a^2*b + 80*a*b^2 + 32*b^3 + 10*(15*a^2*b + 20*a*b^2 + 8*
b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + (10*(5*a^3 + 15*a^2*b + 20*a*b^2 +
8*b^3)*cosh(d*x + c)^3 + 3*(25*a^3 + 75*a^2*b + 80*a*b^2 + 32*b^3)*cosh(d*x
 + c))*sinh(d*x + c)^2 + 10*(5*a^3 + 15*a^2*b + 14*a*b^2 + 4*b^3)*cosh(d*x
+ c) - (5*(15*a^2*b + 20*a*b^2 + 8*b^3)*cosh(d*x + c)^4 + 30*a^2*b + 60*a*b
^2 + 40*b^3 + 3*(45*a^2*b + 80*a*b^2 + 32*b^3)*cosh(d*x + c)^2)*sinh(d*x +
c))/(d*cosh(d*x + c)^7 + 7*d*cosh(d*x + c)*sinh(d*x + c)^6 + d*sinh(d*x + c
)^7 + 3*d*cosh(d*x + c)^5 + (21*d*cosh(d*x + c)^2 + 5*d)*sinh(d*x + c)^5 +
5*(7*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^4 + d*cosh(d*x +
c)^3 + (35*d*cosh(d*x + c)^4 + 50*d*cosh(d*x + c)^2 + 9*d)*sinh(d*x + c)^3
+ 3*(7*d*cosh(d*x + c)^5 + 10*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x
 + c)^2 - 5*d*cosh(d*x + c) + (7*d*cosh(d*x + c)^6 + 25*d*cosh(d*x + c)^4 +
 27*d*cosh(d*x + c)^2 + 5*d)*sinh(d*x + c))
```

giac [B] time = 0.18, size = 249, normalized size = 3.56

$$-\frac{2\left(\frac{5\left(a^3+3a^2b+3ab^2+b^3\right)}{e^{(2dx+2c)}-1}-\frac{15a^2be^{(8dx+8c)}+15ab^2e^{(8dx+8c)}+5b^3e^{(8dx+8c)}+60a^2be^{(6dx+6c)}+90ab^2e^{(6dx+6c)}+30b^3e^{(6dx+6c)}+90a^2be^{(4dx+4c)}+\cdots}{\left(e^{(2dx+2c)}+\cdots\right.}\right.}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -2/5*(5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)/(e^(2*d*x + 2*c) - 1) - (15*a^2*b*e
^(8*d*x + 8*c) + 15*a*b^2*e^(8*d*x + 8*c) + 5*b^3*e^(8*d*x + 8*c) + 60*a^2*
b*e^(6*d*x + 6*c) + 90*a*b^2*e^(6*d*x + 6*c) + 30*b^3*e^(6*d*x + 6*c) + 90*
a^2*b*e^(4*d*x + 4*c) + 160*a*b^2*e^(4*d*x + 4*c) + 80*b^3*e^(4*d*x + 4*c)
+ 60*a^2*b*e^(2*d*x + 2*c) + 110*a*b^2*e^(2*d*x + 2*c) + 50*b^3*e^(2*d*x +
2*c) + 15*a^2*b + 25*a*b^2 + 11*b^3)/(e^(2*d*x + 2*c) + 1)^5)/d
```

maple [B] time = 0.59, size = 148, normalized size = 2.11

$$\frac{-a^3\coth(dx+c)+3a^2b\left(-\frac{1}{\sinh(dx+c)\cosh(dx+c)}-2\tanh(dx+c)\right)+3ab^2\left(-\frac{1}{\sinh(dx+c)\cosh(dx+c)^3}-4\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c\cdots}{3}\right.\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(-a^3*coth(d*x+c)+3*a^2*b*(-1/sinh(d*x+c)/cosh(d*x+c)-2*tanh(d*x+c))+3*
a*b^2*(-1/sinh(d*x+c)/cosh(d*x+c)^3-4*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c))+
b^3*(-1/sinh(d*x+c)/cosh(d*x+c)^5-6*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c
)^2)*tanh(d*x+c)))
```

maxima [B] time = 0.34, size = 358, normalized size = 5.11

$$-\frac{32}{5}\,b^3\left(\frac{4\,e^{(-2\,dx-2\,c)}}{d\left(4\,e^{(-2\,dx-2\,c)}+5\,e^{(-4\,dx-4\,c)}-5\,e^{(-8\,dx-8\,c)}-4\,e^{(-10\,dx-10\,c)}-e^{(-12\,dx-12\,c)}+1\right)}+\frac{}{d\left(4\,e^{(-2\,dx-2\,c)}+5\,e^{(-4\,dx-4\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -32/5*b^3*(4*e^(-2*d*x - 2*c)/(d*(4*e^(-2*d*x - 2*c) + 5*e^(-4*d*x - 4*c) -
 5*e^(-8*d*x - 8*c) - 4*e^(-10*d*x - 10*c) - e^(-12*d*x - 12*c) + 1)) + 5*e
^(-4*d*x - 4*c)/(d*(4*e^(-2*d*x - 2*c) + 5*e^(-4*d*x - 4*c) - 5*e^(-8*d*x -
 8*c) - 4*e^(-10*d*x - 10*c) - e^(-12*d*x - 12*c) + 1)) + 1/(d*(4*e^(-2*d*x
 - 2*c) + 5*e^(-4*d*x - 4*c) - 5*e^(-8*d*x - 8*c) - 4*e^(-10*d*x - 10*c) -
e^(-12*d*x - 12*c) + 1))) - 16*a*b^2*(2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x -
2*c) - 2*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) + 1)) + 1/(d*(2*e^(-2*d*x - 2*
c) - 2*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) + 1))) + 2*a^3/(d*(e^(-2*d*x - 2
*c) - 1)) + 12*a^2*b/(d*(e^(-4*d*x - 4*c) - 1))
```

mupad [B] time = 1.48, size = 644, normalized size = 9.20

$$\frac{\frac{2\left(3\,a^2\,b+6\,a\,b^2+2\,b^3\right)}{5\,d}+\frac{2\,e^{2\,c+2\,d\,x}\left(3\,a^2\,b+3\,a\,b^2+b^3\right)}{5\,d}}{2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1}+\frac{\frac{2\left(3\,a^2\,b+6\,a\,b^2+2\,b^3\right)}{5\,d}+\frac{2\,e^{6\,c+6\,d\,x}\left(3\,a^2\,b+3\,a\,b^2+b^3\right)}{5\,d}+\frac{6\,e^{2\,c+2\,d\,x}\left(3\,a^2\,b+7\,a\,b^2+5\,b^3\right)}{5\,d}+}{4\,e^{2\,c+2\,d\,x}+6\,e^{4\,c+4\,d\,x}+4\,e^{6\,c+6\,d\,x}+e^{8\,c+8\,d\,x}+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cosh(c + d*x)^2)^3/sinh(c + d*x)^2,x)
```

```
[Out] ((2*(6*a*b^2 + 3*a^2*b + 2*b^3))/(5*d) + (2*exp(2*c + 2*d*x)*(3*a*b^2 + 3*a
^2*b + b^3))/(5*d))/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) + ((2*(6*a*
b^2 + 3*a^2*b + 2*b^3))/(5*d) + (2*exp(6*c + 6*d*x)*(3*a*b^2 + 3*a^2*b + b^
3))/(5*d) + (6*exp(2*c + 2*d*x)*(7*a*b^2 + 3*a^2*b + 5*b^3))/(5*d) + (6*exp
(4*c + 4*d*x)*(6*a*b^2 + 3*a^2*b + 2*b^3))/(5*d))/(4*exp(2*c + 2*d*x) + 6*e
xp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) + ((2*(3*a*b^2
 + 3*a^2*b + b^3))/(5*d) + (2*exp(8*c + 8*d*x)*(3*a*b^2 + 3*a^2*b + b^3))/(
5*d) + (8*exp(2*c + 2*d*x)*(6*a*b^2 + 3*a^2*b + 2*b^3))/(5*d) + (12*exp(4*c
 + 4*d*x)*(7*a*b^2 + 3*a^2*b + 5*b^3))/(5*d) + (8*exp(6*c + 6*d*x)*(6*a*b^2
 + 3*a^2*b + 2*b^3))/(5*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*
exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1) + ((2*(7*a*
b^2 + 3*a^2*b + 5*b^3))/(5*d) + (2*exp(4*c + 4*d*x)*(3*a*b^2 + 3*a^2*b + b^
3))/(5*d) + (4*exp(2*c + 2*d*x)*(6*a*b^2 + 3*a^2*b + 2*b^3))/(5*d))/(3*exp(
2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) - (2*(3*a*b^2 + 3
*a^2*b + a^3 + b^3))/(d*(exp(2*c + 2*d*x) - 1)) + (2*(3*a*b^2 + 3*a^2*b + b
^3))/(5*d*(exp(2*c + 2*d*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\,\mathrm{sech}^2\left(c + dx\right)\right)^3 \mathrm{csch}^2\left(c + dx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2*(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**3*csch(c + d*x)**2, x)
```

3.23 $\int \mathrm{csch}^3(c+dx)\left(a+b\,\mathrm{sech}^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=144

$$-\frac{b\left(6a^2+15ab+7b^2\right)\mathrm{sech}^3(c+dx)}{6d}-\frac{b^2(5a+7b)\mathrm{sech}^5(c+dx)}{10d}-\frac{(a+b)^2(a+7b)\mathrm{sech}(c+dx)}{2d}+\frac{(a+b)^2(a+7b)}{}$$

```
[Out] 1/2*(a+b)^2*(a+7*b)*arctanh(cosh(d*x+c))/d-1/2*(a+b)^2*(a+7*b)*sech(d*x+c)/
d-1/6*b*(6*a^2+15*a*b+7*b^2)*sech(d*x+c)^3/d-1/10*b^2*(5*a+7*b)*sech(d*x+c)
^5/d-1/2*(a+b)*(b+a*cosh(d*x+c)^2)^2*csch(d*x+c)^2*sech(d*x+c)^5/d
```

Rubi [A] time = 0.18, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4133, 468, 570, 207}

$$-\frac{b\left(6a^2+15ab+7b^2\right)\mathrm{sech}^3(c+dx)}{6d}-\frac{b^2(5a+7b)\mathrm{sech}^5(c+dx)}{10d}-\frac{(a+b)^2(a+7b)\mathrm{sech}(c+dx)}{2d}+\frac{(a+b)^2(a+7b)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((a + b)^2*(a + 7*b)*ArcTanh[Cosh[c + d*x]])/(2*d) - ((a + b)^2*(a + 7*b)*S
ech[c + d*x])/(2*d) - (b*(6*a^2 + 15*a*b + 7*b^2)*Sech[c + d*x]^3)/(6*d) -
(b^2*(5*a + 7*b)*Sech[c + d*x]^5)/(10*d) - ((a + b)*(b + a*Cosh[c + d*x]^2)
^2*Csch[c + d*x]^2*Sech[c + d*x]^5)/(2*d)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 570

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \operatorname{csch}^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3 dx = \frac{\text{Subst}\left(\int \frac{\left(b+ax^2\right)^3}{x^6\left(1-x^2\right)^2}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{(a+b)\left(b+a\cosh^2(c+dx)\right)^2\operatorname{csch}^2(c+dx)\operatorname{sech}^5(c+dx)}{2d} + \frac{\text{Sub}}{}$$

$$= -\frac{(a+b)\left(b+a\cosh^2(c+dx)\right)^2\operatorname{csch}^2(c+dx)\operatorname{sech}^5(c+dx)}{2d} + \frac{\text{Sub}}{}$$

$$= -\frac{(a+b)^2(a+7b)\operatorname{sech}(c+dx)}{2d} - \frac{b\left(6a^2+15ab+7b^2\right)\operatorname{sech}^3(c+d}{6d}$$

$$= \frac{(a+b)^2(a+7b)\tanh^{-1}(\cosh(c+dx))}{2d} - \frac{(a+b)^2(a+7b)\operatorname{sech}(c+}{2d}$$

Mathematica [A] time = 3.89, size = 224, normalized size = 1.56

$$-\frac{\operatorname{sech}^6(c+dx)\left(a\cosh^2(c+dx)+b\right)^3\left(\frac{3}{4}\left(75a^3+195a^2b+165ab^2+77b^3\right)\sinh(4(c+dx))\operatorname{csch}^3(c+dx)+\operatorname{cot}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] -1/120*((b + a*Cosh[c + d*x]^2)^3*Sech[c + d*x]^6*((150*a^3 + 270*a^2*b - 3
0*a*b^2 - 206*b^3 + 10*(9*a^3 + 45*a^2*b + 75*a*b^2 + 35*b^3)*Cosh[4*(c + d
*x)] + 15*(a + b)^2*(a + 7*b)*Cosh[6*(c + d*x)])*Coth[c + d*x]*Csch[c + d*x
] - 480*(a + b)^2*(a + 7*b)*Cosh[c + d*x]^6*(Log[Cosh[(c + d*x)/2]] - Log[S
inh[(c + d*x)/2]]) + (3*(75*a^3 + 195*a^2*b + 165*a*b^2 + 77*b^3)*Csch[c +
d*x]^3*Sinh[4*(c + d*x)])/4))/(d*(a + 2*b + a*Cosh[2*(c + d*x)])^3)
```

fricas [B] time = 0.60, size = 6717, normalized size = 46.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/30*(30*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^13 + 390*(a^3 +
9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)*sinh(d*x + c)^12 + 30*(a^3 + 9*a^
2*b + 15*a*b^2 + 7*b^3)*sinh(d*x + c)^13 + 20*(9*a^3 + 45*a^2*b + 75*a*b^2
+ 35*b^3)*cosh(d*x + c)^11 + 20*(9*a^3 + 45*a^2*b + 75*a*b^2 + 35*b^3 + 117
*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^11 + 220
*(39*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^3 + (9*a^3 + 45*a^2*b
 + 75*a*b^2 + 35*b^3)*cosh(d*x + c))*sinh(d*x + c)^10 + 6*(75*a^3 + 195*a^2
*b + 165*a*b^2 + 77*b^3)*cosh(d*x + c)^9 + 2*(10725*(a^3 + 9*a^2*b + 15*a*b
^2 + 7*b^3)*cosh(d*x + c)^4 + 225*a^3 + 585*a^2*b + 495*a*b^2 + 231*b^3 + 5
50*(9*a^3 + 45*a^2*b + 75*a*b^2 + 35*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^9
+ 6*(6435*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^5 + 550*(9*a^3 +
 45*a^2*b + 75*a*b^2 + 35*b^3)*cosh(d*x + c)^3 + 9*(75*a^3 + 195*a^2*b + 16
5*a*b^2 + 77*b^3)*cosh(d*x + c))*sinh(d*x + c)^8 + 8*(75*a^3 + 135*a^2*b -
15*a*b^2 - 103*b^3)*cosh(d*x + c)^7 + 8*(6435*(a^3 + 9*a^2*b + 15*a*b^2 + 7
*b^3)*cosh(d*x + c)^6 + 825*(9*a^3 + 45*a^2*b + 75*a*b^2 + 35*b^3)*cosh(d*x
 + c)^4 + 75*a^3 + 135*a^2*b - 15*a*b^2 - 103*b^3 + 27*(75*a^3 + 195*a^2*b
+ 165*a*b^2 + 77*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^7 + 8*(6435*(a^3 + 9*a
^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^7 + 1155*(9*a^3 + 45*a^2*b + 75*a*b^
2 + 35*b^3)*cosh(d*x + c)^5 + 63*(75*a^3 + 195*a^2*b + 165*a*b^2 + 77*b^3)*
cosh(d*x + c)^3 + 7*(75*a^3 + 135*a^2*b - 15*a*b^2 - 103*b^3)*cosh(d*x + c)
)*sinh(d*x + c)^6 + 6*(75*a^3 + 195*a^2*b + 165*a*b^2 + 77*b^3)*cosh(d*x +
c)^5 + 6*(6435*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^8 + 1540*(9
*a^3 + 45*a^2*b + 75*a*b^2 + 35*b^3)*cosh(d*x + c)^6 + 126*(75*a^3 + 195*a^
2*b + 165*a*b^2 + 77*b^3)*cosh(d*x + c)^4 + 75*a^3 + 195*a^2*b + 165*a*b^2
+ 77*b^3 + 28*(75*a^3 + 135*a^2*b - 15*a*b^2 - 103*b^3)*cosh(d*x + c)^2)*si
nh(d*x + c)^5 + 2*(10725*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^9
 + 3300*(9*a^3 + 45*a^2*b + 75*a*b^2 + 35*b^3)*cosh(d*x + c)^7 + 378*(75*a^
3 + 195*a^2*b + 165*a*b^2 + 77*b^3)*cosh(d*x + c)^5 + 140*(75*a^3 + 135*a^2
*b - 15*a*b^2 - 103*b^3)*cosh(d*x + c)^3 + 15*(75*a^3 + 195*a^2*b + 165*a*b
^2 + 77*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 20*(9*a^3 + 45*a^2*b + 75*a*b
^2 + 35*b^3)*cosh(d*x + c)^3 + 4*(2145*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*c
```

osh(d*x + c)^10 + 825*(9*a^3 + 45*a^2*b + 75*a*b^2 + 35*b^3)*cosh(d*x + c)^
8 + 126*(75*a^3 + 195*a^2*b + 165*a*b^2 + 77*b^3)*cosh(d*x + c)^6 + 70*(75*
a^3 + 135*a^2*b - 15*a*b^2 - 103*b^3)*cosh(d*x + c)^4 + 45*a^3 + 225*a^2*b
+ 375*a*b^2 + 175*b^3 + 15*(75*a^3 + 195*a^2*b + 165*a*b^2 + 77*b^3)*cosh(d
*x + c)^2)*sinh(d*x + c)^3 + 4*(585*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh
(d*x + c)^11 + 275*(9*a^3 + 45*a^2*b + 75*a*b^2 + 35*b^3)*cosh(d*x + c)^9 +
54*(75*a^3 + 195*a^2*b + 165*a*b^2 + 77*b^3)*cosh(d*x + c)^7 + 42*(75*a^3
+ 135*a^2*b - 15*a*b^2 - 103*b^3)*cosh(d*x + c)^5 + 15*(75*a^3 + 195*a^2*b
+ 165*a*b^2 + 77*b^3)*cosh(d*x + c)^3 + 15*(9*a^3 + 45*a^2*b + 75*a*b^2 + 3
5*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 30*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^
3)*cosh(d*x + c) - 15*((a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^14
+ 14*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)*sinh(d*x + c)^13 + (a
^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*sinh(d*x + c)^14 + 3*(a^3 + 9*a^2*b + 15*a
*b^2 + 7*b^3)*cosh(d*x + c)^12 + (3*a^3 + 27*a^2*b + 45*a*b^2 + 21*b^3 + 91
*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^12 + 4*(
91*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^3 + 9*(a^3 + 9*a^2*b +
15*a*b^2 + 7*b^3)*cosh(d*x + c))*sinh(d*x + c)^11 + (a^3 + 9*a^2*b + 15*a*b
^2 + 7*b^3)*cosh(d*x + c)^10 + (1001*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cos
h(d*x + c)^4 + a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3 + 198*(a^3 + 9*a^2*b + 15*a
*b^2 + 7*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 2*(1001*(a^3 + 9*a^2*b +
15*a*b^2 + 7*b^3)*cosh(d*x + c)^5 + 330*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*
cosh(d*x + c)^3 + 5*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c))*sinh(
d*x + c)^9 - 5*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^8 + (3003*(
a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^6 + 1485*(a^3 + 9*a^2*b + 1
5*a*b^2 + 7*b^3)*cosh(d*x + c)^4 - 5*a^3 - 45*a^2*b - 75*a*b^2 - 35*b^3 + 4
5*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 8*(
429*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^7 + 297*(a^3 + 9*a^2*b
+ 15*a*b^2 + 7*b^3)*cosh(d*x + c)^5 + 15*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3
)*cosh(d*x + c)^3 - 5*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c))*sin
h(d*x + c)^7 - 5*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^6 + (3003
*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^8 + 2772*(a^3 + 9*a^2*b +
15*a*b^2 + 7*b^3)*cosh(d*x + c)^6 + 210*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)
*cosh(d*x + c)^4 - 5*a^3 - 45*a^2*b - 75*a*b^2 - 35*b^3 - 140*(a^3 + 9*a^2*
b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 2*(1001*(a^3 + 9*a
^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^9 + 1188*(a^3 + 9*a^2*b + 15*a*b^2 +
7*b^3)*cosh(d*x + c)^7 + 126*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x +
c)^5 - 140*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^3 - 15*(a^3 +
9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + (a^3 + 9*a^2*b
+ 15*a*b^2 + 7*b^3)*cosh(d*x + c)^4 + (1001*(a^3 + 9*a^2*b + 15*a*b^2 + 7*
b^3)*cosh(d*x + c)^10 + 1485*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x +
c)^8 + 210*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^6 - 350*(a^3 +
9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^4 + a^3 + 9*a^2*b + 15*a*b^2 + 7*
b^3 - 75*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^
4 + 4*(91*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^11 + 165*(a^3 +
9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^9 + 30*(a^3 + 9*a^2*b + 15*a*b^2

+ 7*b^3)*cosh(d*x + c)^7 - 70*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^5 - 25*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^3 + (a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3 + 3*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^2 + (91*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^12 + 198*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^10 + 45*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^8 - 140*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^6 - 75*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^4 + 3*a^3 + 27*a^2*b + 45*a*b^2 + 21*b^3 + 6*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(7*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^13 + 18*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^11 + 5*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^9 - 20*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^7 - 15*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^5 + 2*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^3 + 3*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) + 15*((a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^14 + 14*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)*sinh(d*x + c)^13 + (a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*sinh(d*x + c)^14 + 3*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^12 + (3*a^3 + 27*a^2*b + 45*a*b^2 + 21*b^3 + 91*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^12 + 4*(91*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^3 + 9*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c))*sinh(d*x + c)^11 + (a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^10 + (1001*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^4 + a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3 + 198*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 2*(1001*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^5 + 330*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^3 + 5*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c))*sinh(d*x + c)^9 - 5*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^8 + (3003*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^6 + 1485*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^4 - 5*a^3 - 45*a^2*b - 75*a*b^2 - 35*b^3 + 45*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 8*(429*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^7 + 297*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^5 + 15*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^3 - 5*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 - 5*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^6 + (3003*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^8 + 2772*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^6 + 210*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^4 - 5*a^3 - 45*a^2*b - 75*a*b^2 - 35*b^3 - 140*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 2*(1001*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^9 + 1188*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^7 + 126*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^5 - 140*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^3 - 15*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + (a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^4 + (1001*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^10 + 1485*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^8

```
+ 210*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^6 - 350*(a^3 + 9*a^2
*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^4 + a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3 -
 75*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4
*(91*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^11 + 165*(a^3 + 9*a^2
*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^9 + 30*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b
^3)*cosh(d*x + c)^7 - 70*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^5
 - 25*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^3 + (a^3 + 9*a^2*b +
 15*a*b^2 + 7*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + a^3 + 9*a^2*b + 15*a*b^
2 + 7*b^3 + 3*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^2 + (91*(a^3
 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^12 + 198*(a^3 + 9*a^2*b + 15*a
*b^2 + 7*b^3)*cosh(d*x + c)^10 + 45*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh
(d*x + c)^8 - 140*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^6 - 75*(
a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^4 + 3*a^3 + 27*a^2*b + 45*a
*b^2 + 21*b^3 + 6*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^2)*sinh(
d*x + c)^2 + 2*(7*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^13 + 18*
(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^11 + 5*(a^3 + 9*a^2*b + 15
*a*b^2 + 7*b^3)*cosh(d*x + c)^9 - 20*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cos
h(d*x + c)^7 - 15*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^5 + 2*(a
^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^3 + 3*(a^3 + 9*a^2*b + 15*a*
b^2 + 7*b^3)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c
) - 1) + 2*(195*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^12 + 110*(
9*a^3 + 45*a^2*b + 75*a*b^2 + 35*b^3)*cosh(d*x + c)^10 + 27*(75*a^3 + 195*a
^2*b + 165*a*b^2 + 77*b^3)*cosh(d*x + c)^8 + 28*(75*a^3 + 135*a^2*b - 15*a*
b^2 - 103*b^3)*cosh(d*x + c)^6 + 15*(75*a^3 + 195*a^2*b + 165*a*b^2 + 77*b^
3)*cosh(d*x + c)^4 + 15*a^3 + 135*a^2*b + 225*a*b^2 + 105*b^3 + 30*(9*a^3 +
 45*a^2*b + 75*a*b^2 + 35*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x
+ c)^14 + 14*d*cosh(d*x + c)*sinh(d*x + c)^13 + d*sinh(d*x + c)^14 + 3*d*co
sh(d*x + c)^12 + (91*d*cosh(d*x + c)^2 + 3*d)*sinh(d*x + c)^12 + 4*(91*d*co
sh(d*x + c)^3 + 9*d*cosh(d*x + c))*sinh(d*x + c)^11 + d*cosh(d*x + c)^10 +
(1001*d*cosh(d*x + c)^4 + 198*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^10 + 2*(
1001*d*cosh(d*x + c)^5 + 330*d*cosh(d*x + c)^3 + 5*d*cosh(d*x + c))*sinh(d*
x + c)^9 - 5*d*cosh(d*x + c)^8 + (3003*d*cosh(d*x + c)^6 + 1485*d*cosh(d*x
+ c)^4 + 45*d*cosh(d*x + c)^2 - 5*d)*sinh(d*x + c)^8 + 8*(429*d*cosh(d*x +
c)^7 + 297*d*cosh(d*x + c)^5 + 15*d*cosh(d*x + c)^3 - 5*d*cosh(d*x + c))*si
nh(d*x + c)^7 - 5*d*cosh(d*x + c)^6 + (3003*d*cosh(d*x + c)^8 + 2772*d*cosh
(d*x + c)^6 + 210*d*cosh(d*x + c)^4 - 140*d*cosh(d*x + c)^2 - 5*d)*sinh(d*x
 + c)^6 + 2*(1001*d*cosh(d*x + c)^9 + 1188*d*cosh(d*x + c)^7 + 126*d*cosh(d
*x + c)^5 - 140*d*cosh(d*x + c)^3 - 15*d*cosh(d*x + c))*sinh(d*x + c)^5 + d
*cosh(d*x + c)^4 + (1001*d*cosh(d*x + c)^10 + 1485*d*cosh(d*x + c)^8 + 210*
d*cosh(d*x + c)^6 - 350*d*cosh(d*x + c)^4 - 75*d*cosh(d*x + c)^2 + d)*sinh(
d*x + c)^4 + 4*(91*d*cosh(d*x + c)^11 + 165*d*cosh(d*x + c)^9 + 30*d*cosh(d
*x + c)^7 - 70*d*cosh(d*x + c)^5 - 25*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*
sinh(d*x + c)^3 + 3*d*cosh(d*x + c)^2 + (91*d*cosh(d*x + c)^12 + 198*d*cosh
(d*x + c)^10 + 45*d*cosh(d*x + c)^8 - 140*d*cosh(d*x + c)^6 - 75*d*cosh(d*x
 + c)^4 + 6*d*cosh(d*x + c)^2 + 3*d)*sinh(d*x + c)^2 + 2*(7*d*cosh(d*x + c)
```

```
^13 + 18*d*cosh(d*x + c)^11 + 5*d*cosh(d*x + c)^9 - 20*d*cosh(d*x + c)^7 -
15*d*cosh(d*x + c)^5 + 2*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x +
c) + d)
```

giac [B] time = 0.20, size = 341, normalized size = 2.37

$$\frac{15\left(a^3+9\,a^2b+15\,ab^2+7\,b^3\right)\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)-15\left(a^3+9\,a^2b+15\,ab^2+7\,b^3\right)\log\left(e^{(dx+c)}+e^{(-dx-c)}-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/60*(15*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*log(e^(d*x + c) + e^(-d*x - c)
+ 2) - 15*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*log(e^(d*x + c) + e^(-d*x - c)
 - 2) - 60*(a^3*(e^(d*x + c) + e^(-d*x - c)) + 3*a^2*b*(e^(d*x + c) + e^(-d
*x - c)) + 3*a*b^2*(e^(d*x + c) + e^(-d*x - c)) + b^3*(e^(d*x + c) + e^(-d*
x - c)))/((e^(d*x + c) + e^(-d*x - c))^2 - 4) - 8*(45*a^2*b*(e^(d*x + c) +
e^(-d*x - c))^4 + 90*a*b^2*(e^(d*x + c) + e^(-d*x - c))^4 + 45*b^3*(e^(d*x
+ c) + e^(-d*x - c))^4 + 60*a*b^2*(e^(d*x + c) + e^(-d*x - c))^2 + 40*b^3*(
e^(d*x + c) + e^(-d*x - c))^2 + 48*b^3)/(e^(d*x + c) + e^(-d*x - c))^5)/d
```

maple [A] time = 0.44, size = 192, normalized size = 1.33

$$\frac{a^3\left(-\frac{\mathrm{csch}(dx+c)\coth(dx+c)}{2}+\mathrm{arctanh}\left(\mathrm{e}^{dx+c}\right)\right)+3a^2b\left(-\frac{1}{2\sinh(dx+c)^2\cosh(dx+c)}-\frac{3}{2\cosh(dx+c)}+3\,\mathrm{arctanh}\left(\mathrm{e}^{dx+c}\right)\right)+3a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(a^3*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))+3*a^2*b*(-1/2/s
inh(d*x+c)^2/cosh(d*x+c)-3/2/cosh(d*x+c)+3*arctanh(exp(d*x+c)))+3*a*b^2*(-1
/2/sinh(d*x+c)^2/cosh(d*x+c)^3-5/6/cosh(d*x+c)^3-5/2/cosh(d*x+c)+5*arctanh(
exp(d*x+c)))+b^3*(-1/2/sinh(d*x+c)^2/cosh(d*x+c)^5-7/10/cosh(d*x+c)^5-7/6/c
osh(d*x+c)^3-7/2/cosh(d*x+c)+7*arctanh(exp(d*x+c))))
```

maxima [B] time = 0.34, size = 556, normalized size = 3.86

$$\frac{1}{30}\,b^3\left(\frac{105\,\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{105\,\log\left(e^{(-dx-c)}-1\right)}{d}-\frac{2\left(105\,e^{(-dx-c)}+350\,e^{(-3\,dx-3\,c)}+231\,e^{(-5\,dx-5\,c)}-412\,e^{(-7\,}\right.}{d\left(3\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}-5\,e^{(-6\,dx-6\,c)}-5\,e^{(-8\,dx-}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/30*b^3*(105*log(e^(-d*x - c) + 1)/d - 105*log(e^(-d*x - c) - 1)/d - 2*(10
5*e^(-d*x - c) + 350*e^(-3*d*x - 3*c) + 231*e^(-5*d*x - 5*c) - 412*e^(-7*d*
x - 7*c) + 231*e^(-9*d*x - 9*c) + 350*e^(-11*d*x - 11*c) + 105*e^(-13*d*x -
 13*c))/(d*(3*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) - 5*e^(-6*d*x - 6*c) - 5*
e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 3*e^(-12*d*x - 12*c) + e^(-14*d*x -
 14*c) + 1))) + 1/2*a*b^2*(15*log(e^(-d*x - c) + 1)/d - 15*log(e^(-d*x - c)
 - 1)/d - 2*(15*e^(-d*x - c) + 20*e^(-3*d*x - 3*c) - 22*e^(-5*d*x - 5*c) +
20*e^(-7*d*x - 7*c) + 15*e^(-9*d*x - 9*c))/(d*(e^(-2*d*x - 2*c) - 2*e^(-4*d
*x - 4*c) - 2*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)
)) + 3/2*a^2*b*(3*log(e^(-d*x - c) + 1)/d - 3*log(e^(-d*x - c) - 1)/d + 2*(
3*e^(-d*x - c) - 2*e^(-3*d*x - 3*c) + 3*e^(-5*d*x - 5*c))/(d*(e^(-2*d*x - 2
*c) + e^(-4*d*x - 4*c) - e^(-6*d*x - 6*c) - 1))) + 1/2*a^3*(log(e^(-d*x - c
) + 1)/d - log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x - c) + e^(-3*d*x - 3*c))/(d
*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1)))
```

mupad [B] time = 1.64, size = 536, normalized size = 3.72

$$\frac{\mathrm{atan}\left(\frac{e^{d\,x}\,e^{c}\,\left(a^3\,\sqrt{-d^2}+7\,b^3\,\sqrt{-d^2}+15\,a\,b^2\,\sqrt{-d^2}+9\,a^2\,b\,\sqrt{-d^2}\right)}{d\,\sqrt{a^6+18\,a^5\,b+111\,a^4\,b^2+284\,a^3\,b^3+351\,a^2\,b^4+210\,a\,b^5+49\,b^6}}\right)\,\sqrt{a^6+18\,a^5\,b+111\,a^4\,b^2+284\,a^3\,b^3+351\,a^2\,b^4+210\,a}{\sqrt{-d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cosh(c + d*x)^2)^3/sinh(c + d*x)^3,x)
```

```
[Out] (atan((exp(d*x)*exp(c)*(a^3*(-d^2)^(1/2) + 7*b^3*(-d^2)^(1/2) + 15*a*b^2*(-
d^2)^(1/2) + 9*a^2*b*(-d^2)^(1/2)))/(d*(210*a*b^5 + 18*a^5*b + a^6 + 49*b^6
 + 351*a^2*b^4 + 284*a^3*b^3 + 111*a^4*b^2)^(1/2)))*(210*a*b^5 + 18*a^5*b +
 a^6 + 49*b^6 + 351*a^2*b^4 + 284*a^3*b^3 + 111*a^4*b^2)^(1/2))/(-d^2)^(1/2
) - (2*exp(c + d*x)*(3*a*b^2 + 3*a^2*b + a^3 + b^3))/(d*(exp(4*c + 4*d*x) -
 2*exp(2*c + 2*d*x) + 1)) - (8*exp(c + d*x)*(3*a*b^2 + 2*b^3))/(3*d*(2*exp(
2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (6*exp(c + d*x)*(2*a*b^2 + a^2*b +
b^3))/(d*(exp(2*c + 2*d*x) + 1)) + (64*b^3*exp(c + d*x))/(5*d*(4*exp(2*c +
2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) +
 (8*exp(c + d*x)*(15*a*b^2 - 2*b^3))/(15*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c
+ 4*d*x) + exp(6*c + 6*d*x) + 1)) - (32*b^3*exp(c + d*x))/(5*d*(5*exp(2*c +
 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) +
exp(10*c + 10*d*x) + 1)) - (exp(c + d*x)*(3*a*b^2 + 3*a^2*b + a^3 + b^3))/(
d*(exp(2*c + 2*d*x) - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\,\mathrm{sech}^2\left(c + dx\right)\right)^3 \mathrm{csch}^3\left(c + dx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3*(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**3*csch(c + d*x)**3, x)
```

3.24 $\int \operatorname{csch}^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=104

$$-\frac{b^2(3a+4b)\tanh^3(c+dx)}{3d}+\frac{3b(a+b)(a+2b)\tanh(c+dx)}{d}-\frac{(a+b)^3\coth^3(c+dx)}{3d}+\frac{(a+b)^2(a+4b)\coth(c+$$

```
[Out] (a+b)^2*(a+4*b)*coth(d*x+c)/d-1/3*(a+b)^3*coth(d*x+c)^3/d+3*b*(a+b)*(a+2*b)
*tanh(d*x+c)/d-1/3*b^2*(3*a+4*b)*tanh(d*x+c)^3/d+1/5*b^3*tanh(d*x+c)^5/d
```

Rubi [A] time = 0.11, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4132, 448}

$$-\frac{b^2(3a+4b)\tanh^3(c+dx)}{3d}+\frac{3b(a+b)(a+2b)\tanh(c+dx)}{d}-\frac{(a+b)^3\coth^3(c+dx)}{3d}+\frac{(a+b)^2(a+4b)\coth(c+$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((a + b)^2*(a + 4*b)*Coth[c + d*x])/d - ((a + b)^3*Coth[c + d*x]^3)/(3*d) +
 (3*b*(a + b)*(a + 2*b)*Tanh[c + d*x])/d - (b^2*(3*a + 4*b)*Tanh[c + d*x]^3
)/(3*d) + (b^3*Tanh[c + d*x]^5)/(5*d)
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \operatorname{csch}^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx = \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)(a+b-bx^2)^3}{x^4}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\operatorname{Subst}\left(\int \left(3b(a+b)(a+2b)+\frac{(a+b)^3}{x^4}-\frac{(a+b)^2(a+4b)}{x^2}-b^2(3a+4b)x^2+\right.\right.}{d}$$
$$= \frac{(a+b)^2(a+4b)\coth(c+dx)}{d}-\frac{(a+b)^3\coth^3(c+dx)}{3d}+\frac{3b(a+b)(}{}$$

Mathematica [B] time = 2.47, size = 213, normalized size = 2.05

$$-\frac{8\tanh(c)\operatorname{sech}^5(c+dx)\left(a\cosh^2(c+dx)+b\right)^3\left(\operatorname{csch}(c)\sinh(dx)\cosh^4(c+dx)\left(5(a+b)^2(2a+11b)\coth(c)\operatorname{cot}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (-8*(b + a*Cosh[c + d*x]^2)^3*Sech[c + d*x]^5*(-3*b^3*Cosh[c + d*x] + Cosh[
c + d*x]^3*(-(b^2*(15*a + 14*b)) + 5*(a + b)^3*Coth[c]^2*Coth[c + d*x]^2) -
 3*b^3*Csch[c]*Sinh[d*x] + Cosh[c + d*x]^4*(-(b*(45*a^2 + 120*a*b + 73*b^2)
) + 5*(a + b)^2*(2*a + 11*b)*Coth[c]*Coth[c + d*x])*Csch[c]*Sinh[d*x] - Cos
h[c + d*x]^2*(b^2*(15*a + 14*b) + 5*(a + b)^3*Coth[c]*Coth[c + d*x]^3)*Csch
[c]*Sinh[d*x])*Tanh[c])/(15*d*(a + 2*b + a*Cosh[2*(c + d*x)])^3)
```

fricas [B] time = 0.59, size = 955, normalized size = 9.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -8/15*((5*a^3 - 30*a^2*b - 60*a*b^2 - 32*b^3)*cosh(d*x + c)^6 + 12*(5*a^3 +
 15*a^2*b + 30*a*b^2 + 16*b^3)*cosh(d*x + c)*sinh(d*x + c)^5 + (5*a^3 - 30*
a^2*b - 60*a*b^2 - 32*b^3)*sinh(d*x + c)^6 + 2*(15*a^3 - 60*a*b^2 - 32*b^3)
*cosh(d*x + c)^4 + (30*a^3 - 120*a*b^2 - 64*b^3 + 15*(5*a^3 - 30*a^2*b - 60
*a*b^2 - 32*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(5*(5*a^3 + 15*a^2*b
+ 30*a*b^2 + 16*b^3)*cosh(d*x + c)^3 + 4*(5*a^3 + 15*a^2*b + 15*a*b^2 + 8*b
^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 50*a^3 + 240*a^2*b + 360*a*b^2 + 192*b
^3 + (75*a^3 + 270*a^2*b + 300*a*b^2 + 64*b^3)*cosh(d*x + c)^2 + (15*(5*a^3
 - 30*a^2*b - 60*a*b^2 - 32*b^3)*cosh(d*x + c)^4 + 75*a^3 + 270*a^2*b + 300
```

```
*a*b^2 + 64*b^3 + 12*(15*a^3 - 60*a*b^2 - 32*b^3)*cosh(d*x + c)^2)*sinh(d*x
 + c)^2 + 4*(3*(5*a^3 + 15*a^2*b + 30*a*b^2 + 16*b^3)*cosh(d*x + c)^5 + 8*(
5*a^3 + 15*a^2*b + 15*a*b^2 + 8*b^3)*cosh(d*x + c)^3 + (25*a^3 + 75*a^2*b +
 30*a*b^2 - 32*b^3)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^10 + 10*
d*cosh(d*x + c)*sinh(d*x + c)^9 + d*sinh(d*x + c)^10 + 2*d*cosh(d*x + c)^8
+ (45*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c)^8 + 8*(15*d*cosh(d*x + c)^3 +
2*d*cosh(d*x + c))*sinh(d*x + c)^7 - 3*d*cosh(d*x + c)^6 + (210*d*cosh(d*x
+ c)^4 + 56*d*cosh(d*x + c)^2 - 3*d)*sinh(d*x + c)^6 + 2*(126*d*cosh(d*x +
c)^5 + 56*d*cosh(d*x + c)^3 - 3*d*cosh(d*x + c))*sinh(d*x + c)^5 - 8*d*cosh
(d*x + c)^4 + (210*d*cosh(d*x + c)^6 + 140*d*cosh(d*x + c)^4 - 45*d*cosh(d*
x + c)^2 - 8*d)*sinh(d*x + c)^4 + 4*(30*d*cosh(d*x + c)^7 + 28*d*cosh(d*x +
 c)^5 - 5*d*cosh(d*x + c)^3 - 4*d*cosh(d*x + c))*sinh(d*x + c)^3 + 2*d*cosh
(d*x + c)^2 + (45*d*cosh(d*x + c)^8 + 56*d*cosh(d*x + c)^6 - 45*d*cosh(d*x
+ c)^4 - 48*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c)^2 + 2*(5*d*cosh(d*x + c)
^9 + 8*d*cosh(d*x + c)^7 - 3*d*cosh(d*x + c)^5 - 8*d*cosh(d*x + c)^3 - 2*d*
cosh(d*x + c))*sinh(d*x + c) + 6*d)
```

giac [B] time = 0.19, size = 355, normalized size = 3.41

$$2\left(\frac{5\left(9\,a^2be^{(4\,dx+4\,c)}+18\,ab^2e^{(4\,dx+4\,c)}+9\,b^3e^{(4\,dx+4\,c)}-6\,a^3e^{(2\,dx+2\,c)}-36\,a^2be^{(2\,dx+2\,c)}-54\,ab^2e^{(2\,dx+2\,c)}-24\,b^3e^{(2\,dx+2\,c)}+2\,a^3+15\,a^2b+24\,ab^2+11\,b^3\right)}{\left(e^{(2\,dx+2\,c)}-1\right)^3} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 2/15*(5*(9*a^2*b*e^(4*d*x + 4*c) + 18*a*b^2*e^(4*d*x + 4*c) + 9*b^3*e^(4*d*
x + 4*c) - 6*a^3*e^(2*d*x + 2*c) - 36*a^2*b*e^(2*d*x + 2*c) - 54*a*b^2*e^(2
*d*x + 2*c) - 24*b^3*e^(2*d*x + 2*c) + 2*a^3 + 15*a^2*b + 24*a*b^2 + 11*b^3
)/(e^(2*d*x + 2*c) - 1)^3 - (45*a^2*b*e^(8*d*x + 8*c) + 90*a*b^2*e^(8*d*x +
 8*c) + 45*b^3*e^(8*d*x + 8*c) + 180*a^2*b*e^(6*d*x + 6*c) + 450*a*b^2*e^(6
*d*x + 6*c) + 240*b^3*e^(6*d*x + 6*c) + 270*a^2*b*e^(4*d*x + 4*c) + 750*a*b
^2*e^(4*d*x + 4*c) + 490*b^3*e^(4*d*x + 4*c) + 180*a^2*b*e^(2*d*x + 2*c) +
510*a*b^2*e^(2*d*x + 2*c) + 320*b^3*e^(2*d*x + 2*c) + 45*a^2*b + 120*a*b^2
+ 73*b^3)/(e^(2*d*x + 2*c) + 1)^5)/d
```

maple [B] time = 0.66, size = 213, normalized size = 2.05

$$a^3\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^2}{3}\right)\coth\left(dx+c\right)+3a^2b\left(-\frac{1}{3\sinh(dx+c)^3\cosh(dx+c)}+\frac{4}{3\sinh(dx+c)\cosh(dx+c)}+\frac{8\tanh(dx+c)}{3}\right)+3a\,b^2\left(-\frac{}{3\,\mathrm{s}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(a^3*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)+3*a^2*b*(-1/3/sinh(d*x+c)^3/co
sh(d*x+c)+4/3/sinh(d*x+c)/cosh(d*x+c)+8/3*tanh(d*x+c))+3*a*b^2*(-1/3/sinh(d
*x+c)^3/cosh(d*x+c)^3+2/sinh(d*x+c)/cosh(d*x+c)^3+8*(2/3+1/3*sech(d*x+c)^2)
*tanh(d*x+c))+b^3*(-1/3/sinh(d*x+c)^3/cosh(d*x+c)^5+8/3/sinh(d*x+c)/cosh(d*
x+c)^5+16*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)*tanh(d*x+c)))
```

maxima [B] time = 0.34, size = 664, normalized size = 6.38

$$\frac{4}{3}a^3\left(\frac{3e^{(-2dx-2c)}}{d\left(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1\right)}-\frac{1}{d\left(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1\right)}\right)+\frac{256}{15}b^3\left(\frac{}{d\left(2e\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 4/3*a^3*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e
^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(
-6*d*x - 6*c) - 1))) + 256/15*b^3*(2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c
) - 2*e^(-4*d*x - 4*c) - 6*e^(-6*d*x - 6*c) + 6*e^(-10*d*x - 10*c) + 2*e^(-
12*d*x - 12*c) - 2*e^(-14*d*x - 14*c) - e^(-16*d*x - 16*c) + 1)) - 2*e^(-4*
d*x - 4*c)/(d*(2*e^(-2*d*x - 2*c) - 2*e^(-4*d*x - 4*c) - 6*e^(-6*d*x - 6*c)
 + 6*e^(-10*d*x - 10*c) + 2*e^(-12*d*x - 12*c) - 2*e^(-14*d*x - 14*c) - e^(
-16*d*x - 16*c) + 1)) - 6*e^(-6*d*x - 6*c)/(d*(2*e^(-2*d*x - 2*c) - 2*e^(-4
*d*x - 4*c) - 6*e^(-6*d*x - 6*c) + 6*e^(-10*d*x - 10*c) + 2*e^(-12*d*x - 12
*c) - 2*e^(-14*d*x - 14*c) - e^(-16*d*x - 16*c) + 1)) + 1/(d*(2*e^(-2*d*x -
 2*c) - 2*e^(-4*d*x - 4*c) - 6*e^(-6*d*x - 6*c) + 6*e^(-10*d*x - 10*c) + 2*
e^(-12*d*x - 12*c) - 2*e^(-14*d*x - 14*c) - e^(-16*d*x - 16*c) + 1))) + 16*
a^2*b*(2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - 2*e^(-6*d*x - 6*c) + e^(
-8*d*x - 8*c) - 1)) - 1/(d*(2*e^(-2*d*x - 2*c) - 2*e^(-6*d*x - 6*c) + e^(-8
*d*x - 8*c) - 1))) + 32*a*b^2*(3*e^(-4*d*x - 4*c)/(d*(3*e^(-4*d*x - 4*c) -
3*e^(-8*d*x - 8*c) + e^(-12*d*x - 12*c) - 1)) - 1/(d*(3*e^(-4*d*x - 4*c) -
3*e^(-8*d*x - 8*c) + e^(-12*d*x - 12*c) - 1)))
```

mupad [B] time = 1.60, size = 745, normalized size = 7.16

$$\frac{6\left(a^2b+2ab^2+b^3\right)}{d\left(e^{2c+2dx}-1\right)}-\frac{\frac{2\left(3a^2b+9ab^2+5b^3\right)}{5d}+\frac{6e^{6c+6dx}\left(a^2b+2ab^2+b^3\right)}{5d}+\frac{6e^{4c+4dx}\left(3a^2b+9ab^2+5b^3\right)}{5d}+\frac{2e^{2c+2dx}\left(9a^2b+30ab^2+2\right.}{5d}}{4e^{2c+2dx}+6e^{4c+4dx}+4e^{6c+6dx}+e^{8c+8dx}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cosh(c + d*x)^2)^3/sinh(c + d*x)^4,x)
```

```
[Out] (6*(2*a*b^2 + a^2*b + b^3))/(d*(exp(2*c + 2*d*x) - 1)) - ((2*(9*a*b^2 + 3*a
^2*b + 5*b^3))/(5*d) + (6*exp(6*c + 6*d*x)*(2*a*b^2 + a^2*b + b^3))/(5*d) +
 (6*exp(4*c + 4*d*x)*(9*a*b^2 + 3*a^2*b + 5*b^3))/(5*d) + (2*exp(2*c + 2*d*
```

```
x)*(30*a*b^2 + 9*a^2*b + 25*b^3))/(5*d))/(4*exp(2*c + 2*d*x) + 6*exp(4*c +
4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) - ((6*(2*a*b^2 + a^2*b
+ b^3))/(5*d) + (6*exp(8*c + 8*d*x)*(2*a*b^2 + a^2*b + b^3))/(5*d) + (8*exp
(2*c + 2*d*x)*(9*a*b^2 + 3*a^2*b + 5*b^3))/(5*d) + (8*exp(6*c + 6*d*x)*(9*a
*b^2 + 3*a^2*b + 5*b^3))/(5*d) + (4*exp(4*c + 4*d*x)*(30*a*b^2 + 9*a^2*b +
25*b^3))/(5*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*
d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1) - ((2*(30*a*b^2 + 9*a^2
*b + 25*b^3))/(15*d) + (6*exp(4*c + 4*d*x)*(2*a*b^2 + a^2*b + b^3))/(5*d) +
 (4*exp(2*c + 2*d*x)*(9*a*b^2 + 3*a^2*b + 5*b^3))/(5*d))/(3*exp(2*c + 2*d*x
) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) - (4*(3*a*b^2 + 3*a^2*b + a^
3 + b^3))/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) - ((2*(9*a*b^2 +
3*a^2*b + 5*b^3))/(5*d) + (6*exp(2*c + 2*d*x)*(2*a*b^2 + a^2*b + b^3))/(5*d
))/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) - (6*(2*a*b^2 + a^2*b + b^3)
)/(5*d*(exp(2*c + 2*d*x) + 1)) - (8*(3*a*b^2 + 3*a^2*b + a^3 + b^3))/(3*d*(
3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\,\mathrm{sech}^2\left(c + dx\right)\right)^3 \mathrm{csch}^4\left(c + dx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4*(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**3*csch(c + d*x)**4, x)
```

3.25 $\int \frac{\sinh^4(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=117

$$-\frac{\sqrt{b}\,(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a^3d}-\frac{(5a+4b)\sinh(c+dx)\cosh(c+dx)}{8a^2d}+\frac{x\left(3a^2+12ab+8b^2\right)}{8a^3}+\frac{\sinh(c+dx)\,c}{4a}$$

```
[Out] 1/8*(3*a^2+12*a*b+8*b^2)*x/a^3-1/8*(5*a+4*b)*cosh(d*x+c)*sinh(d*x+c)/a^2/d+
1/4*cosh(d*x+c)^3*sinh(d*x+c)/a/d-(a+b)^(3/2)*arctanh(b^(1/2)*tanh(d*x+c)/(
a+b)^(1/2))*b^(1/2)/a^3/d
```

Rubi [A] time = 0.20, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4132, 470, 527, 522, 206, 208}

$$\frac{x\left(3a^2+12ab+8b^2\right)}{8a^3}-\frac{\sqrt{b}\,(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a^3d}-\frac{(5a+4b)\sinh(c+dx)\cosh(c+dx)}{8a^2d}+\frac{\sinh(c+dx)\,cc}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^4/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((3*a^2 + 12*a*b + 8*b^2)*x)/(8*a^3) - (Sqrt[b]*(a + b)^(3/2)*ArcTanh[(Sqrt
[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a^3*d) - ((5*a + 4*b)*Cosh[c + d*x]*Sinh[
c + d*x])/(8*a^2*d) + (Cosh[c + d*x]^3*Sinh[c + d*x])/(4*a*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
```

```
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sinh^4(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx = \frac{\operatorname{Subst}\left(\int \frac{x^4}{(1-x^2)^3(a+b-bx^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\cosh^3(c+dx)\sinh(c+dx)}{4ad} - \frac{\operatorname{Subst}\left(\int \frac{a+b+(4a+3b)x^2}{(1-x^2)^2(a+b-bx^2)}\,dx, x, \tanh(c+dx)\right)}{4ad}$$

$$= -\frac{(5a+4b)\cosh(c+dx)\sinh(c+dx)}{8a^2d} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4ad} - \frac{\operatorname{Subst}\left(\int \frac{-(a+}{}\right.}{}$$

$$= -\frac{(5a+4b)\cosh(c+dx)\sinh(c+dx)}{8a^2d} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4ad} - \frac{(b(a+b)^2)\operatorname{Su}}{}$$

$$= \frac{(3a^2+12ab+8b^2)x}{8a^3} - \frac{\sqrt{b}(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a^3d} - \frac{(5a+4b)\cosh(c+dx)}{8a^2d}$$

Mathematica [B] time = 2.48, size = 294, normalized size = 2.51

$$-\frac{\operatorname{sech}^2(c+dx)(a\cosh(2(c+dx))+a+2b)\left(\sqrt{b}\left(3a^3+34a^2b+64ab^2+32b^3\right)(\cosh(2c)-\sinh(2c))\tanh^{-1}\left(\frac{(\mathrm{co}}{}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^4/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -1/64*((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*(Sqrt[b]*(3*a^3 + 34
*a^2*b + 64*a*b^2 + 32*b^3)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*((a
+ 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sin
h[c])^4])]*(Cosh[2*c] - Sinh[2*c]) - Sqrt[b*(Cosh[c] - Sinh[c])^4]*(a^2*(3*
a + 2*b)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]] + Sqrt[b]*Sqrt[a + b]
*(-2*a^2*c + 12*a^2*d*x + 48*a*b*d*x + 32*b^2*d*x - 8*a*(a + b)*Sinh[2*(c +
 d*x)] + a^2*Sinh[4*(c + d*x)]))))/(a^3*Sqrt[b]*Sqrt[a + b]*d*(a + b*Sech[c
 + d*x]^2)*Sqrt[b*(Cosh[c] - Sinh[c])^4])
```

fricas [B] time = 0.59, size = 1681, normalized size = 14.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/64*(a^2*cosh(d*x + c)^8 + 8*a^2*cosh(d*x + c)*sinh(d*x + c)^7 + a^2*sinh
(d*x + c)^8 + 8*(3*a^2 + 12*a*b + 8*b^2)*d*x*cosh(d*x + c)^4 - 8*(a^2 + a*b
)*cosh(d*x + c)^6 + 4*(7*a^2*cosh(d*x + c)^2 - 2*a^2 - 2*a*b)*sinh(d*x + c)
^6 + 8*(7*a^2*cosh(d*x + c)^3 - 6*(a^2 + a*b)*cosh(d*x + c))*sinh(d*x + c)^
5 + 2*(35*a^2*cosh(d*x + c)^4 + 4*(3*a^2 + 12*a*b + 8*b^2)*d*x - 60*(a^2 +
a*b)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*a^2*cosh(d*x + c)^5 + 4*(3*a^2
 + 12*a*b + 8*b^2)*d*x*cosh(d*x + c) - 20*(a^2 + a*b)*cosh(d*x + c)^3)*sinh
(d*x + c)^3 + 8*(a^2 + a*b)*cosh(d*x + c)^2 + 4*(7*a^2*cosh(d*x + c)^6 + 12
*(3*a^2 + 12*a*b + 8*b^2)*d*x*cosh(d*x + c)^2 - 30*(a^2 + a*b)*cosh(d*x + c
)^4 + 2*a^2 + 2*a*b)*sinh(d*x + c)^2 + 32*((a + b)*cosh(d*x + c)^4 + 4*(a +
 b)*cosh(d*x + c)^3*sinh(d*x + c) + 6*(a + b)*cosh(d*x + c)^2*sinh(d*x + c)
^2 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4)*sqr
t(a*b + b^2)*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3
 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*
x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh
(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) + 4*(a*cosh(d*x +
c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(
a*b + b^2))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh
(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*
b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d
*x + c) + a)) - a^2 + 8*(a^2*cosh(d*x + c)^7 + 4*(3*a^2 + 12*a*b + 8*b^2)*d
*x*cosh(d*x + c)^3 - 6*(a^2 + a*b)*cosh(d*x + c)^5 + 2*(a^2 + a*b)*cosh(d*x
 + c))*sinh(d*x + c))/(a^3*d*cosh(d*x + c)^4 + 4*a^3*d*cosh(d*x + c)^3*sinh
(d*x + c) + 6*a^3*d*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*a^3*d*cosh(d*x + c)
*sinh(d*x + c)^3 + a^3*d*sinh(d*x + c)^4), 1/64*(a^2*cosh(d*x + c)^8 + 8*a^
2*cosh(d*x + c)*sinh(d*x + c)^7 + a^2*sinh(d*x + c)^8 + 8*(3*a^2 + 12*a*b +
 8*b^2)*d*x*cosh(d*x + c)^4 - 8*(a^2 + a*b)*cosh(d*x + c)^6 + 4*(7*a^2*cosh
(d*x + c)^2 - 2*a^2 - 2*a*b)*sinh(d*x + c)^6 + 8*(7*a^2*cosh(d*x + c)^3 - 6
*(a^2 + a*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*a^2*cosh(d*x + c)^4 + 4
*(3*a^2 + 12*a*b + 8*b^2)*d*x - 60*(a^2 + a*b)*cosh(d*x + c)^2)*sinh(d*x +
c)^4 + 8*(7*a^2*cosh(d*x + c)^5 + 4*(3*a^2 + 12*a*b + 8*b^2)*d*x*cosh(d*x +
 c) - 20*(a^2 + a*b)*cosh(d*x + c)^3)*sinh(d*x + c)^3 + 8*(a^2 + a*b)*cosh(
d*x + c)^2 + 4*(7*a^2*cosh(d*x + c)^6 + 12*(3*a^2 + 12*a*b + 8*b^2)*d*x*cos
h(d*x + c)^2 - 30*(a^2 + a*b)*cosh(d*x + c)^4 + 2*a^2 + 2*a*b)*sinh(d*x + c
)^2 - 64*((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)^3*sinh(d*x + c)
 + 6*(a + b)*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*(a + b)*cosh(d*x + c)*sinh
(d*x + c)^3 + (a + b)*sinh(d*x + c)^4)*sqrt(-a*b - b^2)*arctan(1/2*(a*cosh(
d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)
*sqrt(-a*b - b^2)/(a*b + b^2)) - a^2 + 8*(a^2*cosh(d*x + c)^7 + 4*(3*a^2 +
12*a*b + 8*b^2)*d*x*cosh(d*x + c)^3 - 6*(a^2 + a*b)*cosh(d*x + c)^5 + 2*(a^
2 + a*b)*cosh(d*x + c))*sinh(d*x + c))/(a^3*d*cosh(d*x + c)^4 + 4*a^3*d*cos
h(d*x + c)^3*sinh(d*x + c) + 6*a^3*d*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*a^
3*d*cosh(d*x + c)*sinh(d*x + c)^3 + a^3*d*sinh(d*x + c)^4)]
```

giac [B] time = 2.88, size = 220, normalized size = 1.88

$$\frac{\frac{8\left(3a^2+12ab+8b^2\right)(dx+c)}{a^3}+\frac{ae^{(4dx+4c)}-8ae^{(2dx+2c)}-8be^{(2dx+2c)}}{a^2}-\frac{\left(18a^2e^{(4dx+4c)}+72abe^{(4dx+4c)}+48b^2e^{(4dx+4c)}-8a^2e^{(2dx+2c)}-8abe^{(2dx+2c)}+\cdots}{a^3}}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/64*(8*(3*a^2 + 12*a*b + 8*b^2)*(d*x + c)/a^3 + (a*e^(4*d*x + 4*c) - 8*a*e
^(2*d*x + 2*c) - 8*b*e^(2*d*x + 2*c))/a^2 - (18*a^2*e^(4*d*x + 4*c) + 72*a*
b*e^(4*d*x + 4*c) + 48*b^2*e^(4*d*x + 4*c) - 8*a^2*e^(2*d*x + 2*c) - 8*a*b*
e^(2*d*x + 2*c) + a^2)*e^(-4*d*x - 4*c)/a^3 - 64*(a^2*b + 2*a*b^2 + b^3)*ar
ctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/(sqrt(-a*b - b^2)*
a^3))/d
```

maple [B] time = 0.41, size = 708, normalized size = 6.05

$$\frac{1}{4da\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}+\frac{1}{2da\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{1}{8da\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{b}{2d\,a^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^4/(a+b*sech(d*x+c)^2),x)
```

```
[Out] 1/4/d/a/(tanh(1/2*d*x+1/2*c)-1)^4+1/2/d/a/(tanh(1/2*d*x+1/2*c)-1)^3-1/8/d/a
/(tanh(1/2*d*x+1/2*c)-1)^2-1/2/d/a^2/(tanh(1/2*d*x+1/2*c)-1)^2*b-3/8/d/a/(t
anh(1/2*d*x+1/2*c)-1)-1/2/d/a^2/(tanh(1/2*d*x+1/2*c)-1)*b-3/8/d/a*ln(tanh(1
/2*d*x+1/2*c)-1)-3/2/d/a^2*ln(tanh(1/2*d*x+1/2*c)-1)*b-1/d/a^3*ln(tanh(1/2*
d*x+1/2*c)-1)*b^2-1/4/d/a/(tanh(1/2*d*x+1/2*c)+1)^4+1/2/d/a/(tanh(1/2*d*x+1
/2*c)+1)^3+1/8/d/a/(tanh(1/2*d*x+1/2*c)+1)^2+1/2/d/a^2/(tanh(1/2*d*x+1/2*c)
+1)^2*b-3/8/d/a/(tanh(1/2*d*x+1/2*c)+1)-1/2/d/a^2/(tanh(1/2*d*x+1/2*c)+1)*b
+3/8/d/a*ln(tanh(1/2*d*x+1/2*c)+1)+3/2/d/a^2*ln(tanh(1/2*d*x+1/2*c)+1)*b+1/
d/a^3*ln(tanh(1/2*d*x+1/2*c)+1)*b^2+1/2/d/a*b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1
/2)*tanh(1/2*d*x+1/2*c)^2-2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))+1/d/a^
2*b^(3/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*b^(1/2)*tanh(1
/2*d*x+1/2*c)+(a+b)^(1/2))+1/2/d/a^3*b^(5/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tan
h(1/2*d*x+1/2*c)^2-2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))-1/2/d/a*b^(1/
2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+
1/2*c)+(a+b)^(1/2))-1/d/a^2*b^(3/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x
+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))-1/2/d/a^3*b^(5/2)/(a+b
)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)+
(a+b)^(1/2))
```

maxima [B] time = 0.46, size = 526, normalized size = 4.50

$$\frac{3\,b\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{16\,\sqrt{(a+b)b}\,ad}+\frac{3\,(dx+c)}{8\,ad}-\frac{\left(8\,be^{(-2\,dx-2\,c)}-a\right)e^{(4\,dx+4\,c)}}{64\,a^2d}-\frac{e^{(2\,dx+2\,c)}}{8\,ad}+\frac{e^{(-2\,dx-2\,c)}}{8\,ad}+\frac{b\log\left(ae^{(4\,dx+}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 3/16*b*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x
- 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*a*d) + 3/8*(d*x + c
)/(a*d) - 1/64*(8*b*e^(-2*d*x - 2*c) - a)*e^(4*d*x + 4*c)/(a^2*d) - 1/8*e^(
2*d*x + 2*c)/(a*d) + 1/8*e^(-2*d*x - 2*c)/(a*d) + 1/4*b*log(a*e^(4*d*x + 4*
c) + 2*(a + 2*b)*e^(2*d*x + 2*c) + a)/(a^2*d) - 1/4*b*log(2*(a + 2*b)*e^(-2
*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/(a^2*d) - 1/8*(a*b + 2*b^2)*log((a*e^
(2*d*x + 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(2*d*x + 2*c) + a + 2*b +
 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*a^2*d) + 1/8*(a*b + 2*b^2)*log((a*e^(
-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b
+ 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*a^2*d) + 1/2*(a*b + 2*b^2)*(d*x + c)
/(a^3*d) + 1/64*(8*b*e^(-2*d*x - 2*c) - a*e^(-4*d*x - 4*c))/(a^2*d) + 1/16*
(a^2*b + 8*a*b^2 + 8*b^3)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b
)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*
a^3*d)
```

mupad [B] time = 2.43, size = 328, normalized size = 2.80

$$\frac{x\left(3\,a^2+12\,a\,b+8\,b^2\right)}{8\,a^3}-\frac{\mathrm{e}^{-4\,c-4\,d\,x}}{64\,a\,d}+\frac{\mathrm{e}^{4\,c+4\,d\,x}}{64\,a\,d}+\frac{\mathrm{e}^{-2\,c-2\,d\,x}\,(a+b)}{8\,a^2\,d}-\frac{\mathrm{e}^{2\,c+2\,d\,x}\,(a+b)}{8\,a^2\,d}+\frac{\sqrt{b}\,\ln\left(\frac{4\,b\,(a+b)^3\,\left(2\,a\,b+a^2+a^2\,\mathrm{e}^{2\,c+}\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^4/(a + b/cosh(c + d*x)^2),x)
```

```
[Out] (x*(12*a*b + 3*a^2 + 8*b^2))/(8*a^3) - exp(- 4*c - 4*d*x)/(64*a*d) + exp(4*
c + 4*d*x)/(64*a*d) + (exp(- 2*c - 2*d*x)*(a + b))/(8*a^2*d) - (exp(2*c + 2
*d*x)*(a + b))/(8*a^2*d) + (b^(1/2)*log((4*b*(a + b)^3*(2*a*b + a^2 + a^2*e
xp(2*c + 2*d*x) + 8*b^2*exp(2*c + 2*d*x) + 8*a*b*exp(2*c + 2*d*x)))/a^8 - (
8*b^(3/2)*(a + b)^(7/2)*(a + 2*a*exp(2*c + 2*d*x) + 4*b*exp(2*c + 2*d*x)))/
a^8)*(a + b)^(3/2))/(2*a^3*d) - (b^(1/2)*log((8*b^(3/2)*(a + b)^(7/2)*(a +
2*a*exp(2*c + 2*d*x) + 4*b*exp(2*c + 2*d*x)))/a^8 + (4*b*(a + b)^3*(2*a*b +
 a^2 + a^2*exp(2*c + 2*d*x) + 8*b^2*exp(2*c + 2*d*x) + 8*a*b*exp(2*c + 2*d*
x)))/a^8)*(a + b)^(3/2))/(2*a^3*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^4(c+dx)}{a+b\,\mathrm{sech}^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**4/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(sinh(c + d*x)**4/(a + b*sech(c + d*x)**2), x)
```

3.26 $\int \frac{\sinh^3(c+dx)}{a+b\,\mathbf{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=71

$$\frac{\sqrt{b}\,(a+b)\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{a^{5/2}d}-\frac{(a+b)\cosh(c+dx)}{a^2d}+\frac{\cosh^3(c+dx)}{3ad}$$

```
[Out] -(a+b)*cosh(d*x+c)/a^2/d+1/3*cosh(d*x+c)^3/a/d+(a+b)*arctan(cosh(d*x+c)*a^(
1/2)/b^(1/2))*b^(1/2)/a^(5/2)/d
```

Rubi [A] time = 0.10, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4133, 459, 321, 205}

$$-\frac{(a+b)\cosh(c+dx)}{a^2d}+\frac{\sqrt{b}\,(a+b)\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{a^{5/2}d}+\frac{\cosh^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] (Sqrt[b]*(a + b)*ArcTan[(Sqrt[a]*Cosh[c + d*x])/Sqrt[b]])/(a^(5/2)*d) - ((a
 + b)*Cosh[c + d*x])/(a^2*d) + Cosh[c + d*x]^3/(3*a*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
```

```
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sinh^3(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{x^2(1-x^2)}{b+ax^2}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= \frac{\cosh^3(c+dx)}{3ad} - \frac{(a+b)\,\text{Subst}\left(\int \frac{x^2}{b+ax^2}\,dx, x, \cosh(c+dx)\right)}{ad}$$

$$= -\frac{(a+b)\cosh(c+dx)}{a^2d} + \frac{\cosh^3(c+dx)}{3ad} + \frac{(b(a+b))\,\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, \cosh(c+d$$

$$= \frac{\sqrt{b}\,(a+b)\tan^{-1}\left(\frac{\sqrt{a}\,\cosh(c+dx)}{\sqrt{b}}\right)}{a^{5/2}d} - \frac{(a+b)\cosh(c+dx)}{a^2d} + \frac{\cosh^3(c+dx)}{3ad}$$

Mathematica [C] time = 2.23, size = 372, normalized size = 5.24

$$\frac{(a\cosh(2(c+dx))+a+2b)\left(2a^{3/2}\sqrt{b}\,\cosh(3(c+dx))+3\left(a^2+8ab+8b^2\right)\tan^{-1}\left(\frac{\sinh(c)\tanh\left(\frac{dx}{2}\right)\left(\sqrt{a}-i\sqrt{a+b}\,\sqrt{(\cosh(}{}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*(3*(a^2 + 8*a*b + 8*b^2)*ArcTan[((Sqrt[a]
- I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c
]*(Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt
[b]] + 3*(a^2 + 8*a*b + 8*b^2)*ArcTan[((Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cosh[
c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] + I*Sqrt[a + b]*
Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]] - 3*a^2*(ArcTan[(Sqrt[
```

```
a] - I*Sqrt[a + b]*Tanh[(c + d*x)/2])/Sqrt[b]] + ArcTan[(Sqrt[a] + I*Sqrt[a
 + b]*Tanh[(c + d*x)/2])/Sqrt[b]]) - 6*Sqrt[a]*Sqrt[b]*(3*a + 4*b)*Cosh[c +
 d*x] + 2*a^(3/2)*Sqrt[b]*Cosh[3*(c + d*x)]))/(48*a^(5/2)*Sqrt[b]*d*(b + a*
Cosh[c + d*x]^2))
```

fricas [B] time = 0.49, size = 1246, normalized size = 17.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/24*(a*cosh(d*x + c)^6 + 6*a*cosh(d*x + c)*sinh(d*x + c)^5 + a*sinh(d*x +
 c)^6 - 3*(3*a + 4*b)*cosh(d*x + c)^4 + 3*(5*a*cosh(d*x + c)^2 - 3*a - 4*b)
*sinh(d*x + c)^4 + 4*(5*a*cosh(d*x + c)^3 - 3*(3*a + 4*b)*cosh(d*x + c))*si
nh(d*x + c)^3 - 3*(3*a + 4*b)*cosh(d*x + c)^2 + 3*(5*a*cosh(d*x + c)^4 - 6*
(3*a + 4*b)*cosh(d*x + c)^2 - 3*a - 4*b)*sinh(d*x + c)^2 + 12*((a + b)*cosh
(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)^2*sinh(d*x + c) + 3*(a + b)*cosh(d*x
+ c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3)*sqrt(-b/a)*log((a*cosh(d*x
+ c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a - 2*b
)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a - 2*b)*sinh(d*x + c)^2 + 4*(
a*cosh(d*x + c)^3 + (a - 2*b)*cosh(d*x + c))*sinh(d*x + c) + 4*(a*cosh(d*x
+ c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 + a*cosh(d*x
 + c) + (3*a*cosh(d*x + c)^2 + a)*sinh(d*x + c))*sqrt(-b/a) + a)/(a*cosh(d*
x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2
*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4
*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) + 6*(a*c
osh(d*x + c)^5 - 2*(3*a + 4*b)*cosh(d*x + c)^3 - (3*a + 4*b)*cosh(d*x + c))
*sinh(d*x + c) + a)/(a^2*d*cosh(d*x + c)^3 + 3*a^2*d*cosh(d*x + c)^2*sinh(d
*x + c) + 3*a^2*d*cosh(d*x + c)*sinh(d*x + c)^2 + a^2*d*sinh(d*x + c)^3), 1
/24*(a*cosh(d*x + c)^6 + 6*a*cosh(d*x + c)*sinh(d*x + c)^5 + a*sinh(d*x + c
)^6 - 3*(3*a + 4*b)*cosh(d*x + c)^4 + 3*(5*a*cosh(d*x + c)^2 - 3*a - 4*b)*s
inh(d*x + c)^4 + 4*(5*a*cosh(d*x + c)^3 - 3*(3*a + 4*b)*cosh(d*x + c))*sinh
(d*x + c)^3 - 3*(3*a + 4*b)*cosh(d*x + c)^2 + 3*(5*a*cosh(d*x + c)^4 - 6*(3
*a + 4*b)*cosh(d*x + c)^2 - 3*a - 4*b)*sinh(d*x + c)^2 - 24*((a + b)*cosh(d
*x + c)^3 + 3*(a + b)*cosh(d*x + c)^2*sinh(d*x + c) + 3*(a + b)*cosh(d*x +
c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3)*sqrt(b/a)*arctan(1/2*(a*cosh(
d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 + (a + 4
*b)*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + a + 4*b)*sinh(d*x + c))*sqrt(b/a
)/b) + 24*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)^2*sinh(d*x + c
) + 3*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3)*sqrt
(b/a)*arctan(1/2*(a*cosh(d*x + c) + a*sinh(d*x + c))*sqrt(b/a)/b) + 6*(a*co
sh(d*x + c)^5 - 2*(3*a + 4*b)*cosh(d*x + c)^3 - (3*a + 4*b)*cosh(d*x + c))*
sinh(d*x + c) + a)/(a^2*d*cosh(d*x + c)^3 + 3*a^2*d*cosh(d*x + c)^2*sinh(d*
x + c) + 3*a^2*d*cosh(d*x + c)*sinh(d*x + c)^2 + a^2*d*sinh(d*x + c)^3)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[31,78]Warning, need to choose a branch for the root of a po
lynomial with parameters. This might be wrong.The choice was done assuming
[a,b]=[-13,-93]Warning, need to choose a branch for the root of a polynomia
l with parameters. This might be wrong.The choice was done assuming [a,b]=[
-65,-82]Warning, need to choose a branch for the root of a polynomial with
parameters. This might be wrong.The choice was done assuming [a,b]=[97,-56]
Warning, need to choose a branch for the root of a polynomial with paramete
rs. This might be wrong.The choice was done assuming [a,b]=[80,44]Warning,
need to choose a branch for the root of a polynomial with parameters. This
might be wrong.The choice was done assuming [a,b]=[22,73]Warning, need to c
hoose a branch for the root of a polynomial with parameters. This might be
wrong.The choice was done assuming [a,b]=[36,86]Warning, need to choose a b
ranch for the root of a polynomial with parameters. This might be wrong.The
 choice was done assuming [a,b]=[-59,-45]Warning, need to choose a branch f
or the root of a polynomial with parameters. This might be wrong.The choice
 was done assuming [a,b]=[15,66]Warning, need to choose a branch for the ro
ot of a polynomial with parameters. This might be wrong.The choice was done
 assuming [a,b]=[55,80]Undef/Unsigned Inf encountered in limitEvaluation ti
me: 1.19Limit: Max order reached or unable to make series expansion Error:
Bad Argument Value
```

maple [B] time = 0.34, size = 261, normalized size = 3.68

$$-\frac{1}{3da\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{2da\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{1}{2da\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{b}{d\,a^{2}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{}{3da\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3/(a+b*sech(d*x+c)^2),x)
```

```
[Out] -1/3/d/a/(tanh(1/2*d*x+1/2*c)-1)^3-1/2/d/a/(tanh(1/2*d*x+1/2*c)-1)^2+1/2/d/
a/(tanh(1/2*d*x+1/2*c)-1)+1/d/a^2/(tanh(1/2*d*x+1/2*c)-1)*b+1/3/d/a/(tanh(1
```

```
/2*d*x+1/2*c)+1)^3-1/2/d/a/(tanh(1/2*d*x+1/2*c)+1)^2-1/2/d/a/(tanh(1/2*d*x+
1/2*c)+1)-1/d/a^2/(tanh(1/2*d*x+1/2*c)+1)*b+1/d*b/a/(a*b)^(1/2)*arctan(1/4*
(2*(a+b)*tanh(1/2*d*x+1/2*c)^2+2*a-2*b)/(a*b)^(1/2))+1/d*b^2/a^2/(a*b)^(1/2
)*arctan(1/4*(2*(a+b)*tanh(1/2*d*x+1/2*c)^2+2*a-2*b)/(a*b)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(3\left(3\,ae^{(4\,c)}+4\,be^{(4\,c)}\right)e^{(4\,dx)}+3\left(3\,ae^{(2\,c)}+4\,be^{(2\,c)}\right)e^{(2\,dx)}-ae^{(6\,dx+6\,c)}-a\right)e^{(-3\,dx-3\,c)}}{24\,a^2d}+\frac{1}{8}\int\frac{16\left(\left(abe^{(3\,c)}+b^2e^{(3\,c}\cdots\right.\right.}{a^3e^{(4\,dx+4\,c)}+a^3+2\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/24*(3*(3*a*e^(4*c) + 4*b*e^(4*c))*e^(4*d*x) + 3*(3*a*e^(2*c) + 4*b*e^(2*
c))*e^(2*d*x) - a*e^(6*d*x + 6*c) - a)*e^(-3*d*x - 3*c)/(a^2*d) + 1/8*integ
rate(16*((a*b*e^(3*c) + b^2*e^(3*c))*e^(3*d*x) - (a*b*e^c + b^2*e^c)*e^(d*x
))/(a^3*e^(4*d*x + 4*c) + a^3 + 2*(a^3*e^(2*c) + 2*a^2*b*e^(2*c))*e^(2*d*x)
), x)
```

mupad [B] time = 1.84, size = 473, normalized size = 6.66

$$\frac{e^{-3\,c-3\,d\,x}}{24\,a\,d}-\frac{\left(2\,\mathrm{atan}\left(\frac{a^6\,e^{d\,x}\,e^{c}\left(\frac{4\left(2\,a^4\,b\,d\,\sqrt{a^2\,b+2\,a\,b^2+b^3}+2\,a^2\,b^3\,d\,\sqrt{a^2\,b+2\,a\,b^2+b^3}+4\,a^3\,b^2\,d\,\sqrt{a^2\,b+2\,a\,b^2+b^3}\right)}{a^{11}\,d^2\,(a+b)}+\frac{2\left(b^4\,\sqrt{a^5\,d^2}+3\,a^2\,b^2\,\sqrt{a^5\,d^2}+3\,a\,b^3\,\sqrt{a^5\,d^2}\cdots\right.}{a^8\,d\,\sqrt{b\,(a+b)^2}\,\sqrt{a^5\,d^2}\cdots}\right.}{4\,a^2\,b+8\,a\,b^2+4\,b^3}\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^3/(a + b/cosh(c + d*x)^2),x)
```

```
[Out] exp(- 3*c - 3*d*x)/(24*a*d) - ((2*atan((a^6*exp(d*x)*exp(c)*((4*(2*a^4*b*d*
(2*a*b^2 + a^2*b + b^3)^(1/2) + 2*a^2*b^3*d*(2*a*b^2 + a^2*b + b^3)^(1/2) +
 4*a^3*b^2*d*(2*a*b^2 + a^2*b + b^3)^(1/2)))/(a^11*d^2*(a + b)) + (2*(b^4*(
a^5*d^2)^(1/2) + 3*a^2*b^2*(a^5*d^2)^(1/2) + 3*a*b^3*(a^5*d^2)^(1/2) + a^3*
b*(a^5*d^2)^(1/2)))/(a^8*d*(b*(a + b)^2)^(1/2)*(a^5*d^2)^(1/2)))*(a^5*d^2)^
(1/2))/(8*a*b^2 + 4*a^2*b + 4*b^3) + (2*exp(3*c)*exp(3*d*x)*(b^4*(a^5*d^2)^
(1/2) + 3*a^2*b^2*(a^5*d^2)^(1/2) + 3*a*b^3*(a^5*d^2)^(1/2) + a^3*b*(a^5*d^
2)^(1/2)))/(a^2*d*(b*(a + b)^2)^(1/2)*(8*a*b^2 + 4*a^2*b + 4*b^3))) - 2*ata
n((exp(d*x)*exp(c)*(a + b)*(a^5*d^2)^(1/2))/(2*a^2*d*(b*(a + b)^2)^(1/2))))
*(2*a*b^2 + a^2*b + b^3)^(1/2))/(2*(a^5*d^2)^(1/2)) + exp(3*c + 3*d*x)/(24*
a*d) - (exp(c + d*x)*(3*a + 4*b))/(8*a^2*d) - (exp(- c - d*x)*(3*a + 4*b))/
(8*a^2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^3(c+dx)}{a+b\,\mathrm{sech}^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(sinh(c + d*x)**3/(a + b*sech(c + d*x)**2), x)
```

3.27 $\int \frac{\sinh^2(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=75

$$\frac{\sqrt{b}\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a^2d}-\frac{x(a+2b)}{2a^2}+\frac{\sinh(c+dx)\cosh(c+dx)}{2ad}$$

```
[Out] -1/2*(a+2*b)*x/a^2+1/2*cosh(d*x+c)*sinh(d*x+c)/a/d+arctanh(b^(1/2)*tanh(d*x
+c)/(a+b)^(1/2))*b^(1/2)*(a+b)^(1/2)/a^2/d
```

Rubi [A] time = 0.11, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4132, 471, 522, 206, 208}

$$\frac{\sqrt{b}\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a^2d}-\frac{x(a+2b)}{2a^2}+\frac{\sinh(c+dx)\cosh(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -((a + 2*b)*x)/(2*a^2) + (Sqrt[b]*Sqrt[a + b]*ArcTanh[(Sqrt[b]*Tanh[c + d*x
])/Sqrt[a + b]])/(a^2*d) + (Cosh[c + d*x]*Sinh[c + d*x])/(2*a*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
```

```
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sinh^2(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1-x^2)^2(a+b-bx^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2ad} - \frac{\text{Subst}\left(\int \frac{a+b+bx^2}{(1-x^2)(a+b-bx^2)}\,dx, x, \tanh(c+dx)\right)}{2ad}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2ad} + \frac{(b(a+b))\,\text{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \tanh(c+dx)\right)}{a^2d} - \frac{(a+$$

$$= -\frac{(a+2b)x}{2a^2} + \frac{\sqrt{b}\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a^2d} + \frac{\cosh(c+dx)\sinh(c+dx)}{2ad}$$

Mathematica [B] time = 1.00, size = 236, normalized size = 3.15

$$\frac{\text{sech}^2(c+dx)(a\cosh(2(c+dx))+a+2b)\left(\frac{\frac{(a^2+8ab+8b^2)(\cosh(2c)-\sinh(2c))\tanh^{-1}\left(\frac{(\cosh(2c)-\sinh(2c))\text{sech}(dx)((a+2b)\sinh(dx)-a\sinh(2c+dx))}{2\sqrt{a+b}\sqrt{b(\cosh(c)-\sinh(c))^4}}\right)}{d\sqrt{a+b}\sqrt{b(\cosh(c)-\sinh(c))^4}}-4}{a^2}\right.}{16\left(a+b\text{sech}^2(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*(-(ArcTanh[(Sqrt[b]*Tanh[c
 + d*x])/Sqrt[a + b]]/(Sqrt[b]*Sqrt[a + b]*d)) + (-4*(a + 2*b)*x + ((a^2 +
8*a*b + 8*b^2)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d
*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(C
osh[2*c] - Sinh[2*c]))/(Sqrt[a + b]*d*Sqrt[b*(Cosh[c] - Sinh[c])^4]) + (2*a
*Cosh[2*d*x]*Sinh[2*c])/d + (2*a*Cosh[2*c]*Sinh[2*d*x])/d)/a^2))/(16*(a + b
*Sech[c + d*x]^2))
```

fricas [B] time = 0.48, size = 805, normalized size = 10.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(a + 2*b)*d*x*cosh(d*x + c)^2 - a*cosh(d*x + c)^4 - 4*a*cosh(d*x +
 c)*sinh(d*x + c)^3 - a*sinh(d*x + c)^4 + 2*(2*(a + 2*b)*d*x - 3*a*cosh(d*x
 + c)^2)*sinh(d*x + c)^2 - 4*sqrt(a*b + b^2)*(cosh(d*x + c)^2 + 2*cosh(d*x
+ c)*sinh(d*x + c) + sinh(d*x + c)^2)*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh
(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x
+ c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*
a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*
x + c) - 4*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*
x + c)^2 + a + 2*b)*sqrt(a*b + b^2))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)
*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a
*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2
*b)*cosh(d*x + c))*sinh(d*x + c) + a)) + 4*(2*(a + 2*b)*d*x*cosh(d*x + c) -
 a*cosh(d*x + c)^3)*sinh(d*x + c) + a)/(a^2*d*cosh(d*x + c)^2 + 2*a^2*d*cos
h(d*x + c)*sinh(d*x + c) + a^2*d*sinh(d*x + c)^2), -1/8*(4*(a + 2*b)*d*x*co
sh(d*x + c)^2 - a*cosh(d*x + c)^4 - 4*a*cosh(d*x + c)*sinh(d*x + c)^3 - a*s
inh(d*x + c)^4 + 2*(2*(a + 2*b)*d*x - 3*a*cosh(d*x + c)^2)*sinh(d*x + c)^2
- 8*sqrt(-a*b - b^2)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sin
h(d*x + c)^2)*arctan(1/2*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x +
c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-a*b - b^2)/(a*b + b^2)) + 4*(2*(a +
 2*b)*d*x*cosh(d*x + c) - a*cosh(d*x + c)^3)*sinh(d*x + c) + a)/(a^2*d*cosh
(d*x + c)^2 + 2*a^2*d*cosh(d*x + c)*sinh(d*x + c) + a^2*d*sinh(d*x + c)^2)]
```

giac [B] time = 1.60, size = 132, normalized size = 1.76

$$-\frac{\frac{4\,(dx+c)(a+2\,b)}{a^2}-\frac{e^{(2\,dx+2\,c)}}{a}-\frac{\left(2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}-a\right)e^{(-2\,dx-2\,c)}}{a^2}-\frac{8\left(ab+b^2\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}\,a^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/8*(4*(d*x + c)*(a + 2*b)/a^2 - e^(2*d*x + 2*c)/a - (2*a*e^(2*d*x + 2*c)
+ 4*b*e^(2*d*x + 2*c) - a)*e^(-2*d*x - 2*c)/a^2 - 8*(a*b + b^2)*arctan(1/2*
(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/(sqrt(-a*b - b^2)*a^2))/d
```

maple [B] time = 0.32, size = 383, normalized size = 5.11

$$\frac{1}{2da\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{1}{2da\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2da}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)b}{d\,a^{2}}-\frac{}{2da\left(\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2/(a+b*sech(d*x+c)^2),x)
```

```
[Out] 1/2/d/a/(tanh(1/2*d*x+1/2*c)-1)^2+1/2/d/a/(tanh(1/2*d*x+1/2*c)-1)+1/2/d/a*l
n(tanh(1/2*d*x+1/2*c)-1)+1/d/a^2*ln(tanh(1/2*d*x+1/2*c)-1)*b-1/2/d/a/(tanh(
1/2*d*x+1/2*c)+1)^2+1/2/d/a/(tanh(1/2*d*x+1/2*c)+1)-1/2/d/a*ln(tanh(1/2*d*x
+1/2*c)+1)-1/d/a^2*ln(tanh(1/2*d*x+1/2*c)+1)*b-1/2/d/a*b^(1/2)/(a+b)^(1/2)*
ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1
/2))+1/2/d/a*b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(
1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))-1/2/d/a^2*b^(3/2)/(a+b)^(1/2)*ln((a+b
)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))+1/
2/d/a^2*b^(3/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*
tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))
```

maxima [B] time = 0.42, size = 352, normalized size = 4.69

$$-\frac{b\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{4\,\sqrt{(a+b)b}\,ad}-\frac{dx+c}{2\,ad}+\frac{e^{(2\,dx+2\,c)}}{8\,ad}-\frac{e^{(-2\,dx-2\,c)}}{8\,ad}-\frac{b\log\left(ae^{(4\,dx+4\,c)}+2\,(a+2\,b)e^{(2\,dx+2\,c)}+a\right)}{4\,a^{2}d}+\frac{b\,\mathrm{lo}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/4*b*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x
- 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*a*d) - 1/2*(d*x + c
)/(a*d) + 1/8*e^(2*d*x + 2*c)/(a*d) - 1/8*e^(-2*d*x - 2*c)/(a*d) - 1/4*b*lo
g(a*e^(4*d*x + 4*c) + 2*(a + 2*b)*e^(2*d*x + 2*c) + a)/(a^2*d) + 1/4*b*log(
2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/(a^2*d) + 1/8*(a*b +
 2*b^2)*log((a*e^(2*d*x + 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(2*d*x +
 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*a^2*d) - 1/8*(a*b +
```

```
2*b^2)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x
- 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*a^2*d)
```

mupad [B] time = 2.03, size = 276, normalized size = 3.68

$$\frac{e^{2c+2dx}}{8ad}-\frac{e^{-2c-2dx}}{8ad}-\frac{x\,(a+2b)}{2a^2}-\frac{\sqrt{b}\,\ln\left(2ab+a^2+a^2\,e^{2c+2dx}+8b^2\,e^{2c+2dx}-2a\sqrt{b}\,\sqrt{a+b}-8b^{3/2}\,e^{2c+2dx}\right.}{2a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^2/(a + b/cosh(c + d*x)^2),x)
```

```
[Out] exp(2*c + 2*d*x)/(8*a*d) - exp(- 2*c - 2*d*x)/(8*a*d) - (x*(a + 2*b))/(2*a^
2) - (b^(1/2)*log(2*a*b + a^2 + a^2*exp(2*c + 2*d*x) + 8*b^2*exp(2*c + 2*d*
x) - 2*a*b^(1/2)*(a + b)^(1/2) - 8*b^(3/2)*exp(2*c + 2*d*x)*(a + b)^(1/2) +
 8*a*b*exp(2*c + 2*d*x) - 4*a*b^(1/2)*exp(2*c + 2*d*x)*(a + b)^(1/2))*(a +
b)^(1/2))/(2*a^2*d) + (b^(1/2)*log(2*a*b + a^2 + a^2*exp(2*c + 2*d*x) + 8*b
^2*exp(2*c + 2*d*x) + 2*a*b^(1/2)*(a + b)^(1/2) + 8*b^(3/2)*exp(2*c + 2*d*x
)*(a + b)^(1/2) + 8*a*b*exp(2*c + 2*d*x) + 4*a*b^(1/2)*exp(2*c + 2*d*x)*(a
+ b)^(1/2))*(a + b)^(1/2))/(2*a^2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^2(c+dx)}{a+b\,\mathrm{sech}^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(sinh(c + d*x)**2/(a + b*sech(c + d*x)**2), x)
```

3.28 $\int \frac{\sinh(c+dx)}{a+b\,\mathbf{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=47

$$\frac{\cosh(c+dx)}{ad}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{a^{3/2}d}$$

```
[Out] cosh(d*x+c)/a/d-arctan(cosh(d*x+c)*a^(1/2)/b^(1/2))*b^(1/2)/a^(3/2)/d
```

Rubi [A] time = 0.05, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4133, 321, 205}

$$\frac{\cosh(c+dx)}{ad}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -((Sqrt[b]*ArcTan[(Sqrt[a]*Cosh[c + d*x])/Sqrt[b]])/(a^(3/2)*d)) + Cosh[c +
 d*x]/(a*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sinh(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx = \frac{\operatorname{Subst}\left(\int \frac{x^2}{b+ax^2}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= \frac{\cosh(c+dx)}{ad} - \frac{b\operatorname{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, \cosh(c+dx)\right)}{ad}$$
$$= -\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{a^{3/2}d} + \frac{\cosh(c+dx)}{ad}$$

Mathematica [C] time = 1.02, size = 328, normalized size = 6.98

$$\frac{\operatorname{sech}^2(c+dx)(a\cosh(2(c+dx))+a+2b)\left(\frac{a\left(\tan^{-1}\left(\frac{\sqrt{a}-i\sqrt{a+b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)+\tan^{-1}\left(\frac{\sqrt{a}+i\sqrt{a+b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)\right)}{\sqrt{b}} - \frac{(a+4b)\left(\tan^{-1}\left(\frac{\sin}{}\right.\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((-(((a + 4*b)*(ArcTan[((Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2
])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] -
 Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]] + ArcTan[((Sqrt[a] + I*Sqrt[a + b]*Sq
rt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] + I*Sqr
t[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]]))/Sqrt[b]) +
(a*(ArcTan[(Sqrt[a] - I*Sqrt[a + b]*Tanh[(c + d*x)/2])/Sqrt[b]] + ArcTan[(S
qrt[a] + I*Sqrt[a + b]*Tanh[(c + d*x)/2])/Sqrt[b]]))/Sqrt[b] + 4*Sqrt[a]*Co
sh[c + d*x])*(a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2)/(8*a^(3/2)*d*
(a + b*Sech[c + d*x]^2))
```

fricas [B] time = 0.51, size = 595, normalized size = 12.66

$$\left[\frac{\sqrt{-\frac{b}{a}}\left(\cosh(dx+c)+\sinh(dx+c)\right)\log\left(\frac{a\cosh(dx+c)^4+4a\cosh(dx+c)\sinh(dx+c)^3+a\sinh(dx+c)^4+2(a-2b)\cosh(dx+c)^2+2\left(3a\cosh\right.}{a\cosh(dx+c)^4+4a\cosh(dx+c)\sinh}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-b/a)*(cosh(d*x + c) + sinh(d*x + c))*log((a*cosh(d*x + c)^4 + 4
*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a - 2*b)*cosh(d*x
 + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a - 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x
 + c)^3 + (a - 2*b)*cosh(d*x + c))*sinh(d*x + c) - 4*(a*cosh(d*x + c)^3 + 3
*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 + a*cosh(d*x + c) + (3
*a*cosh(d*x + c)^2 + a)*sinh(d*x + c))*sqrt(-b/a) + a)/(a*cosh(d*x + c)^4 +
 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d
*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d
*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) + cosh(d*x + c)^2
+ 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)/(a*d*cosh(d*x + c) +
 a*d*sinh(d*x + c)), 1/2*(2*sqrt(b/a)*(cosh(d*x + c) + sinh(d*x + c))*arcta
n(1/2*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x +
 c)^3 + (a + 4*b)*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + a + 4*b)*sinh(d*x
+ c))*sqrt(b/a)/b) - 2*sqrt(b/a)*(cosh(d*x + c) + sinh(d*x + c))*arctan(1/2
*(a*cosh(d*x + c) + a*sinh(d*x + c))*sqrt(b/a)/b) + cosh(d*x + c)^2 + 2*cos
h(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)/(a*d*cosh(d*x + c) + a*d*si
nh(d*x + c))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[31,78]Warning, need to choose a branch for the root of a po
lynomial with parameters. This might be wrong.The choice was done assuming
[a,b]=[-13,-93]Warning, need to choose a branch for the root of a polynomia
l with parameters. This might be wrong.The choice was done assuming [a,b]=[
-65,-82]Warning, need to choose a branch for the root of a polynomial with
parameters. This might be wrong.The choice was done assuming [a,b]=[97,-56]
Warning, need to choose a branch for the root of a polynomial with paramete
rs. This might be wrong.The choice was done assuming [a,b]=[80,44]Warning,
need to choose a branch for the root of a polynomial with parameters. This
might be wrong.The choice was done assuming [a,b]=[22,73]Undef/Unsigned Inf
 encountered in limitEvaluation time: 0.7Limit: Max order reached or unable
 to make series expansion Error: Bad Argument Value
```

maple [A] time = 0.08, size = 44, normalized size = 0.94

$$\frac{b\arctan\left(\frac{\operatorname{sech}(dx+c)b}{\sqrt{ab}}\right)}{da\sqrt{ab}}+\frac{1}{da\operatorname{sech}(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)/(a+b*sech(d*x+c)^2),x)
```

```
[Out] 1/d/a*b/(a*b)^(1/2)*arctan(sech(d*x+c)*b/(a*b)^(1/2))+1/d/a/sech(d*x+c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(e^{(2\,dx+2\,c)}+1\right)e^{(-dx-c)}}{2\,ad}-\frac{1}{2}\int\frac{4\left(be^{(3\,dx+3\,c)}-be^{(dx+c)}\right)}{a^2e^{(4\,dx+4\,c)}+a^2+2\left(a^2e^{(2\,c)}+2\,abe^{(2\,c)}\right)e^{(2\,dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*(e^(2*d*x + 2*c) + 1)*e^(-d*x - c)/(a*d) - 1/2*integrate(4*(b*e^(3*d*x
+ 3*c) - b*e^(d*x + c))/(a^2*e^(4*d*x + 4*c) + a^2 + 2*(a^2*e^(2*c) + 2*a*b
*e^(2*c))*e^(2*d*x)), x)
```

mupad [B] time = 0.14, size = 42, normalized size = 0.89

$$\frac{\cosh\left(c+d\,x\right)}{a\,d}-\frac{b\,\mathrm{atan}\left(\frac{a\cosh(c+d\,x)}{\sqrt{a\,b}}\right)}{a\,d\,\sqrt{a\,b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)/(a + b/cosh(c + d*x)^2),x)
```

```
[Out] cosh(c + d*x)/(a*d) - (b*atan((a*cosh(c + d*x))/(a*b)^(1/2)))/(a*d*(a*b)^(1
/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sinh\left(c+dx\right)}{a+b\operatorname{sech}^2\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(sinh(c + d*x)/(a + b*sech(c + d*x)**2), x)
```

3.29 $\int \frac{\text{csch}(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=55

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{\sqrt{a}\,d(a+b)}-\frac{\tanh^{-1}(\cosh(c+dx))}{d(a+b)}$$

```
[Out] -arctanh(cosh(d*x+c))/(a+b)/d+arctan(cosh(d*x+c)*a^(1/2)/b^(1/2))*b^(1/2)/(
a+b)/d/a^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {4133, 481, 206, 205}

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{\sqrt{a}\,d(a+b)}-\frac{\tanh^{-1}(\cosh(c+dx))}{d(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] (Sqrt[b]*ArcTan[(Sqrt[a]*Cosh[c + d*x])/Sqrt[b]])/(Sqrt[a]*(a + b)*d) - Arc
Tanh[Cosh[c + d*x]]/((a + b)*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1]
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx = -\frac{\operatorname{Subst}\left(\int \frac{x^2}{(1-x^2)(b+ax^2)}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cosh(c+dx)\right)}{(a+b)d} + \frac{b\operatorname{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, \cosh(c+dx)\right)}{(a+b)d}$$

$$= \frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{\sqrt{a}(a+b)d} - \frac{\tanh^{-1}(\cosh(c+dx))}{(a+b)d}$$

Mathematica [C] time = 0.93, size = 232, normalized size = 4.22

$$\frac{\frac{\sqrt{b}\tan^{-1}\left(\frac{\sinh(c)\tanh\left(\frac{dx}{2}\right)\left(\sqrt{a}-i\sqrt{a+b}\sqrt{(\cosh(c)-\sinh(c))^2}\right)+\cosh(c)\left(\sqrt{a}-i\sqrt{a+b}\sqrt{(\cosh(c)-\sinh(c))^2}\tanh\left(\frac{dx}{2}\right)\right)}{\sqrt{b}}\right)}{\sqrt{a}} + \frac{\sqrt{b}\tan^{-1}\left(\frac{\sinh(c)\tanh\left(\frac{dx}{2}\right)\left(\sqrt{a}+i\sqrt{a+b}\sqrt{(c\ldots}\right.}{\ldots}\right.}{\ldots}}{d(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((Sqrt[b]*ArcTan[((Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2])*Sin
h[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[
c])^2]*Tanh[(d*x)/2]))/Sqrt[b]])/Sqrt[a] + (Sqrt[b]*ArcTan[((Sqrt[a] + I*Sq
rt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqr
t[a] + I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]])/
Sqrt[a] - Log[Cosh[(c + d*x)/2]] + Log[Sinh[(c + d*x)/2]])/((a + b)*d)
```

fricas [B] time = 0.51, size = 533, normalized size = 9.69

$$\left[\frac{\sqrt{-\frac{b}{a}}\log\left(\frac{a\cosh(dx+c)^4+4\,a\cosh(dx+c)\sinh(dx+c)^3+a\sinh(dx+c)^4+2\,(a-2\,b)\cosh(dx+c)^2+2\left(3\,a\cosh(dx+c)^2+a-2\,b\right)\sinh(dx+c)^2+4\left(a\cosh(\right.}{a\cosh(dx+c)^4+4\,a\cosh(dx+c)\sinh(dx+c)^3+a\sinh(dx+c)^4+2\,(a+2\,b)\cosh(}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-b/a)*log((a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3
 + a*sinh(d*x + c)^4 + 2*(a - 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2
 + a - 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a - 2*b)*cosh(d*x + c
))*sinh(d*x + c) + 4*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2
 + a*sinh(d*x + c)^3 + a*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + a)*sinh(d*x
 + c))*sqrt(-b/a) + a)/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)
^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)
^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x +
 c))*sinh(d*x + c) + a)) - 2*log(cosh(d*x + c) + sinh(d*x + c) + 1) + 2*log
(cosh(d*x + c) + sinh(d*x + c) - 1))/((a + b)*d), -(sqrt(b/a)*arctan(1/2*(a
*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 +
(a + 4*b)*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + a + 4*b)*sinh(d*x + c))*sq
rt(b/a)/b) - sqrt(b/a)*arctan(1/2*(a*cosh(d*x + c) + a*sinh(d*x + c))*sqrt(
b/a)/b) + log(cosh(d*x + c) + sinh(d*x + c) + 1) - log(cosh(d*x + c) + sinh
(d*x + c) - 1))/((a + b)*d)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[31,78]Warning, need to choose a branch for the root of a po
lynomial with parameters. This might be wrong.The choice was done assuming
[a,b]=[-13,-93]Undef/Unsigned Inf encountered in limitLimit: Max order reac
hed or unable to make series expansion Error: Bad Argument Value
```

maple [A] time = 0.25, size = 67, normalized size = 1.22

$$\frac{b\arctan\left(\frac{2(a+b)\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a-2b}{4\sqrt{ab}}\right)}{d\,(a+b)\,\sqrt{ab}}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)/(a+b*sech(d*x+c)^2),x)
```

```
[Out] 1/d*b/(a+b)/(a*b)^(1/2)*arctan(1/4*(2*(a+b)*tanh(1/2*d*x+1/2*c)^2+2*a-2*b)/
(a*b)^(1/2))+1/d/(a+b)*ln(tanh(1/2*d*x+1/2*c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\log\left(\left(e^{(dx+c)}+1\right)e^{(-c)}\right)}{ad+bd}+\frac{\log\left(\left(e^{(dx+c)}-1\right)e^{(-c)}\right)}{ad+bd}+2\int\frac{be^{(3\,dx+3\,c)}-be^{(dx+c)}}{a^2+ab+\left(a^2e^{(4\,c)}+abe^{(4\,c)}\right)e^{(4\,dx)}+2\left(a^2e^{(2\,c)}+3\,abe^{(2\,c)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -log((e^(d*x + c) + 1)*e^(-c))/(a*d + b*d) + log((e^(d*x + c) - 1)*e^(-c))/
(a*d + b*d) + 2*integrate((b*e^(3*d*x + 3*c) - b*e^(d*x + c))/(a^2 + a*b +
(a^2*e^(4*c) + a*b*e^(4*c))*e^(4*d*x) + 2*(a^2*e^(2*c) + 3*a*b*e^(2*c) + 2*
b^2*e^(2*c))*e^(2*d*x)), x)
```

mupad [B] time = 2.18, size = 616, normalized size = 11.20

$$-\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(b^4\,\sqrt{-a^2\,d^2-2\,a\,b\,d^2-b^2\,d^2}+16\,a^2\,b^2\,\sqrt{-a^2\,d^2-2\,a\,b\,d^2-b^2\,d^2}+8\,a\,b^3\,\sqrt{-a^2\,d^2-2\,a\,b\,d^2-b^2\,d^2}\right)}{16\,d\,a^3\,b^2+24\,d\,a^2\,b^3+9\,d\,a\,b^4+d\,b^5}\right)}{\sqrt{-a^2\,d^2-2\,a\,b\,d^2-b^2\,d^2}}-\frac{\sqrt{b}\,\left(2\,\mathrm{atan}\left(\frac{\left(\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\left(\frac{}{a^5\,(a\cdots}\right.\right.}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)*(a + b/cosh(c + d*x)^2)),x)
```

```
[Out] - (2*atan((exp(d*x)*exp(c)*(b^4*(- a^2*d^2 - b^2*d^2 - 2*a*b*d^2)^(1/2) + 1
6*a^2*b^2*(- a^2*d^2 - b^2*d^2 - 2*a*b*d^2)^(1/2) + 8*a*b^3*(- a^2*d^2 - b^
2*d^2 - 2*a*b*d^2)^(1/2)))/(b^5*d + 24*a^2*b^3*d + 16*a^3*b^2*d + 9*a*b^4*d
)))/(- a^2*d^2 - b^2*d^2 - 2*a*b*d^2)^(1/2) - (b^(1/2)*(2*atan(((exp(d*x)*e
xp(c)*((64*(2*b^(7/2)*d + 8*a^2*b^(3/2)*d + 10*a*b^(5/2)*d))/(a^5*(a + b)*(
```

```
a*d^2*(a + b)^2)^(1/2)*(a^3*d^2 + a*b^2*d^2 + 2*a^2*b*d^2)^(1/2)) + (32*(b^
2*(a^3*d^2 + a*b^2*d^2 + 2*a^2*b*d^2)^(1/2) + 4*a*b*(a^3*d^2 + a*b^2*d^2 +
2*a^2*b*d^2)^(1/2)))/(a^5*b^(1/2)*d*(a + b)^2*(a^3*d^2 + a*b^2*d^2 + 2*a^2*
b*d^2)^(1/2))) + (32*exp(3*c)*exp(3*d*x)*(b^2*(a^3*d^2 + a*b^2*d^2 + 2*a^2*
b*d^2)^(1/2) + 4*a*b*(a^3*d^2 + a*b^2*d^2 + 2*a^2*b*d^2)^(1/2)))/(a^5*b^(1/
2)*d*(a + b)^2*(a^3*d^2 + a*b^2*d^2 + 2*a^2*b*d^2)^(1/2)))*(a^6*(a^3*d^2 +
a*b^2*d^2 + 2*a^2*b*d^2)^(1/2) + a^5*b*(a^3*d^2 + a*b^2*d^2 + 2*a^2*b*d^2)^
(1/2)))/(256*a*b + 64*b^2)) - 2*atan((exp(d*x)*exp(c)*(a*d^2*(a + b)^2)^(1/
2))/(2*b^(1/2)*d*(a + b)))))/(2*(a^3*d^2 + a*b^2*d^2 + 2*a^2*b*d^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(csch(c + d*x)/(a + b*sech(c + d*x)**2), x)
```

3.30 $\int \frac{\textbf{csch}^2(c+dx)}{a+b\,\textbf{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=53

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{d(a+b)^{3/2}}-\frac{\coth(c+dx)}{d(a+b)}$$

```
[Out] -coth(d*x+c)/(a+b)/d+arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))*b^(1/2)/(a+b)
^(3/2)/d
```

Rubi [A] time = 0.07, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4132, 325, 208}

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{d(a+b)^{3/2}}-\frac{\coth(c+dx)}{d(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/((a + b)^(3/2)*d) -
Coth[c + d*x]/((a + b)*d)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
```

```
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx=\frac{\operatorname{Subst}\left(\int \frac{1}{x^2\left(a+b-bx^2\right)}\,dx,x,\tanh(c+dx)\right)}{d}$$
$$=-\frac{\coth(c+dx)}{(a+b)d}+\frac{b\operatorname{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx,x,\tanh(c+dx)\right)}{(a+b)d}$$
$$=\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}d}-\frac{\coth(c+dx)}{(a+b)d}$$

Mathematica [B] time = 0.75, size = 179, normalized size = 3.38

$$\frac{\operatorname{sech}^2(c+dx)(a\cosh(2(c+dx))+a+2b)\left(\sqrt{a+b}\operatorname{csch}(c)\sinh(dx)\sqrt{b(\cosh(c)-\sinh(c))^4}\operatorname{csch}(c+dx)+b(\cos\right.}{2d(a+b)^{3/2}\sqrt{b(\cosh(c)-\sinh(c))^4}\,\left(a+b\operatorname{sech}^2\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*(b*ArcTanh[(Sech[d*x]*(Cos
h[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a +
b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(Cosh[2*c] - Sinh[2*c]) + Sqrt[a + b]*Cs
ch[c]*Csch[c + d*x]*Sqrt[b*(Cosh[c] - Sinh[c])^4]*Sinh[d*x]))/(2*(a + b)^(3
/2)*d*(a + b*Sech[c + d*x]^2)*Sqrt[b*(Cosh[c] - Sinh[c])^4])
```

fricas [B] time = 0.56, size = 588, normalized size = 11.09

$$\left[\frac{\left(\cosh(dx+c)^2+2\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^2-1\right)\sqrt{\frac{b}{a+b}}\log\left(\frac{a^2\cosh(dx+c)^4+4a^2\cosh(dx+c)\sinh(dx+\right.}{\right.}}{2\left((a\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 -
1)*sqrt(b/(a + b))*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x
+ c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*c
osh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^
2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) - 4*((a^2 +
a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a
*b)*sinh(d*x + c)^2 + a^2 + 3*a*b + 2*b^2)*sqrt(b/(a + b)))/(a*cosh(d*x + c
)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*c
osh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*c
osh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) - 4)/((a + b)
*d*cosh(d*x + c)^2 + 2*(a + b)*d*cosh(d*x + c)*sinh(d*x + c) + (a + b)*d*si
nh(d*x + c)^2 - (a + b)*d), ((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x +
c) + sinh(d*x + c)^2 - 1)*sqrt(-b/(a + b))*arctan(1/2*(a*cosh(d*x + c)^2 +
2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-b/(a +
 b))/b) - 2)/((a + b)*d*cosh(d*x + c)^2 + 2*(a + b)*d*cosh(d*x + c)*sinh(d*
x + c) + (a + b)*d*sinh(d*x + c)^2 - (a + b)*d)]
```

giac [A] time = 0.63, size = 75, normalized size = 1.42

$$\frac{\frac{b\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}\,(a+b)} - \frac{2}{(a+b)\left(e^{(2\,dx+2\,c)}-1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] (b*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/(sqrt(-a*b -
b^2)*(a + b)) - 2/((a + b)*(e^(2*d*x + 2*c) - 1)))/d
```

maple [B] time = 0.30, size = 147, normalized size = 2.77

$$-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,(a+b)} - \frac{1}{2d\,(a+b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)} - \frac{\sqrt{b}\,\ln\left(-\sqrt{a+b}\,\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\sqrt{b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\sqrt{a+b}}{2d\,(a+b)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2/(a+b*sech(d*x+c)^2),x)
```

```
[Out] -1/2/d/(a+b)*tanh(1/2*d*x+1/2*c)-1/2/d/(a+b)/tanh(1/2*d*x+1/2*c)-1/2/d*b^(1
/2)/(a+b)^(3/2)*ln(-(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*
x+1/2*c)-(a+b)^(1/2))+1/2/d*b^(1/2)/(a+b)^(3/2)*ln((a+b)^(1/2)*tanh(1/2*d*x
+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))
```

maxima [B] time = 0.43, size = 100, normalized size = 1.89

$$-\frac{b \log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{2\,\sqrt{(a+b)b}\,(a+b)d}+\frac{2}{\left((a+b)e^{(-2\,dx-2\,c)}-a-b\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/2*b*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x
- 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*(a + b)*d) + 2/(((a
 + b)*e^(-2*d*x - 2*c) - a - b)*d)
```

mupad [B] time = 2.47, size = 847, normalized size = 15.98

$$-\frac{2}{\left(e^{2\,c+2\,d\,x}-1\right)\,(a\,d+b\,d)}-\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\left(e^{2\,c}\,e^{2\,d\,x}\left(\frac{2\left(8\,b^{5/2}\,\sqrt{-a^3\,d^2-3\,a^2\,b\,d^2-3\,a\,b^2\,d^2-b^3\,d^2}+8\,a\,b^{3/2}\,\sqrt{-a^3\,d^2-3\,a^2\,b\,d^2-3\,a\,b^2\,d^2-b^3\,d^2}+a^2\,\sqrt{b}\,\sqrt{-a^3 \cdots}\right.}{a^5\,d\,(a+b)^3\left(a^2+2\,a\,b+b^2\right)\sqrt{-a^3\,d^2-3\,a^2\,b\,d^2-3\,a\,b^2\,d^2-b^3\,d \cdots}}\right.\right.}{\cdots}\right)}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^2*(a + b/cosh(c + d*x)^2)),x)
```

```
[Out] - 2/((exp(2*c + 2*d*x) - 1)*(a*d + b*d)) - (b^(1/2)*atan(((exp(2*c)*exp(2*d
*x)*((2*(8*b^(5/2)*(- a^3*d^2 - b^3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2)
+ 8*a*b^(3/2)*(- a^3*d^2 - b^3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2) + a^2
*b^(1/2)*(- a^3*d^2 - b^3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2))*(8*a*b +
a^2 + 8*b^2))/(a^5*d*(a + b)^3*(2*a*b + a^2 + b^2)*(- a^3*d^2 - b^3*d^2 - 3
*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2)) + (4*b^(1/2)*(2*a + 4*b)*(8*b^4*d + 16*a^2
*b^2*d + 20*a*b^3*d + 4*a^3*b*d))/(a^5*(a + b)*(-d^2*(a + b)^3)^(1/2)*(2*a*
b + a^2 + b^2)*(- a^3*d^2 - b^3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2))) +
(2*(2*a*b^(3/2)*(- a^3*d^2 - b^3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2) + a
^2*b^(1/2)*(- a^3*d^2 - b^3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2))*(8*a*b
+ a^2 + 8*b^2))/(a^5*d*(a + b)^3*(2*a*b + a^2 + b^2)*(- a^3*d^2 - b^3*d^2 -
 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2)) + (4*b^(1/2)*(2*a + 4*b)*(4*a^2*b^2*d +
2*a*b^3*d + 2*a^3*b*d))/(a^5*(a + b)*(-d^2*(a + b)^3)^(1/2)*(2*a*b + a^2 +
b^2)*(- a^3*d^2 - b^3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2)))*(a^5*(- a^3*
d^2 - b^3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2) + 3*a^4*b*(- a^3*d^2 - b^3
*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2) + a^2*b^3*(- a^3*d^2 - b^3*d^2 - 3*
a*b^2*d^2 - 3*a^2*b*d^2)^(1/2) + 3*a^3*b^2*(- a^3*d^2 - b^3*d^2 - 3*a*b^2*d
^2 - 3*a^2*b*d^2)^(1/2)))/(4*b)))/(- a^3*d^2 - b^3*d^2 - 3*a*b^2*d^2 - 3*a^
2*b*d^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(csch(c + d*x)**2/(a + b*sech(c + d*x)**2), x)
```

3.31 $\int \frac{\operatorname{csch}^3(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=87

$$-\frac{\sqrt{a}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{d(a+b)^2}+\frac{(a-b)\tanh^{-1}(\cosh(c+dx))}{2d(a+b)^2}-\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2d(a+b)}$$

```
[Out] 1/2*(a-b)*arctanh(cosh(d*x+c))/(a+b)^2/d-1/2*coth(d*x+c)*csch(d*x+c)/(a+b)/
d-arctan(cosh(d*x+c)*a^(1/2)/b^(1/2))*a^(1/2)*b^(1/2)/(a+b)^2/d
```

Rubi [A] time = 0.12, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4133, 471, 522, 206, 205}

$$-\frac{\sqrt{a}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{d(a+b)^2}+\frac{(a-b)\tanh^{-1}(\cosh(c+dx))}{2d(a+b)^2}-\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2d(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -((Sqrt[a]*Sqrt[b]*ArcTan[(Sqrt[a]*Cosh[c + d*x])/Sqrt[b]])/((a + b)^2*d))
+ ((a - b)*ArcTanh[Cosh[c + d*x]])/(2*(a + b)^2*d) - (Coth[c + d*x]*Csch[c
+ d*x])/(2*(a + b)*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
```

```
q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx = \frac{\operatorname{Subst}\left(\int \frac{x^2}{\left(1-x^2\right)^2\left(b+ax^2\right)}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2(a+b)d} - \frac{\operatorname{Subst}\left(\int \frac{b-ax^2}{\left(1-x^2\right)\left(b+ax^2\right)}\,dx, x, \cosh(c+dx)\right)}{2(a+b)d}$$

$$= -\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2(a+b)d} + \frac{(a-b)\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cosh(c+dx)\right)}{2(a+b)^2d} - \frac{(ab)\operatorname{Sub}\cdots}{\cdots}$$

$$= -\frac{\sqrt{a}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{(a+b)^2d} + \frac{(a-b)\tanh^{-1}(\cosh(c+dx))}{2(a+b)^2d} - \frac{\coth(c+dx)\operatorname{csch}(\cdots}{2(a+b)d}$$

Mathematica [C] time = 2.05, size = 338, normalized size = 3.89

$$-\frac{\operatorname{sech}^2(c+dx)(a\cosh(2(c+dx))+a+2b)\left((a+b)\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)+(a+b)\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)+8\sqrt{a}\sqrt{b}\tan\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -1/16*((a + 2*b + a*Cosh[2*(c + d*x)])*(8*Sqrt[a]*Sqrt[b]*ArcTan[((Sqrt[a]
- I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c
]*(Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt
[b]] + 8*Sqrt[a]*Sqrt[b]*ArcTan[((Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cosh[c] - S
inh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] + I*Sqrt[a + b]*Sqrt[(
Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]] + (a + b)*Csch[(c + d*x)/2]^
2 - 4*a*Log[Cosh[(c + d*x)/2]] + 4*b*Log[Cosh[(c + d*x)/2]] + 4*a*Log[Sinh[
(c + d*x)/2]] - 4*b*Log[Sinh[(c + d*x)/2]] + (a + b)*Sech[(c + d*x)/2]^2)*S
ech[c + d*x]^2)/((a + b)^2*d*(a + b*Sech[c + d*x]^2))
```

fricas [B] time = 0.46, size = 1881, normalized size = 21.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(a + b)*cosh(d*x + c)^3 + 6*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2
+ 2*(a + b)*sinh(d*x + c)^3 - (cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x +
 c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - 2*cos
h(d*x + c)^2 + 4*(cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c) + 1)*sqrt(
-a*b)*log((a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d
*x + c)^4 + 2*(a - 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a - 2*b)
*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a - 2*b)*cosh(d*x + c))*sinh(d*x
 + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c
)^3 + (3*cosh(d*x + c)^2 + 1)*sinh(d*x + c) + cosh(d*x + c))*sqrt(-a*b) + a
)/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^
4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*
x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) +
a)) + 2*(a + b)*cosh(d*x + c) - ((a - b)*cosh(d*x + c)^4 + 4*(a - b)*cosh(d
*x + c)*sinh(d*x + c)^3 + (a - b)*sinh(d*x + c)^4 - 2*(a - b)*cosh(d*x + c)
^2 + 2*(3*(a - b)*cosh(d*x + c)^2 - a + b)*sinh(d*x + c)^2 + 4*((a - b)*cos
h(d*x + c)^3 - (a - b)*cosh(d*x + c))*sinh(d*x + c) + a - b)*log(cosh(d*x +
 c) + sinh(d*x + c) + 1) + ((a - b)*cosh(d*x + c)^4 + 4*(a - b)*cosh(d*x +
c)*sinh(d*x + c)^3 + (a - b)*sinh(d*x + c)^4 - 2*(a - b)*cosh(d*x + c)^2 +
2*(3*(a - b)*cosh(d*x + c)^2 - a + b)*sinh(d*x + c)^2 + 4*((a - b)*cosh(d*x
 + c)^3 - (a - b)*cosh(d*x + c))*sinh(d*x + c) + a - b)*log(cosh(d*x + c) +
 sinh(d*x + c) - 1) + 2*(3*(a + b)*cosh(d*x + c)^2 + a + b)*sinh(d*x + c))/
((a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*d*cosh(d*x +
 c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*d*sinh(d*x + c)^4 - 2*(a^2 + 2*a*
b + b^2)*d*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^2 - (
a^2 + 2*a*b + b^2)*d)*sinh(d*x + c)^2 + (a^2 + 2*a*b + b^2)*d + 4*((a^2 + 2
*a*b + b^2)*d*cosh(d*x + c)^3 - (a^2 + 2*a*b + b^2)*d*cosh(d*x + c))*sinh(d
```

```
*x + c)), -1/2*(2*(a + b)*cosh(d*x + c)^3 + 6*(a + b)*cosh(d*x + c)*sinh(d*
x + c)^2 + 2*(a + b)*sinh(d*x + c)^3 + 2*(cosh(d*x + c)^4 + 4*cosh(d*x + c)
*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 - 1)*sinh(d*x + c
)^2 - 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c)
 + 1)*sqrt(a*b)*arctan(1/2*sqrt(a*b)*(cosh(d*x + c) + sinh(d*x + c))/b) - 2
*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(
3*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - 2*cosh(d*x + c)^2 + 4*(cosh(d*x +
c)^3 - cosh(d*x + c))*sinh(d*x + c) + 1)*sqrt(a*b)*arctan(1/2*(a*cosh(d*x +
 c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 + (a + 4*b)*c
osh(d*x + c) + (3*a*cosh(d*x + c)^2 + a + 4*b)*sinh(d*x + c))*sqrt(a*b)/(a*
b)) + 2*(a + b)*cosh(d*x + c) - ((a - b)*cosh(d*x + c)^4 + 4*(a - b)*cosh(d
*x + c)*sinh(d*x + c)^3 + (a - b)*sinh(d*x + c)^4 - 2*(a - b)*cosh(d*x + c)
^2 + 2*(3*(a - b)*cosh(d*x + c)^2 - a + b)*sinh(d*x + c)^2 + 4*((a - b)*cos
h(d*x + c)^3 - (a - b)*cosh(d*x + c))*sinh(d*x + c) + a - b)*log(cosh(d*x +
 c) + sinh(d*x + c) + 1) + ((a - b)*cosh(d*x + c)^4 + 4*(a - b)*cosh(d*x +
c)*sinh(d*x + c)^3 + (a - b)*sinh(d*x + c)^4 - 2*(a - b)*cosh(d*x + c)^2 +
2*(3*(a - b)*cosh(d*x + c)^2 - a + b)*sinh(d*x + c)^2 + 4*((a - b)*cosh(d*x
 + c)^3 - (a - b)*cosh(d*x + c))*sinh(d*x + c) + a - b)*log(cosh(d*x + c) +
 sinh(d*x + c) - 1) + 2*(3*(a + b)*cosh(d*x + c)^2 + a + b)*sinh(d*x + c))/
((a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*d*cosh(d*x +
 c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*d*sinh(d*x + c)^4 - 2*(a^2 + 2*a*
b + b^2)*d*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^2 - (
a^2 + 2*a*b + b^2)*d)*sinh(d*x + c)^2 + (a^2 + 2*a*b + b^2)*d + 4*((a^2 + 2
*a*b + b^2)*d*cosh(d*x + c)^3 - (a^2 + 2*a*b + b^2)*d*cosh(d*x + c))*sinh(d
*x + c))]
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[31,78]Warning, need to choose a branch for the root of a po
lynomial with parameters. This might be wrong.The choice was done assuming
[a,b]=[-13,-93]Warning, need to choose a branch for the root of a polynomia
l with parameters. This might be wrong.The choice was done assuming [a,b]=[
-65,-82]Warning, need to choose a branch for the root of a polynomial with
parameters. This might be wrong.The choice was done assuming [a,b]=[97,-56]
Undef/Unsigned Inf encountered in limitEvaluation time: 0.45Limit: Max orde
r reached or unable to make series expansion Error: Bad Argument Value
```

maple [A] time = 0.32, size = 134, normalized size = 1.54

$$\frac{\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,(a+b)}-\frac{ab\arctan\left(\frac{2(a+b)\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a-2b}{4\sqrt{ab}}\right)}{d\,(a+b)^2\sqrt{ab}}-\frac{1}{8d\,(a+b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a}{2d\,(a+b)^2}+\frac{\ln\left(\tanh\right.}{2d\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3/(a+b*sech(d*x+c)^2),x)
```

```
[Out] 1/8/d*tanh(1/2*d*x+1/2*c)^2/(a+b)-1/d*a*b/(a+b)^2/(a*b)^(1/2)*arctan(1/4*(2
*(a+b)*tanh(1/2*d*x+1/2*c)^2+2*a-2*b)/(a*b)^(1/2))-1/8/d/(a+b)/tanh(1/2*d*x
+1/2*c)^2-1/2/d/(a+b)^2*ln(tanh(1/2*d*x+1/2*c))*a+1/2/d/(a+b)^2*ln(tanh(1/2
*d*x+1/2*c))*b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(a-b)\log\left(\left(e^{(dx+c)}+1\right)e^{(-c)}\right)}{2\left(a^2d+2\,abd+b^2d\right)}-\frac{(a-b)\log\left(\left(e^{(dx+c)}-1\right)e^{(-c)}\right)}{2\left(a^2d+2\,abd+b^2d\right)}-\frac{e^{(3\,dx+3\,c)}+e^{(dx+c)}}{ad+bd+\left(ade^{(4\,c)}+bde^{(4\,c)}\right)e^{(4\,dx)}-2\left(ade^{(2\,c)}+bd\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*(a - b)*log((e^(d*x + c) + 1)*e^(-c))/(a^2*d + 2*a*b*d + b^2*d) - 1/2*(
a - b)*log((e^(d*x + c) - 1)*e^(-c))/(a^2*d + 2*a*b*d + b^2*d) - (e^(3*d*x
+ 3*c) + e^(d*x + c))/(a*d + b*d + (a*d*e^(4*c) + b*d*e^(4*c))*e^(4*d*x) -
2*(a*d*e^(2*c) + b*d*e^(2*c))*e^(2*d*x)) - 8*integrate(1/4*(a*b*e^(3*d*x +
3*c) - a*b*e^(d*x + c))/(a^3 + 2*a^2*b + a*b^2 + (a^3*e^(4*c) + 2*a^2*b*e^(
4*c) + a*b^2*e^(4*c))*e^(4*d*x) + 2*(a^3*e^(2*c) + 4*a^2*b*e^(2*c) + 5*a*b^
2*e^(2*c) + 2*b^3*e^(2*c))*e^(2*d*x)), x)
```

mupad [B] time = 3.07, size = 1586, normalized size = 18.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^3*(a + b/cosh(c + d*x)^2)),x)
```

```
[Out] ((a*b)^(1/2)*(2*atan(((exp(d*x)*exp(c)*((64*(2*b^5*d*(a*b)^(1/2) + 2*a*b^4*
d*(a*b)^(1/2) + 2*a^4*b*d*(a*b)^(1/2) + 2*a^3*b^2*d*(a*b)^(1/2)))/(a^4*(a +
 b)^3*(d^2*(a + b)^4)^(1/2)*(2*a*b + a^2 + b^2)*(a^4*d^2 + b^4*d^2 + 4*a*b^
3*d^2 + 4*a^3*b*d^2 + 6*a^2*b^2*d^2)^(1/2)) + (32*(a*b^3*(a^4*d^2 + b^4*d^2
```

```
 + 4*a*b^3*d^2 + 4*a^3*b*d^2 + 6*a^2*b^2*d^2)^(1/2) + a^3*b*(a^4*d^2 + b^4*
d^2 + 4*a*b^3*d^2 + 4*a^3*b*d^2 + 6*a^2*b^2*d^2)^(1/2) - a^2*b^2*(a^4*d^2 +
 b^4*d^2 + 4*a*b^3*d^2 + 4*a^3*b*d^2 + 6*a^2*b^2*d^2)^(1/2)))/(a^3*d*(a*b)^
(1/2)*(a + b)^5*(2*a*b + a^2 + b^2)*(a^4*d^2 + b^4*d^2 + 4*a*b^3*d^2 + 4*a^
3*b*d^2 + 6*a^2*b^2*d^2)^(1/2))) + (32*exp(3*c)*exp(3*d*x)*(a*b^3*(a^4*d^2
+ b^4*d^2 + 4*a*b^3*d^2 + 4*a^3*b*d^2 + 6*a^2*b^2*d^2)^(1/2) + a^3*b*(a^4*d
^2 + b^4*d^2 + 4*a*b^3*d^2 + 4*a^3*b*d^2 + 6*a^2*b^2*d^2)^(1/2) - a^2*b^2*(
a^4*d^2 + b^4*d^2 + 4*a*b^3*d^2 + 4*a^3*b*d^2 + 6*a^2*b^2*d^2)^(1/2)))/(a^3
*d*(a*b)^(1/2)*(a + b)^5*(2*a*b + a^2 + b^2)*(a^4*d^2 + b^4*d^2 + 4*a*b^3*d
^2 + 4*a^3*b*d^2 + 6*a^2*b^2*d^2)^(1/2)))*(a^8*(a^4*d^2 + b^4*d^2 + 4*a*b^3
*d^2 + 4*a^3*b*d^2 + 6*a^2*b^2*d^2)^(1/2) + 5*a^7*b*(a^4*d^2 + b^4*d^2 + 4*
a*b^3*d^2 + 4*a^3*b*d^2 + 6*a^2*b^2*d^2)^(1/2) + a^3*b^5*(a^4*d^2 + b^4*d^2
 + 4*a*b^3*d^2 + 4*a^3*b*d^2 + 6*a^2*b^2*d^2)^(1/2) + 5*a^4*b^4*(a^4*d^2 +
b^4*d^2 + 4*a*b^3*d^2 + 4*a^3*b*d^2 + 6*a^2*b^2*d^2)^(1/2) + 10*a^5*b^3*(a^
4*d^2 + b^4*d^2 + 4*a*b^3*d^2 + 4*a^3*b*d^2 + 6*a^2*b^2*d^2)^(1/2) + 10*a^6
*b^2*(a^4*d^2 + b^4*d^2 + 4*a*b^3*d^2 + 4*a^3*b*d^2 + 6*a^2*b^2*d^2)^(1/2))
)/(64*a^2*b - 64*a*b^2 + 64*b^3)) - 2*atan((a*exp(d*x)*exp(c)*(d^2*(a + b)^
4)^(1/2))/(2*d*(a*b)^(1/2)*(a + b)^2))))/(2*(a^4*d^2 + b^4*d^2 + 4*a*b^3*d^
2 + 4*a^3*b*d^2 + 6*a^2*b^2*d^2)^(1/2)) - exp(c + d*x)/((exp(2*c + 2*d*x) -
 1)*(a*d + b*d)) - (atan((exp(d*x)*exp(c)*(b^7*(- a^4*d^2 - b^4*d^2 - 4*a*b
^3*d^2 - 4*a^3*b*d^2 - 6*a^2*b^2*d^2)^(1/2) - 3*a*b^6*(- a^4*d^2 - b^4*d^2
- 4*a*b^3*d^2 - 4*a^3*b*d^2 - 6*a^2*b^2*d^2)^(1/2) + 5*a^2*b^5*(- a^4*d^2 -
 b^4*d^2 - 4*a*b^3*d^2 - 4*a^3*b*d^2 - 6*a^2*b^2*d^2)^(1/2) - 5*a^3*b^4*(-
a^4*d^2 - b^4*d^2 - 4*a*b^3*d^2 - 4*a^3*b*d^2 - 6*a^2*b^2*d^2)^(1/2) + 3*a^
4*b^3*(- a^4*d^2 - b^4*d^2 - 4*a*b^3*d^2 - 4*a^3*b*d^2 - 6*a^2*b^2*d^2)^(1/
2) - a^5*b^2*(- a^4*d^2 - b^4*d^2 - 4*a*b^3*d^2 - 4*a^3*b*d^2 - 6*a^2*b^2*d
^2)^(1/2)))/(b^8*d*(a^2 - 2*a*b + b^2)^(1/2) + 2*a^3*b^5*d*(a^2 - 2*a*b + b
^2)^(1/2) + a^6*b^2*d*(a^2 - 2*a*b + b^2)^(1/2)))*(a^2 - 2*a*b + b^2)^(1/2)
)/(- a^4*d^2 - b^4*d^2 - 4*a*b^3*d^2 - 4*a^3*b*d^2 - 6*a^2*b^2*d^2)^(1/2) -
 (2*exp(c + d*x))/((a*d + b*d)*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(csch(c + d*x)**3/(a + b*sech(c + d*x)**2), x)
```

3.32 $\int \frac{\mathbf{csch}^4(c+dx)}{a+b\mathbf{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=75

$$-\frac{a\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{d(a+b)^{5/2}}-\frac{\coth^3(c+dx)}{3d(a+b)}+\frac{a\coth(c+dx)}{d(a+b)^2}$$

```
[Out] a*coth(d*x+c)/(a+b)^2/d-1/3*coth(d*x+c)^3/(a+b)/d-a*arctanh(b^(1/2)*tanh(d*
x+c)/(a+b)^(1/2))*b^(1/2)/(a+b)^(5/2)/d
```

Rubi [A] time = 0.10, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4132, 453, 325, 208}

$$-\frac{a\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{d(a+b)^{5/2}}-\frac{\coth^3(c+dx)}{3d(a+b)}+\frac{a\coth(c+dx)}{d(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -((a*Sqrt[b]*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/((a + b)^(5/2)*d
)) + (a*Coth[c + d*x])/((a + b)^2*d) - Coth[c + d*x]^3/(3*(a + b)*d)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
```

```
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1-x^2}{x^4\left(a+b-bx^2\right)}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{\coth^3(c+dx)}{3(a+b)d} - \frac{a\operatorname{Subst}\left(\int \frac{1}{x^2\left(a+b-bx^2\right)}\,dx, x, \tanh(c+dx)\right)}{(a+b)d}$$

$$= \frac{a\coth(c+dx)}{(a+b)^2d} - \frac{\coth^3(c+dx)}{3(a+b)d} - \frac{(ab)\operatorname{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \tanh(c+dx)\right)}{(a+b)^2d}$$

$$= -\frac{a\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}d} + \frac{a\coth(c+dx)}{(a+b)^2d} - \frac{\coth^3(c+dx)}{3(a+b)d}$$

Mathematica [B] time = 2.08, size = 216, normalized size = 2.88

$$\frac{\operatorname{sech}^2(c+dx)(a\cosh(2(c+dx))+a+2b)\left(\frac{1}{4}\sqrt{a+b}\operatorname{csch}(c)\sqrt{b(\cosh(c)-\sinh(c))^4}\operatorname{csch}^3(c+dx)((b-2a)\sinh}{6d(a+b)^{5/2}\sqrt{b(\cos}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*(3*a*b*ArcTanh[(Sech[d*x]*
(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[
a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(-Cosh[2*c] + Sinh[2*c]) + (Sqrt[a +
 b]*Csch[c]*Csch[c + d*x]^3*Sqrt[b*(Cosh[c] - Sinh[c])^4]*(6*a*Sinh[d*x] -
```

```
3*b*Sinh[2*c + d*x] + (-2*a + b)*Sinh[2*c + 3*d*x]))/4))/(6*(a + b)^(5/2)*d
*(a + b*Sech[c + d*x]^2)*Sqrt[b*(Cosh[c] - Sinh[c])^4])
```

fricas [B] time = 0.52, size = 1753, normalized size = 23.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/6*(12*b*cosh(d*x + c)^4 + 48*b*cosh(d*x + c)*sinh(d*x + c)^3 + 12*b*sin
h(d*x + c)^4 + 24*a*cosh(d*x + c)^2 + 24*(3*b*cosh(d*x + c)^2 + a)*sinh(d*x
 + c)^2 - 3*(a*cosh(d*x + c)^6 + 6*a*cosh(d*x + c)*sinh(d*x + c)^5 + a*sinh
(d*x + c)^6 - 3*a*cosh(d*x + c)^4 + 3*(5*a*cosh(d*x + c)^2 - a)*sinh(d*x +
c)^4 + 4*(5*a*cosh(d*x + c)^3 - 3*a*cosh(d*x + c))*sinh(d*x + c)^3 + 3*a*co
sh(d*x + c)^2 + 3*(5*a*cosh(d*x + c)^4 - 6*a*cosh(d*x + c)^2 + a)*sinh(d*x
+ c)^2 + 6*(a*cosh(d*x + c)^5 - 2*a*cosh(d*x + c)^3 + a*cosh(d*x + c))*sinh
(d*x + c) - a)*sqrt(b/(a + b))*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x +
c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2
+ 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8
*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c)
+ 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c
) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 + 3*a*b + 2*b^2)*sqrt(b/(a + b)))/(a*
cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2
*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c
)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) +
 48*(b*cosh(d*x + c)^3 + a*cosh(d*x + c))*sinh(d*x + c) - 8*a + 4*b)/((a^2
+ 2*a*b + b^2)*d*cosh(d*x + c)^6 + 6*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)*si
nh(d*x + c)^5 + (a^2 + 2*a*b + b^2)*d*sinh(d*x + c)^6 - 3*(a^2 + 2*a*b + b^
2)*d*cosh(d*x + c)^4 + 3*(5*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^2 - (a^2 +
2*a*b + b^2)*d)*sinh(d*x + c)^4 + 3*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^2 +
 4*(5*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^3 - 3*(a^2 + 2*a*b + b^2)*d*cosh(
d*x + c))*sinh(d*x + c)^3 + 3*(5*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^4 - 6*
(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^2 + (a^2 + 2*a*b + b^2)*d)*sinh(d*x + c
)^2 - (a^2 + 2*a*b + b^2)*d + 6*((a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^5 - 2*
(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*d*cosh(d*x + c)
)*sinh(d*x + c)), -1/3*(6*b*cosh(d*x + c)^4 + 24*b*cosh(d*x + c)*sinh(d*x +
 c)^3 + 6*b*sinh(d*x + c)^4 + 12*a*cosh(d*x + c)^2 + 12*(3*b*cosh(d*x + c)^
2 + a)*sinh(d*x + c)^2 + 3*(a*cosh(d*x + c)^6 + 6*a*cosh(d*x + c)*sinh(d*x
+ c)^5 + a*sinh(d*x + c)^6 - 3*a*cosh(d*x + c)^4 + 3*(5*a*cosh(d*x + c)^2 -
 a)*sinh(d*x + c)^4 + 4*(5*a*cosh(d*x + c)^3 - 3*a*cosh(d*x + c))*sinh(d*x
+ c)^3 + 3*a*cosh(d*x + c)^2 + 3*(5*a*cosh(d*x + c)^4 - 6*a*cosh(d*x + c)^2
 + a)*sinh(d*x + c)^2 + 6*(a*cosh(d*x + c)^5 - 2*a*cosh(d*x + c)^3 + a*cosh
(d*x + c))*sinh(d*x + c) - a)*sqrt(-b/(a + b))*arctan(1/2*(a*cosh(d*x + c)^
2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-b/
```

```
(a + b))/b) + 24*(b*cosh(d*x + c)^3 + a*cosh(d*x + c))*sinh(d*x + c) - 4*a
+ 2*b)/((a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^6 + 6*(a^2 + 2*a*b + b^2)*d*cos
h(d*x + c)*sinh(d*x + c)^5 + (a^2 + 2*a*b + b^2)*d*sinh(d*x + c)^6 - 3*(a^2
 + 2*a*b + b^2)*d*cosh(d*x + c)^4 + 3*(5*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c
)^2 - (a^2 + 2*a*b + b^2)*d)*sinh(d*x + c)^4 + 3*(a^2 + 2*a*b + b^2)*d*cosh
(d*x + c)^2 + 4*(5*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^3 - 3*(a^2 + 2*a*b +
 b^2)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*(a^2 + 2*a*b + b^2)*d*cosh(d*
x + c)^4 - 6*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^2 + (a^2 + 2*a*b + b^2)*d)
*sinh(d*x + c)^2 - (a^2 + 2*a*b + b^2)*d + 6*((a^2 + 2*a*b + b^2)*d*cosh(d*
x + c)^5 - 2*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*d*
cosh(d*x + c))*sinh(d*x + c))]
```

giac [A] time = 0.67, size = 123, normalized size = 1.64

$$-\frac{\frac{3\,ab\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{\left(a^2+2\,ab+b^2\right)\sqrt{-ab-b^2}}+\frac{2\left(3\,be^{(4\,dx+4\,c)}+6\,ae^{(2\,dx+2\,c)}-2\,a+b\right)}{\left(a^2+2\,ab+b^2\right)\left(e^{(2\,dx+2\,c)}-1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/3*(3*a*b*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/((a^
2 + 2*a*b + b^2)*sqrt(-a*b - b^2)) + 2*(3*b*e^(4*d*x + 4*c) + 6*a*e^(2*d*x
+ 2*c) - 2*a + b)/((a^2 + 2*a*b + b^2)*(e^(2*d*x + 2*c) - 1)^3))/d
```

maple [B] time = 0.36, size = 258, normalized size = 3.44

$$-\frac{a\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d\,(a+b)^2}-\frac{\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b}{24d\,(a+b)^2}+\frac{3a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,(a+b)^2}-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)b}{8d\,(a+b)^2}+\frac{a\sqrt{b}\,\ln\left(-\sqrt{a+b}\,\left(\tanh^2\left(\frac{dx}{2}+\cdots\right.\right.\right.}{2d\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4/(a+b*sech(d*x+c)^2),x)
```

```
[Out] -1/24/d/(a+b)^2*a*tanh(1/2*d*x+1/2*c)^3-1/24/d/(a+b)^2*tanh(1/2*d*x+1/2*c)^
3*b+3/8/d/(a+b)^2*a*tanh(1/2*d*x+1/2*c)-1/8/d/(a+b)^2*tanh(1/2*d*x+1/2*c)*b
+1/2/d*a*b^(1/2)/(a+b)^(5/2)*ln(-(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2
)*tanh(1/2*d*x+1/2*c)-(a+b)^(1/2))-1/2/d*a*b^(1/2)/(a+b)^(5/2)*ln((a+b)^(1/
2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))-1/24/d/
(a+b)/tanh(1/2*d*x+1/2*c)^3+3/8/d/(a+b)^2/tanh(1/2*d*x+1/2*c)*a-1/8/d/(a+b)
^2/tanh(1/2*d*x+1/2*c)*b
```

maxima [B] time = 0.46, size = 195, normalized size = 2.60

$$\frac{ab\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{2\left(a^2+2\,ab+b^2\right)\sqrt{(a+b)b}\,d}-\frac{2\left(6\,ae^{(-2\,dx-2\,c)}+3\,be^{(-4\,dx-4\,c)}-2\,a+b\right)}{3\left(a^2+2\,ab+b^2-3\left(a^2+2\,ab+b^2\right)e^{(-2\,dx-2\,c)}+3\left(a^2+2\,ab+b^2\right)e^{(-4\,dx-4\,c)}-\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*a*b*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x
 - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^2 + 2*a*b + b^2)*sqrt((a + b)*b
)*d) - 2/3*(6*a*e^(-2*d*x - 2*c) + 3*b*e^(-4*d*x - 4*c) - 2*a + b)/((a^2 +
2*a*b + b^2 - 3*(a^2 + 2*a*b + b^2)*e^(-2*d*x - 2*c) + 3*(a^2 + 2*a*b + b^2
)*e^(-4*d*x - 4*c) - (a^2 + 2*a*b + b^2)*e^(-6*d*x - 6*c))*d)
```

mupad [B] time = 2.17, size = 248, normalized size = 3.31

$$\frac{a\sqrt{b}\,\ln\left(\frac{4\,b\,e^{2\,c+2\,d\,x}}{(a+b)^2}-\frac{2\,\sqrt{b}\,\left(a+a\,e^{2\,c+2\,d\,x}+2\,b\,e^{2\,c+2\,d\,x}\right)}{(a+b)^{5/2}}\right)}{2\,d\,(a+b)^{5/2}}-\frac{8}{3\,(a\,d+b\,d)\,\left(3\,e^{2\,c+2\,d\,x}-3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}-1\right)}-\frac{\cdots}{\left(e^{2\,c+2\,d\,x}\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^4*(a + b/cosh(c + d*x)^2)),x)
```

```
[Out] (a*b^(1/2)*log((4*b*exp(2*c + 2*d*x))/(a + b)^2 - (2*b^(1/2)*(a + a*exp(2*c
 + 2*d*x) + 2*b*exp(2*c + 2*d*x)))/(a + b)^(5/2)))/(2*d*(a + b)^(5/2)) - 8/
(3*(a*d + b*d)*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x)
- 1)) - (2*b)/((exp(2*c + 2*d*x) - 1)*(a + b)*(a*d + b*d)) - 4/((a*d + b*d)
*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) - (a*b^(1/2)*log((4*b*exp(2*c
 + 2*d*x))/(a + b)^2 + (2*b^(1/2)*(a + a*exp(2*c + 2*d*x) + 2*b*exp(2*c + 2
*d*x)))/(a + b)^(5/2)))/(2*d*(a + b)^(5/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{csch}^4(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(csch(c + d*x)**4/(a + b*sech(c + d*x)**2), x)
```

3.33 $\int \frac{\sinh^4(c+dx)}{\left(a+b\,\mathbf{sech}^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=194

$$-\frac{3\sqrt{b}\sqrt{a+b}\,(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^4d}-\frac{3b(3a+4b)\tanh(c+dx)}{8a^3d\left(a-b\tanh^2(c+dx)+b\right)}-\frac{(5a+6b)\sinh(c+dx)\cosh(c+ }{8a^2d\left(a-b\tanh^2(c+dx)+b\right)}$$

```
[Out] 3/8*(a^2+8*a*b+8*b^2)*x/a^4-3/2*(a+2*b)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(
1/2))*b^(1/2)*(a+b)^(1/2)/a^4/d-1/8*(5*a+6*b)*cosh(d*x+c)*sinh(d*x+c)/a^2/d
/(a+b-b*tanh(d*x+c)^2)+1/4*cosh(d*x+c)^3*sinh(d*x+c)/a/d/(a+b-b*tanh(d*x+c)
^2)-3/8*b*(3*a+4*b)*tanh(d*x+c)/a^3/d/(a+b-b*tanh(d*x+c)^2)
```

Rubi [A] time = 0.27, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4132, 470, 527, 522, 206, 208}

$$\frac{3x\left(a^2+8ab+8b^2\right)}{8a^4}-\frac{3\sqrt{b}\sqrt{a+b}\,(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^4d}-\frac{3b(3a+4b)\tanh(c+dx)}{8a^3d\left(a-b\tanh^2(c+dx)+b\right)}-\frac{(5a+6b)\sin }{8a^2d\left(a-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^4/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (3*(a^2 + 8*a*b + 8*b^2)*x)/(8*a^4) - (3*Sqrt[b]*Sqrt[a + b]*(a + 2*b)*ArcT
anh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(2*a^4*d) - ((5*a + 6*b)*Cosh[c +
 d*x]*Sinh[c + d*x])/(8*a^2*d*(a + b - b*Tanh[c + d*x]^2)) + (Cosh[c + d*x]
^3*Sinh[c + d*x])/(4*a*d*(a + b - b*Tanh[c + d*x]^2)) - (3*b*(3*a + 4*b)*Ta
nh[c + d*x])/(8*a^3*d*(a + b - b*Tanh[c + d*x]^2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{\left(1-x^2\right)^3\left(a+b-bx^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\cosh^3(c+dx)\sinh(c+dx)}{4ad\left(a+b-b\tanh^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{a+b+(4a+5b)x^2}{\left(1-x^2\right)^2\left(a+b-bx^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{4ad}$$

$$= -\frac{(5a+6b)\cosh(c+dx)\sinh(c+dx)}{8a^2d\left(a+b-b\tanh^2(c+dx)\right)} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4ad\left(a+b-b\tanh^2(c+dx)\right)} - \frac{\text{Subst}\left(\right.}{}$$

$$= -\frac{(5a+6b)\cosh(c+dx)\sinh(c+dx)}{8a^2d\left(a+b-b\tanh^2(c+dx)\right)} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4ad\left(a+b-b\tanh^2(c+dx)\right)} - \frac{3b(3}{8a^3d\left(a\right.}$$

$$= -\frac{(5a+6b)\cosh(c+dx)\sinh(c+dx)}{8a^2d\left(a+b-b\tanh^2(c+dx)\right)} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4ad\left(a+b-b\tanh^2(c+dx)\right)} - \frac{3b(3}{8a^3d\left(a\right.}$$

$$= \frac{3\left(a^2+8ab+8b^2\right)x}{8a^4} - \frac{3\sqrt{b}\sqrt{a+b}\,(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^4d} - \frac{(5a+6b)\,\mathrm{c}}{8a^2d\left(a\right.}$$

Mathematica [C] time = 14.25, size = 1330, normalized size = 6.86

$$-\frac{(\cosh(2c+2dx)a+a+2b)^2\left(16x + \frac{\left(a^3-6ba^2-24b^2a-16b^3\right)\tanh^{-1}\left(\frac{\operatorname{sech}(dx)(\cosh(2c)-\sinh(2c))((a+2b)\sinh(dx)-a\sinh(2c+dx))}{2\sqrt{a+b}\sqrt{b(\cosh(c)-\sinh(c))^4}}\right)(\cosh(2c)-\mathrm{si}}{b(a+b)^{3/2}d\sqrt{b(\cosh(c)-\sinh(c))^4}}\right.}{256a^2\left(b\operatorname{sech}^2(c+dx)+a\right)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sinh[c + d*x]^4/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] -1/256*((a + 2*b + a*Cosh[2*c + 2*d*x])^2*Sech[c + d*x]^4*(16*x + ((a^3 - 6
*a^2*b - 24*a*b^2 - 16*b^3)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*((a
+ 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sin
h[c])^4])]*(Cosh[2*c] - Sinh[2*c]))/(b*(a + b)^(3/2)*d*Sqrt[b*(Cosh[c] - Si
```

```
nh[c])^4]) + ((a^2 + 8*a*b + 8*b^2)*Sech[2*c]*((a + 2*b)*Sinh[2*c] - a*Sinh
[2*d*x]))/(b*(a + b)*d*(a + 2*b + a*Cosh[2*(c + d*x)]))))/(a^2*(a + b*Sech[
c + d*x]^2)^2) + (3*(a + 2*b + a*Cosh[2*c + 2*d*x])^2*Sech[c + d*x]^4*(((a
+ 2*b)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(8*b^(3/2)*(a + b)^(3/
2)*d) - (a*Sinh[2*(c + d*x)])/(8*b*(a + b)*d*(a + 2*b + a*Cosh[2*(c + d*x)]
))))/(128*(a + b*Sech[c + d*x]^2)^2) + ((a + 2*b + a*Cosh[2*c + 2*d*x])^2*S
ech[c + d*x]^4*(((a^5 - 30*a^4*b - 480*a^3*b^2 - 1600*a^2*b^3 - 1920*a*b^4
- 768*b^5)*(((-1/8*I)*ArcTan[Sech[d*x]*(((-1/2*I)*Cosh[2*c])/(Sqrt[a + b]*S
qrt[b*Cosh[4*c] - b*Sinh[4*c]]) + ((I/2)*Sinh[2*c])/(Sqrt[a + b]*Sqrt[b*Cos
h[4*c] - b*Sinh[4*c]]))*(-(a*Sinh[d*x]) - 2*b*Sinh[d*x] + a*Sinh[2*c + d*x]
)]*Cosh[2*c])/(a^4*b*Sqrt[a + b]*d*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]]) + ((I/8
)*ArcTan[Sech[d*x]*(((-1/2*I)*Cosh[2*c])/(Sqrt[a + b]*Sqrt[b*Cosh[4*c] - b*
Sinh[4*c]]) + ((I/2)*Sinh[2*c])/(Sqrt[a + b]*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]
]))*(-(a*Sinh[d*x]) - 2*b*Sinh[d*x] + a*Sinh[2*c + d*x])]*Sinh[2*c])/(a^4*b
*Sqrt[a + b]*d*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]])))/(a + b) + (Sech[2*c]*(160
*a^4*b*d*x*Cosh[2*c] + 1248*a^3*b^2*d*x*Cosh[2*c] + 3392*a^2*b^3*d*x*Cosh[2
*c] + 3840*a*b^4*d*x*Cosh[2*c] + 1536*b^5*d*x*Cosh[2*c] + 80*a^4*b*d*x*Cosh
[2*d*x] + 464*a^3*b^2*d*x*Cosh[2*d*x] + 768*a^2*b^3*d*x*Cosh[2*d*x] + 384*a
*b^4*d*x*Cosh[2*d*x] + 80*a^4*b*d*x*Cosh[4*c + 2*d*x] + 464*a^3*b^2*d*x*Cos
h[4*c + 2*d*x] + 768*a^2*b^3*d*x*Cosh[4*c + 2*d*x] + 384*a*b^4*d*x*Cosh[4*c
 + 2*d*x] + a^5*Sinh[2*c] + 34*a^4*b*Sinh[2*c] + 224*a^3*b^2*Sinh[2*c] + 57
6*a^2*b^3*Sinh[2*c] + 640*a*b^4*Sinh[2*c] + 256*b^5*Sinh[2*c] - a^5*Sinh[2*
d*x] - 62*a^4*b*Sinh[2*d*x] - 318*a^3*b^2*Sinh[2*d*x] - 512*a^2*b^3*Sinh[2*
d*x] - 256*a*b^4*Sinh[2*d*x] - 30*a^4*b*Sinh[4*c + 2*d*x] - 158*a^3*b^2*Sin
h[4*c + 2*d*x] - 256*a^2*b^3*Sinh[4*c + 2*d*x] - 128*a*b^4*Sinh[4*c + 2*d*x
] - 12*a^4*b*Sinh[2*c + 4*d*x] - 36*a^3*b^2*Sinh[2*c + 4*d*x] - 24*a^2*b^3*
Sinh[2*c + 4*d*x] - 12*a^4*b*Sinh[6*c + 4*d*x] - 36*a^3*b^2*Sinh[6*c + 4*d*
x] - 24*a^2*b^3*Sinh[6*c + 4*d*x] + 2*a^4*b*Sinh[4*c + 6*d*x] + 2*a^3*b^2*S
inh[4*c + 6*d*x] + 2*a^4*b*Sinh[8*c + 6*d*x] + 2*a^3*b^2*Sinh[8*c + 6*d*x])
)/(8*a^4*b*(a + b)*d*(a + 2*b + a*Cosh[2*c + 2*d*x]))))/(128*(a + b*Sech[c
+ d*x]^2)^2)
```

fricas [B] time = 0.56, size = 5169, normalized size = 26.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/64*(a^3*cosh(d*x + c)^12 + 12*a^3*cosh(d*x + c)*sinh(d*x + c)^11 + a^3*s
inh(d*x + c)^12 - 6*(a^3 + 2*a^2*b)*cosh(d*x + c)^10 + 6*(11*a^3*cosh(d*x +
 c)^2 - a^3 - 2*a^2*b)*sinh(d*x + c)^10 + 20*(11*a^3*cosh(d*x + c)^3 - 3*(a
^3 + 2*a^2*b)*cosh(d*x + c))*sinh(d*x + c)^9 - (15*a^3 + 64*a^2*b + 64*a*b^
2 - 24*(a^3 + 8*a^2*b + 8*a*b^2)*d*x)*cosh(d*x + c)^8 + (495*a^3*cosh(d*x +
 c)^4 - 15*a^3 - 64*a^2*b - 64*a*b^2 + 24*(a^3 + 8*a^2*b + 8*a*b^2)*d*x - 2
```

```
70*(a^3 + 2*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 8*(99*a^3*cosh(d*x +
c)^5 - 90*(a^3 + 2*a^2*b)*cosh(d*x + c)^3 - (15*a^3 + 64*a^2*b + 64*a*b^2 -
 24*(a^3 + 8*a^2*b + 8*a*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^7 + 16*(4*a
^2*b + 12*a*b^2 + 8*b^3 + 3*(a^3 + 10*a^2*b + 24*a*b^2 + 16*b^3)*d*x)*cosh(
d*x + c)^6 + 4*(231*a^3*cosh(d*x + c)^6 - 315*(a^3 + 2*a^2*b)*cosh(d*x + c)
^4 + 16*a^2*b + 48*a*b^2 + 32*b^3 + 12*(a^3 + 10*a^2*b + 24*a*b^2 + 16*b^3)
*d*x - 7*(15*a^3 + 64*a^2*b + 64*a*b^2 - 24*(a^3 + 8*a^2*b + 8*a*b^2)*d*x)*
cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(99*a^3*cosh(d*x + c)^7 - 189*(a^3 + 2
*a^2*b)*cosh(d*x + c)^5 - 7*(15*a^3 + 64*a^2*b + 64*a*b^2 - 24*(a^3 + 8*a^2
*b + 8*a*b^2)*d*x)*cosh(d*x + c)^3 + 12*(4*a^2*b + 12*a*b^2 + 8*b^3 + 3*(a^
3 + 10*a^2*b + 24*a*b^2 + 16*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 + (15
*a^3 + 128*a^2*b + 128*a*b^2 + 24*(a^3 + 8*a^2*b + 8*a*b^2)*d*x)*cosh(d*x +
 c)^4 + (495*a^3*cosh(d*x + c)^8 - 1260*(a^3 + 2*a^2*b)*cosh(d*x + c)^6 - 7
0*(15*a^3 + 64*a^2*b + 64*a*b^2 - 24*(a^3 + 8*a^2*b + 8*a*b^2)*d*x)*cosh(d*
x + c)^4 + 15*a^3 + 128*a^2*b + 128*a*b^2 + 24*(a^3 + 8*a^2*b + 8*a*b^2)*d*
x + 240*(4*a^2*b + 12*a*b^2 + 8*b^3 + 3*(a^3 + 10*a^2*b + 24*a*b^2 + 16*b^3
)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(55*a^3*cosh(d*x + c)^9 - 180*(
a^3 + 2*a^2*b)*cosh(d*x + c)^7 - 14*(15*a^3 + 64*a^2*b + 64*a*b^2 - 24*(a^3
 + 8*a^2*b + 8*a*b^2)*d*x)*cosh(d*x + c)^5 + 80*(4*a^2*b + 12*a*b^2 + 8*b^3
 + 3*(a^3 + 10*a^2*b + 24*a*b^2 + 16*b^3)*d*x)*cosh(d*x + c)^3 + (15*a^3 +
128*a^2*b + 128*a*b^2 + 24*(a^3 + 8*a^2*b + 8*a*b^2)*d*x)*cosh(d*x + c))*si
nh(d*x + c)^3 - a^3 + 6*(a^3 + 2*a^2*b)*cosh(d*x + c)^2 + 2*(33*a^3*cosh(d*
x + c)^10 - 135*(a^3 + 2*a^2*b)*cosh(d*x + c)^8 - 14*(15*a^3 + 64*a^2*b + 6
4*a*b^2 - 24*(a^3 + 8*a^2*b + 8*a*b^2)*d*x)*cosh(d*x + c)^6 + 120*(4*a^2*b
+ 12*a*b^2 + 8*b^3 + 3*(a^3 + 10*a^2*b + 24*a*b^2 + 16*b^3)*d*x)*cosh(d*x +
 c)^4 + 3*a^3 + 6*a^2*b + 3*(15*a^3 + 128*a^2*b + 128*a*b^2 + 24*(a^3 + 8*a
^2*b + 8*a*b^2)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 48*((a^2 + 2*a*b)*c
osh(d*x + c)^8 + 8*(a^2 + 2*a*b)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2 + 2*a
*b)*sinh(d*x + c)^8 + 2*(a^2 + 4*a*b + 4*b^2)*cosh(d*x + c)^6 + 2*(14*(a^2
+ 2*a*b)*cosh(d*x + c)^2 + a^2 + 4*a*b + 4*b^2)*sinh(d*x + c)^6 + 4*(14*(a^
2 + 2*a*b)*cosh(d*x + c)^3 + 3*(a^2 + 4*a*b + 4*b^2)*cosh(d*x + c))*sinh(d*
x + c)^5 + (a^2 + 2*a*b)*cosh(d*x + c)^4 + (70*(a^2 + 2*a*b)*cosh(d*x + c)^
4 + 30*(a^2 + 4*a*b + 4*b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^4
 + 4*(14*(a^2 + 2*a*b)*cosh(d*x + c)^5 + 10*(a^2 + 4*a*b + 4*b^2)*cosh(d*x
+ c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(14*(a^2 + 2*a*b)
*cosh(d*x + c)^6 + 15*(a^2 + 4*a*b + 4*b^2)*cosh(d*x + c)^4 + 3*(a^2 + 2*a*
b)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*(2*(a^2 + 2*a*b)*cosh(d*x + c)^7 +
3*(a^2 + 4*a*b + 4*b^2)*cosh(d*x + c)^5 + (a^2 + 2*a*b)*cosh(d*x + c)^3)*si
nh(d*x + c))*sqrt(a*b + b^2)*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)
*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 +
2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b
^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) +
4*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2
+ a + 2*b)*sqrt(a*b + b^2))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x
 + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x
```

```
 + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(
d*x + c))*sinh(d*x + c) + a)) + 4*(3*a^3*cosh(d*x + c)^11 - 15*(a^3 + 2*a^2
*b)*cosh(d*x + c)^9 - 2*(15*a^3 + 64*a^2*b + 64*a*b^2 - 24*(a^3 + 8*a^2*b +
 8*a*b^2)*d*x)*cosh(d*x + c)^7 + 24*(4*a^2*b + 12*a*b^2 + 8*b^3 + 3*(a^3 +
10*a^2*b + 24*a*b^2 + 16*b^3)*d*x)*cosh(d*x + c)^5 + (15*a^3 + 128*a^2*b +
128*a*b^2 + 24*(a^3 + 8*a^2*b + 8*a*b^2)*d*x)*cosh(d*x + c)^3 + 3*(a^3 + 2*
a^2*b)*cosh(d*x + c))*sinh(d*x + c))/(a^5*d*cosh(d*x + c)^8 + 8*a^5*d*cosh(
d*x + c)*sinh(d*x + c)^7 + a^5*d*sinh(d*x + c)^8 + a^5*d*cosh(d*x + c)^4 +
2*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^6 + 2*(14*a^5*d*cosh(d*x + c)^2 + (a^5 +
2*a^4*b)*d)*sinh(d*x + c)^6 + 4*(14*a^5*d*cosh(d*x + c)^3 + 3*(a^5 + 2*a^4*
b)*d*cosh(d*x + c))*sinh(d*x + c)^5 + (70*a^5*d*cosh(d*x + c)^4 + a^5*d + 3
0*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(14*a^5*d*cosh(d*x
 + c)^5 + a^5*d*cosh(d*x + c) + 10*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^3)*sinh(
d*x + c)^3 + 2*(14*a^5*d*cosh(d*x + c)^6 + 3*a^5*d*cosh(d*x + c)^2 + 15*(a^
5 + 2*a^4*b)*d*cosh(d*x + c)^4)*sinh(d*x + c)^2 + 4*(2*a^5*d*cosh(d*x + c)^
7 + a^5*d*cosh(d*x + c)^3 + 3*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^5)*sinh(d*x +
 c)), 1/64*(a^3*cosh(d*x + c)^12 + 12*a^3*cosh(d*x + c)*sinh(d*x + c)^11 +
a^3*sinh(d*x + c)^12 - 6*(a^3 + 2*a^2*b)*cosh(d*x + c)^10 + 6*(11*a^3*cosh(
d*x + c)^2 - a^3 - 2*a^2*b)*sinh(d*x + c)^10 + 20*(11*a^3*cosh(d*x + c)^3 -
 3*(a^3 + 2*a^2*b)*cosh(d*x + c))*sinh(d*x + c)^9 - (15*a^3 + 64*a^2*b + 64
*a*b^2 - 24*(a^3 + 8*a^2*b + 8*a*b^2)*d*x)*cosh(d*x + c)^8 + (495*a^3*cosh(
d*x + c)^4 - 15*a^3 - 64*a^2*b - 64*a*b^2 + 24*(a^3 + 8*a^2*b + 8*a*b^2)*d*
x - 270*(a^3 + 2*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 8*(99*a^3*cosh(d
*x + c)^5 - 90*(a^3 + 2*a^2*b)*cosh(d*x + c)^3 - (15*a^3 + 64*a^2*b + 64*a*
b^2 - 24*(a^3 + 8*a^2*b + 8*a*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^7 + 16
*(4*a^2*b + 12*a*b^2 + 8*b^3 + 3*(a^3 + 10*a^2*b + 24*a*b^2 + 16*b^3)*d*x)*
cosh(d*x + c)^6 + 4*(231*a^3*cosh(d*x + c)^6 - 315*(a^3 + 2*a^2*b)*cosh(d*x
 + c)^4 + 16*a^2*b + 48*a*b^2 + 32*b^3 + 12*(a^3 + 10*a^2*b + 24*a*b^2 + 16
*b^3)*d*x - 7*(15*a^3 + 64*a^2*b + 64*a*b^2 - 24*(a^3 + 8*a^2*b + 8*a*b^2)*
d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(99*a^3*cosh(d*x + c)^7 - 189*(a^
3 + 2*a^2*b)*cosh(d*x + c)^5 - 7*(15*a^3 + 64*a^2*b + 64*a*b^2 - 24*(a^3 +
8*a^2*b + 8*a*b^2)*d*x)*cosh(d*x + c)^3 + 12*(4*a^2*b + 12*a*b^2 + 8*b^3 +
3*(a^3 + 10*a^2*b + 24*a*b^2 + 16*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5
+ (15*a^3 + 128*a^2*b + 128*a*b^2 + 24*(a^3 + 8*a^2*b + 8*a*b^2)*d*x)*cosh(
d*x + c)^4 + (495*a^3*cosh(d*x + c)^8 - 1260*(a^3 + 2*a^2*b)*cosh(d*x + c)^
6 - 70*(15*a^3 + 64*a^2*b + 64*a*b^2 - 24*(a^3 + 8*a^2*b + 8*a*b^2)*d*x)*co
sh(d*x + c)^4 + 15*a^3 + 128*a^2*b + 128*a*b^2 + 24*(a^3 + 8*a^2*b + 8*a*b^
2)*d*x + 240*(4*a^2*b + 12*a*b^2 + 8*b^3 + 3*(a^3 + 10*a^2*b + 24*a*b^2 + 1
6*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(55*a^3*cosh(d*x + c)^9 -
180*(a^3 + 2*a^2*b)*cosh(d*x + c)^7 - 14*(15*a^3 + 64*a^2*b + 64*a*b^2 - 24
*(a^3 + 8*a^2*b + 8*a*b^2)*d*x)*cosh(d*x + c)^5 + 80*(4*a^2*b + 12*a*b^2 +
8*b^3 + 3*(a^3 + 10*a^2*b + 24*a*b^2 + 16*b^3)*d*x)*cosh(d*x + c)^3 + (15*a
^3 + 128*a^2*b + 128*a*b^2 + 24*(a^3 + 8*a^2*b + 8*a*b^2)*d*x)*cosh(d*x + c
))*sinh(d*x + c)^3 - a^3 + 6*(a^3 + 2*a^2*b)*cosh(d*x + c)^2 + 2*(33*a^3*co
sh(d*x + c)^10 - 135*(a^3 + 2*a^2*b)*cosh(d*x + c)^8 - 14*(15*a^3 + 64*a^2*
```

```
b + 64*a*b^2 - 24*(a^3 + 8*a^2*b + 8*a*b^2)*d*x)*cosh(d*x + c)^6 + 120*(4*a
^2*b + 12*a*b^2 + 8*b^3 + 3*(a^3 + 10*a^2*b + 24*a*b^2 + 16*b^3)*d*x)*cosh(
d*x + c)^4 + 3*a^3 + 6*a^2*b + 3*(15*a^3 + 128*a^2*b + 128*a*b^2 + 24*(a^3
+ 8*a^2*b + 8*a*b^2)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 96*((a^2 + 2*a
*b)*cosh(d*x + c)^8 + 8*(a^2 + 2*a*b)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2
+ 2*a*b)*sinh(d*x + c)^8 + 2*(a^2 + 4*a*b + 4*b^2)*cosh(d*x + c)^6 + 2*(14*
(a^2 + 2*a*b)*cosh(d*x + c)^2 + a^2 + 4*a*b + 4*b^2)*sinh(d*x + c)^6 + 4*(1
4*(a^2 + 2*a*b)*cosh(d*x + c)^3 + 3*(a^2 + 4*a*b + 4*b^2)*cosh(d*x + c))*si
nh(d*x + c)^5 + (a^2 + 2*a*b)*cosh(d*x + c)^4 + (70*(a^2 + 2*a*b)*cosh(d*x
+ c)^4 + 30*(a^2 + 4*a*b + 4*b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x +
 c)^4 + 4*(14*(a^2 + 2*a*b)*cosh(d*x + c)^5 + 10*(a^2 + 4*a*b + 4*b^2)*cosh
(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(14*(a^2 + 2
*a*b)*cosh(d*x + c)^6 + 15*(a^2 + 4*a*b + 4*b^2)*cosh(d*x + c)^4 + 3*(a^2 +
 2*a*b)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*(2*(a^2 + 2*a*b)*cosh(d*x + c)
^7 + 3*(a^2 + 4*a*b + 4*b^2)*cosh(d*x + c)^5 + (a^2 + 2*a*b)*cosh(d*x + c)^
3)*sinh(d*x + c))*sqrt(-a*b - b^2)*arctan(1/2*(a*cosh(d*x + c)^2 + 2*a*cosh
(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-a*b - b^2)/(a*
b + b^2)) + 4*(3*a^3*cosh(d*x + c)^11 - 15*(a^3 + 2*a^2*b)*cosh(d*x + c)^9
- 2*(15*a^3 + 64*a^2*b + 64*a*b^2 - 24*(a^3 + 8*a^2*b + 8*a*b^2)*d*x)*cosh(
d*x + c)^7 + 24*(4*a^2*b + 12*a*b^2 + 8*b^3 + 3*(a^3 + 10*a^2*b + 24*a*b^2
+ 16*b^3)*d*x)*cosh(d*x + c)^5 + (15*a^3 + 128*a^2*b + 128*a*b^2 + 24*(a^3
+ 8*a^2*b + 8*a*b^2)*d*x)*cosh(d*x + c)^3 + 3*(a^3 + 2*a^2*b)*cosh(d*x + c)
)*sinh(d*x + c))/(a^5*d*cosh(d*x + c)^8 + 8*a^5*d*cosh(d*x + c)*sinh(d*x +
c)^7 + a^5*d*sinh(d*x + c)^8 + a^5*d*cosh(d*x + c)^4 + 2*(a^5 + 2*a^4*b)*d*
cosh(d*x + c)^6 + 2*(14*a^5*d*cosh(d*x + c)^2 + (a^5 + 2*a^4*b)*d)*sinh(d*x
 + c)^6 + 4*(14*a^5*d*cosh(d*x + c)^3 + 3*(a^5 + 2*a^4*b)*d*cosh(d*x + c))*
sinh(d*x + c)^5 + (70*a^5*d*cosh(d*x + c)^4 + a^5*d + 30*(a^5 + 2*a^4*b)*d*
cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(14*a^5*d*cosh(d*x + c)^5 + a^5*d*cosh
(d*x + c) + 10*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^3)*sinh(d*x + c)^3 + 2*(14*a
^5*d*cosh(d*x + c)^6 + 3*a^5*d*cosh(d*x + c)^2 + 15*(a^5 + 2*a^4*b)*d*cosh(
d*x + c)^4)*sinh(d*x + c)^2 + 4*(2*a^5*d*cosh(d*x + c)^7 + a^5*d*cosh(d*x +
 c)^3 + 3*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^5)*sinh(d*x + c))]
```

giac [A] time = 3.73, size = 323, normalized size = 1.66

$$\frac{\frac{24\left(a^2+8\,ab+8\,b^2\right)(dx+c)}{a^4}-\frac{\left(18\,a^2e^{(4\,dx+4\,c)}+144\,abe^{(4\,dx+4\,c)}+144\,b^2e^{(4\,dx+4\,c)}-8\,a^2e^{(2\,dx+2\,c)}-16\,abe^{(2\,dx+2\,c)}+a^2\right)e^{(-4\,dx-4\,c)}}{a^4}-\frac{96\left(a^2b+3\,ab^2+2\,b^3\right)}{\sqrt{-}}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/64*(24*(a^2 + 8*a*b + 8*b^2)*(d*x + c)/a^4 - (18*a^2*e^(4*d*x + 4*c) + 14
4*a*b*e^(4*d*x + 4*c) + 144*b^2*e^(4*d*x + 4*c) - 8*a^2*e^(2*d*x + 2*c) - 1
```

```
6*a*b*e^(2*d*x + 2*c) + a^2)*e^(-4*d*x - 4*c)/a^4 - 96*(a^2*b + 3*a*b^2 + 2
*b^3)*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/(sqrt(-a*b
 - b^2)*a^4) + (a^2*e^(4*d*x + 4*c) - 8*a^2*e^(2*d*x + 2*c) - 16*a*b*e^(2*d
*x + 2*c))/a^4 + 64*(a^2*b*e^(2*d*x + 2*c) + 3*a*b^2*e^(2*d*x + 2*c) + 2*b^
3*e^(2*d*x + 2*c) + a^2*b + a*b^2)/((a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c
) + 4*b*e^(2*d*x + 2*c) + a)*a^4))/d
```

maple [B] time = 0.40, size = 1034, normalized size = 5.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] -9/4/d/a^3*b^(3/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/
2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))-1/d/a^3/(tanh(1/2*d*x+1/2*c)-1)^2*b-1/d
/a^3/(tanh(1/2*d*x+1/2*c)-1)*b-3/d/a^3*ln(tanh(1/2*d*x+1/2*c)-1)*b-3/d/a^4*
ln(tanh(1/2*d*x+1/2*c)-1)*b^2+1/d/a^3/(tanh(1/2*d*x+1/2*c)+1)^2*b-1/d/a^3/(
tanh(1/2*d*x+1/2*c)+1)*b+3/d/a^3*ln(tanh(1/2*d*x+1/2*c)+1)*b+3/d/a^4*ln(tan
h(1/2*d*x+1/2*c)+1)*b^2+1/4/d/a^2/(tanh(1/2*d*x+1/2*c)-1)^4+1/2/d/a^2/(tanh
(1/2*d*x+1/2*c)-1)^3-1/8/d/a^2/(tanh(1/2*d*x+1/2*c)-1)^2-3/8/d/a^2/(tanh(1/
2*d*x+1/2*c)-1)-3/8/d/a^2*ln(tanh(1/2*d*x+1/2*c)-1)-1/4/d/a^2/(tanh(1/2*d*x
+1/2*c)+1)^4+1/2/d/a^2/(tanh(1/2*d*x+1/2*c)+1)^3+1/8/d/a^2/(tanh(1/2*d*x+1/
2*c)+1)^2-3/8/d/a^2/(tanh(1/2*d*x+1/2*c)+1)+3/8/d/a^2*ln(tanh(1/2*d*x+1/2*c
)+1)-3/2/d/a^4*b^(5/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b
^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))+9/4/d/a^3*b^(3/2)/(a+b)^(1/2)*ln(-(
a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)-(a+b)^(1/2))
-1/d/a^2*b/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+
1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)*tanh(1/2*d*x+1/2*c)^3-1/d/a^3*b^2
/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a
-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)*tanh(1/2*d*x+1/2*c)^3-1/d/a^2*b/(tanh(1/2*d
*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*
d*x+1/2*c)^2*b+a+b)*tanh(1/2*d*x+1/2*c)-1/d/a^3*b^2/(tanh(1/2*d*x+1/2*c)^4*
a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2
*b+a+b)*tanh(1/2*d*x+1/2*c)+3/4/d/a^2*b^(1/2)/(a+b)^(1/2)*ln(-(a+b)^(1/2)*t
anh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)-(a+b)^(1/2))-3/4/d/a^2*b
^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*
d*x+1/2*c)+(a+b)^(1/2))+3/2/d/a^4*b^(5/2)/(a+b)^(1/2)*ln(-(a+b)^(1/2)*tanh(
1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)-(a+b)^(1/2))
```

maxima [B] time = 0.51, size = 1299, normalized size = 6.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/64*(3*a^3*b + 42*a^2*b^2 + 88*a*b^3 + 48*b^4)*log((a*e^(2*d*x + 2*c) + a
 + 2*b - 2*sqrt((a + b)*b))/(a*e^(2*d*x + 2*c) + a + 2*b + 2*sqrt((a + b)*b
)))/((a^5 + a^4*b)*sqrt((a + b)*b)*d) - 1/16*(3*a^2*b + 12*a*b^2 + 8*b^3)*l
og((a*e^(2*d*x + 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(2*d*x + 2*c) + a
 + 2*b + 2*sqrt((a + b)*b)))/((a^4 + a^3*b)*sqrt((a + b)*b)*d) + 1/64*(3*a^
3*b + 42*a^2*b^2 + 88*a*b^3 + 48*b^4)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2
*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^5
 + a^4*b)*sqrt((a + b)*b)*d) + 1/16*(3*a^2*b + 12*a*b^2 + 8*b^3)*log((a*e^(
-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b
+ 2*sqrt((a + b)*b)))/((a^4 + a^3*b)*sqrt((a + b)*b)*d) + 3/32*(3*a*b + 2*b
^2)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2
*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^3 + a^2*b)*sqrt((a + b)*b)*d) + 1/1
6*(a^3*b + 8*a^2*b^2 + 8*a*b^3 + (a^3*b + 18*a^2*b^2 + 48*a*b^3 + 32*b^4)*e
^(2*d*x + 2*c))/((a^6 + a^5*b + (a^6 + a^5*b)*e^(4*d*x + 4*c) + 2*(a^6 + 3*
a^5*b + 2*a^4*b^2)*e^(2*d*x + 2*c))*d) - 1/16*(a^3*b + 8*a^2*b^2 + 8*a*b^3
+ (a^3*b + 18*a^2*b^2 + 48*a*b^3 + 32*b^4)*e^(-2*d*x - 2*c))/((a^6 + a^5*b
+ 2*(a^6 + 3*a^5*b + 2*a^4*b^2)*e^(-2*d*x - 2*c) + (a^6 + a^5*b)*e^(-4*d*x
- 4*c))*d) + 1/4*(a^2*b + 2*a*b^2 + (a^2*b + 8*a*b^2 + 8*b^3)*e^(2*d*x + 2*
c))/((a^5 + a^4*b + (a^5 + a^4*b)*e^(4*d*x + 4*c) + 2*(a^5 + 3*a^4*b + 2*a^
3*b^2)*e^(2*d*x + 2*c))*d) - 1/4*(a^2*b + 2*a*b^2 + (a^2*b + 8*a*b^2 + 8*b^
3)*e^(-2*d*x - 2*c))/((a^5 + a^4*b + 2*(a^5 + 3*a^4*b + 2*a^3*b^2)*e^(-2*d*
x - 2*c) + (a^5 + a^4*b)*e^(-4*d*x - 4*c))*d) - 3/8*(a*b + (a*b + 2*b^2)*e^
(-2*d*x - 2*c))/((a^4 + a^3*b + 2*(a^4 + 3*a^3*b + 2*a^2*b^2)*e^(-2*d*x - 2
*c) + (a^4 + a^3*b)*e^(-4*d*x - 4*c))*d) + 3/8*(d*x + c)/(a^2*d) - 1/8*e^(2
*d*x + 2*c)/(a^2*d) + 1/8*e^(-2*d*x - 2*c)/(a^2*d) + 1/2*b*log(a*e^(4*d*x +
 4*c) + 2*(a + 2*b)*e^(2*d*x + 2*c) + a)/(a^3*d) - 1/2*b*log(2*(a + 2*b)*e^
(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/(a^3*d) + 1/64*(a*e^(4*d*x + 4*c)
- 16*b*e^(2*d*x + 2*c))/(a^3*d) + 1/64*(16*b*e^(-2*d*x - 2*c) - a*e^(-4*d*x
 - 4*c))/(a^3*d) + 1/4*(a*b + 3*b^2)*log(a*e^(4*d*x + 4*c) + 2*(a + 2*b)*e^
(2*d*x + 2*c) + a)/(a^4*d) - 1/4*(a*b + 3*b^2)*log(2*(a + 2*b)*e^(-2*d*x -
2*c) + a*e^(-4*d*x - 4*c) + a)/(a^4*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+dx)^4 \sinh(c+dx)^4}{\left(a\cosh(c+dx)^2+b\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^4/(a + b/cosh(c + d*x)^2)^2,x)
```

```
[Out] int((cosh(c + d*x)^4*sinh(c + d*x)^4)/(b + a*cosh(c + d*x)^2)^2, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**4/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

3.34 $\int \frac{\sinh^3(c+dx)}{\left(a+b\,\mathbf{sech}^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=114

$$\frac{\sqrt{b}\,(3a+5b)\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{2a^{7/2}d}-\frac{b(a+b)\cosh(c+dx)}{2a^3d\left(a\cosh^2(c+dx)+b\right)}-\frac{(a+2b)\cosh(c+dx)}{a^3d}+\frac{\cosh^3(c+dx)}{3a^2d}$$

```
[Out] -(a+2*b)*cosh(d*x+c)/a^3/d+1/3*cosh(d*x+c)^3/a^2/d-1/2*b*(a+b)*cosh(d*x+c)/
a^3/d/(b+a*cosh(d*x+c)^2)+1/2*(3*a+5*b)*arctan(cosh(d*x+c)*a^(1/2)/b^(1/2))
*b^(1/2)/a^(7/2)/d
```

Rubi [A] time = 0.14, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4133, 455, 1153, 205}

$$-\frac{b(a+b)\cosh(c+dx)}{2a^3d\left(a\cosh^2(c+dx)+b\right)}-\frac{(a+2b)\cosh(c+dx)}{a^3d}+\frac{\sqrt{b}\,(3a+5b)\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{2a^{7/2}d}+\frac{\cosh^3(c+dx)}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (Sqrt[b]*(3*a + 5*b)*ArcTan[(Sqrt[a]*Cosh[c + d*x])/Sqrt[b]])/(2*a^(7/2)*d)
 - ((a + 2*b)*Cosh[c + d*x])/(a^3*d) + Cosh[c + d*x]^3/(3*a^2*d) - (b*(a +
b)*Cosh[c + d*x])/(2*a^3*d*(b + a*Cosh[c + d*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx = -\frac{\operatorname{Subst}\left(\int \frac{x^4(1-x^2)}{(b+ax^2)^2}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{b(a+b)\cosh(c+dx)}{2a^3d\left(b+a\cosh^2(c+dx)\right)} + \frac{\operatorname{Subst}\left(\int \frac{b(a+b)-2a(a+b)x^2+2a^2x^4}{b+ax^2}\,dx, x, \cosh(c+dx)\right)}{2a^3d}$$

$$= -\frac{b(a+b)\cosh(c+dx)}{2a^3d\left(b+a\cosh^2(c+dx)\right)} + \frac{\operatorname{Subst}\left(\int \left(-2(a+2b)+2ax^2+\frac{3ab+5b^2}{b+ax^2}\right)dx, x, \cos\right.}{2a^3d}$$

$$= -\frac{(a+2b)\cosh(c+dx)}{a^3d} + \frac{\cosh^3(c+dx)}{3a^2d} - \frac{b(a+b)\cosh(c+dx)}{2a^3d\left(b+a\cosh^2(c+dx)\right)} + \frac{(b(3a+}{}$$

$$= \frac{\sqrt{b}\,(3a+5b)\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{2a^{7/2}d} - \frac{(a+2b)\cosh(c+dx)}{a^3d} + \frac{\cosh^3(c+dx)}{3a^2d} - \frac{}{2a^3}$$

Mathematica [C] time = 4.82, size = 861, normalized size = 7.55

$$\frac{(\cosh(2(c+dx))a+a+2b)^2\operatorname{sech}^4(c+dx)\left(\frac{9\tan^{-1}\left(\frac{\left(\sqrt{a}-i\sqrt{a+b}\sqrt{(\cosh(c)-\sinh(c))^2}\right)\sinh(c)\tanh\left(\frac{dx}{2}\right)+\cosh(c)\left(\sqrt{a}-i\sqrt{a+b}\sqrt{(\cosh(c)-\sinh(c))^2}\,\mathrm{t}\right.}{\sqrt{b}}\right)}{b^{3/2}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sinh[c + d*x]^3/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])^2*Sech[c + d*x]^4*((9*a^3*ArcTan[((Sqrt[a]
 - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[
c]*(Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqr
t[b]])/b^(3/2) + 576*a*Sqrt[b]*ArcTan[((Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[
c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] - I*Sqrt[a + b]*
Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]] + 960*b^(3/2)*ArcTan[(
(Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2]
 + Cosh[c]*(Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/
2]))/Sqrt[b]] + (9*a^3*ArcTan[((Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sin
h[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Co
sh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]])/b^(3/2) + 576*a*Sqrt[b]*ArcTa
n[((Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)
/2] + Cosh[c]*(Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*
x)/2]))/Sqrt[b]] + 960*b^(3/2)*ArcTan[((Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cosh[
c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] + I*Sqrt[a + b]*
Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]] - (9*a^3*ArcTan[(Sqrt[
a] - I*Sqrt[a + b]*Tanh[(c + d*x)/2])/Sqrt[b]])/b^(3/2) - (9*a^3*ArcTan[(Sq
rt[a] + I*Sqrt[a + b]*Tanh[(c + d*x)/2])/Sqrt[b]])/b^(3/2) - 96*Sqrt[a]*(3*
a + 8*b)*Cosh[c]*Cosh[d*x] + 32*a^(3/2)*Cosh[3*c]*Cosh[3*d*x] - (384*a^(3/2
)*b*Cosh[c + d*x])/(a + 2*b + a*Cosh[2*(c + d*x)]) - (384*Sqrt[a]*b^2*Cosh[
c + d*x])/(a + 2*b + a*Cosh[2*(c + d*x)]) - 288*a^(3/2)*Sinh[c]*Sinh[d*x] -
 768*Sqrt[a]*b*Sinh[c]*Sinh[d*x] + 32*a^(3/2)*Sinh[3*c]*Sinh[3*d*x]))/(1536
*a^(7/2)*d*(a + b*Sech[c + d*x]^2)^2)
```

fricas [B] time = 0.49, size = 3804, normalized size = 33.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/24*(a^2*cosh(d*x + c)^10 + 10*a^2*cosh(d*x + c)*sinh(d*x + c)^9 + a^2*si
nh(d*x + c)^10 - (7*a^2 + 20*a*b)*cosh(d*x + c)^8 + (45*a^2*cosh(d*x + c)^2
 - 7*a^2 - 20*a*b)*sinh(d*x + c)^8 + 8*(15*a^2*cosh(d*x + c)^3 - (7*a^2 + 2
0*a*b)*cosh(d*x + c))*sinh(d*x + c)^7 - 2*(13*a^2 + 66*a*b + 60*b^2)*cosh(d
*x + c)^6 + 2*(105*a^2*cosh(d*x + c)^4 - 14*(7*a^2 + 20*a*b)*cosh(d*x + c)^
2 - 13*a^2 - 66*a*b - 60*b^2)*sinh(d*x + c)^6 + 4*(63*a^2*cosh(d*x + c)^5 -
 14*(7*a^2 + 20*a*b)*cosh(d*x + c)^3 - 3*(13*a^2 + 66*a*b + 60*b^2)*cosh(d*
x + c))*sinh(d*x + c)^5 - 2*(13*a^2 + 66*a*b + 60*b^2)*cosh(d*x + c)^4 + 2*
(105*a^2*cosh(d*x + c)^6 - 35*(7*a^2 + 20*a*b)*cosh(d*x + c)^4 - 15*(13*a^2
 + 66*a*b + 60*b^2)*cosh(d*x + c)^2 - 13*a^2 - 66*a*b - 60*b^2)*sinh(d*x +
```

c)^4 + 8*(15*a^2*cosh(d*x + c)^7 - 7*(7*a^2 + 20*a*b)*cosh(d*x + c)^5 - 5*(
13*a^2 + 66*a*b + 60*b^2)*cosh(d*x + c)^3 - (13*a^2 + 66*a*b + 60*b^2)*cosh
(d*x + c))*sinh(d*x + c)^3 - (7*a^2 + 20*a*b)*cosh(d*x + c)^2 + (45*a^2*cos
h(d*x + c)^8 - 28*(7*a^2 + 20*a*b)*cosh(d*x + c)^6 - 30*(13*a^2 + 66*a*b +
60*b^2)*cosh(d*x + c)^4 - 12*(13*a^2 + 66*a*b + 60*b^2)*cosh(d*x + c)^2 - 7
*a^2 - 20*a*b)*sinh(d*x + c)^2 + 6*((3*a^2 + 5*a*b)*cosh(d*x + c)^7 + 7*(3*
a^2 + 5*a*b)*cosh(d*x + c)*sinh(d*x + c)^6 + (3*a^2 + 5*a*b)*sinh(d*x + c)^
7 + 2*(3*a^2 + 11*a*b + 10*b^2)*cosh(d*x + c)^5 + (21*(3*a^2 + 5*a*b)*cosh(
d*x + c)^2 + 6*a^2 + 22*a*b + 20*b^2)*sinh(d*x + c)^5 + 5*(7*(3*a^2 + 5*a*b
)*cosh(d*x + c)^3 + 2*(3*a^2 + 11*a*b + 10*b^2)*cosh(d*x + c))*sinh(d*x + c
)^4 + (3*a^2 + 5*a*b)*cosh(d*x + c)^3 + (35*(3*a^2 + 5*a*b)*cosh(d*x + c)^4
 + 20*(3*a^2 + 11*a*b + 10*b^2)*cosh(d*x + c)^2 + 3*a^2 + 5*a*b)*sinh(d*x +
 c)^3 + (21*(3*a^2 + 5*a*b)*cosh(d*x + c)^5 + 20*(3*a^2 + 11*a*b + 10*b^2)*
cosh(d*x + c)^3 + 3*(3*a^2 + 5*a*b)*cosh(d*x + c))*sinh(d*x + c)^2 + (7*(3*
a^2 + 5*a*b)*cosh(d*x + c)^6 + 10*(3*a^2 + 11*a*b + 10*b^2)*cosh(d*x + c)^4
 + 3*(3*a^2 + 5*a*b)*cosh(d*x + c)^2)*sinh(d*x + c))*sqrt(-b/a)*log((a*cosh
(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a
- 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a - 2*b)*sinh(d*x + c)^2
+ 4*(a*cosh(d*x + c)^3 + (a - 2*b)*cosh(d*x + c))*sinh(d*x + c) + 4*(a*cosh
(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 + a*cos
h(d*x + c) + (3*a*cosh(d*x + c)^2 + a)*sinh(d*x + c))*sqrt(-b/a) + a)/(a*co
sh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(
a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^
2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) + a
^2 + 2*(5*a^2*cosh(d*x + c)^9 - 4*(7*a^2 + 20*a*b)*cosh(d*x + c)^7 - 6*(13*
a^2 + 66*a*b + 60*b^2)*cosh(d*x + c)^5 - 4*(13*a^2 + 66*a*b + 60*b^2)*cosh(
d*x + c)^3 - (7*a^2 + 20*a*b)*cosh(d*x + c))*sinh(d*x + c))/(a^4*d*cosh(d*x
 + c)^7 + 7*a^4*d*cosh(d*x + c)*sinh(d*x + c)^6 + a^4*d*sinh(d*x + c)^7 + a
^4*d*cosh(d*x + c)^3 + 2*(a^4 + 2*a^3*b)*d*cosh(d*x + c)^5 + (21*a^4*d*cosh
(d*x + c)^2 + 2*(a^4 + 2*a^3*b)*d)*sinh(d*x + c)^5 + 5*(7*a^4*d*cosh(d*x +
c)^3 + 2*(a^4 + 2*a^3*b)*d*cosh(d*x + c))*sinh(d*x + c)^4 + (35*a^4*d*cosh(
d*x + c)^4 + a^4*d + 20*(a^4 + 2*a^3*b)*d*cosh(d*x + c)^2)*sinh(d*x + c)^3
+ (21*a^4*d*cosh(d*x + c)^5 + 3*a^4*d*cosh(d*x + c) + 20*(a^4 + 2*a^3*b)*d*
cosh(d*x + c)^3)*sinh(d*x + c)^2 + (7*a^4*d*cosh(d*x + c)^6 + 3*a^4*d*cosh(
d*x + c)^2 + 10*(a^4 + 2*a^3*b)*d*cosh(d*x + c)^4)*sinh(d*x + c)), 1/24*(a^
2*cosh(d*x + c)^10 + 10*a^2*cosh(d*x + c)*sinh(d*x + c)^9 + a^2*sinh(d*x +
c)^10 - (7*a^2 + 20*a*b)*cosh(d*x + c)^8 + (45*a^2*cosh(d*x + c)^2 - 7*a^2
- 20*a*b)*sinh(d*x + c)^8 + 8*(15*a^2*cosh(d*x + c)^3 - (7*a^2 + 20*a*b)*co
sh(d*x + c))*sinh(d*x + c)^7 - 2*(13*a^2 + 66*a*b + 60*b^2)*cosh(d*x + c)^6
 + 2*(105*a^2*cosh(d*x + c)^4 - 14*(7*a^2 + 20*a*b)*cosh(d*x + c)^2 - 13*a^
2 - 66*a*b - 60*b^2)*sinh(d*x + c)^6 + 4*(63*a^2*cosh(d*x + c)^5 - 14*(7*a^
2 + 20*a*b)*cosh(d*x + c)^3 - 3*(13*a^2 + 66*a*b + 60*b^2)*cosh(d*x + c))*s
inh(d*x + c)^5 - 2*(13*a^2 + 66*a*b + 60*b^2)*cosh(d*x + c)^4 + 2*(105*a^2*
cosh(d*x + c)^6 - 35*(7*a^2 + 20*a*b)*cosh(d*x + c)^4 - 15*(13*a^2 + 66*a*b
 + 60*b^2)*cosh(d*x + c)^2 - 13*a^2 - 66*a*b - 60*b^2)*sinh(d*x + c)^4 + 8*

```
(15*a^2*cosh(d*x + c)^7 - 7*(7*a^2 + 20*a*b)*cosh(d*x + c)^5 - 5*(13*a^2 +
66*a*b + 60*b^2)*cosh(d*x + c)^3 - (13*a^2 + 66*a*b + 60*b^2)*cosh(d*x + c)
)*sinh(d*x + c)^3 - (7*a^2 + 20*a*b)*cosh(d*x + c)^2 + (45*a^2*cosh(d*x + c
)^8 - 28*(7*a^2 + 20*a*b)*cosh(d*x + c)^6 - 30*(13*a^2 + 66*a*b + 60*b^2)*c
osh(d*x + c)^4 - 12*(13*a^2 + 66*a*b + 60*b^2)*cosh(d*x + c)^2 - 7*a^2 - 20
*a*b)*sinh(d*x + c)^2 - 12*((3*a^2 + 5*a*b)*cosh(d*x + c)^7 + 7*(3*a^2 + 5*
a*b)*cosh(d*x + c)*sinh(d*x + c)^6 + (3*a^2 + 5*a*b)*sinh(d*x + c)^7 + 2*(3
*a^2 + 11*a*b + 10*b^2)*cosh(d*x + c)^5 + (21*(3*a^2 + 5*a*b)*cosh(d*x + c)
^2 + 6*a^2 + 22*a*b + 20*b^2)*sinh(d*x + c)^5 + 5*(7*(3*a^2 + 5*a*b)*cosh(d
*x + c)^3 + 2*(3*a^2 + 11*a*b + 10*b^2)*cosh(d*x + c))*sinh(d*x + c)^4 + (3
*a^2 + 5*a*b)*cosh(d*x + c)^3 + (35*(3*a^2 + 5*a*b)*cosh(d*x + c)^4 + 20*(3
*a^2 + 11*a*b + 10*b^2)*cosh(d*x + c)^2 + 3*a^2 + 5*a*b)*sinh(d*x + c)^3 +
(21*(3*a^2 + 5*a*b)*cosh(d*x + c)^5 + 20*(3*a^2 + 11*a*b + 10*b^2)*cosh(d*x
 + c)^3 + 3*(3*a^2 + 5*a*b)*cosh(d*x + c))*sinh(d*x + c)^2 + (7*(3*a^2 + 5*
a*b)*cosh(d*x + c)^6 + 10*(3*a^2 + 11*a*b + 10*b^2)*cosh(d*x + c)^4 + 3*(3*
a^2 + 5*a*b)*cosh(d*x + c)^2)*sinh(d*x + c))*sqrt(b/a)*arctan(1/2*(a*cosh(d
*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 + (a + 4*
b)*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + a + 4*b)*sinh(d*x + c))*sqrt(b/a)
/b) + 12*((3*a^2 + 5*a*b)*cosh(d*x + c)^7 + 7*(3*a^2 + 5*a*b)*cosh(d*x + c)
*sinh(d*x + c)^6 + (3*a^2 + 5*a*b)*sinh(d*x + c)^7 + 2*(3*a^2 + 11*a*b + 10
*b^2)*cosh(d*x + c)^5 + (21*(3*a^2 + 5*a*b)*cosh(d*x + c)^2 + 6*a^2 + 22*a*
b + 20*b^2)*sinh(d*x + c)^5 + 5*(7*(3*a^2 + 5*a*b)*cosh(d*x + c)^3 + 2*(3*a
^2 + 11*a*b + 10*b^2)*cosh(d*x + c))*sinh(d*x + c)^4 + (3*a^2 + 5*a*b)*cosh
(d*x + c)^3 + (35*(3*a^2 + 5*a*b)*cosh(d*x + c)^4 + 20*(3*a^2 + 11*a*b + 10
*b^2)*cosh(d*x + c)^2 + 3*a^2 + 5*a*b)*sinh(d*x + c)^3 + (21*(3*a^2 + 5*a*b
)*cosh(d*x + c)^5 + 20*(3*a^2 + 11*a*b + 10*b^2)*cosh(d*x + c)^3 + 3*(3*a^2
 + 5*a*b)*cosh(d*x + c))*sinh(d*x + c)^2 + (7*(3*a^2 + 5*a*b)*cosh(d*x + c)
^6 + 10*(3*a^2 + 11*a*b + 10*b^2)*cosh(d*x + c)^4 + 3*(3*a^2 + 5*a*b)*cosh(
d*x + c)^2)*sinh(d*x + c))*sqrt(b/a)*arctan(1/2*(a*cosh(d*x + c) + a*sinh(d
*x + c))*sqrt(b/a)/b) + a^2 + 2*(5*a^2*cosh(d*x + c)^9 - 4*(7*a^2 + 20*a*b)
*cosh(d*x + c)^7 - 6*(13*a^2 + 66*a*b + 60*b^2)*cosh(d*x + c)^5 - 4*(13*a^2
 + 66*a*b + 60*b^2)*cosh(d*x + c)^3 - (7*a^2 + 20*a*b)*cosh(d*x + c))*sinh(
d*x + c))/(a^4*d*cosh(d*x + c)^7 + 7*a^4*d*cosh(d*x + c)*sinh(d*x + c)^6 +
a^4*d*sinh(d*x + c)^7 + a^4*d*cosh(d*x + c)^3 + 2*(a^4 + 2*a^3*b)*d*cosh(d*
x + c)^5 + (21*a^4*d*cosh(d*x + c)^2 + 2*(a^4 + 2*a^3*b)*d)*sinh(d*x + c)^5
 + 5*(7*a^4*d*cosh(d*x + c)^3 + 2*(a^4 + 2*a^3*b)*d*cosh(d*x + c))*sinh(d*x
 + c)^4 + (35*a^4*d*cosh(d*x + c)^4 + a^4*d + 20*(a^4 + 2*a^3*b)*d*cosh(d*x
 + c)^2)*sinh(d*x + c)^3 + (21*a^4*d*cosh(d*x + c)^5 + 3*a^4*d*cosh(d*x + c
) + 20*(a^4 + 2*a^3*b)*d*cosh(d*x + c)^3)*sinh(d*x + c)^2 + (7*a^4*d*cosh(d
*x + c)^6 + 3*a^4*d*cosh(d*x + c)^2 + 10*(a^4 + 2*a^3*b)*d*cosh(d*x + c)^4)
*sinh(d*x + c))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[6,-20]Warning, need to choose a branch for the root of a po
lynomial with parameters. This might be wrong.The choice was done assuming
[a,b]=[89,-63]Warning, need to choose a branch for the root of a polynomial
 with parameters. This might be wrong.The choice was done assuming [a,b]=[1
2,-32]Warning, need to choose a branch for the root of a polynomial with pa
rameters. This might be wrong.The choice was done assuming [a,b]=[2,72]Warn
ing, need to choose a branch for the root of a polynomial with parameters.
This might be wrong.The choice was done assuming [a,b]=[-37,-59]Warning, ne
ed to choose a branch for the root of a polynomial with parameters. This mi
ght be wrong.The choice was done assuming [a,b]=[-67,22]Warning, need to ch
oose a branch for the root of a polynomial with parameters. This might be w
rong.The choice was done assuming [a,b]=[80,-46]Warning, need to choose a b
ranch for the root of a polynomial with parameters. This might be wrong.The
 choice was done assuming [a,b]=[-72,77]Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
was done assuming [a,b]=[43,41]Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[37,80]Undef/Unsigned Inf encountered in limitEvaluation tim
e: 1.43Limit: Max order reached or unable to make series expansion Error: B
ad Argument Value
```

maple [B] time = 0.37, size = 561, normalized size = 4.92

$$-\frac{1}{3d\,a^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{1}{2d\,a^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{1}{2d\,a^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{2b}{d\,a^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] -1/3/d/a^2/(tanh(1/2*d*x+1/2*c)-1)^3-1/2/d/a^2/(tanh(1/2*d*x+1/2*c)-1)^2+1/
2/d/a^2/(tanh(1/2*d*x+1/2*c)-1)+2/d/a^3/(tanh(1/2*d*x+1/2*c)-1)*b+1/3/d/a^2
/(tanh(1/2*d*x+1/2*c)+1)^3-1/2/d/a^2/(tanh(1/2*d*x+1/2*c)+1)^2-1/2/d/a^2/(t
anh(1/2*d*x+1/2*c)+1)-2/d/a^3/(tanh(1/2*d*x+1/2*c)+1)*b-1/d/a^2*b/(tanh(1/2
*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/
2*d*x+1/2*c)^2*b+a+b)*tanh(1/2*d*x+1/2*c)^2+1/d/a^3*b^2/(tanh(1/2*d*x+1/2*c
```

```
)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*
c)^2*b+a+b)*tanh(1/2*d*x+1/2*c)^2-1/d/a^2*b/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh
(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)-
1/d/a^3*b^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x
+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)+3/2/d/a^2*b/(a*b)^(1/2)*arctan(1
/4*(2*(a+b)*tanh(1/2*d*x+1/2*c)^2+2*a-2*b)/(a*b)^(1/2))+5/2/d/a^3*b^2/(a*b)
^(1/2)*arctan(1/4*(2*(a+b)*tanh(1/2*d*x+1/2*c)^2+2*a-2*b)/(a*b)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^2e^{(10\,dx+10\,c)} + a^2 - \left(7\,a^2e^{(8\,c)} + 20\,abe^{(8\,c)}\right)e^{(8\,dx)} - 2\left(13\,a^2e^{(6\,c)} + 66\,abe^{(6\,c)} + 60\,b^2e^{(6\,c)}\right)e^{(6\,dx)} - 2\left(13\,a^2e^{(4\,c)} + 6\right.}{24\left(a^4de^{(7\,dx+7\,c)} + a^4de^{(3\,dx+3\,c)} + 2\left(a^4de^{(5\,c)} + 2\,a^3bde^{(5\,c)}\right)e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/24*(a^2*e^(10*d*x + 10*c) + a^2 - (7*a^2*e^(8*c) + 20*a*b*e^(8*c))*e^(8*d
*x) - 2*(13*a^2*e^(6*c) + 66*a*b*e^(6*c) + 60*b^2*e^(6*c))*e^(6*d*x) - 2*(1
3*a^2*e^(4*c) + 66*a*b*e^(4*c) + 60*b^2*e^(4*c))*e^(4*d*x) - (7*a^2*e^(2*c)
 + 20*a*b*e^(2*c))*e^(2*d*x))/(a^4*d*e^(7*d*x + 7*c) + a^4*d*e^(3*d*x + 3*c
) + 2*(a^4*d*e^(5*c) + 2*a^3*b*d*e^(5*c))*e^(5*d*x)) + 1/8*integrate(8*((3*
a*b*e^(3*c) + 5*b^2*e^(3*c))*e^(3*d*x) - (3*a*b*e^c + 5*b^2*e^c)*e^(d*x))/(
a^4*e^(4*d*x + 4*c) + a^4 + 2*(a^4*e^(2*c) + 2*a^3*b*e^(2*c))*e^(2*d*x)), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh\left(c+d\,x\right)^4\,\sinh\left(c+d\,x\right)^3}{{\left(a\,\cosh\left(c+d\,x\right)^2+b\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^3/(a + b/cosh(c + d*x)^2)^2,x)
```

```
[Out] int((cosh(c + d*x)^4*sinh(c + d*x)^3)/(b + a*cosh(c + d*x)^2)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

3.35 $\int \frac{\sinh^2(c+dx)}{\left(a+b\,\textbf{sech}^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=131

$$\frac{\sqrt{b}\,(3a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^3d\sqrt{a+b}}-\frac{x(a+4b)}{2a^3}+\frac{b\tanh(c+dx)}{a^2d\left(a-b\tanh^2(c+dx)+b\right)}+\frac{\sinh(c+dx)\cosh(c+dx)}{2ad\left(a-b\tanh^2(c+dx)+b\right)}$$

```
[Out] -1/2*(a+4*b)*x/a^3+1/2*(3*a+4*b)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))*b
^(1/2)/a^3/d/(a+b)^(1/2)+1/2*cosh(d*x+c)*sinh(d*x+c)/a/d/(a+b-b*tanh(d*x+c)
^2)+b*tanh(d*x+c)/a^2/d/(a+b-b*tanh(d*x+c)^2)
```

Rubi [A] time = 0.20, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4132, 471, 527, 522, 206, 208}

$$\frac{\sqrt{b}\,(3a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^3d\sqrt{a+b}}+\frac{b\tanh(c+dx)}{a^2d\left(a-b\tanh^2(c+dx)+b\right)}-\frac{x(a+4b)}{2a^3}+\frac{\sinh(c+dx)\cosh(c+dx)}{2ad\left(a-b\tanh^2(c+dx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] -((a + 4*b)*x)/(2*a^3) + (Sqrt[b]*(3*a + 4*b)*ArcTanh[(Sqrt[b]*Tanh[c + d*x
])/Sqrt[a + b]])/(2*a^3*Sqrt[a + b]*d) + (Cosh[c + d*x]*Sinh[c + d*x])/(2*a
*d*(a + b - b*Tanh[c + d*x]^2)) + (b*Tanh[c + d*x])/(a^2*d*(a + b - b*Tanh[
c + d*x]^2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
```

```
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sinh^2(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1-x^2)^2(a+b-bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2ad\left(a+b-b\tanh^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{a+b+3bx^2}{(1-x^2)(a+b-bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{2ad}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2ad\left(a+b-b\tanh^2(c+dx)\right)} + \frac{b\tanh(c+dx)}{a^2d\left(a+b-b\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{-2(a+b)\cdots}{(1-\cdots}\right.}{\cdots}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2ad\left(a+b-b\tanh^2(c+dx)\right)} + \frac{b\tanh(c+dx)}{a^2d\left(a+b-b\tanh^2(c+dx)\right)} - \frac{(a+4b)\,\text{Subst}\left(\cdots\right.}{\cdots}$$

$$= -\frac{(a+4b)x}{2a^3} + \frac{\sqrt{b}\,(3a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^3\sqrt{a+b}\,d} + \frac{\cosh(c+dx)\sinh(c+dx)}{2ad\left(a+b-b\tanh^2(c+dx)\right)}\cdots$$

Mathematica [B] time = 11.33, size = 791, normalized size = 6.04

$$\frac{\text{sech}^4(c+dx)(a\cosh(2c+2dx)+a+2b)^2\left(\frac{(a^2+8ab+8b^2)\text{sech}(2c)((a+2b)\sinh(2c)-a\sinh(2dx))}{bd(a+b)(a\cosh(2(c+dx))+a+2b)} + \frac{(a^3-6a^2b-24ab^2-16b^3)(\cosh(2c)\cdots}{\cdots}\right.}{128a^2\left(a+b\text{sech}^2(c+dx)\right)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sinh[c + d*x]^2/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*c + 2*d*x])^2*Sech[c + d*x]^4*(16*x + ((a^3 - 6*a^2*b
- 24*a*b^2 - 16*b^3)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*
Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4
])]*(Cosh[2*c] - Sinh[2*c]))/(b*(a + b)^(3/2)*d*Sqrt[b*(Cosh[c] - Sinh[c])^
4]) + ((a^2 + 8*a*b + 8*b^2)*Sech[2*c]*((a + 2*b)*Sinh[2*c] - a*Sinh[2*d*x]
))/(b*(a + b)*d*(a + 2*b + a*Cosh[2*(c + d*x)]))))/(128*a^2*(a + b*Sech[c +
 d*x]^2)^2) + ((a + 2*b + a*Cosh[2*c + 2*d*x])^2*Sech[c + d*x]^4*(-64*(a +
2*b)*x + ((-a^4 + 16*a^3*b + 144*a^2*b^2 + 256*a*b^3 + 128*b^4)*ArcTanh[(Se
ch[d*x]*(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/
```

```
(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(Cosh[2*c] - Sinh[2*c]))/(b*
(a + b)^(3/2)*d*Sqrt[b*(Cosh[c] - Sinh[c])^4]) + (16*a*Cosh[2*d*x]*Sinh[2*c
])/d + (16*a*Cosh[2*c]*Sinh[2*d*x])/d - ((a^3 + 18*a^2*b + 48*a*b^2 + 32*b^
3)*Sech[2*c]*((a + 2*b)*Sinh[2*c] - a*Sinh[2*d*x]))/(b*(a + b)*d*(a + 2*b +
 a*Cosh[2*(c + d*x)]))))/(256*a^3*(a + b*Sech[c + d*x]^2)^2) - ((a + 2*b +
a*Cosh[2*c + 2*d*x])^2*Sech[c + d*x]^4*(-((a*ArcTanh[(Sqrt[b]*Tanh[c + d*x]
)/Sqrt[a + b]])/(a + b)^(3/2)) + (Sqrt[b]*(a + 2*b)*Sinh[2*(c + d*x)])/((a
+ b)*(a + 2*b + a*Cosh[2*(c + d*x)]))))/(256*b^(3/2)*d*(a + b*Sech[c + d*x]
^2)^2) + ((a + 2*b + a*Cosh[2*c + 2*d*x])^2*Sech[c + d*x]^4*(-1/8*((a + 2*b
)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(b^(3/2)*(a + b)^(3/2)*d) +
 (a*Sinh[2*(c + d*x)])/(8*b*(a + b)*d*(a + 2*b + a*Cosh[2*(c + d*x)]))))/(1
6*(a + b*Sech[c + d*x]^2)^2)
```

fricas [B] time = 0.50, size = 2925, normalized size = 22.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(a^2*cosh(d*x + c)^8 + 8*a^2*cosh(d*x + c)*sinh(d*x + c)^7 + a^2*sinh(
d*x + c)^8 - 2*(2*(a^2 + 4*a*b)*d*x - a^2 - 2*a*b)*cosh(d*x + c)^6 + 2*(14*
a^2*cosh(d*x + c)^2 - 2*(a^2 + 4*a*b)*d*x + a^2 + 2*a*b)*sinh(d*x + c)^6 +
4*(14*a^2*cosh(d*x + c)^3 - 3*(2*(a^2 + 4*a*b)*d*x - a^2 - 2*a*b)*cosh(d*x
+ c))*sinh(d*x + c)^5 - 8*((a^2 + 6*a*b + 8*b^2)*d*x + a*b + 2*b^2)*cosh(d*
x + c)^4 + 2*(35*a^2*cosh(d*x + c)^4 - 4*(a^2 + 6*a*b + 8*b^2)*d*x - 15*(2*
(a^2 + 4*a*b)*d*x - a^2 - 2*a*b)*cosh(d*x + c)^2 - 4*a*b - 8*b^2)*sinh(d*x
+ c)^4 + 8*(7*a^2*cosh(d*x + c)^5 - 5*(2*(a^2 + 4*a*b)*d*x - a^2 - 2*a*b)*c
osh(d*x + c)^3 - 4*((a^2 + 6*a*b + 8*b^2)*d*x + a*b + 2*b^2)*cosh(d*x + c))
*sinh(d*x + c)^3 - 2*(2*(a^2 + 4*a*b)*d*x + a^2 + 6*a*b)*cosh(d*x + c)^2 +
2*(14*a^2*cosh(d*x + c)^6 - 15*(2*(a^2 + 4*a*b)*d*x - a^2 - 2*a*b)*cosh(d*x
 + c)^4 - 2*(a^2 + 4*a*b)*d*x - 24*((a^2 + 6*a*b + 8*b^2)*d*x + a*b + 2*b^2
)*cosh(d*x + c)^2 - a^2 - 6*a*b)*sinh(d*x + c)^2 + 2*((3*a^2 + 4*a*b)*cosh(
d*x + c)^6 + 6*(3*a^2 + 4*a*b)*cosh(d*x + c)*sinh(d*x + c)^5 + (3*a^2 + 4*a
*b)*sinh(d*x + c)^6 + 2*(3*a^2 + 10*a*b + 8*b^2)*cosh(d*x + c)^4 + (15*(3*a
^2 + 4*a*b)*cosh(d*x + c)^2 + 6*a^2 + 20*a*b + 16*b^2)*sinh(d*x + c)^4 + 4*
(5*(3*a^2 + 4*a*b)*cosh(d*x + c)^3 + 2*(3*a^2 + 10*a*b + 8*b^2)*cosh(d*x +
c))*sinh(d*x + c)^3 + (3*a^2 + 4*a*b)*cosh(d*x + c)^2 + (15*(3*a^2 + 4*a*b)
*cosh(d*x + c)^4 + 12*(3*a^2 + 10*a*b + 8*b^2)*cosh(d*x + c)^2 + 3*a^2 + 4*
a*b)*sinh(d*x + c)^2 + 2*(3*(3*a^2 + 4*a*b)*cosh(d*x + c)^5 + 4*(3*a^2 + 10
*a*b + 8*b^2)*cosh(d*x + c)^3 + (3*a^2 + 4*a*b)*cosh(d*x + c))*sinh(d*x + c
))*sqrt(b/(a + b))*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x
+ c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*c
osh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^
2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) - 4*((a^2 +
```

a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a
*b)*sinh(d*x + c)^2 + a^2 + 3*a*b + 2*b^2)*sqrt(b/(a + b)))/(a*cosh(d*x + c
)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*c
osh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*c
osh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) - a^2 + 4*(2*
a^2*cosh(d*x + c)^7 - 3*(2*(a^2 + 4*a*b)*d*x - a^2 - 2*a*b)*cosh(d*x + c)^5
 - 8*((a^2 + 6*a*b + 8*b^2)*d*x + a*b + 2*b^2)*cosh(d*x + c)^3 - (2*(a^2 +
4*a*b)*d*x + a^2 + 6*a*b)*cosh(d*x + c))*sinh(d*x + c))/(a^4*d*cosh(d*x + c
)^6 + 6*a^4*d*cosh(d*x + c)*sinh(d*x + c)^5 + a^4*d*sinh(d*x + c)^6 + a^4*d
*cosh(d*x + c)^2 + 2*(a^4 + 2*a^3*b)*d*cosh(d*x + c)^4 + (15*a^4*d*cosh(d*x
 + c)^2 + 2*(a^4 + 2*a^3*b)*d)*sinh(d*x + c)^4 + 4*(5*a^4*d*cosh(d*x + c)^3
 + 2*(a^4 + 2*a^3*b)*d*cosh(d*x + c))*sinh(d*x + c)^3 + (15*a^4*d*cosh(d*x
+ c)^4 + a^4*d + 12*(a^4 + 2*a^3*b)*d*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*
(3*a^4*d*cosh(d*x + c)^5 + a^4*d*cosh(d*x + c) + 4*(a^4 + 2*a^3*b)*d*cosh(d
*x + c)^3)*sinh(d*x + c)), 1/8*(a^2*cosh(d*x + c)^8 + 8*a^2*cosh(d*x + c)*s
inh(d*x + c)^7 + a^2*sinh(d*x + c)^8 - 2*(2*(a^2 + 4*a*b)*d*x - a^2 - 2*a*b
)*cosh(d*x + c)^6 + 2*(14*a^2*cosh(d*x + c)^2 - 2*(a^2 + 4*a*b)*d*x + a^2 +
 2*a*b)*sinh(d*x + c)^6 + 4*(14*a^2*cosh(d*x + c)^3 - 3*(2*(a^2 + 4*a*b)*d*
x - a^2 - 2*a*b)*cosh(d*x + c))*sinh(d*x + c)^5 - 8*((a^2 + 6*a*b + 8*b^2)*
d*x + a*b + 2*b^2)*cosh(d*x + c)^4 + 2*(35*a^2*cosh(d*x + c)^4 - 4*(a^2 + 6
*a*b + 8*b^2)*d*x - 15*(2*(a^2 + 4*a*b)*d*x - a^2 - 2*a*b)*cosh(d*x + c)^2
- 4*a*b - 8*b^2)*sinh(d*x + c)^4 + 8*(7*a^2*cosh(d*x + c)^5 - 5*(2*(a^2 + 4
*a*b)*d*x - a^2 - 2*a*b)*cosh(d*x + c)^3 - 4*((a^2 + 6*a*b + 8*b^2)*d*x + a
*b + 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - 2*(2*(a^2 + 4*a*b)*d*x + a^2 +
 6*a*b)*cosh(d*x + c)^2 + 2*(14*a^2*cosh(d*x + c)^6 - 15*(2*(a^2 + 4*a*b)*d
*x - a^2 - 2*a*b)*cosh(d*x + c)^4 - 2*(a^2 + 4*a*b)*d*x - 24*((a^2 + 6*a*b
+ 8*b^2)*d*x + a*b + 2*b^2)*cosh(d*x + c)^2 - a^2 - 6*a*b)*sinh(d*x + c)^2
+ 4*((3*a^2 + 4*a*b)*cosh(d*x + c)^6 + 6*(3*a^2 + 4*a*b)*cosh(d*x + c)*sinh
(d*x + c)^5 + (3*a^2 + 4*a*b)*sinh(d*x + c)^6 + 2*(3*a^2 + 10*a*b + 8*b^2)*
cosh(d*x + c)^4 + (15*(3*a^2 + 4*a*b)*cosh(d*x + c)^2 + 6*a^2 + 20*a*b + 16
*b^2)*sinh(d*x + c)^4 + 4*(5*(3*a^2 + 4*a*b)*cosh(d*x + c)^3 + 2*(3*a^2 + 1
0*a*b + 8*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + (3*a^2 + 4*a*b)*cosh(d*x +
c)^2 + (15*(3*a^2 + 4*a*b)*cosh(d*x + c)^4 + 12*(3*a^2 + 10*a*b + 8*b^2)*co
sh(d*x + c)^2 + 3*a^2 + 4*a*b)*sinh(d*x + c)^2 + 2*(3*(3*a^2 + 4*a*b)*cosh(
d*x + c)^5 + 4*(3*a^2 + 10*a*b + 8*b^2)*cosh(d*x + c)^3 + (3*a^2 + 4*a*b)*c
osh(d*x + c))*sinh(d*x + c))*sqrt(-b/(a + b))*arctan(1/2*(a*cosh(d*x + c)^2
 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-b/(
a + b))/b) - a^2 + 4*(2*a^2*cosh(d*x + c)^7 - 3*(2*(a^2 + 4*a*b)*d*x - a^2
- 2*a*b)*cosh(d*x + c)^5 - 8*((a^2 + 6*a*b + 8*b^2)*d*x + a*b + 2*b^2)*cosh
(d*x + c)^3 - (2*(a^2 + 4*a*b)*d*x + a^2 + 6*a*b)*cosh(d*x + c))*sinh(d*x +
 c))/(a^4*d*cosh(d*x + c)^6 + 6*a^4*d*cosh(d*x + c)*sinh(d*x + c)^5 + a^4*d
*sinh(d*x + c)^6 + a^4*d*cosh(d*x + c)^2 + 2*(a^4 + 2*a^3*b)*d*cosh(d*x + c
)^4 + (15*a^4*d*cosh(d*x + c)^2 + 2*(a^4 + 2*a^3*b)*d)*sinh(d*x + c)^4 + 4*
(5*a^4*d*cosh(d*x + c)^3 + 2*(a^4 + 2*a^3*b)*d*cosh(d*x + c))*sinh(d*x + c)
^3 + (15*a^4*d*cosh(d*x + c)^4 + a^4*d + 12*(a^4 + 2*a^3*b)*d*cosh(d*x + c)

```
^2)*sinh(d*x + c)^2 + 2*(3*a^4*d*cosh(d*x + c)^5 + a^4*d*cosh(d*x + c) + 4*
(a^4 + 2*a^3*b)*d*cosh(d*x + c)^3)*sinh(d*x + c))]
```

giac [A] time = 2.05, size = 234, normalized size = 1.79

$$-\frac{\frac{12\,(dx+c)(a+4\,b)}{a^3}-\frac{3\,e^{(2\,dx+2\,c)}}{a^2}-\frac{12\left(3\,ab+4\,b^2\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}\,a^3}-\frac{2\,a^2e^{(6\,dx+6\,c)}+8\,abe^{(6\,dx+6\,c)}+a^2e^{(4\,dx+4\,c)}-16\,b^2e^{(4\,dx+4\,c)}-4\,a^2e^{(2\,d}}{\left(ae^{(6\,dx+6\,c)}+2\,ae^{(4\,dx+4\,c)}+4\,be^{(4\,dx+4\,c)}+ae^{(2\,dx+}\right.}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/24*(12*(d*x + c)*(a + 4*b)/a^3 - 3*e^(2*d*x + 2*c)/a^2 - 12*(3*a*b + 4*b
^2)*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/(sqrt(-a*b -
 b^2)*a^3) - (2*a^2*e^(6*d*x + 6*c) + 8*a*b*e^(6*d*x + 6*c) + a^2*e^(4*d*x
+ 4*c) - 16*b^2*e^(4*d*x + 4*c) - 4*a^2*e^(2*d*x + 2*c) - 28*a*b*e^(2*d*x +
 2*c) - 3*a^2)/((a*e^(6*d*x + 6*c) + 2*a*e^(4*d*x + 4*c) + 4*b*e^(4*d*x + 4
*c) + a*e^(2*d*x + 2*c))*a^3))/d
```

maple [B] time = 0.37, size = 543, normalized size = 4.15

$$\frac{1}{2d\,a^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{1}{2d\,a^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2d\,a^2}+\frac{2\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)b}{d\,a^3}-\frac{}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] 1/2/d/a^2/(tanh(1/2*d*x+1/2*c)-1)^2+1/2/d/a^2/(tanh(1/2*d*x+1/2*c)-1)+1/2/d
/a^2*ln(tanh(1/2*d*x+1/2*c)-1)+2/d/a^3*ln(tanh(1/2*d*x+1/2*c)-1)*b-1/2/d/a^
2/(tanh(1/2*d*x+1/2*c)+1)^2+1/2/d/a^2/(tanh(1/2*d*x+1/2*c)+1)-1/2/d/a^2*ln(
tanh(1/2*d*x+1/2*c)+1)-2/d/a^3*ln(tanh(1/2*d*x+1/2*c)+1)*b+1/d/a^2*b/(tanh(
1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh
(1/2*d*x+1/2*c)^2*b+a+b)*tanh(1/2*d*x+1/2*c)^3+1/d/a^2*b/(tanh(1/2*d*x+1/2*
c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2
*c)^2*b+a+b)*tanh(1/2*d*x+1/2*c)-3/4/d/a^2*b^(1/2)/(a+b)^(1/2)*ln(-(a+b)^(1
/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)-(a+b)^(1/2))+3/4/d/
a^2*b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh
(1/2*d*x+1/2*c)+(a+b)^(1/2))-1/d/a^3*b^(3/2)/(a+b)^(1/2)*ln(-(a+b)^(1/2)*ta
nh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)-(a+b)^(1/2))+1/d/a^3*b^(3
/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x
+1/2*c)+(a+b)^(1/2))
```

maxima [B] time = 0.46, size = 696, normalized size = 5.31

$$\frac{\left(3\,a^2b+12\,ab^2+8\,b^3\right)\log\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(2\,dx+2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{16\left(a^4+a^3b\right)\sqrt{(a+b)b}\,d}-\frac{\left(3\,a^2b+12\,ab^2+8\,b^3\right)\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{16\left(a^4+a^3b\right)\sqrt{(a+b)b}\,d}-\frac{(3\,a\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/16*(3*a^2*b + 12*a*b^2 + 8*b^3)*log((a*e^(2*d*x + 2*c) + a + 2*b - 2*sqrt
((a + b)*b))/(a*e^(2*d*x + 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^4 + a^3
*b)*sqrt((a + b)*b)*d) - 1/16*(3*a^2*b + 12*a*b^2 + 8*b^3)*log((a*e^(-2*d*x
 - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sq
rt((a + b)*b)))/((a^4 + a^3*b)*sqrt((a + b)*b)*d) - 1/8*(3*a*b + 2*b^2)*log
((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a
 + 2*b + 2*sqrt((a + b)*b)))/((a^3 + a^2*b)*sqrt((a + b)*b)*d) - 1/4*(a^2*b
 + 2*a*b^2 + (a^2*b + 8*a*b^2 + 8*b^3)*e^(2*d*x + 2*c))/((a^5 + a^4*b + (a^
5 + a^4*b)*e^(4*d*x + 4*c) + 2*(a^5 + 3*a^4*b + 2*a^3*b^2)*e^(2*d*x + 2*c))
*d) + 1/4*(a^2*b + 2*a*b^2 + (a^2*b + 8*a*b^2 + 8*b^3)*e^(-2*d*x - 2*c))/((
a^5 + a^4*b + 2*(a^5 + 3*a^4*b + 2*a^3*b^2)*e^(-2*d*x - 2*c) + (a^5 + a^4*b
)*e^(-4*d*x - 4*c))*d) + 1/2*(a*b + (a*b + 2*b^2)*e^(-2*d*x - 2*c))/((a^4 +
 a^3*b + 2*(a^4 + 3*a^3*b + 2*a^2*b^2)*e^(-2*d*x - 2*c) + (a^4 + a^3*b)*e^(
-4*d*x - 4*c))*d) - 1/2*(d*x + c)/(a^2*d) + 1/8*e^(2*d*x + 2*c)/(a^2*d) - 1
/8*e^(-2*d*x - 2*c)/(a^2*d) - 1/2*b*log(a*e^(4*d*x + 4*c) + 2*(a + 2*b)*e^(
2*d*x + 2*c) + a)/(a^3*d) + 1/2*b*log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-
4*d*x - 4*c) + a)/(a^3*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh\left(c+d\,x\right)^4\sinh\left(c+d\,x\right)^2}{\left(a\cosh\left(c+d\,x\right)^2+b\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^2/(a + b/cosh(c + d*x)^2)^2,x)
```

```
[Out] int((cosh(c + d*x)^4*sinh(c + d*x)^2)/(b + a*cosh(c + d*x)^2)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

3.36 $\int \frac{\sinh(c+dx)}{\left(a+b\mathbf{sech}^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=84

$$-\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{2a^{5/2}d}+\frac{3\cosh(c+dx)}{2a^2d}-\frac{\cosh^3(c+dx)}{2ad\left(a\cosh^2(c+dx)+b\right)}$$

```
[Out] 3/2*cosh(d*x+c)/a^2/d-1/2*cosh(d*x+c)^3/a/d/(b+a*cosh(d*x+c)^2)-3/2*arctan(
cosh(d*x+c)*a^(1/2)/b^(1/2))*b^(1/2)/a^(5/2)/d
```

Rubi [A] time = 0.06, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {4133, 288, 321, 205}

$$-\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{2a^{5/2}d}+\frac{3\cosh(c+dx)}{2a^2d}-\frac{\cosh^3(c+dx)}{2ad\left(a\cosh^2(c+dx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (-3*Sqrt[b]*ArcTan[(Sqrt[a]*Cosh[c + d*x])/Sqrt[b]])/(2*a^(5/2)*d) + (3*Cos
h[c + d*x])/(2*a^2*d) - Cosh[c + d*x]^3/(2*a*d*(b + a*Cosh[c + d*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sinh(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{\left(b+ax^2\right)^2}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{\cosh^3(c+dx)}{2ad\left(b+a\cosh^2(c+dx)\right)} + \frac{3\,\text{Subst}\left(\int \frac{x^2}{b+ax^2}\,dx, x, \cosh(c+dx)\right)}{2ad}$$

$$= \frac{3\cosh(c+dx)}{2a^2d} - \frac{\cosh^3(c+dx)}{2ad\left(b+a\cosh^2(c+dx)\right)} - \frac{(3b)\,\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, \cosh(c+\right.}{2a^2d}$$

$$= -\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{2a^{5/2}d} + \frac{3\cosh(c+dx)}{2a^2d} - \frac{\cosh^3(c+dx)}{2ad\left(b+a\cosh^2(c+dx)\right)}$$

Mathematica [C] time = 2.76, size = 479, normalized size = 5.70

$$\frac{\operatorname{sech}^4(c+dx)(a\cosh(2(c+dx))+a+2b)^2\left(\frac{32b\cosh(c+dx)}{a^2(a\cosh(2(c+dx))+a+2b)} + \frac{32\cosh(c)\cosh(dx)}{a^2} + \frac{2\left(-\left(a^2+24b^2\right)\tan^{-1}\left(\frac{\sinh(c)\tanh\left(\frac{dx}{2}\right.}{}\right.\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])^2*Sech[c + d*x]^4*((32*Cosh[c]*Cosh[d*x])/
a^2 + (32*b*Cosh[c + d*x])/(a^2*(a + 2*b + a*Cosh[2*(c + d*x)])) + (2*(-((a
^2 + 24*b^2)*ArcTan[((Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2])*
Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Si
nh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]]) - a^2*ArcTan[((Sqrt[a] + I*Sqrt[a + b]*
Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] + I*S
qrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]] - 24*b^2*Ar
cTan[((Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d
*x)/2] + Cosh[c]*(Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[
(d*x)/2]))/Sqrt[b]] + a^2*ArcTan[(Sqrt[a] - I*Sqrt[a + b]*Tanh[(c + d*x)/2]
)/Sqrt[b]] + a^2*ArcTan[(Sqrt[a] + I*Sqrt[a + b]*Tanh[(c + d*x)/2])/Sqrt[b]
] + 16*Sqrt[a]*b^(3/2)*Sinh[c]*Sinh[d*x]))/(a^(5/2)*b^(3/2))))/(128*d*(a +
b*Sech[c + d*x]^2)^2)
```

fricas [B] time = 0.54, size = 1780, normalized size = 21.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*cosh(d*x + c)^6 + 12*a*cosh(d*x + c)*sinh(d*x + c)^5 + 2*a*sinh(d
*x + c)^6 + 6*(a + 2*b)*cosh(d*x + c)^4 + 6*(5*a*cosh(d*x + c)^2 + a + 2*b)
*sinh(d*x + c)^4 + 8*(5*a*cosh(d*x + c)^3 + 3*(a + 2*b)*cosh(d*x + c))*sinh
(d*x + c)^3 + 6*(a + 2*b)*cosh(d*x + c)^2 + 6*(5*a*cosh(d*x + c)^4 + 6*(a +
 2*b)*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 3*(a*cosh(d*x + c)^5 + 5
*a*cosh(d*x + c)*sinh(d*x + c)^4 + a*sinh(d*x + c)^5 + 2*(a + 2*b)*cosh(d*x
 + c)^3 + 2*(5*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^3 + 2*(5*a*cosh(d
*x + c)^3 + 3*(a + 2*b)*cosh(d*x + c))*sinh(d*x + c)^2 + a*cosh(d*x + c) +
(5*a*cosh(d*x + c)^4 + 6*(a + 2*b)*cosh(d*x + c)^2 + a)*sinh(d*x + c))*sqrt
(-b/a)*log((a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(
d*x + c)^4 + 2*(a - 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a - 2*b
)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a - 2*b)*cosh(d*x + c))*sinh(d*
x + c) - 4*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(
d*x + c)^3 + a*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + a)*sinh(d*x + c))*sqr
t(-b/a) + a)/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sin
h(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2
*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(
d*x + c) + a)) + 12*(a*cosh(d*x + c)^5 + 2*(a + 2*b)*cosh(d*x + c)^3 + (a +
 2*b)*cosh(d*x + c))*sinh(d*x + c) + 2*a)/(a^3*d*cosh(d*x + c)^5 + 5*a^3*d*
cosh(d*x + c)*sinh(d*x + c)^4 + a^3*d*sinh(d*x + c)^5 + a^3*d*cosh(d*x + c)
 + 2*(a^3 + 2*a^2*b)*d*cosh(d*x + c)^3 + 2*(5*a^3*d*cosh(d*x + c)^2 + (a^3
+ 2*a^2*b)*d)*sinh(d*x + c)^3 + 2*(5*a^3*d*cosh(d*x + c)^3 + 3*(a^3 + 2*a^2
*b)*d*cosh(d*x + c))*sinh(d*x + c)^2 + (5*a^3*d*cosh(d*x + c)^4 + a^3*d + 6
*(a^3 + 2*a^2*b)*d*cosh(d*x + c)^2)*sinh(d*x + c)), 1/2*(a*cosh(d*x + c)^6
```

```
+ 6*a*cosh(d*x + c)*sinh(d*x + c)^5 + a*sinh(d*x + c)^6 + 3*(a + 2*b)*cosh(
d*x + c)^4 + 3*(5*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^4 + 4*(5*a*cos
h(d*x + c)^3 + 3*(a + 2*b)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(a + 2*b)*cos
h(d*x + c)^2 + 3*(5*a*cosh(d*x + c)^4 + 6*(a + 2*b)*cosh(d*x + c)^2 + a + 2
*b)*sinh(d*x + c)^2 + 3*(a*cosh(d*x + c)^5 + 5*a*cosh(d*x + c)*sinh(d*x + c
)^4 + a*sinh(d*x + c)^5 + 2*(a + 2*b)*cosh(d*x + c)^3 + 2*(5*a*cosh(d*x + c
)^2 + a + 2*b)*sinh(d*x + c)^3 + 2*(5*a*cosh(d*x + c)^3 + 3*(a + 2*b)*cosh(
d*x + c))*sinh(d*x + c)^2 + a*cosh(d*x + c) + (5*a*cosh(d*x + c)^4 + 6*(a +
 2*b)*cosh(d*x + c)^2 + a)*sinh(d*x + c))*sqrt(b/a)*arctan(1/2*(a*cosh(d*x
+ c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 + (a + 4*b)*
cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + a + 4*b)*sinh(d*x + c))*sqrt(b/a)/b)
 - 3*(a*cosh(d*x + c)^5 + 5*a*cosh(d*x + c)*sinh(d*x + c)^4 + a*sinh(d*x +
c)^5 + 2*(a + 2*b)*cosh(d*x + c)^3 + 2*(5*a*cosh(d*x + c)^2 + a + 2*b)*sinh
(d*x + c)^3 + 2*(5*a*cosh(d*x + c)^3 + 3*(a + 2*b)*cosh(d*x + c))*sinh(d*x
+ c)^2 + a*cosh(d*x + c) + (5*a*cosh(d*x + c)^4 + 6*(a + 2*b)*cosh(d*x + c)
^2 + a)*sinh(d*x + c))*sqrt(b/a)*arctan(1/2*(a*cosh(d*x + c) + a*sinh(d*x +
 c))*sqrt(b/a)/b) + 6*(a*cosh(d*x + c)^5 + 2*(a + 2*b)*cosh(d*x + c)^3 + (a
 + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)/(a^3*d*cosh(d*x + c)^5 + 5*a^3*d*
cosh(d*x + c)*sinh(d*x + c)^4 + a^3*d*sinh(d*x + c)^5 + a^3*d*cosh(d*x + c)
 + 2*(a^3 + 2*a^2*b)*d*cosh(d*x + c)^3 + 2*(5*a^3*d*cosh(d*x + c)^2 + (a^3
+ 2*a^2*b)*d)*sinh(d*x + c)^3 + 2*(5*a^3*d*cosh(d*x + c)^3 + 3*(a^3 + 2*a^2
*b)*d*cosh(d*x + c))*sinh(d*x + c)^2 + (5*a^3*d*cosh(d*x + c)^4 + a^3*d + 6
*(a^3 + 2*a^2*b)*d*cosh(d*x + c)^2)*sinh(d*x + c))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[6,-20]Warning, need to choose a branch for the root of a po
lynomial with parameters. This might be wrong.The choice was done assuming
[a,b]=[89,-63]Warning, need to choose a branch for the root of a polynomial
 with parameters. This might be wrong.The choice was done assuming [a,b]=[1
2,-32]Warning, need to choose a branch for the root of a polynomial with pa
rameters. This might be wrong.The choice was done assuming [a,b]=[2,72]Warn
ing, need to choose a branch for the root of a polynomial with parameters.
This might be wrong.The choice was done assuming [a,b]=[67,31]Warning, need
 to choose a branch for the root of a polynomial with parameters. This migh
t be wrong.The choice was done assuming [a,b]=[-88,66]Undef/Unsigned Inf en
```

```
countered in limitEvaluation time: 0.78Limit: Max order reached or unable t
o make series expansion Error: Bad Argument Value
```

maple [A] time = 0.17, size = 74, normalized size = 0.88

$$\frac{b\,\mathrm{sech}\,(dx+c)}{2d\,a^2\left(a+b\mathrm{sech}\,(dx+c)^2\right)}+\frac{3b\arctan\left(\frac{\mathrm{sech}(dx+c)b}{\sqrt{ab}}\right)}{2d\,a^2\sqrt{ab}}+\frac{1}{d\,a^2\mathrm{sech}\,(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)/(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] 1/2/d/a^2*b*sech(d*x+c)/(a+b*sech(d*x+c)^2)+3/2/d/a^2*b/(a*b)^(1/2)*arctan(
sech(d*x+c)*b/(a*b)^(1/2))+1/d/a^2/sech(d*x+c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{3\left(ae^{(4c)}+2\,be^{(4c)}\right)e^{(4\,dx)}+3\left(ae^{(2c)}+2\,be^{(2c)}\right)e^{(2\,dx)}+ae^{(6\,dx+6\,c)}+a}{2\left(a^3de^{(5\,dx+5\,c)}+a^3de^{(dx+c)}+2\left(a^3de^{(3\,c)}+2\,a^2bde^{(3\,c)}\right)e^{(3\,dx)}\right)}-\frac{1}{2}\int\frac{6\left(be^{(3\,dx+3\,c)}-be^{(dx+c)}\right)}{a^3e^{(4\,dx+4\,c)}+a^3+2\left(a^3e^{(2\,c)}+2\,a^2be\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(3*(a*e^(4*c) + 2*b*e^(4*c))*e^(4*d*x) + 3*(a*e^(2*c) + 2*b*e^(2*c))*e^
(2*d*x) + a*e^(6*d*x + 6*c) + a)/(a^3*d*e^(5*d*x + 5*c) + a^3*d*e^(d*x + c)
 + 2*(a^3*d*e^(3*c) + 2*a^2*b*d*e^(3*c))*e^(3*d*x)) - 1/2*integrate(6*(b*e^
(3*d*x + 3*c) - b*e^(d*x + c))/(a^3*e^(4*d*x + 4*c) + a^3 + 2*(a^3*e^(2*c)
+ 2*a^2*b*e^(2*c))*e^(2*d*x)), x)
```

mupad [B] time = 1.65, size = 71, normalized size = 0.85

$$\frac{b\cosh\left(c+d\,x\right)}{2\,\left(d\,a^3\cosh\left(c+d\,x\right)^2+b\,d\,a^2\right)}+\frac{\cosh\left(c+d\,x\right)}{a^2\,d}-\frac{3\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\cosh\left(c+d\,x\right)}{\sqrt{b}}\right)}{2\,a^{5/2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)/(a + b/cosh(c + d*x)^2)^2,x)
```

```
[Out] (b*cosh(c + d*x))/(2*(a^3*d*cosh(c + d*x)^2 + a^2*b*d)) + cosh(c + d*x)/(a^
2*d) - (3*b^(1/2)*atan((a^(1/2)*cosh(c + d*x))/b^(1/2)))/(2*a^(5/2)*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

3.37 $\int \frac{\mathbf{csch}(c+dx)}{\left(a+b\mathbf{sech}^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=99

$$\frac{\sqrt{b}\,(3a+b)\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{2a^{3/2}d(a+b)^2}-\frac{b\cosh(c+dx)}{2ad(a+b)\left(a\cosh^2(c+dx)+b\right)}-\frac{\tanh^{-1}(\cosh(c+dx))}{d(a+b)^2}$$

```
[Out] -arctanh(cosh(d*x+c))/(a+b)^2/d-1/2*b*cosh(d*x+c)/a/(a+b)/d/(b+a*cosh(d*x+c
)^2)+1/2*(3*a+b)*arctan(cosh(d*x+c)*a^(1/2)/b^(1/2))*b^(1/2)/a^(3/2)/(a+b)^
2/d
```

Rubi [A] time = 0.13, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4133, 470, 522, 206, 205}

$$\frac{\sqrt{b}\,(3a+b)\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{2a^{3/2}d(a+b)^2}-\frac{b\cosh(c+dx)}{2ad(a+b)\left(a\cosh^2(c+dx)+b\right)}-\frac{\tanh^{-1}(\cosh(c+dx))}{d(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (Sqrt[b]*(3*a + b)*ArcTan[(Sqrt[a]*Cosh[c + d*x])/Sqrt[b]])/(2*a^(3/2)*(a +
 b)^2*d) - ArcTanh[Cosh[c + d*x]]/((a + b)^2*d) - (b*Cosh[c + d*x])/(2*a*(a
 + b)*d*(b + a*Cosh[c + d*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
```

```
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx &= -\frac{\operatorname{Subst}\left(\int \frac{x^4}{(1-x^2)\left(b+ax^2\right)^2}\,dx, x, \cosh(c+dx)\right)}{d} \\
&= -\frac{b\cosh(c+dx)}{2a(a+b)d\left(b+a\cosh^2(c+dx)\right)} + \frac{\operatorname{Subst}\left(\int \frac{b+(-2a-b)x^2}{(1-x^2)\left(b+ax^2\right)}\,dx, x, \cosh(c+dx)\right)}{2a(a+b)d} \\
&= -\frac{b\cosh(c+dx)}{2a(a+b)d\left(b+a\cosh^2(c+dx)\right)} - \frac{\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cosh(c+dx)\right)}{(a+b)^2 d} + \frac{(b(3a+
\\
&= \frac{\sqrt{b}\,(3a+b)\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{2a^{3/2}(a+b)^2 d} - \frac{\tanh^{-1}(\cosh(c+dx))}{(a+b)^2 d} - \frac{b\cosh(c+d}{2a(a+b)d\left(b+a\cos\right.}
\end{aligned}$$

Mathematica [C] time = 1.25, size = 377, normalized size = 3.81

$$\operatorname{sech}^3(c+dx)(a\cosh(2(c+dx))+a+2b)\left(\frac{\sqrt{b}\,(3a+b)\operatorname{sech}(c+dx)(a\cosh(2(c+dx))+a+2b)\tan^{-1}\left(\frac{\sinh(c)\tanh\left(\frac{dx}{2}\right)\left(\sqrt{a}-i\sqrt{a+b}\,\sqrt{(\cosh(c)-s\cdots}\right.}{\cdots}\right.}{a^{3/2}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^3*((-2*b*(a + b))/a + (Sqrt[
b]*(3*a + b)*ArcTan[((Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2])*
Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Si
nh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c +
 d*x])/a^(3/2) + (Sqrt[b]*(3*a + b)*ArcTan[((Sqrt[a] + I*Sqrt[a + b]*Sqrt[(
Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] + I*Sqrt[a
+ b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cosh
[2*(c + d*x)])*Sech[c + d*x])/a^(3/2) - 2*(a + 2*b + a*Cosh[2*(c + d*x)])*L
og[Cosh[(c + d*x)/2]]*Sech[c + d*x] + 2*(a + 2*b + a*Cosh[2*(c + d*x)])*Log
[Sinh[(c + d*x)/2]]*Sech[c + d*x]))/(8*(a + b)^2*d*(a + b*Sech[c + d*x]^2)^
2)
```

fricas [B] time = 0.58, size = 2376, normalized size = 24.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*(a*b + b^2)*cosh(d*x + c)^3 + 12*(a*b + b^2)*cosh(d*x + c)*sinh(d*
x + c)^2 + 4*(a*b + b^2)*sinh(d*x + c)^3 - ((3*a^2 + a*b)*cosh(d*x + c)^4 +
 4*(3*a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a^2 + a*b)*sinh(d*x + c
)^4 + 2*(3*a^2 + 7*a*b + 2*b^2)*cosh(d*x + c)^2 + 2*(3*(3*a^2 + a*b)*cosh(d
*x + c)^2 + 3*a^2 + 7*a*b + 2*b^2)*sinh(d*x + c)^2 + 3*a^2 + a*b + 4*((3*a^
2 + a*b)*cosh(d*x + c)^3 + (3*a^2 + 7*a*b + 2*b^2)*cosh(d*x + c))*sinh(d*x
+ c))*sqrt(-b/a)*log((a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3
 + a*sinh(d*x + c)^4 + 2*(a - 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2
 + a - 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a - 2*b)*cosh(d*x + c
))*sinh(d*x + c) + 4*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2
 + a*sinh(d*x + c)^3 + a*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + a)*sinh(d*x
 + c))*sqrt(-b/a) + a)/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)
```

```
^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)
^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x +
 c))*sinh(d*x + c) + a)) + 4*(a*b + b^2)*cosh(d*x + c) + 4*(a^2*cosh(d*x +
c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 +
 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x
+ c)^2 + a^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d
*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) - 4*(a^2*cosh(d*x + c)^4 +
4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)
*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2
+ a^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c)
)*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 4*(3*(a*b + b^2)*cosh(d*x + c)^2
 + a*b + b^2)*sinh(d*x + c))/((a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^4 +
 4*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 + 2*a^3
*b + a^2*b^2)*d*sinh(d*x + c)^4 + 2*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d
*cosh(d*x + c)^2 + 2*(3*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^2 + (a^4
+ 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d)*sinh(d*x + c)^2 + (a^4 + 2*a^3*b + a^2*
b^2)*d + 4*((a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^3 + (a^4 + 4*a^3*b +
5*a^2*b^2 + 2*a*b^3)*d*cosh(d*x + c))*sinh(d*x + c)), -1/2*(2*(a*b + b^2)*c
osh(d*x + c)^3 + 6*(a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^2 + 2*(a*b + b^2
)*sinh(d*x + c)^3 + ((3*a^2 + a*b)*cosh(d*x + c)^4 + 4*(3*a^2 + a*b)*cosh(d
*x + c)*sinh(d*x + c)^3 + (3*a^2 + a*b)*sinh(d*x + c)^4 + 2*(3*a^2 + 7*a*b
+ 2*b^2)*cosh(d*x + c)^2 + 2*(3*(3*a^2 + a*b)*cosh(d*x + c)^2 + 3*a^2 + 7*a
*b + 2*b^2)*sinh(d*x + c)^2 + 3*a^2 + a*b + 4*((3*a^2 + a*b)*cosh(d*x + c)^
3 + (3*a^2 + 7*a*b + 2*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(b/a)*arctan(
1/2*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c
)^3 + (a + 4*b)*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + a + 4*b)*sinh(d*x +
c))*sqrt(b/a)/b) - ((3*a^2 + a*b)*cosh(d*x + c)^4 + 4*(3*a^2 + a*b)*cosh(d*
x + c)*sinh(d*x + c)^3 + (3*a^2 + a*b)*sinh(d*x + c)^4 + 2*(3*a^2 + 7*a*b +
 2*b^2)*cosh(d*x + c)^2 + 2*(3*(3*a^2 + a*b)*cosh(d*x + c)^2 + 3*a^2 + 7*a*
b + 2*b^2)*sinh(d*x + c)^2 + 3*a^2 + a*b + 4*((3*a^2 + a*b)*cosh(d*x + c)^3
 + (3*a^2 + 7*a*b + 2*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(b/a)*arctan(1
/2*(a*cosh(d*x + c) + a*sinh(d*x + c))*sqrt(b/a)/b) + 2*(a*b + b^2)*cosh(d*
x + c) + 2*(a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2
*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)
^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2
*a*b)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1)
- 2*(a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d
*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^
2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*c
osh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 2*(3*
(a*b + b^2)*cosh(d*x + c)^2 + a*b + b^2)*sinh(d*x + c))/((a^4 + 2*a^3*b + a
^2*b^2)*d*cosh(d*x + c)^4 + 4*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)*sin
h(d*x + c)^3 + (a^4 + 2*a^3*b + a^2*b^2)*d*sinh(d*x + c)^4 + 2*(a^4 + 4*a^3
*b + 5*a^2*b^2 + 2*a*b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^4 + 2*a^3*b + a^2*b^2
)*d*cosh(d*x + c)^2 + (a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d)*sinh(d*x + c
```

```
)^2 + (a^4 + 2*a^3*b + a^2*b^2)*d + 4*((a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x
 + c)^3 + (a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d*cosh(d*x + c))*sinh(d*x +
 c))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[6,-20]Warning, need to choose a branch for the root of a po
lynomial with parameters. This might be wrong.The choice was done assuming
[a,b]=[89,-63]Undef/Unsigned Inf encountered in limitLimit: Max order reach
ed or unable to make series expansion Error: Bad Argument Value
```

maple [B] time = 0.34, size = 431, normalized size = 4.35

$$-\frac{b\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\left(a+b\right)^{2}\left(\left(\tanh^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+b\left(\tanh^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a+b\right)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)/(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] -1/d*b/(a+b)^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*
d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)*tanh(1/2*d*x+1/2*c)^2+1/d*b^2
/(a+b)^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/
2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)/a*tanh(1/2*d*x+1/2*c)^2-1/d*b/(a+b)
^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2
*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)-1/d*b^2/(a+b)^2/(tanh(1/2*d*x+1/2*c)^4*a+
b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b
+a+b)/a+3/2/d*b/(a+b)^2/(a*b)^(1/2)*arctan(1/4*(2*(a+b)*tanh(1/2*d*x+1/2*c)
^2+2*a-2*b)/(a*b)^(1/2))+1/2/d*b^2/(a+b)^2/a/(a*b)^(1/2)*arctan(1/4*(2*(a+b
)*tanh(1/2*d*x+1/2*c)^2+2*a-2*b)/(a*b)^(1/2))+1/d/(a+b)^2*ln(tanh(1/2*d*x+1
/2*c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{be^{(3\,dx+3\,c)}+be^{(dx+c)}}{a^{3}d+a^{2}bd+\left(a^{3}de^{(4\,c)}+a^{2}bde^{(4\,c)}\right)e^{(4\,dx)}+2\left(a^{3}de^{(2\,c)}+3\,a^{2}bde^{(2\,c)}+2\,ab^{2}de^{(2\,c)}\right)e^{(2\,dx)}}-\frac{\log\left(\left(e^{(dx+c)}+1\right)e^{(-c)}\right)}{a^{2}d+2\,abd+b^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -(b*e^(3*d*x + 3*c) + b*e^(d*x + c))/(a^3*d + a^2*b*d + (a^3*d*e^(4*c) + a^
2*b*d*e^(4*c))*e^(4*d*x) + 2*(a^3*d*e^(2*c) + 3*a^2*b*d*e^(2*c) + 2*a*b^2*d
*e^(2*c))*e^(2*d*x)) - log((e^(d*x + c) + 1)*e^(-c))/(a^2*d + 2*a*b*d + b^2
*d) + log((e^(d*x + c) - 1)*e^(-c))/(a^2*d + 2*a*b*d + b^2*d) + 2*integrate
(1/2*((3*a*b*e^(3*c) + b^2*e^(3*c))*e^(3*d*x) - (3*a*b*e^c + b^2*e^c)*e^(d*
x))/(a^4 + 2*a^3*b + a^2*b^2 + (a^4*e^(4*c) + 2*a^3*b*e^(4*c) + a^2*b^2*e^(
4*c))*e^(4*d*x) + 2*(a^4*e^(2*c) + 4*a^3*b*e^(2*c) + 5*a^2*b^2*e^(2*c) + 2*
a*b^3*e^(2*c))*e^(2*d*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh\left(c+d\,x\right)^4}{\sinh\left(c+d\,x\right)\,\left(a\cosh\left(c+d\,x\right)^2+b\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)*(a + b/cosh(c + d*x)^2)^2),x)
```

```
[Out] int(cosh(c + d*x)^4/(sinh(c + d*x)*(b + a*cosh(c + d*x)^2)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}\left(c+dx\right)}{\left(a+b\operatorname{sech}^2\left(c+dx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral(csch(c + d*x)/(a + b*sech(c + d*x)**2)**2, x)
```

3.38 $\int \frac{\mathbf{csch}^2(c+dx)}{\left(a+b\mathbf{sech}^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=92

$$\frac{3\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2d(a+b)^{5/2}}-\frac{3\coth(c+dx)}{2d(a+b)^2}+\frac{\coth(c+dx)}{2d(a+b)\left(a-b\tanh^2(c+dx)+b\right)}$$

```
[Out] -3/2*coth(d*x+c)/(a+b)^2/d+3/2*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))*b^(
1/2)/(a+b)^(5/2)/d+1/2*coth(d*x+c)/(a+b)/d/(a+b-b*tanh(d*x+c)^2)
```

Rubi [A] time = 0.08, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4132, 290, 325, 208}

$$\frac{3\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2d(a+b)^{5/2}}-\frac{3\coth(c+dx)}{2d(a+b)^2}+\frac{\coth(c+dx)}{2d(a+b)\left(a-b\tanh^2(c+dx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (3*Sqrt[b]*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(2*(a + b)^(5/2)*d
) - (3*Coth[c + d*x])/(2*(a + b)^2*d) + Coth[c + d*x]/(2*(a + b)*d*(a + b -
 b*Tanh[c + d*x]^2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
```

```
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2\left(a+b-bx^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\coth(c+dx)}{2(a+b)d\left(a+b-b\tanh^2(c+dx)\right)} + \frac{3\,\text{Subst}\left(\int \frac{1}{x^2\left(a+b-bx^2\right)}\,dx, x, \tanh(c+dx)\right)}{2(a+b)d}$$

$$= -\frac{3\coth(c+dx)}{2(a+b)^2d} + \frac{\coth(c+dx)}{2(a+b)d\left(a+b-b\tanh^2(c+dx)\right)} + \frac{(3b)\,\text{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx\right.}{2(a+b)^{2}}$$

$$= \frac{3\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a+b}}\right)}{2(a+b)^{5/2}d} - \frac{3\coth(c+dx)}{2(a+b)^2d} + \frac{\coth(c+dx)}{2(a+b)d\left(a+b-b\tanh^2(c+dx\right.}$$

Mathematica [B] time = 2.62, size = 220, normalized size = 2.39

$$\frac{\operatorname{sech}^4(c+dx)(a\cosh(2(c+dx))+a+2b)\left(2\operatorname{csch}(c)\sinh(dx)\operatorname{csch}(c+dx)(a\cosh(2(c+dx))+a+2b)+\frac{3b(\cosh(2}{}\right.}{8d(a+b)^2\left(a+\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^4*((3*b*ArcTanh[(Sech[d*x]*(
Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a
 + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(a + 2*b + a*Cosh[2*(c + d*x)])*(Cosh
[2*c] - Sinh[2*c]))/(Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4]) + 2*(a + 2*
b + a*Cosh[2*(c + d*x)])*Csch[c]*Csch[c + d*x]*Sinh[d*x] + b*Sech[2*c]*Sinh
[2*d*x] - (b*(a + 2*b)*Tanh[2*c])/a))/(8*(a + b)^2*d*(a + b*Sech[c + d*x]^2
)^2)
```

fricas [B] time = 0.62, size = 2407, normalized size = 26.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*(2*a^2 + a*b + 2*b^2)*cosh(d*x + c)^4 + 16*(2*a^2 + a*b + 2*b^2)*c
osh(d*x + c)*sinh(d*x + c)^3 + 4*(2*a^2 + a*b + 2*b^2)*sinh(d*x + c)^4 + 8*
(2*a^2 + 4*a*b - b^2)*cosh(d*x + c)^2 + 8*(3*(2*a^2 + a*b + 2*b^2)*cosh(d*x
 + c)^2 + 2*a^2 + 4*a*b - b^2)*sinh(d*x + c)^2 - 3*(a^2*cosh(d*x + c)^6 + 6
*a^2*cosh(d*x + c)*sinh(d*x + c)^5 + a^2*sinh(d*x + c)^6 + (a^2 + 4*a*b)*co
sh(d*x + c)^4 + (15*a^2*cosh(d*x + c)^2 + a^2 + 4*a*b)*sinh(d*x + c)^4 + 4*
(5*a^2*cosh(d*x + c)^3 + (a^2 + 4*a*b)*cosh(d*x + c))*sinh(d*x + c)^3 - (a^
2 + 4*a*b)*cosh(d*x + c)^2 + (15*a^2*cosh(d*x + c)^4 + 6*(a^2 + 4*a*b)*cosh
(d*x + c)^2 - a^2 - 4*a*b)*sinh(d*x + c)^2 - a^2 + 2*(3*a^2*cosh(d*x + c)^5
 + 2*(a^2 + 4*a*b)*cosh(d*x + c)^3 - (a^2 + 4*a*b)*cosh(d*x + c))*sinh(d*x
+ c))*sqrt(b/(a + b))*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d
*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^
2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*
(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) - 4*((a^2
 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2
+ a*b)*sinh(d*x + c)^2 + a^2 + 3*a*b + 2*b^2)*sqrt(b/(a + b)))/(a*cosh(d*x
+ c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b
)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(
a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) + 8*a^2 -
4*a*b + 16*((2*a^2 + a*b + 2*b^2)*cosh(d*x + c)^3 + (2*a^2 + 4*a*b - b^2)*c
osh(d*x + c))*sinh(d*x + c))/((a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^6 +
 6*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a^4 + 2*a^3
*b + a^2*b^2)*d*sinh(d*x + c)^6 + (a^4 + 6*a^3*b + 9*a^2*b^2 + 4*a*b^3)*d*c
osh(d*x + c)^4 + (15*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^2 + (a^4 + 6
*a^3*b + 9*a^2*b^2 + 4*a*b^3)*d)*sinh(d*x + c)^4 - (a^4 + 6*a^3*b + 9*a^2*b
^2 + 4*a*b^3)*d*cosh(d*x + c)^2 + 4*(5*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x
 + c)^3 + (a^4 + 6*a^3*b + 9*a^2*b^2 + 4*a*b^3)*d*cosh(d*x + c))*sinh(d*x +
 c)^3 + (15*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^4 + 6*(a^4 + 6*a^3*b
+ 9*a^2*b^2 + 4*a*b^3)*d*cosh(d*x + c)^2 - (a^4 + 6*a^3*b + 9*a^2*b^2 + 4*a
*b^3)*d)*sinh(d*x + c)^2 - (a^4 + 2*a^3*b + a^2*b^2)*d + 2*(3*(a^4 + 2*a^3*
```

```
b + a^2*b^2)*d*cosh(d*x + c)^5 + 2*(a^4 + 6*a^3*b + 9*a^2*b^2 + 4*a*b^3)*d*
cosh(d*x + c)^3 - (a^4 + 6*a^3*b + 9*a^2*b^2 + 4*a*b^3)*d*cosh(d*x + c))*si
nh(d*x + c)), -1/2*(2*(2*a^2 + a*b + 2*b^2)*cosh(d*x + c)^4 + 8*(2*a^2 + a*
b + 2*b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + 2*(2*a^2 + a*b + 2*b^2)*sinh(d*x
 + c)^4 + 4*(2*a^2 + 4*a*b - b^2)*cosh(d*x + c)^2 + 4*(3*(2*a^2 + a*b + 2*b
^2)*cosh(d*x + c)^2 + 2*a^2 + 4*a*b - b^2)*sinh(d*x + c)^2 - 3*(a^2*cosh(d*
x + c)^6 + 6*a^2*cosh(d*x + c)*sinh(d*x + c)^5 + a^2*sinh(d*x + c)^6 + (a^2
 + 4*a*b)*cosh(d*x + c)^4 + (15*a^2*cosh(d*x + c)^2 + a^2 + 4*a*b)*sinh(d*x
 + c)^4 + 4*(5*a^2*cosh(d*x + c)^3 + (a^2 + 4*a*b)*cosh(d*x + c))*sinh(d*x
+ c)^3 - (a^2 + 4*a*b)*cosh(d*x + c)^2 + (15*a^2*cosh(d*x + c)^4 + 6*(a^2 +
 4*a*b)*cosh(d*x + c)^2 - a^2 - 4*a*b)*sinh(d*x + c)^2 - a^2 + 2*(3*a^2*cos
h(d*x + c)^5 + 2*(a^2 + 4*a*b)*cosh(d*x + c)^3 - (a^2 + 4*a*b)*cosh(d*x + c
))*sinh(d*x + c))*sqrt(-b/(a + b))*arctan(1/2*(a*cosh(d*x + c)^2 + 2*a*cosh
(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-b/(a + b))/b)
+ 4*a^2 - 2*a*b + 8*((2*a^2 + a*b + 2*b^2)*cosh(d*x + c)^3 + (2*a^2 + 4*a*b
 - b^2)*cosh(d*x + c))*sinh(d*x + c))/((a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x
 + c)^6 + 6*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a^
4 + 2*a^3*b + a^2*b^2)*d*sinh(d*x + c)^6 + (a^4 + 6*a^3*b + 9*a^2*b^2 + 4*a
*b^3)*d*cosh(d*x + c)^4 + (15*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^2 +
 (a^4 + 6*a^3*b + 9*a^2*b^2 + 4*a*b^3)*d)*sinh(d*x + c)^4 - (a^4 + 6*a^3*b
+ 9*a^2*b^2 + 4*a*b^3)*d*cosh(d*x + c)^2 + 4*(5*(a^4 + 2*a^3*b + a^2*b^2)*d
*cosh(d*x + c)^3 + (a^4 + 6*a^3*b + 9*a^2*b^2 + 4*a*b^3)*d*cosh(d*x + c))*s
inh(d*x + c)^3 + (15*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^4 + 6*(a^4 +
 6*a^3*b + 9*a^2*b^2 + 4*a*b^3)*d*cosh(d*x + c)^2 - (a^4 + 6*a^3*b + 9*a^2*
b^2 + 4*a*b^3)*d)*sinh(d*x + c)^2 - (a^4 + 2*a^3*b + a^2*b^2)*d + 2*(3*(a^4
 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^5 + 2*(a^4 + 6*a^3*b + 9*a^2*b^2 + 4*
a*b^3)*d*cosh(d*x + c)^3 - (a^4 + 6*a^3*b + 9*a^2*b^2 + 4*a*b^3)*d*cosh(d*x
 + c))*sinh(d*x + c))]
```

giac [B] time = 0.73, size = 239, normalized size = 2.60

$$\frac{\frac{3\,b\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{\left(a^2+2\,ab+b^2\right)\sqrt{-ab-b^2}}-\frac{2\left(2\,a^2e^{(4\,dx+4\,c)}+abe^{(4\,dx+4\,c)}+2\,b^2e^{(4\,dx+4\,c)}+4\,a^2e^{(2\,dx+2\,c)}+8\,abe^{(2\,dx+2\,c)}-2\,b^2e^{(2\,dx+2\,c)}+2\,a^2-ab\right)}{\left(a^3+2\,a^2b+ab^2\right)\left(ae^{(6\,dx+6\,c)}+ae^{(4\,dx+4\,c)}+4\,be^{(4\,dx+4\,c)}-ae^{(2\,dx+2\,c)}-4\,be^{(2\,dx+2\,c)}-a\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(3*b*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/((a^2 +
 2*a*b + b^2)*sqrt(-a*b - b^2)) - 2*(2*a^2*e^(4*d*x + 4*c) + a*b*e^(4*d*x +
 4*c) + 2*b^2*e^(4*d*x + 4*c) + 4*a^2*e^(2*d*x + 2*c) + 8*a*b*e^(2*d*x + 2*
c) - 2*b^2*e^(2*d*x + 2*c) + 2*a^2 - a*b)/((a^3 + 2*a^2*b + a*b^2)*(a*e^(6*
d*x + 6*c) + a*e^(4*d*x + 4*c) + 4*b*e^(4*d*x + 4*c) - a*e^(2*d*x + 2*c) -
4*b*e^(2*d*x + 2*c) - a)))/d
```

maple [B] time = 0.39, size = 313, normalized size = 3.40

$$-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\left(a^{2}+2ab+b^{2}\right)}+\frac{b\left(\tanh^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\left(a+b\right)^{2}\left(\left(\tanh^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+b\left(\tanh^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2\left(\tanh^{2}\left(\frac{dx}{2}\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] -1/2/d/(a^2+2*a*b+b^2)*tanh(1/2*d*x+1/2*c)+1/d*b/(a+b)^2/(tanh(1/2*d*x+1/2*
c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2
*c)^2*b+a+b)*tanh(1/2*d*x+1/2*c)^3+1/d*b/(a+b)^2/(tanh(1/2*d*x+1/2*c)^4*a+b
*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+
a+b)*tanh(1/2*d*x+1/2*c)-3/4/d*b^(1/2)/(a+b)^(5/2)*ln(-(a+b)^(1/2)*tanh(1/2
*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)-(a+b)^(1/2))+3/4/d*b^(1/2)/(a+b
)^(5/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)+
(a+b)^(1/2))-1/2/d/(a+b)^2/tanh(1/2*d*x+1/2*c)
```

maxima [B] time = 0.46, size = 262, normalized size = 2.85

$$-\frac{3\,b\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{4\left(a^{2}+2\,ab+b^{2}\right)\sqrt{(a+b)b}\,d}-\frac{2\,a^{2}-ab+2\left(2\,a^{2}+4\,ab-b^{2}\right)e^{(-2\,dx-2\,c)}+}{\left(a^{4}+2\,a^{3}b+a^{2}b^{2}+\left(a^{4}+6\,a^{3}b+9\,a^{2}b^{2}+4\,ab^{3}\right)e^{(-2\,dx-2\,c)}-\left(a^{4}+6\,a^{3}b+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -3/4*b*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x
- 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^2 + 2*a*b + b^2)*sqrt((a + b)*b)
*d) - (2*a^2 - a*b + 2*(2*a^2 + 4*a*b - b^2)*e^(-2*d*x - 2*c) + (2*a^2 + a*
b + 2*b^2)*e^(-4*d*x - 4*c))/((a^4 + 2*a^3*b + a^2*b^2 + (a^4 + 6*a^3*b + 9
*a^2*b^2 + 4*a*b^3)*e^(-2*d*x - 2*c) - (a^4 + 6*a^3*b + 9*a^2*b^2 + 4*a*b^3
)*e^(-4*d*x - 4*c) - (a^4 + 2*a^3*b + a^2*b^2)*e^(-6*d*x - 6*c))*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh\left(c+d\,x\right)^{4}}{\sinh\left(c+d\,x\right)^{2}\,\left(a\,\cosh\left(c+d\,x\right)^{2}+b\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^2*(a + b/cosh(c + d*x)^2)^2),x)
```

```
[Out] int(cosh(c + d*x)^4/(sinh(c + d*x)^2*(b + a*cosh(c + d*x)^2)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral(csch(c + d*x)**2/(a + b*sech(c + d*x)**2)**2, x)
```

3.39 $\int \frac{\mathbf{csch}^3(c+dx)}{\left(a+b\mathbf{sech}^2(c+dx)\right)^2} dx$

Optimal. Leaf size=147

$$-\frac{(a-b)\cosh(c+dx)}{2d(a+b)^2\left(a\cosh^2(c+dx)+b\right)}-\frac{\sqrt{b}\,(3a-b)\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{2\sqrt{a}\,d(a+b)^3}+\frac{(a-3b)\tanh^{-1}(\cosh(c+dx))}{2d(a+b)^3}-\frac{\coth(c}{2d(a+b)}$$

```
[Out] 1/2*(a-3*b)*arctanh(cosh(d*x+c))/d/(a+b)^3-1/2*(a-b)*cosh(d*x+c)/(a+b)^2/d/
(b+a*cosh(d*x+c)^2)-1/2*coth(d*x+c)*csch(d*x+c)/(a+b)/d/(b+a*cosh(d*x+c)^2)
-1/2*(3*a-b)*arctan(cosh(d*x+c)*a^(1/2)/b^(1/2))*b^(1/2)/(a+b)^3/d/a^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4133, 470, 527, 522, 206, 205}

$$-\frac{(a-b)\cosh(c+dx)}{2d(a+b)^2\left(a\cosh^2(c+dx)+b\right)}-\frac{\sqrt{b}\,(3a-b)\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{2\sqrt{a}\,d(a+b)^3}+\frac{(a-3b)\tanh^{-1}(\cosh(c+dx))}{2d(a+b)^3}-\frac{\coth(c}{2d(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] -((3*a - b)*Sqrt[b]*ArcTan[(Sqrt[a]*Cosh[c + d*x])/Sqrt[b]])/(2*Sqrt[a]*(a
+ b)^3*d) + ((a - 3*b)*ArcTanh[Cosh[c + d*x]])/(2*(a + b)^3*d) - ((a - b)*C
osh[c + d*x])/(2*(a + b)^2*d*(b + a*Cosh[c + d*x]^2)) - (Coth[c + d*x]*Csch
[c + d*x])/(2*(a + b)*d*(b + a*Cosh[c + d*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
```

```
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\text{csch}^3(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)^2(b+ax^2)^2}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{\coth(c+dx)\text{csch}(c+dx)}{2(a+b)d\left(b+a\cosh^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{b+(-a+2b)x^2}{(1-x^2)(b+ax^2)^2}\,dx, x, \cosh(c+dx)\right)}{2(a+b)d}$$

$$= -\frac{(a-b)\cosh(c+dx)}{2(a+b)^2d\left(b+a\cosh^2(c+dx)\right)} - \frac{\coth(c+dx)\text{csch}(c+dx)}{2(a+b)d\left(b+a\cosh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{(a-b)\cosh(c+dx)}{2(a+b)^2d\left(b+a\cosh^2(c+dx)\right)} - \frac{\coth(c+dx)\text{csch}(c+dx)}{2(a+b)d\left(b+a\cosh^2(c+dx)\right)} + \frac{(a-3b)}{}$$

$$= -\frac{(3a-b)\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{a}\,\cosh(c+dx)}{\sqrt{b}}\right)}{2\sqrt{a}\,(a+b)^3d} + \frac{(a-3b)\tanh^{-1}(\cosh(c+dx))}{2(a+b)^3d} - \frac{(a-}{2(a+b)^2d}$$

Mathematica [C] time = 2.23, size = 462, normalized size = 3.14

$$\frac{\text{sech}^3(c+dx)(a\cosh(2(c+dx))+a+2b)\left(-(a+b)\text{sech}^2\left(\frac{1}{2}(c+dx)\right)\text{sech}(c+dx)(a\cosh(2(c+dx))+a+2b)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^3*(8*b*(a + b) + (4*Sqrt[b]*
(-3*a + b)*ArcTan[((Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2])*Si
nh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh
[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d
*x])/Sqrt[a] + (4*Sqrt[b]*(-3*a + b)*ArcTan[((Sqrt[a] + I*Sqrt[a + b]*Sqrt[
(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] + I*Sqrt[a
 + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos
h[2*(c + d*x)])*Sech[c + d*x])/Sqrt[a] - (a + b)*(a + 2*b + a*Cosh[2*(c + d
```

```
*x)])*Csch[(c + d*x)/2]^2*Sech[c + d*x] + 4*(a - 3*b)*(a + 2*b + a*Cosh[2*(
c + d*x)])*Log[Cosh[(c + d*x)/2]]*Sech[c + d*x] - 4*(a - 3*b)*(a + 2*b + a*
Cosh[2*(c + d*x)])*Log[Sinh[(c + d*x)/2]]*Sech[c + d*x] - (a + b)*(a + 2*b
+ a*Cosh[2*(c + d*x)])*Sech[(c + d*x)/2]^2*Sech[c + d*x]))/(32*(a + b)^3*d*
(a + b*Sech[c + d*x]^2)^2)
```

fricas [B] time = 0.73, size = 6878, normalized size = 46.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*(a^2 - b^2)*cosh(d*x + c)^7 + 28*(a^2 - b^2)*cosh(d*x + c)*sinh(d*
x + c)^6 + 4*(a^2 - b^2)*sinh(d*x + c)^7 + 4*(3*a^2 + 8*a*b + 5*b^2)*cosh(d
*x + c)^5 + 4*(21*(a^2 - b^2)*cosh(d*x + c)^2 + 3*a^2 + 8*a*b + 5*b^2)*sinh
(d*x + c)^5 + 20*(7*(a^2 - b^2)*cosh(d*x + c)^3 + (3*a^2 + 8*a*b + 5*b^2)*c
osh(d*x + c))*sinh(d*x + c)^4 + 4*(3*a^2 + 8*a*b + 5*b^2)*cosh(d*x + c)^3 +
 4*(35*(a^2 - b^2)*cosh(d*x + c)^4 + 10*(3*a^2 + 8*a*b + 5*b^2)*cosh(d*x +
c)^2 + 3*a^2 + 8*a*b + 5*b^2)*sinh(d*x + c)^3 + 4*(21*(a^2 - b^2)*cosh(d*x
+ c)^5 + 10*(3*a^2 + 8*a*b + 5*b^2)*cosh(d*x + c)^3 + 3*(3*a^2 + 8*a*b + 5*
b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + ((3*a^2 - a*b)*cosh(d*x + c)^8 + 8*(3
*a^2 - a*b)*cosh(d*x + c)*sinh(d*x + c)^7 + (3*a^2 - a*b)*sinh(d*x + c)^8 +
 4*(3*a*b - b^2)*cosh(d*x + c)^6 + 4*(7*(3*a^2 - a*b)*cosh(d*x + c)^2 + 3*a
*b - b^2)*sinh(d*x + c)^6 + 8*(7*(3*a^2 - a*b)*cosh(d*x + c)^3 + 3*(3*a*b -
 b^2)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(3*a^2 + 11*a*b - 4*b^2)*cosh(d*x
+ c)^4 + 2*(35*(3*a^2 - a*b)*cosh(d*x + c)^4 + 30*(3*a*b - b^2)*cosh(d*x +
c)^2 - 3*a^2 - 11*a*b + 4*b^2)*sinh(d*x + c)^4 + 8*(7*(3*a^2 - a*b)*cosh(d*
x + c)^5 + 10*(3*a*b - b^2)*cosh(d*x + c)^3 - (3*a^2 + 11*a*b - 4*b^2)*cosh
(d*x + c))*sinh(d*x + c)^3 + 4*(3*a*b - b^2)*cosh(d*x + c)^2 + 4*(7*(3*a^2
- a*b)*cosh(d*x + c)^6 + 15*(3*a*b - b^2)*cosh(d*x + c)^4 - 3*(3*a^2 + 11*a
*b - 4*b^2)*cosh(d*x + c)^2 + 3*a*b - b^2)*sinh(d*x + c)^2 + 3*a^2 - a*b +
8*((3*a^2 - a*b)*cosh(d*x + c)^7 + 3*(3*a*b - b^2)*cosh(d*x + c)^5 - (3*a^2
 + 11*a*b - 4*b^2)*cosh(d*x + c)^3 + (3*a*b - b^2)*cosh(d*x + c))*sinh(d*x
+ c))*sqrt(-b/a)*log((a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3
 + a*sinh(d*x + c)^4 + 2*(a - 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2
 + a - 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a - 2*b)*cosh(d*x + c
))*sinh(d*x + c) + 4*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2
 + a*sinh(d*x + c)^3 + a*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + a)*sinh(d*x
 + c))*sqrt(-b/a) + a)/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)
^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)
^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x +
 c))*sinh(d*x + c) + a)) + 4*(a^2 - b^2)*cosh(d*x + c) - 2*((a^2 - 3*a*b)*c
osh(d*x + c)^8 + 8*(a^2 - 3*a*b)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2 - 3*a
*b)*sinh(d*x + c)^8 + 4*(a*b - 3*b^2)*cosh(d*x + c)^6 + 4*(7*(a^2 - 3*a*b)*
```

cosh(d*x + c)^2 + a*b - 3*b^2)*sinh(d*x + c)^6 + 8*(7*(a^2 - 3*a*b)*cosh(d*
x + c)^3 + 3*(a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(a^2 + a*b -
12*b^2)*cosh(d*x + c)^4 + 2*(35*(a^2 - 3*a*b)*cosh(d*x + c)^4 + 30*(a*b - 3
*b^2)*cosh(d*x + c)^2 - a^2 - a*b + 12*b^2)*sinh(d*x + c)^4 + 8*(7*(a^2 - 3
*a*b)*cosh(d*x + c)^5 + 10*(a*b - 3*b^2)*cosh(d*x + c)^3 - (a^2 + a*b - 12*
b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a*b - 3*b^2)*cosh(d*x + c)^2 + 4*(
7*(a^2 - 3*a*b)*cosh(d*x + c)^6 + 15*(a*b - 3*b^2)*cosh(d*x + c)^4 - 3*(a^2
 + a*b - 12*b^2)*cosh(d*x + c)^2 + a*b - 3*b^2)*sinh(d*x + c)^2 + a^2 - 3*a
b + 8((a^2 - 3*a*b)*cosh(d*x + c)^7 + 3*(a*b - 3*b^2)*cosh(d*x + c)^5 - (
a^2 + a*b - 12*b^2)*cosh(d*x + c)^3 + (a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x
 + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) + 2*((a^2 - 3*a*b)*cosh(d*x +
 c)^8 + 8*(a^2 - 3*a*b)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2 - 3*a*b)*sinh(
d*x + c)^8 + 4*(a*b - 3*b^2)*cosh(d*x + c)^6 + 4*(7*(a^2 - 3*a*b)*cosh(d*x
+ c)^2 + a*b - 3*b^2)*sinh(d*x + c)^6 + 8*(7*(a^2 - 3*a*b)*cosh(d*x + c)^3
+ 3*(a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(a^2 + a*b - 12*b^2)*c
osh(d*x + c)^4 + 2*(35*(a^2 - 3*a*b)*cosh(d*x + c)^4 + 30*(a*b - 3*b^2)*cos
h(d*x + c)^2 - a^2 - a*b + 12*b^2)*sinh(d*x + c)^4 + 8*(7*(a^2 - 3*a*b)*cos
h(d*x + c)^5 + 10*(a*b - 3*b^2)*cosh(d*x + c)^3 - (a^2 + a*b - 12*b^2)*cosh
(d*x + c))*sinh(d*x + c)^3 + 4*(a*b - 3*b^2)*cosh(d*x + c)^2 + 4*(7*(a^2 -
3*a*b)*cosh(d*x + c)^6 + 15*(a*b - 3*b^2)*cosh(d*x + c)^4 - 3*(a^2 + a*b -
12*b^2)*cosh(d*x + c)^2 + a*b - 3*b^2)*sinh(d*x + c)^2 + a^2 - 3*a*b + 8*((
a^2 - 3*a*b)*cosh(d*x + c)^7 + 3*(a*b - 3*b^2)*cosh(d*x + c)^5 - (a^2 + a*b
 - 12*b^2)*cosh(d*x + c)^3 + (a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x + c))*lo
g(cosh(d*x + c) + sinh(d*x + c) - 1) + 4*(7*(a^2 - b^2)*cosh(d*x + c)^6 + 5
*(3*a^2 + 8*a*b + 5*b^2)*cosh(d*x + c)^4 + 3*(3*a^2 + 8*a*b + 5*b^2)*cosh(d
*x + c)^2 + a^2 - b^2)*sinh(d*x + c))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*
d*cosh(d*x + c)^8 + 8*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)*s
inh(d*x + c)^7 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*sinh(d*x + c)^8 + 4*
(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d*cosh(d*x + c)^6 + 4*(7*(a^4 + 3*a^3*b
 + 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)^2 + (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^
4)*d)*sinh(d*x + c)^6 - 2*(a^4 + 7*a^3*b + 15*a^2*b^2 + 13*a*b^3 + 4*b^4)*d
*cosh(d*x + c)^4 + 8*(7*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)
^3 + 3*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d*cosh(d*x + c))*sinh(d*x + c)^5
 + 2*(35*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)^4 + 30*(a^3*b
+ 3*a^2*b^2 + 3*a*b^3 + b^4)*d*cosh(d*x + c)^2 - (a^4 + 7*a^3*b + 15*a^2*b^
2 + 13*a*b^3 + 4*b^4)*d)*sinh(d*x + c)^4 + 4*(a^3*b + 3*a^2*b^2 + 3*a*b^3 +
 b^4)*d*cosh(d*x + c)^2 + 8*(7*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*cosh(d
x + c)^5 + 10(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d*cosh(d*x + c)^3 - (a^4
 + 7*a^3*b + 15*a^2*b^2 + 13*a*b^3 + 4*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^
3 + 4*(7*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)^6 + 15*(a^3*b
+ 3*a^2*b^2 + 3*a*b^3 + b^4)*d*cosh(d*x + c)^4 - 3*(a^4 + 7*a^3*b + 15*a^2*
b^2 + 13*a*b^3 + 4*b^4)*d*cosh(d*x + c)^2 + (a^3*b + 3*a^2*b^2 + 3*a*b^3 +
b^4)*d)*sinh(d*x + c)^2 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d + 8*((a^4 +
 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)^7 + 3*(a^3*b + 3*a^2*b^2 + 3*
a*b^3 + b^4)*d*cosh(d*x + c)^5 - (a^4 + 7*a^3*b + 15*a^2*b^2 + 13*a*b^3 + 4

```
*b^4)*d*cosh(d*x + c)^3 + (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d*cosh(d*x +
c))*sinh(d*x + c)), -1/2*(2*(a^2 - b^2)*cosh(d*x + c)^7 + 14*(a^2 - b^2)*co
sh(d*x + c)*sinh(d*x + c)^6 + 2*(a^2 - b^2)*sinh(d*x + c)^7 + 2*(3*a^2 + 8*
a*b + 5*b^2)*cosh(d*x + c)^5 + 2*(21*(a^2 - b^2)*cosh(d*x + c)^2 + 3*a^2 +
8*a*b + 5*b^2)*sinh(d*x + c)^5 + 10*(7*(a^2 - b^2)*cosh(d*x + c)^3 + (3*a^2
 + 8*a*b + 5*b^2)*cosh(d*x + c))*sinh(d*x + c)^4 + 2*(3*a^2 + 8*a*b + 5*b^2
)*cosh(d*x + c)^3 + 2*(35*(a^2 - b^2)*cosh(d*x + c)^4 + 10*(3*a^2 + 8*a*b +
 5*b^2)*cosh(d*x + c)^2 + 3*a^2 + 8*a*b + 5*b^2)*sinh(d*x + c)^3 + 2*(21*(a
^2 - b^2)*cosh(d*x + c)^5 + 10*(3*a^2 + 8*a*b + 5*b^2)*cosh(d*x + c)^3 + 3*
(3*a^2 + 8*a*b + 5*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 - ((3*a^2 - a*b)*cos
h(d*x + c)^8 + 8*(3*a^2 - a*b)*cosh(d*x + c)*sinh(d*x + c)^7 + (3*a^2 - a*b
)*sinh(d*x + c)^8 + 4*(3*a*b - b^2)*cosh(d*x + c)^6 + 4*(7*(3*a^2 - a*b)*co
sh(d*x + c)^2 + 3*a*b - b^2)*sinh(d*x + c)^6 + 8*(7*(3*a^2 - a*b)*cosh(d*x
+ c)^3 + 3*(3*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(3*a^2 + 11*a*b
 - 4*b^2)*cosh(d*x + c)^4 + 2*(35*(3*a^2 - a*b)*cosh(d*x + c)^4 + 30*(3*a*b
 - b^2)*cosh(d*x + c)^2 - 3*a^2 - 11*a*b + 4*b^2)*sinh(d*x + c)^4 + 8*(7*(3
*a^2 - a*b)*cosh(d*x + c)^5 + 10*(3*a*b - b^2)*cosh(d*x + c)^3 - (3*a^2 + 1
1*a*b - 4*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(3*a*b - b^2)*cosh(d*x +
c)^2 + 4*(7*(3*a^2 - a*b)*cosh(d*x + c)^6 + 15*(3*a*b - b^2)*cosh(d*x + c)^
4 - 3*(3*a^2 + 11*a*b - 4*b^2)*cosh(d*x + c)^2 + 3*a*b - b^2)*sinh(d*x + c)
^2 + 3*a^2 - a*b + 8*((3*a^2 - a*b)*cosh(d*x + c)^7 + 3*(3*a*b - b^2)*cosh(
d*x + c)^5 - (3*a^2 + 11*a*b - 4*b^2)*cosh(d*x + c)^3 + (3*a*b - b^2)*cosh(
d*x + c))*sinh(d*x + c))*sqrt(b/a)*arctan(1/2*(a*cosh(d*x + c)^3 + 3*a*cosh
(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 + (a + 4*b)*cosh(d*x + c) + (
3*a*cosh(d*x + c)^2 + a + 4*b)*sinh(d*x + c))*sqrt(b/a)/b) + ((3*a^2 - a*b)
*cosh(d*x + c)^8 + 8*(3*a^2 - a*b)*cosh(d*x + c)*sinh(d*x + c)^7 + (3*a^2 -
 a*b)*sinh(d*x + c)^8 + 4*(3*a*b - b^2)*cosh(d*x + c)^6 + 4*(7*(3*a^2 - a*b
)*cosh(d*x + c)^2 + 3*a*b - b^2)*sinh(d*x + c)^6 + 8*(7*(3*a^2 - a*b)*cosh(
d*x + c)^3 + 3*(3*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(3*a^2 + 11
*a*b - 4*b^2)*cosh(d*x + c)^4 + 2*(35*(3*a^2 - a*b)*cosh(d*x + c)^4 + 30*(3
*a*b - b^2)*cosh(d*x + c)^2 - 3*a^2 - 11*a*b + 4*b^2)*sinh(d*x + c)^4 + 8*(
7*(3*a^2 - a*b)*cosh(d*x + c)^5 + 10*(3*a*b - b^2)*cosh(d*x + c)^3 - (3*a^2
 + 11*a*b - 4*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(3*a*b - b^2)*cosh(d*
x + c)^2 + 4*(7*(3*a^2 - a*b)*cosh(d*x + c)^6 + 15*(3*a*b - b^2)*cosh(d*x +
 c)^4 - 3*(3*a^2 + 11*a*b - 4*b^2)*cosh(d*x + c)^2 + 3*a*b - b^2)*sinh(d*x
+ c)^2 + 3*a^2 - a*b + 8*((3*a^2 - a*b)*cosh(d*x + c)^7 + 3*(3*a*b - b^2)*c
osh(d*x + c)^5 - (3*a^2 + 11*a*b - 4*b^2)*cosh(d*x + c)^3 + (3*a*b - b^2)*c
osh(d*x + c))*sinh(d*x + c))*sqrt(b/a)*arctan(1/2*(a*cosh(d*x + c) + a*sinh
(d*x + c))*sqrt(b/a)/b) + 2*(a^2 - b^2)*cosh(d*x + c) - ((a^2 - 3*a*b)*cosh
(d*x + c)^8 + 8*(a^2 - 3*a*b)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2 - 3*a*b)
*sinh(d*x + c)^8 + 4*(a*b - 3*b^2)*cosh(d*x + c)^6 + 4*(7*(a^2 - 3*a*b)*cos
h(d*x + c)^2 + a*b - 3*b^2)*sinh(d*x + c)^6 + 8*(7*(a^2 - 3*a*b)*cosh(d*x +
 c)^3 + 3*(a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(a^2 + a*b - 12*
b^2)*cosh(d*x + c)^4 + 2*(35*(a^2 - 3*a*b)*cosh(d*x + c)^4 + 30*(a*b - 3*b^
2)*cosh(d*x + c)^2 - a^2 - a*b + 12*b^2)*sinh(d*x + c)^4 + 8*(7*(a^2 - 3*a*
```

```
b)*cosh(d*x + c)^5 + 10*(a*b - 3*b^2)*cosh(d*x + c)^3 - (a^2 + a*b - 12*b^2
)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a*b - 3*b^2)*cosh(d*x + c)^2 + 4*(7*(
a^2 - 3*a*b)*cosh(d*x + c)^6 + 15*(a*b - 3*b^2)*cosh(d*x + c)^4 - 3*(a^2 +
a*b - 12*b^2)*cosh(d*x + c)^2 + a*b - 3*b^2)*sinh(d*x + c)^2 + a^2 - 3*a*b
+ 8*((a^2 - 3*a*b)*cosh(d*x + c)^7 + 3*(a*b - 3*b^2)*cosh(d*x + c)^5 - (a^2
 + a*b - 12*b^2)*cosh(d*x + c)^3 + (a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x +
c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) + ((a^2 - 3*a*b)*cosh(d*x + c)^8
 + 8*(a^2 - 3*a*b)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2 - 3*a*b)*sinh(d*x +
 c)^8 + 4*(a*b - 3*b^2)*cosh(d*x + c)^6 + 4*(7*(a^2 - 3*a*b)*cosh(d*x + c)^
2 + a*b - 3*b^2)*sinh(d*x + c)^6 + 8*(7*(a^2 - 3*a*b)*cosh(d*x + c)^3 + 3*(
a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(a^2 + a*b - 12*b^2)*cosh(d
*x + c)^4 + 2*(35*(a^2 - 3*a*b)*cosh(d*x + c)^4 + 30*(a*b - 3*b^2)*cosh(d*x
 + c)^2 - a^2 - a*b + 12*b^2)*sinh(d*x + c)^4 + 8*(7*(a^2 - 3*a*b)*cosh(d*x
 + c)^5 + 10*(a*b - 3*b^2)*cosh(d*x + c)^3 - (a^2 + a*b - 12*b^2)*cosh(d*x
+ c))*sinh(d*x + c)^3 + 4*(a*b - 3*b^2)*cosh(d*x + c)^2 + 4*(7*(a^2 - 3*a*b
)*cosh(d*x + c)^6 + 15*(a*b - 3*b^2)*cosh(d*x + c)^4 - 3*(a^2 + a*b - 12*b^
2)*cosh(d*x + c)^2 + a*b - 3*b^2)*sinh(d*x + c)^2 + a^2 - 3*a*b + 8*((a^2 -
 3*a*b)*cosh(d*x + c)^7 + 3*(a*b - 3*b^2)*cosh(d*x + c)^5 - (a^2 + a*b - 12
*b^2)*cosh(d*x + c)^3 + (a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x + c))*log(cos
h(d*x + c) + sinh(d*x + c) - 1) + 2*(7*(a^2 - b^2)*cosh(d*x + c)^6 + 5*(3*a
^2 + 8*a*b + 5*b^2)*cosh(d*x + c)^4 + 3*(3*a^2 + 8*a*b + 5*b^2)*cosh(d*x +
c)^2 + a^2 - b^2)*sinh(d*x + c))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*cos
h(d*x + c)^8 + 8*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)*sinh(d
*x + c)^7 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*sinh(d*x + c)^8 + 4*(a^3*
b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d*cosh(d*x + c)^6 + 4*(7*(a^4 + 3*a^3*b + 3*
a^2*b^2 + a*b^3)*d*cosh(d*x + c)^2 + (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d)
*sinh(d*x + c)^6 - 2*(a^4 + 7*a^3*b + 15*a^2*b^2 + 13*a*b^3 + 4*b^4)*d*cosh
(d*x + c)^4 + 8*(7*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)^3 +
3*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*
(35*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)^4 + 30*(a^3*b + 3*a
^2*b^2 + 3*a*b^3 + b^4)*d*cosh(d*x + c)^2 - (a^4 + 7*a^3*b + 15*a^2*b^2 + 1
3*a*b^3 + 4*b^4)*d)*sinh(d*x + c)^4 + 4*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)
*d*cosh(d*x + c)^2 + 8*(7*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*cosh(d*x +
c)^5 + 10*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d*cosh(d*x + c)^3 - (a^4 + 7*
a^3*b + 15*a^2*b^2 + 13*a*b^3 + 4*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4
*(7*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)^6 + 15*(a^3*b + 3*a
^2*b^2 + 3*a*b^3 + b^4)*d*cosh(d*x + c)^4 - 3*(a^4 + 7*a^3*b + 15*a^2*b^2 +
 13*a*b^3 + 4*b^4)*d*cosh(d*x + c)^2 + (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*
d)*sinh(d*x + c)^2 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d + 8*((a^4 + 3*a^
3*b + 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)^7 + 3*(a^3*b + 3*a^2*b^2 + 3*a*b^3
 + b^4)*d*cosh(d*x + c)^5 - (a^4 + 7*a^3*b + 15*a^2*b^2 + 13*a*b^3 + 4*b^4)
*d*cosh(d*x + c)^3 + (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d*cosh(d*x + c))*s
inh(d*x + c))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[6,-20]Warning, need to choose a branch for the root of a po
lynomial with parameters. This might be wrong.The choice was done assuming
[a,b]=[89,-63]Warning, need to choose a branch for the root of a polynomial
 with parameters. This might be wrong.The choice was done assuming [a,b]=[1
2,-32]Warning, need to choose a branch for the root of a polynomial with pa
rameters. This might be wrong.The choice was done assuming [a,b]=[2,72]Unde
f/Unsigned Inf encountered in limitEvaluation time: 0.7Limit: Max order rea
ched or unable to make series expansion Error: Bad Argument Value
```

maple [B] time = 0.42, size = 496, normalized size = 3.37

$$\frac{\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\left(a^2+2ab+b^2\right)}+\frac{ba\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\left(a+b\right)^3\left(\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2\left(\tanh^2\left(\frac{dx}{2}+\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] 1/8/d*tanh(1/2*d*x+1/2*c)^2/(a^2+2*a*b+b^2)+1/d*b/(a+b)^3/(tanh(1/2*d*x+1/2
*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/
2*c)^2*b+a+b)*a*tanh(1/2*d*x+1/2*c)^2-1/d*b^2/(a+b)^3/(tanh(1/2*d*x+1/2*c)^
4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)
^2*b+a+b)*tanh(1/2*d*x+1/2*c)^2+1/d*b/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*ta
nh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b
)*a+1/d*b^2/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh
(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)-3/2/d*b/(a+b)^3/(a*b)^(1
/2)*arctan(1/4*(2*(a+b)*tanh(1/2*d*x+1/2*c)^2+2*a-2*b)/(a*b)^(1/2))*a+1/2/d
*b^2/(a+b)^3/(a*b)^(1/2)*arctan(1/4*(2*(a+b)*tanh(1/2*d*x+1/2*c)^2+2*a-2*b)
/(a*b)^(1/2))-1/8/d/(a+b)^2/tanh(1/2*d*x+1/2*c)^2-1/2/d/(a+b)^3*ln(tanh(1/2
*d*x+1/2*c))*a+3/2/d/(a+b)^3*ln(tanh(1/2*d*x+1/2*c))*b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(a-3\,b)\log\left(\left(e^{(dx+c)}+1\right)e^{(-c)}\right)}{2\left(a^3d+3\,a^2bd+3\,ab^2d+b^3d\right)}-\frac{(a-3\,b)\log\left(\left(e^{(dx+c)}-1\right)e^{(-c)}\right)}{2\left(a^3d+3\,a^2bd+3\,ab^2d+b^3d\right)}-\frac{}{a^3d+2\,a^2bd+ab^2d+\left(a^3de^{(8\,c)}+2\,a^2bde^{(8\,c)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(a - 3*b)*log((e^(d*x + c) + 1)*e^(-c))/(a^3*d + 3*a^2*b*d + 3*a*b^2*d
+ b^3*d) - 1/2*(a - 3*b)*log((e^(d*x + c) - 1)*e^(-c))/(a^3*d + 3*a^2*b*d +
 3*a*b^2*d + b^3*d) - ((a*e^(7*c) - b*e^(7*c))*e^(7*d*x) + (3*a*e^(5*c) + 5
*b*e^(5*c))*e^(5*d*x) + (3*a*e^(3*c) + 5*b*e^(3*c))*e^(3*d*x) + (a*e^c - b*
e^c)*e^(d*x))/(a^3*d + 2*a^2*b*d + a*b^2*d + (a^3*d*e^(8*c) + 2*a^2*b*d*e^(
8*c) + a*b^2*d*e^(8*c))*e^(8*d*x) + 4*(a^2*b*d*e^(6*c) + 2*a*b^2*d*e^(6*c)
+ b^3*d*e^(6*c))*e^(6*d*x) - 2*(a^3*d*e^(4*c) + 6*a^2*b*d*e^(4*c) + 9*a*b^2
*d*e^(4*c) + 4*b^3*d*e^(4*c))*e^(4*d*x) + 4*(a^2*b*d*e^(2*c) + 2*a*b^2*d*e^
(2*c) + b^3*d*e^(2*c))*e^(2*d*x)) - 8*integrate(1/8*((3*a*b*e^(3*c) - b^2*e
^(3*c))*e^(3*d*x) - (3*a*b*e^c - b^2*e^c)*e^(d*x))/(a^4 + 3*a^3*b + 3*a^2*b
^2 + a*b^3 + (a^4*e^(4*c) + 3*a^3*b*e^(4*c) + 3*a^2*b^2*e^(4*c) + a*b^3*e^(
4*c))*e^(4*d*x) + 2*(a^4*e^(2*c) + 5*a^3*b*e^(2*c) + 9*a^2*b^2*e^(2*c) + 7*
a*b^3*e^(2*c) + 2*b^4*e^(2*c))*e^(2*d*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh\left(c+d\,x\right)^4}{\sinh\left(c+d\,x\right)^3\,{\left(a\,\cosh\left(c+d\,x\right)^2+b\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^3*(a + b/cosh(c + d*x)^2)^2),x)
```

```
[Out] int(cosh(c + d*x)^4/(sinh(c + d*x)^3*(b + a*cosh(c + d*x)^2)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^{3}{\left(c+dx\right)}}{\left(a+b\operatorname{sech}^{2}{\left(c+dx\right)}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral(csch(c + d*x)**3/(a + b*sech(c + d*x)**2)**2, x)
```

3.40 $\int \frac{\text{csch}^4(c+dx)}{\left(a+b\,\text{sech}^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=123

$$-\frac{\sqrt{b}(3a-2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2d(a+b)^{7/2}}-\frac{ab\tanh(c+dx)}{2d(a+b)^3\left(a-b\tanh^2(c+dx)+b\right)}-\frac{\coth^3(c+dx)}{3d(a+b)^2}+\frac{(a-b)\coth(c+dx)}{d(a+b)^3}$$

```
[Out] (a-b)*coth(d*x+c)/d/(a+b)^3-1/3*coth(d*x+c)^3/(a+b)^2/d-1/2*(3*a-2*b)*arcta
nh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))*b^(1/2)/(a+b)^(7/2)/d-1/2*a*b*tanh(d*x+
c)/(a+b)^3/d/(a+b-b*tanh(d*x+c)^2)
```

Rubi [A] time = 0.20, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4132, 456, 1261, 208}

$$-\frac{\sqrt{b}(3a-2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2d(a+b)^{7/2}}-\frac{ab\tanh(c+dx)}{2d(a+b)^3\left(a-b\tanh^2(c+dx)+b\right)}-\frac{\coth^3(c+dx)}{3d(a+b)^2}+\frac{(a-b)\coth(c+dx)}{d(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] -((3*a - 2*b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(2*(a +
 b)^(7/2)*d) + ((a - b)*Coth[c + d*x])/((a + b)^3*d) - Coth[c + d*x]^3/(3*(
a + b)^2*d) - (a*b*Tanh[c + d*x])/(2*(a + b)^3*d*(a + b - b*Tanh[c + d*x]^2
))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1-x^2}{x^4\left(a+b-bx^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{ab\tanh(c+dx)}{2(a+b)^3 d\left(a+b-b\tanh^2(c+dx)\right)} + \frac{b\operatorname{Subst}\left(\int \frac{\frac{2}{b(a+b)}-\frac{2ax^2}{b(a+b)^2}-\frac{ax^4}{(a+b)^3}}{x^4\left(a+b-bx^2\right)}\,dx, x, \tanh\right.}{2d}$$

$$= -\frac{ab\tanh(c+dx)}{2(a+b)^3 d\left(a+b-b\tanh^2(c+dx)\right)} + \frac{b\operatorname{Subst}\left(\int\left(\frac{2}{b(a+b)^2x^4} - \frac{2(a-b)}{b(a+b)^3x^2} + \frac{-}{(a+b)^3}\right.\right.}{2d}$$

$$= \frac{(a-b)\coth(c+dx)}{(a+b)^3 d} - \frac{\coth^3(c+dx)}{3(a+b)^2 d} - \frac{ab\tanh(c+dx)}{2(a+b)^3 d\left(a+b-b\tanh^2(c+dx)\right)} - $$

$$= -\frac{(3a-2b)\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2(a+b)^{7/2} d} + \frac{(a-b)\coth(c+dx)}{(a+b)^3 d} - \frac{\coth^3(c+dx)}{3(a+b)^2 d} - $$

Mathematica [B] time = 6.08, size = 295, normalized size = 2.40

$$\frac{\operatorname{sech}^4(c+dx)(a\cosh(2(c+dx))+a+2b)\left(-3ab\operatorname{sech}(2c)\sinh(2dx)-2(a+b)\coth(c)\operatorname{csch}^2(c+dx)(a\cosh(2(c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^4*(-2*(a + b)*(a + 2*b + a*C
osh[2*(c + d*x)])*Coth[c]*Csch[c + d*x]^2 - (3*(3*a - 2*b)*b*ArcTanh[(Sech[
d*x]*(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*
Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(a + 2*b + a*Cosh[2*(c + d*x)])
*(Cosh[2*c] - Sinh[2*c]))/(Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4]) - 4*(
a - 2*b)*(a + 2*b + a*Cosh[2*(c + d*x)])*Csch[c]*Csch[c + d*x]*Sinh[d*x] +
2*(a + b)*(a + 2*b + a*Cosh[2*(c + d*x)])*Csch[c]*Csch[c + d*x]^3*Sinh[d*x]
 - 3*a*b*Sech[2*c]*Sinh[2*d*x] + 3*b*(a + 2*b)*Tanh[2*c]))/(24*(a + b)^3*d*
(a + b*Sech[c + d*x]^2)^2)
```

fricas [B] time = 0.53, size = 6143, normalized size = 49.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/12*(12*(3*a*b - 2*b^2)*cosh(d*x + c)^8 + 96*(3*a*b - 2*b^2)*cosh(d*x +
c)*sinh(d*x + c)^7 + 12*(3*a*b - 2*b^2)*sinh(d*x + c)^8 + 24*(2*a^2 + 3*a*b
 + 11*b^2)*cosh(d*x + c)^6 + 24*(14*(3*a*b - 2*b^2)*cosh(d*x + c)^2 + 2*a^2
 + 3*a*b + 11*b^2)*sinh(d*x + c)^6 + 48*(14*(3*a*b - 2*b^2)*cosh(d*x + c)^3
 + 3*(2*a^2 + 3*a*b + 11*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 8*(10*a^2 +
22*a*b - 33*b^2)*cosh(d*x + c)^4 + 8*(105*(3*a*b - 2*b^2)*cosh(d*x + c)^4 +
 45*(2*a^2 + 3*a*b + 11*b^2)*cosh(d*x + c)^2 + 10*a^2 + 22*a*b - 33*b^2)*si
nh(d*x + c)^4 + 32*(21*(3*a*b - 2*b^2)*cosh(d*x + c)^5 + 15*(2*a^2 + 3*a*b
+ 11*b^2)*cosh(d*x + c)^3 + (10*a^2 + 22*a*b - 33*b^2)*cosh(d*x + c))*sinh(
d*x + c)^3 + 8*(2*a^2 - 9*a*b + 19*b^2)*cosh(d*x + c)^2 + 8*(42*(3*a*b - 2*
b^2)*cosh(d*x + c)^6 + 45*(2*a^2 + 3*a*b + 11*b^2)*cosh(d*x + c)^4 + 6*(10*
a^2 + 22*a*b - 33*b^2)*cosh(d*x + c)^2 + 2*a^2 - 9*a*b + 19*b^2)*sinh(d*x +
 c)^2 + 3*((3*a^2 - 2*a*b)*cosh(d*x + c)^10 + 10*(3*a^2 - 2*a*b)*cosh(d*x +
 c)*sinh(d*x + c)^9 + (3*a^2 - 2*a*b)*sinh(d*x + c)^10 - (3*a^2 - 14*a*b +
8*b^2)*cosh(d*x + c)^8 + (45*(3*a^2 - 2*a*b)*cosh(d*x + c)^2 - 3*a^2 + 14*a
```

*b - 8*b^2)*sinh(d*x + c)^8 + 8*(15*(3*a^2 - 2*a*b)*cosh(d*x + c)^3 - (3*a^
2 - 14*a*b + 8*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 - 2*(3*a^2 + 16*a*b - 12
*b^2)*cosh(d*x + c)^6 + 2*(105*(3*a^2 - 2*a*b)*cosh(d*x + c)^4 - 14*(3*a^2
- 14*a*b + 8*b^2)*cosh(d*x + c)^2 - 3*a^2 - 16*a*b + 12*b^2)*sinh(d*x + c)^
6 + 4*(63*(3*a^2 - 2*a*b)*cosh(d*x + c)^5 - 14*(3*a^2 - 14*a*b + 8*b^2)*cos
h(d*x + c)^3 - 3*(3*a^2 + 16*a*b - 12*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 +
 2*(3*a^2 + 16*a*b - 12*b^2)*cosh(d*x + c)^4 + 2*(105*(3*a^2 - 2*a*b)*cosh(
d*x + c)^6 - 35*(3*a^2 - 14*a*b + 8*b^2)*cosh(d*x + c)^4 - 15*(3*a^2 + 16*a
*b - 12*b^2)*cosh(d*x + c)^2 + 3*a^2 + 16*a*b - 12*b^2)*sinh(d*x + c)^4 + 8
(15(3*a^2 - 2*a*b)*cosh(d*x + c)^7 - 7*(3*a^2 - 14*a*b + 8*b^2)*cosh(d*x
+ c)^5 - 5*(3*a^2 + 16*a*b - 12*b^2)*cosh(d*x + c)^3 + (3*a^2 + 16*a*b - 12
*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + (3*a^2 - 14*a*b + 8*b^2)*cosh(d*x +
c)^2 + (45*(3*a^2 - 2*a*b)*cosh(d*x + c)^8 - 28*(3*a^2 - 14*a*b + 8*b^2)*co
sh(d*x + c)^6 - 30*(3*a^2 + 16*a*b - 12*b^2)*cosh(d*x + c)^4 + 12*(3*a^2 +
16*a*b - 12*b^2)*cosh(d*x + c)^2 + 3*a^2 - 14*a*b + 8*b^2)*sinh(d*x + c)^2
- 3*a^2 + 2*a*b + 2*(5*(3*a^2 - 2*a*b)*cosh(d*x + c)^9 - 4*(3*a^2 - 14*a*b
+ 8*b^2)*cosh(d*x + c)^7 - 6*(3*a^2 + 16*a*b - 12*b^2)*cosh(d*x + c)^5 + 4*
(3*a^2 + 16*a*b - 12*b^2)*cosh(d*x + c)^3 + (3*a^2 - 14*a*b + 8*b^2)*cosh(d
*x + c))*sinh(d*x + c))*sqrt(b/(a + b))*log((a^2*cosh(d*x + c)^4 + 4*a^2*co
sh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*
x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 +
8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(
d*x + c) - 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sin
h(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 + 3*a*b + 2*b^2)*sqrt(b/(a +
 b)))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x +
 c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sin
h(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c
) + a)) - 16*a^2 + 44*a*b + 16*(6*(3*a*b - 2*b^2)*cosh(d*x + c)^7 + 9*(2*a^
2 + 3*a*b + 11*b^2)*cosh(d*x + c)^5 + 2*(10*a^2 + 22*a*b - 33*b^2)*cosh(d*x
 + c)^3 + (2*a^2 - 9*a*b + 19*b^2)*cosh(d*x + c))*sinh(d*x + c))/((a^4 + 3*
a^3*b + 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)^10 + 10*(a^4 + 3*a^3*b + 3*a^2*b
^2 + a*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^9 + (a^4 + 3*a^3*b + 3*a^2*b^2 +
a*b^3)*d*sinh(d*x + c)^10 - (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*
cosh(d*x + c)^8 + (45*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)^2
 - (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d)*sinh(d*x + c)^8 - 2*(a^4
 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*cosh(d*x + c)^6 + 8*(15*(a^4
+ 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)^3 - (a^4 - a^3*b - 9*a^2*b^2
 - 11*a*b^3 - 4*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(105*(a^4 + 3*a^3
*b + 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)^4 - 14*(a^4 - a^3*b - 9*a^2*b^2 - 1
1*a*b^3 - 4*b^4)*d*cosh(d*x + c)^2 - (a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3
 + 6*b^4)*d)*sinh(d*x + c)^6 + 2*(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6
*b^4)*d*cosh(d*x + c)^4 + 4*(63*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*cosh(
d*x + c)^5 - 14*(a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*cosh(d*x + c
)^3 - 3*(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*cosh(d*x + c))*si
nh(d*x + c)^5 + 2*(105*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)^

6 - 35*(a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*cosh(d*x + c)^4 - 15*
(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*cosh(d*x + c)^2 + (a^4 +
9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d)*sinh(d*x + c)^4 + (a^4 - a^3*b
- 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*cosh(d*x + c)^2 + 8*(15*(a^4 + 3*a^3*b +
3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)^7 - 7*(a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^
3 - 4*b^4)*d*cosh(d*x + c)^5 - 5*(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6
*b^4)*d*cosh(d*x + c)^3 + (a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d
*cosh(d*x + c))*sinh(d*x + c)^3 + (45*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d
*cosh(d*x + c)^8 - 28*(a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*cosh(d
x + c)^6 - 30(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*cosh(d*x +
c)^4 + 12*(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*cosh(d*x + c)^
2 + (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d)*sinh(d*x + c)^2 - (a^4
+ 3*a^3*b + 3*a^2*b^2 + a*b^3)*d + 2*(5*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)
*d*cosh(d*x + c)^9 - 4*(a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*cosh(
d*x + c)^7 - 6*(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*cosh(d*x +
c)^5 + 4*(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*cosh(d*x + c)^3
+ (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*cosh(d*x + c))*sinh(d*x +
c)), -1/6*(6*(3*a*b - 2*b^2)*cosh(d*x + c)^8 + 48*(3*a*b - 2*b^2)*cosh(d*x
+ c)*sinh(d*x + c)^7 + 6*(3*a*b - 2*b^2)*sinh(d*x + c)^8 + 12*(2*a^2 + 3*a
*b + 11*b^2)*cosh(d*x + c)^6 + 12*(14*(3*a*b - 2*b^2)*cosh(d*x + c)^2 + 2*a
^2 + 3*a*b + 11*b^2)*sinh(d*x + c)^6 + 24*(14*(3*a*b - 2*b^2)*cosh(d*x + c)
^3 + 3*(2*a^2 + 3*a*b + 11*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 4*(10*a^2
+ 22*a*b - 33*b^2)*cosh(d*x + c)^4 + 4*(105*(3*a*b - 2*b^2)*cosh(d*x + c)^4
+ 45*(2*a^2 + 3*a*b + 11*b^2)*cosh(d*x + c)^2 + 10*a^2 + 22*a*b - 33*b^2)*
sinh(d*x + c)^4 + 16*(21*(3*a*b - 2*b^2)*cosh(d*x + c)^5 + 15*(2*a^2 + 3*a*
b + 11*b^2)*cosh(d*x + c)^3 + (10*a^2 + 22*a*b - 33*b^2)*cosh(d*x + c))*sin
h(d*x + c)^3 + 4*(2*a^2 - 9*a*b + 19*b^2)*cosh(d*x + c)^2 + 4*(42*(3*a*b -
2*b^2)*cosh(d*x + c)^6 + 45*(2*a^2 + 3*a*b + 11*b^2)*cosh(d*x + c)^4 + 6*(1
0*a^2 + 22*a*b - 33*b^2)*cosh(d*x + c)^2 + 2*a^2 - 9*a*b + 19*b^2)*sinh(d*x
+ c)^2 + 3*((3*a^2 - 2*a*b)*cosh(d*x + c)^10 + 10*(3*a^2 - 2*a*b)*cosh(d*x
+ c)*sinh(d*x + c)^9 + (3*a^2 - 2*a*b)*sinh(d*x + c)^10 - (3*a^2 - 14*a*b
+ 8*b^2)*cosh(d*x + c)^8 + (45*(3*a^2 - 2*a*b)*cosh(d*x + c)^2 - 3*a^2 + 14
*a*b - 8*b^2)*sinh(d*x + c)^8 + 8*(15*(3*a^2 - 2*a*b)*cosh(d*x + c)^3 - (3*
a^2 - 14*a*b + 8*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 - 2*(3*a^2 + 16*a*b -
12*b^2)*cosh(d*x + c)^6 + 2*(105*(3*a^2 - 2*a*b)*cosh(d*x + c)^4 - 14*(3*a^
2 - 14*a*b + 8*b^2)*cosh(d*x + c)^2 - 3*a^2 - 16*a*b + 12*b^2)*sinh(d*x + c
)^6 + 4*(63*(3*a^2 - 2*a*b)*cosh(d*x + c)^5 - 14*(3*a^2 - 14*a*b + 8*b^2)*c
osh(d*x + c)^3 - 3*(3*a^2 + 16*a*b - 12*b^2)*cosh(d*x + c))*sinh(d*x + c)^5
+ 2*(3*a^2 + 16*a*b - 12*b^2)*cosh(d*x + c)^4 + 2*(105*(3*a^2 - 2*a*b)*cos
h(d*x + c)^6 - 35*(3*a^2 - 14*a*b + 8*b^2)*cosh(d*x + c)^4 - 15*(3*a^2 + 16
*a*b - 12*b^2)*cosh(d*x + c)^2 + 3*a^2 + 16*a*b - 12*b^2)*sinh(d*x + c)^4 +
8*(15*(3*a^2 - 2*a*b)*cosh(d*x + c)^7 - 7*(3*a^2 - 14*a*b + 8*b^2)*cosh(d*
x + c)^5 - 5*(3*a^2 + 16*a*b - 12*b^2)*cosh(d*x + c)^3 + (3*a^2 + 16*a*b -
12*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + (3*a^2 - 14*a*b + 8*b^2)*cosh(d*x
+ c)^2 + (45*(3*a^2 - 2*a*b)*cosh(d*x + c)^8 - 28*(3*a^2 - 14*a*b + 8*b^2)*

```
cosh(d*x + c)^6 - 30*(3*a^2 + 16*a*b - 12*b^2)*cosh(d*x + c)^4 + 12*(3*a^2
+ 16*a*b - 12*b^2)*cosh(d*x + c)^2 + 3*a^2 - 14*a*b + 8*b^2)*sinh(d*x + c)^
2 - 3*a^2 + 2*a*b + 2*(5*(3*a^2 - 2*a*b)*cosh(d*x + c)^9 - 4*(3*a^2 - 14*a*
b + 8*b^2)*cosh(d*x + c)^7 - 6*(3*a^2 + 16*a*b - 12*b^2)*cosh(d*x + c)^5 +
4*(3*a^2 + 16*a*b - 12*b^2)*cosh(d*x + c)^3 + (3*a^2 - 14*a*b + 8*b^2)*cosh
(d*x + c))*sinh(d*x + c))*sqrt(-b/(a + b))*arctan(1/2*(a*cosh(d*x + c)^2 +
2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-b/(a +
 b))/b) - 8*a^2 + 22*a*b + 8*(6*(3*a*b - 2*b^2)*cosh(d*x + c)^7 + 9*(2*a^2
+ 3*a*b + 11*b^2)*cosh(d*x + c)^5 + 2*(10*a^2 + 22*a*b - 33*b^2)*cosh(d*x +
 c)^3 + (2*a^2 - 9*a*b + 19*b^2)*cosh(d*x + c))*sinh(d*x + c))/((a^4 + 3*a^
3*b + 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)^10 + 10*(a^4 + 3*a^3*b + 3*a^2*b^2
 + a*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^9 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*
b^3)*d*sinh(d*x + c)^10 - (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*co
sh(d*x + c)^8 + (45*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)^2 -
 (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d)*sinh(d*x + c)^8 - 2*(a^4 +
 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*cosh(d*x + c)^6 + 8*(15*(a^4 +
3*a^3*b + 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)^3 - (a^4 - a^3*b - 9*a^2*b^2 -
 11*a*b^3 - 4*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(105*(a^4 + 3*a^3*b
 + 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)^4 - 14*(a^4 - a^3*b - 9*a^2*b^2 - 11*
a*b^3 - 4*b^4)*d*cosh(d*x + c)^2 - (a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 +
 6*b^4)*d)*sinh(d*x + c)^6 + 2*(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b
^4)*d*cosh(d*x + c)^4 + 4*(63*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*cosh(d*
x + c)^5 - 14*(a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*cosh(d*x + c)^
3 - 3*(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*cosh(d*x + c))*sinh
(d*x + c)^5 + 2*(105*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)^6
- 35*(a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*cosh(d*x + c)^4 - 15*(a
^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*cosh(d*x + c)^2 + (a^4 + 9*
a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d)*sinh(d*x + c)^4 + (a^4 - a^3*b -
9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*cosh(d*x + c)^2 + 8*(15*(a^4 + 3*a^3*b + 3*
a^2*b^2 + a*b^3)*d*cosh(d*x + c)^7 - 7*(a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3
- 4*b^4)*d*cosh(d*x + c)^5 - 5*(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b
^4)*d*cosh(d*x + c)^3 + (a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*c
osh(d*x + c))*sinh(d*x + c)^3 + (45*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*c
osh(d*x + c)^8 - 28*(a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*cosh(d*x
 + c)^6 - 30*(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*cosh(d*x + c
)^4 + 12*(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*cosh(d*x + c)^2
+ (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d)*sinh(d*x + c)^2 - (a^4 +
3*a^3*b + 3*a^2*b^2 + a*b^3)*d + 2*(5*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d
*cosh(d*x + c)^9 - 4*(a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*cosh(d*
x + c)^7 - 6*(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*cosh(d*x + c
)^5 + 4*(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*cosh(d*x + c)^3 +
 (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*cosh(d*x + c))*sinh(d*x + c
))]
```

giac [B] time = 0.77, size = 253, normalized size = 2.06

$$-\frac{\frac{3\left(3\,ab-2\,b^2\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\sqrt{-ab-b^2}}-\frac{6\left(abe^{(2\,dx+2\,c)}+2\,b^2e^{(2\,dx+2\,c)}+ab\right)}{\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\left(ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+a\right)}+\frac{8\left(3\,be^{(4\,dx+4\,c)}+3\,ae^{(2\,dx+2\,c)}-3\,be^{(2\,dx}\cdots\right.}{\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\left(e^{(2\,dx+2\,c}\cdots\right.}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/6*(3*(3*a*b - 2*b^2)*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b
- b^2))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(-a*b - b^2)) - 6*(a*b*e^(2*d*
x + 2*c) + 2*b^2*e^(2*d*x + 2*c) + a*b)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(a
*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) + a)) + 8*(3*b
*e^(4*d*x + 4*c) + 3*a*e^(2*d*x + 2*c) - 3*b*e^(2*d*x + 2*c) - a + 2*b)/((a
^3 + 3*a^2*b + 3*a*b^2 + b^3)*(e^(2*d*x + 2*c) - 1)^3))/d
```

maple [B] time = 0.50, size = 577, normalized size = 4.69

$$-\frac{a\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d\left(a+b\right)\left(a^2+2ab+b^2\right)}-\frac{\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b}{24d\left(a+b\right)\left(a^2+2ab+b^2\right)}+\frac{3a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\left(a+b\right)\left(a^2+2ab+b^2\right)}-\frac{5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)b}{8d\left(a+b\right)\left(a^2+2ab+b^2\right)}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] -1/24/d/(a+b)/(a^2+2*a*b+b^2)*a*tanh(1/2*d*x+1/2*c)^3-1/24/d/(a+b)/(a^2+2*a
*b+b^2)*tanh(1/2*d*x+1/2*c)^3*b+3/8/d/(a+b)/(a^2+2*a*b+b^2)*a*tanh(1/2*d*x+
1/2*c)-5/8/d/(a+b)/(a^2+2*a*b+b^2)*tanh(1/2*d*x+1/2*c)*b-1/d*b/(a+b)^3/(tan
h(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*ta
nh(1/2*d*x+1/2*c)^2*b+a+b)*a*tanh(1/2*d*x+1/2*c)^3-1/d*b/(a+b)^3/(tanh(1/2*
d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2
*d*x+1/2*c)^2*b+a+b)*a*tanh(1/2*d*x+1/2*c)+3/4/d*b^(1/2)/(a+b)^(7/2)*a*ln(-
(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)-(a+b)^(1/2)
)-3/4/d*b^(1/2)/(a+b)^(7/2)*a*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2
)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))-1/2/d*b^(3/2)/(a+b)^(7/2)*ln(-(a+b)^(1/2
)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)-(a+b)^(1/2))+1/2/d*b^
(3/2)/(a+b)^(7/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d
*x+1/2*c)+(a+b)^(1/2))-1/24/d/(a+b)^2/tanh(1/2*d*x+1/2*c)^3+3/8/d/(a+b)^3/t
anh(1/2*d*x+1/2*c)*a-5/8/d/(a+b)^3/tanh(1/2*d*x+1/2*c)*b
```

maxima [B] time = 0.52, size = 430, normalized size = 3.50

$$\frac{\left(3\,ab-2\,b^2\right)\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{4\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\sqrt{(a+b)b}\,d}+\frac{4\,a^2-\cdots}{3\left(a^4+3\,a^3b+3\,a^2b^2+ab^3-\left(a^4-a^3b-9\,a^2b^2-11\,ab^3-4\,b^4\right)e^{(-2}\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(3*a*b - 2*b^2)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/
(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^3 + 3*a^2*b + 3*a*b
^2 + b^3)*sqrt((a + b)*b)*d) + 1/3*(4*a^2 - 11*a*b - 2*(2*a^2 - 9*a*b + 19*
b^2)*e^(-2*d*x - 2*c) - 2*(10*a^2 + 22*a*b - 33*b^2)*e^(-4*d*x - 4*c) - 6*(
2*a^2 + 3*a*b + 11*b^2)*e^(-6*d*x - 6*c) - 3*(3*a*b - 2*b^2)*e^(-8*d*x - 8*
c))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3 - (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b
^3 - 4*b^4)*e^(-2*d*x - 2*c) - 2*(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6
*b^4)*e^(-4*d*x - 4*c) + 2*(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*
e^(-6*d*x - 6*c) + (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*e^(-8*d*x -
 8*c) - (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*e^(-10*d*x - 10*c))*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+d\,x)^4}{\sinh(c+d\,x)^4\,\left(a\cosh(c+d\,x)^2+b\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^4*(a + b/cosh(c + d*x)^2)^2),x)
```

```
[Out] int(cosh(c + d*x)^4/(sinh(c + d*x)^4*(b + a*cosh(c + d*x)^2)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral(csch(c + d*x)**4/(a + b*sech(c + d*x)**2)**2, x)
```

3.41 $\int \frac{\sinh^4(c+dx)}{\left(a+b\mathbf{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=242

$$-\frac{3b(a+2b)\tanh(c+dx)}{2a^4d\left(a-b\tanh^2(c+dx)+b\right)}-\frac{b(7a+12b)\tanh(c+dx)}{8a^3d\left(a-b\tanh^2(c+dx)+b\right)^2}-\frac{(5a+8b)\sinh(c+dx)\cosh(c+dx)}{8a^2d\left(a-b\tanh^2(c+dx)+b\right)^2}-\frac{3\sqrt{b}\,(5\ldots}{}$$

```
[Out] 3/8*(a^2+12*a*b+16*b^2)*x/a^5-3/8*(5*a^2+20*a*b+16*b^2)*arctanh(b^(1/2)*tan
h(d*x+c)/(a+b)^(1/2))*b^(1/2)/a^5/d/(a+b)^(1/2)-1/8*(5*a+8*b)*cosh(d*x+c)*s
inh(d*x+c)/a^2/d/(a+b-b*tanh(d*x+c)^2)^2+1/4*cosh(d*x+c)^3*sinh(d*x+c)/a/d/
(a+b-b*tanh(d*x+c)^2)^2-1/8*b*(7*a+12*b)*tanh(d*x+c)/a^3/d/(a+b-b*tanh(d*x+
c)^2)^2-3/2*b*(a+2*b)*tanh(d*x+c)/a^4/d/(a+b-b*tanh(d*x+c)^2)
```

Rubi [A] time = 0.40, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4132, 470, 527, 522, 206, 208}

$$-\frac{3\sqrt{b}\left(5a^2+20ab+16b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^5d\sqrt{a+b}}+\frac{3x\left(a^2+12ab+16b^2\right)}{8a^5}-\frac{3b(a+2b)\tanh(c+dx)}{2a^4d\left(a-b\tanh^2(c+dx)+b\right)}-\frac{b(\ldots}{8a^3d\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^4/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (3*(a^2 + 12*a*b + 16*b^2)*x)/(8*a^5) - (3*Sqrt[b]*(5*a^2 + 20*a*b + 16*b^2
)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(8*a^5*Sqrt[a + b]*d) - ((5
*a + 8*b)*Cosh[c + d*x]*Sinh[c + d*x])/(8*a^2*d*(a + b - b*Tanh[c + d*x]^2)
^2) + (Cosh[c + d*x]^3*Sinh[c + d*x])/(4*a*d*(a + b - b*Tanh[c + d*x]^2)^2)
 - (b*(7*a + 12*b)*Tanh[c + d*x])/(8*a^3*d*(a + b - b*Tanh[c + d*x]^2)^2) -
 (3*b*(a + 2*b)*Tanh[c + d*x])/(2*a^4*d*(a + b - b*Tanh[c + d*x]^2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)^3(a+b-bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\cosh^3(c+dx)\sinh(c+dx)}{4ad\left(a+b-b\tanh^2(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{a+b+(4a+7b)x^2}{(1-x^2)^2(a+b-bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{4ad}$$

$$= -\frac{(5a+8b)\cosh(c+dx)\sinh(c+dx)}{8a^2d\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4ad\left(a+b-b\tanh^2(c+dx)\right)^2} - \frac{\text{Subst}\left(\right.}{}$$

$$= -\frac{(5a+8b)\cosh(c+dx)\sinh(c+dx)}{8a^2d\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4ad\left(a+b-b\tanh^2(c+dx)\right)^2} - \frac{b(7a}{8a^3d\left(a\right.}$$

$$= -\frac{(5a+8b)\cosh(c+dx)\sinh(c+dx)}{8a^2d\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4ad\left(a+b-b\tanh^2(c+dx)\right)^2} - \frac{b(7a}{8a^3d\left(a\right.}$$

$$= -\frac{(5a+8b)\cosh(c+dx)\sinh(c+dx)}{8a^2d\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4ad\left(a+b-b\tanh^2(c+dx)\right)^2} - \frac{b(7a}{8a^3d\left(a\right.}$$

$$= \frac{3\left(a^2+12ab+16b^2\right)x}{8a^5} - \frac{3\sqrt{b}\left(5a^2+20ab+16b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^5\sqrt{a+b}\,d} - \frac{(5a+}{8a^2}$$

Mathematica [B] time = 27.32, size = 3080, normalized size = 12.73

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sinh[c + d*x]^4/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (3*(a + 2*b + a*Cosh[2*c + 2*d*x])^3*Sech[c + d*x]^6*(((3*a^2 + 8*a*b + 8*b
^2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a + b)^(5/2) - (a*Sqrt[b
]*(3*a^2 + 16*a*b + 16*b^2 + 3*a*(a + 2*b)*Cosh[2*(c + d*x)])*Sinh[2*(c + d
*x)])/((a + b)^2*(a + 2*b + a*Cosh[2*(c + d*x)])^2)))/(16384*b^(5/2)*d*(a +
 b*Sech[c + d*x]^2)^3) + ((a + 2*b + a*Cosh[2*c + 2*d*x])^3*Sech[c + d*x]^6
```

```
*((-3*a*(a + 2*b)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a + b)^(5/
2) + (Sqrt[b]*(3*a^3 + 14*a^2*b + 24*a*b^2 + 16*b^3 + a*(3*a^2 + 4*a*b + 4*
b^2)*Cosh[2*(c + d*x)])*Sinh[2*(c + d*x)])/((a + b)^2*(a + 2*b + a*Cosh[2*(
c + d*x)])^2)))/(16384*b^(5/2)*d*(a + b*Sech[c + d*x]^2)^3) - (3*(a + 2*b +
 a*Cosh[2*c + 2*d*x])^3*Sech[c + d*x]^6*((-2*(3*a^5 - 10*a^4*b + 80*a^3*b^2
 + 480*a^2*b^3 + 640*a*b^4 + 256*b^5)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[
2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos
h[c] - Sinh[c])^4])]*(Cosh[2*c] - Sinh[2*c]))/(Sqrt[a + b]*Sqrt[b*(Cosh[c]
- Sinh[c])^4]) + (Sech[2*c]*(256*b^2*(a + b)^2*(3*a^2 + 8*a*b + 8*b^2)*d*x*
Cosh[2*c] + 512*a*b^2*(a + b)^2*(a + 2*b)*d*x*Cosh[2*d*x] + 128*a^4*b^2*d*x
*Cosh[2*(c + 2*d*x)] + 256*a^3*b^3*d*x*Cosh[2*(c + 2*d*x)] + 128*a^2*b^4*d*
x*Cosh[2*(c + 2*d*x)] + 512*a^4*b^2*d*x*Cosh[4*c + 2*d*x] + 2048*a^3*b^3*d*
x*Cosh[4*c + 2*d*x] + 2560*a^2*b^4*d*x*Cosh[4*c + 2*d*x] + 1024*a*b^5*d*x*C
osh[4*c + 2*d*x] + 128*a^4*b^2*d*x*Cosh[6*c + 4*d*x] + 256*a^3*b^3*d*x*Cosh
[6*c + 4*d*x] + 128*a^2*b^4*d*x*Cosh[6*c + 4*d*x] - 9*a^6*Sinh[2*c] + 12*a^
5*b*Sinh[2*c] + 684*a^4*b^2*Sinh[2*c] + 2880*a^3*b^3*Sinh[2*c] + 5280*a^2*b
^4*Sinh[2*c] + 4608*a*b^5*Sinh[2*c] + 1536*b^6*Sinh[2*c] + 9*a^6*Sinh[2*d*x
] - 14*a^5*b*Sinh[2*d*x] - 608*a^4*b^2*Sinh[2*d*x] - 2112*a^3*b^3*Sinh[2*d*
x] - 2560*a^2*b^4*Sinh[2*d*x] - 1024*a*b^5*Sinh[2*d*x] + 3*a^6*Sinh[2*(c +
2*d*x)] - 12*a^5*b*Sinh[2*(c + 2*d*x)] - 204*a^4*b^2*Sinh[2*(c + 2*d*x)] -
384*a^3*b^3*Sinh[2*(c + 2*d*x)] - 192*a^2*b^4*Sinh[2*(c + 2*d*x)] - 3*a^6*S
inh[4*c + 2*d*x] + 10*a^5*b*Sinh[4*c + 2*d*x] + 304*a^4*b^2*Sinh[4*c + 2*d*
x] + 1056*a^3*b^3*Sinh[4*c + 2*d*x] + 1280*a^2*b^4*Sinh[4*c + 2*d*x] + 512*
a*b^5*Sinh[4*c + 2*d*x]))/(a + 2*b + a*Cosh[2*(c + d*x)])^2))/(65536*a^3*b^
2*(a + b)^2*d*(a + b*Sech[c + d*x]^2)^3) + ((a + 2*b + a*Cosh[2*c + 2*d*x])
^3*Sech[c + d*x]^6*((6*(a^6 - 8*a^5*b + 120*a^4*b^2 + 1280*a^3*b^3 + 3200*a
^2*b^4 + 3072*a*b^5 + 1024*b^6)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*
((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] -
 Sinh[c])^4])]*(Cosh[2*c] - Sinh[2*c]))/(Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh
[c])^4]) + (Sech[2*c]*(-1536*b^2*(a + b)^2*(3*a^3 + 14*a^2*b + 24*a*b^2 + 1
6*b^3)*d*x*Cosh[2*c] - 3072*a*b^2*(a^2 + 3*a*b + 2*b^2)^2*d*x*Cosh[2*d*x] -
 768*a^5*b^2*d*x*Cosh[2*(c + 2*d*x)] - 3072*a^4*b^3*d*x*Cosh[2*(c + 2*d*x)]
 - 3840*a^3*b^4*d*x*Cosh[2*(c + 2*d*x)] - 1536*a^2*b^5*d*x*Cosh[2*(c + 2*d*
x)] - 3072*a^5*b^2*d*x*Cosh[4*c + 2*d*x] - 18432*a^4*b^3*d*x*Cosh[4*c + 2*d
*x] - 39936*a^3*b^4*d*x*Cosh[4*c + 2*d*x] - 36864*a^2*b^5*d*x*Cosh[4*c + 2*
d*x] - 12288*a*b^6*d*x*Cosh[4*c + 2*d*x] - 768*a^5*b^2*d*x*Cosh[6*c + 4*d*x
] - 3072*a^4*b^3*d*x*Cosh[6*c + 4*d*x] - 3840*a^3*b^4*d*x*Cosh[6*c + 4*d*x]
 - 1536*a^2*b^5*d*x*Cosh[6*c + 4*d*x] + 9*a^7*Sinh[2*c] - 54*a^6*b*Sinh[2*c
] - 2392*a^5*b^2*Sinh[2*c] - 13968*a^4*b^3*Sinh[2*c] - 36480*a^3*b^4*Sinh[2
*c] - 50432*a^2*b^5*Sinh[2*c] - 35840*a*b^6*Sinh[2*c] - 10240*b^7*Sinh[2*c]
 - 9*a^7*Sinh[2*d*x] + 56*a^6*b*Sinh[2*d*x] + 2552*a^5*b^2*Sinh[2*d*x] + 13
184*a^4*b^3*Sinh[2*d*x] + 27072*a^3*b^4*Sinh[2*d*x] + 24576*a^2*b^5*Sinh[2*
d*x] + 8192*a*b^6*Sinh[2*d*x] - 3*a^7*Sinh[2*(c + 2*d*x)] + 26*a^6*b*Sinh[2
*(c + 2*d*x)] + 992*a^5*b^2*Sinh[2*(c + 2*d*x)] + 3648*a^4*b^3*Sinh[2*(c +
2*d*x)] + 4480*a^3*b^4*Sinh[2*(c + 2*d*x)] + 1792*a^2*b^5*Sinh[2*(c + 2*d*x
```

```
)] + 3*a^7*Sinh[4*c + 2*d*x] - 24*a^6*b*Sinh[4*c + 2*d*x] - 600*a^5*b^2*Sin
h[4*c + 2*d*x] - 3200*a^4*b^3*Sinh[4*c + 2*d*x] - 6720*a^3*b^4*Sinh[4*c + 2
*d*x] - 6144*a^2*b^5*Sinh[4*c + 2*d*x] - 2048*a*b^6*Sinh[4*c + 2*d*x] + 256
*a^5*b^2*Sinh[6*c + 4*d*x] + 1024*a^4*b^3*Sinh[6*c + 4*d*x] + 1280*a^3*b^4*
Sinh[6*c + 4*d*x] + 512*a^2*b^5*Sinh[6*c + 4*d*x] + 64*a^5*b^2*Sinh[4*c + 6
*d*x] + 128*a^4*b^3*Sinh[4*c + 6*d*x] + 64*a^3*b^4*Sinh[4*c + 6*d*x] + 64*a
^5*b^2*Sinh[8*c + 6*d*x] + 128*a^4*b^3*Sinh[8*c + 6*d*x] + 64*a^3*b^4*Sinh[
8*c + 6*d*x]))/(a + 2*b + a*Cosh[2*(c + d*x)])^2))/(32768*a^4*b^2*(a + b)^2
*d*(a + b*Sech[c + d*x]^2)^3) - ((a + 2*b + a*Cosh[2*c + 2*d*x])^3*Sech[c +
 d*x]^6*((6*a^2*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[
d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(
Cosh[2*c] - Sinh[2*c]))/(Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4]) + (a*Se
ch[2*c]*((-9*a^4 - 16*a^3*b + 48*a^2*b^2 + 128*a*b^3 + 64*b^4)*Sinh[2*d*x]
+ a*(-3*a^3 + 2*a^2*b + 24*a*b^2 + 16*b^3)*Sinh[2*(c + 2*d*x)] + (3*a^4 - 6
4*a^2*b^2 - 128*a*b^3 - 64*b^4)*Sinh[4*c + 2*d*x]) + (9*a^5 + 18*a^4*b - 64
*a^3*b^2 - 256*a^2*b^3 - 320*a*b^4 - 128*b^5)*Tanh[2*c])/(a^2*(a + 2*b + a*
Cosh[2*(c + d*x)])^2)))/(8192*b^2*(a + b)^2*d*(a + b*Sech[c + d*x]^2)^3) +
((a + 2*b + a*Cosh[2*c + 2*d*x])^3*Sech[c + d*x]^6*(768*(7*a^2 + 32*a*b + 3
2*b^2)*x - (3*(a^7 - 14*a^6*b + 336*a^5*b^2 + 5600*a^4*b^3 + 22400*a^3*b^4
+ 37632*a^2*b^5 + 28672*a*b^6 + 8192*b^7)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - S
inh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*
(Cosh[c] - Sinh[c])^4])]*(Cosh[2*c] - Sinh[2*c]))/(b^2*(a + b)^(5/2)*d*Sqrt
[b*(Cosh[c] - Sinh[c])^4]) - (4*(a^5 + 50*a^4*b + 400*a^3*b^2 + 1120*a^2*b^
3 + 1280*a*b^4 + 512*b^5)*Sech[2*c]*((a + 2*b)*Sinh[2*c] - a*Sinh[2*d*x]))/
(b*(a + b)*d*(a + 2*b + a*Cosh[2*(c + d*x)])^2) + (768*a*(a + 2*b)*(Cosh[2*
(c + d*x)] - Sinh[2*(c + d*x)]))/d - (768*a*(a + 2*b)*(Cosh[2*(c + d*x)] +
Sinh[2*(c + d*x)]))/d + (128*a^2*Sinh[4*(c + d*x)])/d + (a*(3*a^6 - 44*a^5*
b - 1900*a^4*b^2 - 10880*a^3*b^3 - 23360*a^2*b^4 - 21504*a*b^5 - 7168*b^6)*
Sech[2*c]*Sinh[2*d*x] + (-3*a^7 + 42*a^6*b + 2192*a^5*b^2 + 16480*a^4*b^3 +
 51200*a^3*b^4 + 77824*a^2*b^5 + 57344*a*b^6 + 16384*b^7)*Tanh[2*c])/(b^2*(
a + b)^2*d*(a + 2*b + a*Cosh[2*(c + d*x)]))))/(32768*a^5*(a + b*Sech[c + d*
x]^2)^3)
```

fricas [B] time = 0.64, size = 12353, normalized size = 51.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/64*(a^4*cosh(d*x + c)^16 + 16*a^4*cosh(d*x + c)*sinh(d*x + c)^15 + a^4*s
inh(d*x + c)^16 - 4*(a^4 + 4*a^3*b)*cosh(d*x + c)^14 + 4*(30*a^4*cosh(d*x +
 c)^2 - a^4 - 4*a^3*b)*sinh(d*x + c)^14 + 56*(10*a^4*cosh(d*x + c)^3 - (a^4
 + 4*a^3*b)*cosh(d*x + c))*sinh(d*x + c)^13 - 2*(13*a^4 + 72*a^3*b + 88*a^2
*b^2 - 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*cosh(d*x + c)^12 + 2*(910*a^4*
```

cosh(d*x + c)^4 - 13*a^4 - 72*a^3*b - 88*a^2*b^2 + 12*(a^4 + 12*a^3*b + 16*
a^2*b^2)*d*x - 182*(a^4 + 4*a^3*b)*cosh(d*x + c)^2)*sinh(d*x + c)^12 + 8*(5
46*a^4*cosh(d*x + c)^5 - 182*(a^4 + 4*a^3*b)*cosh(d*x + c)^3 - 3*(13*a^4 +
72*a^3*b + 88*a^2*b^2 - 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*cosh(d*x + c)
)*sinh(d*x + c)^11 - 4*(9*a^4 + 24*a^3*b - 16*a^2*b^2 - 32*a*b^3 - 24*(a^4
+ 14*a^3*b + 40*a^2*b^2 + 32*a*b^3)*d*x)*cosh(d*x + c)^10 + 4*(2002*a^4*cos
h(d*x + c)^6 - 1001*(a^4 + 4*a^3*b)*cosh(d*x + c)^4 - 9*a^4 - 24*a^3*b + 16
*a^2*b^2 + 32*a*b^3 + 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a*b^3)*d*x - 33*
(13*a^4 + 72*a^3*b + 88*a^2*b^2 - 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*cos
h(d*x + c)^2)*sinh(d*x + c)^10 + 8*(1430*a^4*cosh(d*x + c)^7 - 1001*(a^4 +
4*a^3*b)*cosh(d*x + c)^5 - 55*(13*a^4 + 72*a^3*b + 88*a^2*b^2 - 12*(a^4 + 1
2*a^3*b + 16*a^2*b^2)*d*x)*cosh(d*x + c)^3 - 5*(9*a^4 + 24*a^3*b - 16*a^2*b
^2 - 32*a*b^3 - 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a*b^3)*d*x)*cosh(d*x +
 c))*sinh(d*x + c)^9 + 16*(27*a^3*b + 114*a^2*b^2 + 184*a*b^3 + 112*b^4 + 3
*(3*a^4 + 44*a^3*b + 152*a^2*b^2 + 224*a*b^3 + 128*b^4)*d*x)*cosh(d*x + c)^
8 + 2*(6435*a^4*cosh(d*x + c)^8 - 6006*(a^4 + 4*a^3*b)*cosh(d*x + c)^6 - 49
5*(13*a^4 + 72*a^3*b + 88*a^2*b^2 - 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*c
osh(d*x + c)^4 + 216*a^3*b + 912*a^2*b^2 + 1472*a*b^3 + 896*b^4 + 24*(3*a^4
 + 44*a^3*b + 152*a^2*b^2 + 224*a*b^3 + 128*b^4)*d*x - 90*(9*a^4 + 24*a^3*b
 - 16*a^2*b^2 - 32*a*b^3 - 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a*b^3)*d*x)
*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 16*(715*a^4*cosh(d*x + c)^9 - 858*(a^4
+ 4*a^3*b)*cosh(d*x + c)^7 - 99*(13*a^4 + 72*a^3*b + 88*a^2*b^2 - 12*(a^4 +
 12*a^3*b + 16*a^2*b^2)*d*x)*cosh(d*x + c)^5 - 30*(9*a^4 + 24*a^3*b - 16*a^
2*b^2 - 32*a*b^3 - 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a*b^3)*d*x)*cosh(d*
x + c)^3 + 8*(27*a^3*b + 114*a^2*b^2 + 184*a*b^3 + 112*b^4 + 3*(3*a^4 + 44*
a^3*b + 152*a^2*b^2 + 224*a*b^3 + 128*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c
)^7 + 4*(9*a^4 + 168*a^3*b + 496*a^2*b^2 + 416*a*b^3 + 24*(a^4 + 14*a^3*b +
 40*a^2*b^2 + 32*a*b^3)*d*x)*cosh(d*x + c)^6 + 4*(2002*a^4*cosh(d*x + c)^10
 - 3003*(a^4 + 4*a^3*b)*cosh(d*x + c)^8 - 462*(13*a^4 + 72*a^3*b + 88*a^2*b
^2 - 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*cosh(d*x + c)^6 - 210*(9*a^4 + 2
4*a^3*b - 16*a^2*b^2 - 32*a*b^3 - 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a*b^
3)*d*x)*cosh(d*x + c)^4 + 9*a^4 + 168*a^3*b + 496*a^2*b^2 + 416*a*b^3 + 24*
(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a*b^3)*d*x + 112*(27*a^3*b + 114*a^2*b^2
+ 184*a*b^3 + 112*b^4 + 3*(3*a^4 + 44*a^3*b + 152*a^2*b^2 + 224*a*b^3 + 128
*b^4)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(546*a^4*cosh(d*x + c)^11 -
 1001*(a^4 + 4*a^3*b)*cosh(d*x + c)^9 - 198*(13*a^4 + 72*a^3*b + 88*a^2*b^2
 - 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*cosh(d*x + c)^7 - 126*(9*a^4 + 24*
a^3*b - 16*a^2*b^2 - 32*a*b^3 - 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a*b^3)
*d*x)*cosh(d*x + c)^5 + 112*(27*a^3*b + 114*a^2*b^2 + 184*a*b^3 + 112*b^4 +
 3*(3*a^4 + 44*a^3*b + 152*a^2*b^2 + 224*a*b^3 + 128*b^4)*d*x)*cosh(d*x + c
)^3 + 3*(9*a^4 + 168*a^3*b + 496*a^2*b^2 + 416*a*b^3 + 24*(a^4 + 14*a^3*b +
 40*a^2*b^2 + 32*a*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(13*a^4 + 1
44*a^3*b + 200*a^2*b^2 + 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*cosh(d*x + c
)^4 + 2*(910*a^4*cosh(d*x + c)^12 - 2002*(a^4 + 4*a^3*b)*cosh(d*x + c)^10 -
 495*(13*a^4 + 72*a^3*b + 88*a^2*b^2 - 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x

)*cosh(d*x + c)^8 - 420*(9*a^4 + 24*a^3*b - 16*a^2*b^2 - 32*a*b^3 - 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a*b^3)*d*x)*cosh(d*x + c)^6 + 560*(27*a^3*b + 114*a^2*b^2 + 184*a*b^3 + 112*b^4 + 3*(3*a^4 + 44*a^3*b + 152*a^2*b^2 + 224*a*b^3 + 128*b^4)*d*x)*cosh(d*x + c)^4 + 13*a^4 + 144*a^3*b + 200*a^2*b^2 + 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x + 30*(9*a^4 + 168*a^3*b + 496*a^2*b^2 + 416*a*b^3 + 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 - a^4 + 8*(70*a^4*cosh(d*x + c)^13 - 182*(a^4 + 4*a^3*b)*cosh(d*x + c)^11 - 55*(13*a^4 + 72*a^3*b + 88*a^2*b^2 - 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*cosh(d*x + c)^9 - 60*(9*a^4 + 24*a^3*b - 16*a^2*b^2 - 32*a*b^3 - 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a*b^3)*d*x)*cosh(d*x + c)^7 + 112*(27*a^3*b + 114*a^2*b^2 + 184*a*b^3 + 112*b^4 + 3*(3*a^4 + 44*a^3*b + 152*a^2*b^2 + 224*a*b^3 + 128*b^4)*d*x)*cosh(d*x + c)^5 + 10*(9*a^4 + 168*a^3*b + 496*a^2*b^2 + 416*a*b^3 + 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a*b^3)*d*x)*cosh(d*x + c)^3 + (13*a^4 + 144*a^3*b + 200*a^2*b^2 + 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a^4 + 4*a^3*b)*cosh(d*x + c)^2 + 4*(30*a^4*cosh(d*x + c)^14 - 91*(a^4 + 4*a^3*b)*cosh(d*x + c)^12 - 33*(13*a^4 + 72*a^3*b + 88*a^2*b^2 - 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*cosh(d*x + c)^10 - 45*(9*a^4 + 24*a^3*b - 16*a^2*b^2 - 32*a*b^3 - 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a*b^3)*d*x)*cosh(d*x + c)^8 + 112*(27*a^3*b + 114*a^2*b^2 + 184*a*b^3 + 112*b^4 + 3*(3*a^4 + 44*a^3*b + 152*a^2*b^2 + 224*a*b^3 + 128*b^4)*d*x)*cosh(d*x + c)^6 + 15*(9*a^4 + 168*a^3*b + 496*a^2*b^2 + 416*a*b^3 + 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a*b^3)*d*x)*cosh(d*x + c)^4 + a^4 + 4*a^3*b + 3*(13*a^4 + 144*a^3*b + 200*a^2*b^2 + 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 12*((5*a^4 + 20*a^3*b + 16*a^2*b^2)*cosh(d*x + c)^12 + 12*(5*a^4 + 20*a^3*b + 16*a^2*b^2)*cosh(d*x + c)*sinh(d*x + c)^11 + (5*a^4 + 20*a^3*b + 16*a^2*b^2)*sinh(d*x + c)^12 + 4*(5*a^4 + 30*a^3*b + 56*a^2*b^2 + 32*a*b^3)*cosh(d*x + c)^10 + 2*(10*a^4 + 60*a^3*b + 112*a^2*b^2 + 64*a*b^3 + 33*(5*a^4 + 20*a^3*b + 16*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 20*(11*(5*a^4 + 20*a^3*b + 16*a^2*b^2)*cosh(d*x + c)^3 + 2*(5*a^4 + 30*a^3*b + 56*a^2*b^2 + 32*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^9 + 2*(15*a^4 + 100*a^3*b + 248*a^2*b^2 + 288*a*b^3 + 128*b^4)*cosh(d*x + c)^8 + (495*(5*a^4 + 20*a^3*b + 16*a^2*b^2)*cosh(d*x + c)^4 + 30*a^4 + 200*a^3*b + 496*a^2*b^2 + 576*a*b^3 + 256*b^4 + 180*(5*a^4 + 30*a^3*b + 56*a^2*b^2 + 32*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 8*(99*(5*a^4 + 20*a^3*b + 16*a^2*b^2)*cosh(d*x + c)^5 + 60*(5*a^4 + 30*a^3*b + 56*a^2*b^2 + 32*a*b^3)*cosh(d*x + c)^3 + 2*(15*a^4 + 100*a^3*b + 248*a^2*b^2 + 288*a*b^3 + 128*b^4)*cosh(d*x + c))*sinh(d*x + c)^7 + 4*(5*a^4 + 30*a^3*b + 56*a^2*b^2 + 32*a*b^3)*cosh(d*x + c)^6 + 4*(231*(5*a^4 + 20*a^3*b + 16*a^2*b^2)*cosh(d*x + c)^6 + 210*(5*a^4 + 30*a^3*b + 56*a^2*b^2 + 32*a*b^3)*cosh(d*x + c)^4 + 5*a^4 + 30*a^3*b + 56*a^2*b^2 + 32*a*b^3 + 14*(15*a^4 + 100*a^3*b + 248*a^2*b^2 + 288*a*b^3 + 128*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(99*(5*a^4 + 20*a^3*b + 16*a^2*b^2)*cosh(d*x + c)^7 + 126*(5*a^4 + 30*a^3*b + 56*a^2*b^2 + 32*a*b^3)*cosh(d*x + c)^5 + 14*(15*a^4 + 100*a^3*b + 248*a^2*b^2 + 288*a*b^3 + 128*b^4)*cosh(d*x + c)^3 + 3*(5*a^4 + 30*a^3*b + 56*a^2*b^2 + 32*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + (

5*a^4 + 20*a^3*b + 16*a^2*b^2)*cosh(d*x + c)^4 + (495*(5*a^4 + 20*a^3*b + 16*a^2*b^2)*cosh(d*x + c)^8 + 840*(5*a^4 + 30*a^3*b + 56*a^2*b^2 + 32*a*b^3)*cosh(d*x + c)^6 + 140*(15*a^4 + 100*a^3*b + 248*a^2*b^2 + 288*a*b^3 + 128*b^4)*cosh(d*x + c)^4 + 5*a^4 + 20*a^3*b + 16*a^2*b^2 + 60*(5*a^4 + 30*a^3*b + 56*a^2*b^2 + 32*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(55*(5*a^4 + 20*a^3*b + 16*a^2*b^2)*cosh(d*x + c)^9 + 120*(5*a^4 + 30*a^3*b + 56*a^2*b^2 + 32*a*b^3)*cosh(d*x + c)^7 + 28*(15*a^4 + 100*a^3*b + 248*a^2*b^2 + 288*a*b^3 + 128*b^4)*cosh(d*x + c)^5 + 20*(5*a^4 + 30*a^3*b + 56*a^2*b^2 + 32*a*b^3)*cosh(d*x + c)^3 + (5*a^4 + 20*a^3*b + 16*a^2*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(33*(5*a^4 + 20*a^3*b + 16*a^2*b^2)*cosh(d*x + c)^10 + 90*(5*a^4 + 30*a^3*b + 56*a^2*b^2 + 32*a*b^3)*cosh(d*x + c)^8 + 28*(15*a^4 + 100*a^3*b + 248*a^2*b^2 + 288*a*b^3 + 128*b^4)*cosh(d*x + c)^6 + 30*(5*a^4 + 30*a^3*b + 56*a^2*b^2 + 32*a*b^3)*cosh(d*x + c)^4 + 3*(5*a^4 + 20*a^3*b + 16*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*(3*(5*a^4 + 20*a^3*b + 16*a^2*b^2)*cosh(d*x + c)^11 + 10*(5*a^4 + 30*a^3*b + 56*a^2*b^2 + 32*a*b^3)*cosh(d*x + c)^9 + 4*(15*a^4 + 100*a^3*b + 248*a^2*b^2 + 288*a*b^3 + 128*b^4)*cosh(d*x + c)^7 + 6*(5*a^4 + 30*a^3*b + 56*a^2*b^2 + 32*a*b^3)*cosh(d*x + c)^5 + (5*a^4 + 20*a^3*b + 16*a^2*b^2)*cosh(d*x + c)^3)*sinh(d*x + c))*sqrt(b/(a + b))*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) + 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 + 3*a*b + 2*b^2)*sqrt(b/(a + b)))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) + 8*(2*a^4*cosh(d*x + c)^15 - 7*(a^4 + 4*a^3*b)*cosh(d*x + c)^13 - 3*(13*a^4 + 72*a^3*b + 88*a^2*b^2 - 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*cosh(d*x + c)^11 - 5*(9*a^4 + 24*a^3*b - 16*a^2*b^2 - 32*a*b^3 - 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a*b^3)*d*x)*cosh(d*x + c)^9 + 16*(27*a^3*b + 114*a^2*b^2 + 184*a*b^3 + 112*b^4 + 3*(3*a^4 + 44*a^3*b + 152*a^2*b^2 + 224*a*b^3 + 128*b^4)*d*x)*cosh(d*x + c)^7 + 3*(9*a^4 + 168*a^3*b + 496*a^2*b^2 + 416*a*b^3 + 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a*b^3)*d*x)*cosh(d*x + c)^5 + (13*a^4 + 144*a^3*b + 200*a^2*b^2 + 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*cosh(d*x + c)^3 + (a^4 + 4*a^3*b)*cosh(d*x + c))*sinh(d*x + c))/(a^7*d*cosh(d*x + c)^12 + 12*a^7*d*cosh(d*x + c)*sinh(d*x + c)^11 + a^7*d*sinh(d*x + c)^12 + a^7*d*cosh(d*x + c)^4 + 4*(a^7 + 2*a^6*b)*d*cosh(d*x + c)^10 + 2*(33*a^7*d*cosh(d*x + c)^2 + 2*(a^7 + 2*a^6*b)*d)*sinh(d*x + c)^10 + 2*(3*a^7 + 8*a^6*b + 8*a^5*b^2)*d*cosh(d*x + c)^8 + 20*(11*a^7*d*cosh(d*x + c)^3 + 2*(a^7 + 2*a^6*b)*d*cosh(d*x + c))*sinh(d*x + c)^9 + (495*a^7*d*cosh(d*x + c)^4 + 180*(a^7 + 2*a^6*b)*d*cosh(d*x + c)^2 + 2*(3*a^7 + 8*a^6*b + 8*a^5*b^2)*d)*sinh(d*x + c)^8 + 4*(a^7 + 2*a^6*b)*d*cosh(d*x + c)^6 + 8*(99*a^7*d*cosh(d*x + c)^5 + 60*(a^7 + 2*a^6*b)*d*cosh(d*x + c)^3 + 2*(3*a^7 + 8*a^6*b + 8*a^5*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^7 + 4*(231*a^7*d*cosh(d*x + c)^6 + 210*(a^7 + 2*a^6*b)*d*co

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sh(d*x + c)^4 + 14*(3*a^7 + 8*a^6*b + 8*a^5*b^2)*d*cosh(d*x + c)^2 + (a^7 +
 2*a^6*b)*d)*sinh(d*x + c)^6 + 8*(99*a^7*d*cosh(d*x + c)^7 + 126*(a^7 + 2*a
^6*b)*d*cosh(d*x + c)^5 + 14*(3*a^7 + 8*a^6*b + 8*a^5*b^2)*d*cosh(d*x + c)^
3 + 3*(a^7 + 2*a^6*b)*d*cosh(d*x + c))*sinh(d*x + c)^5 + (495*a^7*d*cosh(d*
x + c)^8 + a^7*d + 840*(a^7 + 2*a^6*b)*d*cosh(d*x + c)^6 + 140*(3*a^7 + 8*a
^6*b + 8*a^5*b^2)*d*cosh(d*x + c)^4 + 60*(a^7 + 2*a^6*b)*d*cosh(d*x + c)^2)
*sinh(d*x + c)^4 + 4*(55*a^7*d*cosh(d*x + c)^9 + a^7*d*cosh(d*x + c) + 120*
(a^7 + 2*a^6*b)*d*cosh(d*x + c)^7 + 28*(3*a^7 + 8*a^6*b + 8*a^5*b^2)*d*cosh
(d*x + c)^5 + 20*(a^7 + 2*a^6*b)*d*cosh(d*x + c)^3)*sinh(d*x + c)^3 + 2*(33
*a^7*d*cosh(d*x + c)^10 + 3*a^7*d*cosh(d*x + c)^2 + 90*(a^7 + 2*a^6*b)*d*co
sh(d*x + c)^8 + 28*(3*a^7 + 8*a^6*b + 8*a^5*b^2)*d*cosh(d*x + c)^6 + 30*(a^
7 + 2*a^6*b)*d*cosh(d*x + c)^4)*sinh(d*x + c)^2 + 4*(3*a^7*d*cosh(d*x + c)^
11 + a^7*d*cosh(d*x + c)^3 + 10*(a^7 + 2*a^6*b)*d*cosh(d*x + c)^9 + 4*(3*a^
7 + 8*a^6*b + 8*a^5*b^2)*d*cosh(d*x + c)^7 + 6*(a^7 + 2*a^6*b)*d*cosh(d*x +
 c)^5)*sinh(d*x + c)), 1/64*(a^4*cosh(d*x + c)^16 + 16*a^4*cosh(d*x + c)*si
nh(d*x + c)^15 + a^4*sinh(d*x + c)^16 - 4*(a^4 + 4*a^3*b)*cosh(d*x + c)^14
+ 4*(30*a^4*cosh(d*x + c)^2 - a^4 - 4*a^3*b)*sinh(d*x + c)^14 + 56*(10*a^4*
cosh(d*x + c)^3 - (a^4 + 4*a^3*b)*cosh(d*x + c))*sinh(d*x + c)^13 - 2*(13*a
^4 + 72*a^3*b + 88*a^2*b^2 - 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*cosh(d*x
 + c)^12 + 2*(910*a^4*cosh(d*x + c)^4 - 13*a^4 - 72*a^3*b - 88*a^2*b^2 + 12
*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x - 182*(a^4 + 4*a^3*b)*cosh(d*x + c)^2)*s
inh(d*x + c)^12 + 8*(546*a^4*cosh(d*x + c)^5 - 182*(a^4 + 4*a^3*b)*cosh(d*x
 + c)^3 - 3*(13*a^4 + 72*a^3*b + 88*a^2*b^2 - 12*(a^4 + 12*a^3*b + 16*a^2*b
^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^11 - 4*(9*a^4 + 24*a^3*b - 16*a^2*b^2
 - 32*a*b^3 - 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a*b^3)*d*x)*cosh(d*x + c
)^10 + 4*(2002*a^4*cosh(d*x + c)^6 - 1001*(a^4 + 4*a^3*b)*cosh(d*x + c)^4 -
 9*a^4 - 24*a^3*b + 16*a^2*b^2 + 32*a*b^3 + 24*(a^4 + 14*a^3*b + 40*a^2*b^2
 + 32*a*b^3)*d*x - 33*(13*a^4 + 72*a^3*b + 88*a^2*b^2 - 12*(a^4 + 12*a^3*b
+ 16*a^2*b^2)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 8*(1430*a^4*cosh(d*x
 + c)^7 - 1001*(a^4 + 4*a^3*b)*cosh(d*x + c)^5 - 55*(13*a^4 + 72*a^3*b + 88
*a^2*b^2 - 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*cosh(d*x + c)^3 - 5*(9*a^4
 + 24*a^3*b - 16*a^2*b^2 - 32*a*b^3 - 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*
a*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^9 + 16*(27*a^3*b + 114*a^2*b^2 + 1
84*a*b^3 + 112*b^4 + 3*(3*a^4 + 44*a^3*b + 152*a^2*b^2 + 224*a*b^3 + 128*b^
4)*d*x)*cosh(d*x + c)^8 + 2*(6435*a^4*cosh(d*x + c)^8 - 6006*(a^4 + 4*a^3*b
)*cosh(d*x + c)^6 - 495*(13*a^4 + 72*a^3*b + 88*a^2*b^2 - 12*(a^4 + 12*a^3*
b + 16*a^2*b^2)*d*x)*cosh(d*x + c)^4 + 216*a^3*b + 912*a^2*b^2 + 1472*a*b^3
 + 896*b^4 + 24*(3*a^4 + 44*a^3*b + 152*a^2*b^2 + 224*a*b^3 + 128*b^4)*d*x
- 90*(9*a^4 + 24*a^3*b - 16*a^2*b^2 - 32*a*b^3 - 24*(a^4 + 14*a^3*b + 40*a^
2*b^2 + 32*a*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 16*(715*a^4*cosh(
d*x + c)^9 - 858*(a^4 + 4*a^3*b)*cosh(d*x + c)^7 - 99*(13*a^4 + 72*a^3*b +
88*a^2*b^2 - 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*cosh(d*x + c)^5 - 30*(9*
a^4 + 24*a^3*b - 16*a^2*b^2 - 32*a*b^3 - 24*(a^4 + 14*a^3*b + 40*a^2*b^2 +
32*a*b^3)*d*x)*cosh(d*x + c)^3 + 8*(27*a^3*b + 114*a^2*b^2 + 184*a*b^3 + 11
2*b^4 + 3*(3*a^4 + 44*a^3*b + 152*a^2*b^2 + 224*a*b^3 + 128*b^4)*d*x)*cosh(
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d*x + c))*sinh(d*x + c)^7 + 4*(9*a^4 + 168*a^3*b + 496*a^2*b^2 + 416*a*b^3
+ 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a*b^3)*d*x)*cosh(d*x + c)^6 + 4*(200
2*a^4*cosh(d*x + c)^10 - 3003*(a^4 + 4*a^3*b)*cosh(d*x + c)^8 - 462*(13*a^4
 + 72*a^3*b + 88*a^2*b^2 - 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*cosh(d*x +
 c)^6 - 210*(9*a^4 + 24*a^3*b - 16*a^2*b^2 - 32*a*b^3 - 24*(a^4 + 14*a^3*b
+ 40*a^2*b^2 + 32*a*b^3)*d*x)*cosh(d*x + c)^4 + 9*a^4 + 168*a^3*b + 496*a^2
*b^2 + 416*a*b^3 + 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a*b^3)*d*x + 112*(2
7*a^3*b + 114*a^2*b^2 + 184*a*b^3 + 112*b^4 + 3*(3*a^4 + 44*a^3*b + 152*a^2
*b^2 + 224*a*b^3 + 128*b^4)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(546*
a^4*cosh(d*x + c)^11 - 1001*(a^4 + 4*a^3*b)*cosh(d*x + c)^9 - 198*(13*a^4 +
 72*a^3*b + 88*a^2*b^2 - 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*cosh(d*x + c
)^7 - 126*(9*a^4 + 24*a^3*b - 16*a^2*b^2 - 32*a*b^3 - 24*(a^4 + 14*a^3*b +
40*a^2*b^2 + 32*a*b^3)*d*x)*cosh(d*x + c)^5 + 112*(27*a^3*b + 114*a^2*b^2 +
 184*a*b^3 + 112*b^4 + 3*(3*a^4 + 44*a^3*b + 152*a^2*b^2 + 224*a*b^3 + 128*
b^4)*d*x)*cosh(d*x + c)^3 + 3*(9*a^4 + 168*a^3*b + 496*a^2*b^2 + 416*a*b^3
+ 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a*b^3)*d*x)*cosh(d*x + c))*sinh(d*x
+ c)^5 + 2*(13*a^4 + 144*a^3*b + 200*a^2*b^2 + 12*(a^4 + 12*a^3*b + 16*a^2*
b^2)*d*x)*cosh(d*x + c)^4 + 2*(910*a^4*cosh(d*x + c)^12 - 2002*(a^4 + 4*a^3
*b)*cosh(d*x + c)^10 - 495*(13*a^4 + 72*a^3*b + 88*a^2*b^2 - 12*(a^4 + 12*a
^3*b + 16*a^2*b^2)*d*x)*cosh(d*x + c)^8 - 420*(9*a^4 + 24*a^3*b - 16*a^2*b^
2 - 32*a*b^3 - 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a*b^3)*d*x)*cosh(d*x +
c)^6 + 560*(27*a^3*b + 114*a^2*b^2 + 184*a*b^3 + 112*b^4 + 3*(3*a^4 + 44*a^
3*b + 152*a^2*b^2 + 224*a*b^3 + 128*b^4)*d*x)*cosh(d*x + c)^4 + 13*a^4 + 14
4*a^3*b + 200*a^2*b^2 + 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x + 30*(9*a^4 +
168*a^3*b + 496*a^2*b^2 + 416*a*b^3 + 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*
a*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 - a^4 + 8*(70*a^4*cosh(d*x + c
)^13 - 182*(a^4 + 4*a^3*b)*cosh(d*x + c)^11 - 55*(13*a^4 + 72*a^3*b + 88*a^
2*b^2 - 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*cosh(d*x + c)^9 - 60*(9*a^4 +
 24*a^3*b - 16*a^2*b^2 - 32*a*b^3 - 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a*
b^3)*d*x)*cosh(d*x + c)^7 + 112*(27*a^3*b + 114*a^2*b^2 + 184*a*b^3 + 112*b
^4 + 3*(3*a^4 + 44*a^3*b + 152*a^2*b^2 + 224*a*b^3 + 128*b^4)*d*x)*cosh(d*x
 + c)^5 + 10*(9*a^4 + 168*a^3*b + 496*a^2*b^2 + 416*a*b^3 + 24*(a^4 + 14*a^
3*b + 40*a^2*b^2 + 32*a*b^3)*d*x)*cosh(d*x + c)^3 + (13*a^4 + 144*a^3*b + 2
00*a^2*b^2 + 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*cosh(d*x + c))*sinh(d*x
+ c)^3 + 4*(a^4 + 4*a^3*b)*cosh(d*x + c)^2 + 4*(30*a^4*cosh(d*x + c)^14 - 9
1*(a^4 + 4*a^3*b)*cosh(d*x + c)^12 - 33*(13*a^4 + 72*a^3*b + 88*a^2*b^2 - 1
2*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*cosh(d*x + c)^10 - 45*(9*a^4 + 24*a^3*
b - 16*a^2*b^2 - 32*a*b^3 - 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a*b^3)*d*x
)*cosh(d*x + c)^8 + 112*(27*a^3*b + 114*a^2*b^2 + 184*a*b^3 + 112*b^4 + 3*(
3*a^4 + 44*a^3*b + 152*a^2*b^2 + 224*a*b^3 + 128*b^4)*d*x)*cosh(d*x + c)^6
+ 15*(9*a^4 + 168*a^3*b + 496*a^2*b^2 + 416*a*b^3 + 24*(a^4 + 14*a^3*b + 40
*a^2*b^2 + 32*a*b^3)*d*x)*cosh(d*x + c)^4 + a^4 + 4*a^3*b + 3*(13*a^4 + 144
*a^3*b + 200*a^2*b^2 + 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*cosh(d*x + c)^
2)*sinh(d*x + c)^2 - 24*((5*a^4 + 20*a^3*b + 16*a^2*b^2)*cosh(d*x + c)^12 +
 12*(5*a^4 + 20*a^3*b + 16*a^2*b^2)*cosh(d*x + c)*sinh(d*x + c)^11 + (5*a^4

+ 20*a^3*b + 16*a^2*b^2)*sinh(d*x + c)^12 + 4*(5*a^4 + 30*a^3*b + 56*a^2*b
^2 + 32*a*b^3)*cosh(d*x + c)^10 + 2*(10*a^4 + 60*a^3*b + 112*a^2*b^2 + 64*a
b^3 + 33(5*a^4 + 20*a^3*b + 16*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^10
+ 20*(11*(5*a^4 + 20*a^3*b + 16*a^2*b^2)*cosh(d*x + c)^3 + 2*(5*a^4 + 30*a
^3*b + 56*a^2*b^2 + 32*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^9 + 2*(15*a^4 +
100*a^3*b + 248*a^2*b^2 + 288*a*b^3 + 128*b^4)*cosh(d*x + c)^8 + (495*(5*a^
4 + 20*a^3*b + 16*a^2*b^2)*cosh(d*x + c)^4 + 30*a^4 + 200*a^3*b + 496*a^2*b
^2 + 576*a*b^3 + 256*b^4 + 180*(5*a^4 + 30*a^3*b + 56*a^2*b^2 + 32*a*b^3)*c
osh(d*x + c)^2)*sinh(d*x + c)^8 + 8*(99*(5*a^4 + 20*a^3*b + 16*a^2*b^2)*cos
h(d*x + c)^5 + 60*(5*a^4 + 30*a^3*b + 56*a^2*b^2 + 32*a*b^3)*cosh(d*x + c)^
3 + 2*(15*a^4 + 100*a^3*b + 248*a^2*b^2 + 288*a*b^3 + 128*b^4)*cosh(d*x + c
))*sinh(d*x + c)^7 + 4*(5*a^4 + 30*a^3*b + 56*a^2*b^2 + 32*a*b^3)*cosh(d*x
+ c)^6 + 4*(231*(5*a^4 + 20*a^3*b + 16*a^2*b^2)*cosh(d*x + c)^6 + 210*(5*a^
4 + 30*a^3*b + 56*a^2*b^2 + 32*a*b^3)*cosh(d*x + c)^4 + 5*a^4 + 30*a^3*b +
56*a^2*b^2 + 32*a*b^3 + 14*(15*a^4 + 100*a^3*b + 248*a^2*b^2 + 288*a*b^3 +
128*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(99*(5*a^4 + 20*a^3*b + 16*a^
2*b^2)*cosh(d*x + c)^7 + 126*(5*a^4 + 30*a^3*b + 56*a^2*b^2 + 32*a*b^3)*cos
h(d*x + c)^5 + 14*(15*a^4 + 100*a^3*b + 248*a^2*b^2 + 288*a*b^3 + 128*b^4)*
cosh(d*x + c)^3 + 3*(5*a^4 + 30*a^3*b + 56*a^2*b^2 + 32*a*b^3)*cosh(d*x + c
))*sinh(d*x + c)^5 + (5*a^4 + 20*a^3*b + 16*a^2*b^2)*cosh(d*x + c)^4 + (495
*(5*a^4 + 20*a^3*b + 16*a^2*b^2)*cosh(d*x + c)^8 + 840*(5*a^4 + 30*a^3*b +
56*a^2*b^2 + 32*a*b^3)*cosh(d*x + c)^6 + 140*(15*a^4 + 100*a^3*b + 248*a^2*
b^2 + 288*a*b^3 + 128*b^4)*cosh(d*x + c)^4 + 5*a^4 + 20*a^3*b + 16*a^2*b^2
+ 60*(5*a^4 + 30*a^3*b + 56*a^2*b^2 + 32*a*b^3)*cosh(d*x + c)^2)*sinh(d*x +
c)^4 + 4*(55*(5*a^4 + 20*a^3*b + 16*a^2*b^2)*cosh(d*x + c)^9 + 120*(5*a^4
+ 30*a^3*b + 56*a^2*b^2 + 32*a*b^3)*cosh(d*x + c)^7 + 28*(15*a^4 + 100*a^3*
b + 248*a^2*b^2 + 288*a*b^3 + 128*b^4)*cosh(d*x + c)^5 + 20*(5*a^4 + 30*a^3
*b + 56*a^2*b^2 + 32*a*b^3)*cosh(d*x + c)^3 + (5*a^4 + 20*a^3*b + 16*a^2*b^
2)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(33*(5*a^4 + 20*a^3*b + 16*a^2*b^2)*c
osh(d*x + c)^10 + 90*(5*a^4 + 30*a^3*b + 56*a^2*b^2 + 32*a*b^3)*cosh(d*x +
c)^8 + 28*(15*a^4 + 100*a^3*b + 248*a^2*b^2 + 288*a*b^3 + 128*b^4)*cosh(d*x
+ c)^6 + 30*(5*a^4 + 30*a^3*b + 56*a^2*b^2 + 32*a*b^3)*cosh(d*x + c)^4 + 3
*(5*a^4 + 20*a^3*b + 16*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*(3*(5
*a^4 + 20*a^3*b + 16*a^2*b^2)*cosh(d*x + c)^11 + 10*(5*a^4 + 30*a^3*b + 56*
a^2*b^2 + 32*a*b^3)*cosh(d*x + c)^9 + 4*(15*a^4 + 100*a^3*b + 248*a^2*b^2 +
288*a*b^3 + 128*b^4)*cosh(d*x + c)^7 + 6*(5*a^4 + 30*a^3*b + 56*a^2*b^2 +
32*a*b^3)*cosh(d*x + c)^5 + (5*a^4 + 20*a^3*b + 16*a^2*b^2)*cosh(d*x + c)^3
)*sinh(d*x + c))*sqrt(-b/(a + b))*arctan(1/2*(a*cosh(d*x + c)^2 + 2*a*cosh(
d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-b/(a + b))/b) +
8*(2*a^4*cosh(d*x + c)^15 - 7*(a^4 + 4*a^3*b)*cosh(d*x + c)^13 - 3*(13*a^4
+ 72*a^3*b + 88*a^2*b^2 - 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*cosh(d*x +
c)^11 - 5*(9*a^4 + 24*a^3*b - 16*a^2*b^2 - 32*a*b^3 - 24*(a^4 + 14*a^3*b +
40*a^2*b^2 + 32*a*b^3)*d*x)*cosh(d*x + c)^9 + 16*(27*a^3*b + 114*a^2*b^2 +
184*a*b^3 + 112*b^4 + 3*(3*a^4 + 44*a^3*b + 152*a^2*b^2 + 224*a*b^3 + 128*
b^4)*d*x)*cosh(d*x + c)^7 + 3*(9*a^4 + 168*a^3*b + 496*a^2*b^2 + 416*a*b^3

```
+ 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a*b^3)*d*x)*cosh(d*x + c)^5 + (13*a^
4 + 144*a^3*b + 200*a^2*b^2 + 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*cosh(d*
x + c)^3 + (a^4 + 4*a^3*b)*cosh(d*x + c))*sinh(d*x + c))/(a^7*d*cosh(d*x +
c)^12 + 12*a^7*d*cosh(d*x + c)*sinh(d*x + c)^11 + a^7*d*sinh(d*x + c)^12 +
a^7*d*cosh(d*x + c)^4 + 4*(a^7 + 2*a^6*b)*d*cosh(d*x + c)^10 + 2*(33*a^7*d*
cosh(d*x + c)^2 + 2*(a^7 + 2*a^6*b)*d)*sinh(d*x + c)^10 + 2*(3*a^7 + 8*a^6*
b + 8*a^5*b^2)*d*cosh(d*x + c)^8 + 20*(11*a^7*d*cosh(d*x + c)^3 + 2*(a^7 +
2*a^6*b)*d*cosh(d*x + c))*sinh(d*x + c)^9 + (495*a^7*d*cosh(d*x + c)^4 + 18
0*(a^7 + 2*a^6*b)*d*cosh(d*x + c)^2 + 2*(3*a^7 + 8*a^6*b + 8*a^5*b^2)*d)*si
nh(d*x + c)^8 + 4*(a^7 + 2*a^6*b)*d*cosh(d*x + c)^6 + 8*(99*a^7*d*cosh(d*x
+ c)^5 + 60*(a^7 + 2*a^6*b)*d*cosh(d*x + c)^3 + 2*(3*a^7 + 8*a^6*b + 8*a^5*
b^2)*d*cosh(d*x + c))*sinh(d*x + c)^7 + 4*(231*a^7*d*cosh(d*x + c)^6 + 210*
(a^7 + 2*a^6*b)*d*cosh(d*x + c)^4 + 14*(3*a^7 + 8*a^6*b + 8*a^5*b^2)*d*cosh
(d*x + c)^2 + (a^7 + 2*a^6*b)*d)*sinh(d*x + c)^6 + 8*(99*a^7*d*cosh(d*x + c
)^7 + 126*(a^7 + 2*a^6*b)*d*cosh(d*x + c)^5 + 14*(3*a^7 + 8*a^6*b + 8*a^5*b
^2)*d*cosh(d*x + c)^3 + 3*(a^7 + 2*a^6*b)*d*cosh(d*x + c))*sinh(d*x + c)^5
+ (495*a^7*d*cosh(d*x + c)^8 + a^7*d + 840*(a^7 + 2*a^6*b)*d*cosh(d*x + c)^
6 + 140*(3*a^7 + 8*a^6*b + 8*a^5*b^2)*d*cosh(d*x + c)^4 + 60*(a^7 + 2*a^6*b
)*d*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(55*a^7*d*cosh(d*x + c)^9 + a^7*d*
cosh(d*x + c) + 120*(a^7 + 2*a^6*b)*d*cosh(d*x + c)^7 + 28*(3*a^7 + 8*a^6*b
 + 8*a^5*b^2)*d*cosh(d*x + c)^5 + 20*(a^7 + 2*a^6*b)*d*cosh(d*x + c)^3)*sin
h(d*x + c)^3 + 2*(33*a^7*d*cosh(d*x + c)^10 + 3*a^7*d*cosh(d*x + c)^2 + 90*
(a^7 + 2*a^6*b)*d*cosh(d*x + c)^8 + 28*(3*a^7 + 8*a^6*b + 8*a^5*b^2)*d*cosh
(d*x + c)^6 + 30*(a^7 + 2*a^6*b)*d*cosh(d*x + c)^4)*sinh(d*x + c)^2 + 4*(3*
a^7*d*cosh(d*x + c)^11 + a^7*d*cosh(d*x + c)^3 + 10*(a^7 + 2*a^6*b)*d*cosh(
d*x + c)^9 + 4*(3*a^7 + 8*a^6*b + 8*a^5*b^2)*d*cosh(d*x + c)^7 + 6*(a^7 + 2
*a^6*b)*d*cosh(d*x + c)^5)*sinh(d*x + c))]
```

giac [B] time = 6.13, size = 518, normalized size = 2.14

$$\frac{\frac{24\left(a^2+12ab+16b^2\right)(dx+c)}{a^5} - \frac{24\left(5a^2b+20ab^2+16b^3\right)\arctan\left(\frac{ae^{(2dx+2c)}+a+2b}{2\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}\,a^5} + \frac{a^3e^{(4dx+4c)}-8a^3e^{(2dx+2c)}-24a^2be^{(2dx+2c)}}{a^6} - \frac{6a^4e^{(12dx+12c)}}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/64*(24*(a^2 + 12*a*b + 16*b^2)*(d*x + c)/a^5 - 24*(5*a^2*b + 20*a*b^2 + 1
6*b^3)*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/(sqrt(-a*
b - b^2)*a^5) + (a^3*e^(4*d*x + 4*c) - 8*a^3*e^(2*d*x + 2*c) - 24*a^2*b*e^(
2*d*x + 2*c))/a^6 - (6*a^4*e^(12*d*x + 12*c) + 72*a^3*b*e^(12*d*x + 12*c) +
 96*a^2*b^2*e^(12*d*x + 12*c) + 16*a^4*e^(10*d*x + 10*c) + 168*a^3*b*e^(10*
d*x + 10*c) + 384*a^2*b^2*e^(10*d*x + 10*c) + 256*a*b^3*e^(10*d*x + 10*c) +
 5*a^4*e^(8*d*x + 8*c) - 64*a^3*b*e^(8*d*x + 8*c) - 192*a^2*b^2*e^(8*d*x +
```

```
8*c) - 256*a*b^3*e^(8*d*x + 8*c) - 256*b^4*e^(8*d*x + 8*c) - 20*a^4*e^(6*d*
x + 6*c) - 360*a^3*b*e^(6*d*x + 6*c) - 1024*a^2*b^2*e^(6*d*x + 6*c) - 896*a
*b^3*e^(6*d*x + 6*c) - 20*a^4*e^(4*d*x + 4*c) - 216*a^3*b*e^(4*d*x + 4*c) -
 304*a^2*b^2*e^(4*d*x + 4*c) - 4*a^4*e^(2*d*x + 2*c) - 16*a^3*b*e^(2*d*x +
2*c) + a^4)/((a*e^(6*d*x + 6*c) + 2*a*e^(4*d*x + 4*c) + 4*b*e^(4*d*x + 4*c)
 + a*e^(2*d*x + 2*c))^2*a^5))/d
```

maple [B] time = 0.43, size = 1668, normalized size = 6.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] 3/8/d/a^3*ln(tanh(1/2*d*x+1/2*c)+1)-3/8/d/a^3*ln(tanh(1/2*d*x+1/2*c)-1)+1/8
/d/a^3/(tanh(1/2*d*x+1/2*c)+1)^2-3/8/d/a^3/(tanh(1/2*d*x+1/2*c)+1)+1/4/d/a^
3/(tanh(1/2*d*x+1/2*c)-1)^4+1/2/d/a^3/(tanh(1/2*d*x+1/2*c)-1)^3-1/8/d/a^3/(
tanh(1/2*d*x+1/2*c)-1)^2-3/8/d/a^3/(tanh(1/2*d*x+1/2*c)-1)-1/4/d/a^3/(tanh(
1/2*d*x+1/2*c)+1)^4+1/2/d/a^3/(tanh(1/2*d*x+1/2*c)+1)^3-3/2/d/a^4/(tanh(1/2
*d*x+1/2*c)-1)*b-9/2/d/a^4*ln(tanh(1/2*d*x+1/2*c)-1)*b-6/d/a^5*ln(tanh(1/2*
d*x+1/2*c)-1)*b^2+3/2/d/a^4/(tanh(1/2*d*x+1/2*c)+1)^2*b-3/2/d/a^4/(tanh(1/2
*d*x+1/2*c)+1)*b+9/2/d/a^4*ln(tanh(1/2*d*x+1/2*c)+1)*b+6/d/a^5*ln(tanh(1/2*
d*x+1/2*c)+1)*b^2-15/16/d/a^3*b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d
*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))-35/4/d/a^3*b^2/(tanh
(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tan
h(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^5+3/d/a^4*b^3/(tanh(1/2*d*x
+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*
x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^3-21/4/d/a^3*b^2/(tanh(1/2*d*x+1/2*
c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2
*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)-35/4/d/a^3*b^2/(tanh(1/2*d*x+1/2*c)^4*a+
b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b
+a+b)^2*tanh(1/2*d*x+1/2*c)^3-3/2/d/a^4/(tanh(1/2*d*x+1/2*c)-1)^2*b+15/4/d/
a^4*b^(3/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*b^(1/2)*tanh
(1/2*d*x+1/2*c)+(a+b)^(1/2))-27/4/d/a^2*b/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1
/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*
tanh(1/2*d*x+1/2*c)^5+15/16/d/a^3*b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1
/2*d*x+1/2*c)^2-2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))-9/4/d/a^2*b/(tan
h(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*ta
nh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^7+3/d/a^4*b^3/(tanh(1/2*d*
x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d
*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^5-27/4/d/a^2*b/(tanh(1/2*d*x+1/2*c
)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*
c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^3-3/d/a^4*b^3/(tanh(1/2*d*x+1/2*c)^4*a+b*
tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a
+b)^2*tanh(1/2*d*x+1/2*c)^7-21/4/d/a^3*b^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(
```

```
1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2
*tanh(1/2*d*x+1/2*c)^7-15/4/d/a^4*b^(3/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1
/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))-3/d/a^5*b^(5/2)/
(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2
*c)+(a+b)^(1/2))-9/4/d/a^2*b/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)
^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+
1/2*c)-3/d/a^4*b^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(
1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)+3/d
/a^5*b^(5/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*b^(1/2)*tan
h(1/2*d*x+1/2*c)+(a+b)^(1/2))
```

maxima [B] time = 0.60, size = 2468, normalized size = 10.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -3/256*(5*a^4*b + 100*a^3*b^2 + 320*a^2*b^3 + 352*a*b^4 + 128*b^5)*log((a*e
^(2*d*x + 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(2*d*x + 2*c) + a + 2*b
+ 2*sqrt((a + b)*b)))/((a^7 + 2*a^6*b + a^5*b^2)*sqrt((a + b)*b)*d) - 3/64*
(5*a^3*b + 30*a^2*b^2 + 40*a*b^3 + 16*b^4)*log((a*e^(2*d*x + 2*c) + a + 2*b
 - 2*sqrt((a + b)*b))/(a*e^(2*d*x + 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((
a^6 + 2*a^5*b + a^4*b^2)*sqrt((a + b)*b)*d) + 3/256*(5*a^4*b + 100*a^3*b^2
+ 320*a^2*b^3 + 352*a*b^4 + 128*b^5)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*
sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^7
+ 2*a^6*b + a^5*b^2)*sqrt((a + b)*b)*d) + 3/64*(5*a^3*b + 30*a^2*b^2 + 40*a
*b^3 + 16*b^4)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^
(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^6 + 2*a^5*b + a^4*b^2)*s
qrt((a + b)*b)*d) + 3/128*(15*a^2*b + 20*a*b^2 + 8*b^3)*log((a*e^(-2*d*x -
2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt(
(a + b)*b)))/((a^5 + 2*a^4*b + a^3*b^2)*sqrt((a + b)*b)*d) + 1/64*(9*a^5*b
+ 110*a^4*b^2 + 216*a^3*b^3 + 112*a^2*b^4 + (9*a^5*b + 228*a^4*b^2 + 920*a^
3*b^3 + 1216*a^2*b^4 + 512*a*b^5)*e^(6*d*x + 6*c) + (27*a^5*b + 594*a^4*b^2
 + 2816*a^3*b^3 + 5696*a^2*b^4 + 5248*a*b^5 + 1792*b^6)*e^(4*d*x + 4*c) + (
27*a^5*b + 476*a^4*b^2 + 1720*a^3*b^3 + 2176*a^2*b^4 + 896*a*b^5)*e^(2*d*x
+ 2*c))/((a^9 + 2*a^8*b + a^7*b^2 + (a^9 + 2*a^8*b + a^7*b^2)*e^(8*d*x + 8*
c) + 4*(a^9 + 4*a^8*b + 5*a^7*b^2 + 2*a^6*b^3)*e^(6*d*x + 6*c) + 2*(3*a^9 +
 14*a^8*b + 27*a^7*b^2 + 24*a^6*b^3 + 8*a^5*b^4)*e^(4*d*x + 4*c) + 4*(a^9 +
 4*a^8*b + 5*a^7*b^2 + 2*a^6*b^3)*e^(2*d*x + 2*c))*d) - 1/64*(9*a^5*b + 110
*a^4*b^2 + 216*a^3*b^3 + 112*a^2*b^4 + (27*a^5*b + 476*a^4*b^2 + 1720*a^3*b
^3 + 2176*a^2*b^4 + 896*a*b^5)*e^(-2*d*x - 2*c) + (27*a^5*b + 594*a^4*b^2 +
 2816*a^3*b^3 + 5696*a^2*b^4 + 5248*a*b^5 + 1792*b^6)*e^(-4*d*x - 4*c) + (9
*a^5*b + 228*a^4*b^2 + 920*a^3*b^3 + 1216*a^2*b^4 + 512*a*b^5)*e^(-6*d*x -
6*c))/((a^9 + 2*a^8*b + a^7*b^2 + 4*(a^9 + 4*a^8*b + 5*a^7*b^2 + 2*a^6*b^3)
```

```
*e^(-2*d*x - 2*c) + 2*(3*a^9 + 14*a^8*b + 27*a^7*b^2 + 24*a^6*b^3 + 8*a^5*b
^4)*e^(-4*d*x - 4*c) + 4*(a^9 + 4*a^8*b + 5*a^7*b^2 + 2*a^6*b^3)*e^(-6*d*x
- 6*c) + (a^9 + 2*a^8*b + a^7*b^2)*e^(-8*d*x - 8*c))*d) + 1/16*(9*a^4*b + 3
2*a^3*b^2 + 20*a^2*b^3 + 3*(3*a^4*b + 34*a^3*b^2 + 64*a^2*b^3 + 32*a*b^4)*e
^(6*d*x + 6*c) + (27*a^4*b + 264*a^3*b^2 + 740*a^2*b^3 + 832*a*b^4 + 320*b^
5)*e^(4*d*x + 4*c) + (27*a^4*b + 194*a^3*b^2 + 336*a^2*b^3 + 160*a*b^4)*e^(
2*d*x + 2*c))/((a^8 + 2*a^7*b + a^6*b^2 + (a^8 + 2*a^7*b + a^6*b^2)*e^(8*d*
x + 8*c) + 4*(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*b^3)*e^(6*d*x + 6*c) + 2*(3
*a^8 + 14*a^7*b + 27*a^6*b^2 + 24*a^5*b^3 + 8*a^4*b^4)*e^(4*d*x + 4*c) + 4*
(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*b^3)*e^(2*d*x + 2*c))*d) - 1/16*(9*a^4*b
 + 32*a^3*b^2 + 20*a^2*b^3 + (27*a^4*b + 194*a^3*b^2 + 336*a^2*b^3 + 160*a*
b^4)*e^(-2*d*x - 2*c) + (27*a^4*b + 264*a^3*b^2 + 740*a^2*b^3 + 832*a*b^4 +
 320*b^5)*e^(-4*d*x - 4*c) + 3*(3*a^4*b + 34*a^3*b^2 + 64*a^2*b^3 + 32*a*b^
4)*e^(-6*d*x - 6*c))/((a^8 + 2*a^7*b + a^6*b^2 + 4*(a^8 + 4*a^7*b + 5*a^6*b
^2 + 2*a^5*b^3)*e^(-2*d*x - 2*c) + 2*(3*a^8 + 14*a^7*b + 27*a^6*b^2 + 24*a^
5*b^3 + 8*a^4*b^4)*e^(-4*d*x - 4*c) + 4*(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*
b^3)*e^(-6*d*x - 6*c) + (a^8 + 2*a^7*b + a^6*b^2)*e^(-8*d*x - 8*c))*d) - 3/
32*(9*a^3*b + 6*a^2*b^2 + (27*a^3*b + 68*a^2*b^2 + 32*a*b^3)*e^(-2*d*x - 2*
c) + 3*(9*a^3*b + 30*a^2*b^2 + 40*a*b^3 + 16*b^4)*e^(-4*d*x - 4*c) + (9*a^3
*b + 28*a^2*b^2 + 16*a*b^3)*e^(-6*d*x - 6*c))/((a^7 + 2*a^6*b + a^5*b^2 + 4
*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*e^(-2*d*x - 2*c) + 2*(3*a^7 + 14*a
^6*b + 27*a^5*b^2 + 24*a^4*b^3 + 8*a^3*b^4)*e^(-4*d*x - 4*c) + 4*(a^7 + 4*a
^6*b + 5*a^5*b^2 + 2*a^4*b^3)*e^(-6*d*x - 6*c) + (a^7 + 2*a^6*b + a^5*b^2)*
e^(-8*d*x - 8*c))*d) + 3/8*(d*x + c)/(a^3*d) - 1/8*e^(2*d*x + 2*c)/(a^3*d)
+ 1/8*e^(-2*d*x - 2*c)/(a^3*d) + 3/4*b*log(a*e^(4*d*x + 4*c) + 2*(a + 2*b)*
e^(2*d*x + 2*c) + a)/(a^4*d) - 3/4*b*log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e
^(-4*d*x - 4*c) + a)/(a^4*d) + 1/64*(a*e^(4*d*x + 4*c) - 24*b*e^(2*d*x + 2*
c))/(a^4*d) + 1/64*(24*b*e^(-2*d*x - 2*c) - a*e^(-4*d*x - 4*c))/(a^4*d) + 3
/8*(a*b + 4*b^2)*log(a*e^(4*d*x + 4*c) + 2*(a + 2*b)*e^(2*d*x + 2*c) + a)/(
a^5*d) - 3/8*(a*b + 4*b^2)*log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x -
 4*c) + a)/(a^5*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c+dx)^6 \sinh(c+dx)^4}{\left(a\cosh(c+dx)^2+b\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^4/(a + b/cosh(c + d*x)^2)^3,x)
```

```
[Out] int((cosh(c + d*x)^6*sinh(c + d*x)^4)/(b + a*cosh(c + d*x)^2)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**4/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

3.42 $\int \frac{\sinh^3(c+dx)}{\left(a+b\,\mathbf{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=154

$$\frac{5\sqrt{b}\,(3a+7b)\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{8a^{9/2}d}+\frac{b^2(a+b)\cosh(c+dx)}{4a^4d\left(a\cosh^2(c+dx)+b\right)^2}-\frac{b(9a+13b)\cosh(c+dx)}{8a^4d\left(a\cosh^2(c+dx)+b\right)}-\frac{(a+3b)\cosh(c+}{a^4d}$$

```
[Out] -(a+3*b)*cosh(d*x+c)/a^4/d+1/3*cosh(d*x+c)^3/a^3/d+1/4*b^2*(a+b)*cosh(d*x+c
)/a^4/d/(b+a*cosh(d*x+c)^2)^2-1/8*b*(9*a+13*b)*cosh(d*x+c)/a^4/d/(b+a*cosh(
d*x+c)^2)+5/8*(3*a+7*b)*arctan(cosh(d*x+c)*a^(1/2)/b^(1/2))*b^(1/2)/a^(9/2)
/d
```

Rubi [A] time = 0.22, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4133, 455, 1814, 1153, 205}

$$\frac{b^2(a+b)\cosh(c+dx)}{4a^4d\left(a\cosh^2(c+dx)+b\right)^2}-\frac{b(9a+13b)\cosh(c+dx)}{8a^4d\left(a\cosh^2(c+dx)+b\right)}-\frac{(a+3b)\cosh(c+dx)}{a^4d}+\frac{5\sqrt{b}\,(3a+7b)\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+}{\sqrt{b}}\right.}{8a^{9/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (5*Sqrt[b]*(3*a + 7*b)*ArcTan[(Sqrt[a]*Cosh[c + d*x])/Sqrt[b]])/(8*a^(9/2)*
d) - ((a + 3*b)*Cosh[c + d*x])/(a^4*d) + Cosh[c + d*x]^3/(3*a^3*d) + (b^2*(
a + b)*Cosh[c + d*x])/(4*a^4*d*(b + a*Cosh[c + d*x]^2)^2) - (b*(9*a + 13*b)
*Cosh[c + d*x])/(8*a^4*d*(b + a*Cosh[c + d*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
```

```
(IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{x^6(1-x^2)}{(b+ax^2)^3}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= \frac{b^2(a+b)\cosh(c+dx)}{4a^4d\left(b+a\cosh^2(c+dx)\right)^2} + \frac{\text{Subst}\left(\int \frac{-b^2(a+b)+4ab(a+b)x^2-4a^2(a+b)x^4+4a^3x^6}{(b+ax^2)^2}\,dx, x, \text{co}\right.}{4a^4d}$$

$$= \frac{b^2(a+b)\cosh(c+dx)}{4a^4d\left(b+a\cosh^2(c+dx)\right)^2} - \frac{b(9a+13b)\cosh(c+dx)}{8a^4d\left(b+a\cosh^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{-b^2(7a+11b)+}{}\right.}{}$$

$$= \frac{b^2(a+b)\cosh(c+dx)}{4a^4d\left(b+a\cosh^2(c+dx)\right)^2} - \frac{b(9a+13b)\cosh(c+dx)}{8a^4d\left(b+a\cosh^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \left(8b(a+3b)\right.\right.}{}$$

$$= -\frac{(a+3b)\cosh(c+dx)}{a^4d} + \frac{\cosh^3(c+dx)}{3a^3d} + \frac{b^2(a+b)\cosh(c+dx)}{4a^4d\left(b+a\cosh^2(c+dx)\right)^2} - \frac{b(9a+}{8a^4d\,(b}$$

$$= \frac{5\sqrt{b}\,(3a+7b)\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{8a^{9/2}d} - \frac{(a+3b)\cosh(c+dx)}{a^4d} + \frac{\cosh^3(c+dx)}{3a^3d} + \frac{}{4a}$$

Mathematica [C] time = 10.38, size = 1217, normalized size = 7.90

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sinh[c + d*x]^3/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])^3*Sech[c + d*x]^6*((24*(3*a - 4*b)*(ArcTan
[((Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/
2] + Cosh[c]*(Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x
)/2]))/Sqrt[b]] + ArcTan[((Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])
^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cosh[c]
 - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]]))/(a^(3/2)*b^(5/2)) - (54*(ArcTan[(
Sqrt[a] - I*Sqrt[a + b]*Tanh[(c + d*x)/2])/Sqrt[b]] + ArcTan[(Sqrt[a] + I*S
qrt[a + b]*Tanh[(c + d*x)/2])/Sqrt[b]]))/(Sqrt[a]*b^(5/2)) - (36*Cosh[c + d
*x]*(3*a + 10*b + 3*a*Cosh[2*(c + d*x)]))/(b^2*(a + 2*b + a*Cosh[2*(c + d*x
)])^2) + (48*Cosh[c + d*x]*(3*a^2 + 6*a*b + 8*b^2 + a*(3*a - 4*b)*Cosh[2*(c
```

```
 + d*x)]))/(a*b^2*(a + 2*b + a*Cosh[2*(c + d*x)])^2) + (3*(3*a^4 - 40*a^3*b
 + 720*a^2*b^2 + 6720*a*b^3 + 8960*b^4)*ArcTan[((Sqrt[a] - I*Sqrt[a + b]*Sq
rt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] - I*Sqr
t[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]] + 3*(3*a^4 -
40*a^3*b + 720*a^2*b^2 + 6720*a*b^3 + 8960*b^4)*ArcTan[((Sqrt[a] + I*Sqrt[a
 + b]*Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a]
 + I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]] + (2*
Sqrt[a]*Sqrt[b]*Cosh[c + d*x]*(9*a^5 - 90*a^4*b - 10144*a^3*b^2 - 48672*a^2
*b^3 - 85120*a*b^4 - 53760*b^5 + a*(9*a^4 - 120*a^3*b - 12432*a^2*b^2 - 479
36*a*b^3 - 44800*b^4)*Cosh[2*(c + d*x)] - 128*a^2*b^2*(15*a + 28*b)*Cosh[4*
(c + d*x)] + 128*a^3*b^2*Cosh[6*(c + d*x)]))/(a + 2*b + a*Cosh[2*(c + d*x)]
)^2)/(a^(9/2)*b^(5/2)) + (9*((-3*(a^3 - 8*a^2*b + 80*a*b^2 + 320*b^3)*ArcTa
n[((Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)
/2] + Cosh[c]*(Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*
x)/2]))/Sqrt[b]])/b^(5/2) - (3*(a^3 - 8*a^2*b + 80*a*b^2 + 320*b^3)*ArcTan[
((Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2
] + Cosh[c]*(Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)
/2]))/Sqrt[b]])/b^(5/2) + 512*Sqrt[a]*Cosh[c]*Cosh[d*x] - (8*Sqrt[a]*(a^3 +
 24*a^2*b + 80*a*b^2 + 64*b^3)*Cosh[c + d*x])/(b*(a + 2*b + a*Cosh[2*(c + d
*x)])^2) - (2*Sqrt[a]*(3*a^3 - 24*a^2*b - 400*a*b^2 - 576*b^3)*Cosh[c + d*x
])/(b^2*(a + 2*b + a*Cosh[2*(c + d*x)])) + 512*Sqrt[a]*Sinh[c]*Sinh[d*x]))/
a^(7/2)))/(49152*d*(a + b*Sech[c + d*x]^2)^3)
```

fricas [B] time = 0.56, size = 8667, normalized size = 56.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(2*a^3*cosh(d*x + c)^14 + 28*a^3*cosh(d*x + c)*sinh(d*x + c)^13 + 2*a
^3*sinh(d*x + c)^14 - 2*(5*a^3 + 28*a^2*b)*cosh(d*x + c)^12 + 2*(91*a^3*cos
h(d*x + c)^2 - 5*a^3 - 28*a^2*b)*sinh(d*x + c)^12 + 8*(91*a^3*cosh(d*x + c)
^3 - 3*(5*a^3 + 28*a^2*b)*cosh(d*x + c))*sinh(d*x + c)^11 - 2*(39*a^3 + 290
*a^2*b + 350*a*b^2)*cosh(d*x + c)^10 + 2*(1001*a^3*cosh(d*x + c)^4 - 39*a^3
 - 290*a^2*b - 350*a*b^2 - 66*(5*a^3 + 28*a^2*b)*cosh(d*x + c)^2)*sinh(d*x
+ c)^10 + 4*(1001*a^3*cosh(d*x + c)^5 - 110*(5*a^3 + 28*a^2*b)*cosh(d*x + c
)^3 - 5*(39*a^3 + 290*a^2*b + 350*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^9 - 1
0*(17*a^3 + 146*a^2*b + 282*a*b^2 + 168*b^3)*cosh(d*x + c)^8 + 2*(3003*a^3*
cosh(d*x + c)^6 - 495*(5*a^3 + 28*a^2*b)*cosh(d*x + c)^4 - 85*a^3 - 730*a^2
*b - 1410*a*b^2 - 840*b^3 - 45*(39*a^3 + 290*a^2*b + 350*a*b^2)*cosh(d*x +
c)^2)*sinh(d*x + c)^8 + 16*(429*a^3*cosh(d*x + c)^7 - 99*(5*a^3 + 28*a^2*b)
*cosh(d*x + c)^5 - 15*(39*a^3 + 290*a^2*b + 350*a*b^2)*cosh(d*x + c)^3 - 5*
(17*a^3 + 146*a^2*b + 282*a*b^2 + 168*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 -
 10*(17*a^3 + 146*a^2*b + 282*a*b^2 + 168*b^3)*cosh(d*x + c)^6 + 2*(3003*a^
```

3*cosh(d*x + c)^8 - 924*(5*a^3 + 28*a^2*b)*cosh(d*x + c)^6 - 210*(39*a^3 +
290*a^2*b + 350*a*b^2)*cosh(d*x + c)^4 - 85*a^3 - 730*a^2*b - 1410*a*b^2 -
840*b^3 - 140*(17*a^3 + 146*a^2*b + 282*a*b^2 + 168*b^3)*cosh(d*x + c)^2)*s
inh(d*x + c)^6 + 4*(1001*a^3*cosh(d*x + c)^9 - 396*(5*a^3 + 28*a^2*b)*cosh(
d*x + c)^7 - 126*(39*a^3 + 290*a^2*b + 350*a*b^2)*cosh(d*x + c)^5 - 140*(17
*a^3 + 146*a^2*b + 282*a*b^2 + 168*b^3)*cosh(d*x + c)^3 - 15*(17*a^3 + 146*
a^2*b + 282*a*b^2 + 168*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(39*a^3 + 2
90*a^2*b + 350*a*b^2)*cosh(d*x + c)^4 + 2*(1001*a^3*cosh(d*x + c)^10 - 495*
(5*a^3 + 28*a^2*b)*cosh(d*x + c)^8 - 210*(39*a^3 + 290*a^2*b + 350*a*b^2)*c
osh(d*x + c)^6 - 350*(17*a^3 + 146*a^2*b + 282*a*b^2 + 168*b^3)*cosh(d*x +
c)^4 - 39*a^3 - 290*a^2*b - 350*a*b^2 - 75*(17*a^3 + 146*a^2*b + 282*a*b^2
+ 168*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(91*a^3*cosh(d*x + c)^11 -
55*(5*a^3 + 28*a^2*b)*cosh(d*x + c)^9 - 30*(39*a^3 + 290*a^2*b + 350*a*b^2)
*cosh(d*x + c)^7 - 70*(17*a^3 + 146*a^2*b + 282*a*b^2 + 168*b^3)*cosh(d*x +
c)^5 - 25*(17*a^3 + 146*a^2*b + 282*a*b^2 + 168*b^3)*cosh(d*x + c)^3 - (39
*a^3 + 290*a^2*b + 350*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*a^3 - 2*(5
*a^3 + 28*a^2*b)*cosh(d*x + c)^2 + 2*(91*a^3*cosh(d*x + c)^12 - 66*(5*a^3 +
28*a^2*b)*cosh(d*x + c)^10 - 45*(39*a^3 + 290*a^2*b + 350*a*b^2)*cosh(d*x
+ c)^8 - 140*(17*a^3 + 146*a^2*b + 282*a*b^2 + 168*b^3)*cosh(d*x + c)^6 - 7
5*(17*a^3 + 146*a^2*b + 282*a*b^2 + 168*b^3)*cosh(d*x + c)^4 - 5*a^3 - 28*a
^2*b - 6*(39*a^3 + 290*a^2*b + 350*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2
+ 15*((3*a^3 + 7*a^2*b)*cosh(d*x + c)^11 + 11*(3*a^3 + 7*a^2*b)*cosh(d*x +
c)*sinh(d*x + c)^10 + (3*a^3 + 7*a^2*b)*sinh(d*x + c)^11 + 4*(3*a^3 + 13*a^
2*b + 14*a*b^2)*cosh(d*x + c)^9 + (12*a^3 + 52*a^2*b + 56*a*b^2 + 55*(3*a^3
+ 7*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^9 + 3*(55*(3*a^3 + 7*a^2*b)*cosh
(d*x + c)^3 + 12*(3*a^3 + 13*a^2*b + 14*a*b^2)*cosh(d*x + c))*sinh(d*x + c)
^8 + 2*(9*a^3 + 45*a^2*b + 80*a*b^2 + 56*b^3)*cosh(d*x + c)^7 + 2*(165*(3*a
^3 + 7*a^2*b)*cosh(d*x + c)^4 + 9*a^3 + 45*a^2*b + 80*a*b^2 + 56*b^3 + 72*(
3*a^3 + 13*a^2*b + 14*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^7 + 14*(33*(3*a
^3 + 7*a^2*b)*cosh(d*x + c)^5 + 24*(3*a^3 + 13*a^2*b + 14*a*b^2)*cosh(d*x +
c)^3 + (9*a^3 + 45*a^2*b + 80*a*b^2 + 56*b^3)*cosh(d*x + c))*sinh(d*x + c)
^6 + 4*(3*a^3 + 13*a^2*b + 14*a*b^2)*cosh(d*x + c)^5 + 2*(231*(3*a^3 + 7*a^
2*b)*cosh(d*x + c)^6 + 252*(3*a^3 + 13*a^2*b + 14*a*b^2)*cosh(d*x + c)^4 +
6*a^3 + 26*a^2*b + 28*a*b^2 + 21*(9*a^3 + 45*a^2*b + 80*a*b^2 + 56*b^3)*cos
h(d*x + c)^2)*sinh(d*x + c)^5 + 2*(165*(3*a^3 + 7*a^2*b)*cosh(d*x + c)^7 +
252*(3*a^3 + 13*a^2*b + 14*a*b^2)*cosh(d*x + c)^5 + 35*(9*a^3 + 45*a^2*b +
80*a*b^2 + 56*b^3)*cosh(d*x + c)^3 + 10*(3*a^3 + 13*a^2*b + 14*a*b^2)*cosh(
d*x + c))*sinh(d*x + c)^4 + (3*a^3 + 7*a^2*b)*cosh(d*x + c)^3 + (165*(3*a^3
+ 7*a^2*b)*cosh(d*x + c)^8 + 336*(3*a^3 + 13*a^2*b + 14*a*b^2)*cosh(d*x +
c)^6 + 70*(9*a^3 + 45*a^2*b + 80*a*b^2 + 56*b^3)*cosh(d*x + c)^4 + 3*a^3 +
7*a^2*b + 40*(3*a^3 + 13*a^2*b + 14*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^3
+ (55*(3*a^3 + 7*a^2*b)*cosh(d*x + c)^9 + 144*(3*a^3 + 13*a^2*b + 14*a*b^2
)*cosh(d*x + c)^7 + 42*(9*a^3 + 45*a^2*b + 80*a*b^2 + 56*b^3)*cosh(d*x + c)
^5 + 40*(3*a^3 + 13*a^2*b + 14*a*b^2)*cosh(d*x + c)^3 + 3*(3*a^3 + 7*a^2*b)
*cosh(d*x + c))*sinh(d*x + c)^2 + (11*(3*a^3 + 7*a^2*b)*cosh(d*x + c)^10 +

```
36*(3*a^3 + 13*a^2*b + 14*a*b^2)*cosh(d*x + c)^8 + 14*(9*a^3 + 45*a^2*b + 8
0*a*b^2 + 56*b^3)*cosh(d*x + c)^6 + 20*(3*a^3 + 13*a^2*b + 14*a*b^2)*cosh(d
*x + c)^4 + 3*(3*a^3 + 7*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c))*sqrt(-b/a)*
log((a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c
)^4 + 2*(a - 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a - 2*b)*sinh(
d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a - 2*b)*cosh(d*x + c))*sinh(d*x + c)
 + 4*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c
)^3 + a*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + a)*sinh(d*x + c))*sqrt(-b/a)
 + a)/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x +
 c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sin
h(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c
) + a)) + 4*(7*a^3*cosh(d*x + c)^13 - 6*(5*a^3 + 28*a^2*b)*cosh(d*x + c)^11
 - 5*(39*a^3 + 290*a^2*b + 350*a*b^2)*cosh(d*x + c)^9 - 20*(17*a^3 + 146*a^
2*b + 282*a*b^2 + 168*b^3)*cosh(d*x + c)^7 - 15*(17*a^3 + 146*a^2*b + 282*a
*b^2 + 168*b^3)*cosh(d*x + c)^5 - 2*(39*a^3 + 290*a^2*b + 350*a*b^2)*cosh(d
*x + c)^3 - (5*a^3 + 28*a^2*b)*cosh(d*x + c))*sinh(d*x + c))/(a^6*d*cosh(d*
x + c)^11 + 11*a^6*d*cosh(d*x + c)*sinh(d*x + c)^10 + a^6*d*sinh(d*x + c)^1
1 + 4*(a^6 + 2*a^5*b)*d*cosh(d*x + c)^9 + a^6*d*cosh(d*x + c)^3 + (55*a^6*d
*cosh(d*x + c)^2 + 4*(a^6 + 2*a^5*b)*d)*sinh(d*x + c)^9 + 2*(3*a^6 + 8*a^5*
b + 8*a^4*b^2)*d*cosh(d*x + c)^7 + 3*(55*a^6*d*cosh(d*x + c)^3 + 12*(a^6 +
2*a^5*b)*d*cosh(d*x + c))*sinh(d*x + c)^8 + 2*(165*a^6*d*cosh(d*x + c)^4 +
72*(a^6 + 2*a^5*b)*d*cosh(d*x + c)^2 + (3*a^6 + 8*a^5*b + 8*a^4*b^2)*d)*sin
h(d*x + c)^7 + 4*(a^6 + 2*a^5*b)*d*cosh(d*x + c)^5 + 14*(33*a^6*d*cosh(d*x
+ c)^5 + 24*(a^6 + 2*a^5*b)*d*cosh(d*x + c)^3 + (3*a^6 + 8*a^5*b + 8*a^4*b^
2)*d*cosh(d*x + c))*sinh(d*x + c)^6 + 2*(231*a^6*d*cosh(d*x + c)^6 + 252*(a
^6 + 2*a^5*b)*d*cosh(d*x + c)^4 + 21*(3*a^6 + 8*a^5*b + 8*a^4*b^2)*d*cosh(d
*x + c)^2 + 2*(a^6 + 2*a^5*b)*d)*sinh(d*x + c)^5 + 2*(165*a^6*d*cosh(d*x +
c)^7 + 252*(a^6 + 2*a^5*b)*d*cosh(d*x + c)^5 + 35*(3*a^6 + 8*a^5*b + 8*a^4*
b^2)*d*cosh(d*x + c)^3 + 10*(a^6 + 2*a^5*b)*d*cosh(d*x + c))*sinh(d*x + c)^
4 + (165*a^6*d*cosh(d*x + c)^8 + 336*(a^6 + 2*a^5*b)*d*cosh(d*x + c)^6 + a^
6*d + 70*(3*a^6 + 8*a^5*b + 8*a^4*b^2)*d*cosh(d*x + c)^4 + 40*(a^6 + 2*a^5*
b)*d*cosh(d*x + c)^2)*sinh(d*x + c)^3 + (55*a^6*d*cosh(d*x + c)^9 + 144*(a^
6 + 2*a^5*b)*d*cosh(d*x + c)^7 + 3*a^6*d*cosh(d*x + c) + 42*(3*a^6 + 8*a^5*
b + 8*a^4*b^2)*d*cosh(d*x + c)^5 + 40*(a^6 + 2*a^5*b)*d*cosh(d*x + c)^3)*si
nh(d*x + c)^2 + (11*a^6*d*cosh(d*x + c)^10 + 36*(a^6 + 2*a^5*b)*d*cosh(d*x
+ c)^8 + 3*a^6*d*cosh(d*x + c)^2 + 14*(3*a^6 + 8*a^5*b + 8*a^4*b^2)*d*cosh(
d*x + c)^6 + 20*(a^6 + 2*a^5*b)*d*cosh(d*x + c)^4)*sinh(d*x + c)), 1/24*(a^
3*cosh(d*x + c)^14 + 14*a^3*cosh(d*x + c)*sinh(d*x + c)^13 + a^3*sinh(d*x +
 c)^14 - (5*a^3 + 28*a^2*b)*cosh(d*x + c)^12 + (91*a^3*cosh(d*x + c)^2 - 5*
a^3 - 28*a^2*b)*sinh(d*x + c)^12 + 4*(91*a^3*cosh(d*x + c)^3 - 3*(5*a^3 + 2
8*a^2*b)*cosh(d*x + c))*sinh(d*x + c)^11 - (39*a^3 + 290*a^2*b + 350*a*b^2)
*cosh(d*x + c)^10 + (1001*a^3*cosh(d*x + c)^4 - 39*a^3 - 290*a^2*b - 350*a*
b^2 - 66*(5*a^3 + 28*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 2*(1001*a^3
*cosh(d*x + c)^5 - 110*(5*a^3 + 28*a^2*b)*cosh(d*x + c)^3 - 5*(39*a^3 + 290
*a^2*b + 350*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^9 - 5*(17*a^3 + 146*a^2*b
```

```
+ 282*a*b^2 + 168*b^3)*cosh(d*x + c)^8 + (3003*a^3*cosh(d*x + c)^6 - 495*(5
*a^3 + 28*a^2*b)*cosh(d*x + c)^4 - 85*a^3 - 730*a^2*b - 1410*a*b^2 - 840*b^
3 - 45*(39*a^3 + 290*a^2*b + 350*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^8 +
8*(429*a^3*cosh(d*x + c)^7 - 99*(5*a^3 + 28*a^2*b)*cosh(d*x + c)^5 - 15*(39
*a^3 + 290*a^2*b + 350*a*b^2)*cosh(d*x + c)^3 - 5*(17*a^3 + 146*a^2*b + 282
*a*b^2 + 168*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 - 5*(17*a^3 + 146*a^2*b +
282*a*b^2 + 168*b^3)*cosh(d*x + c)^6 + (3003*a^3*cosh(d*x + c)^8 - 924*(5*a
^3 + 28*a^2*b)*cosh(d*x + c)^6 - 210*(39*a^3 + 290*a^2*b + 350*a*b^2)*cosh(
d*x + c)^4 - 85*a^3 - 730*a^2*b - 1410*a*b^2 - 840*b^3 - 140*(17*a^3 + 146*
a^2*b + 282*a*b^2 + 168*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 2*(1001*a^3
*cosh(d*x + c)^9 - 396*(5*a^3 + 28*a^2*b)*cosh(d*x + c)^7 - 126*(39*a^3 + 2
90*a^2*b + 350*a*b^2)*cosh(d*x + c)^5 - 140*(17*a^3 + 146*a^2*b + 282*a*b^2
 + 168*b^3)*cosh(d*x + c)^3 - 15*(17*a^3 + 146*a^2*b + 282*a*b^2 + 168*b^3)
*cosh(d*x + c))*sinh(d*x + c)^5 - (39*a^3 + 290*a^2*b + 350*a*b^2)*cosh(d*x
 + c)^4 + (1001*a^3*cosh(d*x + c)^10 - 495*(5*a^3 + 28*a^2*b)*cosh(d*x + c)
^8 - 210*(39*a^3 + 290*a^2*b + 350*a*b^2)*cosh(d*x + c)^6 - 350*(17*a^3 + 1
46*a^2*b + 282*a*b^2 + 168*b^3)*cosh(d*x + c)^4 - 39*a^3 - 290*a^2*b - 350*
a*b^2 - 75*(17*a^3 + 146*a^2*b + 282*a*b^2 + 168*b^3)*cosh(d*x + c)^2)*sinh
(d*x + c)^4 + 4*(91*a^3*cosh(d*x + c)^11 - 55*(5*a^3 + 28*a^2*b)*cosh(d*x +
 c)^9 - 30*(39*a^3 + 290*a^2*b + 350*a*b^2)*cosh(d*x + c)^7 - 70*(17*a^3 +
146*a^2*b + 282*a*b^2 + 168*b^3)*cosh(d*x + c)^5 - 25*(17*a^3 + 146*a^2*b +
 282*a*b^2 + 168*b^3)*cosh(d*x + c)^3 - (39*a^3 + 290*a^2*b + 350*a*b^2)*co
sh(d*x + c))*sinh(d*x + c)^3 + a^3 - (5*a^3 + 28*a^2*b)*cosh(d*x + c)^2 + (
91*a^3*cosh(d*x + c)^12 - 66*(5*a^3 + 28*a^2*b)*cosh(d*x + c)^10 - 45*(39*a
^3 + 290*a^2*b + 350*a*b^2)*cosh(d*x + c)^8 - 140*(17*a^3 + 146*a^2*b + 282
*a*b^2 + 168*b^3)*cosh(d*x + c)^6 - 75*(17*a^3 + 146*a^2*b + 282*a*b^2 + 16
8*b^3)*cosh(d*x + c)^4 - 5*a^3 - 28*a^2*b - 6*(39*a^3 + 290*a^2*b + 350*a*b
^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 15*((3*a^3 + 7*a^2*b)*cosh(d*x + c)^
11 + 11*(3*a^3 + 7*a^2*b)*cosh(d*x + c)*sinh(d*x + c)^10 + (3*a^3 + 7*a^2*b
)*sinh(d*x + c)^11 + 4*(3*a^3 + 13*a^2*b + 14*a*b^2)*cosh(d*x + c)^9 + (12*
a^3 + 52*a^2*b + 56*a*b^2 + 55*(3*a^3 + 7*a^2*b)*cosh(d*x + c)^2)*sinh(d*x
+ c)^9 + 3*(55*(3*a^3 + 7*a^2*b)*cosh(d*x + c)^3 + 12*(3*a^3 + 13*a^2*b + 1
4*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^8 + 2*(9*a^3 + 45*a^2*b + 80*a*b^2 +
56*b^3)*cosh(d*x + c)^7 + 2*(165*(3*a^3 + 7*a^2*b)*cosh(d*x + c)^4 + 9*a^3
+ 45*a^2*b + 80*a*b^2 + 56*b^3 + 72*(3*a^3 + 13*a^2*b + 14*a*b^2)*cosh(d*x
+ c)^2)*sinh(d*x + c)^7 + 14*(33*(3*a^3 + 7*a^2*b)*cosh(d*x + c)^5 + 24*(3*
a^3 + 13*a^2*b + 14*a*b^2)*cosh(d*x + c)^3 + (9*a^3 + 45*a^2*b + 80*a*b^2 +
 56*b^3)*cosh(d*x + c))*sinh(d*x + c)^6 + 4*(3*a^3 + 13*a^2*b + 14*a*b^2)*c
osh(d*x + c)^5 + 2*(231*(3*a^3 + 7*a^2*b)*cosh(d*x + c)^6 + 252*(3*a^3 + 13
*a^2*b + 14*a*b^2)*cosh(d*x + c)^4 + 6*a^3 + 26*a^2*b + 28*a*b^2 + 21*(9*a^
3 + 45*a^2*b + 80*a*b^2 + 56*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 2*(165
*(3*a^3 + 7*a^2*b)*cosh(d*x + c)^7 + 252*(3*a^3 + 13*a^2*b + 14*a*b^2)*cosh
(d*x + c)^5 + 35*(9*a^3 + 45*a^2*b + 80*a*b^2 + 56*b^3)*cosh(d*x + c)^3 + 1
0*(3*a^3 + 13*a^2*b + 14*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^4 + (3*a^3 + 7
*a^2*b)*cosh(d*x + c)^3 + (165*(3*a^3 + 7*a^2*b)*cosh(d*x + c)^8 + 336*(3*a
```

^3 + 13*a^2*b + 14*a*b^2)*cosh(d*x + c)^6 + 70*(9*a^3 + 45*a^2*b + 80*a*b^2
 + 56*b^3)*cosh(d*x + c)^4 + 3*a^3 + 7*a^2*b + 40*(3*a^3 + 13*a^2*b + 14*a*
b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + (55*(3*a^3 + 7*a^2*b)*cosh(d*x + c)
^9 + 144*(3*a^3 + 13*a^2*b + 14*a*b^2)*cosh(d*x + c)^7 + 42*(9*a^3 + 45*a^2
*b + 80*a*b^2 + 56*b^3)*cosh(d*x + c)^5 + 40*(3*a^3 + 13*a^2*b + 14*a*b^2)*
cosh(d*x + c)^3 + 3*(3*a^3 + 7*a^2*b)*cosh(d*x + c))*sinh(d*x + c)^2 + (11*
(3*a^3 + 7*a^2*b)*cosh(d*x + c)^10 + 36*(3*a^3 + 13*a^2*b + 14*a*b^2)*cosh(
d*x + c)^8 + 14*(9*a^3 + 45*a^2*b + 80*a*b^2 + 56*b^3)*cosh(d*x + c)^6 + 20
*(3*a^3 + 13*a^2*b + 14*a*b^2)*cosh(d*x + c)^4 + 3*(3*a^3 + 7*a^2*b)*cosh(d
*x + c)^2)*sinh(d*x + c))*sqrt(b/a)*arctan(1/2*(a*cosh(d*x + c)^3 + 3*a*cos
h(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 + (a + 4*b)*cosh(d*x + c) +
(3*a*cosh(d*x + c)^2 + a + 4*b)*sinh(d*x + c))*sqrt(b/a)/b) + 15*((3*a^3 +
7*a^2*b)*cosh(d*x + c)^11 + 11*(3*a^3 + 7*a^2*b)*cosh(d*x + c)*sinh(d*x + c
)^10 + (3*a^3 + 7*a^2*b)*sinh(d*x + c)^11 + 4*(3*a^3 + 13*a^2*b + 14*a*b^2)
*cosh(d*x + c)^9 + (12*a^3 + 52*a^2*b + 56*a*b^2 + 55*(3*a^3 + 7*a^2*b)*cos
h(d*x + c)^2)*sinh(d*x + c)^9 + 3*(55*(3*a^3 + 7*a^2*b)*cosh(d*x + c)^3 + 1
2*(3*a^3 + 13*a^2*b + 14*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^8 + 2*(9*a^3 +
 45*a^2*b + 80*a*b^2 + 56*b^3)*cosh(d*x + c)^7 + 2*(165*(3*a^3 + 7*a^2*b)*c
osh(d*x + c)^4 + 9*a^3 + 45*a^2*b + 80*a*b^2 + 56*b^3 + 72*(3*a^3 + 13*a^2*
b + 14*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^7 + 14*(33*(3*a^3 + 7*a^2*b)*c
osh(d*x + c)^5 + 24*(3*a^3 + 13*a^2*b + 14*a*b^2)*cosh(d*x + c)^3 + (9*a^3
+ 45*a^2*b + 80*a*b^2 + 56*b^3)*cosh(d*x + c))*sinh(d*x + c)^6 + 4*(3*a^3 +
 13*a^2*b + 14*a*b^2)*cosh(d*x + c)^5 + 2*(231*(3*a^3 + 7*a^2*b)*cosh(d*x +
 c)^6 + 252*(3*a^3 + 13*a^2*b + 14*a*b^2)*cosh(d*x + c)^4 + 6*a^3 + 26*a^2*
b + 28*a*b^2 + 21*(9*a^3 + 45*a^2*b + 80*a*b^2 + 56*b^3)*cosh(d*x + c)^2)*s
inh(d*x + c)^5 + 2*(165*(3*a^3 + 7*a^2*b)*cosh(d*x + c)^7 + 252*(3*a^3 + 13
*a^2*b + 14*a*b^2)*cosh(d*x + c)^5 + 35*(9*a^3 + 45*a^2*b + 80*a*b^2 + 56*b
^3)*cosh(d*x + c)^3 + 10*(3*a^3 + 13*a^2*b + 14*a*b^2)*cosh(d*x + c))*sinh(
d*x + c)^4 + (3*a^3 + 7*a^2*b)*cosh(d*x + c)^3 + (165*(3*a^3 + 7*a^2*b)*cos
h(d*x + c)^8 + 336*(3*a^3 + 13*a^2*b + 14*a*b^2)*cosh(d*x + c)^6 + 70*(9*a^
3 + 45*a^2*b + 80*a*b^2 + 56*b^3)*cosh(d*x + c)^4 + 3*a^3 + 7*a^2*b + 40*(3
*a^3 + 13*a^2*b + 14*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + (55*(3*a^3 +
 7*a^2*b)*cosh(d*x + c)^9 + 144*(3*a^3 + 13*a^2*b + 14*a*b^2)*cosh(d*x + c)
^7 + 42*(9*a^3 + 45*a^2*b + 80*a*b^2 + 56*b^3)*cosh(d*x + c)^5 + 40*(3*a^3
+ 13*a^2*b + 14*a*b^2)*cosh(d*x + c)^3 + 3*(3*a^3 + 7*a^2*b)*cosh(d*x + c))
*sinh(d*x + c)^2 + (11*(3*a^3 + 7*a^2*b)*cosh(d*x + c)^10 + 36*(3*a^3 + 13*
a^2*b + 14*a*b^2)*cosh(d*x + c)^8 + 14*(9*a^3 + 45*a^2*b + 80*a*b^2 + 56*b^
3)*cosh(d*x + c)^6 + 20*(3*a^3 + 13*a^2*b + 14*a*b^2)*cosh(d*x + c)^4 + 3*(
3*a^3 + 7*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c))*sqrt(b/a)*arctan(1/2*(a*co
sh(d*x + c) + a*sinh(d*x + c))*sqrt(b/a)/b) + 2*(7*a^3*cosh(d*x + c)^13 - 6
*(5*a^3 + 28*a^2*b)*cosh(d*x + c)^11 - 5*(39*a^3 + 290*a^2*b + 350*a*b^2)*c
osh(d*x + c)^9 - 20*(17*a^3 + 146*a^2*b + 282*a*b^2 + 168*b^3)*cosh(d*x + c
)^7 - 15*(17*a^3 + 146*a^2*b + 282*a*b^2 + 168*b^3)*cosh(d*x + c)^5 - 2*(39
*a^3 + 290*a^2*b + 350*a*b^2)*cosh(d*x + c)^3 - (5*a^3 + 28*a^2*b)*cosh(d*x
 + c))*sinh(d*x + c))/(a^6*d*cosh(d*x + c)^11 + 11*a^6*d*cosh(d*x + c)*sinh

```
(d*x + c)^10 + a^6*d*sinh(d*x + c)^11 + 4*(a^6 + 2*a^5*b)*d*cosh(d*x + c)^9
 + a^6*d*cosh(d*x + c)^3 + (55*a^6*d*cosh(d*x + c)^2 + 4*(a^6 + 2*a^5*b)*d)
*sinh(d*x + c)^9 + 2*(3*a^6 + 8*a^5*b + 8*a^4*b^2)*d*cosh(d*x + c)^7 + 3*(5
5*a^6*d*cosh(d*x + c)^3 + 12*(a^6 + 2*a^5*b)*d*cosh(d*x + c))*sinh(d*x + c)
^8 + 2*(165*a^6*d*cosh(d*x + c)^4 + 72*(a^6 + 2*a^5*b)*d*cosh(d*x + c)^2 +
(3*a^6 + 8*a^5*b + 8*a^4*b^2)*d)*sinh(d*x + c)^7 + 4*(a^6 + 2*a^5*b)*d*cosh
(d*x + c)^5 + 14*(33*a^6*d*cosh(d*x + c)^5 + 24*(a^6 + 2*a^5*b)*d*cosh(d*x
+ c)^3 + (3*a^6 + 8*a^5*b + 8*a^4*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^6 + 2
*(231*a^6*d*cosh(d*x + c)^6 + 252*(a^6 + 2*a^5*b)*d*cosh(d*x + c)^4 + 21*(3
*a^6 + 8*a^5*b + 8*a^4*b^2)*d*cosh(d*x + c)^2 + 2*(a^6 + 2*a^5*b)*d)*sinh(d
*x + c)^5 + 2*(165*a^6*d*cosh(d*x + c)^7 + 252*(a^6 + 2*a^5*b)*d*cosh(d*x +
 c)^5 + 35*(3*a^6 + 8*a^5*b + 8*a^4*b^2)*d*cosh(d*x + c)^3 + 10*(a^6 + 2*a^
5*b)*d*cosh(d*x + c))*sinh(d*x + c)^4 + (165*a^6*d*cosh(d*x + c)^8 + 336*(a
^6 + 2*a^5*b)*d*cosh(d*x + c)^6 + a^6*d + 70*(3*a^6 + 8*a^5*b + 8*a^4*b^2)*
d*cosh(d*x + c)^4 + 40*(a^6 + 2*a^5*b)*d*cosh(d*x + c)^2)*sinh(d*x + c)^3 +
 (55*a^6*d*cosh(d*x + c)^9 + 144*(a^6 + 2*a^5*b)*d*cosh(d*x + c)^7 + 3*a^6*
d*cosh(d*x + c) + 42*(3*a^6 + 8*a^5*b + 8*a^4*b^2)*d*cosh(d*x + c)^5 + 40*(
a^6 + 2*a^5*b)*d*cosh(d*x + c)^3)*sinh(d*x + c)^2 + (11*a^6*d*cosh(d*x + c)
^10 + 36*(a^6 + 2*a^5*b)*d*cosh(d*x + c)^8 + 3*a^6*d*cosh(d*x + c)^2 + 14*(
3*a^6 + 8*a^5*b + 8*a^4*b^2)*d*cosh(d*x + c)^6 + 20*(a^6 + 2*a^5*b)*d*cosh(
d*x + c)^4)*sinh(d*x + c))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[84,-86]Warning, need to choose a branch for the root of a p
olynomial with parameters. This might be wrong.The choice was done assuming
 [a,b]=[-42,-12]Warning, need to choose a branch for the root of a polynomi
al with parameters. This might be wrong.The choice was done assuming [a,b]=
[-43,-99]Warning, need to choose a branch for the root of a polynomial with
 parameters. This might be wrong.The choice was done assuming [a,b]=[-28,94
]Warning, need to choose a branch for the root of a polynomial with paramet
ers. This might be wrong.The choice was done assuming [a,b]=[-7,46]Warning,
 need to choose a branch for the root of a polynomial with parameters. This
 might be wrong.The choice was done assuming [a,b]=[-35,-99]Warning, need t
o choose a branch for the root of a polynomial with parameters. This might
be wrong.The choice was done assuming [a,b]=[7,50]Warning, need to choose a
 branch for the root of a polynomial with parameters. This might be wrong.T
```

```
he choice was done assuming [a,b]=[-63,-70]Warning, need to choose a branch
 for the root of a polynomial with parameters. This might be wrong.The choi
ce was done assuming [a,b]=[-5,48]Warning, need to choose a branch for the
root of a polynomial with parameters. This might be wrong.The choice was do
ne assuming [a,b]=[-33,-84]Warning, need to choose a branch for the root of
 a polynomial with parameters. This might be wrong.The choice was done assu
ming [a,b]=[90,-39]Warning, need to choose a branch for the root of a polyn
omial with parameters. This might be wrong.The choice was done assuming [a,
b]=[70,15]Warning, need to choose a branch for the root of a polynomial wit
h parameters. This might be wrong.The choice was done assuming [a,b]=[-11,2
9]Warning, need to choose a branch for the root of a polynomial with parame
ters. This might be wrong.The choice was done assuming [a,b]=[84,29]Undef/U
nsigned Inf encountered in limitEvaluation time: 3.48Limit: Max order reach
ed or unable to make series expansion Error: Bad Argument Value
```

maple [B] time = 0.38, size = 1296, normalized size = 8.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] -1/3/d/a^3/(tanh(1/2*d*x+1/2*c)-1)^3-1/2/d/a^3/(tanh(1/2*d*x+1/2*c)-1)^2+1/
2/d/a^3/(tanh(1/2*d*x+1/2*c)-1)+3/d/a^4/(tanh(1/2*d*x+1/2*c)-1)*b+1/3/d/a^3
/(tanh(1/2*d*x+1/2*c)+1)^3-1/2/d/a^3/(tanh(1/2*d*x+1/2*c)+1)^2-1/2/d/a^3/(t
anh(1/2*d*x+1/2*c)+1)-3/d/a^4/(tanh(1/2*d*x+1/2*c)+1)*b-9/4/d/a^2*b/(tanh(1
/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(
1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^6+1/2/d/a^3*b^2/(tanh(1/2*d*x
+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*
x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^6+11/4/d/a^4*b^3/(tanh(1/2*d*x+1/2*
c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2
*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^6-27/4/d/a*b/(tanh(1/2*d*x+1/2*c)^4*a+b*
tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a
+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)^4-15/4/d/a^2*b^2/(tanh(1/2*d*x+1/2*c)^4*a+b
*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+
a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)^4-5/4/d/a^3*b^3/(tanh(1/2*d*x+1/2*c)^4*a+b
*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+
a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)^4-33/4/d/a^4*b^4/(tanh(1/2*d*x+1/2*c)^4*a+
b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b
+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)^4-27/4/d/a^2*b/(tanh(1/2*d*x+1/2*c)^4*a+b
*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+
a+b)^2*tanh(1/2*d*x+1/2*c)^2-5/2/d/a^3*b^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(
1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2
*tanh(1/2*d*x+1/2*c)^2+33/4/d/a^4*b^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d
*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh
```

```
(1/2*d*x+1/2*c)^2-9/4/d/a^2*b/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c
)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2-5/d/a^3*b^2/
(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-
2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2-11/4/d/a^4*b^3/(tanh(1/2*d*x+1/2*c)^4*a+b*
tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a
+b)^2+15/8/d/a^3*b/(a*b)^(1/2)*arctan(1/4*(2*(a+b)*tanh(1/2*d*x+1/2*c)^2+2*
a-2*b)/(a*b)^(1/2))+35/8/d/a^4*b^2/(a*b)^(1/2)*arctan(1/4*(2*(a+b)*tanh(1/2
*d*x+1/2*c)^2+2*a-2*b)/(a*b)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh\left(c+d\,x\right)^6\,\sinh\left(c+d\,x\right)^3}{{\left(a\,\cosh\left(c+d\,x\right)^2+b\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^3/(a + b/cosh(c + d*x)^2)^3,x)
```

```
[Out] int((cosh(c + d*x)^6*sinh(c + d*x)^3)/(b + a*cosh(c + d*x)^2)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

3.43 $\int \frac{\sinh^2(c+dx)}{\left(a+b\,\mathbf{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=187

$$-\frac{x(a+6b)}{2a^4}+\frac{b(11a+12b)\tanh(c+dx)}{8a^3d(a+b)\left(a-b\tanh^2(c+dx)+b\right)}+\frac{3b\tanh(c+dx)}{4a^2d\left(a-b\tanh^2(c+dx)+b\right)^2}+\frac{\sqrt{b}\left(15a^2+40ab+24b^2\right)}{8a^4d(a+}$$

```
[Out] -1/2*(a+6*b)*x/a^4+1/8*(15*a^2+40*a*b+24*b^2)*arctanh(b^(1/2)*tanh(d*x+c)/(
a+b)^(1/2))*b^(1/2)/a^4/(a+b)^(3/2)/d+1/2*cosh(d*x+c)*sinh(d*x+c)/a/d/(a+b-
b*tanh(d*x+c)^2)^2+3/4*b*tanh(d*x+c)/a^2/d/(a+b-b*tanh(d*x+c)^2)^2+1/8*b*(1
1*a+12*b)*tanh(d*x+c)/a^3/(a+b)/d/(a+b-b*tanh(d*x+c)^2)
```

Rubi [A] time = 0.29, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4132, 471, 527, 522, 206, 208}

$$\frac{\sqrt{b}\left(15a^2+40ab+24b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^4d(a+b)^{3/2}}+\frac{b(11a+12b)\tanh(c+dx)}{8a^3d(a+b)\left(a-b\tanh^2(c+dx)+b\right)}+\frac{3b\tanh(c+dx}{4a^2d\left(a-b\tanh^2(c+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] -((a + 6*b)*x)/(2*a^4) + (Sqrt[b]*(15*a^2 + 40*a*b + 24*b^2)*ArcTanh[(Sqrt[
b]*Tanh[c + d*x])/Sqrt[a + b]])/(8*a^4*(a + b)^(3/2)*d) + (Cosh[c + d*x]*Si
nh[c + d*x])/(2*a*d*(a + b - b*Tanh[c + d*x]^2)^2) + (3*b*Tanh[c + d*x])/(4
*a^2*d*(a + b - b*Tanh[c + d*x]^2)^2) + (b*(11*a + 12*b)*Tanh[c + d*x])/(8*
a^3*(a + b)*d*(a + b - b*Tanh[c + d*x]^2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sinh^2(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1-x^2)^2(a+b-bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2ad\left(a+b-b\tanh^2(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{a+b+5bx^2}{(1-x^2)(a+b-bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{2ad}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2ad\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{3b\tanh(c+dx)}{4a^2d\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{\text{Subst}\left(\int \frac{-2}{}\right.}{}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2ad\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{3b\tanh(c+dx)}{4a^2d\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{b(11a}{8a^3(a+b)d}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2ad\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{3b\tanh(c+dx)}{4a^2d\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{b(11a}{8a^3(a+b)d}$$

$$= -\frac{(a+6b)x}{2a^4} + \frac{\sqrt{b}\left(15a^2+40ab+24b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^4(a+b)^{3/2}d} + \frac{\cosh(c+dx)}{2ad\left(a+b-b\,t\right.}$$

Mathematica [B] time = 17.84, size = 2544, normalized size = 13.60

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sinh[c + d*x]^2/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (-5*(a + 2*b + a*Cosh[2*c + 2*d*x])^3*Sech[c + d*x]^6*(((3*a^2 + 8*a*b + 8*
b^2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a + b)^(5/2) - (a*Sqrt[
b]*(3*a^2 + 16*a*b + 16*b^2 + 3*a*(a + 2*b)*Cosh[2*(c + d*x)])*Sinh[2*(c +
d*x)])/((a + b)^2*(a + 2*b + a*Cosh[2*(c + d*x)])^2)))/(8192*b^(5/2)*d*(a +
 b*Sech[c + d*x]^2)^3) - ((a + 2*b + a*Cosh[2*c + 2*d*x])^3*Sech[c + d*x]^6
*((-3*a*(a + 2*b)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a + b)^(5/
2) + (Sqrt[b]*(3*a^3 + 14*a^2*b + 24*a*b^2 + 16*b^3 + a*(3*a^2 + 4*a*b + 4*
b^2)*Cosh[2*(c + d*x)])*Sinh[2*(c + d*x)])/((a + b)^2*(a + 2*b + a*Cosh[2*(
c + d*x)])^2)))/(2048*b^(5/2)*d*(a + b*Sech[c + d*x]^2)^3) + ((a + 2*b + a*
```

Cosh[2*c + 2*d*x])^3*Sech[c + d*x]^6*((-2*(3*a^5 - 10*a^4*b + 80*a^3*b^2 + 480*a^2*b^3 + 640*a*b^4 + 256*b^5)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(Cosh[2*c] - Sinh[2*c]))/(Sqrt[a + b]*Sqrt[b*(Cosh[c] - S inh[c])^4]) + (Sech[2*c]*(256*b^2*(a + b)^2*(3*a^2 + 8*a*b + 8*b^2)*d*x*Cos h[2*c] + 512*a*b^2*(a + b)^2*(a + 2*b)*d*x*Cosh[2*d*x] + 128*a^4*b^2*d*x*Co sh[2*(c + 2*d*x)] + 256*a^3*b^3*d*x*Cosh[2*(c + 2*d*x)] + 128*a^2*b^4*d*x*C osh[2*(c + 2*d*x)] + 512*a^4*b^2*d*x*Cosh[4*c + 2*d*x] + 2048*a^3*b^3*d*x*C osh[4*c + 2*d*x] + 2560*a^2*b^4*d*x*Cosh[4*c + 2*d*x] + 1024*a*b^5*d*x*Cosh [4*c + 2*d*x] + 128*a^4*b^2*d*x*Cosh[6*c + 4*d*x] + 256*a^3*b^3*d*x*Cosh[6* c + 4*d*x] + 128*a^2*b^4*d*x*Cosh[6*c + 4*d*x] - 9*a^6*Sinh[2*c] + 12*a^5*b *Sinh[2*c] + 684*a^4*b^2*Sinh[2*c] + 2880*a^3*b^3*Sinh[2*c] + 5280*a^2*b^4* Sinh[2*c] + 4608*a*b^5*Sinh[2*c] + 1536*b^6*Sinh[2*c] + 9*a^6*Sinh[2*d*x] - 14*a^5*b*Sinh[2*d*x] - 608*a^4*b^2*Sinh[2*d*x] - 2112*a^3*b^3*Sinh[2*d*x] - 2560*a^2*b^4*Sinh[2*d*x] - 1024*a*b^5*Sinh[2*d*x] + 3*a^6*Sinh[2*(c + 2*d *x)] - 12*a^5*b*Sinh[2*(c + 2*d*x)] - 204*a^4*b^2*Sinh[2*(c + 2*d*x)] - 384 *a^3*b^3*Sinh[2*(c + 2*d*x)] - 192*a^2*b^4*Sinh[2*(c + 2*d*x)] - 3*a^6*Sinh [4*c + 2*d*x] + 10*a^5*b*Sinh[4*c + 2*d*x] + 304*a^4*b^2*Sinh[4*c + 2*d*x] + 1056*a^3*b^3*Sinh[4*c + 2*d*x] + 1280*a^2*b^4*Sinh[4*c + 2*d*x] + 512*a*b ^5*Sinh[4*c + 2*d*x]))/(a + 2*b + a*Cosh[2*(c + d*x)])^2))/(4096*a^3*b^2*(a + b)^2*d*(a + b*Sech[c + d*x]^2)^3) + ((a + 2*b + a*Cosh[2*c + 2*d*x])^3*S ech[c + d*x]^6*((6*(a^6 - 8*a^5*b + 120*a^4*b^2 + 1280*a^3*b^3 + 3200*a^2*b ^4 + 3072*a*b^5 + 1024*b^6)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sin h[c])^4])]*(Cosh[2*c] - Sinh[2*c]))/(Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c]) ^4]) + (Sech[2*c]*(-1536*b^2*(a + b)^2*(3*a^3 + 14*a^2*b + 24*a*b^2 + 16*b^ 3)*d*x*Cosh[2*c] - 3072*a*b^2*(a^2 + 3*a*b + 2*b^2)^2*d*x*Cosh[2*d*x] - 768 *a^5*b^2*d*x*Cosh[2*(c + 2*d*x)] - 3072*a^4*b^3*d*x*Cosh[2*(c + 2*d*x)] - 3 840*a^3*b^4*d*x*Cosh[2*(c + 2*d*x)] - 1536*a^2*b^5*d*x*Cosh[2*(c + 2*d*x)] - 3072*a^5*b^2*d*x*Cosh[4*c + 2*d*x] - 18432*a^4*b^3*d*x*Cosh[4*c + 2*d*x] - 39936*a^3*b^4*d*x*Cosh[4*c + 2*d*x] - 36864*a^2*b^5*d*x*Cosh[4*c + 2*d*x] - 12288*a*b^6*d*x*Cosh[4*c + 2*d*x] - 768*a^5*b^2*d*x*Cosh[6*c + 4*d*x] - 3072*a^4*b^3*d*x*Cosh[6*c + 4*d*x] - 3840*a^3*b^4*d*x*Cosh[6*c + 4*d*x] - 1 536*a^2*b^5*d*x*Cosh[6*c + 4*d*x] + 9*a^7*Sinh[2*c] - 54*a^6*b*Sinh[2*c] - 2392*a^5*b^2*Sinh[2*c] - 13968*a^4*b^3*Sinh[2*c] - 36480*a^3*b^4*Sinh[2*c] - 50432*a^2*b^5*Sinh[2*c] - 35840*a*b^6*Sinh[2*c] - 10240*b^7*Sinh[2*c] - 9 *a^7*Sinh[2*d*x] + 56*a^6*b*Sinh[2*d*x] + 2552*a^5*b^2*Sinh[2*d*x] + 13184* a^4*b^3*Sinh[2*d*x] + 27072*a^3*b^4*Sinh[2*d*x] + 24576*a^2*b^5*Sinh[2*d*x] + 8192*a*b^6*Sinh[2*d*x] - 3*a^7*Sinh[2*(c + 2*d*x)] + 26*a^6*b*Sinh[2*(c + 2*d*x)] + 992*a^5*b^2*Sinh[2*(c + 2*d*x)] + 3648*a^4*b^3*Sinh[2*(c + 2*d* x)] + 4480*a^3*b^4*Sinh[2*(c + 2*d*x)] + 1792*a^2*b^5*Sinh[2*(c + 2*d*x)] + 3*a^7*Sinh[4*c + 2*d*x] - 24*a^6*b*Sinh[4*c + 2*d*x] - 600*a^5*b^2*Sinh[4* c + 2*d*x] - 3200*a^4*b^3*Sinh[4*c + 2*d*x] - 6720*a^3*b^4*Sinh[4*c + 2*d*x] - 6144*a^2*b^5*Sinh[4*c + 2*d*x] - 2048*a*b^6*Sinh[4*c + 2*d*x] + 256*a^5 *b^2*Sinh[6*c + 4*d*x] + 1024*a^4*b^3*Sinh[6*c + 4*d*x] + 1280*a^3*b^4*Sinh

```
[6*c + 4*d*x] + 512*a^2*b^5*Sinh[6*c + 4*d*x] + 64*a^5*b^2*Sinh[4*c + 6*d*x
] + 128*a^4*b^3*Sinh[4*c + 6*d*x] + 64*a^3*b^4*Sinh[4*c + 6*d*x] + 64*a^5*b
^2*Sinh[8*c + 6*d*x] + 128*a^4*b^3*Sinh[8*c + 6*d*x] + 64*a^3*b^4*Sinh[8*c
+ 6*d*x]))/(a + 2*b + a*Cosh[2*(c + d*x)])^2))/(16384*a^4*b^2*(a + b)^2*d*(
a + b*Sech[c + d*x]^2)^3) + ((a + 2*b + a*Cosh[2*c + 2*d*x])^3*Sech[c + d*x
]^6*((6*a^2*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x]
 - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(Cosh
[2*c] - Sinh[2*c]))/(Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4]) + (a*Sech[2
*c]*((-9*a^4 - 16*a^3*b + 48*a^2*b^2 + 128*a*b^3 + 64*b^4)*Sinh[2*d*x] + a*
(-3*a^3 + 2*a^2*b + 24*a*b^2 + 16*b^3)*Sinh[2*(c + 2*d*x)] + (3*a^4 - 64*a^
2*b^2 - 128*a*b^3 - 64*b^4)*Sinh[4*c + 2*d*x]) + (9*a^5 + 18*a^4*b - 64*a^3
*b^2 - 256*a^2*b^3 - 320*a*b^4 - 128*b^5)*Tanh[2*c])/(a^2*(a + 2*b + a*Cosh
[2*(c + d*x)])^2)))/(4096*b^2*(a + b)^2*d*(a + b*Sech[c + d*x]^2)^3)
```

fricas [B] time = 0.60, size = 9730, normalized size = 52.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(2*(a^4 + a^3*b)*cosh(d*x + c)^12 + 24*(a^4 + a^3*b)*cosh(d*x + c)*si
nh(d*x + c)^11 + 2*(a^4 + a^3*b)*sinh(d*x + c)^12 + 8*(a^4 + 3*a^3*b + 2*a^
2*b^2 - (a^4 + 7*a^3*b + 6*a^2*b^2)*d*x)*cosh(d*x + c)^10 + 4*(2*a^4 + 6*a^
3*b + 4*a^2*b^2 - 2*(a^4 + 7*a^3*b + 6*a^2*b^2)*d*x + 33*(a^4 + a^3*b)*cosh
(d*x + c)^2)*sinh(d*x + c)^10 + 40*(11*(a^4 + a^3*b)*cosh(d*x + c)^3 + 2*(a
^4 + 3*a^3*b + 2*a^2*b^2 - (a^4 + 7*a^3*b + 6*a^2*b^2)*d*x)*cosh(d*x + c))*
sinh(d*x + c)^9 + 2*(5*a^4 + 3*a^3*b - 32*a^2*b^2 - 32*a*b^3 - 16*(a^4 + 9*
a^3*b + 20*a^2*b^2 + 12*a*b^3)*d*x)*cosh(d*x + c)^8 + 2*(495*(a^4 + a^3*b)*
cosh(d*x + c)^4 + 5*a^4 + 3*a^3*b - 32*a^2*b^2 - 32*a*b^3 - 16*(a^4 + 9*a^3
*b + 20*a^2*b^2 + 12*a*b^3)*d*x + 180*(a^4 + 3*a^3*b + 2*a^2*b^2 - (a^4 + 7
*a^3*b + 6*a^2*b^2)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 16*(99*(a^4 + a
^3*b)*cosh(d*x + c)^5 + 60*(a^4 + 3*a^3*b + 2*a^2*b^2 - (a^4 + 7*a^3*b + 6*
a^2*b^2)*d*x)*cosh(d*x + c)^3 + (5*a^4 + 3*a^3*b - 32*a^2*b^2 - 32*a*b^3 -
16*(a^4 + 9*a^3*b + 20*a^2*b^2 + 12*a*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c
)^7 - 4*(27*a^3*b + 102*a^2*b^2 + 152*a*b^3 + 80*b^4 + 4*(3*a^4 + 29*a^3*b
+ 82*a^2*b^2 + 104*a*b^3 + 48*b^4)*d*x)*cosh(d*x + c)^6 + 4*(462*(a^4 + a^3
*b)*cosh(d*x + c)^6 + 420*(a^4 + 3*a^3*b + 2*a^2*b^2 - (a^4 + 7*a^3*b + 6*a
^2*b^2)*d*x)*cosh(d*x + c)^4 - 27*a^3*b - 102*a^2*b^2 - 152*a*b^3 - 80*b^4
- 4*(3*a^4 + 29*a^3*b + 82*a^2*b^2 + 104*a*b^3 + 48*b^4)*d*x + 14*(5*a^4 +
3*a^3*b - 32*a^2*b^2 - 32*a*b^3 - 16*(a^4 + 9*a^3*b + 20*a^2*b^2 + 12*a*b^3
)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(198*(a^4 + a^3*b)*cosh(d*x + c
)^7 + 252*(a^4 + 3*a^3*b + 2*a^2*b^2 - (a^4 + 7*a^3*b + 6*a^2*b^2)*d*x)*cos
h(d*x + c)^5 + 14*(5*a^4 + 3*a^3*b - 32*a^2*b^2 - 32*a*b^3 - 16*(a^4 + 9*a^
3*b + 20*a^2*b^2 + 12*a*b^3)*d*x)*cosh(d*x + c)^3 - 3*(27*a^3*b + 102*a^2*b
```

```
^2 + 152*a*b^3 + 80*b^4 + 4*(3*a^4 + 29*a^3*b + 82*a^2*b^2 + 104*a*b^3 + 48
*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(5*a^4 + 75*a^3*b + 192*a^2*b
^2 + 128*a*b^3 + 16*(a^4 + 9*a^3*b + 20*a^2*b^2 + 12*a*b^3)*d*x)*cosh(d*x +
 c)^4 + 2*(495*(a^4 + a^3*b)*cosh(d*x + c)^8 + 840*(a^4 + 3*a^3*b + 2*a^2*b
^2 - (a^4 + 7*a^3*b + 6*a^2*b^2)*d*x)*cosh(d*x + c)^6 + 70*(5*a^4 + 3*a^3*b
 - 32*a^2*b^2 - 32*a*b^3 - 16*(a^4 + 9*a^3*b + 20*a^2*b^2 + 12*a*b^3)*d*x)*
cosh(d*x + c)^4 - 5*a^4 - 75*a^3*b - 192*a^2*b^2 - 128*a*b^3 - 16*(a^4 + 9*
a^3*b + 20*a^2*b^2 + 12*a*b^3)*d*x - 30*(27*a^3*b + 102*a^2*b^2 + 152*a*b^3
 + 80*b^4 + 4*(3*a^4 + 29*a^3*b + 82*a^2*b^2 + 104*a*b^3 + 48*b^4)*d*x)*cos
h(d*x + c)^2)*sinh(d*x + c)^4 - 2*a^4 - 2*a^3*b + 8*(55*(a^4 + a^3*b)*cosh(
d*x + c)^9 + 120*(a^4 + 3*a^3*b + 2*a^2*b^2 - (a^4 + 7*a^3*b + 6*a^2*b^2)*d
*x)*cosh(d*x + c)^7 + 14*(5*a^4 + 3*a^3*b - 32*a^2*b^2 - 32*a*b^3 - 16*(a^4
 + 9*a^3*b + 20*a^2*b^2 + 12*a*b^3)*d*x)*cosh(d*x + c)^5 - 10*(27*a^3*b + 1
02*a^2*b^2 + 152*a*b^3 + 80*b^4 + 4*(3*a^4 + 29*a^3*b + 82*a^2*b^2 + 104*a*
b^3 + 48*b^4)*d*x)*cosh(d*x + c)^3 - (5*a^4 + 75*a^3*b + 192*a^2*b^2 + 128*
a*b^3 + 16*(a^4 + 9*a^3*b + 20*a^2*b^2 + 12*a*b^3)*d*x)*cosh(d*x + c))*sinh
(d*x + c)^3 - 4*(2*a^4 + 15*a^3*b + 14*a^2*b^2 + 2*(a^4 + 7*a^3*b + 6*a^2*b
^2)*d*x)*cosh(d*x + c)^2 + 4*(33*(a^4 + a^3*b)*cosh(d*x + c)^10 + 90*(a^4 +
 3*a^3*b + 2*a^2*b^2 - (a^4 + 7*a^3*b + 6*a^2*b^2)*d*x)*cosh(d*x + c)^8 + 1
4*(5*a^4 + 3*a^3*b - 32*a^2*b^2 - 32*a*b^3 - 16*(a^4 + 9*a^3*b + 20*a^2*b^2
 + 12*a*b^3)*d*x)*cosh(d*x + c)^6 - 15*(27*a^3*b + 102*a^2*b^2 + 152*a*b^3
+ 80*b^4 + 4*(3*a^4 + 29*a^3*b + 82*a^2*b^2 + 104*a*b^3 + 48*b^4)*d*x)*cosh
(d*x + c)^4 - 2*a^4 - 15*a^3*b - 14*a^2*b^2 - 2*(a^4 + 7*a^3*b + 6*a^2*b^2)
*d*x - 3*(5*a^4 + 75*a^3*b + 192*a^2*b^2 + 128*a*b^3 + 16*(a^4 + 9*a^3*b +
20*a^2*b^2 + 12*a*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((15*a^4 + 4
0*a^3*b + 24*a^2*b^2)*cosh(d*x + c)^10 + 10*(15*a^4 + 40*a^3*b + 24*a^2*b^2
)*cosh(d*x + c)*sinh(d*x + c)^9 + (15*a^4 + 40*a^3*b + 24*a^2*b^2)*sinh(d*x
 + c)^10 + 4*(15*a^4 + 70*a^3*b + 104*a^2*b^2 + 48*a*b^3)*cosh(d*x + c)^8 +
 (60*a^4 + 280*a^3*b + 416*a^2*b^2 + 192*a*b^3 + 45*(15*a^4 + 40*a^3*b + 24
*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 8*(15*(15*a^4 + 40*a^3*b + 24*
a^2*b^2)*cosh(d*x + c)^3 + 4*(15*a^4 + 70*a^3*b + 104*a^2*b^2 + 48*a*b^3)*c
osh(d*x + c))*sinh(d*x + c)^7 + 2*(45*a^4 + 240*a^3*b + 512*a^2*b^2 + 512*a
*b^3 + 192*b^4)*cosh(d*x + c)^6 + 2*(105*(15*a^4 + 40*a^3*b + 24*a^2*b^2)*c
osh(d*x + c)^4 + 45*a^4 + 240*a^3*b + 512*a^2*b^2 + 512*a*b^3 + 192*b^4 + 5
6*(15*a^4 + 70*a^3*b + 104*a^2*b^2 + 48*a*b^3)*cosh(d*x + c)^2)*sinh(d*x +
c)^6 + 4*(63*(15*a^4 + 40*a^3*b + 24*a^2*b^2)*cosh(d*x + c)^5 + 56*(15*a^4
+ 70*a^3*b + 104*a^2*b^2 + 48*a*b^3)*cosh(d*x + c)^3 + 3*(45*a^4 + 240*a^3*
b + 512*a^2*b^2 + 512*a*b^3 + 192*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 4*(
15*a^4 + 70*a^3*b + 104*a^2*b^2 + 48*a*b^3)*cosh(d*x + c)^4 + 2*(105*(15*a^
4 + 40*a^3*b + 24*a^2*b^2)*cosh(d*x + c)^6 + 140*(15*a^4 + 70*a^3*b + 104*a
^2*b^2 + 48*a*b^3)*cosh(d*x + c)^4 + 30*a^4 + 140*a^3*b + 208*a^2*b^2 + 96*
a*b^3 + 15*(45*a^4 + 240*a^3*b + 512*a^2*b^2 + 512*a*b^3 + 192*b^4)*cosh(d*
x + c)^2)*sinh(d*x + c)^4 + 8*(15*(15*a^4 + 40*a^3*b + 24*a^2*b^2)*cosh(d*x
 + c)^7 + 28*(15*a^4 + 70*a^3*b + 104*a^2*b^2 + 48*a*b^3)*cosh(d*x + c)^5 +
 5*(45*a^4 + 240*a^3*b + 512*a^2*b^2 + 512*a*b^3 + 192*b^4)*cosh(d*x + c)^3
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 + 2*(15*a^4 + 70*a^3*b + 104*a^2*b^2 + 48*a*b^3)*cosh(d*x + c))*sinh(d*x +
 c)^3 + (15*a^4 + 40*a^3*b + 24*a^2*b^2)*cosh(d*x + c)^2 + (45*(15*a^4 + 40
*a^3*b + 24*a^2*b^2)*cosh(d*x + c)^8 + 112*(15*a^4 + 70*a^3*b + 104*a^2*b^2
 + 48*a*b^3)*cosh(d*x + c)^6 + 30*(45*a^4 + 240*a^3*b + 512*a^2*b^2 + 512*a
*b^3 + 192*b^4)*cosh(d*x + c)^4 + 15*a^4 + 40*a^3*b + 24*a^2*b^2 + 24*(15*a
^4 + 70*a^3*b + 104*a^2*b^2 + 48*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 +
2*(5*(15*a^4 + 40*a^3*b + 24*a^2*b^2)*cosh(d*x + c)^9 + 16*(15*a^4 + 70*a^3
*b + 104*a^2*b^2 + 48*a*b^3)*cosh(d*x + c)^7 + 6*(45*a^4 + 240*a^3*b + 512*
a^2*b^2 + 512*a*b^3 + 192*b^4)*cosh(d*x + c)^5 + 8*(15*a^4 + 70*a^3*b + 104
*a^2*b^2 + 48*a*b^3)*cosh(d*x + c)^3 + (15*a^4 + 40*a^3*b + 24*a^2*b^2)*cos
h(d*x + c))*sinh(d*x + c))*sqrt(b/(a + b))*log((a^2*cosh(d*x + c)^4 + 4*a^2
*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh
(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2
 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*si
nh(d*x + c) - 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*
sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 + 3*a*b + 2*b^2)*sqrt(b/(
a + b)))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*
x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*
sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x
+ c) + a)) + 8*(3*(a^4 + a^3*b)*cosh(d*x + c)^11 + 10*(a^4 + 3*a^3*b + 2*a^
2*b^2 - (a^4 + 7*a^3*b + 6*a^2*b^2)*d*x)*cosh(d*x + c)^9 + 2*(5*a^4 + 3*a^3
*b - 32*a^2*b^2 - 32*a*b^3 - 16*(a^4 + 9*a^3*b + 20*a^2*b^2 + 12*a*b^3)*d*x
)*cosh(d*x + c)^7 - 3*(27*a^3*b + 102*a^2*b^2 + 152*a*b^3 + 80*b^4 + 4*(3*a
^4 + 29*a^3*b + 82*a^2*b^2 + 104*a*b^3 + 48*b^4)*d*x)*cosh(d*x + c)^5 - (5*
a^4 + 75*a^3*b + 192*a^2*b^2 + 128*a*b^3 + 16*(a^4 + 9*a^3*b + 20*a^2*b^2 +
 12*a*b^3)*d*x)*cosh(d*x + c)^3 - (2*a^4 + 15*a^3*b + 14*a^2*b^2 + 2*(a^4 +
 7*a^3*b + 6*a^2*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a^7 + a^6*b)*d*c
osh(d*x + c)^10 + 10*(a^7 + a^6*b)*d*cosh(d*x + c)*sinh(d*x + c)^9 + (a^7 +
 a^6*b)*d*sinh(d*x + c)^10 + 4*(a^7 + 3*a^6*b + 2*a^5*b^2)*d*cosh(d*x + c)^
8 + (45*(a^7 + a^6*b)*d*cosh(d*x + c)^2 + 4*(a^7 + 3*a^6*b + 2*a^5*b^2)*d)*
sinh(d*x + c)^8 + 2*(3*a^7 + 11*a^6*b + 16*a^5*b^2 + 8*a^4*b^3)*d*cosh(d*x
+ c)^6 + 8*(15*(a^7 + a^6*b)*d*cosh(d*x + c)^3 + 4*(a^7 + 3*a^6*b + 2*a^5*b
^2)*d*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(105*(a^7 + a^6*b)*d*cosh(d*x + c)
^4 + 56*(a^7 + 3*a^6*b + 2*a^5*b^2)*d*cosh(d*x + c)^2 + (3*a^7 + 11*a^6*b +
 16*a^5*b^2 + 8*a^4*b^3)*d)*sinh(d*x + c)^6 + 4*(a^7 + 3*a^6*b + 2*a^5*b^2)
*d*cosh(d*x + c)^4 + 4*(63*(a^7 + a^6*b)*d*cosh(d*x + c)^5 + 56*(a^7 + 3*a^
6*b + 2*a^5*b^2)*d*cosh(d*x + c)^3 + 3*(3*a^7 + 11*a^6*b + 16*a^5*b^2 + 8*a
^4*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(105*(a^7 + a^6*b)*d*cosh(d*x
+ c)^6 + 140*(a^7 + 3*a^6*b + 2*a^5*b^2)*d*cosh(d*x + c)^4 + 15*(3*a^7 + 11
*a^6*b + 16*a^5*b^2 + 8*a^4*b^3)*d*cosh(d*x + c)^2 + 2*(a^7 + 3*a^6*b + 2*a
^5*b^2)*d)*sinh(d*x + c)^4 + (a^7 + a^6*b)*d*cosh(d*x + c)^2 + 8*(15*(a^7 +
 a^6*b)*d*cosh(d*x + c)^7 + 28*(a^7 + 3*a^6*b + 2*a^5*b^2)*d*cosh(d*x + c)^
5 + 5*(3*a^7 + 11*a^6*b + 16*a^5*b^2 + 8*a^4*b^3)*d*cosh(d*x + c)^3 + 2*(a^
7 + 3*a^6*b + 2*a^5*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^3 + (45*(a^7 + a^6*
b)*d*cosh(d*x + c)^8 + 112*(a^7 + 3*a^6*b + 2*a^5*b^2)*d*cosh(d*x + c)^6 +
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30*(3*a^7 + 11*a^6*b + 16*a^5*b^2 + 8*a^4*b^3)*d*cosh(d*x + c)^4 + 24*(a^7 + 3*a^6*b + 2*a^5*b^2)*d*cosh(d*x + c)^2 + (a^7 + a^6*b)*d)*sinh(d*x + c)^2 + 2*(5*(a^7 + a^6*b)*d*cosh(d*x + c)^9 + 16*(a^7 + 3*a^6*b + 2*a^5*b^2)*d*cosh(d*x + c)^7 + 6*(3*a^7 + 11*a^6*b + 16*a^5*b^2 + 8*a^4*b^3)*d*cosh(d*x + c)^5 + 8*(a^7 + 3*a^6*b + 2*a^5*b^2)*d*cosh(d*x + c)^3 + (a^7 + a^6*b)*d*cosh(d*x + c))*sinh(d*x + c)), 1/8*((a^4 + a^3*b)*cosh(d*x + c)^12 + 12*(a^4 + a^3*b)*cosh(d*x + c)*sinh(d*x + c)^11 + (a^4 + a^3*b)*sinh(d*x + c)^12 + 4*(a^4 + 3*a^3*b + 2*a^2*b^2 - (a^4 + 7*a^3*b + 6*a^2*b^2)*d*x)*cosh(d*x + c)^10 + 2*(2*a^4 + 6*a^3*b + 4*a^2*b^2 - 2*(a^4 + 7*a^3*b + 6*a^2*b^2)*d*x + 33*(a^4 + a^3*b)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 20*(11*(a^4 + a^3*b)*cosh(d*x + c)^3 + 2*(a^4 + 3*a^3*b + 2*a^2*b^2 - (a^4 + 7*a^3*b + 6*a^2*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^9 + (5*a^4 + 3*a^3*b - 32*a^2*b^2 - 32*a*b^3 - 16*(a^4 + 9*a^3*b + 20*a^2*b^2 + 12*a*b^3)*d*x)*cosh(d*x + c)^8 + (495*(a^4 + a^3*b)*cosh(d*x + c)^4 + 5*a^4 + 3*a^3*b - 32*a^2*b^2 - 32*a*b^3 - 16*(a^4 + 9*a^3*b + 20*a^2*b^2 + 12*a*b^3)*d*x + 180*(a^4 + 3*a^3*b + 2*a^2*b^2 - (a^4 + 7*a^3*b + 6*a^2*b^2)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 8*(99*(a^4 + a^3*b)*cosh(d*x + c)^5 + 60*(a^4 + 3*a^3*b + 2*a^2*b^2 - (a^4 + 7*a^3*b + 6*a^2*b^2)*d*x)*cosh(d*x + c)^3 + (5*a^4 + 3*a^3*b - 32*a^2*b^2 - 32*a*b^3 - 16*(a^4 + 9*a^3*b + 20*a^2*b^2 + 12*a*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^7 - 2*(27*a^3*b + 102*a^2*b^2 + 152*a*b^3 + 80*b^4 + 4*(3*a^4 + 29*a^3*b + 82*a^2*b^2 + 104*a*b^3 + 48*b^4)*d*x)*cosh(d*x + c)^6 + 2*(462*(a^4 + a^3*b)*cosh(d*x + c)^6 + 420*(a^4 + 3*a^3*b + 2*a^2*b^2 - (a^4 + 7*a^3*b + 6*a^2*b^2)*d*x)*cosh(d*x + c)^4 - 27*a^3*b - 102*a^2*b^2 - 152*a*b^3 - 80*b^4 - 4*(3*a^4 + 29*a^3*b + 82*a^2*b^2 + 104*a*b^3 + 48*b^4)*d*x + 14*(5*a^4 + 3*a^3*b - 32*a^2*b^2 - 32*a*b^3 - 16*(a^4 + 9*a^3*b + 20*a^2*b^2 + 12*a*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(198*(a^4 + a^3*b)*cosh(d*x + c)^7 + 252*(a^4 + 3*a^3*b + 2*a^2*b^2 - (a^4 + 7*a^3*b + 6*a^2*b^2)*d*x)*cosh(d*x + c)^5 + 14*(5*a^4 + 3*a^3*b - 32*a^2*b^2 - 32*a*b^3 - 16*(a^4 + 9*a^3*b + 20*a^2*b^2 + 12*a*b^3)*d*x)*cosh(d*x + c)^3 - 3*(27*a^3*b + 102*a^2*b^2 + 152*a*b^3 + 80*b^4 + 4*(3*a^4 + 29*a^3*b + 82*a^2*b^2 + 104*a*b^3 + 48*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 - (5*a^4 + 75*a^3*b + 192*a^2*b^2 + 128*a*b^3 + 16*(a^4 + 9*a^3*b + 20*a^2*b^2 + 12*a*b^3)*d*x)*cosh(d*x + c)^4 + (495*(a^4 + a^3*b)*cosh(d*x + c)^8 + 840*(a^4 + 3*a^3*b + 2*a^2*b^2 - (a^4 + 7*a^3*b + 6*a^2*b^2)*d*x)*cosh(d*x + c)^6 + 70*(5*a^4 + 3*a^3*b - 32*a^2*b^2 - 32*a*b^3 - 16*(a^4 + 9*a^3*b + 20*a^2*b^2 + 12*a*b^3)*d*x)*cosh(d*x + c)^4 - 5*a^4 - 75*a^3*b - 192*a^2*b^2 - 128*a*b^3 - 16*(a^4 + 9*a^3*b + 20*a^2*b^2 + 12*a*b^3)*d*x - 30*(27*a^3*b + 102*a^2*b^2 + 152*a*b^3 + 80*b^4 + 4*(3*a^4 + 29*a^3*b + 82*a^2*b^2 + 104*a*b^3 + 48*b^4)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 - a^4 - a^3*b + 4*(55*(a^4 + a^3*b)*cosh(d*x + c)^9 + 120*(a^4 + 3*a^3*b + 2*a^2*b^2 - (a^4 + 7*a^3*b + 6*a^2*b^2)*d*x)*cosh(d*x + c)^7 + 14*(5*a^4 + 3*a^3*b - 32*a^2*b^2 - 32*a*b^3 - 16*(a^4 + 9*a^3*b + 20*a^2*b^2 + 12*a*b^3)*d*x)*cosh(d*x + c)^5 - 10*(27*a^3*b + 102*a^2*b^2 + 152*a*b^3 + 80*b^4 + 4*(3*a^4 + 29*a^3*b + 82*a^2*b^2 + 104*a*b^3 + 48*b^4)*d*x)*cosh(d*x + c)^3 - (5*a^4 + 75*a^3*b + 192*a^2*b^2 + 128*a*b^3 + 16*(a^4 + 9*a^3*b + 20*a^2*b^2 + 12*a*b^3)*d*x)*cosh(d*

x + c))*sinh(d*x + c)^3 - 2*(2*a^4 + 15*a^3*b + 14*a^2*b^2 + 2*(a^4 + 7*a^3
*b + 6*a^2*b^2)*d*x)*cosh(d*x + c)^2 + 2*(33*(a^4 + a^3*b)*cosh(d*x + c)^10
 + 90*(a^4 + 3*a^3*b + 2*a^2*b^2 - (a^4 + 7*a^3*b + 6*a^2*b^2)*d*x)*cosh(d*
x + c)^8 + 14*(5*a^4 + 3*a^3*b - 32*a^2*b^2 - 32*a*b^3 - 16*(a^4 + 9*a^3*b
+ 20*a^2*b^2 + 12*a*b^3)*d*x)*cosh(d*x + c)^6 - 15*(27*a^3*b + 102*a^2*b^2
+ 152*a*b^3 + 80*b^4 + 4*(3*a^4 + 29*a^3*b + 82*a^2*b^2 + 104*a*b^3 + 48*b^
4)*d*x)*cosh(d*x + c)^4 - 2*a^4 - 15*a^3*b - 14*a^2*b^2 - 2*(a^4 + 7*a^3*b
+ 6*a^2*b^2)*d*x - 3*(5*a^4 + 75*a^3*b + 192*a^2*b^2 + 128*a*b^3 + 16*(a^4
+ 9*a^3*b + 20*a^2*b^2 + 12*a*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 +
((15*a^4 + 40*a^3*b + 24*a^2*b^2)*cosh(d*x + c)^10 + 10*(15*a^4 + 40*a^3*b
+ 24*a^2*b^2)*cosh(d*x + c)*sinh(d*x + c)^9 + (15*a^4 + 40*a^3*b + 24*a^2*b
^2)*sinh(d*x + c)^10 + 4*(15*a^4 + 70*a^3*b + 104*a^2*b^2 + 48*a*b^3)*cosh(
d*x + c)^8 + (60*a^4 + 280*a^3*b + 416*a^2*b^2 + 192*a*b^3 + 45*(15*a^4 + 4
0*a^3*b + 24*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 8*(15*(15*a^4 + 40
*a^3*b + 24*a^2*b^2)*cosh(d*x + c)^3 + 4*(15*a^4 + 70*a^3*b + 104*a^2*b^2 +
 48*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(45*a^4 + 240*a^3*b + 512*a^2
*b^2 + 512*a*b^3 + 192*b^4)*cosh(d*x + c)^6 + 2*(105*(15*a^4 + 40*a^3*b + 2
4*a^2*b^2)*cosh(d*x + c)^4 + 45*a^4 + 240*a^3*b + 512*a^2*b^2 + 512*a*b^3 +
 192*b^4 + 56*(15*a^4 + 70*a^3*b + 104*a^2*b^2 + 48*a*b^3)*cosh(d*x + c)^2)
*sinh(d*x + c)^6 + 4*(63*(15*a^4 + 40*a^3*b + 24*a^2*b^2)*cosh(d*x + c)^5 +
 56*(15*a^4 + 70*a^3*b + 104*a^2*b^2 + 48*a*b^3)*cosh(d*x + c)^3 + 3*(45*a^
4 + 240*a^3*b + 512*a^2*b^2 + 512*a*b^3 + 192*b^4)*cosh(d*x + c))*sinh(d*x
+ c)^5 + 4*(15*a^4 + 70*a^3*b + 104*a^2*b^2 + 48*a*b^3)*cosh(d*x + c)^4 + 2
(105(15*a^4 + 40*a^3*b + 24*a^2*b^2)*cosh(d*x + c)^6 + 140*(15*a^4 + 70*a
^3*b + 104*a^2*b^2 + 48*a*b^3)*cosh(d*x + c)^4 + 30*a^4 + 140*a^3*b + 208*a
^2*b^2 + 96*a*b^3 + 15*(45*a^4 + 240*a^3*b + 512*a^2*b^2 + 512*a*b^3 + 192*
b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(15*(15*a^4 + 40*a^3*b + 24*a^2*b
^2)*cosh(d*x + c)^7 + 28*(15*a^4 + 70*a^3*b + 104*a^2*b^2 + 48*a*b^3)*cosh(
d*x + c)^5 + 5*(45*a^4 + 240*a^3*b + 512*a^2*b^2 + 512*a*b^3 + 192*b^4)*cos
h(d*x + c)^3 + 2*(15*a^4 + 70*a^3*b + 104*a^2*b^2 + 48*a*b^3)*cosh(d*x + c)
)*sinh(d*x + c)^3 + (15*a^4 + 40*a^3*b + 24*a^2*b^2)*cosh(d*x + c)^2 + (45*
(15*a^4 + 40*a^3*b + 24*a^2*b^2)*cosh(d*x + c)^8 + 112*(15*a^4 + 70*a^3*b +
 104*a^2*b^2 + 48*a*b^3)*cosh(d*x + c)^6 + 30*(45*a^4 + 240*a^3*b + 512*a^2
*b^2 + 512*a*b^3 + 192*b^4)*cosh(d*x + c)^4 + 15*a^4 + 40*a^3*b + 24*a^2*b^
2 + 24*(15*a^4 + 70*a^3*b + 104*a^2*b^2 + 48*a*b^3)*cosh(d*x + c)^2)*sinh(d
x + c)^2 + 2(5*(15*a^4 + 40*a^3*b + 24*a^2*b^2)*cosh(d*x + c)^9 + 16*(15*
a^4 + 70*a^3*b + 104*a^2*b^2 + 48*a*b^3)*cosh(d*x + c)^7 + 6*(45*a^4 + 240*
a^3*b + 512*a^2*b^2 + 512*a*b^3 + 192*b^4)*cosh(d*x + c)^5 + 8*(15*a^4 + 70
*a^3*b + 104*a^2*b^2 + 48*a*b^3)*cosh(d*x + c)^3 + (15*a^4 + 40*a^3*b + 24*
a^2*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b/(a + b))*arctan(1/2*(a*cosh(
d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)
sqrt(-b/(a + b))/b) + 4(3*(a^4 + a^3*b)*cosh(d*x + c)^11 + 10*(a^4 + 3*a^
3*b + 2*a^2*b^2 - (a^4 + 7*a^3*b + 6*a^2*b^2)*d*x)*cosh(d*x + c)^9 + 2*(5*a
^4 + 3*a^3*b - 32*a^2*b^2 - 32*a*b^3 - 16*(a^4 + 9*a^3*b + 20*a^2*b^2 + 12*
a*b^3)*d*x)*cosh(d*x + c)^7 - 3*(27*a^3*b + 102*a^2*b^2 + 152*a*b^3 + 80*b^

```
4 + 4*(3*a^4 + 29*a^3*b + 82*a^2*b^2 + 104*a*b^3 + 48*b^4)*d*x)*cosh(d*x +
c)^5 - (5*a^4 + 75*a^3*b + 192*a^2*b^2 + 128*a*b^3 + 16*(a^4 + 9*a^3*b + 20
*a^2*b^2 + 12*a*b^3)*d*x)*cosh(d*x + c)^3 - (2*a^4 + 15*a^3*b + 14*a^2*b^2
+ 2*(a^4 + 7*a^3*b + 6*a^2*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a^7 +
a^6*b)*d*cosh(d*x + c)^10 + 10*(a^7 + a^6*b)*d*cosh(d*x + c)*sinh(d*x + c)^
9 + (a^7 + a^6*b)*d*sinh(d*x + c)^10 + 4*(a^7 + 3*a^6*b + 2*a^5*b^2)*d*cosh
(d*x + c)^8 + (45*(a^7 + a^6*b)*d*cosh(d*x + c)^2 + 4*(a^7 + 3*a^6*b + 2*a^
5*b^2)*d)*sinh(d*x + c)^8 + 2*(3*a^7 + 11*a^6*b + 16*a^5*b^2 + 8*a^4*b^3)*d
*cosh(d*x + c)^6 + 8*(15*(a^7 + a^6*b)*d*cosh(d*x + c)^3 + 4*(a^7 + 3*a^6*b
 + 2*a^5*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(105*(a^7 + a^6*b)*d*cos
h(d*x + c)^4 + 56*(a^7 + 3*a^6*b + 2*a^5*b^2)*d*cosh(d*x + c)^2 + (3*a^7 +
11*a^6*b + 16*a^5*b^2 + 8*a^4*b^3)*d)*sinh(d*x + c)^6 + 4*(a^7 + 3*a^6*b +
2*a^5*b^2)*d*cosh(d*x + c)^4 + 4*(63*(a^7 + a^6*b)*d*cosh(d*x + c)^5 + 56*(
a^7 + 3*a^6*b + 2*a^5*b^2)*d*cosh(d*x + c)^3 + 3*(3*a^7 + 11*a^6*b + 16*a^5
*b^2 + 8*a^4*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(105*(a^7 + a^6*b)*d
*cosh(d*x + c)^6 + 140*(a^7 + 3*a^6*b + 2*a^5*b^2)*d*cosh(d*x + c)^4 + 15*(
3*a^7 + 11*a^6*b + 16*a^5*b^2 + 8*a^4*b^3)*d*cosh(d*x + c)^2 + 2*(a^7 + 3*a
^6*b + 2*a^5*b^2)*d)*sinh(d*x + c)^4 + (a^7 + a^6*b)*d*cosh(d*x + c)^2 + 8*
(15*(a^7 + a^6*b)*d*cosh(d*x + c)^7 + 28*(a^7 + 3*a^6*b + 2*a^5*b^2)*d*cosh
(d*x + c)^5 + 5*(3*a^7 + 11*a^6*b + 16*a^5*b^2 + 8*a^4*b^3)*d*cosh(d*x + c)
^3 + 2*(a^7 + 3*a^6*b + 2*a^5*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^3 + (45*(
a^7 + a^6*b)*d*cosh(d*x + c)^8 + 112*(a^7 + 3*a^6*b + 2*a^5*b^2)*d*cosh(d*x
 + c)^6 + 30*(3*a^7 + 11*a^6*b + 16*a^5*b^2 + 8*a^4*b^3)*d*cosh(d*x + c)^4
+ 24*(a^7 + 3*a^6*b + 2*a^5*b^2)*d*cosh(d*x + c)^2 + (a^7 + a^6*b)*d)*sinh(
d*x + c)^2 + 2*(5*(a^7 + a^6*b)*d*cosh(d*x + c)^9 + 16*(a^7 + 3*a^6*b + 2*a
^5*b^2)*d*cosh(d*x + c)^7 + 6*(3*a^7 + 11*a^6*b + 16*a^5*b^2 + 8*a^4*b^3)*d
*cosh(d*x + c)^5 + 8*(a^7 + 3*a^6*b + 2*a^5*b^2)*d*cosh(d*x + c)^3 + (a^7 +
 a^6*b)*d*cosh(d*x + c))*sinh(d*x + c))]
```

giac [B] time = 3.70, size = 370, normalized size = 1.98

$$\frac{\frac{\left(15\,a^2b+40\,ab^2+24\,b^3\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{\left(a^5+a^4b\right)\sqrt{-ab-b^2}} - \frac{2\left(9\,a^3be^{(6\,dx+6\,c)}+32\,a^2b^2e^{(6\,dx+6\,c)}+24\,ab^3e^{(6\,dx+6\,c)}+27\,a^3be^{(4\,dx+4\,c)}+102\,a^2b^2e^{(4\,dx+4\,c)}+152\,ab^3 \cdots\right.}{\left(a^5+a^4b\right)\left(ae^{(4\,dx+4\,c)}+2\,ae^{(2 \cdots}\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*((15*a^2*b + 40*a*b^2 + 24*b^3)*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b
)/sqrt(-a*b - b^2))/((a^5 + a^4*b)*sqrt(-a*b - b^2)) - 2*(9*a^3*b*e^(6*d*x
+ 6*c) + 32*a^2*b^2*e^(6*d*x + 6*c) + 24*a*b^3*e^(6*d*x + 6*c) + 27*a^3*b*e
^(4*d*x + 4*c) + 102*a^2*b^2*e^(4*d*x + 4*c) + 152*a*b^3*e^(4*d*x + 4*c) +
80*b^4*e^(4*d*x + 4*c) + 27*a^3*b*e^(2*d*x + 2*c) + 80*a^2*b^2*e^(2*d*x + 2
*c) + 56*a*b^3*e^(2*d*x + 2*c) + 9*a^3*b + 10*a^2*b^2)/((a^5 + a^4*b)*(a*e^
```

```
(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) + a)^2) - 4*(d*x
+ c)*(a + 6*b)/a^4 + e^(2*d*x + 2*c)/a^3 + (2*a*e^(2*d*x + 2*c) + 12*b*e^(2
*d*x + 2*c) - a)*e^(-2*d*x - 2*c)/a^4)/d
```

maple [B] time = 0.39, size = 1329, normalized size = 7.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] 1/2/d/a^3/(tanh(1/2*d*x+1/2*c)-1)^2+1/2/d/a^3/(tanh(1/2*d*x+1/2*c)-1)+1/2/d
/a^3*ln(tanh(1/2*d*x+1/2*c)-1)+3/d/a^4*ln(tanh(1/2*d*x+1/2*c)-1)*b-1/2/d/a^
3/(tanh(1/2*d*x+1/2*c)+1)^2+1/2/d/a^3/(tanh(1/2*d*x+1/2*c)+1)-1/2/d/a^3*ln(
tanh(1/2*d*x+1/2*c)+1)-3/d/a^4*ln(tanh(1/2*d*x+1/2*c)+1)*b+9/4/d/a^2*b/(tan
h(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*ta
nh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^7+2/d/a^3*b^2/(tanh(1/2*d*
x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d
*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^7+27/4/d*b/a/(tanh(1/2*d*x+1/2*c)^
4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)
^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)^5+23/4/d*b^2/a^2/(tanh(1/2*d*x+1/2*c)
^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c
)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)^5-2/d*b^3/a^3/(tanh(1/2*d*x+1/2*c)^4
*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^
2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)^5+27/4/d*b/a/(tanh(1/2*d*x+1/2*c)^4*a+
b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b
+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)^3+23/4/d*b^2/a^2/(tanh(1/2*d*x+1/2*c)^4*a
+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*
b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)^3-2/d*b^3/a^3/(tanh(1/2*d*x+1/2*c)^4*a+b
*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+
a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)^3+9/4/d/a^2*b/(tanh(1/2*d*x+1/2*c)^4*a+b*t
anh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+
b)^2*tanh(1/2*d*x+1/2*c)+2/d/a^3*b^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*
x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(
1/2*d*x+1/2*c)-15/16/d*b^(1/2)/a^2/(a+b)^(3/2)*ln(-(a+b)^(1/2)*tanh(1/2*d*x
+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)-(a+b)^(1/2))-5/2/d*b^(3/2)/a^3/(a+b
)^(3/2)*ln(-(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)
-(a+b)^(1/2))-3/2/d*b^(5/2)/a^4/(a+b)^(3/2)*ln(-(a+b)^(1/2)*tanh(1/2*d*x+1/
2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)-(a+b)^(1/2))+15/16/d*b^(1/2)/a^2/(a+b)
^(3/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(
a+b)^(1/2))+5/2/d*b^(3/2)/a^3/(a+b)^(3/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c
)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))+3/2/d*b^(5/2)/a^4/(a+b)^(3/2
)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^
(1/2))
```

maxima [B] time = 0.53, size = 1373, normalized size = 7.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 3/64*(5*a^3*b + 30*a^2*b^2 + 40*a*b^3 + 16*b^4)*log((a*e^(2*d*x + 2*c) + a
+ 2*b - 2*sqrt((a + b)*b))/(a*e^(2*d*x + 2*c) + a + 2*b + 2*sqrt((a + b)*b)
))/((a^6 + 2*a^5*b + a^4*b^2)*sqrt((a + b)*b)*d) - 3/64*(5*a^3*b + 30*a^2*b
^2 + 40*a*b^3 + 16*b^4)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*
b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^6 + 2*a^5*b + a
^4*b^2)*sqrt((a + b)*b)*d) - 1/32*(15*a^2*b + 20*a*b^2 + 8*b^3)*log((a*e^(-
2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b +
 2*sqrt((a + b)*b)))/((a^5 + 2*a^4*b + a^3*b^2)*sqrt((a + b)*b)*d) - 1/16*(
9*a^4*b + 32*a^3*b^2 + 20*a^2*b^3 + 3*(3*a^4*b + 34*a^3*b^2 + 64*a^2*b^3 +
32*a*b^4)*e^(6*d*x + 6*c) + (27*a^4*b + 264*a^3*b^2 + 740*a^2*b^3 + 832*a*b
^4 + 320*b^5)*e^(4*d*x + 4*c) + (27*a^4*b + 194*a^3*b^2 + 336*a^2*b^3 + 160
*a*b^4)*e^(2*d*x + 2*c))/((a^8 + 2*a^7*b + a^6*b^2 + (a^8 + 2*a^7*b + a^6*b
^2)*e^(8*d*x + 8*c) + 4*(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*b^3)*e^(6*d*x +
6*c) + 2*(3*a^8 + 14*a^7*b + 27*a^6*b^2 + 24*a^5*b^3 + 8*a^4*b^4)*e^(4*d*x
+ 4*c) + 4*(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*b^3)*e^(2*d*x + 2*c))*d) + 1/
16*(9*a^4*b + 32*a^3*b^2 + 20*a^2*b^3 + (27*a^4*b + 194*a^3*b^2 + 336*a^2*b
^3 + 160*a*b^4)*e^(-2*d*x - 2*c) + (27*a^4*b + 264*a^3*b^2 + 740*a^2*b^3 +
832*a*b^4 + 320*b^5)*e^(-4*d*x - 4*c) + 3*(3*a^4*b + 34*a^3*b^2 + 64*a^2*b^
3 + 32*a*b^4)*e^(-6*d*x - 6*c))/((a^8 + 2*a^7*b + a^6*b^2 + 4*(a^8 + 4*a^7*
b + 5*a^6*b^2 + 2*a^5*b^3)*e^(-2*d*x - 2*c) + 2*(3*a^8 + 14*a^7*b + 27*a^6*
b^2 + 24*a^5*b^3 + 8*a^4*b^4)*e^(-4*d*x - 4*c) + 4*(a^8 + 4*a^7*b + 5*a^6*b
^2 + 2*a^5*b^3)*e^(-6*d*x - 6*c) + (a^8 + 2*a^7*b + a^6*b^2)*e^(-8*d*x - 8*
c))*d) + 1/8*(9*a^3*b + 6*a^2*b^2 + (27*a^3*b + 68*a^2*b^2 + 32*a*b^3)*e^(-
2*d*x - 2*c) + 3*(9*a^3*b + 30*a^2*b^2 + 40*a*b^3 + 16*b^4)*e^(-4*d*x - 4*c
) + (9*a^3*b + 28*a^2*b^2 + 16*a*b^3)*e^(-6*d*x - 6*c))/((a^7 + 2*a^6*b + a
^5*b^2 + 4*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*e^(-2*d*x - 2*c) + 2*(3*
a^7 + 14*a^6*b + 27*a^5*b^2 + 24*a^4*b^3 + 8*a^3*b^4)*e^(-4*d*x - 4*c) + 4*
(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*e^(-6*d*x - 6*c) + (a^7 + 2*a^6*b +
 a^5*b^2)*e^(-8*d*x - 8*c))*d) - 1/2*(d*x + c)/(a^3*d) + 1/8*e^(2*d*x + 2*c
)/(a^3*d) - 1/8*e^(-2*d*x - 2*c)/(a^3*d) - 3/4*b*log(a*e^(4*d*x + 4*c) + 2*
(a + 2*b)*e^(2*d*x + 2*c) + a)/(a^4*d) + 3/4*b*log(2*(a + 2*b)*e^(-2*d*x -
2*c) + a*e^(-4*d*x - 4*c) + a)/(a^4*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+d\,x)^6\,\sinh(c+d\,x)^2}{\left(a\cosh(c+d\,x)^2+b\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^2/(a + b/cosh(c + d*x)^2)^3,x)
```

```
[Out] int((cosh(c + d*x)^6*sinh(c + d*x)^2)/(b + a*cosh(c + d*x)^2)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

3.44 $\int \frac{\sinh(c+dx)}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=116

$$-\frac{15\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{8a^{7/2}d}+\frac{15\cosh(c+dx)}{8a^3d}-\frac{5\cosh^3(c+dx)}{8a^2d\left(a\cosh^2(c+dx)+b\right)}-\frac{\cosh^5(c+dx)}{4ad\left(a\cosh^2(c+dx)+b\right)^2}$$

```
[Out] 15/8*cosh(d*x+c)/a^3/d-1/4*cosh(d*x+c)^5/a/d/(b+a*cosh(d*x+c)^2)^2-5/8*cosh
(d*x+c)^3/a^2/d/(b+a*cosh(d*x+c)^2)-15/8*arctan(cosh(d*x+c)*a^(1/2)/b^(1/2)
)*b^(1/2)/a^(7/2)/d
```

Rubi [A] time = 0.08, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {4133, 288, 321, 205}

$$-\frac{5\cosh^3(c+dx)}{8a^2d\left(a\cosh^2(c+dx)+b\right)}-\frac{15\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{8a^{7/2}d}+\frac{15\cosh(c+dx)}{8a^3d}-\frac{\cosh^5(c+dx)}{4ad\left(a\cosh^2(c+dx)+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (-15*Sqrt[b]*ArcTan[(Sqrt[a]*Cosh[c + d*x])/Sqrt[b]])/(8*a^(7/2)*d) + (15*C
osh[c + d*x])/(8*a^3*d) - Cosh[c + d*x]^5/(4*a*d*(b + a*Cosh[c + d*x]^2)^2)
 - (5*Cosh[c + d*x]^3)/(8*a^2*d*(b + a*Cosh[c + d*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sinh(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = \frac{\operatorname{Subst}\left(\int \frac{x^6}{\left(b+ax^2\right)^3}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{\cosh^5(c+dx)}{4ad\left(b+a\cosh^2(c+dx)\right)^2} + \frac{5\operatorname{Subst}\left(\int \frac{x^4}{\left(b+ax^2\right)^2}\,dx, x, \cosh(c+dx)\right)}{4ad}$$

$$= -\frac{\cosh^5(c+dx)}{4ad\left(b+a\cosh^2(c+dx)\right)^2} - \frac{5\cosh^3(c+dx)}{8a^2d\left(b+a\cosh^2(c+dx)\right)} + \frac{15\operatorname{Subst}\left(\int \frac{x^2}{b+ax^2}\,c\right.}{8a}$$

$$= \frac{15\cosh(c+dx)}{8a^3d} - \frac{\cosh^5(c+dx)}{4ad\left(b+a\cosh^2(c+dx)\right)^2} - \frac{5\cosh^3(c+dx)}{8a^2d\left(b+a\cosh^2(c+dx)\right)} - \frac{(1}{}$$

$$= -\frac{15\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{8a^{7/2}d} + \frac{15\cosh(c+dx)}{8a^3d} - \frac{\cosh^5(c+dx)}{4ad\left(b+a\cosh^2(c+dx)\right)^2} -$$

Mathematica [C] time = 9.60, size = 453, normalized size = 3.91

$$\frac{\operatorname{sech}^6(c+dx)(a\cosh(2(c+dx))+a+2b)^3\left(\frac{512\sinh(c)\sinh(dx)}{a^3}+\frac{512\cosh(c)\cosh(dx)}{a^3}-\frac{15\left(\left(a^3+64b^3\right)\tan^{-1}\left(\frac{\sinh(c)\tanh\left(\frac{dx}{2}\right)\left(\sqrt{a}-\cdots\right.}{\cdots}\right.\right.}{\cdots}\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sinh[c + d*x]/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])^3*Sech[c + d*x]^6*((-15*((a^3 + 64*b^3)*Ar
cTan[((Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d
*x)/2] + Cosh[c]*(Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[
(d*x)/2]))/Sqrt[b]] + (a^3 + 64*b^3)*ArcTan[((Sqrt[a] + I*Sqrt[a + b]*Sqrt[
(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] + I*Sqrt[a
 + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]] - a^3*(ArcTan[(S
qrt[a] - I*Sqrt[a + b]*Tanh[(c + d*x)/2])/Sqrt[b]] + ArcTan[(Sqrt[a] + I*Sq
rt[a + b]*Tanh[(c + d*x)/2])/Sqrt[b]])))/(a^(7/2)*b^(5/2)) + (512*Cosh[c]*C
osh[d*x])/a^3 + (8*Cosh[c + d*x]*(16*b^3*(9*a + 14*b) + 3*(a^4 + 48*a*b^3)*
Cosh[2*(c + d*x)]))/(a^3*b^2*(a + 2*b + a*Cosh[2*(c + d*x)])^2) + (512*Sinh
[c]*Sinh[d*x])/a^3 - (6*a*Csch[c + d*x]*Sinh[4*(c + d*x)])/(b^2*(a + 2*b +
a*Cosh[2*(c + d*x)])^2)))/(4096*d*(a + b*Sech[c + d*x]^2)^3)
```

fricas [B] time = 0.51, size = 4829, normalized size = 41.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(8*a^2*cosh(d*x + c)^10 + 80*a^2*cosh(d*x + c)*sinh(d*x + c)^9 + 8*a^
2*sinh(d*x + c)^10 + 20*(2*a^2 + 5*a*b)*cosh(d*x + c)^8 + 20*(18*a^2*cosh(d
*x + c)^2 + 2*a^2 + 5*a*b)*sinh(d*x + c)^8 + 160*(6*a^2*cosh(d*x + c)^3 + (
2*a^2 + 5*a*b)*cosh(d*x + c))*sinh(d*x + c)^7 + 20*(4*a^2 + 15*a*b + 12*b^2
)*cosh(d*x + c)^6 + 20*(84*a^2*cosh(d*x + c)^4 + 28*(2*a^2 + 5*a*b)*cosh(d*
x + c)^2 + 4*a^2 + 15*a*b + 12*b^2)*sinh(d*x + c)^6 + 8*(252*a^2*cosh(d*x +
 c)^5 + 140*(2*a^2 + 5*a*b)*cosh(d*x + c)^3 + 15*(4*a^2 + 15*a*b + 12*b^2)*
cosh(d*x + c))*sinh(d*x + c)^5 + 20*(4*a^2 + 15*a*b + 12*b^2)*cosh(d*x + c)
^4 + 20*(84*a^2*cosh(d*x + c)^6 + 70*(2*a^2 + 5*a*b)*cosh(d*x + c)^4 + 15*(
4*a^2 + 15*a*b + 12*b^2)*cosh(d*x + c)^2 + 4*a^2 + 15*a*b + 12*b^2)*sinh(d*
x + c)^4 + 80*(12*a^2*cosh(d*x + c)^7 + 14*(2*a^2 + 5*a*b)*cosh(d*x + c)^5
```

```
+ 5*(4*a^2 + 15*a*b + 12*b^2)*cosh(d*x + c)^3 + (4*a^2 + 15*a*b + 12*b^2)*c
osh(d*x + c))*sinh(d*x + c)^3 + 20*(2*a^2 + 5*a*b)*cosh(d*x + c)^2 + 20*(18
*a^2*cosh(d*x + c)^8 + 28*(2*a^2 + 5*a*b)*cosh(d*x + c)^6 + 15*(4*a^2 + 15*
a*b + 12*b^2)*cosh(d*x + c)^4 + 6*(4*a^2 + 15*a*b + 12*b^2)*cosh(d*x + c)^2
 + 2*a^2 + 5*a*b)*sinh(d*x + c)^2 + 15*(a^2*cosh(d*x + c)^9 + 9*a^2*cosh(d*
x + c)*sinh(d*x + c)^8 + a^2*sinh(d*x + c)^9 + 4*(a^2 + 2*a*b)*cosh(d*x + c
)^7 + 4*(9*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^7 + 28*(3*a^2*c
osh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c)^6 + 2*(3*a^2 +
8*a*b + 8*b^2)*cosh(d*x + c)^5 + 2*(63*a^2*cosh(d*x + c)^4 + 42*(a^2 + 2*a*
b)*cosh(d*x + c)^2 + 3*a^2 + 8*a*b + 8*b^2)*sinh(d*x + c)^5 + 2*(63*a^2*cos
h(d*x + c)^5 + 70*(a^2 + 2*a*b)*cosh(d*x + c)^3 + 5*(3*a^2 + 8*a*b + 8*b^2)
*cosh(d*x + c))*sinh(d*x + c)^4 + 4*(a^2 + 2*a*b)*cosh(d*x + c)^3 + 4*(21*a
^2*cosh(d*x + c)^6 + 35*(a^2 + 2*a*b)*cosh(d*x + c)^4 + 5*(3*a^2 + 8*a*b +
8*b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^3 + a^2*cosh(d*x + c) +
 4*(9*a^2*cosh(d*x + c)^7 + 21*(a^2 + 2*a*b)*cosh(d*x + c)^5 + 5*(3*a^2 + 8
*a*b + 8*b^2)*cosh(d*x + c)^3 + 3*(a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c
)^2 + (9*a^2*cosh(d*x + c)^8 + 28*(a^2 + 2*a*b)*cosh(d*x + c)^6 + 10*(3*a^2
 + 8*a*b + 8*b^2)*cosh(d*x + c)^4 + 12*(a^2 + 2*a*b)*cosh(d*x + c)^2 + a^2)
*sinh(d*x + c))*sqrt(-b/a)*log((a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(
d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a - 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(
d*x + c)^2 + a - 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a - 2*b)*co
sh(d*x + c))*sinh(d*x + c) - 4*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(
d*x + c)^2 + a*sinh(d*x + c)^3 + a*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + a
)*sinh(d*x + c))*sqrt(-b/a) + a)/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sin
h(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cos
h(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*
cosh(d*x + c))*sinh(d*x + c) + a)) + 8*a^2 + 40*(2*a^2*cosh(d*x + c)^9 + 4*
(2*a^2 + 5*a*b)*cosh(d*x + c)^7 + 3*(4*a^2 + 15*a*b + 12*b^2)*cosh(d*x + c)
^5 + 2*(4*a^2 + 15*a*b + 12*b^2)*cosh(d*x + c)^3 + (2*a^2 + 5*a*b)*cosh(d*x
 + c))*sinh(d*x + c))/(a^5*d*cosh(d*x + c)^9 + 9*a^5*d*cosh(d*x + c)*sinh(d
*x + c)^8 + a^5*d*sinh(d*x + c)^9 + 4*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^7 + 4
*(9*a^5*d*cosh(d*x + c)^2 + (a^5 + 2*a^4*b)*d)*sinh(d*x + c)^7 + a^5*d*cosh
(d*x + c) + 2*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*d*cosh(d*x + c)^5 + 28*(3*a^5*d
*cosh(d*x + c)^3 + (a^5 + 2*a^4*b)*d*cosh(d*x + c))*sinh(d*x + c)^6 + 2*(63
*a^5*d*cosh(d*x + c)^4 + 42*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^2 + (3*a^5 + 8*
a^4*b + 8*a^3*b^2)*d)*sinh(d*x + c)^5 + 4*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^3
 + 2*(63*a^5*d*cosh(d*x + c)^5 + 70*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^3 + 5*(
3*a^5 + 8*a^4*b + 8*a^3*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^4 + 4*(21*a^5*d
*cosh(d*x + c)^6 + 35*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^4 + 5*(3*a^5 + 8*a^4*
b + 8*a^3*b^2)*d*cosh(d*x + c)^2 + (a^5 + 2*a^4*b)*d)*sinh(d*x + c)^3 + 4*(
9*a^5*d*cosh(d*x + c)^7 + 21*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^5 + 5*(3*a^5 +
 8*a^4*b + 8*a^3*b^2)*d*cosh(d*x + c)^3 + 3*(a^5 + 2*a^4*b)*d*cosh(d*x + c)
)*sinh(d*x + c)^2 + (9*a^5*d*cosh(d*x + c)^8 + 28*(a^5 + 2*a^4*b)*d*cosh(d*
x + c)^6 + a^5*d + 10*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*d*cosh(d*x + c)^4 + 12*
(a^5 + 2*a^4*b)*d*cosh(d*x + c)^2)*sinh(d*x + c)), 1/8*(4*a^2*cosh(d*x + c)
```

```
^10 + 40*a^2*cosh(d*x + c)*sinh(d*x + c)^9 + 4*a^2*sinh(d*x + c)^10 + 10*(2
*a^2 + 5*a*b)*cosh(d*x + c)^8 + 10*(18*a^2*cosh(d*x + c)^2 + 2*a^2 + 5*a*b)
*sinh(d*x + c)^8 + 80*(6*a^2*cosh(d*x + c)^3 + (2*a^2 + 5*a*b)*cosh(d*x + c
))*sinh(d*x + c)^7 + 10*(4*a^2 + 15*a*b + 12*b^2)*cosh(d*x + c)^6 + 10*(84*
a^2*cosh(d*x + c)^4 + 28*(2*a^2 + 5*a*b)*cosh(d*x + c)^2 + 4*a^2 + 15*a*b +
 12*b^2)*sinh(d*x + c)^6 + 4*(252*a^2*cosh(d*x + c)^5 + 140*(2*a^2 + 5*a*b)
*cosh(d*x + c)^3 + 15*(4*a^2 + 15*a*b + 12*b^2)*cosh(d*x + c))*sinh(d*x + c
)^5 + 10*(4*a^2 + 15*a*b + 12*b^2)*cosh(d*x + c)^4 + 10*(84*a^2*cosh(d*x +
c)^6 + 70*(2*a^2 + 5*a*b)*cosh(d*x + c)^4 + 15*(4*a^2 + 15*a*b + 12*b^2)*co
sh(d*x + c)^2 + 4*a^2 + 15*a*b + 12*b^2)*sinh(d*x + c)^4 + 40*(12*a^2*cosh(
d*x + c)^7 + 14*(2*a^2 + 5*a*b)*cosh(d*x + c)^5 + 5*(4*a^2 + 15*a*b + 12*b^
2)*cosh(d*x + c)^3 + (4*a^2 + 15*a*b + 12*b^2)*cosh(d*x + c))*sinh(d*x + c)
^3 + 10*(2*a^2 + 5*a*b)*cosh(d*x + c)^2 + 10*(18*a^2*cosh(d*x + c)^8 + 28*(
2*a^2 + 5*a*b)*cosh(d*x + c)^6 + 15*(4*a^2 + 15*a*b + 12*b^2)*cosh(d*x + c)
^4 + 6*(4*a^2 + 15*a*b + 12*b^2)*cosh(d*x + c)^2 + 2*a^2 + 5*a*b)*sinh(d*x
+ c)^2 + 15*(a^2*cosh(d*x + c)^9 + 9*a^2*cosh(d*x + c)*sinh(d*x + c)^8 + a^
2*sinh(d*x + c)^9 + 4*(a^2 + 2*a*b)*cosh(d*x + c)^7 + 4*(9*a^2*cosh(d*x + c
)^2 + a^2 + 2*a*b)*sinh(d*x + c)^7 + 28*(3*a^2*cosh(d*x + c)^3 + (a^2 + 2*a
*b)*cosh(d*x + c))*sinh(d*x + c)^6 + 2*(3*a^2 + 8*a*b + 8*b^2)*cosh(d*x + c
)^5 + 2*(63*a^2*cosh(d*x + c)^4 + 42*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 3*a^2
+ 8*a*b + 8*b^2)*sinh(d*x + c)^5 + 2*(63*a^2*cosh(d*x + c)^5 + 70*(a^2 + 2*
a*b)*cosh(d*x + c)^3 + 5*(3*a^2 + 8*a*b + 8*b^2)*cosh(d*x + c))*sinh(d*x +
c)^4 + 4*(a^2 + 2*a*b)*cosh(d*x + c)^3 + 4*(21*a^2*cosh(d*x + c)^6 + 35*(a^
2 + 2*a*b)*cosh(d*x + c)^4 + 5*(3*a^2 + 8*a*b + 8*b^2)*cosh(d*x + c)^2 + a^
2 + 2*a*b)*sinh(d*x + c)^3 + a^2*cosh(d*x + c) + 4*(9*a^2*cosh(d*x + c)^7 +
 21*(a^2 + 2*a*b)*cosh(d*x + c)^5 + 5*(3*a^2 + 8*a*b + 8*b^2)*cosh(d*x + c)
^3 + 3*(a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c)^2 + (9*a^2*cosh(d*x + c)^
8 + 28*(a^2 + 2*a*b)*cosh(d*x + c)^6 + 10*(3*a^2 + 8*a*b + 8*b^2)*cosh(d*x
+ c)^4 + 12*(a^2 + 2*a*b)*cosh(d*x + c)^2 + a^2)*sinh(d*x + c))*sqrt(b/a)*a
rctan(1/2*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d
*x + c)^3 + (a + 4*b)*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + a + 4*b)*sinh(
d*x + c))*sqrt(b/a)/b) - 15*(a^2*cosh(d*x + c)^9 + 9*a^2*cosh(d*x + c)*sinh
(d*x + c)^8 + a^2*sinh(d*x + c)^9 + 4*(a^2 + 2*a*b)*cosh(d*x + c)^7 + 4*(9*
a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^7 + 28*(3*a^2*cosh(d*x + c
)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c)^6 + 2*(3*a^2 + 8*a*b + 8*b
^2)*cosh(d*x + c)^5 + 2*(63*a^2*cosh(d*x + c)^4 + 42*(a^2 + 2*a*b)*cosh(d*x
 + c)^2 + 3*a^2 + 8*a*b + 8*b^2)*sinh(d*x + c)^5 + 2*(63*a^2*cosh(d*x + c)^
5 + 70*(a^2 + 2*a*b)*cosh(d*x + c)^3 + 5*(3*a^2 + 8*a*b + 8*b^2)*cosh(d*x +
 c))*sinh(d*x + c)^4 + 4*(a^2 + 2*a*b)*cosh(d*x + c)^3 + 4*(21*a^2*cosh(d*x
 + c)^6 + 35*(a^2 + 2*a*b)*cosh(d*x + c)^4 + 5*(3*a^2 + 8*a*b + 8*b^2)*cosh
(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^3 + a^2*cosh(d*x + c) + 4*(9*a^2*c
osh(d*x + c)^7 + 21*(a^2 + 2*a*b)*cosh(d*x + c)^5 + 5*(3*a^2 + 8*a*b + 8*b^
2)*cosh(d*x + c)^3 + 3*(a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c)^2 + (9*a^
2*cosh(d*x + c)^8 + 28*(a^2 + 2*a*b)*cosh(d*x + c)^6 + 10*(3*a^2 + 8*a*b +
8*b^2)*cosh(d*x + c)^4 + 12*(a^2 + 2*a*b)*cosh(d*x + c)^2 + a^2)*sinh(d*x +
```

```
 c))*sqrt(b/a)*arctan(1/2*(a*cosh(d*x + c) + a*sinh(d*x + c))*sqrt(b/a)/b)
+ 4*a^2 + 20*(2*a^2*cosh(d*x + c)^9 + 4*(2*a^2 + 5*a*b)*cosh(d*x + c)^7 + 3
*(4*a^2 + 15*a*b + 12*b^2)*cosh(d*x + c)^5 + 2*(4*a^2 + 15*a*b + 12*b^2)*co
sh(d*x + c)^3 + (2*a^2 + 5*a*b)*cosh(d*x + c))*sinh(d*x + c))/(a^5*d*cosh(d
*x + c)^9 + 9*a^5*d*cosh(d*x + c)*sinh(d*x + c)^8 + a^5*d*sinh(d*x + c)^9 +
 4*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^7 + 4*(9*a^5*d*cosh(d*x + c)^2 + (a^5 +
2*a^4*b)*d)*sinh(d*x + c)^7 + a^5*d*cosh(d*x + c) + 2*(3*a^5 + 8*a^4*b + 8*
a^3*b^2)*d*cosh(d*x + c)^5 + 28*(3*a^5*d*cosh(d*x + c)^3 + (a^5 + 2*a^4*b)*
d*cosh(d*x + c))*sinh(d*x + c)^6 + 2*(63*a^5*d*cosh(d*x + c)^4 + 42*(a^5 +
2*a^4*b)*d*cosh(d*x + c)^2 + (3*a^5 + 8*a^4*b + 8*a^3*b^2)*d)*sinh(d*x + c)
^5 + 4*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^3 + 2*(63*a^5*d*cosh(d*x + c)^5 + 70
*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^3 + 5*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*d*cosh
(d*x + c))*sinh(d*x + c)^4 + 4*(21*a^5*d*cosh(d*x + c)^6 + 35*(a^5 + 2*a^4*
b)*d*cosh(d*x + c)^4 + 5*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*d*cosh(d*x + c)^2 +
(a^5 + 2*a^4*b)*d)*sinh(d*x + c)^3 + 4*(9*a^5*d*cosh(d*x + c)^7 + 21*(a^5 +
 2*a^4*b)*d*cosh(d*x + c)^5 + 5*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*d*cosh(d*x +
c)^3 + 3*(a^5 + 2*a^4*b)*d*cosh(d*x + c))*sinh(d*x + c)^2 + (9*a^5*d*cosh(d
*x + c)^8 + 28*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^6 + a^5*d + 10*(3*a^5 + 8*a^
4*b + 8*a^3*b^2)*d*cosh(d*x + c)^4 + 12*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^2)*
sinh(d*x + c))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[84,-86]Warning, need to choose a branch for the root of a p
olynomial with parameters. This might be wrong.The choice was done assuming
 [a,b]=[-42,-12]Warning, need to choose a branch for the root of a polynomi
al with parameters. This might be wrong.The choice was done assuming [a,b]=
[-43,-99]Warning, need to choose a branch for the root of a polynomial with
 parameters. This might be wrong.The choice was done assuming [a,b]=[-28,94
]Warning, need to choose a branch for the root of a polynomial with paramet
ers. This might be wrong.The choice was done assuming [a,b]=[-7,46]Warning,
 need to choose a branch for the root of a polynomial with parameters. This
 might be wrong.The choice was done assuming [a,b]=[-35,-99]Warning, need t
o choose a branch for the root of a polynomial with parameters. This might
be wrong.The choice was done assuming [a,b]=[7,50]Warning, need to choose a
 branch for the root of a polynomial with parameters. This might be wrong.T
he choice was done assuming [a,b]=[-63,-70]Warning, need to choose a branch
```

```
for the root of a polynomial with parameters. This might be wrong.The choi
ce was done assuming [a,b]=[-82,81]Precision problem choosing root in commo
n_EXT, current precision 14Warning, need to choose a branch for the root of
 a polynomial with parameters. This might be wrong.The choice was done assu
ming [a,b]=[-60,-34]Undef/Unsigned Inf encountered in limitEvaluation time:
 2.06Limit: Max order reached or unable to make series expansion Error: Bad
 Argument Value
```

maple [A] time = 0.17, size = 107, normalized size = 0.92

$$\frac{7b^2\operatorname{sech}(dx+c)^3}{8d\,a^3\left(a+b\operatorname{sech}(dx+c)^2\right)^2}+\frac{9b\operatorname{sech}(dx+c)}{8d\,a^2\left(a+b\operatorname{sech}(dx+c)^2\right)^2}+\frac{15b\arctan\left(\frac{\operatorname{sech}(dx+c)b}{\sqrt{ab}}\right)}{8d\,a^3\sqrt{ab}}+\frac{1}{d\,a^3\operatorname{sech}(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)/(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] 7/8/d/a^3*b^2/(a+b*sech(d*x+c)^2)^2*sech(d*x+c)^3+9/8/d/a^2*b/(a+b*sech(d*x
+c)^2)^2*sech(d*x+c)+15/8/d/a^3*b/(a*b)^(1/2)*arctan(sech(d*x+c)*b/(a*b)^(1
/2))+1/d/a^3/sech(d*x+c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\,a^2e^{(10\,dx+10\,c)}+2\,a^2+5\left(2\,a^2e^{(8\,c)}+5\,abe^{(8\,c)}\right)e^{(8\,dx)}+5\left(4\,a^2e^{(6\,c)}+15\,abe^{(6\,c)}+12\,b^2e^{(6\,c)}\right)e^{(6\,dx)}+5\left(4\,a^2e^{(4\,c)}+1\right.}{4\left(a^5de^{(9\,dx+9\,c)}+a^5de^{(dx+c)}+4\left(a^5de^{(7\,c)}+2\,a^4bde^{(7\,c)}\right)e^{(7\,dx)}+2\left(3\,a^5de^{(5\,c)}+8\,a^4bde^{(5\,c)}+8\,a^3b^2de\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*a^2*e^(10*d*x + 10*c) + 2*a^2 + 5*(2*a^2*e^(8*c) + 5*a*b*e^(8*c))*e^
(8*d*x) + 5*(4*a^2*e^(6*c) + 15*a*b*e^(6*c) + 12*b^2*e^(6*c))*e^(6*d*x) + 5
*(4*a^2*e^(4*c) + 15*a*b*e^(4*c) + 12*b^2*e^(4*c))*e^(4*d*x) + 5*(2*a^2*e^(
2*c) + 5*a*b*e^(2*c))*e^(2*d*x))/(a^5*d*e^(9*d*x + 9*c) + a^5*d*e^(d*x + c)
 + 4*(a^5*d*e^(7*c) + 2*a^4*b*d*e^(7*c))*e^(7*d*x) + 2*(3*a^5*d*e^(5*c) + 8
*a^4*b*d*e^(5*c) + 8*a^3*b^2*d*e^(5*c))*e^(5*d*x) + 4*(a^5*d*e^(3*c) + 2*a^
4*b*d*e^(3*c))*e^(3*d*x)) - 1/2*integrate(15/2*(b*e^(3*d*x + 3*c) - b*e^(d*
x + c))/(a^4*e^(4*d*x + 4*c) + a^4 + 2*(a^4*e^(2*c) + 2*a^3*b*e^(2*c))*e^(2
*d*x)), x)
```

mupad [B] time = 1.61, size = 103, normalized size = 0.89

$$\frac{\frac{7\,b^2\cosh(c+d\,x)}{8}+\frac{9\,a\,b\cosh(c+d\,x)^3}{8}}{d\,a^5\cosh\left(c+d\,x\right)^4+2\,d\,a^4\,b\cosh\left(c+d\,x\right)^2+d\,a^3\,b^2}+\frac{\cosh\left(c+d\,x\right)}{a^3\,d}-\frac{15\,\sqrt{b}\,\operatorname{atan}\left(\frac{\sqrt{a}\,\cosh(c+d\,x)}{\sqrt{b}}\right)}{8\,a^{7/2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)/(a + b/cosh(c + d*x)^2)^3,x)
```

```
[Out] ((7*b^2*cosh(c + d*x))/8 + (9*a*b*cosh(c + d*x)^3)/8)/(a^5*d*cosh(c + d*x)^
4 + a^3*b^2*d + 2*a^4*b*d*cosh(c + d*x)^2) + cosh(c + d*x)/(a^3*d) - (15*b^
(1/2)*atan((a^(1/2)*cosh(c + d*x))/b^(1/2)))/(8*a^(7/2)*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

3.45 $\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=154

$$-\frac{b(7a+3b)\cosh(c+dx)}{8a^2d(a+b)^2\left(a\cosh^2(c+dx)+b\right)}+\frac{\sqrt{b}\left(15a^2+10ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{8a^{5/2}d(a+b)^3}-\frac{b\cosh^3(c+dx)}{4ad(a+b)\left(a\cosh^2(c+dx)\right.}$$

```
[Out] -arctanh(cosh(d*x+c))/d/(a+b)^3-1/4*b*cosh(d*x+c)^3/a/(a+b)/d/(b+a*cosh(d*x
+c)^2)^2-1/8*b*(7*a+3*b)*cosh(d*x+c)/a^2/(a+b)^2/d/(b+a*cosh(d*x+c)^2)+1/8*
(15*a^2+10*a*b+3*b^2)*arctan(cosh(d*x+c)*a^(1/2)/b^(1/2))*b^(1/2)/a^(5/2)/(
a+b)^3/d
```

Rubi [A] time = 0.22, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4133, 470, 578, 522, 206, 205}

$$\frac{\sqrt{b}\left(15a^2+10ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{8a^{5/2}d(a+b)^3}-\frac{b(7a+3b)\cosh(c+dx)}{8a^2d(a+b)^2\left(a\cosh^2(c+dx)+b\right)}-\frac{b\cosh^3(c+dx)}{4ad(a+b)\left(a\cosh^2(c+dx)+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (Sqrt[b]*(15*a^2 + 10*a*b + 3*b^2)*ArcTan[(Sqrt[a]*Cosh[c + d*x])/Sqrt[b]])
/(8*a^(5/2)*(a + b)^3*d) - ArcTanh[Cosh[c + d*x]]/((a + b)^3*d) - (b*Cosh[c
 + d*x]^3)/(4*a*(a + b)*d*(b + a*Cosh[c + d*x]^2)^2) - (b*(7*a + 3*b)*Cosh[
c + d*x])/(8*a^2*(a + b)^2*d*(b + a*Cosh[c + d*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = -\frac{\operatorname{Subst}\left(\int \frac{x^6}{(1-x^2)(b+ax^2)^3}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{b\cosh^3(c+dx)}{4a(a+b)d\left(b+a\cosh^2(c+dx)\right)^2} + \frac{\operatorname{Subst}\left(\int \frac{x^2(3b+(-4a-3b)x^2)}{(1-x^2)(b+ax^2)^2}\,dx, x, \cosh(c+dx\right.}{4a(a+b)d}$$

$$= -\frac{b\cosh^3(c+dx)}{4a(a+b)d\left(b+a\cosh^2(c+dx)\right)^2} - \frac{b(7a+3b)\cosh(c+dx)}{8a^2(a+b)^2d\left(b+a\cosh^2(c+dx)\right)} + \frac{\operatorname{Subst}}{}$$

$$= -\frac{b\cosh^3(c+dx)}{4a(a+b)d\left(b+a\cosh^2(c+dx)\right)^2} - \frac{b(7a+3b)\cosh(c+dx)}{8a^2(a+b)^2d\left(b+a\cosh^2(c+dx)\right)} - \frac{\operatorname{Subst}}{}$$

$$= \frac{\sqrt{b}\left(15a^2+10ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{8a^{5/2}(a+b)^3d} - \frac{\tanh^{-1}(\cosh(c+dx))}{(a+b)^3d} - \frac{}{4a(a+b}$$

Mathematica [C] time = 2.50, size = 440, normalized size = 2.86

$$\frac{\operatorname{sech}^5(c+dx)(a\cosh(2(c+dx))+a+2b)\left(\frac{8b^2(a+b)^2}{a^2} - \frac{2b(9a+5b)(a+b)(a\cosh(2(c+dx))+a+2b)}{a^2} + \frac{\sqrt{b}\left(15a^2+10ab+3b^2\right)\operatorname{sech}(c+dx)}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^5*((8*b^2*(a + b)^2)/a^2 - (
2*b*(a + b)*(9*a + 5*b)*(a + 2*b + a*Cosh[2*(c + d*x)]))/a^2 + (Sqrt[b]*(15
*a^2 + 10*a*b + 3*b^2)*ArcTan[((Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sin
h[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Co
sh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cosh[2*(c + d*x)]
)^2*Sech[c + d*x])/a^(5/2) + (Sqrt[b]*(15*a^2 + 10*a*b + 3*b^2)*ArcTan[((Sq
rt[a] + I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] +
```

```
Cosh[c]*(Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2])
)/Sqrt[b]]*(a + 2*b + a*Cosh[2*(c + d*x)])^2*Sech[c + d*x])/a^(5/2) - 8*(a
+ 2*b + a*Cosh[2*(c + d*x)])^2*Log[Cosh[(c + d*x)/2]]*Sech[c + d*x] + 8*(a
+ 2*b + a*Cosh[2*(c + d*x)])^2*Log[Sinh[(c + d*x)/2]]*Sech[c + d*x]))/(64*(
a + b)^3*d*(a + b*Sech[c + d*x]^2)^3)
```

fricas [B] time = 0.64, size = 8742, normalized size = 56.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*(9*a^3*b + 14*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)^7 + 28*(9*a^3*b +
14*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)*sinh(d*x + c)^6 + 4*(9*a^3*b + 14*a^2*b
^2 + 5*a*b^3)*sinh(d*x + c)^7 + 4*(27*a^3*b + 70*a^2*b^2 + 55*a*b^3 + 12*b^
4)*cosh(d*x + c)^5 + 4*(27*a^3*b + 70*a^2*b^2 + 55*a*b^3 + 12*b^4 + 21*(9*a
^3*b + 14*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 20*(7*(9*a^
3*b + 14*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)^3 + (27*a^3*b + 70*a^2*b^2 + 55*a
*b^3 + 12*b^4)*cosh(d*x + c))*sinh(d*x + c)^4 + 4*(27*a^3*b + 70*a^2*b^2 +
55*a*b^3 + 12*b^4)*cosh(d*x + c)^3 + 4*(35*(9*a^3*b + 14*a^2*b^2 + 5*a*b^3)
*cosh(d*x + c)^4 + 27*a^3*b + 70*a^2*b^2 + 55*a*b^3 + 12*b^4 + 10*(27*a^3*b
 + 70*a^2*b^2 + 55*a*b^3 + 12*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 4*(21
*(9*a^3*b + 14*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)^5 + 10*(27*a^3*b + 70*a^2*b
^2 + 55*a*b^3 + 12*b^4)*cosh(d*x + c)^3 + 3*(27*a^3*b + 70*a^2*b^2 + 55*a*b
^3 + 12*b^4)*cosh(d*x + c))*sinh(d*x + c)^2 - ((15*a^4 + 10*a^3*b + 3*a^2*b
^2)*cosh(d*x + c)^8 + 8*(15*a^4 + 10*a^3*b + 3*a^2*b^2)*cosh(d*x + c)*sinh(
d*x + c)^7 + (15*a^4 + 10*a^3*b + 3*a^2*b^2)*sinh(d*x + c)^8 + 4*(15*a^4 +
40*a^3*b + 23*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^6 + 4*(15*a^4 + 40*a^3*b + 2
3*a^2*b^2 + 6*a*b^3 + 7*(15*a^4 + 10*a^3*b + 3*a^2*b^2)*cosh(d*x + c)^2)*si
nh(d*x + c)^6 + 8*(7*(15*a^4 + 10*a^3*b + 3*a^2*b^2)*cosh(d*x + c)^3 + 3*(1
5*a^4 + 40*a^3*b + 23*a^2*b^2 + 6*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 2
*(45*a^4 + 150*a^3*b + 209*a^2*b^2 + 104*a*b^3 + 24*b^4)*cosh(d*x + c)^4 +
2*(35*(15*a^4 + 10*a^3*b + 3*a^2*b^2)*cosh(d*x + c)^4 + 45*a^4 + 150*a^3*b
+ 209*a^2*b^2 + 104*a*b^3 + 24*b^4 + 30*(15*a^4 + 40*a^3*b + 23*a^2*b^2 + 6
*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 15*a^4 + 10*a^3*b + 3*a^2*b^2 +
8*(7*(15*a^4 + 10*a^3*b + 3*a^2*b^2)*cosh(d*x + c)^5 + 10*(15*a^4 + 40*a^3*
b + 23*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^3 + (45*a^4 + 150*a^3*b + 209*a^2*b
^2 + 104*a*b^3 + 24*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(15*a^4 + 40*a^
3*b + 23*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^2 + 4*(7*(15*a^4 + 10*a^3*b + 3*a
^2*b^2)*cosh(d*x + c)^6 + 15*(15*a^4 + 40*a^3*b + 23*a^2*b^2 + 6*a*b^3)*cos
h(d*x + c)^4 + 15*a^4 + 40*a^3*b + 23*a^2*b^2 + 6*a*b^3 + 3*(45*a^4 + 150*a
^3*b + 209*a^2*b^2 + 104*a*b^3 + 24*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 +
 8*((15*a^4 + 10*a^3*b + 3*a^2*b^2)*cosh(d*x + c)^7 + 3*(15*a^4 + 40*a^3*b
+ 23*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^5 + (45*a^4 + 150*a^3*b + 209*a^2*b^2
```

```
 + 104*a*b^3 + 24*b^4)*cosh(d*x + c)^3 + (15*a^4 + 40*a^3*b + 23*a^2*b^2 +
6*a*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b/a)*log((a*cosh(d*x + c)^4 +
4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a - 2*b)*cosh(d*
x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a - 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*
x + c)^3 + (a - 2*b)*cosh(d*x + c))*sinh(d*x + c) + 4*(a*cosh(d*x + c)^3 +
3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 + a*cosh(d*x + c) + (
3*a*cosh(d*x + c)^2 + a)*sinh(d*x + c))*sqrt(-b/a) + a)/(a*cosh(d*x + c)^4
+ 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(
d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(
d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) + 4*(9*a^3*b + 14
*a^2*b^2 + 5*a*b^3)*cosh(d*x + c) + 16*(a^4*cosh(d*x + c)^8 + 8*a^4*cosh(d*
x + c)*sinh(d*x + c)^7 + a^4*sinh(d*x + c)^8 + 4*(a^4 + 2*a^3*b)*cosh(d*x +
 c)^6 + 4*(7*a^4*cosh(d*x + c)^2 + a^4 + 2*a^3*b)*sinh(d*x + c)^6 + 8*(7*a^
4*cosh(d*x + c)^3 + 3*(a^4 + 2*a^3*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3
*a^4 + 8*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^4 + 2*(35*a^4*cosh(d*x + c)^4 + 3
*a^4 + 8*a^3*b + 8*a^2*b^2 + 30*(a^4 + 2*a^3*b)*cosh(d*x + c)^2)*sinh(d*x +
 c)^4 + a^4 + 8*(7*a^4*cosh(d*x + c)^5 + 10*(a^4 + 2*a^3*b)*cosh(d*x + c)^3
 + (3*a^4 + 8*a^3*b + 8*a^2*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a^4 +
2*a^3*b)*cosh(d*x + c)^2 + 4*(7*a^4*cosh(d*x + c)^6 + 15*(a^4 + 2*a^3*b)*co
sh(d*x + c)^4 + a^4 + 2*a^3*b + 3*(3*a^4 + 8*a^3*b + 8*a^2*b^2)*cosh(d*x +
c)^2)*sinh(d*x + c)^2 + 8*(a^4*cosh(d*x + c)^7 + 3*(a^4 + 2*a^3*b)*cosh(d*x
 + c)^5 + (3*a^4 + 8*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^3 + (a^4 + 2*a^3*b)*c
osh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) - 16*(a
^4*cosh(d*x + c)^8 + 8*a^4*cosh(d*x + c)*sinh(d*x + c)^7 + a^4*sinh(d*x + c
)^8 + 4*(a^4 + 2*a^3*b)*cosh(d*x + c)^6 + 4*(7*a^4*cosh(d*x + c)^2 + a^4 +
2*a^3*b)*sinh(d*x + c)^6 + 8*(7*a^4*cosh(d*x + c)^3 + 3*(a^4 + 2*a^3*b)*cos
h(d*x + c))*sinh(d*x + c)^5 + 2*(3*a^4 + 8*a^3*b + 8*a^2*b^2)*cosh(d*x + c)
^4 + 2*(35*a^4*cosh(d*x + c)^4 + 3*a^4 + 8*a^3*b + 8*a^2*b^2 + 30*(a^4 + 2*
a^3*b)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + a^4 + 8*(7*a^4*cosh(d*x + c)^5 +
10*(a^4 + 2*a^3*b)*cosh(d*x + c)^3 + (3*a^4 + 8*a^3*b + 8*a^2*b^2)*cosh(d*x
 + c))*sinh(d*x + c)^3 + 4*(a^4 + 2*a^3*b)*cosh(d*x + c)^2 + 4*(7*a^4*cosh(
d*x + c)^6 + 15*(a^4 + 2*a^3*b)*cosh(d*x + c)^4 + a^4 + 2*a^3*b + 3*(3*a^4
+ 8*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*(a^4*cosh(d*x +
 c)^7 + 3*(a^4 + 2*a^3*b)*cosh(d*x + c)^5 + (3*a^4 + 8*a^3*b + 8*a^2*b^2)*c
osh(d*x + c)^3 + (a^4 + 2*a^3*b)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x
 + c) + sinh(d*x + c) - 1) + 4*(7*(9*a^3*b + 14*a^2*b^2 + 5*a*b^3)*cosh(d*x
 + c)^6 + 5*(27*a^3*b + 70*a^2*b^2 + 55*a*b^3 + 12*b^4)*cosh(d*x + c)^4 + 9
*a^3*b + 14*a^2*b^2 + 5*a*b^3 + 3*(27*a^3*b + 70*a^2*b^2 + 55*a*b^3 + 12*b^
4)*cosh(d*x + c)^2)*sinh(d*x + c))/((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d
*cosh(d*x + c)^8 + 8*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)*
sinh(d*x + c)^7 + (a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*sinh(d*x + c)^8 +
 4*(a^7 + 5*a^6*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d*cosh(d*x + c)^6 +
4*(7*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^2 + (a^7 + 5*a^6
*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d)*sinh(d*x + c)^6 + 2*(3*a^7 + 17*
a^6*b + 41*a^5*b^2 + 51*a^4*b^3 + 32*a^3*b^4 + 8*a^2*b^5)*d*cosh(d*x + c)^4
```

+ 8*(7*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^3 + 3*(a^7 +
5*a^6*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d*cosh(d*x + c))*sinh(d*x + c)
^5 + 2*(35*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^4 + 30*(a^
7 + 5*a^6*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d*cosh(d*x + c)^2 + (3*a^7
+ 17*a^6*b + 41*a^5*b^2 + 51*a^4*b^3 + 32*a^3*b^4 + 8*a^2*b^5)*d)*sinh(d*x
+ c)^4 + 4*(a^7 + 5*a^6*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d*cosh(d*x
+ c)^2 + 8*(7*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^5 + 10*
(a^7 + 5*a^6*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d*cosh(d*x + c)^3 + (3*
a^7 + 17*a^6*b + 41*a^5*b^2 + 51*a^4*b^3 + 32*a^3*b^4 + 8*a^2*b^5)*d*cosh(d
*x + c))*sinh(d*x + c)^3 + 4*(7*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cos
h(d*x + c)^6 + 15*(a^7 + 5*a^6*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d*cos
h(d*x + c)^4 + 3*(3*a^7 + 17*a^6*b + 41*a^5*b^2 + 51*a^4*b^3 + 32*a^3*b^4 +
8*a^2*b^5)*d*cosh(d*x + c)^2 + (a^7 + 5*a^6*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*
a^3*b^4)*d)*sinh(d*x + c)^2 + (a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d + 8*(
(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^7 + 3*(a^7 + 5*a^6*b
+ 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d*cosh(d*x + c)^5 + (3*a^7 + 17*a^6*b
+ 41*a^5*b^2 + 51*a^4*b^3 + 32*a^3*b^4 + 8*a^2*b^5)*d*cosh(d*x + c)^3 + (a^
7 + 5*a^6*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d*cosh(d*x + c))*sinh(d*x
+ c)), -1/8*(2*(9*a^3*b + 14*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)^7 + 14*(9*a^3
*b + 14*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)*sinh(d*x + c)^6 + 2*(9*a^3*b + 14*
a^2*b^2 + 5*a*b^3)*sinh(d*x + c)^7 + 2*(27*a^3*b + 70*a^2*b^2 + 55*a*b^3 +
12*b^4)*cosh(d*x + c)^5 + 2*(27*a^3*b + 70*a^2*b^2 + 55*a*b^3 + 12*b^4 + 21
*(9*a^3*b + 14*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 10*(7*
(9*a^3*b + 14*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)^3 + (27*a^3*b + 70*a^2*b^2 +
55*a*b^3 + 12*b^4)*cosh(d*x + c))*sinh(d*x + c)^4 + 2*(27*a^3*b + 70*a^2*b
^2 + 55*a*b^3 + 12*b^4)*cosh(d*x + c)^3 + 2*(35*(9*a^3*b + 14*a^2*b^2 + 5*a
*b^3)*cosh(d*x + c)^4 + 27*a^3*b + 70*a^2*b^2 + 55*a*b^3 + 12*b^4 + 10*(27*
a^3*b + 70*a^2*b^2 + 55*a*b^3 + 12*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^3 +
2*(21*(9*a^3*b + 14*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)^5 + 10*(27*a^3*b + 70*
a^2*b^2 + 55*a*b^3 + 12*b^4)*cosh(d*x + c)^3 + 3*(27*a^3*b + 70*a^2*b^2 + 5
5*a*b^3 + 12*b^4)*cosh(d*x + c))*sinh(d*x + c)^2 + ((15*a^4 + 10*a^3*b + 3*
a^2*b^2)*cosh(d*x + c)^8 + 8*(15*a^4 + 10*a^3*b + 3*a^2*b^2)*cosh(d*x + c)*
sinh(d*x + c)^7 + (15*a^4 + 10*a^3*b + 3*a^2*b^2)*sinh(d*x + c)^8 + 4*(15*a
^4 + 40*a^3*b + 23*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^6 + 4*(15*a^4 + 40*a^3*
b + 23*a^2*b^2 + 6*a*b^3 + 7*(15*a^4 + 10*a^3*b + 3*a^2*b^2)*cosh(d*x + c)^
2)*sinh(d*x + c)^6 + 8*(7*(15*a^4 + 10*a^3*b + 3*a^2*b^2)*cosh(d*x + c)^3 +
3*(15*a^4 + 40*a^3*b + 23*a^2*b^2 + 6*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^
5 + 2*(45*a^4 + 150*a^3*b + 209*a^2*b^2 + 104*a*b^3 + 24*b^4)*cosh(d*x + c)
^4 + 2*(35*(15*a^4 + 10*a^3*b + 3*a^2*b^2)*cosh(d*x + c)^4 + 45*a^4 + 150*a
^3*b + 209*a^2*b^2 + 104*a*b^3 + 24*b^4 + 30*(15*a^4 + 40*a^3*b + 23*a^2*b^
2 + 6*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 15*a^4 + 10*a^3*b + 3*a^2*b
^2 + 8*(7*(15*a^4 + 10*a^3*b + 3*a^2*b^2)*cosh(d*x + c)^5 + 10*(15*a^4 + 40
*a^3*b + 23*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^3 + (45*a^4 + 150*a^3*b + 209*
a^2*b^2 + 104*a*b^3 + 24*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(15*a^4 +
40*a^3*b + 23*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^2 + 4*(7*(15*a^4 + 10*a^3*b

+ 3*a^2*b^2)*cosh(d*x + c)^6 + 15*(15*a^4 + 40*a^3*b + 23*a^2*b^2 + 6*a*b^3
)*cosh(d*x + c)^4 + 15*a^4 + 40*a^3*b + 23*a^2*b^2 + 6*a*b^3 + 3*(45*a^4 +
150*a^3*b + 209*a^2*b^2 + 104*a*b^3 + 24*b^4)*cosh(d*x + c)^2)*sinh(d*x + c
)^2 + 8*((15*a^4 + 10*a^3*b + 3*a^2*b^2)*cosh(d*x + c)^7 + 3*(15*a^4 + 40*a
^3*b + 23*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^5 + (45*a^4 + 150*a^3*b + 209*a^
2*b^2 + 104*a*b^3 + 24*b^4)*cosh(d*x + c)^3 + (15*a^4 + 40*a^3*b + 23*a^2*b
^2 + 6*a*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(b/a)*arctan(1/2*(a*cosh(d*
x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 + (a + 4*b
)*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + a + 4*b)*sinh(d*x + c))*sqrt(b/a)/
b) - ((15*a^4 + 10*a^3*b + 3*a^2*b^2)*cosh(d*x + c)^8 + 8*(15*a^4 + 10*a^3*
b + 3*a^2*b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (15*a^4 + 10*a^3*b + 3*a^2*b
^2)*sinh(d*x + c)^8 + 4*(15*a^4 + 40*a^3*b + 23*a^2*b^2 + 6*a*b^3)*cosh(d*x
 + c)^6 + 4*(15*a^4 + 40*a^3*b + 23*a^2*b^2 + 6*a*b^3 + 7*(15*a^4 + 10*a^3*
b + 3*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(15*a^4 + 10*a^3*b +
 3*a^2*b^2)*cosh(d*x + c)^3 + 3*(15*a^4 + 40*a^3*b + 23*a^2*b^2 + 6*a*b^3)*
cosh(d*x + c))*sinh(d*x + c)^5 + 2*(45*a^4 + 150*a^3*b + 209*a^2*b^2 + 104*
a*b^3 + 24*b^4)*cosh(d*x + c)^4 + 2*(35*(15*a^4 + 10*a^3*b + 3*a^2*b^2)*cos
h(d*x + c)^4 + 45*a^4 + 150*a^3*b + 209*a^2*b^2 + 104*a*b^3 + 24*b^4 + 30*(
15*a^4 + 40*a^3*b + 23*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4
+ 15*a^4 + 10*a^3*b + 3*a^2*b^2 + 8*(7*(15*a^4 + 10*a^3*b + 3*a^2*b^2)*cosh
(d*x + c)^5 + 10*(15*a^4 + 40*a^3*b + 23*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^3
 + (45*a^4 + 150*a^3*b + 209*a^2*b^2 + 104*a*b^3 + 24*b^4)*cosh(d*x + c))*s
inh(d*x + c)^3 + 4*(15*a^4 + 40*a^3*b + 23*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)
^2 + 4*(7*(15*a^4 + 10*a^3*b + 3*a^2*b^2)*cosh(d*x + c)^6 + 15*(15*a^4 + 40
*a^3*b + 23*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^4 + 15*a^4 + 40*a^3*b + 23*a^2
*b^2 + 6*a*b^3 + 3*(45*a^4 + 150*a^3*b + 209*a^2*b^2 + 104*a*b^3 + 24*b^4)*
cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((15*a^4 + 10*a^3*b + 3*a^2*b^2)*cosh(
d*x + c)^7 + 3*(15*a^4 + 40*a^3*b + 23*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^5 +
 (45*a^4 + 150*a^3*b + 209*a^2*b^2 + 104*a*b^3 + 24*b^4)*cosh(d*x + c)^3 +
(15*a^4 + 40*a^3*b + 23*a^2*b^2 + 6*a*b^3)*cosh(d*x + c))*sinh(d*x + c))*sq
rt(b/a)*arctan(1/2*(a*cosh(d*x + c) + a*sinh(d*x + c))*sqrt(b/a)/b) + 2*(9*
a^3*b + 14*a^2*b^2 + 5*a*b^3)*cosh(d*x + c) + 8*(a^4*cosh(d*x + c)^8 + 8*a^
4*cosh(d*x + c)*sinh(d*x + c)^7 + a^4*sinh(d*x + c)^8 + 4*(a^4 + 2*a^3*b)*c
osh(d*x + c)^6 + 4*(7*a^4*cosh(d*x + c)^2 + a^4 + 2*a^3*b)*sinh(d*x + c)^6
+ 8*(7*a^4*cosh(d*x + c)^3 + 3*(a^4 + 2*a^3*b)*cosh(d*x + c))*sinh(d*x + c)
^5 + 2*(3*a^4 + 8*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^4 + 2*(35*a^4*cosh(d*x +
 c)^4 + 3*a^4 + 8*a^3*b + 8*a^2*b^2 + 30*(a^4 + 2*a^3*b)*cosh(d*x + c)^2)*s
inh(d*x + c)^4 + a^4 + 8*(7*a^4*cosh(d*x + c)^5 + 10*(a^4 + 2*a^3*b)*cosh(d
*x + c)^3 + (3*a^4 + 8*a^3*b + 8*a^2*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 +
4*(a^4 + 2*a^3*b)*cosh(d*x + c)^2 + 4*(7*a^4*cosh(d*x + c)^6 + 15*(a^4 + 2*
a^3*b)*cosh(d*x + c)^4 + a^4 + 2*a^3*b + 3*(3*a^4 + 8*a^3*b + 8*a^2*b^2)*co
sh(d*x + c)^2)*sinh(d*x + c)^2 + 8*(a^4*cosh(d*x + c)^7 + 3*(a^4 + 2*a^3*b)
*cosh(d*x + c)^5 + (3*a^4 + 8*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^3 + (a^4 + 2
*a^3*b)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1
) - 8*(a^4*cosh(d*x + c)^8 + 8*a^4*cosh(d*x + c)*sinh(d*x + c)^7 + a^4*sinh

```
(d*x + c)^8 + 4*(a^4 + 2*a^3*b)*cosh(d*x + c)^6 + 4*(7*a^4*cosh(d*x + c)^2
+ a^4 + 2*a^3*b)*sinh(d*x + c)^6 + 8*(7*a^4*cosh(d*x + c)^3 + 3*(a^4 + 2*a^
3*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*a^4 + 8*a^3*b + 8*a^2*b^2)*cosh(
d*x + c)^4 + 2*(35*a^4*cosh(d*x + c)^4 + 3*a^4 + 8*a^3*b + 8*a^2*b^2 + 30*(
a^4 + 2*a^3*b)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + a^4 + 8*(7*a^4*cosh(d*x +
 c)^5 + 10*(a^4 + 2*a^3*b)*cosh(d*x + c)^3 + (3*a^4 + 8*a^3*b + 8*a^2*b^2)*
cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a^4 + 2*a^3*b)*cosh(d*x + c)^2 + 4*(7*a
^4*cosh(d*x + c)^6 + 15*(a^4 + 2*a^3*b)*cosh(d*x + c)^4 + a^4 + 2*a^3*b + 3
*(3*a^4 + 8*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*(a^4*co
sh(d*x + c)^7 + 3*(a^4 + 2*a^3*b)*cosh(d*x + c)^5 + (3*a^4 + 8*a^3*b + 8*a^
2*b^2)*cosh(d*x + c)^3 + (a^4 + 2*a^3*b)*cosh(d*x + c))*sinh(d*x + c))*log(
cosh(d*x + c) + sinh(d*x + c) - 1) + 2*(7*(9*a^3*b + 14*a^2*b^2 + 5*a*b^3)*
cosh(d*x + c)^6 + 5*(27*a^3*b + 70*a^2*b^2 + 55*a*b^3 + 12*b^4)*cosh(d*x +
c)^4 + 9*a^3*b + 14*a^2*b^2 + 5*a*b^3 + 3*(27*a^3*b + 70*a^2*b^2 + 55*a*b^3
 + 12*b^4)*cosh(d*x + c)^2)*sinh(d*x + c))/((a^7 + 3*a^6*b + 3*a^5*b^2 + a^
4*b^3)*d*cosh(d*x + c)^8 + 8*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d
*x + c)*sinh(d*x + c)^7 + (a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*sinh(d*x
+ c)^8 + 4*(a^7 + 5*a^6*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d*cosh(d*x +
 c)^6 + 4*(7*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^2 + (a^7
 + 5*a^6*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d)*sinh(d*x + c)^6 + 2*(3*a
^7 + 17*a^6*b + 41*a^5*b^2 + 51*a^4*b^3 + 32*a^3*b^4 + 8*a^2*b^5)*d*cosh(d*
x + c)^4 + 8*(7*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^3 + 3
*(a^7 + 5*a^6*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d*cosh(d*x + c))*sinh(
d*x + c)^5 + 2*(35*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^4
+ 30*(a^7 + 5*a^6*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d*cosh(d*x + c)^2
+ (3*a^7 + 17*a^6*b + 41*a^5*b^2 + 51*a^4*b^3 + 32*a^3*b^4 + 8*a^2*b^5)*d)*
sinh(d*x + c)^4 + 4*(a^7 + 5*a^6*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d*c
osh(d*x + c)^2 + 8*(7*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)
^5 + 10*(a^7 + 5*a^6*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d*cosh(d*x + c)
^3 + (3*a^7 + 17*a^6*b + 41*a^5*b^2 + 51*a^4*b^3 + 32*a^3*b^4 + 8*a^2*b^5)*
d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^
3)*d*cosh(d*x + c)^6 + 15*(a^7 + 5*a^6*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^
4)*d*cosh(d*x + c)^4 + 3*(3*a^7 + 17*a^6*b + 41*a^5*b^2 + 51*a^4*b^3 + 32*a
^3*b^4 + 8*a^2*b^5)*d*cosh(d*x + c)^2 + (a^7 + 5*a^6*b + 9*a^5*b^2 + 7*a^4*
b^3 + 2*a^3*b^4)*d)*sinh(d*x + c)^2 + (a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)
*d + 8*((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^7 + 3*(a^7 +
5*a^6*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d*cosh(d*x + c)^5 + (3*a^7 + 1
7*a^6*b + 41*a^5*b^2 + 51*a^4*b^3 + 32*a^3*b^4 + 8*a^2*b^5)*d*cosh(d*x + c)
^3 + (a^7 + 5*a^6*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d*cosh(d*x + c))*s
inh(d*x + c))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[84,-86]Warning, need to choose a branch for the root of a p
olynomial with parameters. This might be wrong.The choice was done assuming
 [a,b]=[-42,-12]Warning, need to choose a branch for the root of a polynomi
al with parameters. This might be wrong.The choice was done assuming [a,b]=
[-43,-99]Warning, need to choose a branch for the root of a polynomial with
 parameters. This might be wrong.The choice was done assuming [a,b]=[-28,94
]Warning, need to choose a branch for the root of a polynomial with paramet
ers. This might be wrong.The choice was done assuming [a,b]=[-7,46]Warning,
 need to choose a branch for the root of a polynomial with parameters. This
 might be wrong.The choice was done assuming [a,b]=[-35,-99]Undef/Unsigned
Inf encountered in limitEvaluation time: 1.19Limit: Max order reached or un
able to make series expansion Error: Bad Argument Value
```

maple [B] time = 0.36, size = 1476, normalized size = 9.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)/(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] -9/4/d*b/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/
2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*a*tanh(1/2*d*x+1/2*c)^6+1
/4/d*b^2/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/
2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^6+13/
4/d*b^3/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2
*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/a*tanh(1/2*d*x+1/2*c)^6+3/
4/d*b^4/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2
*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/a^2*tanh(1/2*d*x+1/2*c)^6-
27/4/d*b/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/
2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^4*a+9
/4/d*b^2/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/
2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^4-21/
4/d*b^3/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2
*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/a*tanh(1/2*d*x+1/2*c)^4-9/
4/d*b^4/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2
*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/a^2*tanh(1/2*d*x+1/2*c)^4-
27/4/d*b/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/
2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*a*tanh(1/2*d*x+1/2*c)^2-1
3/4/d*b^2/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1
/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^2+23
```

```
/4/d*b^3/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/
2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/a*tanh(1/2*d*x+1/2*c)^2+9
/4/d*b^4/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/
2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/a^2*tanh(1/2*d*x+1/2*c)^2
-9/4/d*b/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/
2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*a-21/4/d*b^2/(a+b)^3/(tan
h(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*ta
nh(1/2*d*x+1/2*c)^2*b+a+b)^2-15/4/d*b^3/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*
tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a
+b)^2/a-3/4/d*b^4/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+
2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/a^2+15/8/d*b/(a+
b)^3/(a*b)^(1/2)*arctan(1/4*(2*(a+b)*tanh(1/2*d*x+1/2*c)^2+2*a-2*b)/(a*b)^(
1/2))+5/4/d*b^2/(a+b)^3/a/(a*b)^(1/2)*arctan(1/4*(2*(a+b)*tanh(1/2*d*x+1/2*
c)^2+2*a-2*b)/(a*b)^(1/2))+3/8/d*b^3/(a+b)^3/a^2/(a*b)^(1/2)*arctan(1/4*(2*
(a+b)*tanh(1/2*d*x+1/2*c)^2+2*a-2*b)/(a*b)^(1/2))+1/d/(a+b)^3*ln(tanh(1/2*d
*x+1/2*c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\big(9\,a^{2}be^{(7\,c)}+5\,ab^{2}e^{(7\,c)}\big)e^{(7\,dx)}+\big(27\,a^{2}be^{(5\,c)}+4}{4\,\big(a^{6}d+2\,a^{5}bd+a^{4}b^{2}d+\big(a^{6}de^{(8\,c)}+2\,a^{5}bde^{(8\,c)}+a^{4}b^{2}de^{(8\,c)}\big)e^{(8\,dx)}+4\,\big(a^{6}de^{(6\,c)}+4\,a^{5}bde^{(6\,c)}+5\,a^{4}b^{2}de^{(6\,c)}+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*((9*a^2*b*e^(7*c) + 5*a*b^2*e^(7*c))*e^(7*d*x) + (27*a^2*b*e^(5*c) + 4
3*a*b^2*e^(5*c) + 12*b^3*e^(5*c))*e^(5*d*x) + (27*a^2*b*e^(3*c) + 43*a*b^2*
e^(3*c) + 12*b^3*e^(3*c))*e^(3*d*x) + (9*a^2*b*e^c + 5*a*b^2*e^c)*e^(d*x))/
(a^6*d + 2*a^5*b*d + a^4*b^2*d + (a^6*d*e^(8*c) + 2*a^5*b*d*e^(8*c) + a^4*b
^2*d*e^(8*c))*e^(8*d*x) + 4*(a^6*d*e^(6*c) + 4*a^5*b*d*e^(6*c) + 5*a^4*b^2*
d*e^(6*c) + 2*a^3*b^3*d*e^(6*c))*e^(6*d*x) + 2*(3*a^6*d*e^(4*c) + 14*a^5*b*
d*e^(4*c) + 27*a^4*b^2*d*e^(4*c) + 24*a^3*b^3*d*e^(4*c) + 8*a^2*b^4*d*e^(4*
c))*e^(4*d*x) + 4*(a^6*d*e^(2*c) + 4*a^5*b*d*e^(2*c) + 5*a^4*b^2*d*e^(2*c)
+ 2*a^3*b^3*d*e^(2*c))*e^(2*d*x)) - log((e^(d*x + c) + 1)*e^(-c))/(a^3*d +
3*a^2*b*d + 3*a*b^2*d + b^3*d) + log((e^(d*x + c) - 1)*e^(-c))/(a^3*d + 3*a
^2*b*d + 3*a*b^2*d + b^3*d) + 2*integrate(1/8*((15*a^2*b*e^(3*c) + 10*a*b^2
*e^(3*c) + 3*b^3*e^(3*c))*e^(3*d*x) - (15*a^2*b*e^c + 10*a*b^2*e^c + 3*b^3*
e^c)*e^(d*x))/(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3 + (a^6*e^(4*c) + 3*a^5*b
*e^(4*c) + 3*a^4*b^2*e^(4*c) + a^3*b^3*e^(4*c))*e^(4*d*x) + 2*(a^6*e^(2*c)
+ 5*a^5*b*e^(2*c) + 9*a^4*b^2*e^(2*c) + 7*a^3*b^3*e^(2*c) + 2*a^2*b^4*e^(2*
c))*e^(2*d*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh\left(c+d\,x\right)^6}{\sinh\left(c+d\,x\right)\,{\left(a\,\cosh\left(c+d\,x\right)^2+b\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)*(a + b/cosh(c + d*x)^2)^3),x)
```

```
[Out] int(cosh(c + d*x)^6/(sinh(c + d*x)*(b + a*cosh(c + d*x)^2)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}{\left(c + dx \right)}}{\left(a + b \operatorname{sech}^{2}{\left(c + dx \right)}\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Integral(csch(c + d*x)/(a + b*sech(c + d*x)**2)**3, x)
```

3.46 $\int \frac{\mathrm{csch}^2(c+dx)}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^3} \, dx$

Optimal. Leaf size=126

$$\frac{15\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8d(a+b)^{7/2}}-\frac{15\coth(c+dx)}{8d(a+b)^3}+\frac{5\coth(c+dx)}{8d(a+b)^2\left(a-b\tanh^2(c+dx)+b\right)}+\frac{\coth(c+dx)}{4d(a+b)\left(a-b\tanh^2(c+\right.}$$

```
[Out] -15/8*coth(d*x+c)/d/(a+b)^3+15/8*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))*b
^(1/2)/(a+b)^(7/2)/d+1/4*coth(d*x+c)/(a+b)/d/(a+b-b*tanh(d*x+c)^2)^2+5/8*co
th(d*x+c)/(a+b)^2/d/(a+b-b*tanh(d*x+c)^2)
```

Rubi [A] time = 0.10, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4132, 290, 325, 208}

$$\frac{15\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8d(a+b)^{7/2}}-\frac{15\coth(c+dx)}{8d(a+b)^3}+\frac{5\coth(c+dx)}{8d(a+b)^2\left(a-b\tanh^2(c+dx)+b\right)}+\frac{\coth(c+dx)}{4d(a+b)\left(a-b\tanh^2(c+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (15*Sqrt[b]*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(8*(a + b)^(7/2)*
d) - (15*Coth[c + d*x])/(8*(a + b)^3*d) + Coth[c + d*x]/(4*(a + b)*d*(a + b
 - b*Tanh[c + d*x]^2)^2) + (5*Coth[c + d*x])/(8*(a + b)^2*d*(a + b - b*Tanh
[c + d*x]^2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{x^2\left(a+b-bx^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\coth(c+dx)}{4(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{5\operatorname{Subst}\left(\int \frac{1}{x^2\left(a+b-bx^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{4(a+b)d}$$

$$= \frac{\coth(c+dx)}{4(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{5\coth(c+dx)}{8(a+b)^2d\left(a+b-b\tanh^2(c+dx)\right)} + \frac{15 \cdots}{\cdots}$$

$$= -\frac{15\coth(c+dx)}{8(a+b)^3d} + \frac{\coth(c+dx)}{4(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{5\coth(c+\cdots}{8(a+b)^2d\left(a+b-b\,\mathrm{t}\cdots\right.}$$

$$= \frac{15\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a+b}}\right)}{8(a+b)^{7/2}d} - \frac{15\coth(c+dx)}{8(a+b)^3d} + \frac{\coth(c+dx)}{4(a+b)d\left(a+b-b\tanh^2(c+\cdots\right.}$$

Mathematica [C] time = 6.92, size = 981, normalized size = 7.79

$$\frac{(\cosh(2c+2dx)a+a+2b)^3\left(\frac{15ib\tan^{-1}\left(\operatorname{sech}(dx)\left(\frac{i\sinh(2c)}{2\sqrt{a+b}\sqrt{b\cosh(4c)-b\sinh(4c)}}-\frac{i\cosh(2c)}{2\sqrt{a+b}\sqrt{b\cosh(4c)-b\sinh(4c)}}\right)(-a\sinh(dx)-2b\sinh(dx)+a\sin}{64\sqrt{a+b}\,d\sqrt{b\cosh(4c)-b\sinh(4c)}}\right.}{(a+}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csch[c + d*x]^2/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*c + 2*d*x])^3*Sech[c + d*x]^6*((((-15*I)/64)*b*ArcTan[
Sech[d*x]*(((-1/2*I)*Cosh[2*c])/(Sqrt[a + b]*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]
]) + ((I/2)*Sinh[2*c])/(Sqrt[a + b]*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]]))*(-(a*
Sinh[d*x]) - 2*b*Sinh[d*x] + a*Sinh[2*c + d*x])]*Cosh[2*c])/(Sqrt[a + b]*d*
Sqrt[b*Cosh[4*c] - b*Sinh[4*c]]) + (((15*I)/64)*b*ArcTan[Sech[d*x]*(((-1/2*
I)*Cosh[2*c])/(Sqrt[a + b]*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]]) + ((I/2)*Sinh[2
*c])/(Sqrt[a + b]*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]]))*(-(a*Sinh[d*x]) - 2*b*S
inh[d*x] + a*Sinh[2*c + d*x])]*Sinh[2*c])/(Sqrt[a + b]*d*Sqrt[b*Cosh[4*c] -
 b*Sinh[4*c]])))/((a + b)^3*(a + b*Sech[c + d*x]^2)^3) + ((a + 2*b + a*Cosh
[2*c + 2*d*x])*Csch[c]*Csch[c + d*x]*Sech[2*c]*Sech[c + d*x]^6*(-32*a^4*Sin
h[d*x] - 64*a^3*b*Sinh[d*x] + 22*a^2*b^2*Sinh[d*x] + 80*a*b^3*Sinh[d*x] + 1
6*b^4*Sinh[d*x] + 32*a^4*Sinh[3*d*x] + 46*a^3*b*Sinh[3*d*x] - 54*a^2*b^2*Si
nh[3*d*x] - 8*a*b^3*Sinh[3*d*x] - 48*a^4*Sinh[2*c - d*x] - 128*a^3*b*Sinh[2
*c - d*x] - 106*a^2*b^2*Sinh[2*c - d*x] + 80*a*b^3*Sinh[2*c - d*x] + 16*b^4
*Sinh[2*c - d*x] + 48*a^4*Sinh[2*c + d*x] + 146*a^3*b*Sinh[2*c + d*x] + 182
*a^2*b^2*Sinh[2*c + d*x] + 80*a*b^3*Sinh[2*c + d*x] + 16*b^4*Sinh[2*c + d*x
] - 32*a^4*Sinh[4*c + d*x] - 82*a^3*b*Sinh[4*c + d*x] - 54*a^2*b^2*Sinh[4*c
 + d*x] - 80*a*b^3*Sinh[4*c + d*x] - 16*b^4*Sinh[4*c + d*x] - 8*a^4*Sinh[2*
c + 3*d*x] + 18*a^3*b*Sinh[2*c + 3*d*x] + 54*a^2*b^2*Sinh[2*c + 3*d*x] + 8*
a*b^3*Sinh[2*c + 3*d*x] + 32*a^4*Sinh[4*c + 3*d*x] + 73*a^3*b*Sinh[4*c + 3*
d*x] + 24*a^2*b^2*Sinh[4*c + 3*d*x] + 8*a*b^3*Sinh[4*c + 3*d*x] - 8*a^4*Sin
h[6*c + 3*d*x] - 9*a^3*b*Sinh[6*c + 3*d*x] - 24*a^2*b^2*Sinh[6*c + 3*d*x] -
 8*a*b^3*Sinh[6*c + 3*d*x] + 8*a^4*Sinh[2*c + 5*d*x] - 9*a^3*b*Sinh[2*c + 5
*d*x] - 2*a^2*b^2*Sinh[2*c + 5*d*x] + 9*a^3*b*Sinh[4*c + 5*d*x] + 2*a^2*b^2
*Sinh[4*c + 5*d*x] + 8*a^4*Sinh[6*c + 5*d*x]))/(512*a^2*(a + b)^3*d*(a + b*
Sech[c + d*x]^2)^3)
```

fricas [B] time = 0.55, size = 7275, normalized size = 57.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*(8*a^4 + 9*a^3*b + 24*a^2*b^2 + 8*a*b^3)*cosh(d*x + c)^8 + 32*(8*
a^4 + 9*a^3*b + 24*a^2*b^2 + 8*a*b^3)*cosh(d*x + c)*sinh(d*x + c)^7 + 4*(8*
a^4 + 9*a^3*b + 24*a^2*b^2 + 8*a*b^3)*sinh(d*x + c)^8 + 8*(16*a^4 + 41*a^3*
b + 27*a^2*b^2 + 40*a*b^3 + 8*b^4)*cosh(d*x + c)^6 + 8*(16*a^4 + 41*a^3*b +
 27*a^2*b^2 + 40*a*b^3 + 8*b^4 + 14*(8*a^4 + 9*a^3*b + 24*a^2*b^2 + 8*a*b^3
)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 16*(14*(8*a^4 + 9*a^3*b + 24*a^2*b^2 +
 8*a*b^3)*cosh(d*x + c)^3 + 3*(16*a^4 + 41*a^3*b + 27*a^2*b^2 + 40*a*b^3 +
8*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 8*(24*a^4 + 64*a^3*b + 53*a^2*b^2 -
 40*a*b^3 - 8*b^4)*cosh(d*x + c)^4 + 8*(35*(8*a^4 + 9*a^3*b + 24*a^2*b^2 +
8*a*b^3)*cosh(d*x + c)^4 + 24*a^4 + 64*a^3*b + 53*a^2*b^2 - 40*a*b^3 - 8*b^
4 + 15*(16*a^4 + 41*a^3*b + 27*a^2*b^2 + 40*a*b^3 + 8*b^4)*cosh(d*x + c)^2)
*sinh(d*x + c)^4 + 32*a^4 - 36*a^3*b - 8*a^2*b^2 + 32*(7*(8*a^4 + 9*a^3*b +
 24*a^2*b^2 + 8*a*b^3)*cosh(d*x + c)^5 + 5*(16*a^4 + 41*a^3*b + 27*a^2*b^2
+ 40*a*b^3 + 8*b^4)*cosh(d*x + c)^3 + (24*a^4 + 64*a^3*b + 53*a^2*b^2 - 40*
a*b^3 - 8*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 8*(16*a^4 + 23*a^3*b - 27*a
^2*b^2 - 4*a*b^3)*cosh(d*x + c)^2 + 8*(14*(8*a^4 + 9*a^3*b + 24*a^2*b^2 + 8
*a*b^3)*cosh(d*x + c)^6 + 15*(16*a^4 + 41*a^3*b + 27*a^2*b^2 + 40*a*b^3 + 8
*b^4)*cosh(d*x + c)^4 + 16*a^4 + 23*a^3*b - 27*a^2*b^2 - 4*a*b^3 + 6*(24*a^
4 + 64*a^3*b + 53*a^2*b^2 - 40*a*b^3 - 8*b^4)*cosh(d*x + c)^2)*sinh(d*x + c
)^2 - 15*(a^4*cosh(d*x + c)^10 + 10*a^4*cosh(d*x + c)*sinh(d*x + c)^9 + a^4
*sinh(d*x + c)^10 + (3*a^4 + 8*a^3*b)*cosh(d*x + c)^8 + (45*a^4*cosh(d*x +
c)^2 + 3*a^4 + 8*a^3*b)*sinh(d*x + c)^8 + 8*(15*a^4*cosh(d*x + c)^3 + (3*a^
4 + 8*a^3*b)*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(a^4 + 4*a^3*b + 8*a^2*b^2)
*cosh(d*x + c)^6 + 2*(105*a^4*cosh(d*x + c)^4 + a^4 + 4*a^3*b + 8*a^2*b^2 +
 14*(3*a^4 + 8*a^3*b)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(63*a^4*cosh(d*x
 + c)^5 + 14*(3*a^4 + 8*a^3*b)*cosh(d*x + c)^3 + 3*(a^4 + 4*a^3*b + 8*a^2*b
^2)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(a^4 + 4*a^3*b + 8*a^2*b^2)*cosh(d*x
 + c)^4 + 2*(105*a^4*cosh(d*x + c)^6 + 35*(3*a^4 + 8*a^3*b)*cosh(d*x + c)^4
 - a^4 - 4*a^3*b - 8*a^2*b^2 + 15*(a^4 + 4*a^3*b + 8*a^2*b^2)*cosh(d*x + c)
^2)*sinh(d*x + c)^4 - a^4 + 8*(15*a^4*cosh(d*x + c)^7 + 7*(3*a^4 + 8*a^3*b)
*cosh(d*x + c)^5 + 5*(a^4 + 4*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^3 - (a^4 + 4
*a^3*b + 8*a^2*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - (3*a^4 + 8*a^3*b)*cosh
(d*x + c)^2 + (45*a^4*cosh(d*x + c)^8 + 28*(3*a^4 + 8*a^3*b)*cosh(d*x + c)^
6 + 30*(a^4 + 4*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^4 - 3*a^4 - 8*a^3*b - 12*(
a^4 + 4*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(5*a^4*cosh
(d*x + c)^9 + 4*(3*a^4 + 8*a^3*b)*cosh(d*x + c)^7 + 6*(a^4 + 4*a^3*b + 8*a^
2*b^2)*cosh(d*x + c)^5 - 4*(a^4 + 4*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^3 - (3
*a^4 + 8*a^3*b)*cosh(d*x + c))*sinh(d*x + c))*sqrt(b/(a + b))*log((a^2*cosh
(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2
*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*si
nh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b
)*cosh(d*x + c))*sinh(d*x + c) - 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 +
a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 + 3*a*
b + 2*b^2)*sqrt(b/(a + b)))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x
 + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x
```

+ c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(
d*x + c))*sinh(d*x + c) + a)) + 16*(2*(8*a^4 + 9*a^3*b + 24*a^2*b^2 + 8*a*b
^3)*cosh(d*x + c)^7 + 3*(16*a^4 + 41*a^3*b + 27*a^2*b^2 + 40*a*b^3 + 8*b^4)
*cosh(d*x + c)^5 + 2*(24*a^4 + 64*a^3*b + 53*a^2*b^2 - 40*a*b^3 - 8*b^4)*co
sh(d*x + c)^3 + (16*a^4 + 23*a^3*b - 27*a^2*b^2 - 4*a*b^3)*cosh(d*x + c))*s
inh(d*x + c))/((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^10 + 1
0*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^9 + (
a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*sinh(d*x + c)^10 + (3*a^7 + 17*a^6*b
+ 33*a^5*b^2 + 27*a^4*b^3 + 8*a^3*b^4)*d*cosh(d*x + c)^8 + (45*(a^7 + 3*a^
6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^2 + (3*a^7 + 17*a^6*b + 33*a^5*b
^2 + 27*a^4*b^3 + 8*a^3*b^4)*d)*sinh(d*x + c)^8 + 2*(a^7 + 7*a^6*b + 23*a^5
*b^2 + 37*a^4*b^3 + 28*a^3*b^4 + 8*a^2*b^5)*d*cosh(d*x + c)^6 + 8*(15*(a^7
+ 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^3 + (3*a^7 + 17*a^6*b + 33
*a^5*b^2 + 27*a^4*b^3 + 8*a^3*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(10
5*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^4 + 14*(3*a^7 + 17*
a^6*b + 33*a^5*b^2 + 27*a^4*b^3 + 8*a^3*b^4)*d*cosh(d*x + c)^2 + (a^7 + 7*a
^6*b + 23*a^5*b^2 + 37*a^4*b^3 + 28*a^3*b^4 + 8*a^2*b^5)*d)*sinh(d*x + c)^6
- 2*(a^7 + 7*a^6*b + 23*a^5*b^2 + 37*a^4*b^3 + 28*a^3*b^4 + 8*a^2*b^5)*d*c
osh(d*x + c)^4 + 4*(63*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c
)^5 + 14*(3*a^7 + 17*a^6*b + 33*a^5*b^2 + 27*a^4*b^3 + 8*a^3*b^4)*d*cosh(d*
x + c)^3 + 3*(a^7 + 7*a^6*b + 23*a^5*b^2 + 37*a^4*b^3 + 28*a^3*b^4 + 8*a^2*
b^5)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(105*(a^7 + 3*a^6*b + 3*a^5*b^2 +
a^4*b^3)*d*cosh(d*x + c)^6 + 35*(3*a^7 + 17*a^6*b + 33*a^5*b^2 + 27*a^4*b^
3 + 8*a^3*b^4)*d*cosh(d*x + c)^4 + 15*(a^7 + 7*a^6*b + 23*a^5*b^2 + 37*a^4*
b^3 + 28*a^3*b^4 + 8*a^2*b^5)*d*cosh(d*x + c)^2 - (a^7 + 7*a^6*b + 23*a^5*b
^2 + 37*a^4*b^3 + 28*a^3*b^4 + 8*a^2*b^5)*d)*sinh(d*x + c)^4 - (3*a^7 + 17*
a^6*b + 33*a^5*b^2 + 27*a^4*b^3 + 8*a^3*b^4)*d*cosh(d*x + c)^2 + 8*(15*(a^7
+ 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^7 + 7*(3*a^7 + 17*a^6*b +
33*a^5*b^2 + 27*a^4*b^3 + 8*a^3*b^4)*d*cosh(d*x + c)^5 + 5*(a^7 + 7*a^6*b
+ 23*a^5*b^2 + 37*a^4*b^3 + 28*a^3*b^4 + 8*a^2*b^5)*d*cosh(d*x + c)^3 - (a^
7 + 7*a^6*b + 23*a^5*b^2 + 37*a^4*b^3 + 28*a^3*b^4 + 8*a^2*b^5)*d*cosh(d*x
+ c))*sinh(d*x + c)^3 + (45*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*
x + c)^8 + 28*(3*a^7 + 17*a^6*b + 33*a^5*b^2 + 27*a^4*b^3 + 8*a^3*b^4)*d*co
sh(d*x + c)^6 + 30*(a^7 + 7*a^6*b + 23*a^5*b^2 + 37*a^4*b^3 + 28*a^3*b^4 +
8*a^2*b^5)*d*cosh(d*x + c)^4 - 12*(a^7 + 7*a^6*b + 23*a^5*b^2 + 37*a^4*b^3
+ 28*a^3*b^4 + 8*a^2*b^5)*d*cosh(d*x + c)^2 - (3*a^7 + 17*a^6*b + 33*a^5*b^
2 + 27*a^4*b^3 + 8*a^3*b^4)*d)*sinh(d*x + c)^2 - (a^7 + 3*a^6*b + 3*a^5*b^2
+ a^4*b^3)*d + 2*(5*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^
9 + 4*(3*a^7 + 17*a^6*b + 33*a^5*b^2 + 27*a^4*b^3 + 8*a^3*b^4)*d*cosh(d*x +
c)^7 + 6*(a^7 + 7*a^6*b + 23*a^5*b^2 + 37*a^4*b^3 + 28*a^3*b^4 + 8*a^2*b^5
)*d*cosh(d*x + c)^5 - 4*(a^7 + 7*a^6*b + 23*a^5*b^2 + 37*a^4*b^3 + 28*a^3*b
^4 + 8*a^2*b^5)*d*cosh(d*x + c)^3 - (3*a^7 + 17*a^6*b + 33*a^5*b^2 + 27*a^4
*b^3 + 8*a^3*b^4)*d*cosh(d*x + c))*sinh(d*x + c)), -1/8*(2*(8*a^4 + 9*a^3*b
+ 24*a^2*b^2 + 8*a*b^3)*cosh(d*x + c)^8 + 16*(8*a^4 + 9*a^3*b + 24*a^2*b^2
+ 8*a*b^3)*cosh(d*x + c)*sinh(d*x + c)^7 + 2*(8*a^4 + 9*a^3*b + 24*a^2*b^2

+ 8*a*b^3)*sinh(d*x + c)^8 + 4*(16*a^4 + 41*a^3*b + 27*a^2*b^2 + 40*a*b^3
+ 8*b^4)*cosh(d*x + c)^6 + 4*(16*a^4 + 41*a^3*b + 27*a^2*b^2 + 40*a*b^3 + 8
b^4 + 14(8*a^4 + 9*a^3*b + 24*a^2*b^2 + 8*a*b^3)*cosh(d*x + c)^2)*sinh(d*
x + c)^6 + 8*(14*(8*a^4 + 9*a^3*b + 24*a^2*b^2 + 8*a*b^3)*cosh(d*x + c)^3 +
3*(16*a^4 + 41*a^3*b + 27*a^2*b^2 + 40*a*b^3 + 8*b^4)*cosh(d*x + c))*sinh(
d*x + c)^5 + 4*(24*a^4 + 64*a^3*b + 53*a^2*b^2 - 40*a*b^3 - 8*b^4)*cosh(d*x
+ c)^4 + 4*(35*(8*a^4 + 9*a^3*b + 24*a^2*b^2 + 8*a*b^3)*cosh(d*x + c)^4 +
24*a^4 + 64*a^3*b + 53*a^2*b^2 - 40*a*b^3 - 8*b^4 + 15*(16*a^4 + 41*a^3*b +
27*a^2*b^2 + 40*a*b^3 + 8*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 16*a^4 -
18*a^3*b - 4*a^2*b^2 + 16*(7*(8*a^4 + 9*a^3*b + 24*a^2*b^2 + 8*a*b^3)*cosh
(d*x + c)^5 + 5*(16*a^4 + 41*a^3*b + 27*a^2*b^2 + 40*a*b^3 + 8*b^4)*cosh(d*
x + c)^3 + (24*a^4 + 64*a^3*b + 53*a^2*b^2 - 40*a*b^3 - 8*b^4)*cosh(d*x + c
))*sinh(d*x + c)^3 + 4*(16*a^4 + 23*a^3*b - 27*a^2*b^2 - 4*a*b^3)*cosh(d*x
+ c)^2 + 4*(14*(8*a^4 + 9*a^3*b + 24*a^2*b^2 + 8*a*b^3)*cosh(d*x + c)^6 + 1
5*(16*a^4 + 41*a^3*b + 27*a^2*b^2 + 40*a*b^3 + 8*b^4)*cosh(d*x + c)^4 + 16*
a^4 + 23*a^3*b - 27*a^2*b^2 - 4*a*b^3 + 6*(24*a^4 + 64*a^3*b + 53*a^2*b^2 -
40*a*b^3 - 8*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 15*(a^4*cosh(d*x + c)
^10 + 10*a^4*cosh(d*x + c)*sinh(d*x + c)^9 + a^4*sinh(d*x + c)^10 + (3*a^4
+ 8*a^3*b)*cosh(d*x + c)^8 + (45*a^4*cosh(d*x + c)^2 + 3*a^4 + 8*a^3*b)*sin
h(d*x + c)^8 + 8*(15*a^4*cosh(d*x + c)^3 + (3*a^4 + 8*a^3*b)*cosh(d*x + c))
*sinh(d*x + c)^7 + 2*(a^4 + 4*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^6 + 2*(105*a
^4*cosh(d*x + c)^4 + a^4 + 4*a^3*b + 8*a^2*b^2 + 14*(3*a^4 + 8*a^3*b)*cosh(
d*x + c)^2)*sinh(d*x + c)^6 + 4*(63*a^4*cosh(d*x + c)^5 + 14*(3*a^4 + 8*a^3
*b)*cosh(d*x + c)^3 + 3*(a^4 + 4*a^3*b + 8*a^2*b^2)*cosh(d*x + c))*sinh(d*x
+ c)^5 - 2*(a^4 + 4*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^4 + 2*(105*a^4*cosh(d
x + c)^6 + 35(3*a^4 + 8*a^3*b)*cosh(d*x + c)^4 - a^4 - 4*a^3*b - 8*a^2*b^
2 + 15*(a^4 + 4*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^4 - a^4 +
8*(15*a^4*cosh(d*x + c)^7 + 7*(3*a^4 + 8*a^3*b)*cosh(d*x + c)^5 + 5*(a^4 +
4*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^3 - (a^4 + 4*a^3*b + 8*a^2*b^2)*cosh(d*
x + c))*sinh(d*x + c)^3 - (3*a^4 + 8*a^3*b)*cosh(d*x + c)^2 + (45*a^4*cosh(
d*x + c)^8 + 28*(3*a^4 + 8*a^3*b)*cosh(d*x + c)^6 + 30*(a^4 + 4*a^3*b + 8*a
^2*b^2)*cosh(d*x + c)^4 - 3*a^4 - 8*a^3*b - 12*(a^4 + 4*a^3*b + 8*a^2*b^2)*
cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(5*a^4*cosh(d*x + c)^9 + 4*(3*a^4 + 8*
a^3*b)*cosh(d*x + c)^7 + 6*(a^4 + 4*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^5 - 4*
(a^4 + 4*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^3 - (3*a^4 + 8*a^3*b)*cosh(d*x +
c))*sinh(d*x + c))*sqrt(-b/(a + b))*arctan(1/2*(a*cosh(d*x + c)^2 + 2*a*cos
h(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-b/(a + b))/b)
+ 8*(2*(8*a^4 + 9*a^3*b + 24*a^2*b^2 + 8*a*b^3)*cosh(d*x + c)^7 + 3*(16*a^
4 + 41*a^3*b + 27*a^2*b^2 + 40*a*b^3 + 8*b^4)*cosh(d*x + c)^5 + 2*(24*a^4 +
64*a^3*b + 53*a^2*b^2 - 40*a*b^3 - 8*b^4)*cosh(d*x + c)^3 + (16*a^4 + 23*a
^3*b - 27*a^2*b^2 - 4*a*b^3)*cosh(d*x + c))*sinh(d*x + c))/((a^7 + 3*a^6*b
+ 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^10 + 10*(a^7 + 3*a^6*b + 3*a^5*b^2 +
a^4*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^9 + (a^7 + 3*a^6*b + 3*a^5*b^2 + a^
4*b^3)*d*sinh(d*x + c)^10 + (3*a^7 + 17*a^6*b + 33*a^5*b^2 + 27*a^4*b^3 + 8
*a^3*b^4)*d*cosh(d*x + c)^8 + (45*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*c

```
osh(d*x + c)^2 + (3*a^7 + 17*a^6*b + 33*a^5*b^2 + 27*a^4*b^3 + 8*a^3*b^4)*d
)*sinh(d*x + c)^8 + 2*(a^7 + 7*a^6*b + 23*a^5*b^2 + 37*a^4*b^3 + 28*a^3*b^4
 + 8*a^2*b^5)*d*cosh(d*x + c)^6 + 8*(15*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^
3)*d*cosh(d*x + c)^3 + (3*a^7 + 17*a^6*b + 33*a^5*b^2 + 27*a^4*b^3 + 8*a^3*
b^4)*d*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(105*(a^7 + 3*a^6*b + 3*a^5*b^2 +
 a^4*b^3)*d*cosh(d*x + c)^4 + 14*(3*a^7 + 17*a^6*b + 33*a^5*b^2 + 27*a^4*b^
3 + 8*a^3*b^4)*d*cosh(d*x + c)^2 + (a^7 + 7*a^6*b + 23*a^5*b^2 + 37*a^4*b^3
 + 28*a^3*b^4 + 8*a^2*b^5)*d)*sinh(d*x + c)^6 - 2*(a^7 + 7*a^6*b + 23*a^5*b
^2 + 37*a^4*b^3 + 28*a^3*b^4 + 8*a^2*b^5)*d*cosh(d*x + c)^4 + 4*(63*(a^7 +
3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^5 + 14*(3*a^7 + 17*a^6*b + 3
3*a^5*b^2 + 27*a^4*b^3 + 8*a^3*b^4)*d*cosh(d*x + c)^3 + 3*(a^7 + 7*a^6*b +
23*a^5*b^2 + 37*a^4*b^3 + 28*a^3*b^4 + 8*a^2*b^5)*d*cosh(d*x + c))*sinh(d*x
 + c)^5 + 2*(105*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^6 +
35*(3*a^7 + 17*a^6*b + 33*a^5*b^2 + 27*a^4*b^3 + 8*a^3*b^4)*d*cosh(d*x + c)
^4 + 15*(a^7 + 7*a^6*b + 23*a^5*b^2 + 37*a^4*b^3 + 28*a^3*b^4 + 8*a^2*b^5)*
d*cosh(d*x + c)^2 - (a^7 + 7*a^6*b + 23*a^5*b^2 + 37*a^4*b^3 + 28*a^3*b^4 +
 8*a^2*b^5)*d)*sinh(d*x + c)^4 - (3*a^7 + 17*a^6*b + 33*a^5*b^2 + 27*a^4*b^
3 + 8*a^3*b^4)*d*cosh(d*x + c)^2 + 8*(15*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b
^3)*d*cosh(d*x + c)^7 + 7*(3*a^7 + 17*a^6*b + 33*a^5*b^2 + 27*a^4*b^3 + 8*a
^3*b^4)*d*cosh(d*x + c)^5 + 5*(a^7 + 7*a^6*b + 23*a^5*b^2 + 37*a^4*b^3 + 28
*a^3*b^4 + 8*a^2*b^5)*d*cosh(d*x + c)^3 - (a^7 + 7*a^6*b + 23*a^5*b^2 + 37*
a^4*b^3 + 28*a^3*b^4 + 8*a^2*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^3 + (45*(a
^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^8 + 28*(3*a^7 + 17*a^6*
b + 33*a^5*b^2 + 27*a^4*b^3 + 8*a^3*b^4)*d*cosh(d*x + c)^6 + 30*(a^7 + 7*a^
6*b + 23*a^5*b^2 + 37*a^4*b^3 + 28*a^3*b^4 + 8*a^2*b^5)*d*cosh(d*x + c)^4 -
 12*(a^7 + 7*a^6*b + 23*a^5*b^2 + 37*a^4*b^3 + 28*a^3*b^4 + 8*a^2*b^5)*d*co
sh(d*x + c)^2 - (3*a^7 + 17*a^6*b + 33*a^5*b^2 + 27*a^4*b^3 + 8*a^3*b^4)*d)
*sinh(d*x + c)^2 - (a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d + 2*(5*(a^7 + 3*
a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^9 + 4*(3*a^7 + 17*a^6*b + 33*a
^5*b^2 + 27*a^4*b^3 + 8*a^3*b^4)*d*cosh(d*x + c)^7 + 6*(a^7 + 7*a^6*b + 23*
a^5*b^2 + 37*a^4*b^3 + 28*a^3*b^4 + 8*a^2*b^5)*d*cosh(d*x + c)^5 - 4*(a^7 +
 7*a^6*b + 23*a^5*b^2 + 37*a^4*b^3 + 28*a^3*b^4 + 8*a^2*b^5)*d*cosh(d*x + c
)^3 - (3*a^7 + 17*a^6*b + 33*a^5*b^2 + 27*a^4*b^3 + 8*a^3*b^4)*d*cosh(d*x +
 c))*sinh(d*x + c))]
```

giac [B] time = 1.76, size = 347, normalized size = 2.75

$$\frac{\frac{15\,b\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\sqrt{-ab-b^2}}-\frac{2\left(9\,a^3be^{(6\,dx+6\,c)}+24\,a^2b^2e^{(6\,dx+6\,c)}+8\,ab^3e^{(6\,dx+6\,c)}+27\,a^3be^{(4\,dx+4\,c)}+78\,a^2b^2e^{(4\,dx+4\,c)}+88\,ab^3e^{(4\,dx+4\,c)}+16\,b^4e^{(4}\right.}{\left(a^5+3\,a^4b+3\,a^3b^2+a^2b^3\right)\left(ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+4\,b\right.}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*(15*b*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/((a^3
+ 3*a^2*b + 3*a*b^2 + b^3)*sqrt(-a*b - b^2)) - 2*(9*a^3*b*e^(6*d*x + 6*c) +
 24*a^2*b^2*e^(6*d*x + 6*c) + 8*a*b^3*e^(6*d*x + 6*c) + 27*a^3*b*e^(4*d*x +
 4*c) + 78*a^2*b^2*e^(4*d*x + 4*c) + 88*a*b^3*e^(4*d*x + 4*c) + 16*b^4*e^(4
*d*x + 4*c) + 27*a^3*b*e^(2*d*x + 2*c) + 56*a^2*b^2*e^(2*d*x + 2*c) + 8*a*b
^3*e^(2*d*x + 2*c) + 9*a^3*b + 2*a^2*b^2)/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2
*b^3)*(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) + a)^2
) - 16/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(e^(2*d*x + 2*c) - 1)))/d
```

maple [B] time = 0.41, size = 816, normalized size = 6.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] -1/2/d/(a^3+3*a^2*b+3*a*b^2+b^3)*tanh(1/2*d*x+1/2*c)+9/4/d*b/(a+b)^3/(tanh(
1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh
(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^7*a+9/4/d*b^2/(a+b)^3/(tanh(
1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh
(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^7+27/4/d*b/(a+b)^3/(tanh(1/2
*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/
2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^5*a-1/4/d*b^2/(a+b)^3/(tanh(1/2
*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/
2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^5+27/4/d*b/(a+b)^3/(tanh(1/2*d*
x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d
*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^3*a-1/4/d*b^2/(a+b)^3/(tanh(1/2*d*
x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d
*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^3+9/4/d*b/(a+b)^3/(tanh(1/2*d*x+1/
2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1
/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)*a+9/4/d*b^2/(a+b)^3/(tanh(1/2*d*x+1/2*
c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2
*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)-15/16/d*b^(1/2)/(a+b)^(7/2)*ln((a+b)^(1/
2)*tanh(1/2*d*x+1/2*c)^2-2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))+15/16/d
*b^(1/2)/(a+b)^(7/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/
2*d*x+1/2*c)+(a+b)^(1/2))-1/2/d/(a+b)^3/tanh(1/2*d*x+1/2*c)
```

maxima [B] time = 0.53, size = 533, normalized size = 4.23

$$-\frac{15\,b\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{16\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\sqrt{(a+b)b}\,d}-\frac{8\,a^4-9\,a^3b-2\,a^2b^2+2}{4\left(a^7+3\,a^6b+3\,a^5b^2+a^4b^3+\left(3\,a^7+17\,a^6b+33\,a^5b^2+27\,a^4b^3+8\,a\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -15/16*b*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*
x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*s
qrt((a + b)*b)*d) - 1/4*(8*a^4 - 9*a^3*b - 2*a^2*b^2 + 2*(16*a^4 + 23*a^3*b
 - 27*a^2*b^2 - 4*a*b^3)*e^(-2*d*x - 2*c) + 2*(24*a^4 + 64*a^3*b + 53*a^2*b
^2 - 40*a*b^3 - 8*b^4)*e^(-4*d*x - 4*c) + 2*(16*a^4 + 41*a^3*b + 27*a^2*b^2
 + 40*a*b^3 + 8*b^4)*e^(-6*d*x - 6*c) + (8*a^4 + 9*a^3*b + 24*a^2*b^2 + 8*a
*b^3)*e^(-8*d*x - 8*c))/((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3 + (3*a^7 + 17
*a^6*b + 33*a^5*b^2 + 27*a^4*b^3 + 8*a^3*b^4)*e^(-2*d*x - 2*c) + 2*(a^7 + 7
*a^6*b + 23*a^5*b^2 + 37*a^4*b^3 + 28*a^3*b^4 + 8*a^2*b^5)*e^(-4*d*x - 4*c)
 - 2*(a^7 + 7*a^6*b + 23*a^5*b^2 + 37*a^4*b^3 + 28*a^3*b^4 + 8*a^2*b^5)*e^(
-6*d*x - 6*c) - (3*a^7 + 17*a^6*b + 33*a^5*b^2 + 27*a^4*b^3 + 8*a^3*b^4)*e^
(-8*d*x - 8*c) - (a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*e^(-10*d*x - 10*c))*
d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh (c + d\, x)^6}{\sinh (c + d\, x)^2 \left(a \cosh (c + d\, x)^2 + b\right)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^2*(a + b/cosh(c + d*x)^2)^3),x)
```

```
[Out] int(cosh(c + d*x)^6/(sinh(c + d*x)^2*(b + a*cosh(c + d*x)^2)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^{2}{\left(c + dx \right)}}{\left(a + b \operatorname{sech}^{2}{\left(c + dx \right)}\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Integral(csch(c + d*x)**2/(a + b*sech(c + d*x)**2)**3, x)
```

3.47 $\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=213

$$-\frac{\left(4a^2-9ab-b^2\right)\cosh(c+dx)}{8ad(a+b)^3\left(a\cosh^2(c+dx)+b\right)}-\frac{\sqrt{b}\left(15a^2-10ab-b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{8a^{3/2}d(a+b)^4}+\frac{b(2a-b)\cosh(c+dx)}{4ad(a+b)^2\left(a\cosh^2(c+dx)+\right.}$$

```
[Out] 1/2*(a-5*b)*arctanh(cosh(d*x+c))/d/(a+b)^4+1/4*(2*a-b)*b*cosh(d*x+c)/a/(a+b
)^2/d/(b+a*cosh(d*x+c)^2)^2-1/8*(4*a^2-9*a*b-b^2)*cosh(d*x+c)/a/(a+b)^3/d/(
b+a*cosh(d*x+c)^2)-1/2*cosh(d*x+c)*coth(d*x+c)^2/(a+b)/d/(b+a*cosh(d*x+c)^2
)^2-1/8*(15*a^2-10*a*b-b^2)*arctan(cosh(d*x+c)*a^(1/2)/b^(1/2))*b^(1/2)/a^(
3/2)/(a+b)^4/d
```

Rubi [A] time = 0.34, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4133, 470, 578, 527, 522, 206, 205}

$$-\frac{\left(4a^2-9ab-b^2\right)\cosh(c+dx)}{8ad(a+b)^3\left(a\cosh^2(c+dx)+b\right)}-\frac{\sqrt{b}\left(15a^2-10ab-b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{8a^{3/2}d(a+b)^4}+\frac{b(2a-b)\cosh(c+dx)}{4ad(a+b)^2\left(a\cosh^2(c+dx)+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] -(Sqrt[b]*(15*a^2 - 10*a*b - b^2)*ArcTan[(Sqrt[a]*Cosh[c + d*x])/Sqrt[b]])/
(8*a^(3/2)*(a + b)^4*d) + ((a - 5*b)*ArcTanh[Cosh[c + d*x]])/(2*(a + b)^4*d
) + ((2*a - b)*b*Cosh[c + d*x])/(4*a*(a + b)^2*d*(b + a*Cosh[c + d*x]^2)^2)
 - ((4*a^2 - 9*a*b - b^2)*Cosh[c + d*x])/(8*a*(a + b)^3*d*(b + a*Cosh[c + d
*x]^2)) - (Cosh[c + d*x]*Coth[c + d*x]^2)/(2*(a + b)*d*(b + a*Cosh[c + d*x]
^2)^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = \frac{\operatorname{Subst}\left(\int \frac{x^6}{\left(1-x^2\right)^2\left(b+ax^2\right)^3}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{\cosh(c+dx)\coth^2(c+dx)}{2(a+b)d\left(b+a\cosh^2(c+dx)\right)^2} - \frac{\operatorname{Subst}\left(\int \frac{x^2\left(3b+(-a+2b)x^2\right)}{\left(1-x^2\right)\left(b+ax^2\right)^3}\,dx, x, \cosh(c+dx)\right)}{2(a+b)d}$$

$$= \frac{(2a-b)b\cosh(c+dx)}{4a(a+b)^2d\left(b+a\cosh^2(c+dx)\right)^2} - \frac{\cosh(c+dx)\coth^2(c+dx)}{2(a+b)d\left(b+a\cosh^2(c+dx)\right)^2} - \frac{\operatorname{Subst}\left(\int\right.}{}$$

$$= \frac{(2a-b)b\cosh(c+dx)}{4a(a+b)^2d\left(b+a\cosh^2(c+dx)\right)^2} - \frac{\left(4a^2-9ab-b^2\right)\cosh(c+dx)}{8a(a+b)^3d\left(b+a\cosh^2(c+dx)\right)} - \frac{\cosh}{2(a+b}$$

$$= \frac{(2a-b)b\cosh(c+dx)}{4a(a+b)^2d\left(b+a\cosh^2(c+dx)\right)^2} - \frac{\left(4a^2-9ab-b^2\right)\cosh(c+dx)}{8a(a+b)^3d\left(b+a\cosh^2(c+dx)\right)} - \frac{\cosh}{2(a+b}$$

$$= -\frac{\sqrt{b}\left(15a^2-10ab-b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{8a^{3/2}(a+b)^4d} + \frac{(a-5b)\tanh^{-1}(\cosh(c+dx))}{2(a+b)^4d} +$$

Mathematica [C] time = 4.12, size = 524, normalized size = 2.46

$$\frac{\operatorname{sech}^5(c+dx)(a\cosh(2(c+dx))+a+2b)\left(\frac{\sqrt{b}\left(-15a^2+10ab+b^2\right)\operatorname{sech}(c+dx)(a\cosh(2(c+dx))+a+2b)^2\tan^{-1}\left(\frac{\sinh(c)\tanh\left(\frac{dx}{2}\right)\left(\sqrt{a}-i\sqrt{a+b}\right.}{}\right.}{a^{3/2}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^5*((-8*b^2*(a + b)^2)/a + (2
*b*(a + b)*(9*a + b)*(a + 2*b + a*Cosh[2*(c + d*x)]))/a + (Sqrt[b]*(-15*a^2
```

```
 + 10*a*b + b^2)*ArcTan[((Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^
2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c]
- Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cosh[2*(c + d*x)])^2*Se
ch[c + d*x])/a^(3/2) + (Sqrt[b]*(-15*a^2 + 10*a*b + b^2)*ArcTan[((Sqrt[a] +
 I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]
*(Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[
b]]*(a + 2*b + a*Cosh[2*(c + d*x)])^2*Sech[c + d*x])/a^(3/2) - (a + b)*(a +
 2*b + a*Cosh[2*(c + d*x)])^2*Csch[(c + d*x)/2]^2*Sech[c + d*x] + 4*(a - 5*
b)*(a + 2*b + a*Cosh[2*(c + d*x)])^2*Log[Cosh[(c + d*x)/2]]*Sech[c + d*x] -
 4*(a - 5*b)*(a + 2*b + a*Cosh[2*(c + d*x)])^2*Log[Sinh[(c + d*x)/2]]*Sech[
c + d*x] - (a + b)*(a + 2*b + a*Cosh[2*(c + d*x)])^2*Sech[(c + d*x)/2]^2*Se
ch[c + d*x]))/(64*(a + b)^4*d*(a + b*Sech[c + d*x]^2)^3)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")

[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[84,-86]Warning, need to choose a branch for the root of a p
olynomial with parameters. This might be wrong.The choice was done assuming
 [a,b]=[-42,-12]Warning, need to choose a branch for the root of a polynomi
al with parameters. This might be wrong.The choice was done assuming [a,b]=
[-43,-99]Warning, need to choose a branch for the root of a polynomial with
 parameters. This might be wrong.The choice was done assuming [a,b]=[-28,94
]Warning, need to choose a branch for the root of a polynomial with paramet
ers. This might be wrong.The choice was done assuming [a,b]=[-7,46]Warning,
 need to choose a branch for the root of a polynomial with parameters. This
 might be wrong.The choice was done assuming [a,b]=[-35,-99]Warning, need t
o choose a branch for the root of a polynomial with parameters. This might
be wrong.The choice was done assuming [a,b]=[7,50]Warning, need to choose a
 branch for the root of a polynomial with parameters. This might be wrong.T
```

```
he choice was done assuming [a,b]=[-63,-70]Undef/Unsigned Inf encountered i
n limitEvaluation time: 1.54Limit: Max order reached or unable to make seri
es expansion Error: Bad Argument Value
```

maple [B] time = 0.45, size = 1555, normalized size = 7.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] 1/8/d*tanh(1/2*d*x+1/2*c)^2/(a^3+3*a^2*b+3*a*b^2+b^3)+9/4/d*b/(a+b)^4/(tanh
(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tan
h(1/2*d*x+1/2*c)^2*b+a+b)^2*a^2*tanh(1/2*d*x+1/2*c)^6-5/4/d*b^2/(a+b)^4/(ta
nh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*t
anh(1/2*d*x+1/2*c)^2*b+a+b)^2*a*tanh(1/2*d*x+1/2*c)^6-13/4/d*b^3/(a+b)^4/(t
anh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*
tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^6+1/4/d*b^4/(a+b)^4/(tan
h(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*ta
nh(1/2*d*x+1/2*c)^2*b+a+b)^2/a*tanh(1/2*d*x+1/2*c)^6+27/4/d*b/(a+b)^4/(tanh
(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tan
h(1/2*d*x+1/2*c)^2*b+a+b)^2*a^2*tanh(1/2*d*x+1/2*c)^4-21/4/d*b^2/(a+b)^4/(t
anh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*
tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*a*tanh(1/2*d*x+1/2*c)^4+29/4/d*b^3/(a+b)^4/(
tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2
*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^4-3/4/d*b^4/(a+b)^4/(ta
nh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*t
anh(1/2*d*x+1/2*c)^2*b+a+b)^2/a*tanh(1/2*d*x+1/2*c)^4+27/4/d*b/(a+b)^4/(tan
h(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*ta
nh(1/2*d*x+1/2*c)^2*b+a+b)^2*a^2*tanh(1/2*d*x+1/2*c)^2+1/4/d*b^2/(a+b)^4/(t
anh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*
tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*a*tanh(1/2*d*x+1/2*c)^2-23/4/d*b^3/(a+b)^4/(
tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2
*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^2+3/4/d*b^4/(a+b)^4/(ta
nh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*t
anh(1/2*d*x+1/2*c)^2*b+a+b)^2/a*tanh(1/2*d*x+1/2*c)^2+9/4/d*b/(a+b)^4/(tanh
(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tan
h(1/2*d*x+1/2*c)^2*b+a+b)^2*a^2+17/4/d*b^2/(a+b)^4/(tanh(1/2*d*x+1/2*c)^4*a
+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*
b+a+b)^2*a+7/4/d*b^3/(a+b)^4/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)
^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2-1/4/d*b^4/(a+
b)^4/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)
^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/a-15/8/d*b/(a+b)^4*a/(a*b)^(1/2)*arct
an(1/4*(2*(a+b)*tanh(1/2*d*x+1/2*c)^2+2*a-2*b)/(a*b)^(1/2))+5/4/d*b^2/(a+b)
^4/(a*b)^(1/2)*arctan(1/4*(2*(a+b)*tanh(1/2*d*x+1/2*c)^2+2*a-2*b)/(a*b)^(1/
```

```
2))+1/8/d*b^3/(a+b)^4/a/(a*b)^(1/2)*arctan(1/4*(2*(a+b)*tanh(1/2*d*x+1/2*c)
^2+2*a-2*b)/(a*b)^(1/2))-1/8/d/(a+b)^3/tanh(1/2*d*x+1/2*c)^2-1/2/d/(a+b)^4*
ln(tanh(1/2*d*x+1/2*c))*a+5/2/d/(a+b)^4*ln(tanh(1/2*d*x+1/2*c))*b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(a - 5*b)*log((e^(d*x + c) + 1)*e^(-c))/(a^4*d + 4*a^3*b*d + 6*a^2*b^2*
d + 4*a*b^3*d + b^4*d) - 1/2*(a - 5*b)*log((e^(d*x + c) - 1)*e^(-c))/(a^4*d
 + 4*a^3*b*d + 6*a^2*b^2*d + 4*a*b^3*d + b^4*d) - 1/4*((4*a^3*e^(11*c) - 9*
a^2*b*e^(11*c) - a*b^2*e^(11*c))*e^(11*d*x) + (20*a^3*e^(9*c) + 23*a^2*b*e^
(9*c) - 29*a*b^2*e^(9*c) + 4*b^3*e^(9*c))*e^(9*d*x) + 2*(20*a^3*e^(7*c) + 5
7*a^2*b*e^(7*c) + 47*a*b^2*e^(7*c) - 2*b^3*e^(7*c))*e^(7*d*x) + 2*(20*a^3*e
^(5*c) + 57*a^2*b*e^(5*c) + 47*a*b^2*e^(5*c) - 2*b^3*e^(5*c))*e^(5*d*x) + (
20*a^3*e^(3*c) + 23*a^2*b*e^(3*c) - 29*a*b^2*e^(3*c) + 4*b^3*e^(3*c))*e^(3*
d*x) + (4*a^3*e^c - 9*a^2*b*e^c - a*b^2*e^c)*e^(d*x))/(a^6*d + 3*a^5*b*d +
3*a^4*b^2*d + a^3*b^3*d + (a^6*d*e^(12*c) + 3*a^5*b*d*e^(12*c) + 3*a^4*b^2*
d*e^(12*c) + a^3*b^3*d*e^(12*c))*e^(12*d*x) + 2*(a^6*d*e^(10*c) + 7*a^5*b*d
*e^(10*c) + 15*a^4*b^2*d*e^(10*c) + 13*a^3*b^3*d*e^(10*c) + 4*a^2*b^4*d*e^(
10*c))*e^(10*d*x) - (a^6*d*e^(8*c) + 3*a^5*b*d*e^(8*c) - 13*a^4*b^2*d*e^(8*
c) - 47*a^3*b^3*d*e^(8*c) - 48*a^2*b^4*d*e^(8*c) - 16*a*b^5*d*e^(8*c))*e^(8
*d*x) - 4*(a^6*d*e^(6*c) + 7*a^5*b*d*e^(6*c) + 23*a^4*b^2*d*e^(6*c) + 37*a^
3*b^3*d*e^(6*c) + 28*a^2*b^4*d*e^(6*c) + 8*a*b^5*d*e^(6*c))*e^(6*d*x) - (a^
6*d*e^(4*c) + 3*a^5*b*d*e^(4*c) - 13*a^4*b^2*d*e^(4*c) - 47*a^3*b^3*d*e^(4*
c) - 48*a^2*b^4*d*e^(4*c) - 16*a*b^5*d*e^(4*c))*e^(4*d*x) + 2*(a^6*d*e^(2*c
) + 7*a^5*b*d*e^(2*c) + 15*a^4*b^2*d*e^(2*c) + 13*a^3*b^3*d*e^(2*c) + 4*a^2
*b^4*d*e^(2*c))*e^(2*d*x)) - 8*integrate(1/32*((15*a^2*b*e^(3*c) - 10*a*b^2
*e^(3*c) - b^3*e^(3*c))*e^(3*d*x) - (15*a^2*b*e^c - 10*a*b^2*e^c - b^3*e^c)
*e^(d*x))/(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4 + (a^6*e^(4*c) +
 4*a^5*b*e^(4*c) + 6*a^4*b^2*e^(4*c) + 4*a^3*b^3*e^(4*c) + a^2*b^4*e^(4*c))
*e^(4*d*x) + 2*(a^6*e^(2*c) + 6*a^5*b*e^(2*c) + 14*a^4*b^2*e^(2*c) + 16*a^3
*b^3*e^(2*c) + 9*a^2*b^4*e^(2*c) + 2*a*b^5*e^(2*c))*e^(2*d*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh\left(c+d\,x\right)^6}{\sinh\left(c+d\,x\right)^3\,{\left(a\,\cosh\left(c+d\,x\right)^2+b\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^3*(a + b/cosh(c + d*x)^2)^3),x)
```

```
[Out] int(cosh(c + d*x)^6/(sinh(c + d*x)^3*(b + a*cosh(c + d*x)^2)^3), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Integral(csch(c + d*x)**3/(a + b*sech(c + d*x)**2)**3, x)
```

3.48 $\int \frac{\mathbf{csch}^4(c+dx)}{\left(a+b\mathbf{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=165

$$-\frac{5\sqrt{b}\,(3a-4b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8d(a+b)^{9/2}}-\frac{b(7a-4b)\tanh(c+dx)}{8d(a+b)^4\left(a-b\tanh^2(c+dx)+b\right)}-\frac{ab\tanh(c+dx)}{4d(a+b)^3\left(a-b\tanh^2(c+dx)+\right.}$$

```
[Out] (a-2*b)*coth(d*x+c)/d/(a+b)^4-1/3*coth(d*x+c)^3/d/(a+b)^3-5/8*(3*a-4*b)*arc
tanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))*b^(1/2)/(a+b)^(9/2)/d-1/4*a*b*tanh(d*
x+c)/(a+b)^3/d/(a+b-b*tanh(d*x+c)^2)^2-1/8*(7*a-4*b)*b*tanh(d*x+c)/(a+b)^4/
d/(a+b-b*tanh(d*x+c)^2)
```

Rubi [**A**] time = 0.27, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4132, 456, 1259, 1261, 208}

$$-\frac{5\sqrt{b}\,(3a-4b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8d(a+b)^{9/2}}-\frac{b(7a-4b)\tanh(c+dx)}{8d(a+b)^4\left(a-b\tanh^2(c+dx)+b\right)}-\frac{ab\tanh(c+dx)}{4d(a+b)^3\left(a-b\tanh^2(c+dx)+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (-5*(3*a - 4*b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(8*(a
 + b)^(9/2)*d) + ((a - 2*b)*Coth[c + d*x])/((a + b)^4*d) - Coth[c + d*x]^3/
(3*(a + b)^3*d) - (a*b*Tanh[c + d*x])/(4*(a + b)^3*d*(a + b - b*Tanh[c + d*
x]^2)^2) - ((7*a - 4*b)*b*Tanh[c + d*x])/(8*(a + b)^4*d*(a + b - b*Tanh[c +
 d*x]^2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
```

```
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1-x^2}{x^4\left(a+b-bx^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{ab\tanh(c+dx)}{4(a+b)^3d\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{b\operatorname{Subst}\left(\int \frac{\frac{4}{b(a+b)}-\frac{4ax^2}{b(a+b)^2}-\frac{3ax^4}{(a+b)^3}}{x^4\left(a+b-bx^2\right)^2}\,dx, x, \tan\right.}{4d}$$

$$= -\frac{ab\tanh(c+dx)}{4(a+b)^3d\left(a+b-b\tanh^2(c+dx)\right)^2} - \frac{(7a-4b)b\tanh(c+dx)}{8(a+b)^4d\left(a+b-b\tanh^2(c+dx)\right)} +$$

$$= -\frac{ab\tanh(c+dx)}{4(a+b)^3d\left(a+b-b\tanh^2(c+dx)\right)^2} - \frac{(7a-4b)b\tanh(c+dx)}{8(a+b)^4d\left(a+b-b\tanh^2(c+dx)\right)} +$$

$$= \frac{(a-2b)\coth(c+dx)}{(a+b)^4d} - \frac{\coth^3(c+dx)}{3(a+b)^3d} - \frac{ab\tanh(c+dx)}{4(a+b)^3d\left(a+b-b\tanh^2(c+dx)\right)^2} -$$

$$= -\frac{5(3a-4b)\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8(a+b)^{9/2}d} + \frac{(a-2b)\coth(c+dx)}{(a+b)^4d} - \frac{\coth^3(c+dx)}{3(a+b)^3d}$$

Mathematica [B] time = 5.53, size = 985, normalized size = 5.97

$$-\frac{(\cosh(2(c+dx))a+a+2b)\operatorname{sech}^6(c+dx)\left(\frac{\operatorname{csch}(c)\operatorname{sech}(2c)\left(224\sinh(2c-dx)a^4-224\sinh(2c+dx)a^4+176\sinh(4c+dx)a^4+48\sinh(2c\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csch[c + d*x]^4/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] -1/6144*((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^6*((480*(3*a - 4*b)*
b*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[
2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(a + 2*b + a*Co
```

```
sh[2*(c + d*x)])^2*(Cosh[2*c] - Sinh[2*c]))/(Sqrt[a + b]*Sqrt[b*(Cosh[c] -
Sinh[c])^4]) + (Csch[c]*Csch[c + d*x]^3*Sech[2*c]*(4*(44*a^4 + 122*a^3*b +
63*a^2*b^2 + 126*a*b^3 + 36*b^4)*Sinh[d*x] + (-96*a^4 - 71*a^3*b + 344*a^2*
b^2 - 1208*a*b^3 + 48*b^4)*Sinh[3*d*x] + 224*a^4*Sinh[2*c - d*x] + 576*a^3*
b*Sinh[2*c - d*x] + 124*a^2*b^2*Sinh[2*c - d*x] - 2184*a*b^3*Sinh[2*c - d*x
] + 144*b^4*Sinh[2*c - d*x] - 224*a^4*Sinh[2*c + d*x] - 657*a^3*b*Sinh[2*c
+ d*x] - 538*a^2*b^2*Sinh[2*c + d*x] + 984*a*b^3*Sinh[2*c + d*x] + 144*b^4*
Sinh[2*c + d*x] + 176*a^4*Sinh[4*c + d*x] + 569*a^3*b*Sinh[4*c + d*x] + 666
*a^2*b^2*Sinh[4*c + d*x] + 1704*a*b^3*Sinh[4*c + d*x] - 144*b^4*Sinh[4*c +
d*x] + 48*a^4*Sinh[2*c + 3*d*x] + 111*a^3*b*Sinh[2*c + 3*d*x] + 360*a^2*b^2
*Sinh[2*c + 3*d*x] + 312*a*b^3*Sinh[2*c + 3*d*x] - 48*b^4*Sinh[2*c + 3*d*x]
 - 96*a^4*Sinh[4*c + 3*d*x] - 152*a^3*b*Sinh[4*c + 3*d*x] + 146*a^2*b^2*Sin
h[4*c + 3*d*x] - 728*a*b^3*Sinh[4*c + 3*d*x] - 48*b^4*Sinh[4*c + 3*d*x] + 4
8*a^4*Sinh[6*c + 3*d*x] + 192*a^3*b*Sinh[6*c + 3*d*x] + 558*a^2*b^2*Sinh[6*
c + 3*d*x] - 168*a*b^3*Sinh[6*c + 3*d*x] + 48*b^4*Sinh[6*c + 3*d*x] + 16*a^
4*Sinh[2*c + 5*d*x] - 598*a^2*b^2*Sinh[2*c + 5*d*x] + 48*a*b^3*Sinh[2*c + 5
*d*x] + 72*a^3*b*Sinh[4*c + 5*d*x] + 150*a^2*b^2*Sinh[4*c + 5*d*x] - 48*a*b
^3*Sinh[4*c + 5*d*x] + 16*a^4*Sinh[6*c + 5*d*x] + 27*a^3*b*Sinh[6*c + 5*d*x
] - 388*a^2*b^2*Sinh[6*c + 5*d*x] + 45*a^3*b*Sinh[8*c + 5*d*x] - 60*a^2*b^2
*Sinh[8*c + 5*d*x] + 16*a^4*Sinh[4*c + 7*d*x] - 83*a^3*b*Sinh[4*c + 7*d*x]
+ 6*a^2*b^2*Sinh[4*c + 7*d*x] + 27*a^3*b*Sinh[6*c + 7*d*x] - 6*a^2*b^2*Sinh
[6*c + 7*d*x] + 16*a^4*Sinh[8*c + 7*d*x] - 56*a^3*b*Sinh[8*c + 7*d*x]))/a))
/((a + b)^4*d*(a + b*Sech[c + d*x]^2)^3)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 1.66, size = 406, normalized size = 2.46

$$-\frac{\frac{15\left(3\,ab-4\,b^{2}\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^{2}}}\right)}{\left(a^{4}+4\,a^{3}b+6\,a^{2}b^{2}+4\,ab^{3}+b^{4}\right)\sqrt{-ab-b^{2}}}-\frac{6\left(9\,a^{3}be^{(6\,dx+6\,c)}+20\,a^{2}b^{2}e^{(6\,dx+6\,c)}+27\,a^{3}be^{(4\,dx+4\,c)}+66\,a^{2}b^{2}e^{(4\,dx+4\,c)}+56\,ab^{3}e^{(4\,dx+4\,c)}-16\,b^{4}e^{(4\,dx+4\,c)}+\cdots}{\left(a^{5}+4\,a^{4}b+6\,a^{3}b^{2}+4\,a^{2}b^{3}+ab^{4}\right)\left(ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}\cdots}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/24*(15*(3*a*b - 4*b^2)*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*
b - b^2))/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*sqrt(-a*b - b^2)) -
```

```
6*(9*a^3*b*e^(6*d*x + 6*c) + 20*a^2*b^2*e^(6*d*x + 6*c) + 27*a^3*b*e^(4*d*x
 + 4*c) + 66*a^2*b^2*e^(4*d*x + 4*c) + 56*a*b^3*e^(4*d*x + 4*c) - 16*b^4*e^
(4*d*x + 4*c) + 27*a^3*b*e^(2*d*x + 2*c) + 44*a^2*b^2*e^(2*d*x + 2*c) - 16*
a*b^3*e^(2*d*x + 2*c) + 9*a^3*b - 2*a^2*b^2)/((a^5 + 4*a^4*b + 6*a^3*b^2 +
4*a^2*b^3 + a*b^4)*(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x
+ 2*c) + a)^2) + 16*(9*b*e^(4*d*x + 4*c) + 6*a*e^(2*d*x + 2*c) - 12*b*e^(2*
d*x + 2*c) - 2*a + 7*b)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*(e^(2*
d*x + 2*c) - 1)^3))/d
```

maple [B] time = 0.48, size = 1443, normalized size = 8.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] -1/24/d/(a^3+3*a^2*b+3*a*b^2+b^3)/(a+b)*a*tanh(1/2*d*x+1/2*c)^3-1/24/d/(a^3
+3*a^2*b+3*a*b^2+b^3)/(a+b)*tanh(1/2*d*x+1/2*c)^3*b+3/8/d/(a^3+3*a^2*b+3*a*
b^2+b^3)/(a+b)*a*tanh(1/2*d*x+1/2*c)-9/8/d/(a^3+3*a^2*b+3*a*b^2+b^3)/(a+b)*
tanh(1/2*d*x+1/2*c)*b-9/4/d*b/(a+b)^4/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d
*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh
(1/2*d*x+1/2*c)^7*a^2-5/4/d*b^2/(a+b)^4/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2
*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*ta
nh(1/2*d*x+1/2*c)^7*a+1/d*b^3/(a+b)^4/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d
*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh
(1/2*d*x+1/2*c)^7-27/4/d*b/(a+b)^4/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+
1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/
2*d*x+1/2*c)^5*a^2+13/4/d*b^2/(a+b)^4/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d
*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh
(1/2*d*x+1/2*c)^5*a-1/d*b^3/(a+b)^4/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x
+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1
/2*d*x+1/2*c)^5-27/4/d*b/(a+b)^4/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/
2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*
d*x+1/2*c)^3*a^2+13/4/d*b^2/(a+b)^4/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x
+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1
/2*d*x+1/2*c)^3*a-1/d*b^3/(a+b)^4/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1
/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2
*d*x+1/2*c)^3-9/4/d*b/(a+b)^4/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c
)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x
+1/2*c)*a^2-5/4/d*b^2/(a+b)^4/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c
)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x
+1/2*c)*a+1/d*b^3/(a+b)^4/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+
2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2
*c)+15/16/d*b^(1/2)/(a+b)^(9/2)*a*ln(-(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b
^(1/2)*tanh(1/2*d*x+1/2*c)-(a+b)^(1/2))-15/16/d*b^(1/2)/(a+b)^(9/2)*a*ln((a
```

```
+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))-
5/4/d*b^(3/2)/(a+b)^(9/2)*ln(-(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*t
anh(1/2*d*x+1/2*c)-(a+b)^(1/2))+5/4/d*b^(3/2)/(a+b)^(9/2)*ln((a+b)^(1/2)*ta
nh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))-1/24/d/(a+b)
^3/tanh(1/2*d*x+1/2*c)^3+3/8/d/(a+b)^4/tanh(1/2*d*x+1/2*c)*a-9/8/d/(a+b)^4/
tanh(1/2*d*x+1/2*c)*b
```

maxima [B] time = 0.60, size = 782, normalized size = 4.74

$$\frac{5\left(3\,ab-4\,b^2\right)\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{16\left(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4\right)\sqrt{(a+b)b}\,d}+\frac{}{12\left(a^7+4\,a^6b+6\,a^5b^2+4\,a^4b^3+a^3b^4+\left(a^7+12\,a^6b+38\,a^5b\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 5/16*(3*a*b - 4*b^2)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))
/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^4 + 4*a^3*b + 6*a^
2*b^2 + 4*a*b^3 + b^4)*sqrt((a + b)*b)*d) + 1/12*(16*a^4 - 83*a^3*b + 6*a^2
*b^2 + 2*(8*a^4 - 299*a^2*b^2 + 24*a*b^3)*e^(-2*d*x - 2*c) - (96*a^4 + 71*a
^3*b - 344*a^2*b^2 + 1208*a*b^3 - 48*b^4)*e^(-4*d*x - 4*c) - 4*(56*a^4 + 14
4*a^3*b + 31*a^2*b^2 - 546*a*b^3 + 36*b^4)*e^(-6*d*x - 6*c) - (176*a^4 + 56
9*a^3*b + 666*a^2*b^2 + 1704*a*b^3 - 144*b^4)*e^(-8*d*x - 8*c) - 6*(8*a^4 +
 32*a^3*b + 93*a^2*b^2 - 28*a*b^3 + 8*b^4)*e^(-10*d*x - 10*c) - 15*(3*a^3*b
 - 4*a^2*b^2)*e^(-12*d*x - 12*c))/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 +
 a^3*b^4 + (a^7 + 12*a^6*b + 38*a^5*b^2 + 52*a^4*b^3 + 33*a^3*b^4 + 8*a^2*b
^5)*e^(-2*d*x - 2*c) - (3*a^7 + 20*a^6*b + 34*a^5*b^2 - 4*a^4*b^3 - 61*a^3*
b^4 - 56*a^2*b^5 - 16*a*b^6)*e^(-4*d*x - 4*c) - (3*a^7 + 28*a^6*b + 130*a^5
*b^2 + 300*a^4*b^3 + 355*a^3*b^4 + 208*a^2*b^5 + 48*a*b^6)*e^(-6*d*x - 6*c)
 + (3*a^7 + 28*a^6*b + 130*a^5*b^2 + 300*a^4*b^3 + 355*a^3*b^4 + 208*a^2*b^
5 + 48*a*b^6)*e^(-8*d*x - 8*c) + (3*a^7 + 20*a^6*b + 34*a^5*b^2 - 4*a^4*b^3
 - 61*a^3*b^4 - 56*a^2*b^5 - 16*a*b^6)*e^(-10*d*x - 10*c) - (a^7 + 12*a^6*b
 + 38*a^5*b^2 + 52*a^4*b^3 + 33*a^3*b^4 + 8*a^2*b^5)*e^(-12*d*x - 12*c) - (
a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*e^(-14*d*x - 14*c))*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh\left(c+d\,x\right)^6}{\sinh\left(c+d\,x\right)^4\,{\left(a\,\cosh\left(c+d\,x\right)^2+b\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^4*(a + b/cosh(c + d*x)^2)^3),x)
```

```
[Out] int(cosh(c + d*x)^6/(sinh(c + d*x)^4*(b + a*cosh(c + d*x)^2)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Integral(csch(c + d*x)**4/(a + b*sech(c + d*x)**2)**3, x)
```

3.49 $\int \cosh^4(c+dx)\left(a+b\,\mathbf{sech}^2(c+dx)\right)\,dx$

Optimal. Leaf size=61

$$\frac{(3a+4b)\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{1}{8}x(3a+4b)+\frac{a\sinh(c+dx)\cosh^3(c+dx)}{4d}$$

```
[Out] 1/8*(3*a+4*b)*x+1/8*(3*a+4*b)*cosh(d*x+c)*sinh(d*x+c)/d+1/4*a*cosh(d*x+c)^3
*sinh(d*x+c)/d
```

Rubi [A] time = 0.05, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4045, 2635, 8}

$$\frac{(3a+4b)\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{1}{8}x(3a+4b)+\frac{a\sinh(c+dx)\cosh^3(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^4*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((3*a + 4*b)*x)/8 + ((3*a + 4*b)*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) + (a*Co
sh[c + d*x]^3*Sinh[c + d*x])/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rubi steps

$$\int \cosh^4(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx = \frac{a\cosh^3(c+dx)\sinh(c+dx)}{4d} + \frac{1}{4}(3a+4b)\int \cosh^2(c+dx)\,dx$$
$$= \frac{(3a+4b)\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{a\cosh^3(c+dx)\sinh(c+dx}{4d}$$
$$= \frac{1}{8}(3a+4b)x + \frac{(3a+4b)\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{a\cosh^3(c+}{}$$

Mathematica [A] time = 0.09, size = 45, normalized size = 0.74

$$\frac{4(3a+4b)(c+dx)+8(a+b)\sinh(2(c+dx))+a\sinh(4(c+dx))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^4*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] (4*(3*a + 4*b)*(c + d*x) + 8*(a + b)*Sinh[2*(c + d*x)] + a*Sinh[4*(c + d*x)
])/(32*d)
```

fricas [A] time = 0.40, size = 61, normalized size = 1.00

$$\frac{a\cosh(dx+c)\sinh(dx+c)^3+(3a+4b)dx+\left(a\cosh(dx+c)^3+4(a+b)\cosh(dx+c)\right)\sinh(dx+c)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4*(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/8*(a*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a + 4*b)*d*x + (a*cosh(d*x + c)^3
 + 4*(a + b)*cosh(d*x + c))*sinh(d*x + c))/d
```

giac [B] time = 0.14, size = 116, normalized size = 1.90

$$\frac{8(dx+c)(3a+4b)+ae^{(4dx+4c)}+8ae^{(2dx+2c)}+8be^{(2dx+2c)}-\left(18ae^{(4dx+4c)}+24be^{(4dx+4c)}+8ae^{(2dx+2c)}+8b\right.}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4*(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/64*(8*(d*x + c)*(3*a + 4*b) + a*e^(4*d*x + 4*c) + 8*a*e^(2*d*x + 2*c) + 8
*b*e^(2*d*x + 2*c) - (18*a*e^(4*d*x + 4*c) + 24*b*e^(4*d*x + 4*c) + 8*a*e^(
2*d*x + 2*c) + 8*b*e^(2*d*x + 2*c) + a)*e^(-4*d*x - 4*c))/d
```

maple [A] time = 0.46, size = 66, normalized size = 1.08

$$\frac{a\left(\left(\frac{\left(\cosh^3(dx+c)\right)}{4}+\frac{3\cosh(dx+c)}{8}\right)\sinh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+b\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^4*(a+b*sech(d*x+c)^2),x)
```

```
[Out] 1/d*(a*((1/4*cosh(d*x+c)^3+3/8*cosh(d*x+c))*sinh(d*x+c)+3/8*d*x+3/8*c)+b*(1
/2*cosh(d*x+c)*sinh(d*x+c)+1/2*d*x+1/2*c))
```

maxima [A] time = 0.31, size = 97, normalized size = 1.59

$$\frac{1}{64}a\left(24x+\frac{e^{(4dx+4c)}}{d}+\frac{8e^{(2dx+2c)}}{d}-\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right)+\frac{1}{8}b\left(4x+\frac{e^{(2dx+2c)}}{d}-\frac{e^{(-2dx-2c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4*(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/64*a*(24*x + e^(4*d*x + 4*c)/d + 8*e^(2*d*x + 2*c)/d - 8*e^(-2*d*x - 2*c)
/d - e^(-4*d*x - 4*c)/d) + 1/8*b*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d*x - 2*c
)/d)
```

mupad [B] time = 0.13, size = 50, normalized size = 0.82

$$\frac{\frac{a\sinh(2c+2dx)}{4}+\frac{a\sinh(4c+4dx)}{32}+\frac{b\sinh(2c+2dx)}{4}}{d}+\frac{3ax}{8}+\frac{bx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^4*(a + b/cosh(c + d*x)^2),x)
```

```
[Out] ((a*sinh(2*c + 2*d*x))/4 + (a*sinh(4*c + 4*d*x))/32 + (b*sinh(2*c + 2*d*x))
/4)/d + (3*a*x)/8 + (b*x)/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\operatorname{sech}^2(c+dx)\right)\cosh^4(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**4*(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)*cosh(c + d*x)**4, x)
```

3.50 $\int \cosh^3(c+dx)\left(a+b\,\mathbf{sech}^2(c+dx)\right)\,dx$

Optimal. Leaf size=30

$$\frac{(a+b)\sinh(c+dx)}{d}+\frac{a\sinh^3(c+dx)}{3d}$$

```
[Out] (a+b)*sinh(d*x+c)/d+1/3*a*sinh(d*x+c)^3/d
```

Rubi [A] time = 0.05, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4044, 3013}

$$\frac{(a+b)\sinh(c+dx)}{d}+\frac{a\sinh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^3*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((a + b)*Sinh[c + d*x])/d + (a*Sinh[c + d*x]^3)/(3*d)
```

Rule 3013

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rule 4044

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)),
 x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \cosh^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)dx &= \int \cosh(c+dx)\left(b+a\cosh^2(c+dx)\right)dx\\ &= \frac{i\,\text{Subst}\left(\int\left(a+b-ax^2\right)dx,x,-i\sinh(c+dx)\right)}{d}\\ &= \frac{(a+b)\sinh(c+dx)}{d}+\frac{a\sinh^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.02, size = 50, normalized size = 1.67

$$\frac{a\sinh^3(c+dx)}{3d}+\frac{a\sinh(c+dx)}{d}+\frac{b\sinh(c)\cosh(dx)}{d}+\frac{b\cosh(c)\sinh(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^3*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] (b*Cosh[d*x]*Sinh[c])/d + (b*Cosh[c]*Sinh[d*x])/d + (a*Sinh[c + d*x])/d + (
a*Sinh[c + d*x]^3)/(3*d)
```

fricas [A] time = 0.39, size = 41, normalized size = 1.37

$$\frac{a\sinh\left(dx+c\right)^3+3\left(a\cosh\left(dx+c\right)^2+3\,a+4\,b\right)\sinh\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/12*(a*sinh(d*x + c)^3 + 3*(a*cosh(d*x + c)^2 + 3*a + 4*b)*sinh(d*x + c))/
d
```

giac [B] time = 0.16, size = 72, normalized size = 2.40

$$\frac{ae^{(3\,dx+3\,c)}+9\,ae^{(dx+c)}+12\,be^{(dx+c)}-\left(9\,ae^{(2\,dx+2\,c)}+12\,be^{(2\,dx+2\,c)}+a\right)e^{(-3\,dx-3\,c)}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/24*(a*e^(3*d*x + 3*c) + 9*a*e^(d*x + c) + 12*b*e^(d*x + c) - (9*a*e^(2*d*
x + 2*c) + 12*b*e^(2*d*x + 2*c) + a)*e^(-3*d*x - 3*c))/d
```

maple [A] time = 0.43, size = 34, normalized size = 1.13

$$\frac{a\left(\frac{2}{3}+\frac{\left(\cosh^2(dx+c)\right)}{3}\right)\sinh\left(dx+c\right)+b\sinh\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3*(a+b*sech(d*x+c)^2),x)
```

```
[Out] 1/d*(a*(2/3+1/3*cosh(d*x+c)^2)*sinh(d*x+c)+b*sinh(d*x+c))
```

maxima [B] time = 0.31, size = 85, normalized size = 2.83

$$\frac{1}{24}\,a\left(\frac{e^{(3\,dx+3\,c)}}{d}+\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}-\frac{e^{(-3\,dx-3\,c)}}{d}\right)+\frac{1}{2}\,b\left(\frac{e^{(dx+c)}}{d}-\frac{e^{(-dx-c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/24*a*(e^(3*d*x + 3*c)/d + 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d - e^(-3*d*x
- 3*c)/d) + 1/2*b*(e^(d*x + c)/d - e^(-d*x - c)/d)
```

mupad [B] time = 0.09, size = 34, normalized size = 1.13

$$\frac{3\,a\sinh\left(c+d\,x\right)+3\,b\sinh\left(c+d\,x\right)+a\sinh\left(c+d\,x\right)^3}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^3*(a + b/cosh(c + d*x)^2),x)
```

```
[Out] (3*a*sinh(c + d*x) + 3*b*sinh(c + d*x) + a*sinh(c + d*x)^3)/(3*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\operatorname{sech}^{2}\left(c+dx\right)\right)\cosh^{3}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3*(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)*cosh(c + d*x)**3, x)
```

3.51 $\int \cosh^2(c+dx)\left(a+b\,\mathbf{sech}^2(c+dx)\right)\,dx$

Optimal. Leaf size=31

$$\frac{1}{2}x(a+2b)+\frac{a\sinh(c+dx)\cosh(c+dx)}{2d}$$

```
[Out] 1/2*(a+2*b)*x+1/2*a*cosh(d*x+c)*sinh(d*x+c)/d
```

Rubi [A] time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4045, 8}

$$\frac{1}{2}x(a+2b)+\frac{a\sinh(c+dx)\cosh(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^2*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((a + 2*b)*x)/2 + (a*Cosh[c + d*x]*Sinh[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cosh^2(c+dx)\left(a+b\,\text{sech}^2(c+dx)\right)\,dx&=\frac{a\cosh(c+dx)\sinh(c+dx)}{2d}+\frac{1}{2}(a+2b)\int 1\,dx\\&=\frac{1}{2}(a+2b)x+\frac{a\cosh(c+dx)\sinh(c+dx)}{2d}\end{aligned}$$

Mathematica [A] time = 0.03, size = 33, normalized size = 1.06

$$\frac{a(c+dx)}{2d}+\frac{a\sinh(2(c+dx))}{4d}+bx$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^2*(a + b*Sech[c + d*x]^2),x]
[Out] b*x + (a*(c + d*x))/(2*d) + (a*Sinh[2*(c + d*x)])/(4*d)
```

fricas [A] time = 0.41, size = 28, normalized size = 0.90

$$\frac{(a+2\,b)dx + a\cosh\left(dx+c\right)\sinh\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*(a+b*sech(d*x+c)^2),x, algorithm="fricas")
[Out] 1/2*((a + 2*b)*d*x + a*cosh(d*x + c)*sinh(d*x + c))/d
```

giac [B] time = 0.15, size = 66, normalized size = 2.13

$$\frac{4\,(dx+c)(a+2\,b) + ae^{(2\,dx+2\,c)} - \left(2\,ae^{(2\,dx+2\,c)} + 4\,be^{(2\,dx+2\,c)} + a\right)e^{(-2\,dx-2\,c)}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*(a+b*sech(d*x+c)^2),x, algorithm="giac")
[Out] 1/8*(4*(d*x + c)*(a + 2*b) + a*e^(2*d*x + 2*c) - (2*a*e^(2*d*x + 2*c) + 4*b
*e^(2*d*x + 2*c) + a)*e^(-2*d*x - 2*c))/d
```

maple [A] time = 0.31, size = 37, normalized size = 1.19

$$\frac{a\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2}\right) + (dx+c)\,b}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2*(a+b*sech(d*x+c)^2),x)
[Out] 1/d*(a*(1/2*cosh(d*x+c)*sinh(d*x+c)+1/2*d*x+1/2*c)+(d*x+c)*b)
```

maxima [A] time = 0.31, size = 38, normalized size = 1.23

$$\frac{1}{8}\,a\left(4\,x + \frac{e^{(2\,dx+2\,c)}}{d} - \frac{e^{(-2\,dx-2\,c)}}{d}\right) + bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

[Out] 1/8*a*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d*x - 2*c)/d) + b*x

mupad [B] time = 0.08, size = 23, normalized size = 0.74

$$\frac{a\,x}{2}+b\,x+\frac{a\,\sinh\left(2\,c+2\,d\,x\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(c + d*x)^2*(a + b/cosh(c + d*x)^2),x)

[Out] (a*x)/2 + b*x + (a*sinh(2*c + 2*d*x))/(4*d)

sympy [A] time = 11.24, size = 60, normalized size = 1.94

$$a\left(\begin{cases} -\frac{x\sinh^2(c+dx)}{2}+\frac{x\cosh^2(c+dx)}{2}+\frac{\sinh(c+dx)\cosh(c+dx)}{2d} & \text{for}\ d\neq 0 \\ x\cosh^2(c) & \text{otherwise} \end{cases}\right)+b\left(\begin{cases} x & \text{for} \\ G_{2,2}^{1,1}\left(\begin{matrix} 1 & 2 \\ 1 & 0 \end{matrix}\middle| x\right)+G_{2,2}^{0,2}\left(\begin{matrix} 2,1 & \\ & 1,0 \end{matrix}\middle| x\right) & \text{othe} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)**2*(a+b*sech(d*x+c)**2),x)

[Out] a*Piecewise((-x*sinh(c + d*x)**2/2 + x*cosh(c + d*x)**2/2 + sinh(c + d*x)*c
osh(c + d*x)/(2*d), Ne(d, 0)), (x*cosh(c)**2, True)) + b*Piecewise((x, Abs(
x) < 1), (meijerg(((1,), (2,)), ((1,), (0,)), x) + meijerg(((2, 1), ()), ((
), (1, 0)), x), True))

3.52 $\int \cosh(c+dx)\left(a+b\mathbf{sech}^2(c+dx)\right)\,dx$

Optimal. Leaf size=24

$$\frac{a\sinh(c+dx)}{d}+\frac{b\tan^{-1}(\sinh(c+dx))}{d}$$

```
[Out] b*arctan(sinh(d*x+c))/d+a*sinh(d*x+c)/d
```

Rubi [A] time = 0.03, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {4045, 3770}

$$\frac{a\sinh(c+dx)}{d}+\frac{b\tan^{-1}(\sinh(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] (b*ArcTan[Sinh[c + d*x]])/d + (a*Sinh[c + d*x])/d
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cosh(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx &= \frac{a\sinh(c+dx)}{d}+b\int \operatorname{sech}(c+dx)\,dx\\ &= \frac{b\tan^{-1}(\sinh(c+dx))}{d}+\frac{a\sinh(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.02, size = 35, normalized size = 1.46

$$\frac{a\sinh(c)\cosh(dx)}{d}+\frac{a\cosh(c)\sinh(dx)}{d}+\frac{b\tan^{-1}(\sinh(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] (b*ArcTan[Sinh[c + d*x]])/d + (a*Cosh[d*x]*Sinh[c])/d + (a*Cosh[c]*Sinh[d*x
])/d
```

fricas [B] time = 0.45, size = 93, normalized size = 3.88

$$\frac{a\cosh\left(dx+c\right)^{2}+2\,a\cosh\left(dx+c\right)\sinh\left(dx+c\right)+a\sinh\left(dx+c\right)^{2}+4\left(b\cosh\left(dx+c\right)+b\sinh\left(dx+c\right)\right)\arctan}{2\left(d\cosh\left(dx+c\right)+d\sinh\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/2*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^
2 + 4*(b*cosh(d*x + c) + b*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x +
 c)) - a)/(d*cosh(d*x + c) + d*sinh(d*x + c))
```

giac [A] time = 0.15, size = 36, normalized size = 1.50

$$\frac{4\,b\arctan\left(e^{\left(dx+c\right)}\right)+ae^{\left(dx+c\right)}-ae^{\left(-dx-c\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/2*(4*b*arctan(e^(d*x + c)) + a*e^(d*x + c) - a*e^(-d*x - c))/d
```

maple [A] time = 0.29, size = 26, normalized size = 1.08

$$\frac{a\sinh\left(dx+c\right)}{d}+\frac{2b\arctan\left(\mathrm{e}^{dx+c}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)*(a+b*sech(d*x+c)^2),x)
```

```
[Out] a*sinh(d*x+c)/d+2/d*b*arctan(exp(d*x+c))
```

maxima [A] time = 0.41, size = 28, normalized size = 1.17

$$-\frac{2\,b\arctan\left(e^{\left(-dx-c\right)}\right)}{d}+\frac{a\sinh\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -2*b*arctan(e^(-d*x - c))/d + a*sinh(d*x + c)/d
```

mupad [B] time = 1.44, size = 62, normalized size = 2.58

$$\frac{2\,\mathrm{atan}\left(\frac{b\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\sqrt{d^2}}{d\,\sqrt{b^2}}\right)\,\sqrt{b^2}}{\sqrt{d^2}}-\frac{a\,\mathrm{e}^{-c-d\,x}}{2\,d}+\frac{a\,\mathrm{e}^{c+d\,x}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)*(a + b/cosh(c + d*x)^2),x)
```

```
[Out] (2*atan((b*exp(d*x)*exp(c)*(d^2)^(1/2))/(d*(b^2)^(1/2)))*(b^2)^(1/2))/(d^2)
^(1/2) - (a*exp(- c - d*x))/(2*d) + (a*exp(c + d*x))/(2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\,\mathrm{sech}^2\,(c + dx)\right) \cosh\,(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)*cosh(c + d*x), x)
```

3.53 $\int \operatorname{sech}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)dx$

Optimal. Leaf size=40

$$\frac{(2a+b)\tan^{-1}(\sinh(c+dx))}{2d}+\frac{b\tanh(c+dx)\operatorname{sech}(c+dx)}{2d}$$

```
[Out] 1/2*(2*a+b)*arctan(sinh(d*x+c))/d+1/2*b*sech(d*x+c)*tanh(d*x+c)/d
```

Rubi [A] time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {4046, 3770}

$$\frac{(2a+b)\tan^{-1}(\sinh(c+dx))}{2d}+\frac{b\tanh(c+dx)\operatorname{sech}(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((2*a + b)*ArcTan[Sinh[c + d*x]])/(2*d) + (b*Sech[c + d*x]*Tanh[c + d*x])/(
2*d)
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)dx &= \frac{b\operatorname{sech}(c+dx)\tanh(c+dx)}{2d}+\frac{1}{2}(2a+b)\int \operatorname{sech}(c+dx)\,dx\\ &= \frac{(2a+b)\tan^{-1}(\sinh(c+dx))}{2d}+\frac{b\operatorname{sech}(c+dx)\tanh(c+dx)}{2d}\end{aligned}$$

Mathematica [A] time = 0.02, size = 48, normalized size = 1.20

$$\frac{a\tan^{-1}(\sinh(c+dx))}{d}+\frac{b\tan^{-1}(\sinh(c+dx))}{2d}+\frac{b\tanh(c+dx)\text{sech}(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] (a*ArcTan[Sinh[c + d*x]])/d + (b*ArcTan[Sinh[c + d*x]])/(2*d) + (b*Sech[c +
 d*x]*Tanh[c + d*x])/(2*d)
```

fricas [B] time = 0.41, size = 321, normalized size = 8.02

$$\frac{b\cosh(dx+c)^3+3\,b\cosh(dx+c)\sinh(dx+c)^2+b\sinh(dx+c)^3+\Big((2\,a+b)\cosh(dx+c)^4+4\,(2\,a+b)\cos}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] (b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x + c)^3
+ ((2*a + b)*cosh(d*x + c)^4 + 4*(2*a + b)*cosh(d*x + c)*sinh(d*x + c)^3 +
(2*a + b)*sinh(d*x + c)^4 + 2*(2*a + b)*cosh(d*x + c)^2 + 2*(3*(2*a + b)*co
sh(d*x + c)^2 + 2*a + b)*sinh(d*x + c)^2 + 4*((2*a + b)*cosh(d*x + c)^3 + (
2*a + b)*cosh(d*x + c))*sinh(d*x + c) + 2*a + b)*arctan(cosh(d*x + c) + sin
h(d*x + c)) - b*cosh(d*x + c) + (3*b*cosh(d*x + c)^2 - b)*sinh(d*x + c))/(d
*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x + c)^3 + d*sinh(d*x + c)^4 +
2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 4*(d*co
sh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

giac [B] time = 0.12, size = 84, normalized size = 2.10

$$\frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)(2\,a+b)+\frac{4\,b\left(e^{(dx+c)}-e^{(-dx-c)}\right)}{\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/4*((pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(2*a + b) + 4*
b*(e^(d*x + c) - e^(-d*x - c))/((e^(d*x + c) - e^(-d*x - c))^2 + 4))/d
```

maple [A] time = 0.30, size = 45, normalized size = 1.12

$$\frac{2a\arctan\left(e^{dx+c}\right)}{d}+\frac{b\,\text{sech}\,(dx+c)\tanh\,(dx+c)}{2d}+\frac{b\arctan\left(e^{dx+c}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)*(a+b*sech(d*x+c)^2),x)
```

```
[Out] 2/d*a*arctan(exp(d*x+c))+1/2*b*sech(d*x+c)*tanh(d*x+c)/d+1/d*b*arctan(exp(d
*x+c))
```

maxima [B] time = 0.40, size = 81, normalized size = 2.02

$$-b\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d\left(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1\right)}\right)+\frac{a\arctan\left(\sinh\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -b*(arctan(e^(-d*x - c))/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*
d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) + a*arctan(sinh(d*x + c))/d
```

mupad [B] time = 0.16, size = 124, normalized size = 3.10

$$\frac{\mathrm{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(2\,a\,\sqrt{d^2}+b\,\sqrt{d^2}\right)}{d\,\sqrt{4\,a^2+4\,a\,b+b^2}}\right)\,\sqrt{4\,a^2+4\,a\,b+b^2}}{\sqrt{d^2}}+\frac{b\,\mathrm{e}^{c+d\,x}}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)}-\frac{2\,b\,\mathrm{e}^{c+d\,x}}{d\,\left(2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cosh(c + d*x)^2)/cosh(c + d*x),x)
```

```
[Out] (atan((exp(d*x)*exp(c)*(2*a*(d^2)^(1/2) + b*(d^2)^(1/2)))/(d*(4*a*b + 4*a^2
 + b^2)^(1/2)))*(4*a*b + 4*a^2 + b^2)^(1/2))/(d^2)^(1/2) + (b*exp(c + d*x))
/(d*(exp(2*c + 2*d*x) + 1)) - (2*b*exp(c + d*x))/(d*(2*exp(2*c + 2*d*x) + e
xp(4*c + 4*d*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\,\mathrm{sech}^2\left(c+dx\right)\right)\mathrm{sech}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)*sech(c + d*x), x)
```

3.54 $\int \operatorname{sech}^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx$

Optimal. Leaf size=30

$$\frac{(a+b)\tanh(c+dx)}{d}-\frac{b\tanh^3(c+dx)}{3d}$$

```
[Out] (a+b)*tanh(d*x+c)/d-1/3*b*tanh(d*x+c)^3/d
```

Rubi [A] time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.43, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4046, 3767, 8}

$$\frac{(3a+2b)\tanh(c+dx)}{3d}+\frac{b\tanh(c+dx)\operatorname{sech}^2(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^2*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((3*a + 2*b)*Tanh[c + d*x])/(3*d) + (b*Sech[c + d*x]^2*Tanh[c + d*x])/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rubi steps

$$\int \operatorname{sech}^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx = \frac{b\operatorname{sech}^2(c+dx)\tanh(c+dx)}{3d}+\frac{1}{3}(3a+2b)\int \operatorname{sech}^2(c+dx)\,dx$$
$$= \frac{b\operatorname{sech}^2(c+dx)\tanh(c+dx)}{3d}+\frac{(i(3a+2b))\operatorname{Subst}(\int 1\,dx,x,-i\tanh(}{3d}$$
$$= \frac{(3a+2b)\tanh(c+dx)}{3d}+\frac{b\operatorname{sech}^2(c+dx)\tanh(c+dx)}{3d}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 1.30

$$\frac{a\tanh(c+dx)}{d}-\frac{b\tanh^3(c+dx)}{3d}+\frac{b\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^2*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] (a*Tanh[c + d*x])/d + (b*Tanh[c + d*x])/d - (b*Tanh[c + d*x]^3)/(3*d)
```

fricas [B] time = 0.38, size = 158, normalized size = 5.27

$$-\frac{4\left((3\,a+b)\cosh\left(dx+c\right)^2-2\,b\cosh\left(dx+c\right)\sinh\left(dx+c\right)+\right.}{3\left(d\cosh\left(dx+c\right)^4+4\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3+d\sinh\left(dx+c\right)^4+4\,d\cosh\left(dx+c\right)^2+2\left(3\,d\cosh\left(dx+\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -4/3*((3*a + b)*cosh(d*x + c)^2 - 2*b*cosh(d*x + c)*sinh(d*x + c) + (3*a +
b)*sinh(d*x + c)^2 + 3*a + 3*b)/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh
(d*x + c)^3 + d*sinh(d*x + c)^4 + 4*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c
)^2 + 2*d)*sinh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d
*x + c) + 3*d)
```

giac [B] time = 0.12, size = 61, normalized size = 2.03

$$-\frac{2\left(3\,ae^{(4\,dx+4\,c)}+6\,ae^{(2\,dx+2\,c)}+6\,be^{(2\,dx+2\,c)}+3\,a+2\,b\right)}{3\,d\left(e^{(2\,dx+2\,c)}+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -2/3*(3*a*e^(4*d*x + 4*c) + 6*a*e^(2*d*x + 2*c) + 6*b*e^(2*d*x + 2*c) + 3*a
 + 2*b)/(d*(e^(2*d*x + 2*c) + 1)^3)
```

maple [A] time = 0.34, size = 34, normalized size = 1.13

$$\frac{a\tanh(dx+c)+b\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^{2}}{3}\right)\tanh(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^2*(a+b*sech(d*x+c)^2),x)
```

```
[Out] 1/d*(a*tanh(d*x+c)+b*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c))
```

maxima [B] time = 0.32, size = 112, normalized size = 3.73

$$\frac{4}{3}b\left(\frac{3e^{(-2dx-2c)}}{d\left(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1\right)}+\frac{1}{d\left(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1\right)}\right)+\frac{2a}{d\left(e^{(-2dx-2c)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 4/3*b*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(
-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6
*d*x - 6*c) + 1))) + 2*a/(d*(e^(-2*d*x - 2*c) + 1))
```

mupad [B] time = 1.38, size = 61, normalized size = 2.03

$$-\frac{2\left(3a+2b+6a\,e^{2c+2dx}+3a\,e^{4c+4dx}+6b\,e^{2c+2dx}\right)}{3d\left(e^{2c+2dx}+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cosh(c + d*x)^2)/cosh(c + d*x)^2,x)
```

```
[Out] -(2*(3*a + 2*b + 6*a*exp(2*c + 2*d*x) + 3*a*exp(4*c + 4*d*x) + 6*b*exp(2*c
+ 2*d*x)))/(3*d*(exp(2*c + 2*d*x) + 1)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\operatorname{sech}^{2}(c+dx)\right)\operatorname{sech}^{2}(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**2*(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)*sech(c + d*x)**2, x)
```

3.55 $\int \text{sech}^3(c+dx)\left(a+b\text{sech}^2(c+dx)\right)dx$

Optimal. Leaf size=70

$$\frac{(4a+3b)\tan^{-1}(\sinh(c+dx))}{8d}+\frac{(4a+3b)\tanh(c+dx)\text{sech}(c+dx)}{8d}+\frac{b\tanh(c+dx)\text{sech}^3(c+dx)}{4d}$$

```
[Out] 1/8*(4*a+3*b)*arctan(sinh(d*x+c))/d+1/8*(4*a+3*b)*sech(d*x+c)*tanh(d*x+c)/d
+1/4*b*sech(d*x+c)^3*tanh(d*x+c)/d
```

Rubi [A] time = 0.05, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4046, 3768, 3770}

$$\frac{(4a+3b)\tan^{-1}(\sinh(c+dx))}{8d}+\frac{(4a+3b)\tanh(c+dx)\text{sech}(c+dx)}{8d}+\frac{b\tanh(c+dx)\text{sech}^3(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^3*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((4*a + 3*b)*ArcTan[Sinh[c + d*x]])/(8*d) + ((4*a + 3*b)*Sech[c + d*x]*Tanh
[c + d*x])/(8*d) + (b*Sech[c + d*x]^3*Tanh[c + d*x])/(4*d)
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \operatorname{sech}^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx = \frac{b\operatorname{sech}^3(c+dx)\tanh(c+dx)}{4d} + \frac{1}{4}(4a+3b)\int \operatorname{sech}^3(c+dx)\,dx$$
$$= \frac{(4a+3b)\operatorname{sech}(c+dx)\tanh(c+dx)}{8d} + \frac{b\operatorname{sech}^3(c+dx)\tanh(c+dx)}{4d}$$
$$= \frac{(4a+3b)\tan^{-1}(\sinh(c+dx))}{8d} + \frac{(4a+3b)\operatorname{sech}(c+dx)\tanh(c+dx)}{8d}$$

Mathematica [A] time = 0.11, size = 60, normalized size = 0.86

$$\frac{(4a+3b)\tan^{-1}(\sinh(c+dx)) + (4a+3b)\tanh(c+dx)\operatorname{sech}(c+dx) + 2b\tanh(c+dx)\operatorname{sech}^3(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^3*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((4*a + 3*b)*ArcTan[Sinh[c + d*x]] + (4*a + 3*b)*Sech[c + d*x]*Tanh[c + d*x
] + 2*b*Sech[c + d*x]^3*Tanh[c + d*x])/(8*d)
```

fricas [B] time = 0.42, size = 1112, normalized size = 15.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/4*((4*a + 3*b)*cosh(d*x + c)^7 + 7*(4*a + 3*b)*cosh(d*x + c)*sinh(d*x + c
)^6 + (4*a + 3*b)*sinh(d*x + c)^7 + (4*a + 11*b)*cosh(d*x + c)^5 + (21*(4*a
 + 3*b)*cosh(d*x + c)^2 + 4*a + 11*b)*sinh(d*x + c)^5 + 5*(7*(4*a + 3*b)*co
sh(d*x + c)^3 + (4*a + 11*b)*cosh(d*x + c))*sinh(d*x + c)^4 - (4*a + 11*b)*
cosh(d*x + c)^3 + (35*(4*a + 3*b)*cosh(d*x + c)^4 + 10*(4*a + 11*b)*cosh(d*
x + c)^2 - 4*a - 11*b)*sinh(d*x + c)^3 + (21*(4*a + 3*b)*cosh(d*x + c)^5 +
10*(4*a + 11*b)*cosh(d*x + c)^3 - 3*(4*a + 11*b)*cosh(d*x + c))*sinh(d*x +
c)^2 + ((4*a + 3*b)*cosh(d*x + c)^8 + 8*(4*a + 3*b)*cosh(d*x + c)*sinh(d*x
+ c)^7 + (4*a + 3*b)*sinh(d*x + c)^8 + 4*(4*a + 3*b)*cosh(d*x + c)^6 + 4*(7
*(4*a + 3*b)*cosh(d*x + c)^2 + 4*a + 3*b)*sinh(d*x + c)^6 + 8*(7*(4*a + 3*b
)*cosh(d*x + c)^3 + 3*(4*a + 3*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 6*(4*a +
 3*b)*cosh(d*x + c)^4 + 2*(35*(4*a + 3*b)*cosh(d*x + c)^4 + 30*(4*a + 3*b)*
cosh(d*x + c)^2 + 12*a + 9*b)*sinh(d*x + c)^4 + 8*(7*(4*a + 3*b)*cosh(d*x +
 c)^5 + 10*(4*a + 3*b)*cosh(d*x + c)^3 + 3*(4*a + 3*b)*cosh(d*x + c))*sinh(
d*x + c)^3 + 4*(4*a + 3*b)*cosh(d*x + c)^2 + 4*(7*(4*a + 3*b)*cosh(d*x + c)
```

```
^6 + 15*(4*a + 3*b)*cosh(d*x + c)^4 + 9*(4*a + 3*b)*cosh(d*x + c)^2 + 4*a +
 3*b)*sinh(d*x + c)^2 + 8*((4*a + 3*b)*cosh(d*x + c)^7 + 3*(4*a + 3*b)*cosh
(d*x + c)^5 + 3*(4*a + 3*b)*cosh(d*x + c)^3 + (4*a + 3*b)*cosh(d*x + c))*si
nh(d*x + c) + 4*a + 3*b)*arctan(cosh(d*x + c) + sinh(d*x + c)) - (4*a + 3*b
)*cosh(d*x + c) + (7*(4*a + 3*b)*cosh(d*x + c)^6 + 5*(4*a + 11*b)*cosh(d*x
+ c)^4 - 3*(4*a + 11*b)*cosh(d*x + c)^2 - 4*a - 3*b)*sinh(d*x + c))/(d*cosh
(d*x + c)^8 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 + 4*d*c
osh(d*x + c)^6 + 4*(7*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^6 + 8*(7*d*cosh(
d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^5 + 6*d*cosh(d*x + c)^4 + 2*(
35*d*cosh(d*x + c)^4 + 30*d*cosh(d*x + c)^2 + 3*d)*sinh(d*x + c)^4 + 8*(7*d
*cosh(d*x + c)^5 + 10*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^
3 + 4*d*cosh(d*x + c)^2 + 4*(7*d*cosh(d*x + c)^6 + 15*d*cosh(d*x + c)^4 + 9
*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 8*(d*cosh(d*x + c)^7 + 3*d*cosh(d
*x + c)^5 + 3*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

giac [B] time = 0.16, size = 156, normalized size = 2.23

$$\frac{\left(\pi + 2\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)} - 1\right)e^{(-dx-c)}\right)\right)(4\,a + 3\,b) + \frac{4\left(4\,a\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3+3\,b\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3+16\,a\left(e^{(dx+c)}-e^{(-dx-c)}\right)+20\,b\left(e\right.\right.}{\left(\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4\right)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/16*((pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(4*a + 3*b) +
 4*(4*a*(e^(d*x + c) - e^(-d*x - c))^3 + 3*b*(e^(d*x + c) - e^(-d*x - c))^3
 + 16*a*(e^(d*x + c) - e^(-d*x - c)) + 20*b*(e^(d*x + c) - e^(-d*x - c)))/(
(e^(d*x + c) - e^(-d*x - c))^2 + 4)^2)/d
```

maple [A] time = 0.42, size = 83, normalized size = 1.19

$$\frac{a\,\mathrm{sech}\,(dx+c)\tanh\,(dx+c)}{2d}+\frac{a\arctan\left(\mathrm{e}^{dx+c}\right)}{d}+\frac{b\mathrm{sech}\,(dx+c)^3\tanh\,(dx+c)}{4d}+\frac{3b\,\mathrm{sech}\,(dx+c)\tanh\,(dx+c)}{8d}+\frac{3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^3*(a+b*sech(d*x+c)^2),x)
```

```
[Out] 1/2/d*a*sech(d*x+c)*tanh(d*x+c)+1/d*a*arctan(exp(d*x+c))+1/4*b*sech(d*x+c)^
3*tanh(d*x+c)/d+3/8*b*sech(d*x+c)*tanh(d*x+c)/d+3/4/d*b*arctan(exp(d*x+c))
```

maxima [B] time = 0.41, size = 184, normalized size = 2.63

$$-\frac{1}{4}\,b\left(\frac{3\,\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{3\,e^{(-dx-c)} + 11\,e^{(-3\,dx-3\,c)} - 11\,e^{(-5\,dx-5\,c)} - 3\,e^{(-7\,dx-7\,c)}}{d\left(4\,e^{(-2\,dx-2\,c)} + 6\,e^{(-4\,dx-4\,c)} + 4\,e^{(-6\,dx-6\,c)} + e^{(-8\,dx-8\,c)} + 1\right)}\right) - a\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{}{d(}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/4*b*(3*arctan(e^(-d*x - c))/d - (3*e^(-d*x - c) + 11*e^(-3*d*x - 3*c) -
11*e^(-5*d*x - 5*c) - 3*e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*
d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) - a*(arctan(e^(-d
*x - c))/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) + e^(
-4*d*x - 4*c) + 1)))
```

mupad [B] time = 1.38, size = 283, normalized size = 4.04

$$\frac{\operatorname{atan}\left(\frac{e^{dx} e^{c} \left(4 a \sqrt{d^2} + 3 b \sqrt{d^2}\right)}{d \sqrt{16 a^2 + 24 a b + 9 b^2}}\right) \sqrt{16 a^2 + 24 a b + 9 b^2}}{4 \sqrt{d^2}} - \frac{\frac{a e^{5c+5dx}}{d} + \frac{2 e^{3c+3dx} (a+2b)}{d} + \frac{a e^{c+dx}}{d}}{4 e^{2c+2dx} + 6 e^{4c+4dx} + 4 e^{6c+6dx} + e^{8c+8dx} + 1} - \frac{e}{2 d \left(2 e^{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cosh(c + d*x)^2)/cosh(c + d*x)^3,x)
```

```
[Out] (atan((exp(d*x)*exp(c)*(4*a*(d^2)^(1/2) + 3*b*(d^2)^(1/2)))/(d*(24*a*b + 16
*a^2 + 9*b^2)^(1/2)))*(24*a*b + 16*a^2 + 9*b^2)^(1/2))/(4*(d^2)^(1/2)) - ((
a*exp(5*c + 5*d*x))/d + (2*exp(3*c + 3*d*x)*(a + 2*b))/d + (a*exp(c + d*x))
/d)/(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c
 + 8*d*x) + 1) - (exp(c + d*x)*(2*a - b))/(2*d*(2*exp(2*c + 2*d*x) + exp(4*
c + 4*d*x) + 1)) - (2*b*exp(c + d*x))/(d*(3*exp(2*c + 2*d*x) + 3*exp(4*c +
4*d*x) + exp(6*c + 6*d*x) + 1)) + (exp(c + d*x)*(4*a + 3*b))/(4*d*(exp(2*c
+ 2*d*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \operatorname{sech}^{2}{\left(c + d x \right)}\right) \operatorname{sech}^{3}{\left(c + d x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**3*(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)*sech(c + d*x)**3, x)
```

3.56 $\int \text{sech}^4(c+dx)\left(a+b\,\text{sech}^2(c+dx)\right)dx$

Optimal. Leaf size=50

$$-\frac{(a+2b)\tanh^3(c+dx)}{3d}+\frac{(a+b)\tanh(c+dx)}{d}+\frac{b\tanh^5(c+dx)}{5d}$$

```
[Out] (a+b)*tanh(d*x+c)/d-1/3*(a+2*b)*tanh(d*x+c)^3/d+1/5*b*tanh(d*x+c)^5/d
```

Rubi [A] time = 0.05, antiderivative size = 65, normalized size of antiderivative = 1.30, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4046, 3767}

$$-\frac{(5a+4b)\tanh^3(c+dx)}{15d}+\frac{(5a+4b)\tanh(c+dx)}{5d}+\frac{b\tanh(c+dx)\text{sech}^4(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^4*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((5*a + 4*b)*Tanh[c + d*x])/(5*d) + (b*Sech[c + d*x]^4*Tanh[c + d*x])/(5*d)
 - ((5*a + 4*b)*Tanh[c + d*x]^3)/(15*d)
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \text{sech}^4(c+dx)\left(a+b\,\text{sech}^2(c+dx)\right)dx=\frac{b\,\text{sech}^4(c+dx)\tanh(c+dx)}{5d}+\frac{1}{5}(5a+4b)\int \text{sech}^4(c+dx)\,dx$$

$$=\frac{b\,\text{sech}^4(c+dx)\tanh(c+dx)}{5d}+\frac{(i(5a+4b))\,\text{Subst}\left(\int\left(1+x^2\right)dx,x,\right.}{5d}$$

$$=\frac{(5a+4b)\tanh(c+dx)}{5d}+\frac{b\,\text{sech}^4(c+dx)\tanh(c+dx)}{5d}-\frac{(5a+4b)\,\text{t}}{1}$$

Mathematica [A] time = 0.01, size = 71, normalized size = 1.42

$$-\frac{a\tanh^3(c+dx)}{3d}+\frac{a\tanh(c+dx)}{d}+\frac{b\tanh^5(c+dx)}{5d}-\frac{2b\tanh^3(c+dx)}{3d}+\frac{b\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^4*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] (a*Tanh[c + d*x])/d + (b*Tanh[c + d*x])/d - (a*Tanh[c + d*x]^3)/(3*d) - (2*
b*Tanh[c + d*x]^3)/(3*d) + (b*Tanh[c + d*x]^5)/(5*d)
```

fricas [B] time = 0.38, size = 343, normalized size = 6.86

$$-\frac{}{15\left(d\cosh\left(dx+c\right)^7+7\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^6+d\sinh\left(dx+c\right)^7+5\,d\cosh\left(dx+c\right)^5+\left(21\,d\cosh\left(dx\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4*(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -8/15*(2*(5*a + b)*cosh(d*x + c)^3 + 6*(5*a + b)*cosh(d*x + c)*sinh(d*x + c
)^2 + (5*a - 2*b)*sinh(d*x + c)^3 + 30*(a + b)*cosh(d*x + c) + (3*(5*a - 2*
b)*cosh(d*x + c)^2 + 5*a + 10*b)*sinh(d*x + c))/(d*cosh(d*x + c)^7 + 7*d*co
sh(d*x + c)*sinh(d*x + c)^6 + d*sinh(d*x + c)^7 + 5*d*cosh(d*x + c)^5 + (21
*d*cosh(d*x + c)^2 + 5*d)*sinh(d*x + c)^5 + 5*(7*d*cosh(d*x + c)^3 + 5*d*co
sh(d*x + c))*sinh(d*x + c)^4 + 11*d*cosh(d*x + c)^3 + (35*d*cosh(d*x + c)^4
 + 50*d*cosh(d*x + c)^2 + 9*d)*sinh(d*x + c)^3 + (21*d*cosh(d*x + c)^5 + 50
*d*cosh(d*x + c)^3 + 33*d*cosh(d*x + c))*sinh(d*x + c)^2 + 15*d*cosh(d*x +
c) + (7*d*cosh(d*x + c)^6 + 25*d*cosh(d*x + c)^4 + 27*d*cosh(d*x + c)^2 + 5
*d)*sinh(d*x + c))
```

giac [A] time = 0.15, size = 85, normalized size = 1.70

$$-\frac{4\left(15\,ae^{(6\,dx+6\,c)}+35\,ae^{(4\,dx+4\,c)}+40\,be^{(4\,dx+4\,c)}+25\,ae^{(2\,dx+2\,c)}+20\,be^{(2\,dx+2\,c)}+5\,a+4\,b\right)}{15\,d\left(e^{(2\,dx+2\,c)}+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4*(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -4/15*(15*a*e^(6*d*x + 6*c) + 35*a*e^(4*d*x + 4*c) + 40*b*e^(4*d*x + 4*c) +
 25*a*e^(2*d*x + 2*c) + 20*b*e^(2*d*x + 2*c) + 5*a + 4*b)/(d*(e^(2*d*x + 2*
c) + 1)^5)
```

maple [A] time = 0.37, size = 56, normalized size = 1.12

$$\frac{a\left(\frac{2}{3}+\frac{\mathrm{sech}(dx+c)^2}{3}\right)\tanh(dx+c)+b\left(\frac{8}{15}+\frac{\mathrm{sech}(dx+c)^4}{5}+\frac{4\mathrm{sech}(dx+c)^2}{15}\right)\tanh(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^4*(a+b*sech(d*x+c)^2),x)
```

```
[Out] 1/d*(a*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c)+b*(8/15+1/5*sech(d*x+c)^4+4/15*s
ech(d*x+c)^2)*tanh(d*x+c))
```

maxima [B] time = 0.32, size = 300, normalized size = 6.00

$$\frac{16}{15}b\left(\frac{5e^{(-2dx-2c)}}{d\left(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1\right)}+\frac{\cdots}{d\left(5e^{(-2dx-2c)}+10e^{(-4dx-4c}\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4*(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 16/15*b*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) +
10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 10*e^
(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x
- 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*d*x
- 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e
^(-10*d*x - 10*c) + 1))) + 4/3*a*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c)
 + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) +
 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)))
```

mupad [B] time = 1.43, size = 292, normalized size = 5.84

$$-\frac{\frac{8(a+2b)}{15d}+\frac{4ae^{2c+2dx}}{5d}}{3e^{2c+2dx}+3e^{4c+4dx}+e^{6c+6dx}+1}-\frac{\frac{8ae^{2c+2dx}}{5d}+\frac{8ae^{6c+6dx}}{5d}+\frac{16e^{4c+4dx}(a+2b)}{5d}}{5e^{2c+2dx}+10e^{4c+4dx}+10e^{6c+6dx}+5e^{8c+8dx}+e^{10c+10dx}+1}-\frac{\cdots}{4e^{2c+}\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cosh(c + d*x)^2)/cosh(c + d*x)^4,x)
```

```
[Out] - ((8*(a + 2*b))/(15*d) + (4*a*exp(2*c + 2*d*x))/(5*d))/(3*exp(2*c + 2*d*x)
 + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) - ((8*a*exp(2*c + 2*d*x))/(5*
d) + (8*a*exp(6*c + 6*d*x))/(5*d) + (16*exp(4*c + 4*d*x)*(a + 2*b))/(5*d))/
(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c
 + 8*d*x) + exp(10*c + 10*d*x) + 1) - ((2*a)/(5*d) + (6*a*exp(4*c + 4*d*x))
```

```
/(5*d) + (8*exp(2*c + 2*d*x)*(a + 2*b))/(5*d))/(4*exp(2*c + 2*d*x) + 6*exp(
4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) - (2*a)/(5*d*(2*e
xp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\operatorname{sech}^2(c + dx)\right)\operatorname{sech}^4(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**4*(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)*sech(c + d*x)**4, x)
```

3.57 $\int \cosh^4(c+dx)\left(a+b\mathbf{sech}^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=82

$$\frac{1}{8}x\left(3a^2+8ab+8b^2\right)+\frac{3a(a+2b)\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{a\sinh(c+dx)\cosh^3(c+dx)\left(a-b\tanh^2(c+dx)\right.}{4d}$$

```
[Out] 1/8*(3*a^2+8*a*b+8*b^2)*x+3/8*a*(a+2*b)*cosh(d*x+c)*sinh(d*x+c)/d+1/4*a*cos
h(d*x+c)^3*sinh(d*x+c)*(a+b-b*tanh(d*x+c)^2)/d
```

Rubi [A] time = 0.09, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4146, 413, 385, 206}

$$\frac{1}{8}x\left(3a^2+8ab+8b^2\right)+\frac{3a(a+2b)\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{a\sinh(c+dx)\cosh^3(c+dx)\left(a-b\tanh^2(c+dx)\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^4*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((3*a^2 + 8*a*b + 8*b^2)*x)/8 + (3*a*(a + 2*b)*Cosh[c + d*x]*Sinh[c + d*x])
/(8*d) + (a*Cosh[c + d*x]^3*Sinh[c + d*x]*(a + b - b*Tanh[c + d*x]^2))/(4*d
)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
```

```
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \cosh^4(c+dx)\left(a+b\,\text{sech}^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b-bx^2\right)^2}{\left(1-x^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{a\cosh^3(c+dx)\sinh(c+dx)\left(a+b-b\tanh^2(c+dx)\right)}{4d} - \frac{\text{Subst}\left(\right.}{}$$

$$= \frac{3a(a+2b)\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{a\cosh^3(c+dx)\sinh(c+}{}$$

$$= \frac{1}{8}\left(3a^2+8ab+8b^2\right)x + \frac{3a(a+2b)\cosh(c+dx)\sinh(c+dx)}{8d} + $$

Mathematica [A] time = 0.13, size = 58, normalized size = 0.71

$$\frac{4\left(3a^2+8ab+8b^2\right)(c+dx)+a^2\sinh(4(c+dx))+8a(a+2b)\sinh(2(c+dx))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^4*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (4*(3*a^2 + 8*a*b + 8*b^2)*(c + d*x) + 8*a*(a + 2*b)*Sinh[2*(c + d*x)] + a^
2*Sinh[4*(c + d*x)])/(32*d)
```

fricas [A] time = 0.40, size = 78, normalized size = 0.95

$$\frac{a^2\cosh(dx+c)\sinh(dx+c)^3 + \left(3a^2+8ab+8b^2\right)dx + \left(a^2\cosh(dx+c)^3 + 4\left(a^2+2ab\right)\cosh(dx+c)\right)\sinh}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(a^2*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a^2 + 8*a*b + 8*b^2)*d*x + (a^2
*cosh(d*x + c)^3 + 4*(a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c))/d
```

giac [A] time = 0.15, size = 151, normalized size = 1.84

$$\frac{a^2e^{(4\,dx+4\,c)} + 8\,a^2e^{(2\,dx+2\,c)} + 16\,abe^{(2\,dx+2\,c)} + 8\left(3\,a^2 + 8\,ab + 8\,b^2\right)(dx+c) - \left(18\,a^2e^{(4\,dx+4\,c)} + 48\,abe^{(4\,dx+4\,c)} + 48}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/64*(a^2*e^(4*d*x + 4*c) + 8*a^2*e^(2*d*x + 2*c) + 16*a*b*e^(2*d*x + 2*c)
+ 8*(3*a^2 + 8*a*b + 8*b^2)*(d*x + c) - (18*a^2*e^(4*d*x + 4*c) + 48*a*b*e^
(4*d*x + 4*c) + 48*b^2*e^(4*d*x + 4*c) + 8*a^2*e^(2*d*x + 2*c) + 16*a*b*e^(
2*d*x + 2*c) + a^2)*e^(-4*d*x - 4*c))/d
```

maple [A] time = 0.40, size = 79, normalized size = 0.96

$$\frac{a^2\left(\left(\frac{\left(\cosh^3(dx+c)\right)}{4}+\frac{3\cosh(dx+c)}{8}\right)\sinh\left(dx+c\right)+\frac{3dx}{8}+\frac{3c}{8}\right)+2ab\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+b^2\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(a^2*((1/4*cosh(d*x+c)^3+3/8*cosh(d*x+c))*sinh(d*x+c)+3/8*d*x+3/8*c)+2*
a*b*(1/2*cosh(d*x+c)*sinh(d*x+c)+1/2*d*x+1/2*c)+b^2*(d*x+c))
```

maxima [A] time = 0.31, size = 105, normalized size = 1.28

$$\frac{1}{64}\,a^2\left(24\,x+\frac{e^{(4\,dx+4\,c)}}{d}+\frac{8\,e^{(2\,dx+2\,c)}}{d}-\frac{8\,e^{(-2\,dx-2\,c)}}{d}-\frac{e^{(-4\,dx-4\,c)}}{d}\right)+\frac{1}{4}\,ab\left(4\,x+\frac{e^{(2\,dx+2\,c)}}{d}-\frac{e^{(-2\,dx-2\,c)}}{d}\right)+b^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/64*a^2*(24*x + e^(4*d*x + 4*c)/d + 8*e^(2*d*x + 2*c)/d - 8*e^(-2*d*x - 2*
c)/d - e^(-4*d*x - 4*c)/d) + 1/4*a*b*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d*x -
 2*c)/d) + b^2*x
```

mupad [B] time = 1.37, size = 66, normalized size = 0.80

$$\frac{3a^2x}{8}+b^2x+abx+\frac{a^2\sinh(2c+2dx)}{4d}+\frac{a^2\sinh(4c+4dx)}{32d}+\frac{ab\sinh(2c+2dx)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^4*(a + b/cosh(c + d*x)^2)^2,x)
```

```
[Out] (3*a^2*x)/8 + b^2*x + a*b*x + (a^2*sinh(2*c + 2*d*x))/(4*d) + (a^2*sinh(4*c
 + 4*d*x))/(32*d) + (a*b*sinh(2*c + 2*d*x))/(2*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**4*(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

3.58 $\int \cosh^3(c+dx)\left(a+b\,\mathbf{sech}^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=49

$$\frac{a^2\sinh^3(c+dx)}{3d}+\frac{a(a+2b)\sinh(c+dx)}{d}+\frac{b^2\tan^{-1}(\sinh(c+dx))}{d}$$

```
[Out] b^2*arctan(sinh(d*x+c))/d+a*(a+2*b)*sinh(d*x+c)/d+1/3*a^2*sinh(d*x+c)^3/d
```

Rubi [A] time = 0.06, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4147, 390, 203}

$$\frac{a^2\sinh^3(c+dx)}{3d}+\frac{a(a+2b)\sinh(c+dx)}{d}+\frac{b^2\tan^{-1}(\sinh(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^3*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (b^2*ArcTan[Sinh[c + d*x]])/d + (a*(a + 2*b)*Sinh[c + d*x])/d + (a^2*Sinh[c
 + d*x]^3)/(3*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \cosh^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx = \frac{\operatorname{Subst}\left(\int \frac{\left(a+b+ax^2\right)^2}{1+x^2}\,dx,x,\sinh(c+dx)\right)}{d}$$
$$= \frac{\operatorname{Subst}\left(\int \left(a(a+2b)+a^2x^2+\frac{b^2}{1+x^2}\right)dx,x,\sinh(c+dx)\right)}{d}$$
$$= \frac{a(a+2b)\sinh(c+dx)}{d}+\frac{a^2\sinh^3(c+dx)}{3d}+\frac{b^2\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx,\right.}{d}$$
$$= \frac{b^2\tan^{-1}(\sinh(c+dx))}{d}+\frac{a(a+2b)\sinh(c+dx)}{d}+\frac{a^2\sinh^3(c+d}{3d}$$

Mathematica [A] time = 0.02, size = 72, normalized size = 1.47

$$\frac{a^2\sinh^3(c+dx)}{3d}+\frac{a^2\sinh(c+dx)}{d}+\frac{2ab\sinh(c)\cosh(dx)}{d}+\frac{2ab\cosh(c)\sinh(dx)}{d}+\frac{b^2\tan^{-1}(\sinh(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^3*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (b^2*ArcTan[Sinh[c + d*x]])/d + (2*a*b*Cosh[d*x]*Sinh[c])/d + (2*a*b*Cosh[c
]*Sinh[d*x])/d + (a^2*Sinh[c + d*x])/d + (a^2*Sinh[c + d*x]^3)/(3*d)
```

fricas [B] time = 0.41, size = 414, normalized size = 8.45

$$\frac{a^2\cosh(dx+c)^6+6a^2\cosh(dx+c)\sinh(dx+c)^5+a^2\sinh(dx+c)^6+3\left(3a^2+8ab\right)\cosh(dx+c)^4+3\left(5a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(a^2*cosh(d*x + c)^6 + 6*a^2*cosh(d*x + c)*sinh(d*x + c)^5 + a^2*sinh(
d*x + c)^6 + 3*(3*a^2 + 8*a*b)*cosh(d*x + c)^4 + 3*(5*a^2*cosh(d*x + c)^2 +
 3*a^2 + 8*a*b)*sinh(d*x + c)^4 + 4*(5*a^2*cosh(d*x + c)^3 + 3*(3*a^2 + 8*a
*b)*cosh(d*x + c))*sinh(d*x + c)^3 - 3*(3*a^2 + 8*a*b)*cosh(d*x + c)^2 + 3*
(5*a^2*cosh(d*x + c)^4 + 6*(3*a^2 + 8*a*b)*cosh(d*x + c)^2 - 3*a^2 - 8*a*b)
*sinh(d*x + c)^2 - a^2 + 48*(b^2*cosh(d*x + c)^3 + 3*b^2*cosh(d*x + c)^2*si
nh(d*x + c) + 3*b^2*cosh(d*x + c)*sinh(d*x + c)^2 + b^2*sinh(d*x + c)^3)*ar
ctan(cosh(d*x + c) + sinh(d*x + c)) + 6*(a^2*cosh(d*x + c)^5 + 2*(3*a^2 + 8
*a*b)*cosh(d*x + c)^3 - (3*a^2 + 8*a*b)*cosh(d*x + c))*sinh(d*x + c))/(d*co
```

```
sh(d*x + c)^3 + 3*d*cosh(d*x + c)^2*sinh(d*x + c) + 3*d*cosh(d*x + c)*sinh(
d*x + c)^2 + d*sinh(d*x + c)^3)
```

giac [A] time = 0.15, size = 94, normalized size = 1.92

$$\frac{48\,b^2\arctan\left(e^{(dx+c)}\right)+a^2e^{(3\,dx+3\,c)}+9\,a^2e^{(dx+c)}+24\,abe^{(dx+c)}-\left(9\,a^2e^{(2\,dx+2\,c)}+24\,abe^{(2\,dx+2\,c)}+a^2\right)e^{(-3\,dx-3\,c)}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/24*(48*b^2*arctan(e^(d*x + c)) + a^2*e^(3*d*x + 3*c) + 9*a^2*e^(d*x + c)
+ 24*a*b*e^(d*x + c) - (9*a^2*e^(2*d*x + 2*c) + 24*a*b*e^(2*d*x + 2*c) + a^
2)*e^(-3*d*x - 3*c))/d
```

maple [A] time = 0.36, size = 66, normalized size = 1.35

$$\frac{2a^2\sinh(dx+c)}{3d}+\frac{a^2\sinh(dx+c)\left(\cosh^2(dx+c)\right)}{3d}+\frac{2ab\sinh(dx+c)}{d}+\frac{2b^2\arctan\left(\mathrm{e}^{dx+c}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] 2/3*a^2*sinh(d*x+c)/d+1/3/d*a^2*sinh(d*x+c)*cosh(d*x+c)^2+2*a*b*sinh(d*x+c)
/d+2/d*b^2*arctan(exp(d*x+c))
```

maxima [B] time = 0.41, size = 105, normalized size = 2.14

$$\frac{1}{24}\,a^2\left(\frac{e^{(3\,dx+3\,c)}}{d}+\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}-\frac{e^{(-3\,dx-3\,c)}}{d}\right)+ab\left(\frac{e^{(dx+c)}}{d}-\frac{e^{(-dx-c)}}{d}\right)-\frac{2\,b^2\arctan\left(e^{(-dx-c)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/24*a^2*(e^(3*d*x + 3*c)/d + 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d - e^(-3*d*
x - 3*c)/d) + a*b*(e^(d*x + c)/d - e^(-d*x - c)/d) - 2*b^2*arctan(e^(-d*x -
 c))/d
```

mupad [B] time = 0.17, size = 114, normalized size = 2.33

$$\frac{2\,\mathrm{atan}\left(\frac{b^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\sqrt{d^2}}{d\,\sqrt{b^4}}\right)\,\sqrt{b^4}}{\sqrt{d^2}}-\frac{\mathrm{e}^{-c-d\,x}\,\left(3\,a^2+8\,b\,a\right)}{8\,d}-\frac{a^2\,\mathrm{e}^{-3\,c-3\,d\,x}}{24\,d}+\frac{a^2\,\mathrm{e}^{3\,c+3\,d\,x}}{24\,d}+\frac{a\,\mathrm{e}^{c+d\,x}\,\left(3\,a+8\,b\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^3*(a + b/cosh(c + d*x)^2)^2,x)
```

```
[Out] (2*atan((b^2*exp(d*x)*exp(c)*(d^2)^(1/2))/(d*(b^4)^(1/2)))*(b^4)^(1/2))/(d^
2)^(1/2) - (exp(- c - d*x)*(8*a*b + 3*a^2))/(8*d) - (a^2*exp(- 3*c - 3*d*x)
)/(24*d) + (a^2*exp(3*c + 3*d*x))/(24*d) + (a*exp(c + d*x)*(3*a + 8*b))/(8*
d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3*(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

3.59 $\int \cosh^2(c+dx)\left(a+b\,\mathbf{sech}^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=47

$$\frac{a^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{1}{2}ax(a+4b)+\frac{b^2\tanh(c+dx)}{d}$$

```
[Out] 1/2*a*(a+4*b)*x+1/2*a^2*cosh(d*x+c)*sinh(d*x+c)/d+b^2*tanh(d*x+c)/d
```

Rubi [A] time = 0.08, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4146, 390, 385, 206}

$$\frac{a^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{1}{2}ax(a+4b)+\frac{b^2\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^2*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (a*(a + 4*b)*x)/2 + (a^2*Cosh[c + d*x]*Sinh[c + d*x])/(2*d) + (b^2*Tanh[c +
 d*x])/d
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \cosh^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx &= \frac{\operatorname{Subst}\left(\int \frac{\left(a+b-bx^2\right)^2}{\left(1-x^2\right)^2}\,dx,x,\tanh(c+dx)\right)}{d}\\
&= \frac{\operatorname{Subst}\left(\int \left(b^2+\frac{a(a+2b)-2abx^2}{\left(1-x^2\right)^2}\right)dx,x,\tanh(c+dx)\right)}{d}\\
&= \frac{b^2\tanh(c+dx)}{d}+\frac{\operatorname{Subst}\left(\int \frac{a(a+2b)-2abx^2}{\left(1-x^2\right)^2}\,dx,x,\tanh(c+dx)\right)}{d}\\
&= \frac{a^2\cosh(c+dx)\sinh(c+dx)}{2d}+\frac{b^2\tanh(c+dx)}{d}+\frac{(a(a+4b))\operatorname{Sub}}{}\\
&= \frac{1}{2}a(a+4b)x+\frac{a^2\cosh(c+dx)\sinh(c+dx)}{2d}+\frac{b^2\tanh(c+dx)}{d}
\end{aligned}
$$

Mathematica [A] time = 0.15, size = 52, normalized size = 1.11

$$\frac{a^2(c+dx)}{2d}+\frac{a^2\sinh(2(c+dx))}{4d}+2abx+\frac{b^2\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^2*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] 2*a*b*x + (a^2*(c + d*x))/(2*d) + (a^2*Sinh[2*(c + d*x)])/(4*d) + (b^2*Tanh
[c + d*x])/d
```

fricas [A] time = 0.39, size = 80, normalized size = 1.70

$$\frac{a^2\sinh(dx+c)^3+4\left(\left(a^2+4\,ab\right)dx-2\,b^2\right)\cosh(dx+c)+\left(3\,a^2\cosh(dx+c)^2+a^2+8\,b^2\right)\sinh(dx+c)}{8\,d\cosh(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(a^2*sinh(d*x + c)^3 + 4*((a^2 + 4*a*b)*d*x - 2*b^2)*cosh(d*x + c) + (3
*a^2*cosh(d*x + c)^2 + a^2 + 8*b^2)*sinh(d*x + c))/(d*cosh(d*x + c))
```

giac [B] time = 0.17, size = 128, normalized size = 2.72

$$\frac{a^2 e^{(2\,dx+2\,c)} + 4\left(a^2 + 4\,ab\right)(dx+c) - \frac{a^2 e^{(4\,dx+4\,c)} + 4\,abe^{(4\,dx+4\,c)} + 2\,a^2 e^{(2\,dx+2\,c)} + 4\,abe^{(2\,dx+2\,c)} + 16\,b^2 e^{(2\,dx+2\,c)} + a^2}{e^{(4\,dx+4\,c)} + e^{(2\,dx+2\,c)}}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/8*(a^2*e^(2*d*x + 2*c) + 4*(a^2 + 4*a*b)*(d*x + c) - (a^2*e^(4*d*x + 4*c)
 + 4*a*b*e^(4*d*x + 4*c) + 2*a^2*e^(2*d*x + 2*c) + 4*a*b*e^(2*d*x + 2*c) +
16*b^2*e^(2*d*x + 2*c) + a^2)/(e^(4*d*x + 4*c) + e^(2*d*x + 2*c)))/d
```

maple [A] time = 0.36, size = 51, normalized size = 1.09

$$\frac{a^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2}\right) + 2ab\,(dx+c) + b^2\tanh\,(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(a^2*(1/2*cosh(d*x+c)*sinh(d*x+c)+1/2*d*x+1/2*c)+2*a*b*(d*x+c)+b^2*tanh
(d*x+c))
```

maxima [A] time = 0.32, size = 63, normalized size = 1.34

$$\frac{1}{8}\,a^2\left(4\,x + \frac{e^{(2\,dx+2\,c)}}{d} - \frac{e^{(-2\,dx-2\,c)}}{d}\right) + 2\,abx + \frac{2\,b^2}{d\left(e^{(-2\,dx-2\,c)} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/8*a^2*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d*x - 2*c)/d) + 2*a*b*x + 2*b^2/(d
*(e^(-2*d*x - 2*c) + 1))
```

mupad [B] time = 0.16, size = 65, normalized size = 1.38

$$\frac{a^2\,\mathrm{e}^{2\,c+2\,d\,x}}{8\,d} - \frac{a^2\,\mathrm{e}^{-2\,c-2\,d\,x}}{8\,d} - \frac{2\,b^2}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x} + 1\right)} + \frac{a\,x\,\left(a + 4\,b\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^2*(a + b/cosh(c + d*x)^2)^2,x)
```

```
[Out] (a^2*exp(2*c + 2*d*x))/(8*d) - (a^2*exp(- 2*c - 2*d*x))/(8*d) - (2*b^2)/(d*
(exp(2*c + 2*d*x) + 1)) + (a*x*(a + 4*b))/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\,\mathrm{sech}^2\left(c + dx\right)\right)^2 \cosh^2\left(c + dx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2*(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**2*cosh(c + d*x)**2, x)
```

3.60 $\int \cosh(c+dx)\left(a+b\,\mathbf{sech}^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=56

$$\frac{a^2\sinh(c+dx)}{d}+\frac{b(4a+b)\tan^{-1}(\sinh(c+dx))}{2d}+\frac{b^2\tanh(c+dx)\text{sech}(c+dx)}{2d}$$

```
[Out] 1/2*b*(4*a+b)*arctan(sinh(d*x+c))/d+a^2*sinh(d*x+c)/d+1/2*b^2*sech(d*x+c)*t
anh(d*x+c)/d
```

Rubi [A] time = 0.07, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {4147, 390, 385, 203}

$$\frac{a^2\sinh(c+dx)}{d}+\frac{b(4a+b)\tan^{-1}(\sinh(c+dx))}{2d}+\frac{b^2\tanh(c+dx)\text{sech}(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (b*(4*a + b)*ArcTan[Sinh[c + d*x]])/(2*d) + (a^2*Sinh[c + d*x])/d + (b^2*Se
ch[c + d*x]*Tanh[c + d*x])/(2*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \cosh(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{(a+b+ax^2)^2}{(1+x^2)^2}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(a^2+\frac{b(2a+b)+2abx^2}{(1+x^2)^2}\right)dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{a^2\sinh(c+dx)}{d}+\frac{\text{Subst}\left(\int \frac{b(2a+b)+2abx^2}{(1+x^2)^2}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{a^2\sinh(c+dx)}{d}+\frac{b^2\text{sech}(c+dx)\tanh(c+dx)}{2d}+\frac{(b(4a+b))\,\text{Subs}}{}$$

$$= \frac{b(4a+b)\tan^{-1}(\sinh(c+dx))}{2d}+\frac{a^2\sinh(c+dx)}{d}+\frac{b^2\text{sech}(c+dx)}{2d}$$

Mathematica [A] time = 0.04, size = 80, normalized size = 1.43

$$\frac{a^2\sinh(c)\cosh(dx)}{d}+\frac{a^2\cosh(c)\sinh(dx)}{d}+\frac{2ab\tan^{-1}(\sinh(c+dx))}{d}+\frac{b^2\tan^{-1}(\sinh(c+dx))}{2d}+\frac{b^2\tanh(c+dx)\text{se}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (2*a*b*ArcTan[Sinh[c + d*x]])/d + (b^2*ArcTan[Sinh[c + d*x]])/(2*d) + (a^2*
Cosh[d*x]*Sinh[c])/d + (a^2*Cosh[c]*Sinh[d*x])/d + (b^2*Sech[c + d*x]*Tanh[
c + d*x])/(2*d)
```

fricas [B] time = 0.42, size = 653, normalized size = 11.66

$$\frac{a^2\cosh(dx+c)^6+6a^2\cosh(dx+c)\sinh(dx+c)^5+a^2\sinh(dx+c)^6+\left(a^2+2b^2\right)\cosh(dx+c)^4+\left(15a^2\text{co}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*cosh(d*x + c)^6 + 6*a^2*cosh(d*x + c)*sinh(d*x + c)^5 + a^2*sinh(d
*x + c)^6 + (a^2 + 2*b^2)*cosh(d*x + c)^4 + (15*a^2*cosh(d*x + c)^2 + a^2 +
 2*b^2)*sinh(d*x + c)^4 + 4*(5*a^2*cosh(d*x + c)^3 + (a^2 + 2*b^2)*cosh(d*x
 + c))*sinh(d*x + c)^3 - (a^2 + 2*b^2)*cosh(d*x + c)^2 + (15*a^2*cosh(d*x +
 c)^4 + 6*(a^2 + 2*b^2)*cosh(d*x + c)^2 - a^2 - 2*b^2)*sinh(d*x + c)^2 - a^
2 + 2*((4*a*b + b^2)*cosh(d*x + c)^5 + 5*(4*a*b + b^2)*cosh(d*x + c)*sinh(d
*x + c)^4 + (4*a*b + b^2)*sinh(d*x + c)^5 + 2*(4*a*b + b^2)*cosh(d*x + c)^3
 + 2*(5*(4*a*b + b^2)*cosh(d*x + c)^2 + 4*a*b + b^2)*sinh(d*x + c)^3 + 2*(5
*(4*a*b + b^2)*cosh(d*x + c)^3 + 3*(4*a*b + b^2)*cosh(d*x + c))*sinh(d*x +
c)^2 + (4*a*b + b^2)*cosh(d*x + c) + (5*(4*a*b + b^2)*cosh(d*x + c)^4 + 6*(
4*a*b + b^2)*cosh(d*x + c)^2 + 4*a*b + b^2)*sinh(d*x + c))*arctan(cosh(d*x
+ c) + sinh(d*x + c)) + 2*(3*a^2*cosh(d*x + c)^5 + 2*(a^2 + 2*b^2)*cosh(d*x
 + c)^3 - (a^2 + 2*b^2)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^5 +
5*d*cosh(d*x + c)*sinh(d*x + c)^4 + d*sinh(d*x + c)^5 + 2*d*cosh(d*x + c)^3
 + 2*(5*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^3 + 2*(5*d*cosh(d*x + c)^3 + 3
*d*cosh(d*x + c))*sinh(d*x + c)^2 + d*cosh(d*x + c) + (5*d*cosh(d*x + c)^4
+ 6*d*cosh(d*x + c)^2 + d)*sinh(d*x + c))
```

giac [B] time = 0.14, size = 112, normalized size = 2.00

$$\frac{2\,a^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)+\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)\left(4\,ab+b^2\right)+\frac{4\,b^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)}{\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/4*(2*a^2*(e^(d*x + c) - e^(-d*x - c)) + (pi + 2*arctan(1/2*(e^(2*d*x + 2*
c) - 1)*e^(-d*x - c)))*(4*a*b + b^2) + 4*b^2*(e^(d*x + c) - e^(-d*x - c))/(
(e^(d*x + c) - e^(-d*x - c))^2 + 4))/d
```

maple [A] time = 0.36, size = 63, normalized size = 1.12

$$\frac{a^2\sinh(dx+c)}{d}+\frac{4ab\arctan\left(e^{dx+c}\right)}{d}+\frac{b^2\mathrm{sech}(dx+c)\tanh(dx+c)}{2d}+\frac{b^2\arctan\left(e^{dx+c}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)*(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] a^2*sinh(d*x+c)/d+4/d*a*b*arctan(exp(d*x+c))+1/2/d*b^2*sech(d*x+c)*tanh(d*x
+c)+1/d*b^2*arctan(exp(d*x+c))
```

maxima [A] time = 0.42, size = 101, normalized size = 1.80

$$-b^2\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{e^{(-dx-c)} - e^{(-3\,dx-3\,c)}}{d\left(2\,e^{(-2\,dx-2\,c)} + e^{(-4\,dx-4\,c)} + 1\right)}\right) - \frac{4\,ab\arctan\left(e^{(-dx-c)}\right)}{d} + \frac{a^2\sinh\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -b^2*(arctan(e^(-d*x - c))/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-
2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) - 4*a*b*arctan(e^(-d*x - c))/d + a^2
*sinh(d*x + c)/d
```

mupad [B] time = 1.44, size = 172, normalized size = 3.07

$$\frac{\mathrm{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(b^2\,\sqrt{d^2}+4\,a\,b\,\sqrt{d^2}\right)}{d\,\sqrt{16\,a^2\,b^2+8\,a\,b^3+b^4}}\right)\,\sqrt{16\,a^2\,b^2+8\,a\,b^3+b^4}}{\sqrt{d^2}} + \frac{a^2\,\mathrm{e}^{c+d\,x}}{2\,d} - \frac{a^2\,\mathrm{e}^{-c-d\,x}}{2\,d} + \frac{b^2\,\mathrm{e}^{c+d\,x}}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)} - \frac{2\,b^2\,\mathrm{e}^{c+d\,x}}{d\,\left(2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)*(a + b/cosh(c + d*x)^2)^2,x)
```

```
[Out] (atan((exp(d*x)*exp(c)*(b^2*(d^2)^(1/2) + 4*a*b*(d^2)^(1/2)))/(d*(8*a*b^3 +
 b^4 + 16*a^2*b^2)^(1/2)))*(8*a*b^3 + b^4 + 16*a^2*b^2)^(1/2))/(d^2)^(1/2)
+ (a^2*exp(c + d*x))/(2*d) - (a^2*exp(- c - d*x))/(2*d) + (b^2*exp(c + d*x)
)/(d*(exp(2*c + 2*d*x) + 1)) - (2*b^2*exp(c + d*x))/(d*(2*exp(2*c + 2*d*x)
+ exp(4*c + 4*d*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\operatorname{sech}^2\left(c+dx\right)\right)^2\cosh\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**2*cosh(c + d*x), x)
```

3.61 $\int \mathbf{sech}(c+dx)\left(a+b\mathbf{sech}^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=90

$$\frac{\left(8a^2+8ab+3b^2\right)\tan^{-1}(\sinh(c+dx))}{8d}+\frac{3b(2a+b)\tanh(c+dx)\text{sech}(c+dx)}{8d}+\frac{b\tanh(c+dx)\text{sech}^3(c+dx)\left(a\sinh\right.}{4d}$$

```
[Out] 1/8*(8*a^2+8*a*b+3*b^2)*arctan(sinh(d*x+c))/d+3/8*b*(2*a+b)*sech(d*x+c)*tan
h(d*x+c)/d+1/4*b*sech(d*x+c)^3*(a+b+a*sinh(d*x+c)^2)*tanh(d*x+c)/d
```

Rubi [A] time = 0.08, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {4147, 413, 385, 203}

$$\frac{\left(8a^2+8ab+3b^2\right)\tan^{-1}(\sinh(c+dx))}{8d}+\frac{3b(2a+b)\tanh(c+dx)\text{sech}(c+dx)}{8d}+\frac{b\tanh(c+dx)\text{sech}^3(c+dx)\left(a\sinh\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((8*a^2 + 8*a*b + 3*b^2)*ArcTan[Sinh[c + d*x]])/(8*d) + (3*b*(2*a + b)*Sech
[c + d*x]*Tanh[c + d*x])/(8*d) + (b*Sech[c + d*x]^3*(a + b + a*Sinh[c + d*x
]^2)*Tanh[c + d*x])/(4*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
```

```
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \text{sech}(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b+ax^2\right)^2}{\left(1+x^2\right)^3}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{b\text{sech}^3(c+dx)\left(a+b+a\sinh^2(c+dx)\right)\tanh(c+dx)}{4d} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= \frac{3b(2a+b)\text{sech}(c+dx)\tanh(c+dx)}{8d} + \frac{b\text{sech}^3(c+dx)\left(a+b+a\,\text{s}\right.}{4}$$

$$= \frac{\left(8a^2+8ab+3b^2\right)\tan^{-1}(\sinh(c+dx))}{8d} + \frac{3b(2a+b)\text{sech}(c+dx)\,\text{ta}}{8d}$$

Mathematica [A] time = 0.13, size = 71, normalized size = 0.79

$$\frac{\left(8a^2+8ab+3b^2\right)\tan^{-1}(\sinh(c+dx))+b(8a+3b)\tanh(c+dx)\text{sech}(c+dx)+2b^2\tanh(c+dx)\text{sech}^3(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((8*a^2 + 8*a*b + 3*b^2)*ArcTan[Sinh[c + d*x]] + b*(8*a + 3*b)*Sech[c + d*x
]*Tanh[c + d*x] + 2*b^2*Sech[c + d*x]^3*Tanh[c + d*x])/(8*d)
```

fricas [B] time = 0.43, size = 1372, normalized size = 15.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/4*((8*a*b + 3*b^2)*cosh(d*x + c)^7 + 7*(8*a*b + 3*b^2)*cosh(d*x + c)*sinh
(d*x + c)^6 + (8*a*b + 3*b^2)*sinh(d*x + c)^7 + (8*a*b + 11*b^2)*cosh(d*x +
 c)^5 + (21*(8*a*b + 3*b^2)*cosh(d*x + c)^2 + 8*a*b + 11*b^2)*sinh(d*x + c)
^5 + 5*(7*(8*a*b + 3*b^2)*cosh(d*x + c)^3 + (8*a*b + 11*b^2)*cosh(d*x + c))
*sinh(d*x + c)^4 - (8*a*b + 11*b^2)*cosh(d*x + c)^3 + (35*(8*a*b + 3*b^2)*c
osh(d*x + c)^4 + 10*(8*a*b + 11*b^2)*cosh(d*x + c)^2 - 8*a*b - 11*b^2)*sinh
(d*x + c)^3 + (21*(8*a*b + 3*b^2)*cosh(d*x + c)^5 + 10*(8*a*b + 11*b^2)*cos
h(d*x + c)^3 - 3*(8*a*b + 11*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + ((8*a^2
+ 8*a*b + 3*b^2)*cosh(d*x + c)^8 + 8*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)*
sinh(d*x + c)^7 + (8*a^2 + 8*a*b + 3*b^2)*sinh(d*x + c)^8 + 4*(8*a^2 + 8*a*
b + 3*b^2)*cosh(d*x + c)^6 + 4*(7*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^2 +
 8*a^2 + 8*a*b + 3*b^2)*sinh(d*x + c)^6 + 8*(7*(8*a^2 + 8*a*b + 3*b^2)*cosh
(d*x + c)^3 + 3*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 6*
(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^4 + 2*(35*(8*a^2 + 8*a*b + 3*b^2)*cos
h(d*x + c)^4 + 30*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^2 + 24*a^2 + 24*a*b
 + 9*b^2)*sinh(d*x + c)^4 + 8*(7*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^5 +
10*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^3 + 3*(8*a^2 + 8*a*b + 3*b^2)*cosh
(d*x + c))*sinh(d*x + c)^3 + 4*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^2 + 4*
(7*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^6 + 15*(8*a^2 + 8*a*b + 3*b^2)*cos
h(d*x + c)^4 + 9*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^2 + 8*a^2 + 8*a*b +
3*b^2)*sinh(d*x + c)^2 + 8*a^2 + 8*a*b + 3*b^2 + 8*((8*a^2 + 8*a*b + 3*b^2)
*cosh(d*x + c)^7 + 3*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^5 + 3*(8*a^2 + 8
*a*b + 3*b^2)*cosh(d*x + c)^3 + (8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c))*sinh
(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) - (8*a*b + 3*b^2)*cosh(d*x
 + c) + (7*(8*a*b + 3*b^2)*cosh(d*x + c)^6 + 5*(8*a*b + 11*b^2)*cosh(d*x +
c)^4 - 3*(8*a*b + 11*b^2)*cosh(d*x + c)^2 - 8*a*b - 3*b^2)*sinh(d*x + c))/(
d*cosh(d*x + c)^8 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 +
 4*d*cosh(d*x + c)^6 + 4*(7*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^6 + 8*(7*d
*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^5 + 6*d*cosh(d*x + c)^4
 + 2*(35*d*cosh(d*x + c)^4 + 30*d*cosh(d*x + c)^2 + 3*d)*sinh(d*x + c)^4 +
8*(7*d*cosh(d*x + c)^5 + 10*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x
 + c)^3 + 4*d*cosh(d*x + c)^2 + 4*(7*d*cosh(d*x + c)^6 + 15*d*cosh(d*x + c)
^4 + 9*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 8*(d*cosh(d*x + c)^7 + 3*d*
cosh(d*x + c)^5 + 3*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

giac [B] time = 0.14, size = 170, normalized size = 1.89

$$\frac{\left(\pi + 2\,\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)} - 1\right)e^{(-dx-c)}\right)\right)\left(8\,a^2 + 8\,ab + 3\,b^2\right) + \frac{4\left(8\,ab\left(e^{(dx+c)} - e^{(-dx-c)}\right)^3 + 3\,b^2\left(e^{(dx+c)} - e^{(-dx-c)}\right)^3 + 32\,ab\left(e^{(dx+c)} - e\right.\right.}{\left(\left(e^{(dx+c)} - e^{(-dx-c)}\right)^2 + 4\right)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/16*((pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(8*a^2 + 8*a*
b + 3*b^2) + 4*(8*a*b*(e^(d*x + c) - e^(-d*x - c))^3 + 3*b^2*(e^(d*x + c) -
 e^(-d*x - c))^3 + 32*a*b*(e^(d*x + c) - e^(-d*x - c)) + 20*b^2*(e^(d*x + c
) - e^(-d*x - c)))/((e^(d*x + c) - e^(-d*x - c))^2 + 4)^2)/d
```

maple [A] time = 0.38, size = 106, normalized size = 1.18

$$\frac{2a^2\arctan\left(e^{dx+c}\right)}{d}+\frac{ab\,\mathrm{sech}\,(dx+c)\tanh\,(dx+c)}{d}+\frac{2ab\arctan\left(e^{dx+c}\right)}{d}+\frac{b^2\tanh\,(dx+c)\,\mathrm{sech}\,(dx+c)^3}{4d}+\frac{3b^2\mathrm{se}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)*(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] 2/d*a^2*arctan(exp(d*x+c))+1/d*a*b*sech(d*x+c)*tanh(d*x+c)+2/d*a*b*arctan(e
xp(d*x+c))+1/4/d*b^2*tanh(d*x+c)*sech(d*x+c)^3+3/8/d*b^2*sech(d*x+c)*tanh(d
*x+c)+3/4/d*b^2*arctan(exp(d*x+c))
```

maxima [B] time = 0.42, size = 201, normalized size = 2.23

$$-\frac{1}{4}b^2\left(\frac{3\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{3e^{(-dx-c)}+11e^{(-3dx-3c)}-11e^{(-5dx-5c)}-3e^{(-7dx-7c)}}{d\left(4e^{(-2dx-2c)}+6e^{(-4dx-4c)}+4e^{(-6dx-6c)}+e^{(-8dx-8c)}+1\right)}\right)-2ab\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/4*b^2*(3*arctan(e^(-d*x - c))/d - (3*e^(-d*x - c) + 11*e^(-3*d*x - 3*c)
- 11*e^(-5*d*x - 5*c) - 3*e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-
4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) - 2*a*b*(arctan
(e^(-d*x - c))/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c)
 + e^(-4*d*x - 4*c) + 1))) + a^2*arctan(sinh(d*x + c))/d
```

mupad [B] time = 1.52, size = 303, normalized size = 3.37

$$\frac{\mathrm{atan}\left(\frac{e^{d\,x}\,e^c\,\left(8\,a^2\,\sqrt{d^2}+3\,b^2\,\sqrt{d^2}+8\,a\,b\,\sqrt{d^2}\right)}{d\,\sqrt{64\,a^4+128\,a^3\,b+112\,a^2\,b^2+48\,a\,b^3+9\,b^4}}\right)\,\sqrt{64\,a^4+128\,a^3\,b+112\,a^2\,b^2+48\,a\,b^3+9\,b^4}}{4\,\sqrt{d^2}}-\frac{6\,b^2\,e^{c+}}{d\,\left(3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cosh(c + d*x)^2)^2/cosh(c + d*x),x)
```

```
[Out] (atan((exp(d*x)*exp(c)*(8*a^2*(d^2)^(1/2) + 3*b^2*(d^2)^(1/2) + 8*a*b*(d^2)
^(1/2)))/(d*(48*a*b^3 + 128*a^3*b + 64*a^4 + 9*b^4 + 112*a^2*b^2)^(1/2)))*(
48*a*b^3 + 128*a^3*b + 64*a^4 + 9*b^4 + 112*a^2*b^2)^(1/2))/(4*(d^2)^(1/2))
 - (6*b^2*exp(c + d*x))/(d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6
*c + 6*d*x) + 1)) + (4*b^2*exp(c + d*x))/(d*(4*exp(2*c + 2*d*x) + 6*exp(4*c
 + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) + (exp(c + d*x)*(8*
a*b + 3*b^2))/(4*d*(exp(2*c + 2*d*x) + 1)) - (exp(c + d*x)*(8*a*b - b^2))/(
2*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\,\mathrm{sech}^2\left(c + dx\right)\right)^2 \mathrm{sech}\left(c + dx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**2*sech(c + d*x), x)
```

3.62 $\int \text{sech}^2(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=53

$$-\frac{2b(a+b)\tanh^3(c+dx)}{3d}+\frac{(a+b)^2\tanh(c+dx)}{d}+\frac{b^2\tanh^5(c+dx)}{5d}$$

```
[Out] (a+b)^2*tanh(d*x+c)/d-2/3*b*(a+b)*tanh(d*x+c)^3/d+1/5*b^2*tanh(d*x+c)^5/d
```

Rubi [A] time = 0.07, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4146, 194}

$$-\frac{2b(a+b)\tanh^3(c+dx)}{3d}+\frac{(a+b)^2\tanh(c+dx)}{d}+\frac{b^2\tanh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^2*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + b)^2*Tanh[c + d*x])/d - (2*b*(a + b)*Tanh[c + d*x]^3)/(3*d) + (b^2*Ta
nh[c + d*x]^5)/(5*d)
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \operatorname{sech}^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2 dx = \frac{\text{Subst}\left(\int \left(a+b-bx^2\right)^2 dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(a^2\left(1+\frac{b(2a+b)}{a^2}\right)-2ab\left(1+\frac{b}{a}\right)x^2+b^2x^4\right) dx, x, \tanh(c+ \ldots}{d}$$

$$= \frac{(a+b)^2\tanh(c+dx)}{d} - \frac{2b(a+b)\tanh^3(c+dx)}{3d} + \frac{b^2\tanh^5(c+dx)}{5d}$$

Mathematica [A] time = 0.03, size = 93, normalized size = 1.75

$$\frac{a^2\tanh(c+dx)}{d} - \frac{2ab\tanh^3(c+dx)}{3d} + \frac{2ab\tanh(c+dx)}{d} + \frac{b^2\tanh^5(c+dx)}{5d} - \frac{2b^2\tanh^3(c+dx)}{3d} + \frac{b^2\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^2*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (a^2*Tanh[c + d*x])/d + (2*a*b*Tanh[c + d*x])/d + (b^2*Tanh[c + d*x])/d - (
2*a*b*Tanh[c + d*x]^3)/(3*d) - (2*b^2*Tanh[c + d*x]^3)/(3*d) + (b^2*Tanh[c
+ d*x]^5)/(5*d)
```

fricas [B] time = 0.41, size = 404, normalized size = 7.62

$$-\frac{4\left(\left(15\,a^2+10\,ab+4\,b^2\right)\cosh\left(dx+c\right)^4-8\left(5\,ab+2\,b^2\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3+\left(15\,a^2+10 \ldots\right.\right.}{15\left(d\cosh\left(dx+c\right)^6+6\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^5+d\sinh\left(dx+c\right)^6+6\,d\cosh\left(dx+c\right)^4+3\left(5\,d\cosh\left(dx \ldots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -4/15*((15*a^2 + 10*a*b + 4*b^2)*cosh(d*x + c)^4 - 8*(5*a*b + 2*b^2)*cosh(d
*x + c)*sinh(d*x + c)^3 + (15*a^2 + 10*a*b + 4*b^2)*sinh(d*x + c)^4 + 20*(3
*a^2 + 4*a*b + b^2)*cosh(d*x + c)^2 + 2*(3*(15*a^2 + 10*a*b + 4*b^2)*cosh(d
*x + c)^2 + 30*a^2 + 40*a*b + 10*b^2)*sinh(d*x + c)^2 + 45*a^2 + 70*a*b + 4
0*b^2 - 8*((5*a*b + 2*b^2)*cosh(d*x + c)^3 + 5*(a*b + b^2)*cosh(d*x + c))*s
inh(d*x + c))/(d*cosh(d*x + c)^6 + 6*d*cosh(d*x + c)*sinh(d*x + c)^5 + d*si
nh(d*x + c)^6 + 6*d*cosh(d*x + c)^4 + 3*(5*d*cosh(d*x + c)^2 + 2*d)*sinh(d*
x + c)^4 + 4*(5*d*cosh(d*x + c)^3 + 4*d*cosh(d*x + c))*sinh(d*x + c)^3 + 15
*d*cosh(d*x + c)^2 + 3*(5*d*cosh(d*x + c)^4 + 12*d*cosh(d*x + c)^2 + 5*d)*s
inh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^5 + 8*d*cosh(d*x + c)^3 + 5*d*cosh(d*
x + c))*sinh(d*x + c) + 10*d)
```

giac [B] time = 0.17, size = 156, normalized size = 2.94

$$-\frac{2\left(15\,a^2e^{(8\,dx+8\,c)}+60\,a^2e^{(6\,dx+6\,c)}+60\,abe^{(6\,dx+6\,c)}+90\,a^2e^{(4\,dx+4\,c)}+140\,abe^{(4\,dx+4\,c)}+80\,b^2e^{(4\,dx+4\,c)}+60\,a^2e^{(2\cdots}}{15\,d\left(e^{(2\,dx+2\,c)}+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -2/15*(15*a^2*e^(8*d*x + 8*c) + 60*a^2*e^(6*d*x + 6*c) + 60*a*b*e^(6*d*x +
6*c) + 90*a^2*e^(4*d*x + 4*c) + 140*a*b*e^(4*d*x + 4*c) + 80*b^2*e^(4*d*x +
 4*c) + 60*a^2*e^(2*d*x + 2*c) + 100*a*b*e^(2*d*x + 2*c) + 40*b^2*e^(2*d*x
+ 2*c) + 15*a^2 + 20*a*b + 8*b^2)/(d*(e^(2*d*x + 2*c) + 1)^5)
```

maple [A] time = 0.42, size = 70, normalized size = 1.32

$$\frac{a^2\tanh\left(dx+c\right)+2ab\left(\frac{2}{3}+\frac{\mathrm{sech}(dx+c)^2}{3}\right)\tanh\left(dx+c\right)+b^2\left(\frac{8}{15}+\frac{\mathrm{sech}(dx+c)^4}{5}+\frac{4\mathrm{sech}(dx+c)^2}{15}\right)\tanh\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(a^2*tanh(d*x+c)+2*a*b*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c)+b^2*(8/15+1/
5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)*tanh(d*x+c))
```

maxima [B] time = 0.33, size = 324, normalized size = 6.11

$$\frac{16}{15}\,b^2\left(\frac{5\,e^{(-2\,dx-2\,c)}}{d\left(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}+5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}+1\right)}+\frac{\cdots}{d\left(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx\cdots}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 16/15*b^2*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c)
+ 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 10*
e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*
x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*d*
x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) +
 e^(-10*d*x - 10*c) + 1))) + 8/3*a*b*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x -
2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*
c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + 2*a^2/(d*(e^(-2*d*x - 2
*c) + 1))
```

mupad [B] time = 1.46, size = 452, normalized size = 8.53

$$-\frac{\frac{2a(a+2b)}{5d}+\frac{2a^2e^{2c+2dx}}{5d}}{2e^{2c+2dx}+e^{4c+4dx}+1}-\frac{\frac{2a^2}{5d}+\frac{2a^2e^{8c+8dx}}{5d}+\frac{4e^{4c+4dx}\left(3a^2+8ab+8b^2\right)}{5d}+\frac{8ae^{2c+2dx}(a+2b)}{5d}+\frac{8ae^{6c+6dx}(a+2b)}{5d}}{5e^{2c+2dx}+10e^{4c+4dx}+10e^{6c+6dx}+5e^{8c+8dx}+e^{10c+10dx}+1}-\frac{\frac{2a(a+2b)}{5d}\cdots}{4\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cosh(c + d*x)^2)^2/cosh(c + d*x)^2,x)
```

```
[Out] - ((2*a*(a + 2*b))/(5*d) + (2*a^2*exp(2*c + 2*d*x))/(5*d))/(2*exp(2*c + 2*d
*x) + exp(4*c + 4*d*x) + 1) - ((2*a^2)/(5*d) + (2*a^2*exp(8*c + 8*d*x))/(5*
d) + (4*exp(4*c + 4*d*x)*(8*a*b + 3*a^2 + 8*b^2))/(5*d) + (8*a*exp(2*c + 2*
d*x)*(a + 2*b))/(5*d) + (8*a*exp(6*c + 6*d*x)*(a + 2*b))/(5*d))/(5*exp(2*c
+ 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) +
 exp(10*c + 10*d*x) + 1) - ((2*a*(a + 2*b))/(5*d) + (2*a^2*exp(6*c + 6*d*x)
)/(5*d) + (2*exp(2*c + 2*d*x)*(8*a*b + 3*a^2 + 8*b^2))/(5*d) + (6*a*exp(4*c
 + 4*d*x)*(a + 2*b))/(5*d))/(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*ex
p(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) - ((2*(8*a*b + 3*a^2 + 8*b^2))/(15*d
) + (2*a^2*exp(4*c + 4*d*x))/(5*d) + (4*a*exp(2*c + 2*d*x)*(a + 2*b))/(5*d)
)/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) - (2*a^2
)/(5*d*(exp(2*c + 2*d*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\operatorname{sech}^2(c+dx)\right)^2 \operatorname{sech}^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**2*(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**2*sech(c + d*x)**2, x)
```

3.63 $\int \text{sech}^3(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=128

$$\frac{\left(8a^2+12ab+5b^2\right)\tan^{-1}(\sinh(c+dx))}{16d}+\frac{\left(8a^2+12ab+5b^2\right)\tanh(c+dx)\text{sech}(c+dx)}{16d}+\frac{b(8a+5b)\tanh(c+d}{24d}$$

```
[Out] 1/16*(8*a^2+12*a*b+5*b^2)*arctan(sinh(d*x+c))/d+1/16*(8*a^2+12*a*b+5*b^2)*s
ech(d*x+c)*tanh(d*x+c)/d+1/24*b*(8*a+5*b)*sech(d*x+c)^3*tanh(d*x+c)/d+1/6*b
*sech(d*x+c)^5*(a+b+a*sinh(d*x+c)^2)*tanh(d*x+c)/d
```

Rubi [A] time = 0.15, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4147, 413, 385, 199, 203}

$$\frac{\left(8a^2+12ab+5b^2\right)\tan^{-1}(\sinh(c+dx))}{16d}+\frac{\left(8a^2+12ab+5b^2\right)\tanh(c+dx)\text{sech}(c+dx)}{16d}+\frac{b(8a+5b)\tanh(c+d}{24d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^3*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((8*a^2 + 12*a*b + 5*b^2)*ArcTan[Sinh[c + d*x]])/(16*d) + ((8*a^2 + 12*a*b
+ 5*b^2)*Sech[c + d*x]*Tanh[c + d*x])/(16*d) + (b*(8*a + 5*b)*Sech[c + d*x]
^3*Tanh[c + d*x])/(24*d) + (b*Sech[c + d*x]^5*(a + b + a*Sinh[c + d*x]^2)*T
anh[c + d*x])/(6*d)
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
```

```
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \operatorname{sech}^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx = \frac{\operatorname{Subst}\left(\int \frac{\left(a+b+ax^2\right)^2}{\left(1+x^2\right)^4}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{b\operatorname{sech}^5(c+dx)\left(a+b+a\sinh^2(c+dx)\right)\tanh(c+dx)}{6d} + \frac{\operatorname{Subst}\left(\int \right.}{}$$

$$= \frac{b(8a+5b)\operatorname{sech}^3(c+dx)\tanh(c+dx)}{24d} + \frac{b\operatorname{sech}^5(c+dx)\left(a+b+a\,\mathrm{s}\right.}{6}$$

$$= \frac{\left(8a^2+12ab+5b^2\right)\operatorname{sech}(c+dx)\tanh(c+dx)}{16d} + \frac{b(8a+5b)\operatorname{sech}^3(c}{2}$$

$$= \frac{\left(8a^2+12ab+5b^2\right)\tan^{-1}(\sinh(c+dx))}{16d} + \frac{\left(8a^2+12ab+5b^2\right)\operatorname{sech}}{16d}$$

Mathematica [A] time = 0.24, size = 104, normalized size = 0.81

$$\frac{3\left(8a^2+12ab+5b^2\right)\tan^{-1}(\sinh(c+dx))+3\left(8a^2+12ab+5b^2\right)\tanh(c+dx)\text{sech}(c+dx)+2b(12a+5b)\tanh}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^3*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (3*(8*a^2 + 12*a*b + 5*b^2)*ArcTan[Sinh[c + d*x]] + 3*(8*a^2 + 12*a*b + 5*b
^2)*Sech[c + d*x]*Tanh[c + d*x] + 2*b*(12*a + 5*b)*Sech[c + d*x]^3*Tanh[c +
 d*x] + 8*b^2*Sech[c + d*x]^5*Tanh[c + d*x])/(48*d)
```

fricas [B] time = 0.43, size = 2946, normalized size = 23.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(3*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^11 + 33*(8*a^2 + 12*a*b + 5*
b^2)*cosh(d*x + c)*sinh(d*x + c)^10 + 3*(8*a^2 + 12*a*b + 5*b^2)*sinh(d*x +
 c)^11 + (72*a^2 + 204*a*b + 85*b^2)*cosh(d*x + c)^9 + (165*(8*a^2 + 12*a*b
 + 5*b^2)*cosh(d*x + c)^2 + 72*a^2 + 204*a*b + 85*b^2)*sinh(d*x + c)^9 + 9*
(55*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^3 + (72*a^2 + 204*a*b + 85*b^2)*
cosh(d*x + c))*sinh(d*x + c)^8 + 6*(8*a^2 + 28*a*b + 33*b^2)*cosh(d*x + c)^
7 + 6*(165*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^4 + 6*(72*a^2 + 204*a*b +
 85*b^2)*cosh(d*x + c)^2 + 8*a^2 + 28*a*b + 33*b^2)*sinh(d*x + c)^7 + 42*(3
3*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^5 + 2*(72*a^2 + 204*a*b + 85*b^2)*
cosh(d*x + c)^3 + (8*a^2 + 28*a*b + 33*b^2)*cosh(d*x + c))*sinh(d*x + c)^6
- 6*(8*a^2 + 28*a*b + 33*b^2)*cosh(d*x + c)^5 + 6*(231*(8*a^2 + 12*a*b + 5*
b^2)*cosh(d*x + c)^6 + 21*(72*a^2 + 204*a*b + 85*b^2)*cosh(d*x + c)^4 + 21*
(8*a^2 + 28*a*b + 33*b^2)*cosh(d*x + c)^2 - 8*a^2 - 28*a*b - 33*b^2)*sinh(d
*x + c)^5 + 6*(165*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^7 + 21*(72*a^2 +
204*a*b + 85*b^2)*cosh(d*x + c)^5 + 35*(8*a^2 + 28*a*b + 33*b^2)*cosh(d*x +
 c)^3 - 5*(8*a^2 + 28*a*b + 33*b^2)*cosh(d*x + c))*sinh(d*x + c)^4 - (72*a^
2 + 204*a*b + 85*b^2)*cosh(d*x + c)^3 + (495*(8*a^2 + 12*a*b + 5*b^2)*cosh(
d*x + c)^8 + 84*(72*a^2 + 204*a*b + 85*b^2)*cosh(d*x + c)^6 + 210*(8*a^2 +
28*a*b + 33*b^2)*cosh(d*x + c)^4 - 60*(8*a^2 + 28*a*b + 33*b^2)*cosh(d*x +
c)^2 - 72*a^2 - 204*a*b - 85*b^2)*sinh(d*x + c)^3 + 3*(55*(8*a^2 + 12*a*b +
 5*b^2)*cosh(d*x + c)^9 + 12*(72*a^2 + 204*a*b + 85*b^2)*cosh(d*x + c)^7 +
42*(8*a^2 + 28*a*b + 33*b^2)*cosh(d*x + c)^5 - 20*(8*a^2 + 28*a*b + 33*b^2)
*cosh(d*x + c)^3 - (72*a^2 + 204*a*b + 85*b^2)*cosh(d*x + c))*sinh(d*x + c)
^2 + 3*((8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^12 + 12*(8*a^2 + 12*a*b + 5*
b^2)*cosh(d*x + c)*sinh(d*x + c)^11 + (8*a^2 + 12*a*b + 5*b^2)*sinh(d*x + c
```

)^12 + 6*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^10 + 6*(11*(8*a^2 + 12*a*b
+ 5*b^2)*cosh(d*x + c)^2 + 8*a^2 + 12*a*b + 5*b^2)*sinh(d*x + c)^10 + 20*(1
1*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^3 + 3*(8*a^2 + 12*a*b + 5*b^2)*cos
h(d*x + c))*sinh(d*x + c)^9 + 15*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^8 +
 15*(33*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^4 + 18*(8*a^2 + 12*a*b + 5*b
^2)*cosh(d*x + c)^2 + 8*a^2 + 12*a*b + 5*b^2)*sinh(d*x + c)^8 + 24*(33*(8*a
^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^5 + 30*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x
 + c)^3 + 5*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 + 20*(8
*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^6 + 4*(231*(8*a^2 + 12*a*b + 5*b^2)*co
sh(d*x + c)^6 + 315*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^4 + 105*(8*a^2 +
 12*a*b + 5*b^2)*cosh(d*x + c)^2 + 40*a^2 + 60*a*b + 25*b^2)*sinh(d*x + c)^
6 + 24*(33*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^7 + 63*(8*a^2 + 12*a*b +
5*b^2)*cosh(d*x + c)^5 + 35*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^3 + 5*(8
*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 15*(8*a^2 + 12*a*b
+ 5*b^2)*cosh(d*x + c)^4 + 15*(33*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^8
+ 84*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^6 + 70*(8*a^2 + 12*a*b + 5*b^2)
*cosh(d*x + c)^4 + 20*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^2 + 8*a^2 + 12
*a*b + 5*b^2)*sinh(d*x + c)^4 + 20*(11*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x +
c)^9 + 36*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^7 + 42*(8*a^2 + 12*a*b + 5
*b^2)*cosh(d*x + c)^5 + 20*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^3 + 3*(8*
a^2 + 12*a*b + 5*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 6*(8*a^2 + 12*a*b +
5*b^2)*cosh(d*x + c)^2 + 6*(11*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^10 +
45*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^8 + 70*(8*a^2 + 12*a*b + 5*b^2)*c
osh(d*x + c)^6 + 50*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^4 + 15*(8*a^2 +
12*a*b + 5*b^2)*cosh(d*x + c)^2 + 8*a^2 + 12*a*b + 5*b^2)*sinh(d*x + c)^2 +
 8*a^2 + 12*a*b + 5*b^2 + 12*((8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^11 + 5
*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^9 + 10*(8*a^2 + 12*a*b + 5*b^2)*cos
h(d*x + c)^7 + 10*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^5 + 5*(8*a^2 + 12*
a*b + 5*b^2)*cosh(d*x + c)^3 + (8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c))*sinh
(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) - 3*(8*a^2 + 12*a*b + 5*b^
2)*cosh(d*x + c) + 3*(11*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^10 + 3*(72*
a^2 + 204*a*b + 85*b^2)*cosh(d*x + c)^8 + 14*(8*a^2 + 28*a*b + 33*b^2)*cosh
(d*x + c)^6 - 10*(8*a^2 + 28*a*b + 33*b^2)*cosh(d*x + c)^4 - (72*a^2 + 204*
a*b + 85*b^2)*cosh(d*x + c)^2 - 8*a^2 - 12*a*b - 5*b^2)*sinh(d*x + c))/(d*c
osh(d*x + c)^12 + 12*d*cosh(d*x + c)*sinh(d*x + c)^11 + d*sinh(d*x + c)^12
+ 6*d*cosh(d*x + c)^10 + 6*(11*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^10 + 20
*(11*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^9 + 15*d*cosh(d*x
 + c)^8 + 15*(33*d*cosh(d*x + c)^4 + 18*d*cosh(d*x + c)^2 + d)*sinh(d*x + c
)^8 + 24*(33*d*cosh(d*x + c)^5 + 30*d*cosh(d*x + c)^3 + 5*d*cosh(d*x + c))*
sinh(d*x + c)^7 + 20*d*cosh(d*x + c)^6 + 4*(231*d*cosh(d*x + c)^6 + 315*d*c
osh(d*x + c)^4 + 105*d*cosh(d*x + c)^2 + 5*d)*sinh(d*x + c)^6 + 24*(33*d*co
sh(d*x + c)^7 + 63*d*cosh(d*x + c)^5 + 35*d*cosh(d*x + c)^3 + 5*d*cosh(d*x
+ c))*sinh(d*x + c)^5 + 15*d*cosh(d*x + c)^4 + 15*(33*d*cosh(d*x + c)^8 + 8
4*d*cosh(d*x + c)^6 + 70*d*cosh(d*x + c)^4 + 20*d*cosh(d*x + c)^2 + d)*sinh
(d*x + c)^4 + 20*(11*d*cosh(d*x + c)^9 + 36*d*cosh(d*x + c)^7 + 42*d*cosh(d

```
*x + c)^5 + 20*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^3 + 6*d
*cosh(d*x + c)^2 + 6*(11*d*cosh(d*x + c)^10 + 45*d*cosh(d*x + c)^8 + 70*d*c
osh(d*x + c)^6 + 50*d*cosh(d*x + c)^4 + 15*d*cosh(d*x + c)^2 + d)*sinh(d*x
+ c)^2 + 12*(d*cosh(d*x + c)^11 + 5*d*cosh(d*x + c)^9 + 10*d*cosh(d*x + c)^
7 + 10*d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x
+ c) + d)
```

giac [B] time = 0.16, size = 293, normalized size = 2.29

$$\frac{3\left(\pi+2\arctan\left(\frac{1}{2}\left(e^{(2dx+2c)}-1\right)e^{(-dx-c)}\right)\right)\left(8a^2+12ab+5b^2\right)+\frac{4\left(24a^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)^5+36ab\left(e^{(dx+c)}-e^{(-dx-c)}\right)^5+15b^2\left(e\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/96*(3*(pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(8*a^2 + 12
*a*b + 5*b^2) + 4*(24*a^2*(e^(d*x + c) - e^(-d*x - c))^5 + 36*a*b*(e^(d*x +
 c) - e^(-d*x - c))^5 + 15*b^2*(e^(d*x + c) - e^(-d*x - c))^5 + 192*a^2*(e^
(d*x + c) - e^(-d*x - c))^3 + 384*a*b*(e^(d*x + c) - e^(-d*x - c))^3 + 160*
b^2*(e^(d*x + c) - e^(-d*x - c))^3 + 384*a^2*(e^(d*x + c) - e^(-d*x - c)) +
 960*a*b*(e^(d*x + c) - e^(-d*x - c)) + 528*b^2*(e^(d*x + c) - e^(-d*x - c)
))/((e^(d*x + c) - e^(-d*x - c))^2 + 4)^3)/d
```

maple [A] time = 0.46, size = 169, normalized size = 1.32

$$\frac{a^2\operatorname{sech}(dx+c)\tanh(dx+c)}{2d}+\frac{a^2\arctan\left(e^{dx+c}\right)}{d}+\frac{ab\tanh(dx+c)\operatorname{sech}(dx+c)^3}{2d}+\frac{3ab\operatorname{sech}(dx+c)\tanh(dx+}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] 1/2/d*a^2*sech(d*x+c)*tanh(d*x+c)+1/d*a^2*arctan(exp(d*x+c))+1/2/d*a*b*tanh
(d*x+c)*sech(d*x+c)^3+3/4/d*a*b*sech(d*x+c)*tanh(d*x+c)+3/2/d*a*b*arctan(ex
p(d*x+c))+1/6/d*b^2*tanh(d*x+c)*sech(d*x+c)^5+5/24/d*b^2*tanh(d*x+c)*sech(d
*x+c)^3+5/16/d*b^2*sech(d*x+c)*tanh(d*x+c)+5/8/d*b^2*arctan(exp(d*x+c))
```

maxima [B] time = 0.42, size = 348, normalized size = 2.72

$$-\frac{1}{24}b^2\left(\frac{15\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{15e^{(-dx-c)}+85e^{(-3dx-3c)}+198e^{(-5dx-5c)}-198e^{(-7dx-7c)}-85e^{(-9dx-9c)}-15e}{d\left(6e^{(-2dx-2c)}+15e^{(-4dx-4c)}+20e^{(-6dx-6c)}+15e^{(-8dx-8c)}+6e^{(-10dx-10c)}+e^{(-1}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/24*b^2*(15*arctan(e^(-d*x - c))/d - (15*e^(-d*x - c) + 85*e^(-3*d*x - 3*
c) + 198*e^(-5*d*x - 5*c) - 198*e^(-7*d*x - 7*c) - 85*e^(-9*d*x - 9*c) - 15
*e^(-11*d*x - 11*c))/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x - 4*c) + 20*e^(-
6*d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) + e^(-12*d*x - 12
*c) + 1))) - 1/2*a*b*(3*arctan(e^(-d*x - c))/d - (3*e^(-d*x - c) + 11*e^(-3
*d*x - 3*c) - 11*e^(-5*d*x - 5*c) - 3*e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2
*c) + 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) - a
^2*(arctan(e^(-d*x - c))/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*
d*x - 2*c) + e^(-4*d*x - 4*c) + 1)))
```

mupad [B] time = 1.57, size = 569, normalized size = 4.45

$$\frac{\operatorname{atan}\left(\frac{e^{dx}\,e^{c}\left(8\,a^2\,\sqrt{d^2}+5\,b^2\,\sqrt{d^2}+12\,a\,b\,\sqrt{d^2}\right)}{d\,\sqrt{64\,a^4+192\,a^3\,b+224\,a^2\,b^2+120\,a\,b^3+25\,b^4}}\right)\sqrt{64\,a^4+192\,a^3\,b+224\,a^2\,b^2+120\,a\,b^3+25\,b^4}}{8\,\sqrt{d^2}}-\frac{\frac{2\,a^2\,e^{c+d\,x}}{3\,d}+\frac{2\,a^2\,e^{9\,c+9\,d\,x}}{3\,d}}{6\,e^{2\,c+2\,d\,x}+15\,e^{4\,c+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cosh(c + d*x)^2)^2/cosh(c + d*x)^3,x)
```

```
[Out] (atan((exp(d*x)*exp(c)*(8*a^2*(d^2)^(1/2) + 5*b^2*(d^2)^(1/2) + 12*a*b*(d^2
)^(1/2)))/(d*(120*a*b^3 + 192*a^3*b + 64*a^4 + 25*b^4 + 224*a^2*b^2)^(1/2))
)*(120*a*b^3 + 192*a^3*b + 64*a^4 + 25*b^4 + 224*a^2*b^2)^(1/2))/(8*(d^2)^(
1/2)) - ((2*a^2*exp(c + d*x))/(3*d) + (2*a^2*exp(9*c + 9*d*x))/(3*d) + (4*e
xp(5*c + 5*d*x)*(8*a*b + 3*a^2 + 8*b^2))/(3*d) + (8*a*exp(3*c + 3*d*x)*(a +
 2*b))/(3*d) + (8*a*exp(7*c + 7*d*x)*(a + 2*b))/(3*d))/(6*exp(2*c + 2*d*x)
+ 15*exp(4*c + 4*d*x) + 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*exp(1
0*c + 10*d*x) + exp(12*c + 12*d*x) + 1) + (2*exp(c + d*x)*(4*a*b - 11*b^2))
/(3*d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8
*c + 8*d*x) + 1)) + (16*b^2*exp(c + d*x))/(3*d*(5*exp(2*c + 2*d*x) + 10*exp
(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*
x) + 1)) + (exp(c + d*x)*(12*a*b + 8*a^2 + 5*b^2))/(8*d*(exp(2*c + 2*d*x) +
 1)) + (exp(c + d*x)*(12*a*b - 16*a^2 + 5*b^2))/(12*d*(2*exp(2*c + 2*d*x) +
 exp(4*c + 4*d*x) + 1)) - (exp(c + d*x)*(20*a*b - b^2))/(3*d*(3*exp(2*c + 2
*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\,\mathrm{sech}^2\left(c + dx\right)\right)^2 \mathrm{sech}^3\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**3*(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**2*sech(c + d*x)**3, x)
```

3.64 $\int \text{sech}^4(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=80

$$\frac{b(2a+3b)\tanh^5(c+dx)}{5d}-\frac{(a+b)(a+3b)\tanh^3(c+dx)}{3d}+\frac{(a+b)^2\tanh(c+dx)}{d}-\frac{b^2\tanh^7(c+dx)}{7d}$$

```
[Out] (a+b)^2*tanh(d*x+c)/d-1/3*(a+b)*(a+3*b)*tanh(d*x+c)^3/d+1/5*b*(2*a+3*b)*tan
h(d*x+c)^5/d-1/7*b^2*tanh(d*x+c)^7/d
```

Rubi [A] time = 0.08, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4146, 373}

$$\frac{b(2a+3b)\tanh^5(c+dx)}{5d}-\frac{(a+b)(a+3b)\tanh^3(c+dx)}{3d}+\frac{(a+b)^2\tanh(c+dx)}{d}-\frac{b^2\tanh^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^4*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + b)^2*Tanh[c + d*x])/d - ((a + b)*(a + 3*b)*Tanh[c + d*x]^3)/(3*d) + (
b*(2*a + 3*b)*Tanh[c + d*x]^5)/(5*d) - (b^2*Tanh[c + d*x]^7)/(7*d)
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \operatorname{sech}^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int\left(1-x^2\right)\left(a+b-bx^2\right)^2\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int\left((a+b)^2+(-a-3b)(a+b)x^2+b(2a+3b)x^4-b^2x^6\right)\,dx,\right.}{d}$$
$$= \frac{(a+b)^2\tanh(c+dx)}{d} - \frac{(a+b)(a+3b)\tanh^3(c+dx)}{3d} + \frac{b(2a+3b)}{}$$

Mathematica [A] time = 0.02, size = 144, normalized size = 1.80

$$-\frac{a^2\tanh^3(c+dx)}{3d}+\frac{a^2\tanh(c+dx)}{d}+\frac{2ab\tanh^5(c+dx)}{5d}-\frac{4ab\tanh^3(c+dx)}{3d}+\frac{2ab\tanh(c+dx)}{d}-\frac{b^2\tanh^7(c+dx}{7d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^4*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (a^2*Tanh[c + d*x])/d + (2*a*b*Tanh[c + d*x])/d + (b^2*Tanh[c + d*x])/d - (
a^2*Tanh[c + d*x]^3)/(3*d) - (4*a*b*Tanh[c + d*x]^3)/(3*d) - (b^2*Tanh[c +
d*x]^3)/d + (2*a*b*Tanh[c + d*x]^5)/(5*d) + (3*b^2*Tanh[c + d*x]^5)/(5*d) -
 (b^2*Tanh[c + d*x]^7)/(7*d)
```

fricas [B] time = 0.39, size = 677, normalized size = 8.46

$$-\frac{8\left(2\left(35a^2+14\right.\right.}{105\left(d\cosh(dx+c)^9+9d\cosh(dx+c)\sinh(dx+c)^8+d\sinh(dx+c)^9+7d\cosh(dx+c)^7+\left(36d\cosh(dx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -8/105*(2*(35*a^2 + 14*a*b + 6*b^2)*cosh(d*x + c)^5 + 10*(35*a^2 + 14*a*b +
 6*b^2)*cosh(d*x + c)*sinh(d*x + c)^4 + (35*a^2 - 28*a*b - 12*b^2)*sinh(d*x
 + c)^5 + 14*(25*a^2 + 34*a*b + 6*b^2)*cosh(d*x + c)^3 + (10*(35*a^2 - 28*a
*b - 12*b^2)*cosh(d*x + c)^2 + 105*a^2 + 84*a*b - 84*b^2)*sinh(d*x + c)^3 +
 2*(10*(35*a^2 + 14*a*b + 6*b^2)*cosh(d*x + c)^3 + 21*(25*a^2 + 34*a*b + 6*
b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 28*(25*a^2 + 46*a*b + 24*b^2)*cosh(d*
x + c) + (5*(35*a^2 - 28*a*b - 12*b^2)*cosh(d*x + c)^4 + 63*(5*a^2 + 4*a*b
- 4*b^2)*cosh(d*x + c)^2 + 70*a^2 + 112*a*b + 168*b^2)*sinh(d*x + c))/(d*co
sh(d*x + c)^9 + 9*d*cosh(d*x + c)*sinh(d*x + c)^8 + d*sinh(d*x + c)^9 + 7*d
*cosh(d*x + c)^7 + (36*d*cosh(d*x + c)^2 + 7*d)*sinh(d*x + c)^7 + 7*(12*d*c
osh(d*x + c)^3 + 7*d*cosh(d*x + c))*sinh(d*x + c)^6 + 22*d*cosh(d*x + c)^5
```

```
+ (126*d*cosh(d*x + c)^4 + 147*d*cosh(d*x + c)^2 + 20*d)*sinh(d*x + c)^5 +
(126*d*cosh(d*x + c)^5 + 245*d*cosh(d*x + c)^3 + 110*d*cosh(d*x + c))*sinh(
d*x + c)^4 + 42*d*cosh(d*x + c)^3 + (84*d*cosh(d*x + c)^6 + 245*d*cosh(d*x
+ c)^4 + 200*d*cosh(d*x + c)^2 + 28*d)*sinh(d*x + c)^3 + (36*d*cosh(d*x + c
)^7 + 147*d*cosh(d*x + c)^5 + 220*d*cosh(d*x + c)^3 + 126*d*cosh(d*x + c))*
sinh(d*x + c)^2 + 56*d*cosh(d*x + c) + (9*d*cosh(d*x + c)^8 + 49*d*cosh(d*x
 + c)^6 + 100*d*cosh(d*x + c)^4 + 84*d*cosh(d*x + c)^2 + 14*d)*sinh(d*x + c
))
```

giac [B] time = 0.16, size = 197, normalized size = 2.46

$$-\frac{4\left(105\,a^{2}e^{(10\,dx+10\,c)}+455\,a^{2}e^{(8\,dx+8\,c)}+560\,abe^{(8\,dx+8\,c)}+770\,a^{2}e^{(6\,dx+6\,c)}+1400\,abe^{(6\,dx+6\,c)}+840\,b^{2}e^{(6\,dx+6\,c)}\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -4/105*(105*a^2*e^(10*d*x + 10*c) + 455*a^2*e^(8*d*x + 8*c) + 560*a*b*e^(8*
d*x + 8*c) + 770*a^2*e^(6*d*x + 6*c) + 1400*a*b*e^(6*d*x + 6*c) + 840*b^2*e
^(6*d*x + 6*c) + 630*a^2*e^(4*d*x + 4*c) + 1176*a*b*e^(4*d*x + 4*c) + 504*b
^2*e^(4*d*x + 4*c) + 245*a^2*e^(2*d*x + 2*c) + 392*a*b*e^(2*d*x + 2*c) + 16
8*b^2*e^(2*d*x + 2*c) + 35*a^2 + 56*a*b + 24*b^2)/(d*(e^(2*d*x + 2*c) + 1)^
7)
```

maple [A] time = 0.44, size = 102, normalized size = 1.28

$$\frac{a^{2}\left(\frac{2}{3}+\frac{\mathrm{sech}(dx+c)^{2}}{3}\right)\tanh\left(dx+c\right)+2ab\left(\frac{8}{15}+\frac{\mathrm{sech}(dx+c)^{4}}{5}+\frac{4\mathrm{sech}(dx+c)^{2}}{15}\right)\tanh\left(dx+c\right)+b^{2}\left(\frac{16}{35}+\frac{\mathrm{sech}(dx+c)^{6}}{7}+\frac{6\mathrm{sec}\cdots}{}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(a^2*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c)+2*a*b*(8/15+1/5*sech(d*x+c)^4+
4/15*sech(d*x+c)^2)*tanh(d*x+c)+b^2*(16/35+1/7*sech(d*x+c)^6+6/35*sech(d*x+
c)^4+8/35*sech(d*x+c)^2)*tanh(d*x+c))
```

maxima [B] time = 0.33, size = 671, normalized size = 8.39

$$\frac{32}{35}\,b^{2}\left(\frac{7\,e^{(-2\,dx-2\,c)}}{d\left(7\,e^{(-2\,dx-2\,c)}+21\,e^{(-4\,dx-4\,c)}+35\,e^{(-6\,dx-6\,c)}+35\,e^{(-8\,dx-8\,c)}+21\,e^{(-10\,dx-10\,c)}+7\,e^{(-12\,dx-12\,c)}+e^{(-14\,dx-14\,c}\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 32/35*b^2*(7*e^(-2*d*x - 2*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c)
+ 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(
-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 21*e^(-4*d*x - 4*c)/(d*(7*e^(-
2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8
*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1
)) + 35*e^(-6*d*x - 6*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*
e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d
*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 1/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4
*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 1
0*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1))) + 32/15*a*b*(5*e^(-
2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x -
6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 10*e^(-4*d*x - 4*c)/
(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-
8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*d*x - 2*c) + 10*e^(
-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*
c) + 1))) + 4/3*a^2*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*
x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x
- 4*c) + e^(-6*d*x - 6*c) + 1)))
```

mupad [B] time = 1.42, size = 692, normalized size = 8.65

$$-\frac{\frac{32\,a\,(a+2\,b)}{105\,d}+\frac{8\,a^2\,e^{2\,c+2\,d\,x}}{21\,d}}{3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1}-\frac{\frac{8\,a^2\,e^{2\,c+2\,d\,x}}{7\,d}+\frac{8\,a^2\,e^{10\,c+10\,d\,x}}{7\,d}+\frac{16\,e^{6\,c+6\,d\,x}\left(3\,a^2+8\,a\,b+8\,b^2\right)}{7\,d}+\frac{32\,a\,e^{4\,c+4\,d\,x}\,(a+2\,b)}{7\,d}}{7\,e^{2\,c+2\,d\,x}+21\,e^{4\,c+4\,d\,x}+35\,e^{6\,c+6\,d\,x}+35\,e^{8\,c+8\,d\,x}+21\,e^{10\,c+10\,d\,x}+7\,e^{12\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cosh(c + d*x)^2)^2/cosh(c + d*x)^4,x)
```

```
[Out] - ((32*a*(a + 2*b))/(105*d) + (8*a^2*exp(2*c + 2*d*x))/(21*d))/(3*exp(2*c +
 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) - ((8*a^2*exp(2*c + 2*
d*x))/(7*d) + (8*a^2*exp(10*c + 10*d*x))/(7*d) + (16*exp(6*c + 6*d*x)*(8*a*
b + 3*a^2 + 8*b^2))/(7*d) + (32*a*exp(4*c + 4*d*x)*(a + 2*b))/(7*d) + (32*a
*exp(8*c + 8*d*x)*(a + 2*b))/(7*d))/(7*exp(2*c + 2*d*x) + 21*exp(4*c + 4*d*
x) + 35*exp(6*c + 6*d*x) + 35*exp(8*c + 8*d*x) + 21*exp(10*c + 10*d*x) + 7*
exp(12*c + 12*d*x) + exp(14*c + 14*d*x) + 1) - ((4*a^2)/(21*d) + (20*a^2*ex
p(8*c + 8*d*x))/(21*d) + (8*exp(4*c + 4*d*x)*(8*a*b + 3*a^2 + 8*b^2))/(7*d)
 + (32*a*exp(2*c + 2*d*x)*(a + 2*b))/(21*d) + (64*a*exp(6*c + 6*d*x)*(a + 2
*b))/(21*d))/(6*exp(2*c + 2*d*x) + 15*exp(4*c + 4*d*x) + 20*exp(6*c + 6*d*x
) + 15*exp(8*c + 8*d*x) + 6*exp(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1) -
((32*a*(a + 2*b))/(105*d) + (16*a^2*exp(6*c + 6*d*x))/(21*d) + (16*exp(2*c
+ 2*d*x)*(8*a*b + 3*a^2 + 8*b^2))/(35*d) + (64*a*exp(4*c + 4*d*x)*(a + 2*b)
)/(35*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) +
 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1) - ((4*(8*a*b + 3*a^2 + 8*b^2)
```

```
)/(35*d) + (4*a^2*exp(4*c + 4*d*x))/(7*d) + (32*a*exp(2*c + 2*d*x)*(a + 2*b
))/(35*d))/(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) +
exp(8*c + 8*d*x) + 1) - (4*a^2)/(21*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x
) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\,\mathrm{sech}^2\left(c + dx\right)\right)^2 \mathrm{sech}^4\left(c + dx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**4*(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**2*sech(c + d*x)**4, x)
```

3.65 $\int \cosh^4(c+dx)\left(a+b\,\mathbf{sech}^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=84

$$\frac{a^3\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{8}ax\left(a^2+4ab+8b^2\right)+\frac{3a^2(a+4b)\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{b^3\tanh(c+dx)}{d}$$

```
[Out] 3/8*a*(a^2+4*a*b+8*b^2)*x+3/8*a^2*(a+4*b)*cosh(d*x+c)*sinh(d*x+c)/d+1/4*a^3
*cosh(d*x+c)^3*sinh(d*x+c)/d+b^3*tanh(d*x+c)/d
```

Rubi [A] time = 0.11, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4146, 390, 1157, 385, 206}

$$\frac{3}{8}ax\left(a^2+4ab+8b^2\right)+\frac{3a^2(a+4b)\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{a^3\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{b^3\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^4*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (3*a*(a^2 + 4*a*b + 8*b^2)*x)/8 + (3*a^2*(a + 4*b)*Cosh[c + d*x]*Sinh[c + d
*x])/(8*d) + (a^3*Cosh[c + d*x]^3*Sinh[c + d*x])/(4*d) + (b^3*Tanh[c + d*x]
)/d
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \cosh^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx = \frac{\operatorname{Subst}\left(\int \frac{\left(a+b-bx^2\right)^3}{\left(1-x^2\right)^3}\,dx,x,\tanh(c+dx)\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \left(b^3+\frac{a\left(a^2+3ab+3b^2\right)-3ab(a+2b)x^2+3ab^2x^4}{\left(1-x^2\right)^3}\right)dx,x,\tanh(c+dx)\right.}{d}$$

$$= \frac{b^3\tanh(c+dx)}{d}+\frac{\operatorname{Subst}\left(\int \frac{a\left(a^2+3ab+3b^2\right)-3ab(a+2b)x^2+3ab^2x^4}{\left(1-x^2\right)^3}\,dx,x,\text{ta}\right.}{d}$$

$$= \frac{a^3\cosh^3(c+dx)\sinh(c+dx)}{4d}+\frac{b^3\tanh(c+dx)}{d}-\frac{\operatorname{Subst}\left(\int \frac{-3a(a}{}\right.}{}$$

$$= \frac{3a^2(a+4b)\cosh(c+dx)\sinh(c+dx)}{8d}+\frac{a^3\cosh^3(c+dx)\sinh(c}{4d}$$

$$= \frac{3}{8}a\left(a^2+4ab+8b^2\right)x+\frac{3a^2(a+4b)\cosh(c+dx)\sinh(c+dx)}{8d}+$$

Mathematica [A] time = 0.43, size = 70, normalized size = 0.83

$$\frac{a^3\sinh(4(c+dx))+12a\left(a^2+4ab+8b^2\right)(c+dx)+8a^2(a+3b)\sinh(2(c+dx))+32b^3\tanh(c+dx)}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^4*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (12*a*(a^2 + 4*a*b + 8*b^2)*(c + d*x) + 8*a^2*(a + 3*b)*Sinh[2*(c + d*x)] +
 a^3*Sinh[4*(c + d*x)] + 32*b^3*Tanh[c + d*x])/(32*d)
```

fricas [A] time = 0.40, size = 153, normalized size = 1.82

$$\frac{a^3\sinh(dx+c)^5+\left(10\,a^3\cosh(dx+c)^2+9\,a^3+24\,a^2b\right)\sinh(dx+c)^3-8\left(8\,b^3-3\left(a^3+4\,a^2b+8\,ab^2\right)dx\right)\cosh}{64\,d\cosh(dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(a^3*sinh(d*x + c)^5 + (10*a^3*cosh(d*x + c)^2 + 9*a^3 + 24*a^2*b)*sin
h(d*x + c)^3 - 8*(8*b^3 - 3*(a^3 + 4*a^2*b + 8*a*b^2)*d*x)*cosh(d*x + c) +
(5*a^3*cosh(d*x + c)^4 + 8*a^3 + 24*a^2*b + 64*b^3 + 9*(3*a^3 + 8*a^2*b)*co
sh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c))
```

giac [B] time = 0.21, size = 177, normalized size = 2.11

$$\frac{a^3e^{(4\,dx+4\,c)}+8\,a^3e^{(2\,dx+2\,c)}+24\,a^2be^{(2\,dx+2\,c)}+24\left(a^3+4\,a^2b+8\,ab^2\right)(dx+c)-\frac{128\,b^3}{e^{(2\,dx+2\,c)}+1}-\left(18\,a^3e^{(4\,dx+4\,c)}+72\,a\right.}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/64*(a^3*e^(4*d*x + 4*c) + 8*a^3*e^(2*d*x + 2*c) + 24*a^2*b*e^(2*d*x + 2*c
) + 24*(a^3 + 4*a^2*b + 8*a*b^2)*(d*x + c) - 128*b^3/(e^(2*d*x + 2*c) + 1)
- (18*a^3*e^(4*d*x + 4*c) + 72*a^2*b*e^(4*d*x + 4*c) + 144*a*b^2*e^(4*d*x +
 4*c) + 8*a^3*e^(2*d*x + 2*c) + 24*a^2*b*e^(2*d*x + 2*c) + a^3)*e^(-4*d*x -
 4*c))/d
```

maple [A] time = 0.48, size = 93, normalized size = 1.11

$$\frac{a^3\left(\left(\frac{\left(\cosh^3(dx+c)\right)}{4}+\frac{3\cosh(dx+c)}{8}\right)\sinh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+3a^2b\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3a\,b^2\,(dx+c)+b}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(a^3*((1/4*cosh(d*x+c)^3+3/8*cosh(d*x+c))*sinh(d*x+c)+3/8*d*x+3/8*c)+3*
a^2*b*(1/2*cosh(d*x+c)*sinh(d*x+c)+1/2*d*x+1/2*c)+3*a*b^2*(d*x+c)+b^3*tanh(
d*x+c))
```

maxima [A] time = 0.32, size = 130, normalized size = 1.55

$$\frac{1}{64}a^3\left(24x+\frac{e^{(4dx+4c)}}{d}+\frac{8e^{(2dx+2c)}}{d}-\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right)+\frac{3}{8}a^2b\left(4x+\frac{e^{(2dx+2c)}}{d}-\frac{e^{(-2dx-2c)}}{d}\right)+3ab^2x+\frac{}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/64*a^3*(24*x + e^(4*d*x + 4*c)/d + 8*e^(2*d*x + 2*c)/d - 8*e^(-2*d*x - 2*
c)/d - e^(-4*d*x - 4*c)/d) + 3/8*a^2*b*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d*x
 - 2*c)/d) + 3*a*b^2*x + 2*b^3/(d*(e^(-2*d*x - 2*c) + 1))
```

mupad [B] time = 1.53, size = 117, normalized size = 1.39

$$\frac{3ax\left(a^2+4ab+8b^2\right)}{8}-\frac{2b^3}{d\left(e^{2c+2dx}+1\right)}-\frac{a^3e^{-4c-4dx}}{64d}+\frac{a^3e^{4c+4dx}}{64d}-\frac{a^2e^{-2c-2dx}(a+3b)}{8d}+\frac{a^2e^{2c+2dx}(a+3b)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^4*(a + b/cosh(c + d*x)^2)^3,x)
```

```
[Out] (3*a*x*(4*a*b + a^2 + 8*b^2))/8 - (2*b^3)/(d*(exp(2*c + 2*d*x) + 1)) - (a^3
*exp(- 4*c - 4*d*x))/(64*d) + (a^3*exp(4*c + 4*d*x))/(64*d) - (a^2*exp(- 2*
c - 2*d*x)*(a + 3*b))/(8*d) + (a^2*exp(2*c + 2*d*x)*(a + 3*b))/(8*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**4*(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

3.66 $\int \cosh^3(c+dx)\left(a+b\,\mathbf{sech}^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=81

$$\frac{a^3\sinh^3(c+dx)}{3d}+\frac{a^2(a+3b)\sinh(c+dx)}{d}+\frac{b^2(6a+b)\tan^{-1}(\sinh(c+dx))}{2d}+\frac{b^3\tanh(c+dx)\text{sech}(c+dx)}{2d}$$

```
[Out] 1/2*b^2*(6*a+b)*arctan(sinh(d*x+c))/d+a^2*(a+3*b)*sinh(d*x+c)/d+1/3*a^3*sin
h(d*x+c)^3/d+1/2*b^3*sech(d*x+c)*tanh(d*x+c)/d
```

Rubi [A] time = 0.09, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4147, 390, 385, 203}

$$\frac{a^2(a+3b)\sinh(c+dx)}{d}+\frac{a^3\sinh^3(c+dx)}{3d}+\frac{b^2(6a+b)\tan^{-1}(\sinh(c+dx))}{2d}+\frac{b^3\tanh(c+dx)\text{sech}(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^3*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (b^2*(6*a + b)*ArcTan[Sinh[c + d*x]])/(2*d) + (a^2*(a + 3*b)*Sinh[c + d*x])
/d + (a^3*Sinh[c + d*x]^3)/(3*d) + (b^3*Sech[c + d*x]*Tanh[c + d*x])/(2*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \cosh^3(c+dx)\left(a+b\,\text{sech}^2(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b+ax^2\right)^3}{\left(1+x^2\right)^2}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(a^2(a+3b)+a^3x^2+\frac{b^2(3a+b)+3ab^2x^2}{\left(1+x^2\right)^2}\right)dx, x, \sinh(c+dx)\right.}{d}$$

$$= \frac{a^2(a+3b)\sinh(c+dx)}{d}+\frac{a^3\sinh^3(c+dx)}{3d}+\frac{\text{Subst}\left(\int \frac{b^2(3a+b)+3a}{\left(1+x^2\right)^2}\right.}{}$$

$$= \frac{a^2(a+3b)\sinh(c+dx)}{d}+\frac{a^3\sinh^3(c+dx)}{3d}+\frac{b^3\text{sech}(c+dx)\tanh}{2d}$$

$$= \frac{b^2(6a+b)\tan^{-1}(\sinh(c+dx))}{2d}+\frac{a^2(a+3b)\sinh(c+dx)}{d}+\frac{a^3\sin}{}$$

Mathematica [C] time = 6.88, size = 483, normalized size = 5.96

$$\frac{\coth^3(c+dx)\text{csch}^2(c+dx)(a\cosh(c+dx)+b\,\text{sech}(c+dx))^3\left(-256\sinh^8(c+dx)\left(a\sinh^2(c+dx)+a+b\right)^3\ {}_5F\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cosh[c + d*x]^3*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (Coth[c + d*x]^3*Csch[c + d*x]^2*(a*Cosh[c + d*x] + b*Sech[c + d*x])^3*(-25
6*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*S
inh[c + d*x]^8*(a + b + a*Sinh[c + d*x]^2)^3 - (315*ArcTanh[Sqrt[-Sinh[c +
d*x]^2]]*(b^3*(2401 + 1875*Sinh[c + d*x]^2 + 243*Sinh[c + d*x]^4 - 47*Sinh[
c + d*x]^6) + 3*a^2*b*Cosh[c + d*x]^4*(2401 + 1875*Sinh[c + d*x]^2 + 243*Si
```

```
nh[c + d*x]^4 + Sinh[c + d*x]^6) + a^3*Cosh[c + d*x]^6*(2401 + 1875*Sinh[c
+ d*x]^2 + 243*Sinh[c + d*x]^4 + Sinh[c + d*x]^6) + 3*a*b^2*(2401 + 4276*Si
nh[c + d*x]^2 + 2118*Sinh[c + d*x]^4 + 148*Sinh[c + d*x]^6 + Sinh[c + d*x]^
8)))/Sqrt[-Sinh[c + d*x]^2] + 21*(b^3*(36015 + 16120*Sinh[c + d*x]^2 + 1473
*Sinh[c + d*x]^4) + 3*a*b^2*(36015 + 52135*Sinh[c + d*x]^2 + 17593*Sinh[c +
 d*x]^4 + 753*Sinh[c + d*x]^6) + 3*a^2*b*(36015 + 88150*Sinh[c + d*x]^2 + 6
9728*Sinh[c + d*x]^4 + 19786*Sinh[c + d*x]^6 + 753*Sinh[c + d*x]^8) + a^3*(
36015 + 124165*Sinh[c + d*x]^2 + 157878*Sinh[c + d*x]^4 + 89514*Sinh[c + d*
x]^6 + 19579*Sinh[c + d*x]^8 + 753*Sinh[c + d*x]^10))))/(3780*d*(a + 2*b +
a*Cosh[2*c + 2*d*x])^3)
```

fricas [B] time = 0.42, size = 1409, normalized size = 17.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(a^3*cosh(d*x + c)^10 + 10*a^3*cosh(d*x + c)*sinh(d*x + c)^9 + a^3*sin
h(d*x + c)^10 + (11*a^3 + 36*a^2*b)*cosh(d*x + c)^8 + (45*a^3*cosh(d*x + c)
^2 + 11*a^3 + 36*a^2*b)*sinh(d*x + c)^8 + 8*(15*a^3*cosh(d*x + c)^3 + (11*a
^3 + 36*a^2*b)*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(5*a^3 + 18*a^2*b + 12*b^
3)*cosh(d*x + c)^6 + 2*(105*a^3*cosh(d*x + c)^4 + 5*a^3 + 18*a^2*b + 12*b^3
 + 14*(11*a^3 + 36*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(63*a^3*cosh
(d*x + c)^5 + 14*(11*a^3 + 36*a^2*b)*cosh(d*x + c)^3 + 3*(5*a^3 + 18*a^2*b
+ 12*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(5*a^3 + 18*a^2*b + 12*b^3)*co
sh(d*x + c)^4 + 2*(105*a^3*cosh(d*x + c)^6 + 35*(11*a^3 + 36*a^2*b)*cosh(d*
x + c)^4 - 5*a^3 - 18*a^2*b - 12*b^3 + 15*(5*a^3 + 18*a^2*b + 12*b^3)*cosh(
d*x + c)^2)*sinh(d*x + c)^4 + 8*(15*a^3*cosh(d*x + c)^7 + 7*(11*a^3 + 36*a^
2*b)*cosh(d*x + c)^5 + 5*(5*a^3 + 18*a^2*b + 12*b^3)*cosh(d*x + c)^3 - (5*a
^3 + 18*a^2*b + 12*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 - a^3 - (11*a^3 + 36
*a^2*b)*cosh(d*x + c)^2 + (45*a^3*cosh(d*x + c)^8 + 28*(11*a^3 + 36*a^2*b)*
cosh(d*x + c)^6 + 30*(5*a^3 + 18*a^2*b + 12*b^3)*cosh(d*x + c)^4 - 11*a^3 -
 36*a^2*b - 12*(5*a^3 + 18*a^2*b + 12*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2
 + 24*((6*a*b^2 + b^3)*cosh(d*x + c)^7 + 7*(6*a*b^2 + b^3)*cosh(d*x + c)*si
nh(d*x + c)^6 + (6*a*b^2 + b^3)*sinh(d*x + c)^7 + 2*(6*a*b^2 + b^3)*cosh(d*
x + c)^5 + (12*a*b^2 + 2*b^3 + 21*(6*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x
 + c)^5 + 5*(7*(6*a*b^2 + b^3)*cosh(d*x + c)^3 + 2*(6*a*b^2 + b^3)*cosh(d*x
 + c))*sinh(d*x + c)^4 + (6*a*b^2 + b^3)*cosh(d*x + c)^3 + (35*(6*a*b^2 + b
^3)*cosh(d*x + c)^4 + 6*a*b^2 + b^3 + 20*(6*a*b^2 + b^3)*cosh(d*x + c)^2)*s
inh(d*x + c)^3 + (21*(6*a*b^2 + b^3)*cosh(d*x + c)^5 + 20*(6*a*b^2 + b^3)*c
osh(d*x + c)^3 + 3*(6*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + (7*(6*a
*b^2 + b^3)*cosh(d*x + c)^6 + 10*(6*a*b^2 + b^3)*cosh(d*x + c)^4 + 3*(6*a*b
^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x +
 c)) + 2*(5*a^3*cosh(d*x + c)^9 + 4*(11*a^3 + 36*a^2*b)*cosh(d*x + c)^7 + 6
```

```
*(5*a^3 + 18*a^2*b + 12*b^3)*cosh(d*x + c)^5 - 4*(5*a^3 + 18*a^2*b + 12*b^3
)*cosh(d*x + c)^3 - (11*a^3 + 36*a^2*b)*cosh(d*x + c))*sinh(d*x + c))/(d*co
sh(d*x + c)^7 + 7*d*cosh(d*x + c)*sinh(d*x + c)^6 + d*sinh(d*x + c)^7 + 2*d
*cosh(d*x + c)^5 + (21*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c)^5 + 5*(7*d*co
sh(d*x + c)^3 + 2*d*cosh(d*x + c))*sinh(d*x + c)^4 + d*cosh(d*x + c)^3 + (3
5*d*cosh(d*x + c)^4 + 20*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^3 + (21*d*cos
h(d*x + c)^5 + 20*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 +
(7*d*cosh(d*x + c)^6 + 10*d*cosh(d*x + c)^4 + 3*d*cosh(d*x + c)^2)*sinh(d*x
 + c))
```

giac [B] time = 0.21, size = 163, normalized size = 2.01

$$\frac{a^3\left(e^{(dx+c)} - e^{(-dx-c)}\right)^3 + 12\,a^3\left(e^{(dx+c)} - e^{(-dx-c)}\right) + 36\,a^2b\left(e^{(dx+c)} - e^{(-dx-c)}\right) + \frac{24\,b^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)}{\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4} + 6\left(\pi + 2\,\arctan\right.}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/24*(a^3*(e^(d*x + c) - e^(-d*x - c))^3 + 12*a^3*(e^(d*x + c) - e^(-d*x -
c)) + 36*a^2*b*(e^(d*x + c) - e^(-d*x - c)) + 24*b^3*(e^(d*x + c) - e^(-d*x
 - c))/((e^(d*x + c) - e^(-d*x - c))^2 + 4) + 6*(pi + 2*arctan(1/2*(e^(2*d*
x + 2*c) - 1)*e^(-d*x - c)))*(6*a*b^2 + b^3))/d
```

maple [A] time = 0.45, size = 103, normalized size = 1.27

$$\frac{2a^3 \sinh(dx+c)}{3d} + \frac{a^3 \sinh(dx+c)\left(\cosh^2(dx+c)\right)}{3d} + \frac{3a^2b \sinh(dx+c)}{d} + \frac{6a\,b^2 \arctan\left(e^{dx+c}\right)}{d} + \frac{b^3 \mathrm{sech}(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] 2/3*a^3*sinh(d*x+c)/d+1/3/d*a^3*sinh(d*x+c)*cosh(d*x+c)^2+3/d*a^2*b*sinh(d*
x+c)+6/d*a*b^2*arctan(exp(d*x+c))+1/2/d*b^3*sech(d*x+c)*tanh(d*x+c)+1/d*b^3
*arctan(exp(d*x+c))
```

maxima [B] time = 0.42, size = 179, normalized size = 2.21

$$-b^3\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{e^{(-dx-c)} - e^{(-3\,dx-3\,c)}}{d\left(2\,e^{(-2\,dx-2\,c)} + e^{(-4\,dx-4\,c)} + 1\right)}\right) + \frac{1}{24}\,a^3\left(\frac{e^{(3\,dx+3\,c)}}{d} + \frac{9\,e^{(dx+c)}}{d} - \frac{9\,e^{(-dx-c)}}{d} - \frac{e^{(-3\,dx-3\,c)}}{d}\right) +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -b^3*(arctan(e^(-d*x - c))/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-
2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) + 1/24*a^3*(e^(3*d*x + 3*c)/d + 9*e^
(d*x + c)/d - 9*e^(-d*x - c)/d - e^(-3*d*x - 3*c)/d) + 3/2*a^2*b*(e^(d*x +
c)/d - e^(-d*x - c)/d) - 6*a*b^2*arctan(e^(-d*x - c))/d
```

mupad [B] time = 0.22, size = 218, normalized size = 2.69

$$\frac{\operatorname{atan}\left(\frac{e^{d\,x}\,e^{c}\,\left(b^3\,\sqrt{d^2}+6\,a\,b^2\,\sqrt{d^2}\right)}{d\,\sqrt{36\,a^2\,b^4+12\,a\,b^5+b^6}}\right)\,\sqrt{36\,a^2\,b^4+12\,a\,b^5+b^6}}{\sqrt{d^2}}-\frac{a^3\,e^{-3\,c-3\,d\,x}}{24\,d}+\frac{a^3\,e^{3\,c+3\,d\,x}}{24\,d}-\frac{3\,a^2\,e^{-c-d\,x}\,\left(a+4\,b\right)}{8\,d}+\frac{3\,a^2\,e^{c+d}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^3*(a + b/cosh(c + d*x)^2)^3,x)
```

```
[Out] (atan((exp(d*x)*exp(c)*(b^3*(d^2)^(1/2) + 6*a*b^2*(d^2)^(1/2)))/(d*(12*a*b^
5 + b^6 + 36*a^2*b^4)^(1/2)))*(12*a*b^5 + b^6 + 36*a^2*b^4)^(1/2))/(d^2)^(1
/2) - (a^3*exp(- 3*c - 3*d*x))/(24*d) + (a^3*exp(3*c + 3*d*x))/(24*d) - (3*
a^2*exp(- c - d*x)*(a + 4*b))/(8*d) + (3*a^2*exp(c + d*x)*(a + 4*b))/(8*d)
+ (b^3*exp(c + d*x))/(d*(exp(2*c + 2*d*x) + 1)) - (2*b^3*exp(c + d*x))/(d*(
2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3*(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

3.67 $\int \cosh^2(c+dx)\left(a+b\,\mathbf{sech}^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=72

$$\frac{a^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{1}{2}a^2x(a+6b)+\frac{b^2(3a+b)\tanh(c+dx)}{d}-\frac{b^3\tanh^3(c+dx)}{3d}$$

```
[Out] 1/2*a^2*(a+6*b)*x+1/2*a^3*cosh(d*x+c)*sinh(d*x+c)/d+b^2*(3*a+b)*tanh(d*x+c)
/d-1/3*b^3*tanh(d*x+c)^3/d
```

Rubi [A] time = 0.09, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4146, 390, 385, 206}

$$\frac{1}{2}a^2x(a+6b)+\frac{a^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{b^2(3a+b)\tanh(c+dx)}{d}-\frac{b^3\tanh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^2*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (a^2*(a + 6*b)*x)/2 + (a^3*Cosh[c + d*x]*Sinh[c + d*x])/(2*d) + (b^2*(3*a +
 b)*Tanh[c + d*x])/d - (b^3*Tanh[c + d*x]^3)/(3*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \cosh^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx=\frac{\operatorname{Subst}\left(\int\frac{\left(a+b-bx^2\right)^3}{\left(1-x^2\right)^2}\,dx,x,\tanh(c+dx)\right)}{d}$$

$$=\frac{\operatorname{Subst}\left(\int\left(b^2(3a+b)-b^3x^2+\frac{a^2(a+3b)-3a^2bx^2}{\left(1-x^2\right)^2}\right)dx,x,\tanh(c+dx)\right)}{d}$$

$$=\frac{b^2(3a+b)\tanh(c+dx)}{d}-\frac{b^3\tanh^3(c+dx)}{3d}+\frac{\operatorname{Subst}\left(\int\frac{a^2(a+3b)-3a^2\cdots}{\left(1-x^2\right)^2}\cdots\right.}{\cdots}$$

$$=\frac{a^3\cosh(c+dx)\sinh(c+dx)}{2d}+\frac{b^2(3a+b)\tanh(c+dx)}{d}-\frac{b^3\tanh^3\cdots}{3\cdots}$$

$$=\frac{1}{2}a^2(a+6b)x+\frac{a^3\cosh(c+dx)\sinh(c+dx)}{2d}+\frac{b^2(3a+b)\tanh(c+\cdots}{d}$$

Mathematica [A] time = 0.51, size = 64, normalized size = 0.89

$$\frac{3a^3\sinh(2(c+dx))+6a^2(a+6b)(c+dx)+4b^2\tanh(c+dx)\left(9a+b\operatorname{sech}^2(c+dx)+2b\right)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^2*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (6*a^2*(a + 6*b)*(c + d*x) + 3*a^3*Sinh[2*(c + d*x)] + 4*b^2*(9*a + 2*b + b
*Sech[c + d*x]^2)*Tanh[c + d*x])/(12*d)
```

fricas [B] time = 0.43, size = 270, normalized size = 3.75

$$\frac{3\,a^3\sinh(dx+c)^5-4\left(18\,ab^2+4\,b^3-3\left(a^3+6\,a^2b\right)dx\right)\cosh(dx+c)^3-12\left(18\,ab^2+4\,b^3-3\left(a^3+6\,a^2b\right)dx\right)\operatorname{co}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(3*a^3*sinh(d*x + c)^5 - 4*(18*a*b^2 + 4*b^3 - 3*(a^3 + 6*a^2*b)*d*x)*
cosh(d*x + c)^3 - 12*(18*a*b^2 + 4*b^3 - 3*(a^3 + 6*a^2*b)*d*x)*cosh(d*x +
c)*sinh(d*x + c)^2 + (30*a^3*cosh(d*x + c)^2 + 9*a^3 + 72*a*b^2 + 16*b^3)*s
inh(d*x + c)^3 - 12*(18*a*b^2 + 4*b^3 - 3*(a^3 + 6*a^2*b)*d*x)*cosh(d*x + c
) + 3*(5*a^3*cosh(d*x + c)^4 + 2*a^3 + 24*a*b^2 + 16*b^3 + (9*a^3 + 72*a*b^
2 + 16*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^3 + 3*d*cosh(d
*x + c)*sinh(d*x + c)^2 + 3*d*cosh(d*x + c))
```

giac [B] time = 0.20, size = 152, normalized size = 2.11

$$\frac{3\,a^3e^{(2\,dx+2\,c)}+12\left(a^3+6\,a^2b\right)(dx+c)-3\left(2\,a^3e^{(2\,dx+2\,c)}+12\,a^2be^{(2\,dx+2\,c)}+a^3\right)e^{(-2\,dx-2\,c)}-\frac{16\left(9\,ab^2e^{(4\,dx+4\,c)}+18\,ab^2e\cdots\right.}{\left(e^{\cdots}\right.}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/24*(3*a^3*e^(2*d*x + 2*c) + 12*(a^3 + 6*a^2*b)*(d*x + c) - 3*(2*a^3*e^(2*
d*x + 2*c) + 12*a^2*b*e^(2*d*x + 2*c) + a^3)*e^(-2*d*x - 2*c) - 16*(9*a*b^2
*e^(4*d*x + 4*c) + 18*a*b^2*e^(2*d*x + 2*c) + 6*b^3*e^(2*d*x + 2*c) + 9*a*b
^2 + 2*b^3)/(e^(2*d*x + 2*c) + 1)^3)/d
```

maple [A] time = 0.46, size = 77, normalized size = 1.07

$$\frac{a^3\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3a^2b\,(dx+c)+3a\,b^2\tanh\,(dx+c)+b^3\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh\,(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(a^3*(1/2*cosh(d*x+c)*sinh(d*x+c)+1/2*d*x+1/2*c)+3*a^2*b*(d*x+c)+3*a*b^
2*tanh(d*x+c)+b^3*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c))
```

maxima [B] time = 0.32, size = 160, normalized size = 2.22

$$\frac{1}{8}\,a^3\left(4\,x+\frac{e^{(2\,dx+2\,c)}}{d}-\frac{e^{(-2\,dx-2\,c)}}{d}\right)+3\,a^2bx+\frac{4}{3}\,b^3\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d\left(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1\right)}+\frac{\cdots}{d\left(3\,e^{(-2\,dx-2\,c)}\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*a^3*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d*x - 2*c)/d) + 3*a^2*b*x + 4/3*b^
3*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d
*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x
 - 6*c) + 1))) + 6*a*b^2/(d*(e^(-2*d*x - 2*c) + 1))
```

mupad [B] time = 0.17, size = 221, normalized size = 3.07

$$\frac{a^2\,x\,\left(a+6\,b\right)}{2}-\frac{\frac{2\,a\,b^2}{d}+\frac{4\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(2\,b^3+3\,a\,b^2\right)}{3\,d}+\frac{2\,a\,b^2\,{\mathrm{e}}^{4\,c+4\,d\,x}}{d}}{3\,{\mathrm{e}}^{2\,c+2\,d\,x}+3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}+1}-\frac{\frac{2\,\left(2\,b^3+3\,a\,b^2\right)}{3\,d}+\frac{2\,a\,b^2\,{\mathrm{e}}^{2\,c+2\,d\,x}}{d}}{2\,{\mathrm{e}}^{2\,c+2\,d\,x}+{\mathrm{e}}^{4\,c+4\,d\,x}+1}-\frac{a^3\,{\mathrm{e}}^{-2\,c-2\,d\,x}}{8\,d}+\frac{a^3\,{\mathrm{e}}^{2\,c+2\,d\,x}}{8\,d}-\frac{}{d\,\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^2*(a + b/cosh(c + d*x)^2)^3,x)
```

```
[Out] (a^2*x*(a + 6*b))/2 - ((2*a*b^2)/d + (4*exp(2*c + 2*d*x)*(3*a*b^2 + 2*b^3))
/(3*d) + (2*a*b^2*exp(4*c + 4*d*x))/d)/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*
d*x) + exp(6*c + 6*d*x) + 1) - ((2*(3*a*b^2 + 2*b^3))/(3*d) + (2*a*b^2*exp(
2*c + 2*d*x))/d)/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) - (a^3*exp(- 2
*c - 2*d*x))/(8*d) + (a^3*exp(2*c + 2*d*x))/(8*d) - (2*a*b^2)/(d*(exp(2*c +
 2*d*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2*(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

3.68 $\int \cosh(c+dx)\left(a+b\,\mathbf{sech}^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=93

$$\frac{a^3\sinh(c+dx)}{d}+\frac{3b\left(8a^2+4ab+b^2\right)\tan^{-1}(\sinh(c+dx))}{8d}+\frac{3b^2(4a+b)\tanh(c+dx)\mathrm{sech}(c+dx)}{8d}+\frac{b^3\tanh(c+$$

```
[Out] 3/8*b*(8*a^2+4*a*b+b^2)*arctan(sinh(d*x+c))/d+a^3*sinh(d*x+c)/d+3/8*b^2*(4*
a+b)*sech(d*x+c)*tanh(d*x+c)/d+1/4*b^3*sech(d*x+c)^3*tanh(d*x+c)/d
```

Rubi [A] time = 0.10, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4147, 390, 1157, 385, 203}

$$\frac{3b\left(8a^2+4ab+b^2\right)\tan^{-1}(\sinh(c+dx))}{8d}+\frac{a^3\sinh(c+dx)}{d}+\frac{3b^2(4a+b)\tanh(c+dx)\mathrm{sech}(c+dx)}{8d}+\frac{b^3\tanh(c+$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (3*b*(8*a^2 + 4*a*b + b^2)*ArcTan[Sinh[c + d*x]])/(8*d) + (a^3*Sinh[c + d*x
])/d + (3*b^2*(4*a + b)*Sech[c + d*x]*Tanh[c + d*x])/(8*d) + (b^3*Sech[c +
d*x]^3*Tanh[c + d*x])/(4*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
```

```
0] && GeQ[p, -q]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \cosh(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx=\frac{\text{Subst}\left(\int \frac{\left(a+b+ax^2\right)^3}{\left(1+x^2\right)^3}\,dx,x,\sinh(c+dx)\right)}{d}$$

$$=\frac{\text{Subst}\left(\int \left(a^3+\frac{b\left(3a^2+3ab+b^2\right)+3ab(2a+b)x^2+3a^2bx^4}{\left(1+x^2\right)^3}\right)dx,x,\sinh(c+dx)\right)}{d}$$

$$=\frac{a^3\sinh(c+dx)}{d}+\frac{\text{Subst}\left(\int \frac{b\left(3a^2+3ab+b^2\right)+3ab(2a+b)x^2+3a^2bx^4}{\left(1+x^2\right)^3}\,dx,x,\sinh\right.}{d}$$

$$=\frac{a^3\sinh(c+dx)}{d}+\frac{b^3\operatorname{sech}^3(c+dx)\tanh(c+dx)}{4d}-\frac{\text{Subst}\left(\int \frac{-3b(2a+b}{(1+}\right.}{}$$

$$=\frac{a^3\sinh(c+dx)}{d}+\frac{3b^2(4a+b)\operatorname{sech}(c+dx)\tanh(c+dx)}{8d}+\frac{b^3\operatorname{sech}^3(}{}$$

$$=\frac{3b\left(8a^2+4ab+b^2\right)\tan^{-1}(\sinh(c+dx))}{8d}+\frac{a^3\sinh(c+dx)}{d}+\frac{3b^2(4a}{}$$

Mathematica [C] time = 7.88, size = 575, normalized size = 6.18

$$-\frac{\cosh(c+dx)\coth^5(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\left(256\sinh^8(c+dx)\left(a\sinh^2(c+dx)+a+b\right)^3{}_6F_5\left(\frac{3}{2},2,2,2,\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cosh[c + d*x]*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] -1/7560*(Cosh[c + d*x]*Coth[c + d*x]^5*(a + b*Sech[c + d*x]^2)^3*(256*Hyper
geometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Si
nh[c + d*x]^8*(a + b + a*Sinh[c + d*x]^2)^3 + 384*HypergeometricPFQ[{3/2, 2
, 2, 2, 2}, {1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^8*(a + b + a*S
inh[c + d*x]^2)^2*(7*b + a*(7 + 5*Sinh[c + d*x]^2)) + (315*ArcTanh[Sqrt[-Si
nh[c + d*x]^2]]*(b^3*(16807 + 15000*Sinh[c + d*x]^2 + 2187*Sinh[c + d*x]^4
- 62*Sinh[c + d*x]^6) + a^3*Cosh[c + d*x]^4*(16807 + 24604*Sinh[c + d*x]^2
+ 11562*Sinh[c + d*x]^4 + 1468*Sinh[c + d*x]^6 + 7*Sinh[c + d*x]^8) + 3*a*b
^2*(16807 + 29406*Sinh[c + d*x]^2 + 15312*Sinh[c + d*x]^4 + 1858*Sinh[c + d
*x]^6 + 9*Sinh[c + d*x]^8) + 3*a^2*b*(16807 + 43812*Sinh[c + d*x]^2 + 40442
*Sinh[c + d*x]^4 + 14956*Sinh[c + d*x]^6 + 1719*Sinh[c + d*x]^8 + 8*Sinh[c
+ d*x]^10)))/Sqrt[-Sinh[c + d*x]^2] - 21*(b^3*(252105 + 140965*Sinh[c + d*x
]^2 + 8226*Sinh[c + d*x]^4) + 3*a*b^2*(252105 + 357055*Sinh[c + d*x]^2 + 13
3071*Sinh[c + d*x]^4 + 6393*Sinh[c + d*x]^6) + 3*a^2*b*(252105 + 573145*Sin
h[c + d*x]^2 + 437991*Sinh[c + d*x]^4 + 120431*Sinh[c + d*x]^6 + 5640*Sinh[
c + d*x]^8) + a^3*(252105 + 789235*Sinh[c + d*x]^2 + 922986*Sinh[c + d*x]^4
 + 491574*Sinh[c + d*x]^6 + 107725*Sinh[c + d*x]^8 + 4887*Sinh[c + d*x]^10)
)))/(d*(a + 2*b + a*Cosh[2*c + 2*d*x])^3)
```

fricas [B] time = 0.47, size = 1992, normalized size = 21.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*a^3*cosh(d*x + c)^10 + 20*a^3*cosh(d*x + c)*sinh(d*x + c)^9 + 2*a^3*
sinh(d*x + c)^10 + 3*(2*a^3 + 4*a*b^2 + b^3)*cosh(d*x + c)^8 + 3*(30*a^3*co
sh(d*x + c)^2 + 2*a^3 + 4*a*b^2 + b^3)*sinh(d*x + c)^8 + 24*(10*a^3*cosh(d*
x + c)^3 + (2*a^3 + 4*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + (4*a^3
+ 12*a*b^2 + 11*b^3)*cosh(d*x + c)^6 + (420*a^3*cosh(d*x + c)^4 + 4*a^3 + 1
2*a*b^2 + 11*b^3 + 84*(2*a^3 + 4*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c
)^6 + 6*(84*a^3*cosh(d*x + c)^5 + 28*(2*a^3 + 4*a*b^2 + b^3)*cosh(d*x + c)^
3 + (4*a^3 + 12*a*b^2 + 11*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 - (4*a^3 + 1
```

```
2*a*b^2 + 11*b^3)*cosh(d*x + c)^4 + (420*a^3*cosh(d*x + c)^6 + 210*(2*a^3 +
 4*a*b^2 + b^3)*cosh(d*x + c)^4 - 4*a^3 - 12*a*b^2 - 11*b^3 + 15*(4*a^3 + 1
2*a*b^2 + 11*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(60*a^3*cosh(d*x + c
)^7 + 42*(2*a^3 + 4*a*b^2 + b^3)*cosh(d*x + c)^5 + 5*(4*a^3 + 12*a*b^2 + 11
*b^3)*cosh(d*x + c)^3 - (4*a^3 + 12*a*b^2 + 11*b^3)*cosh(d*x + c))*sinh(d*x
 + c)^3 - 2*a^3 - 3*(2*a^3 + 4*a*b^2 + b^3)*cosh(d*x + c)^2 + 3*(30*a^3*cos
h(d*x + c)^8 + 28*(2*a^3 + 4*a*b^2 + b^3)*cosh(d*x + c)^6 + 5*(4*a^3 + 12*a
*b^2 + 11*b^3)*cosh(d*x + c)^4 - 2*a^3 - 4*a*b^2 - b^3 - 2*(4*a^3 + 12*a*b^
2 + 11*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 3*((8*a^2*b + 4*a*b^2 + b^3)
*cosh(d*x + c)^9 + 9*(8*a^2*b + 4*a*b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^
8 + (8*a^2*b + 4*a*b^2 + b^3)*sinh(d*x + c)^9 + 4*(8*a^2*b + 4*a*b^2 + b^3)
*cosh(d*x + c)^7 + 4*(8*a^2*b + 4*a*b^2 + b^3 + 9*(8*a^2*b + 4*a*b^2 + b^3)
*cosh(d*x + c)^2)*sinh(d*x + c)^7 + 28*(3*(8*a^2*b + 4*a*b^2 + b^3)*cosh(d*
x + c)^3 + (8*a^2*b + 4*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c)^6 + 6*(8*
a^2*b + 4*a*b^2 + b^3)*cosh(d*x + c)^5 + 6*(21*(8*a^2*b + 4*a*b^2 + b^3)*co
sh(d*x + c)^4 + 8*a^2*b + 4*a*b^2 + b^3 + 14*(8*a^2*b + 4*a*b^2 + b^3)*cosh
(d*x + c)^2)*sinh(d*x + c)^5 + 2*(63*(8*a^2*b + 4*a*b^2 + b^3)*cosh(d*x + c
)^5 + 70*(8*a^2*b + 4*a*b^2 + b^3)*cosh(d*x + c)^3 + 15*(8*a^2*b + 4*a*b^2
+ b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 4*(8*a^2*b + 4*a*b^2 + b^3)*cosh(d*
x + c)^3 + 4*(21*(8*a^2*b + 4*a*b^2 + b^3)*cosh(d*x + c)^6 + 35*(8*a^2*b +
4*a*b^2 + b^3)*cosh(d*x + c)^4 + 8*a^2*b + 4*a*b^2 + b^3 + 15*(8*a^2*b + 4*
a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 12*(3*(8*a^2*b + 4*a*b^2 +
b^3)*cosh(d*x + c)^7 + 7*(8*a^2*b + 4*a*b^2 + b^3)*cosh(d*x + c)^5 + 5*(8*a
^2*b + 4*a*b^2 + b^3)*cosh(d*x + c)^3 + (8*a^2*b + 4*a*b^2 + b^3)*cosh(d*x
+ c))*sinh(d*x + c)^2 + (8*a^2*b + 4*a*b^2 + b^3)*cosh(d*x + c) + (9*(8*a^2
*b + 4*a*b^2 + b^3)*cosh(d*x + c)^8 + 28*(8*a^2*b + 4*a*b^2 + b^3)*cosh(d*x
 + c)^6 + 30*(8*a^2*b + 4*a*b^2 + b^3)*cosh(d*x + c)^4 + 8*a^2*b + 4*a*b^2
+ b^3 + 12*(8*a^2*b + 4*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c))*arctan
(cosh(d*x + c) + sinh(d*x + c)) + 2*(10*a^3*cosh(d*x + c)^9 + 12*(2*a^3 + 4
*a*b^2 + b^3)*cosh(d*x + c)^7 + 3*(4*a^3 + 12*a*b^2 + 11*b^3)*cosh(d*x + c)
^5 - 2*(4*a^3 + 12*a*b^2 + 11*b^3)*cosh(d*x + c)^3 - 3*(2*a^3 + 4*a*b^2 + b
^3)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^9 + 9*d*cosh(d*x + c)*si
nh(d*x + c)^8 + d*sinh(d*x + c)^9 + 4*d*cosh(d*x + c)^7 + 4*(9*d*cosh(d*x +
 c)^2 + d)*sinh(d*x + c)^7 + 28*(3*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sin
h(d*x + c)^6 + 6*d*cosh(d*x + c)^5 + 6*(21*d*cosh(d*x + c)^4 + 14*d*cosh(d*
x + c)^2 + d)*sinh(d*x + c)^5 + 2*(63*d*cosh(d*x + c)^5 + 70*d*cosh(d*x + c
)^3 + 15*d*cosh(d*x + c))*sinh(d*x + c)^4 + 4*d*cosh(d*x + c)^3 + 4*(21*d*c
osh(d*x + c)^6 + 35*d*cosh(d*x + c)^4 + 15*d*cosh(d*x + c)^2 + d)*sinh(d*x
+ c)^3 + 12*(3*d*cosh(d*x + c)^7 + 7*d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)^
3 + d*cosh(d*x + c))*sinh(d*x + c)^2 + d*cosh(d*x + c) + (9*d*cosh(d*x + c)
^8 + 28*d*cosh(d*x + c)^6 + 30*d*cosh(d*x + c)^4 + 12*d*cosh(d*x + c)^2 + d
)*sinh(d*x + c))
```

giac [B] time = 0.19, size = 199, normalized size = 2.14

$$\frac{8\,a^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)+3\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)\left(8\,a^2b+4\,ab^2+b^3\right)+\frac{4\left(12\,ab^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3\right.}{}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/16*(8*a^3*(e^(d*x + c) - e^(-d*x - c)) + 3*(pi + 2*arctan(1/2*(e^(2*d*x +
 2*c) - 1)*e^(-d*x - c)))*(8*a^2*b + 4*a*b^2 + b^3) + 4*(12*a*b^2*(e^(d*x +
 c) - e^(-d*x - c))^3 + 3*b^3*(e^(d*x + c) - e^(-d*x - c))^3 + 48*a*b^2*(e^
(d*x + c) - e^(-d*x - c)) + 20*b^3*(e^(d*x + c) - e^(-d*x - c)))/((e^(d*x +
 c) - e^(-d*x - c))^2 + 4)^2)/d
```

maple [A] time = 0.51, size = 125, normalized size = 1.34

$$\frac{a^3\sinh(dx+c)}{d}+\frac{6a^2b\arctan\left(e^{dx+c}\right)}{d}+\frac{3a\,b^2\mathrm{sech}(dx+c)\tanh(dx+c)}{2d}+\frac{3a\,b^2\arctan\left(e^{dx+c}\right)}{d}+\frac{b^3\tanh(dx+c)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)*(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] a^3*sinh(d*x+c)/d+6/d*a^2*b*arctan(exp(d*x+c))+3/2/d*a*b^2*sech(d*x+c)*tanh
(d*x+c)+3/d*a*b^2*arctan(exp(d*x+c))+1/4/d*b^3*tanh(d*x+c)*sech(d*x+c)^3+3/
8/d*b^3*sech(d*x+c)*tanh(d*x+c)+3/4/d*b^3*arctan(exp(d*x+c))
```

maxima [B] time = 0.41, size = 221, normalized size = 2.38

$$-\frac{1}{4}\,b^3\left(\frac{3\,\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{3\,e^{(-dx-c)}+11\,e^{(-3\,dx-3\,c)}-11\,e^{(-5\,dx-5\,c)}-3\,e^{(-7\,dx-7\,c)}}{d\left(4\,e^{(-2\,dx-2\,c)}+6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}+e^{(-8\,dx-8\,c)}+1\right)}\right)-3\,ab^2\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*b^3*(3*arctan(e^(-d*x - c))/d - (3*e^(-d*x - c) + 11*e^(-3*d*x - 3*c)
- 11*e^(-5*d*x - 5*c) - 3*e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-
4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) - 3*a*b^2*(arct
an(e^(-d*x - c))/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*
c) + e^(-4*d*x - 4*c) + 1))) - 6*a^2*b*arctan(e^(-d*x - c))/d + a^3*sinh(d*
x + c)/d
```

mupad [B] time = 0.19, size = 344, normalized size = 3.70

$$\frac{a^3 e^{c+dx}}{2d} - \frac{a^3 e^{-c-dx}}{2d} + \frac{3\,\mathrm{atan}\left(\frac{e^{dx}\,e^{c}\left(b^3\sqrt{d^2}+4\,a\,b^2\sqrt{d^2}+8\,a^2\,b\,\sqrt{d^2}\right)}{d\,\sqrt{64\,a^4\,b^2+64\,a^3\,b^3+32\,a^2\,b^4+8\,a\,b^5+b^6}}\right)\sqrt{64\,a^4\,b^2+64\,a^3\,b^3+32\,a^2\,b^4+8\,a\,b^5+b^6}}{4\,\sqrt{d^2}} - \frac{\ldots}{d\,(3\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)*(a + b/cosh(c + d*x)^2)^3,x)
```

```
[Out] (a^3*exp(c + d*x))/(2*d) - (a^3*exp(- c - d*x))/(2*d) + (3*atan((exp(d*x)*e
xp(c)*(b^3*(d^2)^(1/2) + 4*a*b^2*(d^2)^(1/2) + 8*a^2*b*(d^2)^(1/2)))/(d*(8*
a*b^5 + b^6 + 32*a^2*b^4 + 64*a^3*b^3 + 64*a^4*b^2)^(1/2)))*(8*a*b^5 + b^6
+ 32*a^2*b^4 + 64*a^3*b^3 + 64*a^4*b^2)^(1/2))/(4*(d^2)^(1/2)) - (6*b^3*exp
(c + d*x))/(d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) +
 1)) - (exp(c + d*x)*(12*a*b^2 - b^3))/(2*d*(2*exp(2*c + 2*d*x) + exp(4*c +
 4*d*x) + 1)) + (4*b^3*exp(c + d*x))/(d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4
*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) + (3*exp(c + d*x)*(4*a*
b^2 + b^3))/(4*d*(exp(2*c + 2*d*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\,\mathrm{sech}^2\,(c+dx)\right)^3 \cosh\,(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**3*cosh(c + d*x), x)
```

3.69 $\int \mathbf{sech}(c+dx)\left(a+b\mathbf{sech}^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=147

$$\frac{(2a+b)\left(8a^2+8ab+5b^2\right)\tan^{-1}(\sinh(c+dx))}{16d}+\frac{b\left(44a^2+44ab+15b^2\right)\tanh(c+dx)\text{sech}(c+dx)}{48d}+\frac{b\tanh(c+$$

```
[Out] 1/16*(2*a+b)*(8*a^2+8*a*b+5*b^2)*arctan(sinh(d*x+c))/d+1/48*b*(44*a^2+44*a*
b+15*b^2)*sech(d*x+c)*tanh(d*x+c)/d+5/24*b*(2*a+b)*sech(d*x+c)^3*(a+b+a*sin
h(d*x+c)^2)*tanh(d*x+c)/d+1/6*b*sech(d*x+c)^5*(a+b+a*sinh(d*x+c)^2)^2*tanh(
d*x+c)/d
```

Rubi [A] time = 0.14, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4147, 413, 526, 385, 203}

$$\frac{(2a+b)\left(8a^2+8ab+5b^2\right)\tan^{-1}(\sinh(c+dx))}{16d}+\frac{b\left(44a^2+44ab+15b^2\right)\tanh(c+dx)\text{sech}(c+dx)}{48d}+\frac{b\tanh(c+$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((2*a + b)*(8*a^2 + 8*a*b + 5*b^2)*ArcTan[Sinh[c + d*x]])/(16*d) + (b*(44*a
^2 + 44*a*b + 15*b^2)*Sech[c + d*x]*Tanh[c + d*x])/(48*d) + (5*b*(2*a + b)*
Sech[c + d*x]^3*(a + b + a*Sinh[c + d*x]^2)*Tanh[c + d*x])/(24*d) + (b*Sech
[c + d*x]^5*(a + b + a*Sinh[c + d*x]^2)^2*Tanh[c + d*x])/(6*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 526

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^q)/(a*b*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p +
 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \text{sech}(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b+ax^2\right)^3}{\left(1+x^2\right)^4}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{b\text{sech}^5(c+dx)\left(a+b+a\sinh^2(c+dx)\right)^2\tanh(c+dx)}{6d} + \frac{\text{Subst}\left(\int \right.}{}$$

$$= \frac{5b(2a+b)\text{sech}^3(c+dx)\left(a+b+a\sinh^2(c+dx)\right)\tanh(c+dx)}{24d} + \frac{b\text{s}}{}$$

$$= \frac{b\left(44a^2+44ab+15b^2\right)\text{sech}(c+dx)\tanh(c+dx)}{48d} + \frac{5b(2a+b)\text{sech}}{}$$

$$= \frac{(2a+b)\left(8a^2+8ab+5b^2\right)\tan^{-1}(\sinh(c+dx))}{16d} + \frac{b\left(44a^2+44ab+1\right.}{}$$

Mathematica [C] time = 9.84, size = 1430, normalized size = 9.73

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sech[c + d*x]*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (Coth[c + d*x]^6*Csch[c + d*x]*(a + b*Sech[c + d*x]^2)^3*(117228825*(a + b)
^3*Sinh[c + d*x]^2 + 274542345*a*(a + b)^2*Sinh[c + d*x]^4 + 70189350*(a +
b)^3*Sinh[c + d*x]^4 + 215549775*a^2*(a + b)*Sinh[c + d*x]^6 + 168951510*a*
(a + b)^2*Sinh[c + d*x]^6 + 4093425*(a + b)^3*Sinh[c + d*x]^6 + 58009455*a^
3*Sinh[c + d*x]^8 + 135323370*a^2*(a + b)*Sinh[c + d*x]^8 + 9514449*a*(a +
b)^2*Sinh[c + d*x]^8 + 36772890*a^3*Sinh[c + d*x]^10 + 7808535*a^2*(a + b)*
Sinh[c + d*x]^10 - 75520*(a + b)^3*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1,
 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^10 - 13824*(a + b)^3*Hypergeo
metricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[
c + d*x]^10 - 1024*(a + b)^3*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2, 2}, {1,
 1, 1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^10 + 2160711*a^3*Sinh[c
 + d*x]^12 - 189696*a*(a + b)^2*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1,
 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^12 - 38400*a*(a + b)^2*Hypergeom
etricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c
 + d*x]^12 - 3072*a*(a + b)^2*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2, 2}, {1
, 1, 1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^12 - 158976*a^2*(a + b
)*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*S
inh[c + d*x]^14 - 35328*a^2*(a + b)*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2},
 {1, 1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^14 - 3072*a^2*(a + b)*
HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 1, 11/2}, -Sinh[c +
 d*x]^2]*Sinh[c + d*x]^14 - 44800*a^3*HypergeometricPFQ[{3/2, 2, 2, 2, 2},
{1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^16 - 10752*a^3*Hypergeomet
ricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c +
 d*x]^16 - 1024*a^3*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2, 2}, {1, 1, 1, 1,
 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^16 + 117228825*(a + b)^3*ArcTanh
[Sqrt[-Sinh[c + d*x]^2]]*Sqrt[-Sinh[c + d*x]^2] + 215549775*a^2*(a + b)*Arc
Tanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^4*Sqrt[-Sinh[c + d*x]^2] + 26046
5625*a*(a + b)^2*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^4*Sqrt[-Sinh
[c + d*x]^2] + 17069535*(a + b)^3*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c +
d*x]^4*Sqrt[-Sinh[c + d*x]^2] + 58009455*a^3*ArcTanh[Sqrt[-Sinh[c + d*x]^2]
]*Sinh[c + d*x]^6*Sqrt[-Sinh[c + d*x]^2] + 207173295*a^2*(a + b)*ArcTanh[Sq
rt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^6*Sqrt[-Sinh[c + d*x]^2] + 41427855*a*(
a + b)^2*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^6*Sqrt[-Sinh[c + d*x
]^2] + 56109375*a^3*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^8*Sqrt[-S
inh[c + d*x]^2] + 33756345*a^2*(a + b)*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh
[c + d*x]^8*Sqrt[-Sinh[c + d*x]^2] + 210735*a*(a + b)^2*ArcTanh[Sqrt[-Sinh[
c + d*x]^2]]*Sinh[c + d*x]^8*Sqrt[-Sinh[c + d*x]^2] + 9261945*a^3*ArcTanh[S
```

```
qrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^10*Sqrt[-Sinh[c + d*x]^2] + 174825*a^2
*(a + b)*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^10*Sqrt[-Sinh[c + d*
x]^2] + 48825*a^3*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^12*Sqrt[-Si
nh[c + d*x]^2] - 274542345*a*(a + b)^2*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*(-Si
nh[c + d*x]^2)^(3/2) - 109265625*(a + b)^3*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*
(-Sinh[c + d*x]^2)^(3/2) + 142065*(a + b)^3*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]
*Sinh[c + d*x]^4*(-Sinh[c + d*x]^2)^(3/2)))/(90720*d*(a + 2*b + a*Cosh[2*c
+ 2*d*x])^3)
```

fricas [B] time = 0.47, size = 3465, normalized size = 23.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(3*(24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^11 + 33*(24*a^2*b + 18*
a*b^2 + 5*b^3)*cosh(d*x + c)*sinh(d*x + c)^10 + 3*(24*a^2*b + 18*a*b^2 + 5*
b^3)*sinh(d*x + c)^11 + (216*a^2*b + 306*a*b^2 + 85*b^3)*cosh(d*x + c)^9 +
(216*a^2*b + 306*a*b^2 + 85*b^3 + 165*(24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*
x + c)^2)*sinh(d*x + c)^9 + 9*(55*(24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x +
c)^3 + (216*a^2*b + 306*a*b^2 + 85*b^3)*cosh(d*x + c))*sinh(d*x + c)^8 + 18
*(8*a^2*b + 14*a*b^2 + 11*b^3)*cosh(d*x + c)^7 + 18*(55*(24*a^2*b + 18*a*b^
2 + 5*b^3)*cosh(d*x + c)^4 + 8*a^2*b + 14*a*b^2 + 11*b^3 + 2*(216*a^2*b + 3
06*a*b^2 + 85*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^7 + 42*(33*(24*a^2*b + 18
*a*b^2 + 5*b^3)*cosh(d*x + c)^5 + 2*(216*a^2*b + 306*a*b^2 + 85*b^3)*cosh(d
*x + c)^3 + 3*(8*a^2*b + 14*a*b^2 + 11*b^3)*cosh(d*x + c))*sinh(d*x + c)^6
- 18*(8*a^2*b + 14*a*b^2 + 11*b^3)*cosh(d*x + c)^5 + 18*(77*(24*a^2*b + 18*
a*b^2 + 5*b^3)*cosh(d*x + c)^6 + 7*(216*a^2*b + 306*a*b^2 + 85*b^3)*cosh(d*
x + c)^4 - 8*a^2*b - 14*a*b^2 - 11*b^3 + 21*(8*a^2*b + 14*a*b^2 + 11*b^3)*c
osh(d*x + c)^2)*sinh(d*x + c)^5 + 18*(55*(24*a^2*b + 18*a*b^2 + 5*b^3)*cosh
(d*x + c)^7 + 7*(216*a^2*b + 306*a*b^2 + 85*b^3)*cosh(d*x + c)^5 + 35*(8*a^
2*b + 14*a*b^2 + 11*b^3)*cosh(d*x + c)^3 - 5*(8*a^2*b + 14*a*b^2 + 11*b^3)*
cosh(d*x + c))*sinh(d*x + c)^4 - (216*a^2*b + 306*a*b^2 + 85*b^3)*cosh(d*x
+ c)^3 + (495*(24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^8 + 84*(216*a^2*b
 + 306*a*b^2 + 85*b^3)*cosh(d*x + c)^6 + 630*(8*a^2*b + 14*a*b^2 + 11*b^3)*
cosh(d*x + c)^4 - 216*a^2*b - 306*a*b^2 - 85*b^3 - 180*(8*a^2*b + 14*a*b^2
+ 11*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 3*(55*(24*a^2*b + 18*a*b^2 + 5
*b^3)*cosh(d*x + c)^9 + 12*(216*a^2*b + 306*a*b^2 + 85*b^3)*cosh(d*x + c)^7
 + 126*(8*a^2*b + 14*a*b^2 + 11*b^3)*cosh(d*x + c)^5 - 60*(8*a^2*b + 14*a*b
^2 + 11*b^3)*cosh(d*x + c)^3 - (216*a^2*b + 306*a*b^2 + 85*b^3)*cosh(d*x +
c))*sinh(d*x + c)^2 + 3*((16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x +
c)^12 + 12*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)*sinh(d*x +
c)^11 + (16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*sinh(d*x + c)^12 + 6*(16*a^3
 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^10 + 6*(16*a^3 + 24*a^2*b + 1
```

```
8*a*b^2 + 5*b^3 + 11*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^2
)*sinh(d*x + c)^10 + 20*(11*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x
 + c)^3 + 3*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x
+ c)^9 + 15*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^8 + 15*(33
*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^4 + 16*a^3 + 24*a^2*b
 + 18*a*b^2 + 5*b^3 + 18*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x +
c)^2)*sinh(d*x + c)^8 + 24*(33*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(
d*x + c)^5 + 30*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + 5*
(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + 20*
(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^6 + 4*(231*(16*a^3 + 2
4*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^6 + 315*(16*a^3 + 24*a^2*b + 18*a
*b^2 + 5*b^3)*cosh(d*x + c)^4 + 80*a^3 + 120*a^2*b + 90*a*b^2 + 25*b^3 + 10
5*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 +
 24*(33*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^7 + 63*(16*a^3
 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^5 + 35*(16*a^3 + 24*a^2*b + 1
8*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + 5*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)
*cosh(d*x + c))*sinh(d*x + c)^5 + 15*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)
*cosh(d*x + c)^4 + 15*(33*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x +
 c)^8 + 84*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^6 + 70*(16*
a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^4 + 16*a^3 + 24*a^2*b + 18
*a*b^2 + 5*b^3 + 20*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^2)
*sinh(d*x + c)^4 + 20*(11*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x +
 c)^9 + 36*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^7 + 42*(16*
a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^5 + 20*(16*a^3 + 24*a^2*b
+ 18*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + 3*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b
^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3 +
 6*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^2 + 6*(11*(16*a^3 +
 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^10 + 45*(16*a^3 + 24*a^2*b + 18
*a*b^2 + 5*b^3)*cosh(d*x + c)^8 + 70*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)
*cosh(d*x + c)^6 + 50*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^
4 + 16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3 + 15*(16*a^3 + 24*a^2*b + 18*a*b^2
 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 12*((16*a^3 + 24*a^2*b + 18*a*
b^2 + 5*b^3)*cosh(d*x + c)^11 + 5*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*co
sh(d*x + c)^9 + 10*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^7 +
 10*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^5 + 5*(16*a^3 + 24
*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + (16*a^3 + 24*a^2*b + 18*a*b^2
+ 5*b^3)*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)
) - 3*(24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c) + 3*(11*(24*a^2*b + 18*a*
b^2 + 5*b^3)*cosh(d*x + c)^10 + 3*(216*a^2*b + 306*a*b^2 + 85*b^3)*cosh(d*x
 + c)^8 + 42*(8*a^2*b + 14*a*b^2 + 11*b^3)*cosh(d*x + c)^6 - 30*(8*a^2*b +
14*a*b^2 + 11*b^3)*cosh(d*x + c)^4 - 24*a^2*b - 18*a*b^2 - 5*b^3 - (216*a^2
*b + 306*a*b^2 + 85*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^1
2 + 12*d*cosh(d*x + c)*sinh(d*x + c)^11 + d*sinh(d*x + c)^12 + 6*d*cosh(d*x
 + c)^10 + 6*(11*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^10 + 20*(11*d*cosh(d*
x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^9 + 15*d*cosh(d*x + c)^8 + 15*(
```

```
33*d*cosh(d*x + c)^4 + 18*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^8 + 24*(33*d
*cosh(d*x + c)^5 + 30*d*cosh(d*x + c)^3 + 5*d*cosh(d*x + c))*sinh(d*x + c)^
7 + 20*d*cosh(d*x + c)^6 + 4*(231*d*cosh(d*x + c)^6 + 315*d*cosh(d*x + c)^4
 + 105*d*cosh(d*x + c)^2 + 5*d)*sinh(d*x + c)^6 + 24*(33*d*cosh(d*x + c)^7
+ 63*d*cosh(d*x + c)^5 + 35*d*cosh(d*x + c)^3 + 5*d*cosh(d*x + c))*sinh(d*x
 + c)^5 + 15*d*cosh(d*x + c)^4 + 15*(33*d*cosh(d*x + c)^8 + 84*d*cosh(d*x +
 c)^6 + 70*d*cosh(d*x + c)^4 + 20*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^4 +
20*(11*d*cosh(d*x + c)^9 + 36*d*cosh(d*x + c)^7 + 42*d*cosh(d*x + c)^5 + 20
*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^3 + 6*d*cosh(d*x + c)
^2 + 6*(11*d*cosh(d*x + c)^10 + 45*d*cosh(d*x + c)^8 + 70*d*cosh(d*x + c)^6
 + 50*d*cosh(d*x + c)^4 + 15*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 12*(d
*cosh(d*x + c)^11 + 5*d*cosh(d*x + c)^9 + 10*d*cosh(d*x + c)^7 + 10*d*cosh(
d*x + c)^5 + 5*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

giac [B] time = 0.19, size = 310, normalized size = 2.11

$$\frac{3\left(\pi + 2\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)} - 1\right)e^{(-dx-c)}\right)\right)\left(16\,a^3 + 24\,a^2b + 18\,ab^2 + 5\,b^3\right) + \frac{4\left(72\,a^2b\left(e^{(dx+c)}-e^{(-dx-c)}\right)^5+54\,ab^2\left(e^{(dx+c)}-e^{(-\cdots\right.\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/96*(3*(pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(16*a^3 + 2
4*a^2*b + 18*a*b^2 + 5*b^3) + 4*(72*a^2*b*(e^(d*x + c) - e^(-d*x - c))^5 +
54*a*b^2*(e^(d*x + c) - e^(-d*x - c))^5 + 15*b^3*(e^(d*x + c) - e^(-d*x - c
))^5 + 576*a^2*b*(e^(d*x + c) - e^(-d*x - c))^3 + 576*a*b^2*(e^(d*x + c) -
e^(-d*x - c))^3 + 160*b^3*(e^(d*x + c) - e^(-d*x - c))^3 + 1152*a^2*b*(e^(d
*x + c) - e^(-d*x - c)) + 1440*a*b^2*(e^(d*x + c) - e^(-d*x - c)) + 528*b^3
*(e^(d*x + c) - e^(-d*x - c)))/((e^(d*x + c) - e^(-d*x - c))^2 + 4)^3)/d
```

maple [A] time = 0.46, size = 193, normalized size = 1.31

$$\frac{2a^3\arctan\left(\mathrm{e}^{dx+c}\right)}{d} + \frac{3a^2b\,\mathrm{sech}\,(dx+c)\tanh\,(dx+c)}{2d} + \frac{3a^2b\arctan\left(\mathrm{e}^{dx+c}\right)}{d} + \frac{3a\,b^2\tanh\,(dx+c)\,\mathrm{sech}\,(dx+c)^3}{4d} + \frac{9\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)*(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] 2/d*a^3*arctan(exp(d*x+c))+3/2/d*a^2*b*sech(d*x+c)*tanh(d*x+c)+3/d*a^2*b*ar
ctan(exp(d*x+c))+3/4/d*a*b^2*tanh(d*x+c)*sech(d*x+c)^3+9/8/d*a*b^2*sech(d*x
+c)*tanh(d*x+c)+9/4/d*a*b^2*arctan(exp(d*x+c))+1/6/d*b^3*tanh(d*x+c)*sech(d
*x+c)^5+5/24/d*b^3*tanh(d*x+c)*sech(d*x+c)^3+5/16/d*b^3*sech(d*x+c)*tanh(d*
x+c)+5/8/d*b^3*arctan(exp(d*x+c))
```

maxima [B] time = 0.41, size = 365, normalized size = 2.48

$$-\frac{1}{24}b^3\left(\frac{15\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{15e^{(-dx-c)}+85e^{(-3dx-3c)}+198e^{(-5dx-5c)}-198e^{(-7dx-7c)}-85e^{(-9dx-9c)}-15e^{\cdots}}{d\left(6e^{(-2dx-2c)}+15e^{(-4dx-4c)}+20e^{(-6dx-6c)}+15e^{(-8dx-8c)}+6e^{(-10dx-10c)}+e^{(-1\cdots}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/24*b^3*(15*arctan(e^(-d*x - c))/d - (15*e^(-d*x - c) + 85*e^(-3*d*x - 3*
c) + 198*e^(-5*d*x - 5*c) - 198*e^(-7*d*x - 7*c) - 85*e^(-9*d*x - 9*c) - 15
*e^(-11*d*x - 11*c))/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x - 4*c) + 20*e^(-
6*d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) + e^(-12*d*x - 12
*c) + 1))) - 3/4*a*b^2*(3*arctan(e^(-d*x - c))/d - (3*e^(-d*x - c) + 11*e^(
-3*d*x - 3*c) - 11*e^(-5*d*x - 5*c) - 3*e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x -
 2*c) + 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) -
 3*a^2*b*(arctan(e^(-d*x - c))/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*
e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) + a^3*arctan(sinh(d*x + c))/d
```

mupad [B] time = 1.47, size = 535, normalized size = 3.64

$$\frac{\operatorname{atan}\left(\frac{e^{dx}e^{c}\left(16a^3\sqrt{d^2}+5b^3\sqrt{d^2}+18ab^2\sqrt{d^2}+24a^2b\sqrt{d^2}\right)}{d\sqrt{256a^6+768a^5b+1152a^4b^2+1024a^3b^3+564a^2b^4+180ab^5+25b^6}}\right)\sqrt{256a^6+768a^5b+1152a^4b^2+1024a^3b^3+56\cdots}}{8\sqrt{d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cosh(c + d*x)^2)^3/cosh(c + d*x),x)
```

```
[Out] (atan((exp(d*x)*exp(c)*(16*a^3*(d^2)^(1/2) + 5*b^3*(d^2)^(1/2) + 18*a*b^2*(
d^2)^(1/2) + 24*a^2*b*(d^2)^(1/2)))/(d*(180*a*b^5 + 768*a^5*b + 256*a^6 + 2
5*b^6 + 564*a^2*b^4 + 1024*a^3*b^3 + 1152*a^4*b^2)^(1/2)))*(180*a*b^5 + 768
*a^5*b + 256*a^6 + 25*b^6 + 564*a^2*b^4 + 1024*a^3*b^3 + 1152*a^4*b^2)^(1/2
))/(8*(d^2)^(1/2)) - (exp(c + d*x)*(54*a*b^2 - b^3))/(3*d*(3*exp(2*c + 2*d*
x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) + (80*b^3*exp(c + d*x))/(3
*d*(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(
8*c + 8*d*x) + exp(10*c + 10*d*x) + 1)) + (6*exp(c + d*x)*(2*a*b^2 - 3*b^3)
)/(d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*
c + 8*d*x) + 1)) - (32*b^3*exp(c + d*x))/(3*d*(6*exp(2*c + 2*d*x) + 15*exp(
4*c + 4*d*x) + 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*exp(10*c + 10*
d*x) + exp(12*c + 12*d*x) + 1)) + (exp(c + d*x)*(18*a*b^2 + 24*a^2*b + 5*b^
3))/(8*d*(exp(2*c + 2*d*x) + 1)) + (exp(c + d*x)*(18*a*b^2 - 72*a^2*b + 5*b
^3))/(12*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

<u>sympy</u> [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\,\mathrm{sech}^2\left(c + dx\right)\right)^3 \mathrm{sech}\left(c + dx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**3*sech(c + d*x), x)
```

3.70 $\int \mathrm{sech}^2(c+dx)\left(a+b\,\mathrm{sech}^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=74

$$\frac{3b^2(a+b)\tanh^5(c+dx)}{5d}-\frac{b(a+b)^2\tanh^3(c+dx)}{d}+\frac{(a+b)^3\tanh(c+dx)}{d}-\frac{b^3\tanh^7(c+dx)}{7d}$$

```
[Out] (a+b)^3*tanh(d*x+c)/d-b*(a+b)^2*tanh(d*x+c)^3/d+3/5*b^2*(a+b)*tanh(d*x+c)^5
/d-1/7*b^3*tanh(d*x+c)^7/d
```

Rubi [A] time = 0.08, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4146, 194}

$$\frac{3b^2(a+b)\tanh^5(c+dx)}{5d}-\frac{b(a+b)^2\tanh^3(c+dx)}{d}+\frac{(a+b)^3\tanh(c+dx)}{d}-\frac{b^3\tanh^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^2*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((a + b)^3*Tanh[c + d*x])/d - (b*(a + b)^2*Tanh[c + d*x]^3)/d + (3*b^2*(a +
 b)*Tanh[c + d*x]^5)/(5*d) - (b^3*Tanh[c + d*x]^7)/(7*d)
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \operatorname{sech}^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx = \frac{\operatorname{Subst}\left(\int\left(a+b-bx^2\right)^3\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\operatorname{Subst}\left(\int\left(a^3\left(1+\frac{b\left(3a^2+3ab+b^2\right)}{a^3}\right)-3b(a+b)^2x^2+3b^2(a+b)x^4-b^3x\right.\right.}{d}$$
$$= \frac{(a+b)^3\tanh(c+dx)}{d} - \frac{b(a+b)^2\tanh^3(c+dx)}{d} + \frac{3b^2(a+b)\tanh^5}{5d}$$

Mathematica [B] time = 1.49, size = 319, normalized size = 4.31

$$\frac{\operatorname{sech}(c)\operatorname{sech}(c+dx)\left(525a^3\sinh(2c+3dx)-210a^3\sinh(4c+3dx)+210a^3\sinh(4c+5dx)-35a^3\sinh(6c+5dx\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^2*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (Sech[c]*Sech[c + d*x]*(a + b*Sech[c + d*x]^2)^3*(140*(5*a^3 + 11*a^2*b + 1
0*a*b^2 + 4*b^3)*Sinh[d*x] - 35*a*(15*a^2 + 26*a*b + 16*b^2)*Sinh[2*c + d*x
] + 525*a^3*Sinh[2*c + 3*d*x] + 1260*a^2*b*Sinh[2*c + 3*d*x] + 1176*a*b^2*S
inh[2*c + 3*d*x] + 336*b^3*Sinh[2*c + 3*d*x] - 210*a^3*Sinh[4*c + 3*d*x] -
210*a^2*b*Sinh[4*c + 3*d*x] + 210*a^3*Sinh[4*c + 5*d*x] + 490*a^2*b*Sinh[4*
c + 5*d*x] + 392*a*b^2*Sinh[4*c + 5*d*x] + 112*b^3*Sinh[4*c + 5*d*x] - 35*a
^3*Sinh[6*c + 5*d*x] + 35*a^3*Sinh[6*c + 7*d*x] + 70*a^2*b*Sinh[6*c + 7*d*x
] + 56*a*b^2*Sinh[6*c + 7*d*x] + 16*b^3*Sinh[6*c + 7*d*x]))/(280*d*(a + 2*b
 + a*Cosh[2*(c + d*x)])^3)
```

fricas [B] time = 0.44, size = 816, normalized size = 11.03

$$-\frac{4\left(\left(35\,a^3+35\,a^2b+28\,ab^2+8\,b^3\right)\cosh\left(dx+c\right)^6-6\left(35\,a^2b+28\,ab^2+8\,b^3\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^5+\left(35\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -4/35*((35*a^3 + 35*a^2*b + 28*a*b^2 + 8*b^3)*cosh(d*x + c)^6 - 6*(35*a^2*b
 + 28*a*b^2 + 8*b^3)*cosh(d*x + c)*sinh(d*x + c)^5 + (35*a^3 + 35*a^2*b + 2
8*a*b^2 + 8*b^3)*sinh(d*x + c)^6 + 14*(15*a^3 + 25*a^2*b + 14*a*b^2 + 4*b^3
)*cosh(d*x + c)^4 + (210*a^3 + 350*a^2*b + 196*a*b^2 + 56*b^3 + 15*(35*a^3
+ 35*a^2*b + 28*a*b^2 + 8*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 - 4*(5*(35*
```

```
a^2*b + 28*a*b^2 + 8*b^3)*cosh(d*x + c)^3 + 28*(5*a^2*b + 7*a*b^2 + 2*b^3)*
cosh(d*x + c))*sinh(d*x + c)^3 + 350*a^3 + 770*a^2*b + 700*a*b^2 + 280*b^3
+ 7*(75*a^3 + 155*a^2*b + 124*a*b^2 + 24*b^3)*cosh(d*x + c)^2 + (15*(35*a^3
 + 35*a^2*b + 28*a*b^2 + 8*b^3)*cosh(d*x + c)^4 + 525*a^3 + 1085*a^2*b + 86
8*a*b^2 + 168*b^3 + 84*(15*a^3 + 25*a^2*b + 14*a*b^2 + 4*b^3)*cosh(d*x + c)
^2)*sinh(d*x + c)^2 - 2*(3*(35*a^2*b + 28*a*b^2 + 8*b^3)*cosh(d*x + c)^5 +
56*(5*a^2*b + 7*a*b^2 + 2*b^3)*cosh(d*x + c)^3 + 7*(25*a^2*b + 44*a*b^2 + 2
4*b^3)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^8 + 8*d*cosh(d*x + c)
*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 + 8*d*cosh(d*x + c)^6 + 4*(7*d*cosh(d*
x + c)^2 + 2*d)*sinh(d*x + c)^6 + 4*(14*d*cosh(d*x + c)^3 + 9*d*cosh(d*x +
c))*sinh(d*x + c)^5 + 28*d*cosh(d*x + c)^4 + 2*(35*d*cosh(d*x + c)^4 + 60*d
*cosh(d*x + c)^2 + 14*d)*sinh(d*x + c)^4 + 8*(7*d*cosh(d*x + c)^5 + 15*d*co
sh(d*x + c)^3 + 7*d*cosh(d*x + c))*sinh(d*x + c)^3 + 56*d*cosh(d*x + c)^2 +
 4*(7*d*cosh(d*x + c)^6 + 30*d*cosh(d*x + c)^4 + 42*d*cosh(d*x + c)^2 + 14*
d)*sinh(d*x + c)^2 + 4*(2*d*cosh(d*x + c)^7 + 9*d*cosh(d*x + c)^5 + 14*d*co
sh(d*x + c)^3 + 7*d*cosh(d*x + c))*sinh(d*x + c) + 35*d)
```

giac [B] time = 0.19, size = 302, normalized size = 4.08

$$-\frac{2\left(35\,a^3e^{(12\,dx+12\,c)}+210\,a^3e^{(10\,dx+10\,c)}+210\,a^2be^{(10\,dx+10\,c)}+525\,a^3e^{(8\,dx+8\,c)}+910\,a^2be^{(8\,dx+8\,c)}+560\,ab^2e^{(8\,dx+\cdots}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -2/35*(35*a^3*e^(12*d*x + 12*c) + 210*a^3*e^(10*d*x + 10*c) + 210*a^2*b*e^(
10*d*x + 10*c) + 525*a^3*e^(8*d*x + 8*c) + 910*a^2*b*e^(8*d*x + 8*c) + 560*
a*b^2*e^(8*d*x + 8*c) + 700*a^3*e^(6*d*x + 6*c) + 1540*a^2*b*e^(6*d*x + 6*c
) + 1400*a*b^2*e^(6*d*x + 6*c) + 560*b^3*e^(6*d*x + 6*c) + 525*a^3*e^(4*d*x
 + 4*c) + 1260*a^2*b*e^(4*d*x + 4*c) + 1176*a*b^2*e^(4*d*x + 4*c) + 336*b^3
*e^(4*d*x + 4*c) + 210*a^3*e^(2*d*x + 2*c) + 490*a^2*b*e^(2*d*x + 2*c) + 39
2*a*b^2*e^(2*d*x + 2*c) + 112*b^3*e^(2*d*x + 2*c) + 35*a^3 + 70*a^2*b + 56*
a*b^2 + 16*b^3)/(d*(e^(2*d*x + 2*c) + 1)^7)
```

maple [A] time = 0.57, size = 116, normalized size = 1.57

$$\frac{a^3\tanh\left(dx+c\right)+3a^2b\left(\frac{2}{3}+\frac{\mathrm{sech}(dx+c)^2}{3}\right)\tanh\left(dx+c\right)+3a\,b^2\left(\frac{8}{15}+\frac{\mathrm{sech}(dx+c)^4}{5}+\frac{4\mathrm{sech}(dx+c)^2}{15}\right)\tanh\left(dx+c\right)+b^3\cdots}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(a^3*tanh(d*x+c)+3*a^2*b*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c)+3*a*b^2*(8
/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)*tanh(d*x+c)+b^3*(16/35+1/7*sech(d
*x+c)^6+6/35*sech(d*x+c)^4+8/35*sech(d*x+c)^2)*tanh(d*x+c))
```

maxima [B] time = 0.33, size = 695, normalized size = 9.39

$$\frac{32}{35}\,b^3\Bigg(\frac{7\,e^{(-2\,dx-2\,c)}}{d\Big(7\,e^{(-2\,dx-2\,c)}+21\,e^{(-4\,dx-4\,c)}+35\,e^{(-6\,dx-6\,c)}+35\,e^{(-8\,dx-8\,c)}+21\,e^{(-10\,dx-10\,c)}+7\,e^{(-12\,dx-12\,c)}+e^{(-14\,dx-14\,c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 32/35*b^3*(7*e^(-2*d*x - 2*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c)
+ 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(
-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 21*e^(-4*d*x - 4*c)/(d*(7*e^(-
2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8
*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1
)) + 35*e^(-6*d*x - 6*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*
e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d
*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 1/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4
*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 1
0*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1))) + 16/5*a*b^2*(5*e^(
-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x -
 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 10*e^(-4*d*x - 4*c)
/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(
-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*d*x - 2*c) + 10*e^
(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10
*c) + 1))) + 4*a^2*b*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d
*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x
 - 4*c) + e^(-6*d*x - 6*c) + 1))) + 2*a^3/(d*(e^(-2*d*x - 2*c) + 1))
```

mupad [B] time = 1.50, size = 978, normalized size = 13.22

$$-\frac{\frac{2\left(5\,a^3+18\,a^2\,b+24\,a\,b^2+16\,b^3\right)}{35\,d}+\frac{2\,a^3\,e^{6\,c+6\,d\,x}}{7\,d}+\frac{6\,a\,e^{2\,c+2\,d\,x}\left(5\,a^2+16\,a\,b+16\,b^2\right)}{35\,d}+\frac{6\,a^2\,e^{4\,c+4\,d\,x}\,(a+2\,b)}{7\,d}}{4\,e^{2\,c+2\,d\,x}+6\,e^{4\,c+4\,d\,x}+4\,e^{6\,c+6\,d\,x}+e^{8\,c+8\,d\,x}+1}-\frac{\frac{2\,a^2\,(a+2\,b)}{7\,d}+\frac{2\,a^3\,e^{2\,c+2\,d\,x}}{7\,d}}{2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cosh(c + d*x)^2)^3/cosh(c + d*x)^2,x)
```

```
[Out] - ((2*(24*a*b^2 + 18*a^2*b + 5*a^3 + 16*b^3))/(35*d) + (2*a^3*exp(6*c + 6*d
*x))/(7*d) + (6*a*exp(2*c + 2*d*x)*(16*a*b + 5*a^2 + 16*b^2))/(35*d) + (6*a
^2*exp(4*c + 4*d*x)*(a + 2*b))/(7*d))/(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d
*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) - ((2*a^2*(a + 2*b))/(7*d)
```

```
 + (2*a^3*exp(2*c + 2*d*x))/(7*d))/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) +
 1) - ((2*a*(16*a*b + 5*a^2 + 16*b^2))/(35*d) + (8*exp(2*c + 2*d*x)*(24*a*b
^2 + 18*a^2*b + 5*a^3 + 16*b^3))/(35*d) + (2*a^3*exp(8*c + 8*d*x))/(7*d) +
(12*a*exp(4*c + 4*d*x)*(16*a*b + 5*a^2 + 16*b^2))/(35*d) + (8*a^2*exp(6*c +
 6*d*x)*(a + 2*b))/(7*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*ex
p(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1) - ((2*a^3)/(7
*d) + (8*exp(6*c + 6*d*x)*(24*a*b^2 + 18*a^2*b + 5*a^3 + 16*b^3))/(7*d) + (
2*a^3*exp(12*c + 12*d*x))/(7*d) + (6*a*exp(4*c + 4*d*x)*(16*a*b + 5*a^2 + 1
6*b^2))/(7*d) + (6*a*exp(8*c + 8*d*x)*(16*a*b + 5*a^2 + 16*b^2))/(7*d) + (1
2*a^2*exp(2*c + 2*d*x)*(a + 2*b))/(7*d) + (12*a^2*exp(10*c + 10*d*x)*(a + 2
*b))/(7*d))/(7*exp(2*c + 2*d*x) + 21*exp(4*c + 4*d*x) + 35*exp(6*c + 6*d*x)
 + 35*exp(8*c + 8*d*x) + 21*exp(10*c + 10*d*x) + 7*exp(12*c + 12*d*x) + exp
(14*c + 14*d*x) + 1) - ((2*a*(16*a*b + 5*a^2 + 16*b^2))/(35*d) + (2*a^3*exp
(4*c + 4*d*x))/(7*d) + (4*a^2*exp(2*c + 2*d*x)*(a + 2*b))/(7*d))/(3*exp(2*c
 + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) - ((2*a^2*(a + 2*b))
/(7*d) + (4*exp(4*c + 4*d*x)*(24*a*b^2 + 18*a^2*b + 5*a^3 + 16*b^3))/(7*d)
+ (2*a^3*exp(10*c + 10*d*x))/(7*d) + (2*a*exp(2*c + 2*d*x)*(16*a*b + 5*a^2
+ 16*b^2))/(7*d) + (4*a*exp(6*c + 6*d*x)*(16*a*b + 5*a^2 + 16*b^2))/(7*d) +
 (10*a^2*exp(8*c + 8*d*x)*(a + 2*b))/(7*d))/(6*exp(2*c + 2*d*x) + 15*exp(4*
c + 4*d*x) + 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*exp(10*c + 10*d*
x) + exp(12*c + 12*d*x) + 1) - (2*a^3)/(7*d*(exp(2*c + 2*d*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\,\text{sech}^2\left(c + dx\right)\right)^3 \text{sech}^2\left(c + dx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**2*(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**3*sech(c + d*x)**2, x)
```

3.71 $\int \mathbf{sech}^3(c+dx)\left(a+b\mathbf{sech}^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=196

$$\frac{b\left(72a^2+92ab+35b^2\right)\tanh(c+dx)\text{sech}^3(c+dx)}{192d}+\frac{\left(64a^3+144a^2b+120ab^2+35b^3\right)\tan^{-1}(\sinh(c+dx))}{128d}+\frac{(64a}{}$$

```
[Out] 1/128*(64*a^3+144*a^2*b+120*a*b^2+35*b^3)*arctan(sinh(d*x+c))/d+1/128*(64*a
^3+144*a^2*b+120*a*b^2+35*b^3)*sech(d*x+c)*tanh(d*x+c)/d+1/192*b*(72*a^2+92
*a*b+35*b^2)*sech(d*x+c)^3*tanh(d*x+c)/d+1/48*b*(12*a+7*b)*sech(d*x+c)^5*(a
+b+a*sinh(d*x+c)^2)*tanh(d*x+c)/d+1/8*b*sech(d*x+c)^7*(a+b+a*sinh(d*x+c)^2)
^2*tanh(d*x+c)/d
```

Rubi [A] time = 0.23, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4147, 413, 526, 385, 199, 203}

$$\frac{\left(144a^2b+64a^3+120ab^2+35b^3\right)\tan^{-1}(\sinh(c+dx))}{128d}+\frac{b\left(72a^2+92ab+35b^2\right)\tanh(c+dx)\text{sech}^3(c+dx)}{192d}+\frac{(144}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^3*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*ArcTan[Sinh[c + d*x]])/(128*d) +
 ((64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*Sech[c + d*x]*Tanh[c + d*x])/(1
28*d) + (b*(72*a^2 + 92*a*b + 35*b^2)*Sech[c + d*x]^3*Tanh[c + d*x])/(192*d
) + (b*(12*a + 7*b)*Sech[c + d*x]^5*(a + b + a*Sinh[c + d*x]^2)*Tanh[c + d*
x])/(48*d) + (b*Sech[c + d*x]^7*(a + b + a*Sinh[c + d*x]^2)^2*Tanh[c + d*x]
)/(8*d)
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 526

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^q)/(a*b*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p +
 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \operatorname{sech}^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx = \frac{\operatorname{Subst}\left(\int \frac{\left(a+b+ax^2\right)^3}{\left(1+x^2\right)^5}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{b\operatorname{sech}^7(c+dx)\left(a+b+a\sinh^2(c+dx)\right)^2\tanh(c+dx)}{8d} + \frac{\operatorname{Subst}\left(\int\right.}{}$$

$$= \frac{b(12a+7b)\operatorname{sech}^5(c+dx)\left(a+b+a\sinh^2(c+dx)\right)\tanh(c+dx)}{48d} +$$

$$= \frac{b\left(72a^2+92ab+35b^2\right)\operatorname{sech}^3(c+dx)\tanh(c+dx)}{192d} + \frac{b(12a+7b)\mathrm{se}}{}$$

$$= \frac{\left(64a^3+144a^2b+120ab^2+35b^3\right)\operatorname{sech}(c+dx)\tanh(c+dx)}{128d} + \frac{b\left(7\right.}{}$$

$$= \frac{\left(64a^3+144a^2b+120ab^2+35b^3\right)\tan^{-1}(\sinh(c+dx))}{128d} + \frac{\left(64a^3+14\right.}{}$$

Mathematica [A] time = 9.40, size = 297, normalized size = 1.52

$$\frac{\operatorname{sech}^8(c+dx)\left(a\cosh^2(c+dx)+b\right)^3\left(2b\left(144a^2+120ab+35b^2\right)\tanh(c)\cosh^5(c+dx)+2b\left(144a^2+120ab+3\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^3*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((b + a*Cosh[c + d*x]^2)^3*Sech[c + d*x]^8*(6*(64*a^3 + 144*a^2*b + 120*a*b
^2 + 35*b^3)*ArcTan[Tanh[(c + d*x)/2]]*Cosh[c + d*x]^8 + 48*b^3*Sech[c]*Sin
h[d*x] + 8*b^2*(24*a + 7*b)*Cosh[c + d*x]^2*Sech[c]*Sinh[d*x] + 2*b*(144*a^
2 + 120*a*b + 35*b^2)*Cosh[c + d*x]^4*Sech[c]*Sinh[d*x] + 3*(64*a^3 + 144*a
^2*b + 120*a*b^2 + 35*b^3)*Cosh[c + d*x]^6*Sech[c]*Sinh[d*x] + 48*b^3*Cosh[
c + d*x]*Tanh[c] + 8*b^2*(24*a + 7*b)*Cosh[c + d*x]^3*Tanh[c] + 2*b*(144*a^
2 + 120*a*b + 35*b^2)*Cosh[c + d*x]^5*Tanh[c] + 3*(64*a^3 + 144*a^2*b + 120
*a*b^2 + 35*b^3)*Cosh[c + d*x]^7*Tanh[c]))/(48*d*(a + 2*b + a*Cosh[2*(c + d
*x)])^3)
```

fricas [B] time = 0.50, size = 6114, normalized size = 31.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/192*(3*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^15 + 45*(6
4*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)*sinh(d*x + c)^14 + 3*
(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*sinh(d*x + c)^15 + (960*a^3 + 331
2*a^2*b + 2760*a*b^2 + 805*b^3)*cosh(d*x + c)^13 + (960*a^3 + 3312*a^2*b +
2760*a*b^2 + 805*b^3 + 315*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d
*x + c)^2)*sinh(d*x + c)^13 + 13*(105*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*
b^3)*cosh(d*x + c)^3 + (960*a^3 + 3312*a^2*b + 2760*a*b^2 + 805*b^3)*cosh(d
*x + c))*sinh(d*x + c)^12 + (1728*a^3 + 7344*a^2*b + 9192*a*b^2 + 2681*b^3)
*cosh(d*x + c)^11 + (4095*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*
x + c)^4 + 1728*a^3 + 7344*a^2*b + 9192*a*b^2 + 2681*b^3 + 78*(960*a^3 + 33
12*a^2*b + 2760*a*b^2 + 805*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^11 + 11*(81
9*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^5 + 26*(960*a^3 +
 3312*a^2*b + 2760*a*b^2 + 805*b^3)*cosh(d*x + c)^3 + (1728*a^3 + 7344*a^2*
b + 9192*a*b^2 + 2681*b^3)*cosh(d*x + c))*sinh(d*x + c)^10 + (960*a^3 + 446
4*a^2*b + 6792*a*b^2 + 5053*b^3)*cosh(d*x + c)^9 + (15015*(64*a^3 + 144*a^2
*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^6 + 715*(960*a^3 + 3312*a^2*b + 2760
*a*b^2 + 805*b^3)*cosh(d*x + c)^4 + 960*a^3 + 4464*a^2*b + 6792*a*b^2 + 505
3*b^3 + 55*(1728*a^3 + 7344*a^2*b + 9192*a*b^2 + 2681*b^3)*cosh(d*x + c)^2)
*sinh(d*x + c)^9 + 3*(6435*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d
*x + c)^7 + 429*(960*a^3 + 3312*a^2*b + 2760*a*b^2 + 805*b^3)*cosh(d*x + c)
^5 + 55*(1728*a^3 + 7344*a^2*b + 9192*a*b^2 + 2681*b^3)*cosh(d*x + c)^3 + 3
*(960*a^3 + 4464*a^2*b + 6792*a*b^2 + 5053*b^3)*cosh(d*x + c))*sinh(d*x + c
)^8 - (960*a^3 + 4464*a^2*b + 6792*a*b^2 + 5053*b^3)*cosh(d*x + c)^7 + (193
05*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^8 + 1716*(960*a^
3 + 3312*a^2*b + 2760*a*b^2 + 805*b^3)*cosh(d*x + c)^6 + 330*(1728*a^3 + 73
44*a^2*b + 9192*a*b^2 + 2681*b^3)*cosh(d*x + c)^4 - 960*a^3 - 4464*a^2*b -
6792*a*b^2 - 5053*b^3 + 36*(960*a^3 + 4464*a^2*b + 6792*a*b^2 + 5053*b^3)*c
osh(d*x + c)^2)*sinh(d*x + c)^7 + (15015*(64*a^3 + 144*a^2*b + 120*a*b^2 +
35*b^3)*cosh(d*x + c)^9 + 1716*(960*a^3 + 3312*a^2*b + 2760*a*b^2 + 805*b^3
)*cosh(d*x + c)^7 + 462*(1728*a^3 + 7344*a^2*b + 9192*a*b^2 + 2681*b^3)*cos
h(d*x + c)^5 + 84*(960*a^3 + 4464*a^2*b + 6792*a*b^2 + 5053*b^3)*cosh(d*x +
 c)^3 - 7*(960*a^3 + 4464*a^2*b + 6792*a*b^2 + 5053*b^3)*cosh(d*x + c))*sin
h(d*x + c)^6 - (1728*a^3 + 7344*a^2*b + 9192*a*b^2 + 2681*b^3)*cosh(d*x + c
)^5 + (9009*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^10 + 12
87*(960*a^3 + 3312*a^2*b + 2760*a*b^2 + 805*b^3)*cosh(d*x + c)^8 + 462*(172
8*a^3 + 7344*a^2*b + 9192*a*b^2 + 2681*b^3)*cosh(d*x + c)^6 + 126*(960*a^3
+ 4464*a^2*b + 6792*a*b^2 + 5053*b^3)*cosh(d*x + c)^4 - 1728*a^3 - 7344*a^2
*b - 9192*a*b^2 - 2681*b^3 - 21*(960*a^3 + 4464*a^2*b + 6792*a*b^2 + 5053*b
^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + (4095*(64*a^3 + 144*a^2*b + 120*a*b^
2 + 35*b^3)*cosh(d*x + c)^11 + 715*(960*a^3 + 3312*a^2*b + 2760*a*b^2 + 805
*b^3)*cosh(d*x + c)^9 + 330*(1728*a^3 + 7344*a^2*b + 9192*a*b^2 + 2681*b^3)
```

*cosh(d*x + c)^7 + 126*(960*a^3 + 4464*a^2*b + 6792*a*b^2 + 5053*b^3)*cosh(
d*x + c)^5 - 35*(960*a^3 + 4464*a^2*b + 6792*a*b^2 + 5053*b^3)*cosh(d*x + c
)^3 - 5*(1728*a^3 + 7344*a^2*b + 9192*a*b^2 + 2681*b^3)*cosh(d*x + c))*sinh
(d*x + c)^4 - (960*a^3 + 3312*a^2*b + 2760*a*b^2 + 805*b^3)*cosh(d*x + c)^3
 + (1365*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^12 + 286*(
960*a^3 + 3312*a^2*b + 2760*a*b^2 + 805*b^3)*cosh(d*x + c)^10 + 165*(1728*a
^3 + 7344*a^2*b + 9192*a*b^2 + 2681*b^3)*cosh(d*x + c)^8 + 84*(960*a^3 + 44
64*a^2*b + 6792*a*b^2 + 5053*b^3)*cosh(d*x + c)^6 - 35*(960*a^3 + 4464*a^2*
b + 6792*a*b^2 + 5053*b^3)*cosh(d*x + c)^4 - 960*a^3 - 3312*a^2*b - 2760*a*
b^2 - 805*b^3 - 10*(1728*a^3 + 7344*a^2*b + 9192*a*b^2 + 2681*b^3)*cosh(d*x
 + c)^2)*sinh(d*x + c)^3 + (315*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*c
osh(d*x + c)^13 + 78*(960*a^3 + 3312*a^2*b + 2760*a*b^2 + 805*b^3)*cosh(d*x
 + c)^11 + 55*(1728*a^3 + 7344*a^2*b + 9192*a*b^2 + 2681*b^3)*cosh(d*x + c)
^9 + 36*(960*a^3 + 4464*a^2*b + 6792*a*b^2 + 5053*b^3)*cosh(d*x + c)^7 - 21
*(960*a^3 + 4464*a^2*b + 6792*a*b^2 + 5053*b^3)*cosh(d*x + c)^5 - 10*(1728*
a^3 + 7344*a^2*b + 9192*a*b^2 + 2681*b^3)*cosh(d*x + c)^3 - 3*(960*a^3 + 33
12*a^2*b + 2760*a*b^2 + 805*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 3*((64*a^
3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^16 + 16*(64*a^3 + 144*a^2
*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)*sinh(d*x + c)^15 + (64*a^3 + 144*a^2
*b + 120*a*b^2 + 35*b^3)*sinh(d*x + c)^16 + 8*(64*a^3 + 144*a^2*b + 120*a*b
^2 + 35*b^3)*cosh(d*x + c)^14 + 8*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3
+ 15*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^2)*sinh(d*x +
c)^14 + 112*(5*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^3 +
(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c))*sinh(d*x + c)^13 +
 28*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^12 + 28*(65*(64
*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^4 + 64*a^3 + 144*a^2*b
 + 120*a*b^2 + 35*b^3 + 26*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d
*x + c)^2)*sinh(d*x + c)^12 + 112*(39*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*
b^3)*cosh(d*x + c)^5 + 26*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*
x + c)^3 + 3*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c))*sinh(
d*x + c)^11 + 56*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^10
 + 56*(143*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^6 + 143*
(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^4 + 64*a^3 + 144*a^
2*b + 120*a*b^2 + 35*b^3 + 33*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cos
h(d*x + c)^2)*sinh(d*x + c)^10 + 16*(715*(64*a^3 + 144*a^2*b + 120*a*b^2 +
35*b^3)*cosh(d*x + c)^7 + 1001*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*co
sh(d*x + c)^5 + 385*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)
^3 + 35*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c))*sinh(d*x +
 c)^9 + 70*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^8 + 2*(6
435*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^8 + 12012*(64*a
^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^6 + 6930*(64*a^3 + 144*a
^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^4 + 2240*a^3 + 5040*a^2*b + 4200*a
*b^2 + 1225*b^3 + 1260*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x +
 c)^2)*sinh(d*x + c)^8 + 16*(715*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*
cosh(d*x + c)^9 + 1716*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x +

c)^7 + 1386*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^5 + 42
0*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^3 + 35*(64*a^3 +
144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + 56*(64*a^3
 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^6 + 56*(143*(64*a^3 + 144*
a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^10 + 429*(64*a^3 + 144*a^2*b + 12
0*a*b^2 + 35*b^3)*cosh(d*x + c)^8 + 462*(64*a^3 + 144*a^2*b + 120*a*b^2 + 3
5*b^3)*cosh(d*x + c)^6 + 210*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh
(d*x + c)^4 + 64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3 + 35*(64*a^3 + 144*a^
2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 112*(39*(64*a^
3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^11 + 143*(64*a^3 + 144*a^
2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^9 + 198*(64*a^3 + 144*a^2*b + 120*a
*b^2 + 35*b^3)*cosh(d*x + c)^7 + 126*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b
^3)*cosh(d*x + c)^5 + 35*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x
 + c)^3 + 3*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c))*sinh(d
x + c)^5 + 28(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^4 +
28*(65*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^12 + 286*(64
*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^10 + 495*(64*a^3 + 144
*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^8 + 420*(64*a^3 + 144*a^2*b + 12
0*a*b^2 + 35*b^3)*cosh(d*x + c)^6 + 175*(64*a^3 + 144*a^2*b + 120*a*b^2 + 3
5*b^3)*cosh(d*x + c)^4 + 64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3 + 30*(64*a
^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 112
(5(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^13 + 26*(64*a^3
 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^11 + 55*(64*a^3 + 144*a^2*
b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^9 + 60*(64*a^3 + 144*a^2*b + 120*a*b^
2 + 35*b^3)*cosh(d*x + c)^7 + 35*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*
cosh(d*x + c)^5 + 10*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c
)^3 + (64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c))*sinh(d*x + c
)^3 + 64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3 + 8*(64*a^3 + 144*a^2*b + 120
*a*b^2 + 35*b^3)*cosh(d*x + c)^2 + 8*(15*(64*a^3 + 144*a^2*b + 120*a*b^2 +
35*b^3)*cosh(d*x + c)^14 + 91*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cos
h(d*x + c)^12 + 231*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)
^10 + 315*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^8 + 245*(
64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^6 + 105*(64*a^3 + 14
4*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^4 + 64*a^3 + 144*a^2*b + 120*a*
b^2 + 35*b^3 + 21*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^2
)*sinh(d*x + c)^2 + 16*((64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x
+ c)^15 + 7*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^13 + 21
*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^11 + 35*(64*a^3 +
144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^9 + 35*(64*a^3 + 144*a^2*b +
120*a*b^2 + 35*b^3)*cosh(d*x + c)^7 + 21*(64*a^3 + 144*a^2*b + 120*a*b^2 +
35*b^3)*cosh(d*x + c)^5 + 7*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(
d*x + c)^3 + (64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c))*sinh(
d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) - 3*(64*a^3 + 144*a^2*b + 1
20*a*b^2 + 35*b^3)*cosh(d*x + c) + (45*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35
*b^3)*cosh(d*x + c)^14 + 13*(960*a^3 + 3312*a^2*b + 2760*a*b^2 + 805*b^3)*c

```
osh(d*x + c)^12 + 11*(1728*a^3 + 7344*a^2*b + 9192*a*b^2 + 2681*b^3)*cosh(d
*x + c)^10 + 9*(960*a^3 + 4464*a^2*b + 6792*a*b^2 + 5053*b^3)*cosh(d*x + c)
^8 - 7*(960*a^3 + 4464*a^2*b + 6792*a*b^2 + 5053*b^3)*cosh(d*x + c)^6 - 5*(
1728*a^3 + 7344*a^2*b + 9192*a*b^2 + 2681*b^3)*cosh(d*x + c)^4 - 192*a^3 -
432*a^2*b - 360*a*b^2 - 105*b^3 - 3*(960*a^3 + 3312*a^2*b + 2760*a*b^2 + 80
5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^16 + 16*d*cosh(d*x
+ c)*sinh(d*x + c)^15 + d*sinh(d*x + c)^16 + 8*d*cosh(d*x + c)^14 + 8*(15*d
*cosh(d*x + c)^2 + d)*sinh(d*x + c)^14 + 112*(5*d*cosh(d*x + c)^3 + d*cosh(
d*x + c))*sinh(d*x + c)^13 + 28*d*cosh(d*x + c)^12 + 28*(65*d*cosh(d*x + c)
^4 + 26*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^12 + 112*(39*d*cosh(d*x + c)^5
 + 26*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^11 + 56*d*cosh(d
*x + c)^10 + 56*(143*d*cosh(d*x + c)^6 + 143*d*cosh(d*x + c)^4 + 33*d*cosh(
d*x + c)^2 + d)*sinh(d*x + c)^10 + 16*(715*d*cosh(d*x + c)^7 + 1001*d*cosh(
d*x + c)^5 + 385*d*cosh(d*x + c)^3 + 35*d*cosh(d*x + c))*sinh(d*x + c)^9 +
70*d*cosh(d*x + c)^8 + 2*(6435*d*cosh(d*x + c)^8 + 12012*d*cosh(d*x + c)^6
+ 6930*d*cosh(d*x + c)^4 + 1260*d*cosh(d*x + c)^2 + 35*d)*sinh(d*x + c)^8 +
 16*(715*d*cosh(d*x + c)^9 + 1716*d*cosh(d*x + c)^7 + 1386*d*cosh(d*x + c)^
5 + 420*d*cosh(d*x + c)^3 + 35*d*cosh(d*x + c))*sinh(d*x + c)^7 + 56*d*cosh
(d*x + c)^6 + 56*(143*d*cosh(d*x + c)^10 + 429*d*cosh(d*x + c)^8 + 462*d*co
sh(d*x + c)^6 + 210*d*cosh(d*x + c)^4 + 35*d*cosh(d*x + c)^2 + d)*sinh(d*x
+ c)^6 + 112*(39*d*cosh(d*x + c)^11 + 143*d*cosh(d*x + c)^9 + 198*d*cosh(d*
x + c)^7 + 126*d*cosh(d*x + c)^5 + 35*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)
)*sinh(d*x + c)^5 + 28*d*cosh(d*x + c)^4 + 28*(65*d*cosh(d*x + c)^12 + 286*
d*cosh(d*x + c)^10 + 495*d*cosh(d*x + c)^8 + 420*d*cosh(d*x + c)^6 + 175*d*
cosh(d*x + c)^4 + 30*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^4 + 112*(5*d*cosh
(d*x + c)^13 + 26*d*cosh(d*x + c)^11 + 55*d*cosh(d*x + c)^9 + 60*d*cosh(d*x
 + c)^7 + 35*d*cosh(d*x + c)^5 + 10*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*si
nh(d*x + c)^3 + 8*d*cosh(d*x + c)^2 + 8*(15*d*cosh(d*x + c)^14 + 91*d*cosh(
d*x + c)^12 + 231*d*cosh(d*x + c)^10 + 315*d*cosh(d*x + c)^8 + 245*d*cosh(d
*x + c)^6 + 105*d*cosh(d*x + c)^4 + 21*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)
^2 + 16*(d*cosh(d*x + c)^15 + 7*d*cosh(d*x + c)^13 + 21*d*cosh(d*x + c)^11
+ 35*d*cosh(d*x + c)^9 + 35*d*cosh(d*x + c)^7 + 21*d*cosh(d*x + c)^5 + 7*d*
cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

giac [B] time = 0.18, size = 485, normalized size = 2.47

$$\frac{3\left(\pi + 2\,\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)\left(64\,a^3+144\,a^2b+120\,ab^2+35\,b^3\right)+\frac{4\left(192\,a^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)^7+432\,a^2b\left(e^{(dx+\cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/768*(3*(pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(64*a^3 +
```

```
144*a^2*b + 120*a*b^2 + 35*b^3) + 4*(192*a^3*(e^(d*x + c) - e^(-d*x - c))^7
 + 432*a^2*b*(e^(d*x + c) - e^(-d*x - c))^7 + 360*a*b^2*(e^(d*x + c) - e^(-
d*x - c))^7 + 105*b^3*(e^(d*x + c) - e^(-d*x - c))^7 + 2304*a^3*(e^(d*x + c
) - e^(-d*x - c))^5 + 6336*a^2*b*(e^(d*x + c) - e^(-d*x - c))^5 + 5280*a*b^
2*(e^(d*x + c) - e^(-d*x - c))^5 + 1540*b^3*(e^(d*x + c) - e^(-d*x - c))^5
+ 9216*a^3*(e^(d*x + c) - e^(-d*x - c))^3 + 29952*a^2*b*(e^(d*x + c) - e^(-
d*x - c))^3 + 28032*a*b^2*(e^(d*x + c) - e^(-d*x - c))^3 + 8176*b^3*(e^(d*x
 + c) - e^(-d*x - c))^3 + 12288*a^3*(e^(d*x + c) - e^(-d*x - c)) + 46080*a^
2*b*(e^(d*x + c) - e^(-d*x - c)) + 50688*a*b^2*(e^(d*x + c) - e^(-d*x - c))
 + 17856*b^3*(e^(d*x + c) - e^(-d*x - c)))/((e^(d*x + c) - e^(-d*x - c))^2
+ 4)^4)/d
```

maple [A] time = 0.47, size = 280, normalized size = 1.43

$$\frac{a^3 \mathrm{sech}\,(dx+c)\tanh\,(dx+c)}{2d}+\frac{a^3\arctan\left(\mathrm{e}^{dx+c}\right)}{d}+\frac{3a^2b\tanh\,(dx+c)\,\mathrm{sech}\,(dx+c)^3}{4d}+\frac{9a^2b\,\mathrm{sech}\,(dx+c)\tanh\,(c}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] 1/2/d*a^3*sech(d*x+c)*tanh(d*x+c)+1/d*a^3*arctan(exp(d*x+c))+3/4/d*a^2*b*ta
nh(d*x+c)*sech(d*x+c)^3+9/8/d*a^2*b*sech(d*x+c)*tanh(d*x+c)+9/4/d*a^2*b*arc
tan(exp(d*x+c))+1/2/d*a*b^2*tanh(d*x+c)*sech(d*x+c)^5+5/8/d*a*b^2*tanh(d*x+
c)*sech(d*x+c)^3+15/16/d*a*b^2*sech(d*x+c)*tanh(d*x+c)+15/8/d*a*b^2*arctan(
exp(d*x+c))+1/8/d*b^3*tanh(d*x+c)*sech(d*x+c)^7+7/48/d*b^3*tanh(d*x+c)*sech
(d*x+c)^5+35/192/d*b^3*tanh(d*x+c)*sech(d*x+c)^3+35/128/d*b^3*sech(d*x+c)*t
anh(d*x+c)+35/64/d*b^3*arctan(exp(d*x+c))
```

maxima [B] time = 0.42, size = 556, normalized size = 2.84

$$-\frac{1}{192}\,b^3\left(\frac{105\,\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{105\,e^{(-dx-c)}+805\,e^{(-3\,dx-3\,c)}+2681\,e^{(-5\,dx-5\,c)}+5053\,e^{(-7\,dx-7\,c)}-5053\,e^{(-9\,dx-9}}{d\left(8\,e^{(-2\,dx-2\,c)}+28\,e^{(-4\,dx-4\,c)}+56\,e^{(-6\,dx-6\,c)}+70\,e^{(-8\,dx-8\,c)}+56\,e^{(-10\,dx-10}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/192*b^3*(105*arctan(e^(-d*x - c))/d - (105*e^(-d*x - c) + 805*e^(-3*d*x
- 3*c) + 2681*e^(-5*d*x - 5*c) + 5053*e^(-7*d*x - 7*c) - 5053*e^(-9*d*x - 9
*c) - 2681*e^(-11*d*x - 11*c) - 805*e^(-13*d*x - 13*c) - 105*e^(-15*d*x - 1
5*c))/(d*(8*e^(-2*d*x - 2*c) + 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x - 6*c) +
70*e^(-8*d*x - 8*c) + 56*e^(-10*d*x - 10*c) + 28*e^(-12*d*x - 12*c) + 8*e^(
-14*d*x - 14*c) + e^(-16*d*x - 16*c) + 1))) - 1/8*a*b^2*(15*arctan(e^(-d*x
- c))/d - (15*e^(-d*x - c) + 85*e^(-3*d*x - 3*c) + 198*e^(-5*d*x - 5*c) - 1
98*e^(-7*d*x - 7*c) - 85*e^(-9*d*x - 9*c) - 15*e^(-11*d*x - 11*c))/(d*(6*e^
```

```
(-2*d*x - 2*c) + 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) + 15*e^(-8*d*x -
 8*c) + 6*e^(-10*d*x - 10*c) + e^(-12*d*x - 12*c) + 1))) - 3/4*a^2*b*(3*arc
tan(e^(-d*x - c))/d - (3*e^(-d*x - c) + 11*e^(-3*d*x - 3*c) - 11*e^(-5*d*x
- 5*c) - 3*e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*d*x - 4*c) +
4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) - a^3*(arctan(e^(-d*x - c))/d
- (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*
c) + 1)))
```

mupad [B] time = 1.60, size = 931, normalized size = 4.75

$$\frac{\operatorname{atan}\left(\frac{e^{d\,x}\,e^{c}\left(64\,a^3\,\sqrt{d^2}+35\,b^3\,\sqrt{d^2}+120\,a\,b^2\,\sqrt{d^2}+144\,a^2\,b\,\sqrt{d^2}\right)}{d\,\sqrt{4096\,a^6+18432\,a^5\,b+36096\,a^4\,b^2+39040\,a^3\,b^3+24480\,a^2\,b^4+8400\,a\,b^5+1225\,b^6}}\right)\sqrt{4096\,a^6+18432\,a^5\,b+36096\,a^4\,b^2+390}{64\,\sqrt{d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cosh(c + d*x)^2)^3/cosh(c + d*x)^3,x)
```

```
[Out] (atan((exp(d*x)*exp(c)*(64*a^3*(d^2)^(1/2) + 35*b^3*(d^2)^(1/2) + 120*a*b^2
*(d^2)^(1/2) + 144*a^2*b*(d^2)^(1/2)))/(d*(8400*a*b^5 + 18432*a^5*b + 4096*
a^6 + 1225*b^6 + 24480*a^2*b^4 + 39040*a^3*b^3 + 36096*a^4*b^2)^(1/2)))*(84
00*a*b^5 + 18432*a^5*b + 4096*a^6 + 1225*b^6 + 24480*a^2*b^4 + 39040*a^3*b^
3 + 36096*a^4*b^2)^(1/2))/(64*(d^2)^(1/2)) - ((a^3*exp(c + d*x))/(2*d) + (2
*exp(7*c + 7*d*x)*(24*a*b^2 + 18*a^2*b + 5*a^3 + 16*b^3))/d + (a^3*exp(13*c
 + 13*d*x))/(2*d) + (3*a*exp(5*c + 5*d*x)*(16*a*b + 5*a^2 + 16*b^2))/(2*d)
+ (3*a*exp(9*c + 9*d*x)*(16*a*b + 5*a^2 + 16*b^2))/(2*d) + (3*a^2*exp(3*c +
 3*d*x)*(a + 2*b))/d + (3*a^2*exp(11*c + 11*d*x)*(a + 2*b))/d)/(8*exp(2*c +
 2*d*x) + 28*exp(4*c + 4*d*x) + 56*exp(6*c + 6*d*x) + 70*exp(8*c + 8*d*x) +
 56*exp(10*c + 10*d*x) + 28*exp(12*c + 12*d*x) + 8*exp(14*c + 14*d*x) + exp
(16*c + 16*d*x) + 1) + (2*exp(c + d*x)*(48*a*b^2 - 37*b^3))/(3*d*(5*exp(2*c
 + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x)
+ exp(10*c + 10*d*x) + 1)) + (exp(c + d*x)*(24*a^2*b - 120*a*b^2 + b^3))/(4
*d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c
+ 8*d*x) + 1)) - (16*b^3*exp(c + d*x))/(d*(7*exp(2*c + 2*d*x) + 21*exp(4*c
+ 4*d*x) + 35*exp(6*c + 6*d*x) + 35*exp(8*c + 8*d*x) + 21*exp(10*c + 10*d*x
) + 7*exp(12*c + 12*d*x) + exp(14*c + 14*d*x) + 1)) + (exp(c + d*x)*(120*a*
b^2 + 144*a^2*b + 64*a^3 + 35*b^3))/(64*d*(exp(2*c + 2*d*x) + 1)) - (4*exp(
c + d*x)*(6*a*b^2 - 29*b^3))/(3*d*(6*exp(2*c + 2*d*x) + 15*exp(4*c + 4*d*x)
 + 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*exp(10*c + 10*d*x) + exp(1
2*c + 12*d*x) + 1)) + (exp(c + d*x)*(120*a*b^2 + 144*a^2*b - 144*a^3 + 35*b
^3))/(96*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) + (exp(c + d*x)*(24
*a*b^2 - 288*a^2*b + 7*b^3))/(24*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x)
 + exp(6*c + 6*d*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\,\mathrm{sech}^{2}\left(c+dx\right)\right)^{3}\mathrm{sech}^{3}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**3*(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**3*sech(c + d*x)**3, x)
```

3.72 $\int \mathbf{sech}^4(c+dx)\left(a+b\mathbf{sech}^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=108

$$-\frac{b^2(3a+4b)\tanh^7(c+dx)}{7d}+\frac{3b(a+b)(a+2b)\tanh^5(c+dx)}{5d}-\frac{(a+b)^2(a+4b)\tanh^3(c+dx)}{3d}+\frac{(a+b)^3\tanh(c+}{d}$$

```
[Out] (a+b)^3*tanh(d*x+c)/d-1/3*(a+b)^2*(a+4*b)*tanh(d*x+c)^3/d+3/5*b*(a+b)*(a+2*
b)*tanh(d*x+c)^5/d-1/7*b^2*(3*a+4*b)*tanh(d*x+c)^7/d+1/9*b^3*tanh(d*x+c)^9/
d
```

Rubi [A] time = 0.09, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4146, 373}

$$-\frac{b^2(3a+4b)\tanh^7(c+dx)}{7d}+\frac{3b(a+b)(a+2b)\tanh^5(c+dx)}{5d}-\frac{(a+b)^2(a+4b)\tanh^3(c+dx)}{3d}+\frac{(a+b)^3\tanh(c+}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^4*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((a + b)^3*Tanh[c + d*x])/d - ((a + b)^2*(a + 4*b)*Tanh[c + d*x]^3)/(3*d) +
 (3*b*(a + b)*(a + 2*b)*Tanh[c + d*x]^5)/(5*d) - (b^2*(3*a + 4*b)*Tanh[c +
d*x]^7)/(7*d) + (b^3*Tanh[c + d*x]^9)/(9*d)
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \operatorname{sech}^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx = \frac{\operatorname{Subst}\left(\int \left(1-x^2\right)\left(a+b-bx^2\right)^3\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\operatorname{Subst}\left(\int \left((a+b)^3-(a+b)^2(a+4b)x^2+3b(a+b)(a+2b)x^4-b^2\right.\right.}{d}$$
$$= \frac{(a+b)^3\tanh(c+dx)}{d} - \frac{(a+b)^2(a+4b)\tanh^3(c+dx)}{3d} + \frac{3b(a+b}{}$$

Mathematica [B] time = 1.76, size = 348, normalized size = 3.22

$$\frac{\operatorname{sech}(c)\operatorname{sech}^9(c+dx)\left(6825a^3\sinh(2c+3dx)-1995a^3\sinh(4c+3dx)+3465a^3\sinh(4c+5dx)-315a^3\sinh(6\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^4*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (Sech[c]*Sech[c + d*x]^9*(63*(125*a^3 + 324*a^2*b + 312*a*b^2 + 128*b^3)*Si
nh[d*x] - 315*a*(17*a^2 + 36*a*b + 24*b^2)*Sinh[2*c + d*x] + 6825*a^3*Sinh[
2*c + 3*d*x] + 18648*a^2*b*Sinh[2*c + 3*d*x] + 18144*a*b^2*Sinh[2*c + 3*d*x
] + 5376*b^3*Sinh[2*c + 3*d*x] - 1995*a^3*Sinh[4*c + 3*d*x] - 2520*a^2*b*Si
nh[4*c + 3*d*x] + 3465*a^3*Sinh[4*c + 5*d*x] + 9072*a^2*b*Sinh[4*c + 5*d*x]
 + 7776*a*b^2*Sinh[4*c + 5*d*x] + 2304*b^3*Sinh[4*c + 5*d*x] - 315*a^3*Sinh
[6*c + 5*d*x] + 945*a^3*Sinh[6*c + 7*d*x] + 2268*a^2*b*Sinh[6*c + 7*d*x] +
1944*a*b^2*Sinh[6*c + 7*d*x] + 576*b^3*Sinh[6*c + 7*d*x] + 105*a^3*Sinh[8*c
 + 9*d*x] + 252*a^2*b*Sinh[8*c + 9*d*x] + 216*a*b^2*Sinh[8*c + 9*d*x] + 64*
b^3*Sinh[8*c + 9*d*x]))/(40320*d)
```

fricas [B] time = 0.42, size = 1190, normalized size = 11.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -8/315*(2*(105*a^3 + 63*a^2*b + 54*a*b^2 + 16*b^3)*cosh(d*x + c)^7 + 14*(10
5*a^3 + 63*a^2*b + 54*a*b^2 + 16*b^3)*cosh(d*x + c)*sinh(d*x + c)^6 + (105*
a^3 - 126*a^2*b - 108*a*b^2 - 32*b^3)*sinh(d*x + c)^7 + 6*(245*a^3 + 399*a^
2*b + 162*a*b^2 + 48*b^3)*cosh(d*x + c)^5 + 3*(175*a^3 + 42*a^2*b - 324*a*b
^2 - 96*b^3 + 7*(105*a^3 - 126*a^2*b - 108*a*b^2 - 32*b^3)*cosh(d*x + c)^2)
*sinh(d*x + c)^5 + 10*(7*(105*a^3 + 63*a^2*b + 54*a*b^2 + 16*b^3)*cosh(d*x
+ c)^3 + 3*(245*a^3 + 399*a^2*b + 162*a*b^2 + 48*b^3)*cosh(d*x + c))*sinh(d
```

```
*x + c)^4 + 18*(245*a^3 + 567*a^2*b + 426*a*b^2 + 64*b^3)*cosh(d*x + c)^3 +
 (35*(105*a^3 - 126*a^2*b - 108*a*b^2 - 32*b^3)*cosh(d*x + c)^4 + 945*a^3 +
 1134*a^2*b - 108*a*b^2 - 1152*b^3 + 30*(175*a^3 + 42*a^2*b - 324*a*b^2 - 9
6*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 6*(7*(105*a^3 + 63*a^2*b + 54*a*b
^2 + 16*b^3)*cosh(d*x + c)^5 + 10*(245*a^3 + 399*a^2*b + 162*a*b^2 + 48*b^3
)*cosh(d*x + c)^3 + 9*(245*a^3 + 567*a^2*b + 426*a*b^2 + 64*b^3)*cosh(d*x +
 c))*sinh(d*x + c)^2 + 210*(35*a^3 + 93*a^2*b + 90*a*b^2 + 32*b^3)*cosh(d*x
 + c) + (7*(105*a^3 - 126*a^2*b - 108*a*b^2 - 32*b^3)*cosh(d*x + c)^6 + 15*
(175*a^3 + 42*a^2*b - 324*a*b^2 - 96*b^3)*cosh(d*x + c)^4 + 525*a^3 + 882*a
^2*b + 756*a*b^2 + 1344*b^3 + 27*(105*a^3 + 126*a^2*b - 12*a*b^2 - 128*b^3)
*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^11 + 11*d*cosh(d*x + c)*s
inh(d*x + c)^10 + d*sinh(d*x + c)^11 + 9*d*cosh(d*x + c)^9 + (55*d*cosh(d*x
 + c)^2 + 9*d)*sinh(d*x + c)^9 + 3*(55*d*cosh(d*x + c)^3 + 27*d*cosh(d*x +
c))*sinh(d*x + c)^8 + 37*d*cosh(d*x + c)^7 + (330*d*cosh(d*x + c)^4 + 324*d
*cosh(d*x + c)^2 + 35*d)*sinh(d*x + c)^7 + 7*(66*d*cosh(d*x + c)^5 + 108*d*
cosh(d*x + c)^3 + 37*d*cosh(d*x + c))*sinh(d*x + c)^6 + 93*d*cosh(d*x + c)^
5 + 3*(154*d*cosh(d*x + c)^6 + 378*d*cosh(d*x + c)^4 + 245*d*cosh(d*x + c)^
2 + 25*d)*sinh(d*x + c)^5 + (330*d*cosh(d*x + c)^7 + 1134*d*cosh(d*x + c)^5
 + 1295*d*cosh(d*x + c)^3 + 465*d*cosh(d*x + c))*sinh(d*x + c)^4 + 162*d*co
sh(d*x + c)^3 + (165*d*cosh(d*x + c)^8 + 756*d*cosh(d*x + c)^6 + 1225*d*cos
h(d*x + c)^4 + 750*d*cosh(d*x + c)^2 + 90*d)*sinh(d*x + c)^3 + (55*d*cosh(d
*x + c)^9 + 324*d*cosh(d*x + c)^7 + 777*d*cosh(d*x + c)^5 + 930*d*cosh(d*x
+ c)^3 + 486*d*cosh(d*x + c))*sinh(d*x + c)^2 + 210*d*cosh(d*x + c) + (11*d
*cosh(d*x + c)^10 + 81*d*cosh(d*x + c)^8 + 245*d*cosh(d*x + c)^6 + 375*d*co
sh(d*x + c)^4 + 270*d*cosh(d*x + c)^2 + 42*d)*sinh(d*x + c))
```

giac [B] time = 0.18, size = 360, normalized size = 3.33

$$-\frac{4\left(315\,a^3e^{(14\,dx+14\,c)} + 1995\,a^3e^{(12\,dx+12\,c)} + 2520\,a^2be^{(12\,dx+12\,c)} + 5355\,a^3e^{(10\,dx+10\,c)} + 11340\,a^2be^{(10\,dx+10\,c)} + 756\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -4/315*(315*a^3*e^(14*d*x + 14*c) + 1995*a^3*e^(12*d*x + 12*c) + 2520*a^2*b
*e^(12*d*x + 12*c) + 5355*a^3*e^(10*d*x + 10*c) + 11340*a^2*b*e^(10*d*x + 1
0*c) + 7560*a*b^2*e^(10*d*x + 10*c) + 7875*a^3*e^(8*d*x + 8*c) + 20412*a^2*
b*e^(8*d*x + 8*c) + 19656*a*b^2*e^(8*d*x + 8*c) + 8064*b^3*e^(8*d*x + 8*c)
+ 6825*a^3*e^(6*d*x + 6*c) + 18648*a^2*b*e^(6*d*x + 6*c) + 18144*a*b^2*e^(6
*d*x + 6*c) + 5376*b^3*e^(6*d*x + 6*c) + 3465*a^3*e^(4*d*x + 4*c) + 9072*a^
2*b*e^(4*d*x + 4*c) + 7776*a*b^2*e^(4*d*x + 4*c) + 2304*b^3*e^(4*d*x + 4*c)
 + 945*a^3*e^(2*d*x + 2*c) + 2268*a^2*b*e^(2*d*x + 2*c) + 1944*a*b^2*e^(2*d
*x + 2*c) + 576*b^3*e^(2*d*x + 2*c) + 105*a^3 + 252*a^2*b + 216*a*b^2 + 64*
b^3)/(d*(e^(2*d*x + 2*c) + 1)^9)
```

maple [A] time = 0.52, size = 158, normalized size = 1.46

$$a^3\left(\frac{2}{3}+\frac{\mathrm{sech}(dx+c)^2}{3}\right)\tanh(dx+c)+3a^2b\left(\frac{8}{15}+\frac{\mathrm{sech}(dx+c)^4}{5}+\frac{4\mathrm{sech}(dx+c)^2}{15}\right)\tanh(dx+c)+3a\,b^2\left(\frac{16}{35}+\frac{\mathrm{sech}(dx+c)^6}{7}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(a^3*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c)+3*a^2*b*(8/15+1/5*sech(d*x+c)^
4+4/15*sech(d*x+c)^2)*tanh(d*x+c)+3*a*b^2*(16/35+1/7*sech(d*x+c)^6+6/35*sec
h(d*x+c)^4+8/35*sech(d*x+c)^2)*tanh(d*x+c)+b^3*(128/315+1/9*sech(d*x+c)^8+8
/63*sech(d*x+c)^6+16/105*sech(d*x+c)^4+64/315*sech(d*x+c)^2)*tanh(d*x+c))
```

maxima [B] time = 0.34, size = 1245, normalized size = 11.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 256/315*b^3*(9*e^(-2*d*x - 2*c)/(d*(9*e^(-2*d*x - 2*c) + 36*e^(-4*d*x - 4*c
) + 84*e^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) + 8
4*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) + 9*e^(-16*d*x - 16*c) + e^(-1
8*d*x - 18*c) + 1)) + 36*e^(-4*d*x - 4*c)/(d*(9*e^(-2*d*x - 2*c) + 36*e^(-4
*d*x - 4*c) + 84*e^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x -
 10*c) + 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) + 9*e^(-16*d*x - 16*
c) + e^(-18*d*x - 18*c) + 1)) + 84*e^(-6*d*x - 6*c)/(d*(9*e^(-2*d*x - 2*c)
+ 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*c) + 126*e^
(-10*d*x - 10*c) + 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) + 9*e^(-16
*d*x - 16*c) + e^(-18*d*x - 18*c) + 1)) + 126*e^(-8*d*x - 8*c)/(d*(9*e^(-2*
d*x - 2*c) + 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*
c) + 126*e^(-10*d*x - 10*c) + 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c)
 + 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) + 1)) + 1/(d*(9*e^(-2*d*x - 2*
c) + 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*c) + 126
*e^(-10*d*x - 10*c) + 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) + 9*e^(
-16*d*x - 16*c) + e^(-18*d*x - 18*c) + 1))) + 96/35*a*b^2*(7*e^(-2*d*x - 2*
c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*
e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*
x - 14*c) + 1)) + 21*e^(-4*d*x - 4*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x
 - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c)
 + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 35*e^(-6*d*x - 6*c)/(d
*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8
*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 1
```

```
4*c) + 1)) + 1/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x
- 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c)
 + e^(-14*d*x - 14*c) + 1))) + 16/5*a^2*b*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d
*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c)
+ e^(-10*d*x - 10*c) + 1)) + 10*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 1
0*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x
- 10*c) + 1)) + 1/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d
*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) + 4/3*a^3*(3*e^(
-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c
) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c)
+ 1)))
```

mupad [B] time = 1.47, size = 1333, normalized size = 12.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cosh(c + d*x)^2)^3/cosh(c + d*x)^4,x)
```

```
[Out] - ((16*(24*a*b^2 + 18*a^2*b + 5*a^3 + 16*b^3))/(315*d) + (4*a^3*exp(6*c + 6
*d*x))/(9*d) + (4*a*exp(2*c + 2*d*x)*(16*a*b + 5*a^2 + 16*b^2))/(21*d) + (8
*a^2*exp(4*c + 4*d*x)*(a + 2*b))/(7*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c +
4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1)
 - ((32*exp(8*c + 8*d*x)*(24*a*b^2 + 18*a^2*b + 5*a^3 + 16*b^3))/(9*d) + (8
*a^3*exp(2*c + 2*d*x))/(9*d) + (8*a^3*exp(14*c + 14*d*x))/(9*d) + (8*a*exp(
6*c + 6*d*x)*(16*a*b + 5*a^2 + 16*b^2))/(3*d) + (8*a*exp(10*c + 10*d*x)*(16
*a*b + 5*a^2 + 16*b^2))/(3*d) + (16*a^2*exp(4*c + 4*d*x)*(a + 2*b))/(3*d) +
 (16*a^2*exp(12*c + 12*d*x)*(a + 2*b))/(3*d))/(9*exp(2*c + 2*d*x) + 36*exp(
4*c + 4*d*x) + 84*exp(6*c + 6*d*x) + 126*exp(8*c + 8*d*x) + 126*exp(10*c +
10*d*x) + 84*exp(12*c + 12*d*x) + 36*exp(14*c + 14*d*x) + 9*exp(16*c + 16*d
*x) + exp(18*c + 18*d*x) + 1) - ((4*a^2*(a + 2*b))/(21*d) + (2*a^3*exp(2*c
+ 2*d*x))/(9*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x
) + 1) - ((a*(16*a*b + 5*a^2 + 16*b^2))/(21*d) + (16*exp(2*c + 2*d*x)*(24*a
*b^2 + 18*a^2*b + 5*a^3 + 16*b^3))/(63*d) + (5*a^3*exp(8*c + 8*d*x))/(9*d)
+ (10*a*exp(4*c + 4*d*x)*(16*a*b + 5*a^2 + 16*b^2))/(21*d) + (40*a^2*exp(6*
c + 6*d*x)*(a + 2*b))/(21*d))/(6*exp(2*c + 2*d*x) + 15*exp(4*c + 4*d*x) + 2
0*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*exp(10*c + 10*d*x) + exp(12*c
+ 12*d*x) + 1) - (a^3/(9*d) + (16*exp(6*c + 6*d*x)*(24*a*b^2 + 18*a^2*b + 5
*a^3 + 16*b^3))/(9*d) + (7*a^3*exp(12*c + 12*d*x))/(9*d) + (a*exp(4*c + 4*d
*x)*(16*a*b + 5*a^2 + 16*b^2))/d + (5*a*exp(8*c + 8*d*x)*(16*a*b + 5*a^2 +
16*b^2))/(3*d) + (4*a^2*exp(2*c + 2*d*x)*(a + 2*b))/(3*d) + (4*a^2*exp(10*c
 + 10*d*x)*(a + 2*b))/d)/(8*exp(2*c + 2*d*x) + 28*exp(4*c + 4*d*x) + 56*exp
(6*c + 6*d*x) + 70*exp(8*c + 8*d*x) + 56*exp(10*c + 10*d*x) + 28*exp(12*c +
 12*d*x) + 8*exp(14*c + 14*d*x) + exp(16*c + 16*d*x) + 1) - ((a*(16*a*b + 5
*a^2 + 16*b^2))/(21*d) + (a^3*exp(4*c + 4*d*x))/(3*d) + (4*a^2*exp(2*c + 2*
```

```
d*x)*(a + 2*b))/(7*d))/(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c
 + 6*d*x) + exp(8*c + 8*d*x) + 1) - ((4*a^2*(a + 2*b))/(21*d) + (16*exp(4*c
 + 4*d*x)*(24*a*b^2 + 18*a^2*b + 5*a^3 + 16*b^3))/(21*d) + (2*a^3*exp(10*c
+ 10*d*x))/(3*d) + (2*a*exp(2*c + 2*d*x)*(16*a*b + 5*a^2 + 16*b^2))/(7*d) +
 (20*a*exp(6*c + 6*d*x)*(16*a*b + 5*a^2 + 16*b^2))/(21*d) + (20*a^2*exp(8*c
 + 8*d*x)*(a + 2*b))/(7*d))/(7*exp(2*c + 2*d*x) + 21*exp(4*c + 4*d*x) + 35*
exp(6*c + 6*d*x) + 35*exp(8*c + 8*d*x) + 21*exp(10*c + 10*d*x) + 7*exp(12*c
 + 12*d*x) + exp(14*c + 14*d*x) + 1) - a^3/(9*d*(2*exp(2*c + 2*d*x) + exp(4
*c + 4*d*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\,\text{sech}^2\,(c + dx)\right)^3 \text{sech}^4\,(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**4*(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**3*sech(c + d*x)**4, x)
```

3.73 $\int \frac{\cosh^4(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=117

$$-\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a^3d\sqrt{a+b}}+\frac{(3a-4b)\sinh(c+dx)\cosh(c+dx)}{8a^2d}+\frac{x\left(3a^2-4ab+8b^2\right)}{8a^3}+\frac{\sinh(c+dx)\cosh^3(c+$$

```
[Out] 1/8*(3*a^2-4*a*b+8*b^2)*x/a^3+1/8*(3*a-4*b)*cosh(d*x+c)*sinh(d*x+c)/a^2/d+1
/4*cosh(d*x+c)^3*sinh(d*x+c)/a/d-b^(5/2)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^
(1/2))/a^3/d/(a+b)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4146, 414, 527, 522, 206, 208}

$$-\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a^3d\sqrt{a+b}}+\frac{x\left(3a^2-4ab+8b^2\right)}{8a^3}+\frac{(3a-4b)\sinh(c+dx)\cosh(c+dx)}{8a^2d}+\frac{\sinh(c+dx)\cosh^3(c+$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^4/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((3*a^2 - 4*a*b + 8*b^2)*x)/(8*a^3) - (b^(5/2)*ArcTanh[(Sqrt[b]*Tanh[c + d*
x])/Sqrt[a + b]])/(a^3*Sqrt[a + b]*d) + ((3*a - 4*b)*Cosh[c + d*x]*Sinh[c +
 d*x])/(8*a^2*d) + (Cosh[c + d*x]^3*Sinh[c + d*x])/(4*a*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
```

```
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\cosh^4(c+dx)}{a+b\,\text{sech}^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^3(a+b-bx^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\cosh^3(c+dx)\sinh(c+dx)}{4ad} + \frac{\text{Subst}\left(\int \frac{3a-b-3bx^2}{(1-x^2)^2(a+b-bx^2)}\,dx, x, \tanh(c+dx)\right)}{4ad}$$

$$= \frac{(3a-4b)\cosh(c+dx)\sinh(c+dx)}{8a^2d} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4ad} + \frac{\text{Subst}\left(\int \frac{3a^2-ab}{(1-}\right.}{}$$

$$= \frac{(3a-4b)\cosh(c+dx)\sinh(c+dx)}{8a^2d} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4ad} - \frac{b^3\,\text{Subst}\left(\int \frac{}{a+b}\right.}{}$$

$$= \frac{(3a^2-4ab+8b^2)\,x}{8a^3} - \frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a^3\sqrt{a+b}\,d} + \frac{(3a-4b)\cosh(c+dx)\sinh(c+}{8a^2d}$$

Mathematica [A] time = 0.50, size = 95, normalized size = 0.81

$$\frac{4\left(3a^2-4ab+8b^2\right)(c+dx)+a^2\sinh(4(c+dx))-\frac{32b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{a+b}}+8a(a-b)\sinh(2(c+dx))}{32a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^4/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] (4*(3*a^2 - 4*a*b + 8*b^2)*(c + d*x) - (32*b^(5/2)*ArcTanh[(Sqrt[b]*Tanh[c
+ d*x])/Sqrt[a + b]])/Sqrt[a + b] + 8*a*(a - b)*Sinh[2*(c + d*x)] + a^2*Sin
h[4*(c + d*x)])/(32*a^3*d)
```

fricas [B] time = 0.46, size = 1713, normalized size = 14.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/64*(a^2*cosh(d*x + c)^8 + 8*a^2*cosh(d*x + c)*sinh(d*x + c)^7 + a^2*sinh
(d*x + c)^8 + 8*(3*a^2 - 4*a*b + 8*b^2)*d*x*cosh(d*x + c)^4 + 8*(a^2 - a*b)
*cosh(d*x + c)^6 + 4*(7*a^2*cosh(d*x + c)^2 + 2*a^2 - 2*a*b)*sinh(d*x + c)^
```

```
6 + 8*(7*a^2*cosh(d*x + c)^3 + 6*(a^2 - a*b)*cosh(d*x + c))*sinh(d*x + c)^5
 + 2*(35*a^2*cosh(d*x + c)^4 + 4*(3*a^2 - 4*a*b + 8*b^2)*d*x + 60*(a^2 - a*
b)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*a^2*cosh(d*x + c)^5 + 4*(3*a^2 -
 4*a*b + 8*b^2)*d*x*cosh(d*x + c) + 20*(a^2 - a*b)*cosh(d*x + c)^3)*sinh(d*
x + c)^3 - 8*(a^2 - a*b)*cosh(d*x + c)^2 + 4*(7*a^2*cosh(d*x + c)^6 + 12*(3
*a^2 - 4*a*b + 8*b^2)*d*x*cosh(d*x + c)^2 + 30*(a^2 - a*b)*cosh(d*x + c)^4
- 2*a^2 + 2*a*b)*sinh(d*x + c)^2 + 32*(b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x
 + c)^3*sinh(d*x + c) + 6*b^2*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b^2*cosh(
d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4)*sqrt(b/(a + b))*log((a^2*co
sh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 +
 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*
sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a
*b)*cosh(d*x + c))*sinh(d*x + c) + 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2
+ a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 + 3*
a*b + 2*b^2)*sqrt(b/(a + b)))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d
*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d
*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cos
h(d*x + c))*sinh(d*x + c) + a)) - a^2 + 8*(a^2*cosh(d*x + c)^7 + 4*(3*a^2 -
 4*a*b + 8*b^2)*d*x*cosh(d*x + c)^3 + 6*(a^2 - a*b)*cosh(d*x + c)^5 - 2*(a^
2 - a*b)*cosh(d*x + c))*sinh(d*x + c))/(a^3*d*cosh(d*x + c)^4 + 4*a^3*d*cos
h(d*x + c)^3*sinh(d*x + c) + 6*a^3*d*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*a^
3*d*cosh(d*x + c)*sinh(d*x + c)^3 + a^3*d*sinh(d*x + c)^4), 1/64*(a^2*cosh(
d*x + c)^8 + 8*a^2*cosh(d*x + c)*sinh(d*x + c)^7 + a^2*sinh(d*x + c)^8 + 8*
(3*a^2 - 4*a*b + 8*b^2)*d*x*cosh(d*x + c)^4 + 8*(a^2 - a*b)*cosh(d*x + c)^6
 + 4*(7*a^2*cosh(d*x + c)^2 + 2*a^2 - 2*a*b)*sinh(d*x + c)^6 + 8*(7*a^2*cos
h(d*x + c)^3 + 6*(a^2 - a*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*a^2*cos
h(d*x + c)^4 + 4*(3*a^2 - 4*a*b + 8*b^2)*d*x + 60*(a^2 - a*b)*cosh(d*x + c)
^2)*sinh(d*x + c)^4 + 8*(7*a^2*cosh(d*x + c)^5 + 4*(3*a^2 - 4*a*b + 8*b^2)*
d*x*cosh(d*x + c) + 20*(a^2 - a*b)*cosh(d*x + c)^3)*sinh(d*x + c)^3 - 8*(a^
2 - a*b)*cosh(d*x + c)^2 + 4*(7*a^2*cosh(d*x + c)^6 + 12*(3*a^2 - 4*a*b + 8
*b^2)*d*x*cosh(d*x + c)^2 + 30*(a^2 - a*b)*cosh(d*x + c)^4 - 2*a^2 + 2*a*b)
*sinh(d*x + c)^2 - 64*(b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)^3*sinh(d*x
 + c) + 6*b^2*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b^2*cosh(d*x + c)*sinh(d*
x + c)^3 + b^2*sinh(d*x + c)^4)*sqrt(-b/(a + b))*arctan(1/2*(a*cosh(d*x + c
)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-
b/(a + b))/b) - a^2 + 8*(a^2*cosh(d*x + c)^7 + 4*(3*a^2 - 4*a*b + 8*b^2)*d*
x*cosh(d*x + c)^3 + 6*(a^2 - a*b)*cosh(d*x + c)^5 - 2*(a^2 - a*b)*cosh(d*x
+ c))*sinh(d*x + c))/(a^3*d*cosh(d*x + c)^4 + 4*a^3*d*cosh(d*x + c)^3*sinh(
d*x + c) + 6*a^3*d*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*a^3*d*cosh(d*x + c)*
sinh(d*x + c)^3 + a^3*d*sinh(d*x + c)^4)]
```

giac [B] time = 2.62, size = 208, normalized size = 1.78

$$-\frac{\frac{64\,b^3\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}\,a^3}-\frac{8\left(3\,a^2-4\,ab+8\,b^2\right)(dx+c)}{a^3}-\frac{ae^{(4\,dx+4\,c)}+8\,ae^{(2\,dx+2\,c)}-8\,be^{(2\,dx+2\,c)}}{a^2}+\frac{\left(18\,a^2e^{(4\,dx+4\,c)}-24\,abe^{(4\,dx+4\,c)}+48\,b^2e^{(4\,dx}}{}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/64*(64*b^3*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/(s
qrt(-a*b - b^2)*a^3) - 8*(3*a^2 - 4*a*b + 8*b^2)*(d*x + c)/a^3 - (a*e^(4*d*
x + 4*c) + 8*a*e^(2*d*x + 2*c) - 8*b*e^(2*d*x + 2*c))/a^2 + (18*a^2*e^(4*d*
x + 4*c) - 24*a*b*e^(4*d*x + 4*c) + 48*b^2*e^(4*d*x + 4*c) + 8*a^2*e^(2*d*x
 + 2*c) - 8*a*b*e^(2*d*x + 2*c) + a^2)*e^(-4*d*x - 4*c)/a^3)/d
```

maple [B] time = 0.54, size = 493, normalized size = 4.21

$$\frac{1}{4da\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}+\frac{1}{2da\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}+\frac{7}{8da\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{b}{2d\,a^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^4/(a+b*sech(d*x+c)^2),x)
```

```
[Out] 1/4/d/a/(tanh(1/2*d*x+1/2*c)-1)^4+1/2/d/a/(tanh(1/2*d*x+1/2*c)-1)^3+7/8/d/a
/(tanh(1/2*d*x+1/2*c)-1)^2-1/2/d/a^2/(tanh(1/2*d*x+1/2*c)-1)^2*b+5/8/d/a/(t
anh(1/2*d*x+1/2*c)-1)-1/2/d/a^2/(tanh(1/2*d*x+1/2*c)-1)*b-3/8/d/a*ln(tanh(1
/2*d*x+1/2*c)-1)+1/2/d/a^2*ln(tanh(1/2*d*x+1/2*c)-1)*b-1/d/a^3*ln(tanh(1/2*
d*x+1/2*c)-1)*b^2-1/4/d/a/(tanh(1/2*d*x+1/2*c)+1)^4+1/2/d/a/(tanh(1/2*d*x+1
/2*c)+1)^3+5/8/d/a/(tanh(1/2*d*x+1/2*c)+1)-1/2/d/a^2/(tanh(1/2*d*x+1/2*c)+1
)*b-7/8/d/a/(tanh(1/2*d*x+1/2*c)+1)^2+1/2/d/a^2/(tanh(1/2*d*x+1/2*c)+1)^2*b
+3/8/d/a*ln(tanh(1/2*d*x+1/2*c)+1)-1/2/d/a^2*ln(tanh(1/2*d*x+1/2*c)+1)*b+1/
d/a^3*ln(tanh(1/2*d*x+1/2*c)+1)*b^2+1/2/d*b^(5/2)/a^3/(a+b)^(1/2)*ln((a+b)^
(1/2)*tanh(1/2*d*x+1/2*c)^2-2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))-1/2/
d*b^(5/2)/a^3/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*ta
nh(1/2*d*x+1/2*c)+(a+b)^(1/2))
```

maxima [B] time = 0.46, size = 526, normalized size = 4.50

$$\frac{3\,b\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{16\,\sqrt{(a+b)b}\,ad}+\frac{3\,(dx+c)}{8\,ad}-\frac{\left(8\,be^{(-2\,dx-2\,c)}-a\right)e^{(4\,dx+4\,c)}}{64\,a^2d}+\frac{e^{(2\,dx+2\,c)}}{8\,ad}-\frac{e^{(-2\,dx-2\,c)}}{8\,ad}-\frac{b\log\left(ae^{(4\,dx+4\,c}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 3/16*b*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x
- 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*a*d) + 3/8*(d*x + c
)/(a*d) - 1/64*(8*b*e^(-2*d*x - 2*c) - a)*e^(4*d*x + 4*c)/(a^2*d) + 1/8*e^(
2*d*x + 2*c)/(a*d) - 1/8*e^(-2*d*x - 2*c)/(a*d) - 1/4*b*log(a*e^(4*d*x + 4*
c) + 2*(a + 2*b)*e^(2*d*x + 2*c) + a)/(a^2*d) + 1/4*b*log(2*(a + 2*b)*e^(-2
*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/(a^2*d) + 1/8*(a*b + 2*b^2)*log((a*e^
(2*d*x + 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(2*d*x + 2*c) + a + 2*b +
 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*a^2*d) - 1/8*(a*b + 2*b^2)*log((a*e^(
-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b
+ 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*a^2*d) + 1/2*(a*b + 2*b^2)*(d*x + c)
/(a^3*d) + 1/64*(8*b*e^(-2*d*x - 2*c) - a*e^(-4*d*x - 4*c))/(a^2*d) + 1/16*
(a^2*b + 8*a*b^2 + 8*b^3)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b
)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*
a^3*d)
```

mupad [B] time = 2.00, size = 260, normalized size = 2.22

$$\frac{x\left(3\,a^2-4\,a\,b+8\,b^2\right)}{8\,a^3}-\frac{\mathrm{e}^{-4\,c-4\,d\,x}}{64\,a\,d}+\frac{\mathrm{e}^{4\,c+4\,d\,x}}{64\,a\,d}-\frac{\mathrm{e}^{-2\,c-2\,d\,x}\,(a-b)}{8\,a^2\,d}+\frac{\mathrm{e}^{2\,c+2\,d\,x}\,(a-b)}{8\,a^2\,d}+\frac{b^{5/2}\,\ln\left(\frac{4\,b^3\,\mathrm{e}^{2\,c+2\,d\,x}}{a^4}-\frac{2\,b^{5/2}\,\left(a\,d+a\,\cdots\right.}{\cdots}\right.}{2\,a^3\,d\,\sqrt{a+\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^4/(a + b/cosh(c + d*x)^2),x)
```

```
[Out] (x*(3*a^2 - 4*a*b + 8*b^2))/(8*a^3) - exp(- 4*c - 4*d*x)/(64*a*d) + exp(4*c
 + 4*d*x)/(64*a*d) - (exp(- 2*c - 2*d*x)*(a - b))/(8*a^2*d) + (exp(2*c + 2*
d*x)*(a - b))/(8*a^2*d) + (b^(5/2)*log((4*b^3*exp(2*c + 2*d*x))/a^4 - (2*b^
(5/2)*(a*d + a*d*exp(2*c + 2*d*x) + 2*b*d*exp(2*c + 2*d*x)))/(a^4*d*(a + b)
^(1/2))))/(2*a^3*d*(a + b)^(1/2)) - (b^(5/2)*log((4*b^3*exp(2*c + 2*d*x))/a
^4 + (2*b^(5/2)*(a*d + a*d*exp(2*c + 2*d*x) + 2*b*d*exp(2*c + 2*d*x)))/(a^4
*d*(a + b)^(1/2))))/(2*a^3*d*(a + b)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^4(c+dx)}{a+b\,\mathrm{sech}^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**4/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(cosh(c + d*x)**4/(a + b*sech(c + d*x)**2), x)
```

3.74 $\int \frac{\cosh^3(c+dx)}{a+b\,\mathbf{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=76

$$\frac{b^2\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{a^{5/2}d\sqrt{a+b}}+\frac{(a-b)\sinh(c+dx)}{a^2d}+\frac{\sinh^3(c+dx)}{3ad}$$

```
[Out] (a-b)*sinh(d*x+c)/a^2/d+1/3*sinh(d*x+c)^3/a/d+b^2*arctan(sinh(d*x+c)*a^(1/2
)/(a+b)^(1/2))/a^(5/2)/d/(a+b)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4147, 390, 205}

$$\frac{b^2\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{a^{5/2}d\sqrt{a+b}}+\frac{(a-b)\sinh(c+dx)}{a^2d}+\frac{\sinh^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^3/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] (b^2*ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]])/(a^(5/2)*Sqrt[a + b]*d) +
 ((a - b)*Sinh[c + d*x])/(a^2*d) + Sinh[c + d*x]^3/(3*a*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
```

```
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^3(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{\left(1+x^2\right)^2}{a+b+ax^2}\,dx, x, \sinh(c+dx)\right)}{d}\\ &= \frac{\operatorname{Subst}\left(\int \left(\frac{a-b}{a^2}+\frac{x^2}{a}+\frac{b^2}{a^2\left(a+b+ax^2\right)}\right)dx, x, \sinh(c+dx)\right)}{d}\\ &= \frac{(a-b)\sinh(c+dx)}{a^2d}+\frac{\sinh^3(c+dx)}{3ad}+\frac{b^2\operatorname{Subst}\left(\int \frac{1}{a+b+ax^2}\,dx, x, \sinh(c+dx)\right)}{a^2d}\\ &= \frac{b^2\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{a^{5/2}\sqrt{a+b}\,d}+\frac{(a-b)\sinh(c+dx)}{a^2d}+\frac{\sinh^3(c+dx)}{3ad}\end{aligned}$$

Mathematica [A] time = 0.27, size = 79, normalized size = 1.04

$$\frac{a^{3/2}\sinh(3(c+dx))-\frac{12b^2\tan^{-1}\left(\frac{\sqrt{a+b}\operatorname{csch}(c+dx)}{\sqrt{a}}\right)}{\sqrt{a+b}}+3\sqrt{a}\,(3a-4b)\sinh(c+dx)}{12a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^3/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((-12*b^2*ArcTan[(Sqrt[a + b]*Csch[c + d*x])/Sqrt[a]])/Sqrt[a + b] + 3*Sqrt
[a]*(3*a - 4*b)*Sinh[c + d*x] + a^(3/2)*Sinh[3*(c + d*x)])/(12*a^(5/2)*d)
```

fricas [B] time = 0.47, size = 1616, normalized size = 21.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/24*((a^3 + a^2*b)*cosh(d*x + c)^6 + 6*(a^3 + a^2*b)*cosh(d*x + c)*sinh(d
*x + c)^5 + (a^3 + a^2*b)*sinh(d*x + c)^6 + 3*(3*a^3 - a^2*b - 4*a*b^2)*cos
h(d*x + c)^4 + 3*(3*a^3 - a^2*b - 4*a*b^2 + 5*(a^3 + a^2*b)*cosh(d*x + c)^2
)*sinh(d*x + c)^4 + 4*(5*(a^3 + a^2*b)*cosh(d*x + c)^3 + 3*(3*a^3 - a^2*b -
```

```
 4*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - a^3 - a^2*b - 3*(3*a^3 - a^2*b -
 4*a*b^2)*cosh(d*x + c)^2 + 3*(5*(a^3 + a^2*b)*cosh(d*x + c)^4 - 3*a^3 + a^
2*b + 4*a*b^2 + 6*(3*a^3 - a^2*b - 4*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^
2 - 12*(b^2*cosh(d*x + c)^3 + 3*b^2*cosh(d*x + c)^2*sinh(d*x + c) + 3*b^2*c
osh(d*x + c)*sinh(d*x + c)^2 + b^2*sinh(d*x + c)^3)*sqrt(-a^2 - a*b)*log((a
*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 -
2*(3*a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 - 3*a - 2*b)*sinh(d*
x + c)^2 + 4*(a*cosh(d*x + c)^3 - (3*a + 2*b)*cosh(d*x + c))*sinh(d*x + c)
- 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 +
(3*cosh(d*x + c)^2 - 1)*sinh(d*x + c) - cosh(d*x + c))*sqrt(-a^2 - a*b) + a
)/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^
4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*
x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) +
a)) + 6*((a^3 + a^2*b)*cosh(d*x + c)^5 + 2*(3*a^3 - a^2*b - 4*a*b^2)*cosh(d
*x + c)^3 - (3*a^3 - a^2*b - 4*a*b^2)*cosh(d*x + c))*sinh(d*x + c))/((a^4 +
 a^3*b)*d*cosh(d*x + c)^3 + 3*(a^4 + a^3*b)*d*cosh(d*x + c)^2*sinh(d*x + c)
 + 3*(a^4 + a^3*b)*d*cosh(d*x + c)*sinh(d*x + c)^2 + (a^4 + a^3*b)*d*sinh(d
*x + c)^3), 1/24*((a^3 + a^2*b)*cosh(d*x + c)^6 + 6*(a^3 + a^2*b)*cosh(d*x
+ c)*sinh(d*x + c)^5 + (a^3 + a^2*b)*sinh(d*x + c)^6 + 3*(3*a^3 - a^2*b - 4
*a*b^2)*cosh(d*x + c)^4 + 3*(3*a^3 - a^2*b - 4*a*b^2 + 5*(a^3 + a^2*b)*cosh
(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(a^3 + a^2*b)*cosh(d*x + c)^3 + 3*(3*a^
3 - a^2*b - 4*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - a^3 - a^2*b - 3*(3*a^
3 - a^2*b - 4*a*b^2)*cosh(d*x + c)^2 + 3*(5*(a^3 + a^2*b)*cosh(d*x + c)^4 -
 3*a^3 + a^2*b + 4*a*b^2 + 6*(3*a^3 - a^2*b - 4*a*b^2)*cosh(d*x + c)^2)*sin
h(d*x + c)^2 + 24*(b^2*cosh(d*x + c)^3 + 3*b^2*cosh(d*x + c)^2*sinh(d*x + c
) + 3*b^2*cosh(d*x + c)*sinh(d*x + c)^2 + b^2*sinh(d*x + c)^3)*sqrt(a^2 + a
*b)*arctan(1/2*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*s
inh(d*x + c)^3 + (3*a + 4*b)*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + 3*a + 4
*b)*sinh(d*x + c))/sqrt(a^2 + a*b)) + 24*(b^2*cosh(d*x + c)^3 + 3*b^2*cosh(
d*x + c)^2*sinh(d*x + c) + 3*b^2*cosh(d*x + c)*sinh(d*x + c)^2 + b^2*sinh(d
*x + c)^3)*sqrt(a^2 + a*b)*arctan(1/2*sqrt(a^2 + a*b)*(cosh(d*x + c) + sinh
(d*x + c))/(a + b)) + 6*((a^3 + a^2*b)*cosh(d*x + c)^5 + 2*(3*a^3 - a^2*b -
 4*a*b^2)*cosh(d*x + c)^3 - (3*a^3 - a^2*b - 4*a*b^2)*cosh(d*x + c))*sinh(d
*x + c))/((a^4 + a^3*b)*d*cosh(d*x + c)^3 + 3*(a^4 + a^3*b)*d*cosh(d*x + c)
^2*sinh(d*x + c) + 3*(a^4 + a^3*b)*d*cosh(d*x + c)*sinh(d*x + c)^2 + (a^4 +
 a^3*b)*d*sinh(d*x + c)^3)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
```

```
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[31,78]Warning, need to choose a branch for the root of a po
lynomial with parameters. This might be wrong.The choice was done assuming
[a,b]=[-13,-93]Warning, need to choose a branch for the root of a polynomia
l with parameters. This might be wrong.The choice was done assuming [a,b]=[
-65,-82]Warning, need to choose a branch for the root of a polynomial with
parameters. This might be wrong.The choice was done assuming [a,b]=[97,-56]
Warning, need to choose a branch for the root of a polynomial with paramete
rs. This might be wrong.The choice was done assuming [a,b]=[80,44]Warning,
need to choose a branch for the root of a polynomial with parameters. This
might be wrong.The choice was done assuming [a,b]=[22,73]Warning, need to c
hoose a branch for the root of a polynomial with parameters. This might be
wrong.The choice was done assuming [a,b]=[36,86]Warning, need to choose a b
ranch for the root of a polynomial with parameters. This might be wrong.The
 choice was done assuming [a,b]=[-59,-45]Warning, need to choose a branch f
or the root of a polynomial with parameters. This might be wrong.The choice
 was done assuming [a,b]=[15,66]Warning, need to choose a branch for the ro
ot of a polynomial with parameters. This might be wrong.The choice was done
 assuming [a,b]=[55,80]Undef/Unsigned Inf encountered in limitEvaluation ti
me: 1.25Limit: Max order reached or unable to make series expansion Error:
Bad Argument Value
```

maple [B] time = 0.49, size = 256, normalized size = 3.37

$$-\frac{1}{3da\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{2da\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{1}{da\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{b}{d\,a^{2}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{}{3da\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3/(a+b*sech(d*x+c)^2),x)
```

```
[Out] -1/3/d/a/(tanh(1/2*d*x+1/2*c)-1)^3-1/2/d/a/(tanh(1/2*d*x+1/2*c)-1)^2-1/d/a/
(tanh(1/2*d*x+1/2*c)-1)+1/d/a^2/(tanh(1/2*d*x+1/2*c)-1)*b-1/3/d/a/(tanh(1/2
*d*x+1/2*c)+1)^3+1/2/d/a/(tanh(1/2*d*x+1/2*c)+1)^2-1/d/a/(tanh(1/2*d*x+1/2*
c)+1)+1/d/a^2/(tanh(1/2*d*x+1/2*c)+1)*b+1/d*b^2/a^(5/2)/(a+b)^(1/2)*arctan(
1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)-2*b^(1/2))/a^(1/2))+1/d*b^2/a^(5/2)/
(a+b)^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)+2*b^(1/2))/a^(1/2
))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(3\left(3\,ae^{(4\,c)}-4\,be^{(4\,c)}\right)e^{(4\,dx)}-3\left(3\,ae^{(2\,c)}-4\,be^{(2\,c)}\right)e^{(2\,dx)}+ae^{(6\,dx+6\,c)}-a\right)e^{(-3\,dx-3\,c)}}{24\,a^{2}d}+\frac{1}{8}\int\frac{16\left(b^{2}e^{(3\,dx+}\right.}{a^{3}e^{(4\,dx+4\,c)}+a^{3}+2\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/24*(3*(3*a*e^(4*c) - 4*b*e^(4*c))*e^(4*d*x) - 3*(3*a*e^(2*c) - 4*b*e^(2*c
))*e^(2*d*x) + a*e^(6*d*x + 6*c) - a)*e^(-3*d*x - 3*c)/(a^2*d) + 1/8*integr
ate(16*(b^2*e^(3*d*x + 3*c) + b^2*e^(d*x + c))/(a^3*e^(4*d*x + 4*c) + a^3 +
 2*(a^3*e^(2*c) + 2*a^2*b*e^(2*c))*e^(2*d*x)), x)
```

mupad [B] time = 2.21, size = 332, normalized size = 4.37

$$\frac{e^{3c+3dx}}{24ad} - \frac{e^{-3c-3dx}}{24ad} - \frac{\sqrt{b^4}\left(2\,\mathrm{atan}\left(\left(\left(e^{dx}\,e^{c}\left(\frac{2b^2}{a^8\,d\,(a+b)^2\,\sqrt{b^4}} - \frac{4\left(2a^2b^4d\sqrt{b^4}+2a^3b^3d\sqrt{b^4}\right)}{a^6b^5(a+b)\sqrt{a^6d^2+b\,a^5d^2}\,\sqrt{a^5d^2(a+b)}}\right) - \frac{2b^2e^{3c}e^{3dx}}{a^8\,d\,(a+b)^2\,\sqrt{b^4}}\right)\left(\frac{a^7\sqrt{\cdots}}{\cdots}\cdots\right.\right.\right.\right.}{2\sqrt{a^6d^2+b\,a^5d^2}\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^3/(a + b/cosh(c + d*x)^2),x)
```

```
[Out] exp(3*c + 3*d*x)/(24*a*d) - exp(- 3*c - 3*d*x)/(24*a*d) - ((b^4)^(1/2)*(2*a
tan((exp(d*x)*exp(c)*((2*b^2)/(a^8*d*(a + b)^2*(b^4)^(1/2)) - (4*(2*a^2*b^4
*d*(b^4)^(1/2) + 2*a^3*b^3*d*(b^4)^(1/2)))/(a^6*b^5*(a + b)*(a^6*d^2 + a^5*
b*d^2)^(1/2)*(a^5*d^2*(a + b))^(1/2))) - (2*b^2*exp(3*c)*exp(3*d*x))/(a^8*d
*(a + b)^2*(b^4)^(1/2)))*((a^7*(a^6*d^2 + a^5*b*d^2)^(1/2))/4 + (a^6*b*(a^6
*d^2 + a^5*b*d^2)^(1/2))/4)) - 2*atan((b^2*exp(d*x)*exp(c)*(a^5*d^2*(a + b)
)^(1/2))/(2*a^2*d*(a + b)*(b^4)^(1/2)))))/(2*(a^6*d^2 + a^5*b*d^2)^(1/2)) +
 (exp(c + d*x)*(3*a - 4*b))/(8*a^2*d) - (exp(- c - d*x)*(3*a - 4*b))/(8*a^2
*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^3(c+dx)}{a+b\,\mathrm{sech}^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(cosh(c + d*x)**3/(a + b*sech(c + d*x)**2), x)
```

3.75 $\int \frac{\cosh^2(c+dx)}{a+b\,\mathbf{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=75

$$\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a^2d\sqrt{a+b}}+\frac{x(a-2b)}{2a^2}+\frac{\sinh(c+dx)\cosh(c+dx)}{2ad}$$

```
[Out] 1/2*(a-2*b)*x/a^2+1/2*cosh(d*x+c)*sinh(d*x+c)/a/d+b^(3/2)*arctanh(b^(1/2)*t
anh(d*x+c)/(a+b)^(1/2))/a^2/d/(a+b)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4146, 414, 522, 206, 208}

$$\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a^2d\sqrt{a+b}}+\frac{x(a-2b)}{2a^2}+\frac{\sinh(c+dx)\cosh(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^2/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((a - 2*b)*x)/(2*a^2) + (b^(3/2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b
]])/(a^2*Sqrt[a + b]*d) + (Cosh[c + d*x]*Sinh[c + d*x])/(2*a*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
```

```
d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\cosh^2(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^2(a+b-bx^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2ad} + \frac{\text{Subst}\left(\int \frac{a-b-bx^2}{(1-x^2)(a+b-bx^2)}\,dx, x, \tanh(c+dx)\right)}{2ad}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2ad} + \frac{(a-2b)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{2a^2d} + \frac{b^2\,\text{Subst}(}{}$$

$$= \frac{(a-2b)x}{2a^2} + \frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a^2\sqrt{a+b}\,d} + \frac{\cosh(c+dx)\sinh(c+dx)}{2ad}$$

Mathematica [A] time = 0.24, size = 67, normalized size = 0.89

$$\frac{\frac{4b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{a+b}} + 2(a-2b)(c+dx) + a\sinh(2(c+dx))}{4a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^2/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] (2*(a - 2*b)*(c + d*x) + (4*b^(3/2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a
+ b]])/Sqrt[a + b] + a*Sinh[2*(c + d*x)])/(4*a^2*d)
```

fricas [B] time = 0.46, size = 829, normalized size = 11.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*(a - 2*b)*d*x*cosh(d*x + c)^2 + a*cosh(d*x + c)^4 + 4*a*cosh(d*x +
c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(2*(a - 2*b)*d*x + 3*a*cosh(d*x
+ c)^2)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x
 + c) + b*sinh(d*x + c)^2)*sqrt(b/(a + b))*log((a^2*cosh(d*x + c)^4 + 4*a^2
*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh
(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2
 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*si
nh(d*x + c) - 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*
sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 + 3*a*b + 2*b^2)*sqrt(b/(
a + b)))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*
x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*
sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x
+ c) + a)) + 4*(2*(a - 2*b)*d*x*cosh(d*x + c) + a*cosh(d*x + c)^3)*sinh(d*x
 + c) - a)/(a^2*d*cosh(d*x + c)^2 + 2*a^2*d*cosh(d*x + c)*sinh(d*x + c) + a
^2*d*sinh(d*x + c)^2), 1/8*(4*(a - 2*b)*d*x*cosh(d*x + c)^2 + a*cosh(d*x +
c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(2*(a - 2*
b)*d*x + 3*a*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*(b*cosh(d*x + c)^2 + 2*b*
cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2)*sqrt(-b/(a + b))*arctan(1/
2*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2
+ a + 2*b)*sqrt(-b/(a + b))/b) + 4*(2*(a - 2*b)*d*x*cosh(d*x + c) + a*cosh(
d*x + c)^3)*sinh(d*x + c) - a)/(a^2*d*cosh(d*x + c)^2 + 2*a^2*d*cosh(d*x +
c)*sinh(d*x + c) + a^2*d*sinh(d*x + c)^2)]
```

giac [A] time = 1.57, size = 125, normalized size = 1.67

$$\frac{\frac{8\,b^2\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}\,a^2}+\frac{4\,(dx+c)(a-2\,b)}{a^2}+\frac{e^{(2\,dx+2\,c)}}{a}-\frac{\left(2\,ae^{(2\,dx+2\,c)}-4\,be^{(2\,dx+2\,c)}+a\right)e^{(-2\,dx-2\,c)}}{a^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/8*(8*b^2*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/(sqrt
(-a*b - b^2)*a^2) + 4*(d*x + c)*(a - 2*b)/a^2 + e^(2*d*x + 2*c)/a - (2*a*e^
(2*d*x + 2*c) - 4*b*e^(2*d*x + 2*c) + a)*e^(-2*d*x - 2*c)/a^2)/d
```

maple [B] time = 0.46, size = 278, normalized size = 3.71

$$\frac{1}{2da\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{1}{2da\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2da}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)b}{d\,a^{2}}-\frac{}{2da\left(\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2/(a+b*sech(d*x+c)^2),x)
```

```
[Out] 1/2/d/a/(tanh(1/2*d*x+1/2*c)-1)^2+1/2/d/a/(tanh(1/2*d*x+1/2*c)-1)-1/2/d/a*l
n(tanh(1/2*d*x+1/2*c)-1)+1/d/a^2*ln(tanh(1/2*d*x+1/2*c)-1)*b-1/2/d/a/(tanh(
1/2*d*x+1/2*c)+1)^2+1/2/d/a/(tanh(1/2*d*x+1/2*c)+1)+1/2/d/a*ln(tanh(1/2*d*x
+1/2*c)+1)-1/d/a^2*ln(tanh(1/2*d*x+1/2*c)+1)*b-1/2/d*b^(3/2)/a^2/(a+b)^(1/2
)*ln(-(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)-(a+b)
^(1/2))+1/2/d/a^2*b^(3/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+
2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))
```

maxima [B] time = 0.42, size = 352, normalized size = 4.69

$$\frac{b\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{4\,\sqrt{(a+b)b}\,ad}+\frac{dx+c}{2\,ad}+\frac{e^{(2\,dx+2\,c)}}{8\,ad}-\frac{e^{(-2\,dx-2\,c)}}{8\,ad}-\frac{b\log\left(ae^{(4\,dx+4\,c)}+2\,(a+2\,b)e^{(2\,dx+2\,c)}+a\right)}{4\,a^{2}d}+\frac{b\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/4*b*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x -
 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*a*d) + 1/2*(d*x + c)
/(a*d) + 1/8*e^(2*d*x + 2*c)/(a*d) - 1/8*e^(-2*d*x - 2*c)/(a*d) - 1/4*b*log
(a*e^(4*d*x + 4*c) + 2*(a + 2*b)*e^(2*d*x + 2*c) + a)/(a^2*d) + 1/4*b*log(2
*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/(a^2*d) + 1/8*(a*b +
2*b^2)*log((a*e^(2*d*x + 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(2*d*x +
2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*a^2*d) - 1/8*(a*b + 2
*b^2)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x -
 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*a^2*d)
```

mupad [B] time = 1.97, size = 206, normalized size = 2.75

$$\frac{x\,\left(a-2\,b\right)}{2\,a^{2}}-\frac{\mathrm{e}^{-2\,c-2\,d\,x}}{8\,a\,d}+\frac{\mathrm{e}^{2\,c+2\,d\,x}}{8\,a\,d}+\frac{b^{3/2}\,\ln\left(-\frac{4\,b^{2}\,\mathrm{e}^{2\,c+2\,d\,x}}{a^{3}}-\frac{2\,b^{3/2}\,\left(a\,d+a\,d\,\mathrm{e}^{2\,c+2\,d\,x}+2\,b\,d\,\mathrm{e}^{2\,c+2\,d\,x}\right)}{a^{3}\,d\,\sqrt{a+b}}\right)}{2\,a^{2}\,d\,\sqrt{a+b}}-\frac{b^{3/2}\,\ln\left(\frac{2\,b^{3/2}\,\left(a\,d+a\,d\,\mathrm{e}^{2\,c+2\,d}\right.}{a^{3}\,d\,\sqrt{a}}\right.}{2\,a^{2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^2/(a + b/cosh(c + d*x)^2),x)
```

```
[Out] (x*(a - 2*b))/(2*a^2) - exp(- 2*c - 2*d*x)/(8*a*d) + exp(2*c + 2*d*x)/(8*a*
d) + (b^(3/2)*log(- (4*b^2*exp(2*c + 2*d*x))/a^3 - (2*b^(3/2)*(a*d + a*d*ex
p(2*c + 2*d*x) + 2*b*d*exp(2*c + 2*d*x)))/(a^3*d*(a + b)^(1/2))))/(2*a^2*d*
(a + b)^(1/2)) - (b^(3/2)*log((2*b^(3/2)*(a*d + a*d*exp(2*c + 2*d*x) + 2*b*
d*exp(2*c + 2*d*x)))/(a^3*d*(a + b)^(1/2)) - (4*b^2*exp(2*c + 2*d*x))/a^3))
/(2*a^2*d*(a + b)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^2(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(cosh(c + d*x)**2/(a + b*sech(c + d*x)**2), x)
```

3.76 $\int \frac{\cosh(c+dx)}{a+b\,\mathbf{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=52

$$\frac{\sinh(c+dx)}{ad}-\frac{b\tan^{-1}\left(\frac{\sqrt{a}\,\sinh(c+dx)}{\sqrt{a+b}}\right)}{a^{3/2}d\sqrt{a+b}}$$

```
[Out] sinh(d*x+c)/a/d-b*arctan(sinh(d*x+c)*a^(1/2)/(a+b)^(1/2))/a^(3/2)/d/(a+b)^(
1/2)
```

Rubi [A] time = 0.06, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4147, 388, 205}

$$\frac{\sinh(c+dx)}{ad}-\frac{b\tan^{-1}\left(\frac{\sqrt{a}\,\sinh(c+dx)}{\sqrt{a+b}}\right)}{a^{3/2}d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -((b*ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]])/(a^(3/2)*Sqrt[a + b]*d))
+ Sinh[c + d*x]/(a*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cosh(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{a+b+ax^2}\,dx, x, \sinh(c+dx)\right)}{d}$$
$$= \frac{\sinh(c+dx)}{ad} - \frac{b\,\text{Subst}\left(\int \frac{1}{a+b+ax^2}\,dx, x, \sinh(c+dx)\right)}{ad}$$
$$= -\frac{b\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{a^{3/2}\sqrt{a+b}\,d} + \frac{\sinh(c+dx)}{ad}$$

Mathematica [A] time = 0.12, size = 52, normalized size = 1.00

$$\frac{\sqrt{a}\sinh(c+dx) - \frac{b\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{a+b}}}{a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] (-((b*ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]])/Sqrt[a + b]) + Sqrt[a]*S
inh[c + d*x])/(a^(3/2)*d)
```

fricas [B] time = 0.46, size = 718, normalized size = 13.81

$$\left[\frac{\left(a^2+ab\right)\cosh\left(dx+c\right)^2 + 2\left(a^2+ab\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right) + \left(a^2+ab\right)\sinh\left(dx+c\right)^2 - \sqrt{-a^2-ab}\,(b\,\text{co}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x +
c) + (a^2 + a*b)*sinh(d*x + c)^2 - sqrt(-a^2 - a*b)*(b*cosh(d*x + c) + b*si
nh(d*x + c))*log((a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a
*sinh(d*x + c)^4 - 2*(3*a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 -
 3*a - 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 - (3*a + 2*b)*cosh(d*x +
 c))*sinh(d*x + c) + 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 +
 sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1)*sinh(d*x + c) - cosh(d*x + c))*s
```

```
qrt(-a^2 - a*b) + a)/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3
 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2
 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c
))*sinh(d*x + c) + a)) - a^2 - a*b)/((a^3 + a^2*b)*d*cosh(d*x + c) + (a^3 +
 a^2*b)*d*sinh(d*x + c)), 1/2*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*
cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 - 2*sqrt(a^2 + a*
b)*(b*cosh(d*x + c) + b*sinh(d*x + c))*arctan(1/2*(a*cosh(d*x + c)^3 + 3*a*
cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 + (3*a + 4*b)*cosh(d*x +
c) + (3*a*cosh(d*x + c)^2 + 3*a + 4*b)*sinh(d*x + c))/sqrt(a^2 + a*b)) - 2*
sqrt(a^2 + a*b)*(b*cosh(d*x + c) + b*sinh(d*x + c))*arctan(1/2*sqrt(a^2 + a
*b)*(cosh(d*x + c) + sinh(d*x + c))/(a + b)) - a^2 - a*b)/((a^3 + a^2*b)*d*
cosh(d*x + c) + (a^3 + a^2*b)*d*sinh(d*x + c))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[31,78]Warning, need to choose a branch for the root of a po
lynomial with parameters. This might be wrong.The choice was done assuming
[a,b]=[-13,-93]Warning, need to choose a branch for the root of a polynomia
l with parameters. This might be wrong.The choice was done assuming [a,b]=[
-65,-82]Warning, need to choose a branch for the root of a polynomial with
parameters. This might be wrong.The choice was done assuming [a,b]=[97,-56]
Warning, need to choose a branch for the root of a polynomial with paramete
rs. This might be wrong.The choice was done assuming [a,b]=[80,44]Warning,
need to choose a branch for the root of a polynomial with parameters. This
might be wrong.The choice was done assuming [a,b]=[22,73]Undef/Unsigned Inf
 encountered in limitEvaluation time: 0.84Limit: Max order reached or unabl
e to make series expansion Error: Bad Argument Value
```

maple [B] time = 0.40, size = 128, normalized size = 2.46

$$-\frac{1}{da\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{b\arctan\left(\frac{2\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sqrt{b}}{2\sqrt{a}}\right)}{d\,a^{\frac{3}{2}}\sqrt{a+b}}-\frac{b\arctan\left(\frac{2\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\sqrt{b}}{2\sqrt{a}}\right)}{d\,a^{\frac{3}{2}}\sqrt{a+b}}-\frac{1}{da\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/(a+b*sech(d*x+c)^2),x)
```

```
[Out] -1/d/a/(tanh(1/2*d*x+1/2*c)+1)-1/d/a^(3/2)*b/(a+b)^(1/2)*arctan(1/2*(2*(a+b
)^(1/2)*tanh(1/2*d*x+1/2*c)-2*b^(1/2))/a^(1/2))-1/d/a^(3/2)*b/(a+b)^(1/2)*a
rctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)+2*b^(1/2))/a^(1/2))-1/d/a/(tan
h(1/2*d*x+1/2*c)-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}}{2\,ad}-\frac{1}{2}\int\frac{4\left(be^{(3\,dx+3\,c)}+be^{(dx+c)}\right)}{a^2e^{(4\,dx+4\,c)}+a^2+2\left(a^2e^{(2\,c)}+2\,abe^{(2\,c)}\right)e^{(2\,dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)/(a*d) - 1/2*integrate(4*(b*e^(3*d*x
 + 3*c) + b*e^(d*x + c))/(a^2*e^(4*d*x + 4*c) + a^2 + 2*(a^2*e^(2*c) + 2*a*b
*e^(2*c))*e^(2*d*x)), x)
```

mupad [B] time = 1.74, size = 277, normalized size = 5.33

$$\frac{e^{c+d\,x}}{2\,a\,d}-\frac{e^{-c-d\,x}}{2\,a\,d}-\frac{\left(2\,\mathrm{atan}\left(\frac{b^3\,e^{d\,x}\,e^{c}\,\sqrt{a^3\,d^2\,(a+b)}}{2\,a\,d\,(a+b)\,{\left(b^2\right)}^{3/2}}\right)-2\,\mathrm{atan}\left(\left(e^{d\,x}\,e^{c}\,\left(\frac{2\,b^3}{a^5\,d\,{(a+b)}^2\,{\left(b^2\right)}^{3/2}}-\frac{4\left(2\,a^2\,d\,{\left(b^2\right)}^{3/2}+2\,a\,b\,d\,{\left(b^2\right)}^{3/2}\right)}{a^4\,b^3\,(a+b)\,\sqrt{a^4\,d^2+b\,a^3\,d^2}\,\sqrt{a^3\,d^2\,(a+b)}}\right)\right.\right.\cdots}{2\,\sqrt{a^4\,d^2+b\,a^3\,d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)/(a + b/cosh(c + d*x)^2),x)
```

```
[Out] exp(c + d*x)/(2*a*d) - exp(- c - d*x)/(2*a*d) - ((2*atan((b^3*exp(d*x)*exp(
c)*(a^3*d^2*(a + b))^(1/2))/(2*a*d*(a + b)*(b^2)^(3/2))) - 2*atan((exp(d*x)
*exp(c)*((2*b^3)/(a^5*d*(a + b)^2*(b^2)^(3/2)) - (4*(2*a^2*d*(b^2)^(3/2) +
2*a*b*d*(b^2)^(3/2)))/(a^4*b^3*(a + b)*(a^4*d^2 + a^3*b*d^2)^(1/2)*(a^3*d^2
*(a + b))^(1/2))) - (2*b^3*exp(3*c)*exp(3*d*x))/(a^5*d*(a + b)^2*(b^2)^(3/2
)))*((a^5*(a^4*d^2 + a^3*b*d^2)^(1/2))/4 + (a^4*b*(a^4*d^2 + a^3*b*d^2)^(1/
2))/4)))*(b^2)^(1/2))/(2*(a^4*d^2 + a^3*b*d^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cosh\left(c+dx\right)}{a+b\,\mathrm{sech}^2\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(cosh(c + d*x)/(a + b*sech(c + d*x)**2), x)
```

3.77 $\int \frac{\text{sech}(c+dx)}{a+b\,\text{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=36

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{a}\,d\sqrt{a+b}}$$

```
[Out] arctan(sinh(d*x+c)*a^(1/2)/(a+b)^(1/2))/d/a^(1/2)/(a+b)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4147, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{a}\,d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]]/(Sqrt[a]*Sqrt[a + b]*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{sech}(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{a+b+ax^2}\,dx, x, \sinh(c+dx)\right)}{d}$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{a}\sqrt{a+b}\,d}$$

Mathematica [A] time = 0.07, size = 36, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{a}\,d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]]/(Sqrt[a]*Sqrt[a + b]*d)
```

fricas [B] time = 0.42, size = 487, normalized size = 13.53

$$\left[-\frac{\sqrt{-a^2-ab}\,\log\left(\frac{a\cosh(dx+c)^4+4\,a\cosh(dx+c)\sinh(dx+c)^3+a\sinh(dx+c)^4-2\,(3\,a+2\,b)\cosh(dx+c)^2+2\left(3\,a\cosh(dx+c)^2-3\,a-2\,b\right)\sinh(dx+ \cdots}{a\cosh(dx+c)^4+4\,a\cosh(dx+c)\sinh(dx+c)^3+a\sinh(dx+c)^4+2\,(\cdots}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a^2 - a*b)*log((a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x
+ c)^3 + a*sinh(d*x + c)^4 - 2*(3*a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*
x + c)^2 - 3*a - 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 - (3*a + 2*b)*
cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*
x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1)*sinh(d*x + c) - cosh(d
*x + c))*sqrt(-a^2 - a*b) + a)/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(
d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(
d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*co
sh(d*x + c))*sinh(d*x + c) + a))/((a^2 + a*b)*d), (sqrt(a^2 + a*b)*arctan(1
/2*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)
^3 + (3*a + 4*b)*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + 3*a + 4*b)*sinh(d*x
 + c))/sqrt(a^2 + a*b)) + sqrt(a^2 + a*b)*arctan(1/2*sqrt(a^2 + a*b)*(cosh(
d*x + c) + sinh(d*x + c))/(a + b)))/((a^2 + a*b)*d)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[31,78]Warning, need to choose a branch for the root of a po
lynomial with parameters. This might be wrong.The choice was done assuming
[a,b]=[-13,-93]Undef/Unsigned Inf encountered in limitLimit: Max order reac
hed or unable to make series expansion Error: Bad Argument Value
```

maple [B] time = 0.28, size = 82, normalized size = 2.28

$$\frac{\arctan\left(\frac{2\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sqrt{b}}{2\sqrt{a}}\right)}{d\sqrt{a+b}\,\sqrt{a}}+\frac{\arctan\left(\frac{2\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\sqrt{b}}{2\sqrt{a}}\right)}{d\sqrt{a+b}\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)/(a+b*sech(d*x+c)^2),x)
```

```
[Out] 1/d/(a+b)^(1/2)/a^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)-2*b^(
1/2))/a^(1/2))+1/d/(a+b)^(1/2)/a^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d
*x+1/2*c)+2*b^(1/2))/a^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(dx+c)}{b\operatorname{sech}(dx+c)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] integrate(sech(d*x + c)/(b*sech(d*x + c)^2 + a), x)
```

mupad [B] time = 1.90, size = 108, normalized size = 3.00

$$-\frac{\ln\left(-\frac{4\left(b-b\,e^{2c+2dx}\right)}{a^2\,(a+b)}-\frac{8\,b\,e^{c+dx}}{(-a)^{5/2}\,\sqrt{a+b}}\right)-\ln\left(\frac{8\,b\,e^{c+dx}}{(-a)^{5/2}\,\sqrt{a+b}}-\frac{4\left(b-b\,e^{2c+2dx}\right)}{a^2\,(a+b)}\right)}{2\sqrt{-a}\,d\,\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)*(a + b/cosh(c + d*x)^2)),x)
```

```
[Out] -(log(- (4*(b - b*exp(2*c + 2*d*x)))/(a^2*(a + b)) - (8*b*exp(c + d*x))/((-
a)^(5/2)*(a + b)^(1/2))) - log((8*b*exp(c + d*x))/((-a)^(5/2)*(a + b)^(1/2)
) - (4*(b - b*exp(2*c + 2*d*x)))/(a^2*(a + b))))/(2*(-a)^(1/2)*d*(a + b)^(1
/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(sech(c + d*x)/(a + b*sech(c + d*x)**2), x)
```

3.78 $\int \frac{\text{sech}^2(c+dx)}{a+b\,\text{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=36

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{b}\,d\sqrt{a+b}}$$

```
[Out] arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))/d/b^(1/2)/(a+b)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4146, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{b}\,d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^2/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]]/(Sqrt[b]*Sqrt[a + b]*d)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{b}\,\sqrt{a+b}\,d}$$

Mathematica [A] time = 0.08, size = 36, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{b}\,d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^2/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]]/(Sqrt[b]*Sqrt[a + b]*d)
```

fricas [B] time = 0.43, size = 411, normalized size = 11.42

$$\left[\frac{\log\left(\frac{a^2\cosh(dx+c)^4+4a^2\cosh(dx+c)\sinh(dx+c)^3+a^2\sinh(dx+c)^4+2\left(a^2+2ab\right)\cosh(dx+c)^2+2\left(3a^2\cosh(dx+c)^2+a^2+2ab\right)\sinh(dx+c)^2+a^2+8a}{a\cosh(dx+c)^4+4a\cosh(dx+c)\sinh(dx+c)^3+a\sinh(dx+c)^4+2(a+2b)\cosh(dx+c)^2+2(3a}\right)}{2\sqrt{ab+}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*s
inh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2
 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c
)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) - 4*(a*cosh(d*x + c)^2 + 2
*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(a*b + b^
2))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c
)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(
d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c)
+ a))/(sqrt(a*b + b^2)*d), sqrt(-a*b - b^2)*arctan(1/2*(a*cosh(d*x + c)^2 +
 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-a*b -
 b^2)/(a*b + b^2))/((a*b + b^2)*d)]
```

giac [A]   time = 0.64, size = 47, normalized size = 1.31

$$\frac{\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/(sqrt(-a*b - b^2
)*d)
```

maple [B]   time = 0.25, size = 104, normalized size = 2.89

$$-\frac{\ln\left(\sqrt{a+b}\,\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\sqrt{b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{a+b}\right)}{2d\sqrt{b}\,\sqrt{a+b}}+\frac{\ln\left(\sqrt{a+b}\,\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\sqrt{b}\,\tanh\left(\frac{dx}{2}\right.\right.}{2d\sqrt{b}\,\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^2/(a+b*sech(d*x+c)^2),x)
```

```
[Out] -1/2/d/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*b^(1/2)*t
anh(1/2*d*x+1/2*c)+(a+b)^(1/2))+1/2/d/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*ta
nh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))
```

maxima [B]   time = 0.42, size = 66, normalized size = 1.83

$$-\frac{\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{2\,\sqrt{(a+b)b}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/2*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x -
2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*d)
```

mupad [B]   time = 0.57, size = 125, normalized size = 3.47

$$\frac{\mathrm{atan}\left(\frac{d\,(a+2\,b)}{2\,\sqrt{-b\,d^2\,(a+b)}}+\frac{a\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}\,\left(\frac{4}{a^2\,d}+\frac{(a+2\,b)\,(a\,d+2\,b\,d)}{a^2\,\sqrt{-b^2\,d^2-a\,b\,d^2}\,\sqrt{-b\,d^2\,(a+b)}}\right)\,\sqrt{-b^2\,d^2-a\,b\,d^2}}{2}\right)}{\sqrt{-b^2\,d^2-a\,b\,d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^2*(a + b/cosh(c + d*x)^2)),x)
```

```
[Out] atan((d*(a + 2*b))/(2*(-b*d^2*(a + b))^(1/2)) + (a*exp(2*c)*exp(2*d*x)*(4/(
a^2*d) + ((a + 2*b)*(a*d + 2*b*d))/(a^2*(- b^2*d^2 - a*b*d^2)^(1/2)*(-b*d^2
*(a + b))^(1/2)))*(- b^2*d^2 - a*b*d^2)^(1/2))/2)/(- b^2*d^2 - a*b*d^2)^(1/
2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^2(c + dx)}{a + b\operatorname{sech}^2(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**2/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(sech(c + d*x)**2/(a + b*sech(c + d*x)**2), x)
```

3.79 $\int \frac{\text{sech}^3(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=55

$$\frac{\tan^{-1}(\sinh(c+dx))}{bd}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{bd\sqrt{a+b}}$$

```
[Out] arctan(sinh(d*x+c))/b/d-arctan(sinh(d*x+c)*a^(1/2)/(a+b)^(1/2))*a^(1/2)/b/d
/(a+b)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4147, 391, 203, 205}

$$\frac{\tan^{-1}(\sinh(c+dx))}{bd}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{bd\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^3/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ArcTan[Sinh[c + d*x]]/(b*d) - (Sqrt[a]*ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[
a + b]])/(b*Sqrt[a + b]*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{(1+x^2)(a+b+ax^2)}\,dx, x, \sinh(c+dx)\right)}{d}$$
$$= \frac{\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(c+dx)\right)}{bd} - \frac{a\operatorname{Subst}\left(\int \frac{1}{a+b+ax^2}\,dx, x, \sinh(c+dx)\right)}{bd}$$
$$= \frac{\tan^{-1}(\sinh(c+dx))}{bd} - \frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{b\sqrt{a+b}\,d}$$

Mathematica [B] time = 0.71, size = 194, normalized size = 3.53

$$\frac{\operatorname{sech}^2(c+dx)(a\cosh(2(c+dx))+a+2b)\left(2\sqrt{a+b}\sqrt{(\cosh(c)-\sinh(c))^2}\tan^{-1}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)+\sqrt{a}\cosh\right.}{2bd\sqrt{a+b}\sqrt{(\cosh(c)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^3/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*(Sqrt[a]*ArcTan[(Sqrt[a +
b]*Csch[c + d*x]*Sqrt[(Cosh[c] - Sinh[c])^2]*(Cosh[c] + Sinh[c]))/Sqrt[a]]*
Cosh[c] + 2*Sqrt[a + b]*ArcTan[Tanh[(c + d*x)/2]]*Sqrt[(Cosh[c] - Sinh[c])^
2] - Sqrt[a]*ArcTan[(Sqrt[a + b]*Csch[c + d*x]*Sqrt[(Cosh[c] - Sinh[c])^2]*
(Cosh[c] + Sinh[c]))/Sqrt[a]]*Sinh[c]))/(2*b*Sqrt[a + b]*d*(a + b*Sech[c +
d*x]^2)*Sqrt[(Cosh[c] - Sinh[c])^2])
```

fricas [B] time = 0.44, size = 526, normalized size = 9.56

$$\left[\frac{\sqrt{-\frac{a}{a+b}}\log\left(\frac{a\cosh(dx+c)^4+4a\cosh(dx+c)\sinh(dx+c)^3+a\sinh(dx+c)^4-2(3a+2b)\cosh(dx+c)^2+2\left(3a\cosh(dx+c)^2-3a-2b\right)\sinh(dx+c)^2+4(}{a\cosh(dx+c)^4+4a\cosh(dx+c)\sinh(dx+c)^3+a\sinh(dx+}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a/(a + b))*log((a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x
+ c)^3 + a*sinh(d*x + c)^4 - 2*(3*a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*
x + c)^2 - 3*a - 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 - (3*a + 2*b)*
cosh(d*x + c))*sinh(d*x + c) - 4*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(
d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 - (a + b)*cosh(d*x + c)
+ (3*(a + b)*cosh(d*x + c)^2 - a - b)*sinh(d*x + c))*sqrt(-a/(a + b)) + a)/
(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4
+ 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x
+ c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)
) + 4*arctan(cosh(d*x + c) + sinh(d*x + c)))/(b*d), -(sqrt(a/(a + b))*arcta
n(1/2*sqrt(a/(a + b))*(cosh(d*x + c) + sinh(d*x + c))) + sqrt(a/(a + b))*ar
ctan(1/2*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*
x + c)^3 + (3*a + 4*b)*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + 3*a + 4*b)*si
nh(d*x + c))*sqrt(a/(a + b))/a) - 2*arctan(cosh(d*x + c) + sinh(d*x + c)))/
(b*d)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[31,78]Warning, need to choose a branch for the root of a po
lynomial with parameters. This might be wrong.The choice was done assuming
[a,b]=[-13,-93]Warning, need to choose a branch for the root of a polynomia
l with parameters. This might be wrong.The choice was done assuming [a,b]=[
-65,-82]Warning, need to choose a branch for the root of a polynomial with
parameters. This might be wrong.The choice was done assuming [a,b]=[97,-56]
Undef/Unsigned Inf encountered in limitEvaluation time: 0.57Limit: Max orde
r reached or unable to make series expansion Error: Bad Argument Value
```

maple [B] time = 0.25, size = 108, normalized size = 1.96

$$-\frac{\sqrt{a}\,\arctan\left(\frac{2\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sqrt{b}}{2\sqrt{a}}\right)}{db\sqrt{a+b}}-\frac{\sqrt{a}\,\arctan\left(\frac{2\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\sqrt{b}}{2\sqrt{a}}\right)}{db\sqrt{a+b}}+\frac{2\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{db}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^3/(a+b*sech(d*x+c)^2),x)
```

```
[Out] -1/d*a^(1/2)/b/(a+b)^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)-2*
b^(1/2))/a^(1/2))-1/d*a^(1/2)/b/(a+b)^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(
1/2*d*x+1/2*c)+2*b^(1/2))/a^(1/2))+2/d/b*arctan(tanh(1/2*d*x+1/2*c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\arctan\left(e^{(dx+c)}\right)}{bd} - 8\int \frac{ae^{(3\,dx+3\,c)} + ae^{(dx+c)}}{4\left(abe^{(4\,dx+4\,c)} + ab + 2\left(abe^{(2\,c)} + 2\,b^2e^{(2\,c)}\right)e^{(2\,dx)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 2*arctan(e^(d*x + c))/(b*d) - 8*integrate(1/4*(a*e^(3*d*x + 3*c) + a*e^(d*x
 + c))/(a*b*e^(4*d*x + 4*c) + a*b + 2*(a*b*e^(2*c) + 2*b^2*e^(2*c))*e^(2*d*
x)), x)
```

mupad [B] time = 1.81, size = 307, normalized size = 5.58

$$\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(9\,a^2\,\sqrt{b^2\,d^2}+16\,b^2\,\sqrt{b^2\,d^2}+24\,a\,b\,\sqrt{b^2\,d^2}\right)}{9\,d\,a^2\,b+24\,d\,a\,b^2+16\,d\,b^3}\right)}{\sqrt{b^2\,d^2}}-\frac{\sqrt{a}\,\left(2\,\mathrm{atan}\left(\frac{\sqrt{a}\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\sqrt{b^2\,d^2\,(a+b)}}{2\,b\,d\,(a+b)}\right)+2\,\mathrm{atan}\left(\frac{4\,b^4\,d^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}+4\,a^2\,b^2\,d^2\,\mathrm{e}\cdots}{\cdots}\right)\cdots\right)}{2\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^3*(a + b/cosh(c + d*x)^2)),x)
```

```
[Out] (2*atan((exp(d*x)*exp(c)*(9*a^2*(b^2*d^2)^(1/2) + 16*b^2*(b^2*d^2)^(1/2) +
24*a*b*(b^2*d^2)^(1/2)))/(16*b^3*d + 24*a*b^2*d + 9*a^2*b*d)))/(b^2*d^2)^(1
/2) - (a^(1/2)*(2*atan((a^(1/2)*exp(d*x)*exp(c)*(b^2*d^2*(a + b))^(1/2))/(2
*b*d*(a + b))) + 2*atan((4*b^4*d^2*exp(d*x)*exp(c) + 4*a^2*b^2*d^2*exp(d*x)
*exp(c) - a*exp(d*x)*exp(c)*(b^3*d^2 + a*b^2*d^2)^(1/2)*(b^2*d^2*(a + b))^(
1/2) + 8*a*b^3*d^2*exp(d*x)*exp(c) + a*exp(3*c)*exp(3*d*x)*(b^3*d^2 + a*b^2
*d^2)^(1/2)*(b^2*d^2*(a + b))^(1/2))/(a^(1/2)*d*(2*a*b + 2*b^2)*(b^2*d^2*(a
 + b))^(1/2)))))/(2*(b^3*d^2 + a*b^2*d^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^{3}{\left(c + dx \right)}}{a + b\operatorname{sech}^{2}{\left(c + dx \right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**3/(a+b*sech(d*x+c)**2),x)
[Out] Integral(sech(c + d*x)**3/(a + b*sech(c + d*x)**2), x)
```

3.80 $\int \frac{\text{sech}^4(c+dx)}{a+b\,\text{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=52

$$\frac{\tanh(c+dx)}{bd}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{b^{3/2}d\sqrt{a+b}}$$

```
[Out] -a*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))/b^(3/2)/d/(a+b)^(1/2)+tanh(d*x+
c)/b/d
```

Rubi [A] time = 0.07, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4146, 388, 208}

$$\frac{\tanh(c+dx)}{bd}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{b^{3/2}d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^4/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -((a*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(b^(3/2)*Sqrt[a + b]*d))
 + Tanh[c + d*x]/(b*d)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
```

```
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\operatorname{sech}^4(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1-x^2}{a+b-bx^2}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\tanh(c+dx)}{bd} - \frac{a\operatorname{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \tanh(c+dx)\right)}{bd}$$
$$= -\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{b^{3/2}\sqrt{a+b}\,d} + \frac{\tanh(c+dx)}{bd}$$

Mathematica [B] time = 0.63, size = 182, normalized size = 3.50

$$\frac{\operatorname{sech}^2(c+dx)(a\cosh(2(c+dx))+a+2b)\left(\sqrt{a+b}\operatorname{sech}(c)\sinh(dx)\sqrt{b(\cosh(c)-\sinh(c))^4}\operatorname{sech}(c+dx)+a(\sin\right.}{2bd\sqrt{a+b}\sqrt{b(\cosh(c)-\sinh(c))^4}\left(a+b\operatorname{sech}^2\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^4/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*(a*ArcTanh[(Sech[d*x]*(Cos
h[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a +
b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(-Cosh[2*c] + Sinh[2*c]) + Sqrt[a + b]*S
ech[c]*Sech[c + d*x]*Sqrt[b*(Cosh[c] - Sinh[c])^4]*Sinh[d*x]))/(2*b*Sqrt[a
+ b]*d*(a + b*Sech[c + d*x]^2)*Sqrt[b*(Cosh[c] - Sinh[c])^4])
```

fricas [B] time = 0.43, size = 645, normalized size = 12.40

$$\left[\frac{\left(a\cosh(dx+c)^2+2a\cosh(dx+c)\sinh(dx+c)+a\sinh(dx+c)^2+a\right)\sqrt{ab+b^2}\log\left(\frac{a^2\cosh(dx+c)^4+4a^2\cosh(dx+\right.}{}\right.}{2\left(\left(ab^2+b^3\right)\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*((a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c
)^2 + a)*sqrt(a*b + b^2)*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sin
```

```
h(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3
*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 +
 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) + 4*(a
*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a
+ 2*b)*sqrt(a*b + b^2))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c
)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c
)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x
+ c))*sinh(d*x + c) + a)) - 4*a*b - 4*b^2)/((a*b^2 + b^3)*d*cosh(d*x + c)^2
 + 2*(a*b^2 + b^3)*d*cosh(d*x + c)*sinh(d*x + c) + (a*b^2 + b^3)*d*sinh(d*x
 + c)^2 + (a*b^2 + b^3)*d), -((a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d
*x + c) + a*sinh(d*x + c)^2 + a)*sqrt(-a*b - b^2)*arctan(1/2*(a*cosh(d*x +
c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(
-a*b - b^2)/(a*b + b^2)) + 2*a*b + 2*b^2)/((a*b^2 + b^3)*d*cosh(d*x + c)^2
+ 2*(a*b^2 + b^3)*d*cosh(d*x + c)*sinh(d*x + c) + (a*b^2 + b^3)*d*sinh(d*x
+ c)^2 + (a*b^2 + b^3)*d)]
```

giac [A] time = 0.62, size = 72, normalized size = 1.38

$$-\frac{\frac{a\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}\,b}+\frac{2}{b\left(e^{(2\,dx+2\,c)}+1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -(a*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/(sqrt(-a*b -
 b^2)*b) + 2/(b*(e^(2*d*x + 2*c) + 1)))/d
```

maple [B] time = 0.22, size = 141, normalized size = 2.71

$$\frac{a\ln\left(-\sqrt{a+b}\,\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\sqrt{b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\sqrt{a+b}\right)}{2d\,b^{\frac{3}{2}}\sqrt{a+b}}-\frac{a\ln\left(\sqrt{a+b}\,\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\sqrt{b}\,\tanh\right.}{2d\,b^{\frac{3}{2}}\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^4/(a+b*sech(d*x+c)^2),x)
```

```
[Out] 1/2/d*a/b^(3/2)/(a+b)^(1/2)*ln(-(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)
*tanh(1/2*d*x+1/2*c)-(a+b)^(1/2))-1/2/d*a/b^(3/2)/(a+b)^(1/2)*ln((a+b)^(1/2
)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))+2/d/b*ta
nh(1/2*d*x+1/2*c)/(tanh(1/2*d*x+1/2*c)^2+1)
```

maxima [B] time = 0.43, size = 91, normalized size = 1.75

$$\frac{a\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{2\,\sqrt{(a+b)b}\,bd}+\frac{2}{\left(be^{(-2\,dx-2\,c)}+b\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*a*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x -
 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*b*d) + 2/((b*e^(-2*d
*x - 2*c) + b)*d)
```

mupad [B] time = 0.48, size = 166, normalized size = 3.19

$$\frac{a\,\ln\left(\frac{4\,e^{2\,c+2\,d\,x}}{b}-\frac{2\,\left(a\,d+a\,d\,e^{2\,c+2\,d\,x}+2\,b\,d\,e^{2\,c+2\,d\,x}\right)}{b^{3/2}\,d\,\sqrt{a+b}}\right)}{2\,b^{3/2}\,d\,\sqrt{a+b}}-\frac{2}{b\,d\,\left(e^{2\,c+2\,d\,x}+1\right)}-\frac{a\,\ln\left(\frac{4\,e^{2\,c+2\,d\,x}}{b}+\frac{2\,\left(a\,d+a\,d\,e^{2\,c+2\,d\,x}+2\,b\,d\,e^{2\,c+2\,d\,x}\right)}{b^{3/2}\,d\,\sqrt{a+b}}\right)}{2\,b^{3/2}\,d\,\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^4*(a + b/cosh(c + d*x)^2)),x)
```

```
[Out] (a*log((4*exp(2*c + 2*d*x))/b - (2*(a*d + a*d*exp(2*c + 2*d*x) + 2*b*d*exp(
2*c + 2*d*x)))/(b^(3/2)*d*(a + b)^(1/2))))/(2*b^(3/2)*d*(a + b)^(1/2)) - 2/
(b*d*(exp(2*c + 2*d*x) + 1)) - (a*log((4*exp(2*c + 2*d*x))/b + (2*(a*d + a*
d*exp(2*c + 2*d*x) + 2*b*d*exp(2*c + 2*d*x)))/(b^(3/2)*d*(a + b)^(1/2))))/(
2*b^(3/2)*d*(a + b)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{sech}^4(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**4/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(sech(c + d*x)**4/(a + b*sech(c + d*x)**2), x)
```

3.81 $\int \frac{\operatorname{sech}^5(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=86

$$\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{b^2d\sqrt{a+b}}-\frac{(2a-b)\tan^{-1}(\sinh(c+dx))}{2b^2d}+\frac{\tanh(c+dx)\operatorname{sech}(c+dx)}{2bd}$$

```
[Out] -1/2*(2*a-b)*arctan(sinh(d*x+c))/b^2/d+a^(3/2)*arctan(sinh(d*x+c)*a^(1/2)/(
a+b)^(1/2))/b^2/d/(a+b)^(1/2)+1/2*sech(d*x+c)*tanh(d*x+c)/b/d
```

Rubi [A] time = 0.10, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4147, 414, 522, 203, 205}

$$\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{b^2d\sqrt{a+b}}-\frac{(2a-b)\tan^{-1}(\sinh(c+dx))}{2b^2d}+\frac{\tanh(c+dx)\operatorname{sech}(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^5/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -((2*a - b)*ArcTan[Sinh[c + d*x]])/(2*b^2*d) + (a^(3/2)*ArcTan[(Sqrt[a]*Sin
h[c + d*x])/Sqrt[a + b]])/(b^2*Sqrt[a + b]*d) + (Sech[c + d*x]*Tanh[c + d*x
])/(2*b*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
```

```
&& !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{sech}^5(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{(1+x^2)^2(a+b+ax^2)}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{2bd} - \frac{\operatorname{Subst}\left(\int \frac{a-b-ax^2}{(1+x^2)(a+b+ax^2)}\,dx, x, \sinh(c+dx)\right)}{2bd}$$

$$= \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{2bd} + \frac{a^2\operatorname{Subst}\left(\int \frac{1}{a+b+ax^2}\,dx, x, \sinh(c+dx)\right)}{b^2 d} - \frac{(2a-b)\operatorname{Sub}}{}$$

$$= -\frac{(2a-b)\tan^{-1}(\sinh(c+dx))}{2b^2 d} + \frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{b^2\sqrt{a+b}\,d} + \frac{\operatorname{sech}(c+dx)\tanh(c+d}{2bd}$$

Mathematica [B] time = 1.84, size = 213, normalized size = 2.48

$$\frac{\cosh(c)\operatorname{sech}^2(c+dx)(a\cosh(2(c+dx))+a+2b)\left(2a^{3/2}(\tanh(c)-1)\tan^{-1}\left(\frac{\sqrt{a+b}\sqrt{(\cosh(c)-\sinh(c))^2}(\sinh(c)+\cosh(c))\operatorname{cs}}{\sqrt{a}}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^5/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] (Cosh[c]*(a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*(b*Sqrt[a + b]*Sec
h[c]^2*Sech[c + d*x]^2*Sqrt[(Cosh[c] - Sinh[c])^2]*Sinh[d*x] + 2*a^(3/2)*Ar
cTan[(Sqrt[a + b]*Csch[c + d*x]*Sqrt[(Cosh[c] - Sinh[c])^2]*(Cosh[c] + Sinh
[c]))/Sqrt[a]]*(-1 + Tanh[c]) - Sqrt[a + b]*Sech[c]*Sqrt[(Cosh[c] - Sinh[c]
)^2]*(2*(2*a - b)*ArcTan[Tanh[(c + d*x)/2]] - b*Sech[c + d*x]*Tanh[c])))/(4
*b^2*Sqrt[a + b]*d*(a + b*Sech[c + d*x]^2)*Sqrt[(Cosh[c] - Sinh[c])^2])
```

fricas [B] time = 0.49, size = 1518, normalized size = 17.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*b*cosh(d*x + c)^3 + 6*b*cosh(d*x + c)*sinh(d*x + c)^2 + 2*b*sinh(d*
x + c)^3 + (a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(
d*x + c)^4 + 2*a*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a)*sinh(d*x + c
)^2 + 4*(a*cosh(d*x + c)^3 + a*cosh(d*x + c))*sinh(d*x + c) + a)*sqrt(-a/(a
 + b))*log((a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(
d*x + c)^4 - 2*(3*a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 - 3*a -
 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 - (3*a + 2*b)*cosh(d*x + c))*s
inh(d*x + c) + 4*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)*sinh(d*
x + c)^2 + (a + b)*sinh(d*x + c)^3 - (a + b)*cosh(d*x + c) + (3*(a + b)*cos
h(d*x + c)^2 - a - b)*sinh(d*x + c))*sqrt(-a/(a + b)) + a)/(a*cosh(d*x + c)
^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*co
sh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*co
sh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) - 2*((2*a - b)
*cosh(d*x + c)^4 + 4*(2*a - b)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a - b)*si
nh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*(2*a - b)*cosh(d*x + c)^
2 + 2*a - b)*sinh(d*x + c)^2 + 4*((2*a - b)*cosh(d*x + c)^3 + (2*a - b)*cos
h(d*x + c))*sinh(d*x + c) + 2*a - b)*arctan(cosh(d*x + c) + sinh(d*x + c))
- 2*b*cosh(d*x + c) + 2*(3*b*cosh(d*x + c)^2 - b)*sinh(d*x + c))/(b^2*d*cos
h(d*x + c)^4 + 4*b^2*d*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*d*sinh(d*x + c)^
4 + 2*b^2*d*cosh(d*x + c)^2 + b^2*d + 2*(3*b^2*d*cosh(d*x + c)^2 + b^2*d)*s
inh(d*x + c)^2 + 4*(b^2*d*cosh(d*x + c)^3 + b^2*d*cosh(d*x + c))*sinh(d*x +
 c)), (b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x +
 c)^3 + (a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x
 + c)^4 + 2*a*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a)*sinh(d*x + c)^2
 + 4*(a*cosh(d*x + c)^3 + a*cosh(d*x + c))*sinh(d*x + c) + a)*sqrt(a/(a + b
))*arctan(1/2*sqrt(a/(a + b))*(cosh(d*x + c) + sinh(d*x + c))) + (a*cosh(d*
x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*a*cosh
(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a)*sinh(d*x + c)^2 + 4*(a*cosh(d*x +
 c)^3 + a*cosh(d*x + c))*sinh(d*x + c) + a)*sqrt(a/(a + b))*arctan(1/2*(a*c
```

```
osh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 + (3
*a + 4*b)*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + 3*a + 4*b)*sinh(d*x + c))*
sqrt(a/(a + b))/a) - ((2*a - b)*cosh(d*x + c)^4 + 4*(2*a - b)*cosh(d*x + c)
*sinh(d*x + c)^3 + (2*a - b)*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2
+ 2*(3*(2*a - b)*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*((2*a - b)*
cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + 2*a - b)*arctan(
cosh(d*x + c) + sinh(d*x + c)) - b*cosh(d*x + c) + (3*b*cosh(d*x + c)^2 - b
)*sinh(d*x + c))/(b^2*d*cosh(d*x + c)^4 + 4*b^2*d*cosh(d*x + c)*sinh(d*x +
c)^3 + b^2*d*sinh(d*x + c)^4 + 2*b^2*d*cosh(d*x + c)^2 + b^2*d + 2*(3*b^2*d
*cosh(d*x + c)^2 + b^2*d)*sinh(d*x + c)^2 + 4*(b^2*d*cosh(d*x + c)^3 + b^2*
d*cosh(d*x + c))*sinh(d*x + c))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[31,78]Warning, need to choose a branch for the root of a po
lynomial with parameters. This might be wrong.The choice was done assuming
[a,b]=[-13,-93]Warning, need to choose a branch for the root of a polynomia
l with parameters. This might be wrong.The choice was done assuming [a,b]=[
-65,-82]Warning, need to choose a branch for the root of a polynomial with
parameters. This might be wrong.The choice was done assuming [a,b]=[97,-56]
Undef/Unsigned Inf encountered in limitEvaluation time: 0.48Limit: Max orde
r reached or unable to make series expansion Error: Bad Argument Value
```

maple [B] time = 0.28, size = 189, normalized size = 2.20

$$\frac{a^{\frac{3}{2}} \arctan\left(\frac{2\sqrt{a+b}\ \tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sqrt{b}}{2\sqrt{a}}\right)}{d\,b^{2}\sqrt{a+b}}+\frac{a^{\frac{3}{2}} \arctan\left(\frac{2\sqrt{a+b}\ \tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\sqrt{b}}{2\sqrt{a}}\right)}{d\,b^{2}\sqrt{a+b}}-\frac{\tanh^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)}{db\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}+\frac{\tanh\left(\frac{dx}{2}\right.}{db\left(\tanh^{2}\left(\frac{dx}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^5/(a+b*sech(d*x+c)^2),x)
```

```
[Out] 1/d*a^(3/2)/b^2/(a+b)^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)-2
*b^(1/2))/a^(1/2))+1/d*a^(3/2)/b^2/(a+b)^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*ta
nh(1/2*d*x+1/2*c)+2*b^(1/2))/a^(1/2))-1/d/b/(tanh(1/2*d*x+1/2*c)^2+1)^2*tan
```

```
h(1/2*d*x+1/2*c)^3+1/d/b/(tanh(1/2*d*x+1/2*c)^2+1)^2*tanh(1/2*d*x+1/2*c)+1/
d/b*arctan(tanh(1/2*d*x+1/2*c))-2/d/b^2*arctan(tanh(1/2*d*x+1/2*c))*a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{e^{(3\,dx+3\,c)}-e^{(dx+c)}}{bde^{(4\,dx+4\,c)}+2\,bde^{(2\,dx+2\,c)}+bd}-\frac{(2\,ae^{c}-be^{c})\arctan\left(e^{(dx+c)}\right)e^{(-c)}}{b^2d}+32\int\frac{a^2e^{(3\,dx+3\,c)}+a^2e^{(dx+c)}}{16\left(ab^2e^{(4\,dx+4\,c)}+ab^2+2\left(ab^2e^{(2\,c)}+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] (e^(3*d*x + 3*c) - e^(d*x + c))/(b*d*e^(4*d*x + 4*c) + 2*b*d*e^(2*d*x + 2*c
) + b*d) - (2*a*e^c - b*e^c)*arctan(e^(d*x + c))*e^(-c)/(b^2*d) + 32*integr
ate(1/16*(a^2*e^(3*d*x + 3*c) + a^2*e^(d*x + c))/(a*b^2*e^(4*d*x + 4*c) + a
*b^2 + 2*(a*b^2*e^(2*c) + 2*b^3*e^(2*c))*e^(2*d*x)), x)
```

mupad [B] time = 3.50, size = 946, normalized size = 11.00

$$\frac{\sqrt{a^3}\left(2\,\mathrm{atan}\left(\left(\left(\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\left(\frac{64\left(6\,b^3\,d\left(a^3\right)^{3/2}+2\,b^6\,d\,\sqrt{a^3}+6\,a\,b^2\,d\left(a^3\right)^{3/2}-4\,a\,b^5\,d\,\sqrt{a^3}-6\,a^2\,b^4\,d\,\sqrt{a^3}\right)}{a^4\,b^4\,(a+b)\left(b^2+a\,b\right)\sqrt{b^5\,d^2+a\,b^4\,d^2}\,\sqrt{b^4\,d^2\,(a+b)}\left(3\,a^3-3\,a\,b^2+b^3\right)}-\frac{32\left(3\,a^5\,\sqrt{b^5\,d^2+a\,b^4\,d^2}+a^2\,b^3\,\sqrt{b^5\,d^2+\right.}{a^2\,b^6\,d\,(a+b)^2\left(b^2+a\,b\right)\sqrt{b^5\,d^2+a\,b}}\right.\right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^5*(a + b/cosh(c + d*x)^2)),x)
```

```
[Out] ((a^3)^(1/2)*(2*atan((exp(d*x)*exp(c)*((64*(6*b^3*d*(a^3)^(3/2) + 2*b^6*d*(
a^3)^(1/2) + 6*a*b^2*d*(a^3)^(3/2) - 4*a*b^5*d*(a^3)^(1/2) - 6*a^2*b^4*d*(a
^3)^(1/2)))/(a^4*b^4*(a + b)*(a*b + b^2)*(b^5*d^2 + a*b^4*d^2)^(1/2)*(b^4*d
^2*(a + b))^(1/2)*(3*a^3 - 3*a*b^2 + b^3)) - (32*(3*a^5*(b^5*d^2 + a*b^4*d^
2)^(1/2) + a^2*b^3*(b^5*d^2 + a*b^4*d^2)^(1/2) - 3*a^3*b^2*(b^5*d^2 + a*b^4
*d^2)^(1/2)))/(a^2*b^6*d*(a + b)^2*(a*b + b^2)*(b^5*d^2 + a*b^4*d^2)^(1/2)*
(a^3)^(1/2)*(3*a^3 - 3*a*b^2 + b^3))) + (32*exp(3*c)*exp(3*d*x)*(3*a^5*(b^5
*d^2 + a*b^4*d^2)^(1/2) + a^2*b^3*(b^5*d^2 + a*b^4*d^2)^(1/2) - 3*a^3*b^2*(
b^5*d^2 + a*b^4*d^2)^(1/2)))/(a^2*b^6*d*(a + b)^2*(a*b + b^2)*(b^5*d^2 + a*
b^4*d^2)^(1/2)*(a^3)^(1/2)*(3*a^3 - 3*a*b^2 + b^3)))*((a^2*b^7*(b^5*d^2 + a
*b^4*d^2)^(1/2))/64 + (a^3*b^6*(b^5*d^2 + a*b^4*d^2)^(1/2))/32 + (a^4*b^5*(
b^5*d^2 + a*b^4*d^2)^(1/2))/64)) + 2*atan((a^2*exp(d*x)*exp(c)*(b^4*d^2*(a
+ b))^(1/2))/(2*b^2*d*(a + b)*(a^3)^(1/2)))))/(2*(b^5*d^2 + a*b^4*d^2)^(1/2
)) - (atan((exp(d*x)*exp(c)*(18*a^7*(b^4*d^2)^(1/2) - b^7*(b^4*d^2)^(1/2) -
 21*a^2*b^5*(b^4*d^2)^(1/2) + 12*a^3*b^4*(b^4*d^2)^(1/2) + 30*a^4*b^3*(b^4*
d^2)^(1/2) - 36*a^5*b^2*(b^4*d^2)^(1/2) + 8*a*b^6*(b^4*d^2)^(1/2) - 9*a^6*b
*(b^4*d^2)^(1/2)))/(b^8*d*(4*a^2 - 4*a*b + b^2)^(1/2) + 9*a^2*b^6*d*(4*a^2
- 4*a*b + b^2)^(1/2) + 6*a^3*b^5*d*(4*a^2 - 4*a*b + b^2)^(1/2) - 18*a^4*b^4
```

```
*d*(4*a^2 - 4*a*b + b^2)^(1/2) + 9*a^6*b^2*d*(4*a^2 - 4*a*b + b^2)^(1/2) -
6*a*b^7*d*(4*a^2 - 4*a*b + b^2)^(1/2)))*(4*a^2 - 4*a*b + b^2)^(1/2))/(b^4*d
^2)^(1/2) + exp(c + d*x)/(b*d*(exp(2*c + 2*d*x) + 1)) - (2*exp(c + d*x))/(b
*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^5(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**5/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(sech(c + d*x)**5/(a + b*sech(c + d*x)**2), x)
```

3.82 $\int \frac{\text{sech}^6(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=77

$$\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{b^{5/2}d\sqrt{a+b}}-\frac{(a-b)\tanh(c+dx)}{b^2d}-\frac{\tanh^3(c+dx)}{3bd}$$

```
[Out] a^2*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))/b^(5/2)/d/(a+b)^(1/2)-(a-b)*ta
nh(d*x+c)/b^2/d-1/3*tanh(d*x+c)^3/b/d
```

Rubi [A] time = 0.09, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4146, 390, 208}

$$\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{b^{5/2}d\sqrt{a+b}}-\frac{(a-b)\tanh(c+dx)}{b^2d}-\frac{\tanh^3(c+dx)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^6/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] (a^2*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(b^(5/2)*Sqrt[a + b]*d)
- ((a - b)*Tanh[c + d*x])/(b^2*d) - Tanh[c + d*x]^3/(3*b*d)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
```

m/2] && IntegerQ[n/2]

Rubi steps

$$\int \frac{\operatorname{sech}^6(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx = \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)^2}{a+b-bx^2}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\operatorname{Subst}\left(\int \left(-\frac{a-b}{b^2}-\frac{x^2}{b}+\frac{a^2}{b^2(a+b-bx^2)}\right)dx, x, \tanh(c+dx)\right)}{d}$$
$$= -\frac{(a-b)\tanh(c+dx)}{b^2d}-\frac{\tanh^3(c+dx)}{3bd}+\frac{a^2\operatorname{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \tanh(c+dx)\right)}{b^2d}$$
$$= \frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{b^{5/2}\sqrt{a+b}\,d}-\frac{(a-b)\tanh(c+dx)}{b^2d}-\frac{\tanh^3(c+dx)}{3bd}$$

Mathematica [B] time = 2.18, size = 214, normalized size = 2.78

$$\frac{\operatorname{sech}^2(c+dx)(a\cosh(2(c+dx))+a+2b)\left(3a^2(\cosh(2c)-\sinh(2c))\tanh^{-1}\left(\frac{(\cosh(2c)-\sinh(2c))\operatorname{sech}(dx)((a+2b)\sinh(dx)-}{2\sqrt{a+b}\sqrt{b(\cosh(c)-\sinh(c))^4}}\right.\right.}{6b^2d\sqrt{a+b}\sqrt{b(\operatorname{co}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^6/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*(3*a^2*ArcTanh[(Sech[d*x]*
(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[
a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(Cosh[2*c] - Sinh[2*c]) + Sqrt[a + b
]*Sech[c + d*x]*Sqrt[b*(Cosh[c] - Sinh[c])^4]*(Sech[c]*(-3*a + 2*b + b*Sech
[c + d*x]^2)*Sinh[d*x] + b*Sech[c + d*x]*Tanh[c])))/(6*b^2*Sqrt[a + b]*d*(a
 + b*Sech[c + d*x]^2)*Sqrt[b*(Cosh[c] - Sinh[c])^4])
```

fricas [B] time = 0.49, size = 1905, normalized size = 24.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/6*(12*(a^2*b + a*b^2)*cosh(d*x + c)^4 + 48*(a^2*b + a*b^2)*cosh(d*x + c)
*sinh(d*x + c)^3 + 12*(a^2*b + a*b^2)*sinh(d*x + c)^4 + 12*a^2*b + 4*a*b^2
- 8*b^3 + 24*(a^2*b - b^3)*cosh(d*x + c)^2 + 24*(a^2*b - b^3 + 3*(a^2*b + a
*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 3*(a^2*cosh(d*x + c)^6 + 6*a^2*cos
h(d*x + c)*sinh(d*x + c)^5 + a^2*sinh(d*x + c)^6 + 3*a^2*cosh(d*x + c)^4 +
3*(5*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^4 + 3*a^2*cosh(d*x + c)^2 + 4
*(5*a^2*cosh(d*x + c)^3 + 3*a^2*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*a^2*c
osh(d*x + c)^4 + 6*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^2 + a^2 + 6*(a^
2*cosh(d*x + c)^5 + 2*a^2*cosh(d*x + c)^3 + a^2*cosh(d*x + c))*sinh(d*x + c
))*sqrt(a*b + b^2)*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x
+ c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*c
osh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^
2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) - 4*(a*cosh(
d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)
*sqrt(a*b + b^2))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 +
a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 +
a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*
sinh(d*x + c) + a)) + 48*((a^2*b + a*b^2)*cosh(d*x + c)^3 + (a^2*b - b^3)*c
osh(d*x + c))*sinh(d*x + c))/((a*b^3 + b^4)*d*cosh(d*x + c)^6 + 6*(a*b^3 +
b^4)*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a*b^3 + b^4)*d*sinh(d*x + c)^6 + 3*
(a*b^3 + b^4)*d*cosh(d*x + c)^4 + 3*(5*(a*b^3 + b^4)*d*cosh(d*x + c)^2 + (a
*b^3 + b^4)*d)*sinh(d*x + c)^4 + 3*(a*b^3 + b^4)*d*cosh(d*x + c)^2 + 4*(5*(
a*b^3 + b^4)*d*cosh(d*x + c)^3 + 3*(a*b^3 + b^4)*d*cosh(d*x + c))*sinh(d*x
+ c)^3 + 3*(5*(a*b^3 + b^4)*d*cosh(d*x + c)^4 + 6*(a*b^3 + b^4)*d*cosh(d*x
+ c)^2 + (a*b^3 + b^4)*d)*sinh(d*x + c)^2 + (a*b^3 + b^4)*d + 6*((a*b^3 + b
^4)*d*cosh(d*x + c)^5 + 2*(a*b^3 + b^4)*d*cosh(d*x + c)^3 + (a*b^3 + b^4)*d
*cosh(d*x + c))*sinh(d*x + c)), 1/3*(6*(a^2*b + a*b^2)*cosh(d*x + c)^4 + 24
*(a^2*b + a*b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + 6*(a^2*b + a*b^2)*sinh(d*x
 + c)^4 + 6*a^2*b + 2*a*b^2 - 4*b^3 + 12*(a^2*b - b^3)*cosh(d*x + c)^2 + 12
*(a^2*b - b^3 + 3*(a^2*b + a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 3*(a^2
*cosh(d*x + c)^6 + 6*a^2*cosh(d*x + c)*sinh(d*x + c)^5 + a^2*sinh(d*x + c)^
6 + 3*a^2*cosh(d*x + c)^4 + 3*(5*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^4
 + 3*a^2*cosh(d*x + c)^2 + 4*(5*a^2*cosh(d*x + c)^3 + 3*a^2*cosh(d*x + c))*
sinh(d*x + c)^3 + 3*(5*a^2*cosh(d*x + c)^4 + 6*a^2*cosh(d*x + c)^2 + a^2)*s
inh(d*x + c)^2 + a^2 + 6*(a^2*cosh(d*x + c)^5 + 2*a^2*cosh(d*x + c)^3 + a^2
*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a*b - b^2)*arctan(1/2*(a*cosh(d*x + c)
^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-a
*b - b^2)/(a*b + b^2)) + 24*((a^2*b + a*b^2)*cosh(d*x + c)^3 + (a^2*b - b^3
)*cosh(d*x + c))*sinh(d*x + c))/((a*b^3 + b^4)*d*cosh(d*x + c)^6 + 6*(a*b^3
 + b^4)*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a*b^3 + b^4)*d*sinh(d*x + c)^6 +
 3*(a*b^3 + b^4)*d*cosh(d*x + c)^4 + 3*(5*(a*b^3 + b^4)*d*cosh(d*x + c)^2 +
 (a*b^3 + b^4)*d)*sinh(d*x + c)^4 + 3*(a*b^3 + b^4)*d*cosh(d*x + c)^2 + 4*(
5*(a*b^3 + b^4)*d*cosh(d*x + c)^3 + 3*(a*b^3 + b^4)*d*cosh(d*x + c))*sinh(d
*x + c)^3 + 3*(5*(a*b^3 + b^4)*d*cosh(d*x + c)^4 + 6*(a*b^3 + b^4)*d*cosh(d
*x + c)^2 + (a*b^3 + b^4)*d)*sinh(d*x + c)^2 + (a*b^3 + b^4)*d + 6*((a*b^3
```

```
+ b^4)*d*cosh(d*x + c)^5 + 2*(a*b^3 + b^4)*d*cosh(d*x + c)^3 + (a*b^3 + b^4
)*d*cosh(d*x + c))*sinh(d*x + c))]
```

giac [A] time = 0.64, size = 118, normalized size = 1.53

$$\frac{\frac{3\,a^2\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}\,b^2}+\frac{2\left(3\,ae^{(4\,dx+4\,c)}+6\,ae^{(2\,dx+2\,c)}-6\,be^{(2\,dx+2\,c)}+3\,a-2\,b\right)}{b^2\left(e^{(2\,dx+2\,c)}+1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/3*(3*a^2*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/(sqrt
(-a*b - b^2)*b^2) + 2*(3*a*e^(4*d*x + 4*c) + 6*a*e^(2*d*x + 2*c) - 6*b*e^(2
*d*x + 2*c) + 3*a - 2*b)/(b^2*(e^(2*d*x + 2*c) + 1)^3))/d
```

maple [B] time = 0.28, size = 316, normalized size = 4.10

$$-\frac{a^2\ln\left(-\sqrt{a+b}\,\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\sqrt{b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\sqrt{a+b}\right)}{2d\,b^{\frac{5}{2}}\sqrt{a+b}}+\frac{a^2\ln\left(\sqrt{a+b}\,\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\sqrt{b}\,\text{ta}\right.}{2d\,b^{\frac{5}{2}}\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^6/(a+b*sech(d*x+c)^2),x)
```

```
[Out] -1/2/d*a^2/b^(5/2)/(a+b)^(1/2)*ln(-(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1
/2)*tanh(1/2*d*x+1/2*c)-(a+b)^(1/2))+1/2/d*a^2/b^(5/2)/(a+b)^(1/2)*ln((a+b)
^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))-2/d
/b^2/(tanh(1/2*d*x+1/2*c)^2+1)^3*tanh(1/2*d*x+1/2*c)^5*a+2/d/b/(tanh(1/2*d*
x+1/2*c)^2+1)^3*tanh(1/2*d*x+1/2*c)^5-4/d/b^2/(tanh(1/2*d*x+1/2*c)^2+1)^3*t
anh(1/2*d*x+1/2*c)^3*a+4/3/d/b/(tanh(1/2*d*x+1/2*c)^2+1)^3*tanh(1/2*d*x+1/2
*c)^3-2/d/b^2/(tanh(1/2*d*x+1/2*c)^2+1)^3*tanh(1/2*d*x+1/2*c)*a+2/d/b/(tanh
(1/2*d*x+1/2*c)^2+1)^3*tanh(1/2*d*x+1/2*c)
```

maxima [B] time = 0.46, size = 160, normalized size = 2.08

$$-\frac{a^2\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{2\,\sqrt{(a+b)b}\,b^2d}-\frac{2\left(6\,(a-b)e^{(-2\,dx-2\,c)}+3\,ae^{(-4\,dx-4\,c)}+3\,a-2\,b\right)}{3\left(3\,b^2e^{(-2\,dx-2\,c)}+3\,b^2e^{(-4\,dx-4\,c)}+b^2e^{(-6\,dx-6\,c)}+b^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/2*a^2*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*
x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*b^2*d) - 2/3*(6*(
a - b)*e^(-2*d*x - 2*c) + 3*a*e^(-4*d*x - 4*c) + 3*a - 2*b)/((3*b^2*e^(-2*d
*x - 2*c) + 3*b^2*e^(-4*d*x - 4*c) + b^2*e^(-6*d*x - 6*c) + b^2)*d)
```

mupad [B] time = 1.99, size = 334, normalized size = 4.34

$$\frac{8}{3\,b\,d\,\left(3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1\right)}-\frac{4}{b\,d\,\left(2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1\right)}+\frac{2\,a}{b^2\,d\,\left(e^{2\,c+2\,d\,x}+1\right)}-\frac{a^2\,\ln\left(\frac{4\,a^2\,(2\,a\ldots}{\ldots}\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^6*(a + b/cosh(c + d*x)^2)),x)
```

```
[Out] 8/(3*b*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1))
- 4/(b*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) + (2*a)/(b^2*d*(exp(2
*c + 2*d*x) + 1)) - (a^2*log((4*a^2*(2*a*b + a^2 + a^2*exp(2*c + 2*d*x) + 8
*b^2*exp(2*c + 2*d*x) + 8*a*b*exp(2*c + 2*d*x)))/(b^5*(a + b)) - (8*a^2*(a
+ 2*a*exp(2*c + 2*d*x) + 4*b*exp(2*c + 2*d*x)))/(b^(9/2)*(a + b)^(1/2))))/(
2*b^(5/2)*d*(a + b)^(1/2)) + (a^2*log((8*a^2*(a + 2*a*exp(2*c + 2*d*x) + 4*
b*exp(2*c + 2*d*x)))/(b^(9/2)*(a + b)^(1/2)) + (4*a^2*(2*a*b + a^2 + a^2*ex
p(2*c + 2*d*x) + 8*b^2*exp(2*c + 2*d*x) + 8*a*b*exp(2*c + 2*d*x)))/(b^5*(a
+ b))))/(2*b^(5/2)*d*(a + b)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^6(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**6/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(sech(c + d*x)**6/(a + b*sech(c + d*x)**2), x)
```

3.83 $\int \frac{\cosh^3(c+dx)}{\left(a+b\,\mathbf{sech}^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=125

$$\frac{b^2(6a+5b)\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{2a^{7/2}d(a+b)^{3/2}}-\frac{b^3\sinh(c+dx)}{2a^3d(a+b)\left(a\sinh^2(c+dx)+a+b\right)}+\frac{(a-2b)\sinh(c+dx)}{a^3d}+\frac{\sinh^3(c+dx)}{3a^2d}$$

```
[Out] 1/2*b^2*(6*a+5*b)*arctan(sinh(d*x+c)*a^(1/2)/(a+b)^(1/2))/a^(7/2)/(a+b)^(3/
2)/d+(a-2*b)*sinh(d*x+c)/a^3/d+1/3*sinh(d*x+c)^3/a^2/d-1/2*b^3*sinh(d*x+c)/
a^3/(a+b)/d/(a+b+a*sinh(d*x+c)^2)
```

Rubi [A] time = 0.15, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4147, 390, 385, 205}

$$-\frac{b^3\sinh(c+dx)}{2a^3d(a+b)\left(a\sinh^2(c+dx)+a+b\right)}+\frac{b^2(6a+5b)\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{2a^{7/2}d(a+b)^{3/2}}+\frac{(a-2b)\sinh(c+dx)}{a^3d}+\frac{\sinh^3(c+dx)}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^3/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (b^2*(6*a + 5*b)*ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]])/(2*a^(7/2)*(a
 + b)^(3/2)*d) + ((a - 2*b)*Sinh[c + d*x])/(a^3*d) + Sinh[c + d*x]^3/(3*a^2
*d) - (b^3*Sinh[c + d*x])/(2*a^3*(a + b)*d*(a + b + a*Sinh[c + d*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
```

```
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cosh^3(c+dx)}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^2}\,dx = \frac{\mathrm{Subst}\left(\int \frac{\left(1+x^2\right)^3}{\left(a+b+ax^2\right)^2}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\mathrm{Subst}\left(\int \left(\frac{a-2b}{a^3}+\frac{x^2}{a^2}+\frac{b^2(3a+2b)+3ab^2x^2}{a^3\left(a+b+ax^2\right)^2}\right)dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{(a-2b)\sinh(c+dx)}{a^3d}+\frac{\sinh^3(c+dx)}{3a^2d}+\frac{\mathrm{Subst}\left(\int \frac{b^2(3a+2b)+3ab^2x^2}{\left(a+b+ax^2\right)^2}\,dx, x, \sinh(c+\right.}{a^3d}$$

$$= \frac{(a-2b)\sinh(c+dx)}{a^3d}+\frac{\sinh^3(c+dx)}{3a^2d}-\frac{b^3\sinh(c+dx)}{2a^3(a+b)d\left(a+b+a\sinh^2(c+dx)\right)}+$$

$$= \frac{b^2(6a+5b)\tan^{-1}\left(\frac{\sqrt{a}\,\sinh(c+dx)}{\sqrt{a+b}}\right)}{2a^{7/2}(a+b)^{3/2}d}+\frac{(a-2b)\sinh(c+dx)}{a^3d}+\frac{\sinh^3(c+dx)}{3a^2d}-\frac{}{2a^3}$$

Mathematica [A] time = 0.92, size = 113, normalized size = 0.90

$$\frac{a^{3/2}\sinh(3(c+dx))+3\sqrt{a}\,\sinh(c+dx)\left(-\frac{4b^3}{(a+b)(a\cosh(2(c+dx))+a+2b)}+3a-8b\right)-\frac{6b^2(6a+5b)\tan^{-1}\left(\frac{\sqrt{a+b}\,\mathrm{csch}(c+dx)}{\sqrt{a}}\right)}{(a+b)^{3/2}}}{12a^{7/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^3/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((-6*b^2*(6*a + 5*b)*ArcTan[(Sqrt[a + b]*Csch[c + d*x])/Sqrt[a]])/(a + b)^(
3/2) + 3*Sqrt[a]*(3*a - 8*b - (4*b^3)/((a + b)*(a + 2*b + a*Cosh[2*(c + d*x
)])))*Sinh[c + d*x] + a^(3/2)*Sinh[3*(c + d*x)])/(12*a^(7/2)*d)
```

fricas [B] time = 0.54, size = 5842, normalized size = 46.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/24*((a^5 + 2*a^4*b + a^3*b^2)*cosh(d*x + c)^10 + 10*(a^5 + 2*a^4*b + a^3
*b^2)*cosh(d*x + c)*sinh(d*x + c)^9 + (a^5 + 2*a^4*b + a^3*b^2)*sinh(d*x +
c)^10 + (11*a^5 + 2*a^4*b - 29*a^3*b^2 - 20*a^2*b^3)*cosh(d*x + c)^8 + (11*
a^5 + 2*a^4*b - 29*a^3*b^2 - 20*a^2*b^3 + 45*(a^5 + 2*a^4*b + a^3*b^2)*cosh
(d*x + c)^2)*sinh(d*x + c)^8 + 8*(15*(a^5 + 2*a^4*b + a^3*b^2)*cosh(d*x + c
)^3 + (11*a^5 + 2*a^4*b - 29*a^3*b^2 - 20*a^2*b^3)*cosh(d*x + c))*sinh(d*x
+ c)^7 + 2*(5*a^5 + 16*a^4*b - 31*a^3*b^2 - 102*a^2*b^3 - 60*a*b^4)*cosh(d*
x + c)^6 + 2*(5*a^5 + 16*a^4*b - 31*a^3*b^2 - 102*a^2*b^3 - 60*a*b^4 + 105*
(a^5 + 2*a^4*b + a^3*b^2)*cosh(d*x + c)^4 + 14*(11*a^5 + 2*a^4*b - 29*a^3*b
^2 - 20*a^2*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(63*(a^5 + 2*a^4*b +
a^3*b^2)*cosh(d*x + c)^5 + 14*(11*a^5 + 2*a^4*b - 29*a^3*b^2 - 20*a^2*b^3)*
cosh(d*x + c)^3 + 3*(5*a^5 + 16*a^4*b - 31*a^3*b^2 - 102*a^2*b^3 - 60*a*b^4
)*cosh(d*x + c))*sinh(d*x + c)^5 - a^5 - 2*a^4*b - a^3*b^2 - 2*(5*a^5 + 16*
a^4*b - 31*a^3*b^2 - 102*a^2*b^3 - 60*a*b^4)*cosh(d*x + c)^4 + 2*(105*(a^5
+ 2*a^4*b + a^3*b^2)*cosh(d*x + c)^6 - 5*a^5 - 16*a^4*b + 31*a^3*b^2 + 102*
a^2*b^3 + 60*a*b^4 + 35*(11*a^5 + 2*a^4*b - 29*a^3*b^2 - 20*a^2*b^3)*cosh(d
*x + c)^4 + 15*(5*a^5 + 16*a^4*b - 31*a^3*b^2 - 102*a^2*b^3 - 60*a*b^4)*cos
h(d*x + c)^2)*sinh(d*x + c)^4 + 8*(15*(a^5 + 2*a^4*b + a^3*b^2)*cosh(d*x +
c)^7 + 7*(11*a^5 + 2*a^4*b - 29*a^3*b^2 - 20*a^2*b^3)*cosh(d*x + c)^5 + 5*(
5*a^5 + 16*a^4*b - 31*a^3*b^2 - 102*a^2*b^3 - 60*a*b^4)*cosh(d*x + c)^3 - (
5*a^5 + 16*a^4*b - 31*a^3*b^2 - 102*a^2*b^3 - 60*a*b^4)*cosh(d*x + c))*sinh
(d*x + c)^3 - (11*a^5 + 2*a^4*b - 29*a^3*b^2 - 20*a^2*b^3)*cosh(d*x + c)^2
+ (45*(a^5 + 2*a^4*b + a^3*b^2)*cosh(d*x + c)^8 + 28*(11*a^5 + 2*a^4*b - 29
*a^3*b^2 - 20*a^2*b^3)*cosh(d*x + c)^6 - 11*a^5 - 2*a^4*b + 29*a^3*b^2 + 20
*a^2*b^3 + 30*(5*a^5 + 16*a^4*b - 31*a^3*b^2 - 102*a^2*b^3 - 60*a*b^4)*cosh
(d*x + c)^4 - 12*(5*a^5 + 16*a^4*b - 31*a^3*b^2 - 102*a^2*b^3 - 60*a*b^4)*c
osh(d*x + c)^2)*sinh(d*x + c)^2 - 6*((6*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)^7
+ 7*(6*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)*sinh(d*x + c)^6 + (6*a^2*b^2 + 5*a*
b^3)*sinh(d*x + c)^7 + 2*(6*a^2*b^2 + 17*a*b^3 + 10*b^4)*cosh(d*x + c)^5 +
(12*a^2*b^2 + 34*a*b^3 + 20*b^4 + 21*(6*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)^2)
*sinh(d*x + c)^5 + 5*(7*(6*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)^3 + 2*(6*a^2*b^
2 + 17*a*b^3 + 10*b^4)*cosh(d*x + c))*sinh(d*x + c)^4 + (6*a^2*b^2 + 5*a*b^
3)*cosh(d*x + c)^3 + (35*(6*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)^4 + 6*a^2*b^2
+ 5*a*b^3 + 20*(6*a^2*b^2 + 17*a*b^3 + 10*b^4)*cosh(d*x + c)^2)*sinh(d*x +
```

c)^3 + (21*(6*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)^5 + 20*(6*a^2*b^2 + 17*a*b^3
 + 10*b^4)*cosh(d*x + c)^3 + 3*(6*a^2*b^2 + 5*a*b^3)*cosh(d*x + c))*sinh(d*
x + c)^2 + (7*(6*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)^6 + 10*(6*a^2*b^2 + 17*a*
b^3 + 10*b^4)*cosh(d*x + c)^4 + 3*(6*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)^2)*si
nh(d*x + c))*sqrt(-a^2 - a*b)*log((a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*si
nh(d*x + c)^3 + a*sinh(d*x + c)^4 - 2*(3*a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*
cosh(d*x + c)^2 - 3*a - 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 - (3*a
+ 2*b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*
sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1)*sinh(d*x + c) -
 cosh(d*x + c))*sqrt(-a^2 - a*b) + a)/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c
)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*
a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a +
2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) + 2*(5*(a^5 + 2*a^4*b + a^3*b^2)*co
sh(d*x + c)^9 + 4*(11*a^5 + 2*a^4*b - 29*a^3*b^2 - 20*a^2*b^3)*cosh(d*x + c
)^7 + 6*(5*a^5 + 16*a^4*b - 31*a^3*b^2 - 102*a^2*b^3 - 60*a*b^4)*cosh(d*x +
 c)^5 - 4*(5*a^5 + 16*a^4*b - 31*a^3*b^2 - 102*a^2*b^3 - 60*a*b^4)*cosh(d*x
 + c)^3 - (11*a^5 + 2*a^4*b - 29*a^3*b^2 - 20*a^2*b^3)*cosh(d*x + c))*sinh(
d*x + c))/((a^7 + 2*a^6*b + a^5*b^2)*d*cosh(d*x + c)^7 + 7*(a^7 + 2*a^6*b +
 a^5*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^6 + (a^7 + 2*a^6*b + a^5*b^2)*d*sin
h(d*x + c)^7 + 2*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*d*cosh(d*x + c)^5
+ (21*(a^7 + 2*a^6*b + a^5*b^2)*d*cosh(d*x + c)^2 + 2*(a^7 + 4*a^6*b + 5*a^
5*b^2 + 2*a^4*b^3)*d)*sinh(d*x + c)^5 + (a^7 + 2*a^6*b + a^5*b^2)*d*cosh(d*
x + c)^3 + 5*(7*(a^7 + 2*a^6*b + a^5*b^2)*d*cosh(d*x + c)^3 + 2*(a^7 + 4*a^
6*b + 5*a^5*b^2 + 2*a^4*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^4 + (35*(a^7 +
2*a^6*b + a^5*b^2)*d*cosh(d*x + c)^4 + 20*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^
4*b^3)*d*cosh(d*x + c)^2 + (a^7 + 2*a^6*b + a^5*b^2)*d)*sinh(d*x + c)^3 + (
21*(a^7 + 2*a^6*b + a^5*b^2)*d*cosh(d*x + c)^5 + 20*(a^7 + 4*a^6*b + 5*a^5*
b^2 + 2*a^4*b^3)*d*cosh(d*x + c)^3 + 3*(a^7 + 2*a^6*b + a^5*b^2)*d*cosh(d*x
 + c))*sinh(d*x + c)^2 + (7*(a^7 + 2*a^6*b + a^5*b^2)*d*cosh(d*x + c)^6 + 1
0*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*d*cosh(d*x + c)^4 + 3*(a^7 + 2*a^
6*b + a^5*b^2)*d*cosh(d*x + c)^2)*sinh(d*x + c)), 1/24*((a^5 + 2*a^4*b + a^
3*b^2)*cosh(d*x + c)^10 + 10*(a^5 + 2*a^4*b + a^3*b^2)*cosh(d*x + c)*sinh(d
*x + c)^9 + (a^5 + 2*a^4*b + a^3*b^2)*sinh(d*x + c)^10 + (11*a^5 + 2*a^4*b
- 29*a^3*b^2 - 20*a^2*b^3)*cosh(d*x + c)^8 + (11*a^5 + 2*a^4*b - 29*a^3*b^2
 - 20*a^2*b^3 + 45*(a^5 + 2*a^4*b + a^3*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)
^8 + 8*(15*(a^5 + 2*a^4*b + a^3*b^2)*cosh(d*x + c)^3 + (11*a^5 + 2*a^4*b -
29*a^3*b^2 - 20*a^2*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(5*a^5 + 16*a^4
*b - 31*a^3*b^2 - 102*a^2*b^3 - 60*a*b^4)*cosh(d*x + c)^6 + 2*(5*a^5 + 16*a
^4*b - 31*a^3*b^2 - 102*a^2*b^3 - 60*a*b^4 + 105*(a^5 + 2*a^4*b + a^3*b^2)*
cosh(d*x + c)^4 + 14*(11*a^5 + 2*a^4*b - 29*a^3*b^2 - 20*a^2*b^3)*cosh(d*x
+ c)^2)*sinh(d*x + c)^6 + 4*(63*(a^5 + 2*a^4*b + a^3*b^2)*cosh(d*x + c)^5 +
 14*(11*a^5 + 2*a^4*b - 29*a^3*b^2 - 20*a^2*b^3)*cosh(d*x + c)^3 + 3*(5*a^5
 + 16*a^4*b - 31*a^3*b^2 - 102*a^2*b^3 - 60*a*b^4)*cosh(d*x + c))*sinh(d*x
+ c)^5 - a^5 - 2*a^4*b - a^3*b^2 - 2*(5*a^5 + 16*a^4*b - 31*a^3*b^2 - 102*a
^2*b^3 - 60*a*b^4)*cosh(d*x + c)^4 + 2*(105*(a^5 + 2*a^4*b + a^3*b^2)*cosh(

d*x + c)^6 - 5*a^5 - 16*a^4*b + 31*a^3*b^2 + 102*a^2*b^3 + 60*a*b^4 + 35*(1
1*a^5 + 2*a^4*b - 29*a^3*b^2 - 20*a^2*b^3)*cosh(d*x + c)^4 + 15*(5*a^5 + 16
*a^4*b - 31*a^3*b^2 - 102*a^2*b^3 - 60*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c
)^4 + 8*(15*(a^5 + 2*a^4*b + a^3*b^2)*cosh(d*x + c)^7 + 7*(11*a^5 + 2*a^4*b
 - 29*a^3*b^2 - 20*a^2*b^3)*cosh(d*x + c)^5 + 5*(5*a^5 + 16*a^4*b - 31*a^3*
b^2 - 102*a^2*b^3 - 60*a*b^4)*cosh(d*x + c)^3 - (5*a^5 + 16*a^4*b - 31*a^3*
b^2 - 102*a^2*b^3 - 60*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 - (11*a^5 + 2*
a^4*b - 29*a^3*b^2 - 20*a^2*b^3)*cosh(d*x + c)^2 + (45*(a^5 + 2*a^4*b + a^3
*b^2)*cosh(d*x + c)^8 + 28*(11*a^5 + 2*a^4*b - 29*a^3*b^2 - 20*a^2*b^3)*cos
h(d*x + c)^6 - 11*a^5 - 2*a^4*b + 29*a^3*b^2 + 20*a^2*b^3 + 30*(5*a^5 + 16*
a^4*b - 31*a^3*b^2 - 102*a^2*b^3 - 60*a*b^4)*cosh(d*x + c)^4 - 12*(5*a^5 +
16*a^4*b - 31*a^3*b^2 - 102*a^2*b^3 - 60*a*b^4)*cosh(d*x + c)^2)*sinh(d*x +
 c)^2 + 12*((6*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)^7 + 7*(6*a^2*b^2 + 5*a*b^3)
*cosh(d*x + c)*sinh(d*x + c)^6 + (6*a^2*b^2 + 5*a*b^3)*sinh(d*x + c)^7 + 2*
(6*a^2*b^2 + 17*a*b^3 + 10*b^4)*cosh(d*x + c)^5 + (12*a^2*b^2 + 34*a*b^3 +
20*b^4 + 21*(6*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 5*(7*(
6*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)^3 + 2*(6*a^2*b^2 + 17*a*b^3 + 10*b^4)*co
sh(d*x + c))*sinh(d*x + c)^4 + (6*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)^3 + (35*
(6*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)^4 + 6*a^2*b^2 + 5*a*b^3 + 20*(6*a^2*b^2
 + 17*a*b^3 + 10*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + (21*(6*a^2*b^2 + 5
*a*b^3)*cosh(d*x + c)^5 + 20*(6*a^2*b^2 + 17*a*b^3 + 10*b^4)*cosh(d*x + c)^
3 + 3*(6*a^2*b^2 + 5*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + (7*(6*a^2*b^2
+ 5*a*b^3)*cosh(d*x + c)^6 + 10*(6*a^2*b^2 + 17*a*b^3 + 10*b^4)*cosh(d*x +
c)^4 + 3*(6*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))*sqrt(a^2 + a
*b)*arctan(1/2*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*s
inh(d*x + c)^3 + (3*a + 4*b)*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + 3*a + 4
*b)*sinh(d*x + c))/sqrt(a^2 + a*b)) + 12*((6*a^2*b^2 + 5*a*b^3)*cosh(d*x +
c)^7 + 7*(6*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)*sinh(d*x + c)^6 + (6*a^2*b^2 +
 5*a*b^3)*sinh(d*x + c)^7 + 2*(6*a^2*b^2 + 17*a*b^3 + 10*b^4)*cosh(d*x + c)
^5 + (12*a^2*b^2 + 34*a*b^3 + 20*b^4 + 21*(6*a^2*b^2 + 5*a*b^3)*cosh(d*x +
c)^2)*sinh(d*x + c)^5 + 5*(7*(6*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)^3 + 2*(6*a
^2*b^2 + 17*a*b^3 + 10*b^4)*cosh(d*x + c))*sinh(d*x + c)^4 + (6*a^2*b^2 + 5
*a*b^3)*cosh(d*x + c)^3 + (35*(6*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)^4 + 6*a^2
*b^2 + 5*a*b^3 + 20*(6*a^2*b^2 + 17*a*b^3 + 10*b^4)*cosh(d*x + c)^2)*sinh(d
x + c)^3 + (21(6*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)^5 + 20*(6*a^2*b^2 + 17*
a*b^3 + 10*b^4)*cosh(d*x + c)^3 + 3*(6*a^2*b^2 + 5*a*b^3)*cosh(d*x + c))*si
nh(d*x + c)^2 + (7*(6*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)^6 + 10*(6*a^2*b^2 +
17*a*b^3 + 10*b^4)*cosh(d*x + c)^4 + 3*(6*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)^
2)*sinh(d*x + c))*sqrt(a^2 + a*b)*arctan(1/2*sqrt(a^2 + a*b)*(cosh(d*x + c)
 + sinh(d*x + c))/(a + b)) + 2*(5*(a^5 + 2*a^4*b + a^3*b^2)*cosh(d*x + c)^9
 + 4*(11*a^5 + 2*a^4*b - 29*a^3*b^2 - 20*a^2*b^3)*cosh(d*x + c)^7 + 6*(5*a^
5 + 16*a^4*b - 31*a^3*b^2 - 102*a^2*b^3 - 60*a*b^4)*cosh(d*x + c)^5 - 4*(5*
a^5 + 16*a^4*b - 31*a^3*b^2 - 102*a^2*b^3 - 60*a*b^4)*cosh(d*x + c)^3 - (11
*a^5 + 2*a^4*b - 29*a^3*b^2 - 20*a^2*b^3)*cosh(d*x + c))*sinh(d*x + c))/((a
^7 + 2*a^6*b + a^5*b^2)*d*cosh(d*x + c)^7 + 7*(a^7 + 2*a^6*b + a^5*b^2)*d*c

```
osh(d*x + c)*sinh(d*x + c)^6 + (a^7 + 2*a^6*b + a^5*b^2)*d*sinh(d*x + c)^7
+ 2*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*d*cosh(d*x + c)^5 + (21*(a^7 +
2*a^6*b + a^5*b^2)*d*cosh(d*x + c)^2 + 2*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4
*b^3)*d)*sinh(d*x + c)^5 + (a^7 + 2*a^6*b + a^5*b^2)*d*cosh(d*x + c)^3 + 5*
(7*(a^7 + 2*a^6*b + a^5*b^2)*d*cosh(d*x + c)^3 + 2*(a^7 + 4*a^6*b + 5*a^5*b
^2 + 2*a^4*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^4 + (35*(a^7 + 2*a^6*b + a^5
*b^2)*d*cosh(d*x + c)^4 + 20*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*d*cosh
(d*x + c)^2 + (a^7 + 2*a^6*b + a^5*b^2)*d)*sinh(d*x + c)^3 + (21*(a^7 + 2*a
^6*b + a^5*b^2)*d*cosh(d*x + c)^5 + 20*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b
^3)*d*cosh(d*x + c)^3 + 3*(a^7 + 2*a^6*b + a^5*b^2)*d*cosh(d*x + c))*sinh(d
*x + c)^2 + (7*(a^7 + 2*a^6*b + a^5*b^2)*d*cosh(d*x + c)^6 + 10*(a^7 + 4*a^
6*b + 5*a^5*b^2 + 2*a^4*b^3)*d*cosh(d*x + c)^4 + 3*(a^7 + 2*a^6*b + a^5*b^2
)*d*cosh(d*x + c)^2)*sinh(d*x + c))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[6,-20]Warning, need to choose a branch for the root of a po
lynomial with parameters. This might be wrong.The choice was done assuming
[a,b]=[89,-63]Warning, need to choose a branch for the root of a polynomial
 with parameters. This might be wrong.The choice was done assuming [a,b]=[1
2,-32]Warning, need to choose a branch for the root of a polynomial with pa
rameters. This might be wrong.The choice was done assuming [a,b]=[2,72]Warn
ing, need to choose a branch for the root of a polynomial with parameters.
This might be wrong.The choice was done assuming [a,b]=[-37,-59]Warning, ne
ed to choose a branch for the root of a polynomial with parameters. This mi
ght be wrong.The choice was done assuming [a,b]=[-67,22]Warning, need to ch
oose a branch for the root of a polynomial with parameters. This might be w
rong.The choice was done assuming [a,b]=[62,70]Warning, need to choose a br
anch for the root of a polynomial with parameters. This might be wrong.The
choice was done assuming [a,b]=[50,35]Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done assuming [a,b]=[43,41]Warning, need to choose a branch for the root
of a polynomial with parameters. This might be wrong.The choice was done as
suming [a,b]=[37,80]Undef/Unsigned Inf encountered in limitEvaluation time:
 1.64Limit: Max order reached or unable to make series expansion Error: Bad
 Argument Value
```

maple [B] time = 0.53, size = 517, normalized size = 4.14

$$-\frac{1}{3d\,a^{2}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{2d\,a^{2}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{1}{d\,a^{2}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{2b}{d\,a^{3}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] -1/3/d/a^2/(tanh(1/2*d*x+1/2*c)-1)^3-1/2/d/a^2/(tanh(1/2*d*x+1/2*c)-1)^2-1/
d/a^2/(tanh(1/2*d*x+1/2*c)-1)+2/d/a^3/(tanh(1/2*d*x+1/2*c)-1)*b-1/3/d/a^2/(
tanh(1/2*d*x+1/2*c)+1)^3+1/2/d/a^2/(tanh(1/2*d*x+1/2*c)+1)^2-1/d/a^2/(tanh(
1/2*d*x+1/2*c)+1)+2/d/a^3/(tanh(1/2*d*x+1/2*c)+1)*b+1/d/a^3*b^3/(tanh(1/2*d
*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*
d*x+1/2*c)^2*b+a+b)/(a+b)*tanh(1/2*d*x+1/2*c)^3-1/d/a^3*b^3/(tanh(1/2*d*x+1
/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+
1/2*c)^2*b+a+b)/(a+b)*tanh(1/2*d*x+1/2*c)+3/d/a^(5/2)*b^2/(a+b)^(3/2)*arcta
n(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)-2*b^(1/2))/a^(1/2))+3/d/a^(5/2)*b^
2/(a+b)^(3/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)+2*b^(1/2))/a^(1
/2))+5/2/d/a^(7/2)*b^3/(a+b)^(3/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1
/2*c)-2*b^(1/2))/a^(1/2))+5/2/d/a^(7/2)*b^3/(a+b)^(3/2)*arctan(1/2*(2*(a+b)
^(1/2)*tanh(1/2*d*x+1/2*c)+2*b^(1/2))/a^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{a^{3}+a^{2}b-\left(a^{3}e^{(10\,c)}+a^{2}be^{(10\,c)}\right)e^{(10\,dx)}-\left(11\,a^{3}e^{(8\,c)}-9\,a^{2}be^{(8\,c)}-20\,ab^{2}e^{(8\,c)}\right)e^{(8\,dx)}-2\left(5\,a^{3}e^{(6\,c)}+11\,a^{2}be^{(6\,c)}-\right.}{24\left(\left(a^{5}de^{(7\,c)}+a^{4}bde^{(7\,c)}\right)e^{(7\,dx)}+2\left(a^{5}de^{(5\,c)}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/24*(a^3 + a^2*b - (a^3*e^(10*c) + a^2*b*e^(10*c))*e^(10*d*x) - (11*a^3*e
^(8*c) - 9*a^2*b*e^(8*c) - 20*a*b^2*e^(8*c))*e^(8*d*x) - 2*(5*a^3*e^(6*c) +
 11*a^2*b*e^(6*c) - 42*a*b^2*e^(6*c) - 60*b^3*e^(6*c))*e^(6*d*x) + 2*(5*a^3
*e^(4*c) + 11*a^2*b*e^(4*c) - 42*a*b^2*e^(4*c) - 60*b^3*e^(4*c))*e^(4*d*x)
+ (11*a^3*e^(2*c) - 9*a^2*b*e^(2*c) - 20*a*b^2*e^(2*c))*e^(2*d*x))/((a^5*d*
e^(7*c) + a^4*b*d*e^(7*c))*e^(7*d*x) + 2*(a^5*d*e^(5*c) + 3*a^4*b*d*e^(5*c)
 + 2*a^3*b^2*d*e^(5*c))*e^(5*d*x) + (a^5*d*e^(3*c) + a^4*b*d*e^(3*c))*e^(3*
d*x)) + 1/8*integrate(8*((6*a*b^2*e^(3*c) + 5*b^3*e^(3*c))*e^(3*d*x) + (6*a
*b^2*e^c + 5*b^3*e^c)*e^(d*x))/(a^5 + a^4*b + (a^5*e^(4*c) + a^4*b*e^(4*c))
*e^(4*d*x) + 2*(a^5*e^(2*c) + 3*a^4*b*e^(2*c) + 2*a^3*b^2*e^(2*c))*e^(2*d*x
)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cosh\left(c+d\,x\right)}^3}{{\left(a+\frac{b}{{\cosh\left(c+d\,x\right)}^2}\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^3/(a + b/cosh(c + d*x)^2)^2,x)
[Out] int(cosh(c + d*x)^3/(a + b/cosh(c + d*x)^2)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3/(a+b*sech(d*x+c)**2)**2,x)
[Out] Timed out
```

3.84 $\int \frac{\cosh^2(c+dx)}{\left(a+b\,\mathbf{sech}^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=144

$$\frac{b^{3/2}(5a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^3d(a+b)^{3/2}}+\frac{x(a-4b)}{2a^3}+\frac{b(a+2b)\tanh(c+dx)}{2a^2d(a+b)\left(a-b\tanh^2(c+dx)+b\right)}+\frac{\sinh(c+dx)\cosh(c+a}{2ad\left(a-b\tanh^2(c+dx)\right.}$$

```
[Out] 1/2*(a-4*b)*x/a^3+1/2*b^(3/2)*(5*a+4*b)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(
1/2))/a^3/(a+b)^(3/2)/d+1/2*cosh(d*x+c)*sinh(d*x+c)/a/d/(a+b-b*tanh(d*x+c)^
2)+1/2*b*(a+2*b)*tanh(d*x+c)/a^2/(a+b)/d/(a+b-b*tanh(d*x+c)^2)
```

Rubi [A] time = 0.24, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4146, 414, 527, 522, 206, 208}

$$\frac{b^{3/2}(5a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^3d(a+b)^{3/2}}+\frac{b(a+2b)\tanh(c+dx)}{2a^2d(a+b)\left(a-b\tanh^2(c+dx)+b\right)}+\frac{x(a-4b)}{2a^3}+\frac{\sinh(c+dx)\cosh(c+a}{2ad\left(a-b\tanh^2(c+dx)\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^2/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a - 4*b)*x)/(2*a^3) + (b^(3/2)*(5*a + 4*b)*ArcTanh[(Sqrt[b]*Tanh[c + d*x]
)/Sqrt[a + b]])/(2*a^3*(a + b)^(3/2)*d) + (Cosh[c + d*x]*Sinh[c + d*x])/(2*
a*d*(a + b - b*Tanh[c + d*x]^2)) + (b*(a + 2*b)*Tanh[c + d*x])/(2*a^2*(a +
b)*d*(a + b - b*Tanh[c + d*x]^2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
```

```
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\mathrm{sech}^2(c+dx)\right)^2}\,dx = \frac{\mathrm{Subst}\left(\int \frac{1}{(1-x^2)^2(a+b-bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2ad\left(a+b-b\tanh^2(c+dx)\right)} + \frac{\mathrm{Subst}\left(\int \frac{a-b-3bx^2}{(1-x^2)(a+b-bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{2ad}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2ad\left(a+b-b\tanh^2(c+dx)\right)} + \frac{b(a+2b)\tanh(c+dx)}{2a^2(a+b)d\left(a+b-b\tanh^2(c+dx)\right)} - \frac{\mathrm{Subst}\Big(\int}{}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2ad\left(a+b-b\tanh^2(c+dx)\right)} + \frac{b(a+2b)\tanh(c+dx)}{2a^2(a+b)d\left(a+b-b\tanh^2(c+dx)\right)} + \frac{(a-4b)}{}$$

$$= \frac{(a-4b)x}{2a^3} + \frac{b^{3/2}(5a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^3(a+b)^{3/2}d} + \frac{\cosh(c+dx)\sinh(c+dx)}{2ad\left(a+b-b\tanh^2(c+dx)\right)}$$

Mathematica [A] time = 1.38, size = 103, normalized size = 0.72

$$\frac{\frac{2b^{3/2}(5a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}} + \sinh(2(c+dx))\left(\frac{2ab^2}{(a+b)(a\cosh(2(c+dx))+a+2b)}+a\right) + 2(a-4b)(c+dx)}{4a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^2/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (2*(a - 4*b)*(c + d*x) + (2*b^(3/2)*(5*a + 4*b)*ArcTanh[(Sqrt[b]*Tanh[c + d
*x])/Sqrt[a + b]])/(a + b)^(3/2) + (a + (2*a*b^2)/((a + b)*(a + 2*b + a*Cos
h[2*(c + d*x)])))*Sinh[2*(c + d*x)])/(4*a^3*d)
```

fricas [B] time = 0.49, size = 3739, normalized size = 25.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*((a^3 + a^2*b)*cosh(d*x + c)^8 + 8*(a^3 + a^2*b)*cosh(d*x + c)*sinh(d*
x + c)^7 + (a^3 + a^2*b)*sinh(d*x + c)^8 + 2*(a^3 + 3*a^2*b + 2*a*b^2 + 2*(
```

```
a^3 - 3*a^2*b - 4*a*b^2)*d*x)*cosh(d*x + c)^6 + 2*(a^3 + 3*a^2*b + 2*a*b^2
+ 2*(a^3 - 3*a^2*b - 4*a*b^2)*d*x + 14*(a^3 + a^2*b)*cosh(d*x + c)^2)*sinh(
d*x + c)^6 + 4*(14*(a^3 + a^2*b)*cosh(d*x + c)^3 + 3*(a^3 + 3*a^2*b + 2*a*b
^2 + 2*(a^3 - 3*a^2*b - 4*a*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 - 8*(a
*b^2 + 2*b^3 - (a^3 - a^2*b - 10*a*b^2 - 8*b^3)*d*x)*cosh(d*x + c)^4 + 2*(3
5*(a^3 + a^2*b)*cosh(d*x + c)^4 - 4*a*b^2 - 8*b^3 + 4*(a^3 - a^2*b - 10*a*b
^2 - 8*b^3)*d*x + 15*(a^3 + 3*a^2*b + 2*a*b^2 + 2*(a^3 - 3*a^2*b - 4*a*b^2)
*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(a^3 + a^2*b)*cosh(d*x + c)^5
 + 5*(a^3 + 3*a^2*b + 2*a*b^2 + 2*(a^3 - 3*a^2*b - 4*a*b^2)*d*x)*cosh(d*x +
 c)^3 - 4*(a*b^2 + 2*b^3 - (a^3 - a^2*b - 10*a*b^2 - 8*b^3)*d*x)*cosh(d*x +
 c))*sinh(d*x + c)^3 - a^3 - a^2*b - 2*(a^3 + 3*a^2*b + 6*a*b^2 - 2*(a^3 -
3*a^2*b - 4*a*b^2)*d*x)*cosh(d*x + c)^2 + 2*(14*(a^3 + a^2*b)*cosh(d*x + c)
^6 + 15*(a^3 + 3*a^2*b + 2*a*b^2 + 2*(a^3 - 3*a^2*b - 4*a*b^2)*d*x)*cosh(d*
x + c)^4 - a^3 - 3*a^2*b - 6*a*b^2 + 2*(a^3 - 3*a^2*b - 4*a*b^2)*d*x - 24*(
a*b^2 + 2*b^3 - (a^3 - a^2*b - 10*a*b^2 - 8*b^3)*d*x)*cosh(d*x + c)^2)*sinh
(d*x + c)^2 + 2*((5*a^2*b + 4*a*b^2)*cosh(d*x + c)^6 + 6*(5*a^2*b + 4*a*b^2
)*cosh(d*x + c)*sinh(d*x + c)^5 + (5*a^2*b + 4*a*b^2)*sinh(d*x + c)^6 + 2*(
5*a^2*b + 14*a*b^2 + 8*b^3)*cosh(d*x + c)^4 + (10*a^2*b + 28*a*b^2 + 16*b^3
 + 15*(5*a^2*b + 4*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(5*a^2*b
+ 4*a*b^2)*cosh(d*x + c)^3 + 2*(5*a^2*b + 14*a*b^2 + 8*b^3)*cosh(d*x + c))*
sinh(d*x + c)^3 + (5*a^2*b + 4*a*b^2)*cosh(d*x + c)^2 + (15*(5*a^2*b + 4*a*
b^2)*cosh(d*x + c)^4 + 5*a^2*b + 4*a*b^2 + 12*(5*a^2*b + 14*a*b^2 + 8*b^3)*
cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(3*(5*a^2*b + 4*a*b^2)*cosh(d*x + c)^5
 + 4*(5*a^2*b + 14*a*b^2 + 8*b^3)*cosh(d*x + c)^3 + (5*a^2*b + 4*a*b^2)*cos
h(d*x + c))*sinh(d*x + c))*sqrt(b/(a + b))*log((a^2*cosh(d*x + c)^4 + 4*a^2
*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh
(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2
 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*si
nh(d*x + c) - 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*
sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 + 3*a*b + 2*b^2)*sqrt(b/(
a + b)))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*
x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*
sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x
+ c) + a)) + 4*(2*(a^3 + a^2*b)*cosh(d*x + c)^7 + 3*(a^3 + 3*a^2*b + 2*a*b^
2 + 2*(a^3 - 3*a^2*b - 4*a*b^2)*d*x)*cosh(d*x + c)^5 - 8*(a*b^2 + 2*b^3 - (
a^3 - a^2*b - 10*a*b^2 - 8*b^3)*d*x)*cosh(d*x + c)^3 - (a^3 + 3*a^2*b + 6*a
*b^2 - 2*(a^3 - 3*a^2*b - 4*a*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a^5
 + a^4*b)*d*cosh(d*x + c)^6 + 6*(a^5 + a^4*b)*d*cosh(d*x + c)*sinh(d*x + c)
^5 + (a^5 + a^4*b)*d*sinh(d*x + c)^6 + 2*(a^5 + 3*a^4*b + 2*a^3*b^2)*d*cosh
(d*x + c)^4 + (15*(a^5 + a^4*b)*d*cosh(d*x + c)^2 + 2*(a^5 + 3*a^4*b + 2*a^
3*b^2)*d)*sinh(d*x + c)^4 + (a^5 + a^4*b)*d*cosh(d*x + c)^2 + 4*(5*(a^5 + a
^4*b)*d*cosh(d*x + c)^3 + 2*(a^5 + 3*a^4*b + 2*a^3*b^2)*d*cosh(d*x + c))*si
nh(d*x + c)^3 + (15*(a^5 + a^4*b)*d*cosh(d*x + c)^4 + 12*(a^5 + 3*a^4*b + 2
*a^3*b^2)*d*cosh(d*x + c)^2 + (a^5 + a^4*b)*d)*sinh(d*x + c)^2 + 2*(3*(a^5
+ a^4*b)*d*cosh(d*x + c)^5 + 4*(a^5 + 3*a^4*b + 2*a^3*b^2)*d*cosh(d*x + c)^
```

```
3 + (a^5 + a^4*b)*d*cosh(d*x + c))*sinh(d*x + c)), 1/8*((a^3 + a^2*b)*cosh(
d*x + c)^8 + 8*(a^3 + a^2*b)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^3 + a^2*b)*
sinh(d*x + c)^8 + 2*(a^3 + 3*a^2*b + 2*a*b^2 + 2*(a^3 - 3*a^2*b - 4*a*b^2)*
d*x)*cosh(d*x + c)^6 + 2*(a^3 + 3*a^2*b + 2*a*b^2 + 2*(a^3 - 3*a^2*b - 4*a*
b^2)*d*x + 14*(a^3 + a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(14*(a^3 +
 a^2*b)*cosh(d*x + c)^3 + 3*(a^3 + 3*a^2*b + 2*a*b^2 + 2*(a^3 - 3*a^2*b - 4
*a*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 - 8*(a*b^2 + 2*b^3 - (a^3 - a^2
*b - 10*a*b^2 - 8*b^3)*d*x)*cosh(d*x + c)^4 + 2*(35*(a^3 + a^2*b)*cosh(d*x
+ c)^4 - 4*a*b^2 - 8*b^3 + 4*(a^3 - a^2*b - 10*a*b^2 - 8*b^3)*d*x + 15*(a^3
 + 3*a^2*b + 2*a*b^2 + 2*(a^3 - 3*a^2*b - 4*a*b^2)*d*x)*cosh(d*x + c)^2)*si
nh(d*x + c)^4 + 8*(7*(a^3 + a^2*b)*cosh(d*x + c)^5 + 5*(a^3 + 3*a^2*b + 2*a
*b^2 + 2*(a^3 - 3*a^2*b - 4*a*b^2)*d*x)*cosh(d*x + c)^3 - 4*(a*b^2 + 2*b^3
- (a^3 - a^2*b - 10*a*b^2 - 8*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 - a^
3 - a^2*b - 2*(a^3 + 3*a^2*b + 6*a*b^2 - 2*(a^3 - 3*a^2*b - 4*a*b^2)*d*x)*c
osh(d*x + c)^2 + 2*(14*(a^3 + a^2*b)*cosh(d*x + c)^6 + 15*(a^3 + 3*a^2*b +
2*a*b^2 + 2*(a^3 - 3*a^2*b - 4*a*b^2)*d*x)*cosh(d*x + c)^4 - a^3 - 3*a^2*b
- 6*a*b^2 + 2*(a^3 - 3*a^2*b - 4*a*b^2)*d*x - 24*(a*b^2 + 2*b^3 - (a^3 - a^
2*b - 10*a*b^2 - 8*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((5*a^2*b
 + 4*a*b^2)*cosh(d*x + c)^6 + 6*(5*a^2*b + 4*a*b^2)*cosh(d*x + c)*sinh(d*x
+ c)^5 + (5*a^2*b + 4*a*b^2)*sinh(d*x + c)^6 + 2*(5*a^2*b + 14*a*b^2 + 8*b^
3)*cosh(d*x + c)^4 + (10*a^2*b + 28*a*b^2 + 16*b^3 + 15*(5*a^2*b + 4*a*b^2)
*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(5*a^2*b + 4*a*b^2)*cosh(d*x + c)^
3 + 2*(5*a^2*b + 14*a*b^2 + 8*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + (5*a^2*
b + 4*a*b^2)*cosh(d*x + c)^2 + (15*(5*a^2*b + 4*a*b^2)*cosh(d*x + c)^4 + 5*
a^2*b + 4*a*b^2 + 12*(5*a^2*b + 14*a*b^2 + 8*b^3)*cosh(d*x + c)^2)*sinh(d*x
 + c)^2 + 2*(3*(5*a^2*b + 4*a*b^2)*cosh(d*x + c)^5 + 4*(5*a^2*b + 14*a*b^2
+ 8*b^3)*cosh(d*x + c)^3 + (5*a^2*b + 4*a*b^2)*cosh(d*x + c))*sinh(d*x + c)
)*sqrt(-b/(a + b))*arctan(1/2*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d
*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-b/(a + b))/b) + 4*(2*(a^3 + a^
2*b)*cosh(d*x + c)^7 + 3*(a^3 + 3*a^2*b + 2*a*b^2 + 2*(a^3 - 3*a^2*b - 4*a*
b^2)*d*x)*cosh(d*x + c)^5 - 8*(a*b^2 + 2*b^3 - (a^3 - a^2*b - 10*a*b^2 - 8*
b^3)*d*x)*cosh(d*x + c)^3 - (a^3 + 3*a^2*b + 6*a*b^2 - 2*(a^3 - 3*a^2*b - 4
*a*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a^5 + a^4*b)*d*cosh(d*x + c)^6
 + 6*(a^5 + a^4*b)*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a^5 + a^4*b)*d*sinh(d
*x + c)^6 + 2*(a^5 + 3*a^4*b + 2*a^3*b^2)*d*cosh(d*x + c)^4 + (15*(a^5 + a^
4*b)*d*cosh(d*x + c)^2 + 2*(a^5 + 3*a^4*b + 2*a^3*b^2)*d)*sinh(d*x + c)^4 +
 (a^5 + a^4*b)*d*cosh(d*x + c)^2 + 4*(5*(a^5 + a^4*b)*d*cosh(d*x + c)^3 + 2
*(a^5 + 3*a^4*b + 2*a^3*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^3 + (15*(a^5 +
a^4*b)*d*cosh(d*x + c)^4 + 12*(a^5 + 3*a^4*b + 2*a^3*b^2)*d*cosh(d*x + c)^2
 + (a^5 + a^4*b)*d)*sinh(d*x + c)^2 + 2*(3*(a^5 + a^4*b)*d*cosh(d*x + c)^5
+ 4*(a^5 + 3*a^4*b + 2*a^3*b^2)*d*cosh(d*x + c)^3 + (a^5 + a^4*b)*d*cosh(d*
x + c))*sinh(d*x + c))]
```

giac [B] time = 2.02, size = 323, normalized size = 2.24

$$\frac{\frac{12\left(5\,ab^2+4\,b^3\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{\left(a^4+a^3b\right)\sqrt{-ab-b^2}} - \frac{2\,a^3e^{(6\,dx+6\,c)}-6\,a^2be^{(6\,dx+6\,c)}-8\,ab^2e^{(6\,dx+6\,c)}+7\,a^3e^{(4\,dx+4\,c)}-a^2be^{(4\,dx+4\,c)}-16\,ab^2e^{(4\,dx+4\,c)}+16\,b^3e^{(4\,dx+}}{\left(a^4+a^3b\right)\left(ae^{(6\,dx+6\,c)}+2\,ae^{(4\,dx+4\,c)}+4\,be^{(4\,dx+4\,c)}+\right.}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/24*(12*(5*a*b^2 + 4*b^3)*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a
*b - b^2))/((a^4 + a^3*b)*sqrt(-a*b - b^2)) - (2*a^3*e^(6*d*x + 6*c) - 6*a^
2*b*e^(6*d*x + 6*c) - 8*a*b^2*e^(6*d*x + 6*c) + 7*a^3*e^(4*d*x + 4*c) - a^2
*b*e^(4*d*x + 4*c) - 16*a*b^2*e^(4*d*x + 4*c) + 16*b^3*e^(4*d*x + 4*c) + 8*
a^3*e^(2*d*x + 2*c) + 12*a^2*b*e^(2*d*x + 2*c) + 28*a*b^2*e^(2*d*x + 2*c) +
 3*a^3 + 3*a^2*b)/((a^4 + a^3*b)*(a*e^(6*d*x + 6*c) + 2*a*e^(4*d*x + 4*c) +
 4*b*e^(4*d*x + 4*c) + a*e^(2*d*x + 2*c))) + 12*(d*x + c)*(a - 4*b)/a^3 + 3
*e^(2*d*x + 2*c)/a^2)/d
```

maple [B] time = 0.51, size = 557, normalized size = 3.87

$$\frac{1}{2d\,a^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{1}{2d\,a^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2d\,a^2}+\frac{2\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)b}{d\,a^3}-\frac{}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] 1/2/d/a^2/(tanh(1/2*d*x+1/2*c)-1)^2+1/2/d/a^2/(tanh(1/2*d*x+1/2*c)-1)-1/2/d
/a^2*ln(tanh(1/2*d*x+1/2*c)-1)+2/d/a^3*ln(tanh(1/2*d*x+1/2*c)-1)*b-1/2/d/a^
2/(tanh(1/2*d*x+1/2*c)+1)^2+1/2/d/a^2/(tanh(1/2*d*x+1/2*c)+1)+1/2/d/a^2*ln(
tanh(1/2*d*x+1/2*c)+1)-2/d/a^3*ln(tanh(1/2*d*x+1/2*c)+1)*b+1/d/a^2*b^2/(tan
h(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*ta
nh(1/2*d*x+1/2*c)^2*b+a+b)/(a+b)*tanh(1/2*d*x+1/2*c)^3+1/d/a^2*b^2/(tanh(1/
2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1
/2*d*x+1/2*c)^2*b+a+b)/(a+b)*tanh(1/2*d*x+1/2*c)-5/4/d/a^2*b^(3/2)/(a+b)^(3
/2)*ln(-(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)-(a+
b)^(1/2))+5/4/d/a^2*b^(3/2)/(a+b)^(3/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^
2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))-1/d/a^3*b^(5/2)/(a+b)^(3/2)*ln
(-(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)-(a+b)^(1/
2))+1/d/a^3*b^(5/2)/(a+b)^(3/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1
/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))
```

maxima [B] time = 0.47, size = 696, normalized size = 4.83

$$\frac{\left(3\,a^2b+12\,ab^2+8\,b^3\right)\log\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(2\,dx+2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{16\left(a^4+a^3b\right)\sqrt{(a+b)b}\,d}-\frac{\left(3\,a^2b+12\,ab^2+8\,b^3\right)\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{16\left(a^4+a^3b\right)\sqrt{(a+b)b}\,d}+\frac{(3\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/16*(3*a^2*b + 12*a*b^2 + 8*b^3)*log((a*e^(2*d*x + 2*c) + a + 2*b - 2*sqrt
((a + b)*b))/(a*e^(2*d*x + 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^4 + a^3
*b)*sqrt((a + b)*b)*d) - 1/16*(3*a^2*b + 12*a*b^2 + 8*b^3)*log((a*e^(-2*d*x
 - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sq
rt((a + b)*b)))/((a^4 + a^3*b)*sqrt((a + b)*b)*d) + 1/8*(3*a*b + 2*b^2)*log
((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a
 + 2*b + 2*sqrt((a + b)*b)))/((a^3 + a^2*b)*sqrt((a + b)*b)*d) - 1/4*(a^2*b
 + 2*a*b^2 + (a^2*b + 8*a*b^2 + 8*b^3)*e^(2*d*x + 2*c))/((a^5 + a^4*b + (a^
5 + a^4*b)*e^(4*d*x + 4*c) + 2*(a^5 + 3*a^4*b + 2*a^3*b^2)*e^(2*d*x + 2*c))
*d) + 1/4*(a^2*b + 2*a*b^2 + (a^2*b + 8*a*b^2 + 8*b^3)*e^(-2*d*x - 2*c))/((
a^5 + a^4*b + 2*(a^5 + 3*a^4*b + 2*a^3*b^2)*e^(-2*d*x - 2*c) + (a^5 + a^4*b
)*e^(-4*d*x - 4*c))*d) - 1/2*(a*b + (a*b + 2*b^2)*e^(-2*d*x - 2*c))/((a^4 +
 a^3*b + 2*(a^4 + 3*a^3*b + 2*a^2*b^2)*e^(-2*d*x - 2*c) + (a^4 + a^3*b)*e^(
-4*d*x - 4*c))*d) + 1/2*(d*x + c)/(a^2*d) + 1/8*e^(2*d*x + 2*c)/(a^2*d) - 1
/8*e^(-2*d*x - 2*c)/(a^2*d) - 1/2*b*log(a*e^(4*d*x + 4*c) + 2*(a + 2*b)*e^(
2*d*x + 2*c) + a)/(a^3*d) + 1/2*b*log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-
4*d*x - 4*c) + a)/(a^3*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh\left(c+d\,x\right)^2}{\left(a+\frac{b}{\cosh(c+d\,x)^2}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^2/(a + b/cosh(c + d*x)^2)^2,x)
```

```
[Out] int(cosh(c + d*x)^2/(a + b/cosh(c + d*x)^2)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

3.85 $\int \frac{\cosh(c+dx)}{\left(a+b\,\mathbf{sech}^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=100

$$-\frac{b(4a+3b)\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{2a^{5/2}d(a+b)^{3/2}}+\frac{b^2\sinh(c+dx)}{2a^2d(a+b)\left(a\sinh^2(c+dx)+a+b\right)}+\frac{\sinh(c+dx)}{a^2d}$$

```
[Out] -1/2*b*(4*a+3*b)*arctan(sinh(d*x+c)*a^(1/2)/(a+b)^(1/2))/a^(5/2)/(a+b)^(3/2
)/d+sinh(d*x+c)/a^2/d+1/2*b^2*sinh(d*x+c)/a^2/(a+b)/d/(a+b+a*sinh(d*x+c)^2)
```

Rubi [A] time = 0.13, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {4147, 390, 385, 205}

$$\frac{b^2\sinh(c+dx)}{2a^2d(a+b)\left(a\sinh^2(c+dx)+a+b\right)}-\frac{b(4a+3b)\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{2a^{5/2}d(a+b)^{3/2}}+\frac{\sinh(c+dx)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] -(b*(4*a + 3*b)*ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]])/(2*a^(5/2)*(a
+ b)^(3/2)*d) + Sinh[c + d*x]/(a^2*d) + (b^2*Sinh[c + d*x])/(2*a^2*(a + b)*
d*(a + b + a*Sinh[c + d*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
```

```
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cosh(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx = \frac{\operatorname{Subst}\left(\int \frac{\left(1+x^2\right)^2}{\left(a+b+ax^2\right)^2}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \left(\frac{1}{a^2}-\frac{b(2a+b)+2abx^2}{a^2\left(a+b+ax^2\right)^2}\right)dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\sinh(c+dx)}{a^2d}-\frac{\operatorname{Subst}\left(\int \frac{b(2a+b)+2abx^2}{\left(a+b+ax^2\right)^2}\,dx, x, \sinh(c+dx)\right)}{a^2d}$$

$$= \frac{\sinh(c+dx)}{a^2d}+\frac{b^2\sinh(c+dx)}{2a^2(a+b)d\left(a+b+a\sinh^2(c+dx)\right)}-\frac{(b(4a+3b))\operatorname{Subst}\left(\int \frac{\ }{a+b \ldots}\right.}{2a^2(a \ldots}$$

$$= -\frac{b(4a+3b)\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{2a^{5/2}(a+b)^{3/2}d}+\frac{\sinh(c+dx)}{a^2d}+\frac{b^2\sinh(c+dx)}{2a^2(a+b)d\left(a+b+a\sinh^2(c \ldots\right.}$$

Mathematica [B] time = 1.81, size = 234, normalized size = 2.34

$$\frac{\operatorname{sech}^3(c+dx)(a\cosh(2(c+dx))+a+2b)\left(\frac{2\sqrt{a}\,b^2\tanh(c+dx)}{a+b}+2\sqrt{a}\sinh(c)\cosh(dx)\operatorname{sech}(c+dx)(a\cosh(2(c+dx \ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^3*((b*(4*a + 3*b)*ArcTan[(Sq
rt[a + b]*Csch[c + d*x]*Sqrt[(Cosh[c] - Sinh[c])^2]*(Cosh[c] + Sinh[c]))/Sq
rt[a]]*(a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]*(Cosh[c] - Sinh[c]))/(
(a + b)^(3/2)*Sqrt[(Cosh[c] - Sinh[c])^2]) + 2*Sqrt[a]*Cosh[d*x]*(a + 2*b +
 a*Cosh[2*(c + d*x)])*Sech[c + d*x]*Sinh[c] + 2*Sqrt[a]*Cosh[c]*(a + 2*b +
a*Cosh[2*(c + d*x)])*Sech[c + d*x]*Sinh[d*x] + (2*Sqrt[a]*b^2*Tanh[c + d*x]
)/(a + b)))/(8*a^(5/2)*d*(a + b*Sech[c + d*x]^2)^2)
```

fricas [B] time = 0.49, size = 3154, normalized size = 31.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)^6 + 12*(a^4 + 2*a^3*b + a^2
*b^2)*cosh(d*x + c)*sinh(d*x + c)^5 + 2*(a^4 + 2*a^3*b + a^2*b^2)*sinh(d*x
+ c)^6 + 2*(a^4 + 6*a^3*b + 11*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^4 + 2*(a^4
+ 6*a^3*b + 11*a^2*b^2 + 6*a*b^3 + 15*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x +
c)^2)*sinh(d*x + c)^4 - 2*a^4 - 4*a^3*b - 2*a^2*b^2 + 8*(5*(a^4 + 2*a^3*b +
 a^2*b^2)*cosh(d*x + c)^3 + (a^4 + 6*a^3*b + 11*a^2*b^2 + 6*a*b^3)*cosh(d*x
 + c))*sinh(d*x + c)^3 - 2*(a^4 + 6*a^3*b + 11*a^2*b^2 + 6*a*b^3)*cosh(d*x
+ c)^2 + 2*(15*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)^4 - a^4 - 6*a^3*b -
11*a^2*b^2 - 6*a*b^3 + 6*(a^4 + 6*a^3*b + 11*a^2*b^2 + 6*a*b^3)*cosh(d*x +
c)^2)*sinh(d*x + c)^2 - ((4*a^2*b + 3*a*b^2)*cosh(d*x + c)^5 + 5*(4*a^2*b +
 3*a*b^2)*cosh(d*x + c)*sinh(d*x + c)^4 + (4*a^2*b + 3*a*b^2)*sinh(d*x + c)
^5 + 2*(4*a^2*b + 11*a*b^2 + 6*b^3)*cosh(d*x + c)^3 + 2*(4*a^2*b + 11*a*b^2
 + 6*b^3 + 5*(4*a^2*b + 3*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 2*(5*(4
*a^2*b + 3*a*b^2)*cosh(d*x + c)^3 + 3*(4*a^2*b + 11*a*b^2 + 6*b^3)*cosh(d*x
 + c))*sinh(d*x + c)^2 + (4*a^2*b + 3*a*b^2)*cosh(d*x + c) + (5*(4*a^2*b +
3*a*b^2)*cosh(d*x + c)^4 + 4*a^2*b + 3*a*b^2 + 6*(4*a^2*b + 11*a*b^2 + 6*b^
3)*cosh(d*x + c)^2)*sinh(d*x + c))*sqrt(-a^2 - a*b)*log((a*cosh(d*x + c)^4
+ 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 - 2*(3*a + 2*b)*cos
h(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 - 3*a - 2*b)*sinh(d*x + c)^2 + 4*(a*c
osh(d*x + c)^3 - (3*a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + 4*(cosh(d*x + c
)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^
2 - 1)*sinh(d*x + c) - cosh(d*x + c))*sqrt(-a^2 - a*b) + a)/(a*cosh(d*x + c
)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*c
osh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*c
osh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) + 4*(3*(a^4 +
 2*a^3*b + a^2*b^2)*cosh(d*x + c)^5 + 2*(a^4 + 6*a^3*b + 11*a^2*b^2 + 6*a*b
^3)*cosh(d*x + c)^3 - (a^4 + 6*a^3*b + 11*a^2*b^2 + 6*a*b^3)*cosh(d*x + c))
*sinh(d*x + c))/((a^6 + 2*a^5*b + a^4*b^2)*d*cosh(d*x + c)^5 + 5*(a^6 + 2*a
```

^5*b + a^4*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^4 + (a^6 + 2*a^5*b + a^4*b^2)
*d*sinh(d*x + c)^5 + 2*(a^6 + 4*a^5*b + 5*a^4*b^2 + 2*a^3*b^3)*d*cosh(d*x +
c)^3 + 2*(5*(a^6 + 2*a^5*b + a^4*b^2)*d*cosh(d*x + c)^2 + (a^6 + 4*a^5*b +
5*a^4*b^2 + 2*a^3*b^3)*d)*sinh(d*x + c)^3 + (a^6 + 2*a^5*b + a^4*b^2)*d*co
sh(d*x + c) + 2*(5*(a^6 + 2*a^5*b + a^4*b^2)*d*cosh(d*x + c)^3 + 3*(a^6 + 4
*a^5*b + 5*a^4*b^2 + 2*a^3*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^2 + (5*(a^6
+ 2*a^5*b + a^4*b^2)*d*cosh(d*x + c)^4 + 6*(a^6 + 4*a^5*b + 5*a^4*b^2 + 2*a
^3*b^3)*d*cosh(d*x + c)^2 + (a^6 + 2*a^5*b + a^4*b^2)*d)*sinh(d*x + c)), 1/
2*((a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)^6 + 6*(a^4 + 2*a^3*b + a^2*b^2)*
cosh(d*x + c)*sinh(d*x + c)^5 + (a^4 + 2*a^3*b + a^2*b^2)*sinh(d*x + c)^6 +
(a^4 + 6*a^3*b + 11*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^4 + (a^4 + 6*a^3*b +
11*a^2*b^2 + 6*a*b^3 + 15*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)^2)*sinh(d
*x + c)^4 - a^4 - 2*a^3*b - a^2*b^2 + 4*(5*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d
*x + c)^3 + (a^4 + 6*a^3*b + 11*a^2*b^2 + 6*a*b^3)*cosh(d*x + c))*sinh(d*x
+ c)^3 - (a^4 + 6*a^3*b + 11*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^2 + (15*(a^4
+ 2*a^3*b + a^2*b^2)*cosh(d*x + c)^4 - a^4 - 6*a^3*b - 11*a^2*b^2 - 6*a*b^3
+ 6*(a^4 + 6*a^3*b + 11*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^
2 - ((4*a^2*b + 3*a*b^2)*cosh(d*x + c)^5 + 5*(4*a^2*b + 3*a*b^2)*cosh(d*x +
c)*sinh(d*x + c)^4 + (4*a^2*b + 3*a*b^2)*sinh(d*x + c)^5 + 2*(4*a^2*b + 11
*a*b^2 + 6*b^3)*cosh(d*x + c)^3 + 2*(4*a^2*b + 11*a*b^2 + 6*b^3 + 5*(4*a^2*
b + 3*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 2*(5*(4*a^2*b + 3*a*b^2)*co
sh(d*x + c)^3 + 3*(4*a^2*b + 11*a*b^2 + 6*b^3)*cosh(d*x + c))*sinh(d*x + c)
^2 + (4*a^2*b + 3*a*b^2)*cosh(d*x + c) + (5*(4*a^2*b + 3*a*b^2)*cosh(d*x +
c)^4 + 4*a^2*b + 3*a*b^2 + 6*(4*a^2*b + 11*a*b^2 + 6*b^3)*cosh(d*x + c)^2)*
sinh(d*x + c))*sqrt(a^2 + a*b)*arctan(1/2*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x
+ c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 + (3*a + 4*b)*cosh(d*x + c) + (3*
a*cosh(d*x + c)^2 + 3*a + 4*b)*sinh(d*x + c))/sqrt(a^2 + a*b)) - ((4*a^2*b
+ 3*a*b^2)*cosh(d*x + c)^5 + 5*(4*a^2*b + 3*a*b^2)*cosh(d*x + c)*sinh(d*x +
c)^4 + (4*a^2*b + 3*a*b^2)*sinh(d*x + c)^5 + 2*(4*a^2*b + 11*a*b^2 + 6*b^3
)*cosh(d*x + c)^3 + 2*(4*a^2*b + 11*a*b^2 + 6*b^3 + 5*(4*a^2*b + 3*a*b^2)*c
osh(d*x + c)^2)*sinh(d*x + c)^3 + 2*(5*(4*a^2*b + 3*a*b^2)*cosh(d*x + c)^3
+ 3*(4*a^2*b + 11*a*b^2 + 6*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + (4*a^2*b
+ 3*a*b^2)*cosh(d*x + c) + (5*(4*a^2*b + 3*a*b^2)*cosh(d*x + c)^4 + 4*a^2*b
+ 3*a*b^2 + 6*(4*a^2*b + 11*a*b^2 + 6*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))
*sqrt(a^2 + a*b)*arctan(1/2*sqrt(a^2 + a*b)*(cosh(d*x + c) + sinh(d*x + c))
/(a + b)) + 2*(3*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)^5 + 2*(a^4 + 6*a^3
*b + 11*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^3 - (a^4 + 6*a^3*b + 11*a^2*b^2 +
6*a*b^3)*cosh(d*x + c))*sinh(d*x + c))/((a^6 + 2*a^5*b + a^4*b^2)*d*cosh(d*
x + c)^5 + 5*(a^6 + 2*a^5*b + a^4*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^4 + (a
^6 + 2*a^5*b + a^4*b^2)*d*sinh(d*x + c)^5 + 2*(a^6 + 4*a^5*b + 5*a^4*b^2 +
2*a^3*b^3)*d*cosh(d*x + c)^3 + 2*(5*(a^6 + 2*a^5*b + a^4*b^2)*d*cosh(d*x +
c)^2 + (a^6 + 4*a^5*b + 5*a^4*b^2 + 2*a^3*b^3)*d)*sinh(d*x + c)^3 + (a^6 +
2*a^5*b + a^4*b^2)*d*cosh(d*x + c) + 2*(5*(a^6 + 2*a^5*b + a^4*b^2)*d*cosh(
d*x + c)^3 + 3*(a^6 + 4*a^5*b + 5*a^4*b^2 + 2*a^3*b^3)*d*cosh(d*x + c))*sin
h(d*x + c)^2 + (5*(a^6 + 2*a^5*b + a^4*b^2)*d*cosh(d*x + c)^4 + 6*(a^6 + 4*

```
a^5*b + 5*a^4*b^2 + 2*a^3*b^3)*d*cosh(d*x + c)^2 + (a^6 + 2*a^5*b + a^4*b^2
)*d)*sinh(d*x + c))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[6,-20]Warning, need to choose a branch for the root of a po
lynomial with parameters. This might be wrong.The choice was done assuming
[a,b]=[89,-63]Warning, need to choose a branch for the root of a polynomial
 with parameters. This might be wrong.The choice was done assuming [a,b]=[1
2,-32]Warning, need to choose a branch for the root of a polynomial with pa
rameters. This might be wrong.The choice was done assuming [a,b]=[2,72]Warn
ing, need to choose a branch for the root of a polynomial with parameters.
This might be wrong.The choice was done assuming [a,b]=[67,31]Warning, need
 to choose a branch for the root of a polynomial with parameters. This migh
t be wrong.The choice was done assuming [a,b]=[-88,66]Undef/Unsigned Inf en
countered in limitEvaluation time: 0.99Limit: Max order reached or unable t
o make series expansion Error: Bad Argument Value
```

maple [B] time = 0.51, size = 385, normalized size = 3.85

$$-\frac{1}{d\,a^{2}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{1}{d\,a^{2}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{b^{2}\left(\tanh^{3}\left(\frac{dx}{2}\right.\right.}{d\,a^{2}\left(\left(\tanh^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+b\left(\tanh^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\left(\tanh^{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] -1/d/a^2/(tanh(1/2*d*x+1/2*c)-1)-1/d/a^2/(tanh(1/2*d*x+1/2*c)+1)-1/d/a^2*b^
2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*
a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)/(a+b)*tanh(1/2*d*x+1/2*c)^3+1/d/a^2*b^2/(t
anh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*
tanh(1/2*d*x+1/2*c)^2*b+a+b)/(a+b)*tanh(1/2*d*x+1/2*c)-2/d/a^(3/2)*b/(a+b)^
(3/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)-2*b^(1/2))/a^(1/2))-2/d
/a^(3/2)*b/(a+b)^(3/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)+2*b^(1
/2))/a^(1/2))-3/2/d/a^(5/2)*b^2/(a+b)^(3/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(
```

```
1/2*d*x+1/2*c)-2*b^(1/2))/a^(1/2))-3/2/d/a^(5/2)*b^2/(a+b)^(3/2)*arctan(1/2
*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)+2*b^(1/2))/a^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{a^2 + ab - \left(a^2e^{(6\,c)} + abe^{(6\,c)}\right)e^{(6\,dx)} - \left(a^2e^{(4\,c)} + 5\,abe^{(4\,c)} + 6\,b^2e^{(4\,c)}\right)e^{(4\,dx)} + \left(a^2e^{(2\,c)} + 5\,abe^{(2\,c)} + 6\,b^2e^{(2\,c)}\right)e^{(2\,dx)}}{2\left(\left(a^4de^{(5\,c)} + a^3bde^{(5\,c)}\right)e^{(5\,dx)} + 2\left(a^4de^{(3\,c)} + 3\,a^3bde^{(3\,c)} + 2\,a^2b^2de^{(3\,c)}\right)e^{(3\,dx)} + \left(a^4de^c + a^3bde^c\right)e^{(dx)}\right)} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(a^2 + a*b - (a^2*e^(6*c) + a*b*e^(6*c))*e^(6*d*x) - (a^2*e^(4*c) + 5*
a*b*e^(4*c) + 6*b^2*e^(4*c))*e^(4*d*x) + (a^2*e^(2*c) + 5*a*b*e^(2*c) + 6*b
^2*e^(2*c))*e^(2*d*x))/((a^4*d*e^(5*c) + a^3*b*d*e^(5*c))*e^(5*d*x) + 2*(a^
4*d*e^(3*c) + 3*a^3*b*d*e^(3*c) + 2*a^2*b^2*d*e^(3*c))*e^(3*d*x) + (a^4*d*e
^c + a^3*b*d*e^c)*e^(d*x)) - 1/2*integrate(2*((4*a*b*e^(3*c) + 3*b^2*e^(3*c
))*e^(3*d*x) + (4*a*b*e^c + 3*b^2*e^c)*e^(d*x))/(a^4 + a^3*b + (a^4*e^(4*c)
 + a^3*b*e^(4*c))*e^(4*d*x) + 2*(a^4*e^(2*c) + 3*a^3*b*e^(2*c) + 2*a^2*b^2*
e^(2*c))*e^(2*d*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh\left(c+d\,x\right)}{{\left(a+\frac{b}{{\cosh\left(c+d\,x\right)}^2}\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)/(a + b/cosh(c + d*x)^2)^2,x)
```

```
[Out] int(cosh(c + d*x)/(a + b/cosh(c + d*x)^2)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

3.86 $\int \frac{\operatorname{sech}(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2}\,dx$

Optimal. Leaf size=82

$$\frac{(2a+b)\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{2a^{3/2}d(a+b)^{3/2}}-\frac{b\sinh(c+dx)}{2ad(a+b)\left(a\sinh^2(c+dx)+a+b\right)}$$

```
[Out] 1/2*(2*a+b)*arctan(sinh(d*x+c)*a^(1/2)/(a+b)^(1/2))/a^(3/2)/(a+b)^(3/2)/d-1
/2*b*sinh(d*x+c)/a/(a+b)/d/(a+b+a*sinh(d*x+c)^2)
```

Rubi [A] time = 0.07, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4147, 385, 205}

$$\frac{(2a+b)\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{2a^{3/2}d(a+b)^{3/2}}-\frac{b\sinh(c+dx)}{2ad(a+b)\left(a\sinh^2(c+dx)+a+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((2*a + b)*ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]])/(2*a^(3/2)*(a + b)^
(3/2)*d) - (b*Sinh[c + d*x])/(2*a*(a + b)*d*(a + b + a*Sinh[c + d*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
```

```
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{\left(a+b+ax^2\right)^2}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= -\frac{b\sinh(c+dx)}{2a(a+b)d\left(a+b+a\sinh^2(c+dx)\right)} + \frac{(2a+b)\,\text{Subst}\left(\int \frac{1}{a+b+ax^2}\,dx, x, \sinh(c+dx)\right)}{2a(a+b)d}$$

$$= \frac{(2a+b)\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{2a^{3/2}(a+b)^{3/2}d} - \frac{b\sinh(c+dx)}{2a(a+b)d\left(a+b+a\sinh^2(c+dx)\right)}$$

Mathematica [A] time = 0.31, size = 124, normalized size = 1.51

$$\frac{\left(2a^2+3ab+b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right) - \sqrt{a}\,b\sqrt{a+b}\,\sinh(c+dx) + a(2a+b)\sinh^2(c+dx)\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{a^{3/2}d(a+b)^{3/2}(a\cosh(2(c+dx))+a+2b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((2*a^2 + 3*a*b + b^2)*ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]] - Sqrt[a
]*b*Sqrt[a + b]*Sinh[c + d*x] + a*(2*a + b)*ArcTan[(Sqrt[a]*Sinh[c + d*x])/
Sqrt[a + b]]*Sinh[c + d*x]^2)/(a^(3/2)*(a + b)^(3/2)*d*(a + 2*b + a*Cosh[2*
(c + d*x)]))
```

fricas [B] time = 0.45, size = 1856, normalized size = 22.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*(a^2*b + a*b^2)*cosh(d*x + c)^3 + 12*(a^2*b + a*b^2)*cosh(d*x + c)
*sinh(d*x + c)^2 + 4*(a^2*b + a*b^2)*sinh(d*x + c)^3 + ((2*a^2 + a*b)*cosh(
d*x + c)^4 + 4*(2*a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a^2 + a*b)*
```

```
sinh(d*x + c)^4 + 2*(2*a^2 + 5*a*b + 2*b^2)*cosh(d*x + c)^2 + 2*(3*(2*a^2 +
 a*b)*cosh(d*x + c)^2 + 2*a^2 + 5*a*b + 2*b^2)*sinh(d*x + c)^2 + 2*a^2 + a*
b + 4*((2*a^2 + a*b)*cosh(d*x + c)^3 + (2*a^2 + 5*a*b + 2*b^2)*cosh(d*x + c
))*sinh(d*x + c))*sqrt(-a^2 - a*b)*log((a*cosh(d*x + c)^4 + 4*a*cosh(d*x +
c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 - 2*(3*a + 2*b)*cosh(d*x + c)^2 + 2*
(3*a*cosh(d*x + c)^2 - 3*a - 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 -
(3*a + 2*b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x
+ c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1)*sinh(d*x +
 c) - cosh(d*x + c))*sqrt(-a^2 - a*b) + a)/(a*cosh(d*x + c)^4 + 4*a*cosh(d*
x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 +
2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 +
(a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) - 4*(a^2*b + a*b^2)*cosh(d*x +
 c) - 4*(a^2*b + a*b^2 - 3*(a^2*b + a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c))/
((a^5 + 2*a^4*b + a^3*b^2)*d*cosh(d*x + c)^4 + 4*(a^5 + 2*a^4*b + a^3*b^2)*
d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^5 + 2*a^4*b + a^3*b^2)*d*sinh(d*x + c)
^4 + 2*(a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^
5 + 2*a^4*b + a^3*b^2)*d*cosh(d*x + c)^2 + (a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a
^2*b^3)*d)*sinh(d*x + c)^2 + (a^5 + 2*a^4*b + a^3*b^2)*d + 4*((a^5 + 2*a^4*
b + a^3*b^2)*d*cosh(d*x + c)^3 + (a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3)*d*
cosh(d*x + c))*sinh(d*x + c)), -1/2*(2*(a^2*b + a*b^2)*cosh(d*x + c)^3 + 6*
(a^2*b + a*b^2)*cosh(d*x + c)*sinh(d*x + c)^2 + 2*(a^2*b + a*b^2)*sinh(d*x
+ c)^3 - ((2*a^2 + a*b)*cosh(d*x + c)^4 + 4*(2*a^2 + a*b)*cosh(d*x + c)*sin
h(d*x + c)^3 + (2*a^2 + a*b)*sinh(d*x + c)^4 + 2*(2*a^2 + 5*a*b + 2*b^2)*co
sh(d*x + c)^2 + 2*(3*(2*a^2 + a*b)*cosh(d*x + c)^2 + 2*a^2 + 5*a*b + 2*b^2)
*sinh(d*x + c)^2 + 2*a^2 + a*b + 4*((2*a^2 + a*b)*cosh(d*x + c)^3 + (2*a^2
+ 5*a*b + 2*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 + a*b)*arctan(1/2*(
a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 +
 (3*a + 4*b)*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + 3*a + 4*b)*sinh(d*x + c
))/sqrt(a^2 + a*b)) - ((2*a^2 + a*b)*cosh(d*x + c)^4 + 4*(2*a^2 + a*b)*cosh
(d*x + c)*sinh(d*x + c)^3 + (2*a^2 + a*b)*sinh(d*x + c)^4 + 2*(2*a^2 + 5*a*
b + 2*b^2)*cosh(d*x + c)^2 + 2*(3*(2*a^2 + a*b)*cosh(d*x + c)^2 + 2*a^2 + 5
*a*b + 2*b^2)*sinh(d*x + c)^2 + 2*a^2 + a*b + 4*((2*a^2 + a*b)*cosh(d*x + c
)^3 + (2*a^2 + 5*a*b + 2*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 + a*b)
*arctan(1/2*sqrt(a^2 + a*b)*(cosh(d*x + c) + sinh(d*x + c))/(a + b)) - 2*(a
^2*b + a*b^2)*cosh(d*x + c) - 2*(a^2*b + a*b^2 - 3*(a^2*b + a*b^2)*cosh(d*x
 + c)^2)*sinh(d*x + c))/((a^5 + 2*a^4*b + a^3*b^2)*d*cosh(d*x + c)^4 + 4*(a
^5 + 2*a^4*b + a^3*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^5 + 2*a^4*b +
a^3*b^2)*d*sinh(d*x + c)^4 + 2*(a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3)*d*co
sh(d*x + c)^2 + 2*(3*(a^5 + 2*a^4*b + a^3*b^2)*d*cosh(d*x + c)^2 + (a^5 + 4
*a^4*b + 5*a^3*b^2 + 2*a^2*b^3)*d)*sinh(d*x + c)^2 + (a^5 + 2*a^4*b + a^3*b
^2)*d + 4*((a^5 + 2*a^4*b + a^3*b^2)*d*cosh(d*x + c)^3 + (a^5 + 4*a^4*b + 5
*a^3*b^2 + 2*a^2*b^3)*d*cosh(d*x + c))*sinh(d*x + c))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[6,-20]Warning, need to choose a branch for the root of a po
lynomial with parameters. This might be wrong.The choice was done assuming
[a,b]=[89,-63]Undef/Unsigned Inf encountered in limitLimit: Max order reach
ed or unable to make series expansion Error: Bad Argument Value
```

maple [B] time = 0.34, size = 331, normalized size = 4.04

$$\frac{b\left(\tanh^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\left(\left(\tanh^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+b\left(\tanh^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a+b\right)a\left(a+b\right)}-\frac{}{d\left(\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)/(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] 1/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^
2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)*b/a/(a+b)*tanh(1/2*d*x+1/2*c)^3-1/d/(tan
h(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*ta
nh(1/2*d*x+1/2*c)^2*b+a+b)*b/a/(a+b)*tanh(1/2*d*x+1/2*c)+1/d/(a+b)^(3/2)/a^
(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)-2*b^(1/2))/a^(1/2))+1/d
/(a+b)^(3/2)/a^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)+2*b^(1/2
))/a^(1/2))+1/2/d/a^(3/2)*b/(a+b)^(3/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*
d*x+1/2*c)-2*b^(1/2))/a^(1/2))+1/2/d/a^(3/2)*b/(a+b)^(3/2)*arctan(1/2*(2*(a
+b)^(1/2)*tanh(1/2*d*x+1/2*c)+2*b^(1/2))/a^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{be^{(3\,dx+3\,c)}-be^{(dx+c)}}{a^{3}d+a^{2}bd+\left(a^{3}de^{(4\,c)}+a^{2}bde^{(4\,c)}\right)e^{(4\,dx)}+2\left(a^{3}de^{(2\,c)}+3\,a^{2}bde^{(2\,c)}+2\,ab^{2}de^{(2\,c)}\right)e^{(2\,dx)}}+2\int\frac{}{2\left(a^{3}+a^{2}b+\left(a^{3}e\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -(b*e^(3*d*x + 3*c) - b*e^(d*x + c))/(a^3*d + a^2*b*d + (a^3*d*e^(4*c) + a^
2*b*d*e^(4*c))*e^(4*d*x) + 2*(a^3*d*e^(2*c) + 3*a^2*b*d*e^(2*c) + 2*a*b^2*d
*e^(2*c))*e^(2*d*x)) + 2*integrate(1/2*((2*a*e^(3*c) + b*e^(3*c))*e^(3*d*x)
 + (2*a*e^c + b*e^c)*e^(d*x))/(a^3 + a^2*b + (a^3*e^(4*c) + a^2*b*e^(4*c))*
```

```
e^(4*d*x) + 2*(a^3*e^(2*c) + 3*a^2*b*e^(2*c) + 2*a*b^2*e^(2*c))*e^(2*d*x)),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh(c+dx)\left(a+\frac{b}{\cosh(c+dx)^2}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)*(a + b/cosh(c + d*x)^2)^2),x)
```

```
[Out] int(1/(cosh(c + d*x)*(a + b/cosh(c + d*x)^2)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral(sech(c + d*x)/(a + b*sech(c + d*x)**2)**2, x)
```

3.87 $\int \frac{\text{sech}^2(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2} dx$

Optimal. Leaf size=74

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2\sqrt{b}\,d(a+b)^{3/2}}+\frac{\tanh(c+dx)}{2d(a+b)\left(a-b\tanh^2(c+dx)+b\right)}$$

```
[Out] 1/2*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))/(a+b)^(3/2)/d/b^(1/2)+1/2*tanh
(d*x+c)/(a+b)/d/(a+b-b*tanh(d*x+c)^2)
```

Rubi [A] time = 0.07, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4146, 199, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2\sqrt{b}\,d(a+b)^{3/2}}+\frac{\tanh(c+dx)}{2d(a+b)\left(a-b\tanh^2(c+dx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^2/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]]/(2*Sqrt[b]*(a + b)^(3/2)*d) +
Tanh[c + d*x]/(2*(a + b)*d*(a + b - b*Tanh[c + d*x]^2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
```

```
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{\left(a+b-bx^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\tanh(c+dx)}{2(a+b)d\left(a+b-b\tanh^2(c+dx)\right)} + \frac{\operatorname{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \tanh(c+dx)\right)}{2(a+b)d}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2\sqrt{b}\,(a+b)^{3/2}d} + \frac{\tanh(c+dx)}{2(a+b)d\left(a+b-b\tanh^2(c+dx)\right)}$$

Mathematica [B] time = 1.01, size = 187, normalized size = 2.53

$$\frac{\operatorname{sech}^4(c+dx)(a\cosh(2(c+dx))+a+2b)\left(\frac{(\cosh(2c)-\sinh(2c))(a\cosh(2(c+dx))+a+2b)\tanh^{-1}\left(\frac{(\cosh(2c)-\sinh(2c))\operatorname{sech}(dx)((a+2b)\sinh(dx \ldots}{2\sqrt{a+b}\sqrt{b(\cosh(c)-\sinh(c))\ldots}}\right.}{\sqrt{a+b}\sqrt{b(\cosh(c)-\sinh(c))^4}} \ldots\right.}{8d(a+b)\left(a+b\operatorname{sech}^2(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^2/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^4*((ArcTanh[(Sech[d*x]*(Cosh
[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b
]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(a + 2*b + a*Cosh[2*(c + d*x)])*(Cosh[2*c
] - Sinh[2*c]))/(Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4]) + Sech[2*c]*Sin
h[2*d*x] - ((a + 2*b)*Tanh[2*c])/a))/(8*(a + b)*d*(a + b*Sech[c + d*x]^2)^2
)
```

fricas [B] time = 0.44, size = 1489, normalized size = 20.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*a^2*b + 4*a*b^2 + 4*(a^2*b + 3*a*b^2 + 2*b^3)*cosh(d*x + c)^2 + 8*
(a^2*b + 3*a*b^2 + 2*b^3)*cosh(d*x + c)*sinh(d*x + c) + 4*(a^2*b + 3*a*b^2
+ 2*b^3)*sinh(d*x + c)^2 - (a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(
d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a
^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 4*(a^2*cosh(d*x +
 c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a*b + b^2)*log((a^
2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)
^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a
*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 +
 2*a*b)*cosh(d*x + c))*sinh(d*x + c) - 4*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x
+ c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(a*b + b^2))/(a*cosh(
d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a +
 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 +
 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)))/((a^4
*b + 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^4 + 4*(a^4*b + 2*a^3*b^2 + a^2*b^
3)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*d*sinh(d
*x + c)^4 + 2*(a^4*b + 4*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4)*d*cosh(d*x + c)^2 +
 2*(3*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^2 + (a^4*b + 4*a^3*b^2
+ 5*a^2*b^3 + 2*a*b^4)*d)*sinh(d*x + c)^2 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*d
 + 4*((a^4*b + 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^3 + (a^4*b + 4*a^3*b^2
+ 5*a^2*b^3 + 2*a*b^4)*d*cosh(d*x + c))*sinh(d*x + c)), -1/2*(2*a^2*b + 2*a
*b^2 + 2*(a^2*b + 3*a*b^2 + 2*b^3)*cosh(d*x + c)^2 + 4*(a^2*b + 3*a*b^2 + 2
*b^3)*cosh(d*x + c)*sinh(d*x + c) + 2*(a^2*b + 3*a*b^2 + 2*b^3)*sinh(d*x +
c)^2 - (a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sin
h(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 +
 a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b
)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a*b - b^2)*arctan(1/2*(a*cosh(d*x + c
)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-
a*b - b^2)/(a*b + b^2)))/((a^4*b + 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^4 +
 4*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4*b +
 2*a^3*b^2 + a^2*b^3)*d*sinh(d*x + c)^4 + 2*(a^4*b + 4*a^3*b^2 + 5*a^2*b^3
+ 2*a*b^4)*d*cosh(d*x + c)^2 + 2*(3*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*cosh(d*
x + c)^2 + (a^4*b + 4*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4)*d)*sinh(d*x + c)^2 + (
a^4*b + 2*a^3*b^2 + a^2*b^3)*d + 4*((a^4*b + 2*a^3*b^2 + a^2*b^3)*d*cosh(d*
x + c)^3 + (a^4*b + 4*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4)*d*cosh(d*x + c))*sinh(
d*x + c))]
```

giac [A] time = 0.74, size = 130, normalized size = 1.76

$$\frac{\frac{\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}\,(a+b)}-\frac{2\left(ae^{(2\,dx+2\,c)}+2\,be^{(2\,dx+2\,c)}+a\right)}{\left(a^2+ab\right)\left(ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+a\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/(sqrt(-a*b
- b^2)*(a + b)) - 2*(a*e^(2*d*x + 2*c) + 2*b*e^(2*d*x + 2*c) + a)/((a^2 + a
*b)*(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) + a)))/d
```

maple [B] time = 0.25, size = 263, normalized size = 3.55

$$\frac{\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\left(\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a+b\right)(a+b)}+\frac{}{d\left(\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] 1/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^
2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)/(a+b)*tanh(1/2*d*x+1/2*c)^3+1/d/(tanh(1/
2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1
/2*d*x+1/2*c)^2*b+a+b)/(a+b)*tanh(1/2*d*x+1/2*c)-1/4/d/(a+b)^(3/2)/b^(1/2)*
ln(-(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)-(a+b)^(
1/2))+1/4/d/(a+b)^(3/2)/b^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1
/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))
```

maxima [B] time = 0.46, size = 150, normalized size = 2.03

$$\frac{(a+2b)e^{(-2dx-2c)}+a}{\left(a^3+a^2b+2\left(a^3+3a^2b+2ab^2\right)e^{(-2dx-2c)}+\left(a^3+a^2b\right)e^{(-4dx-4c)}\right)d}-\frac{\log\left(\frac{ae^{(-2dx-2c)}+a+2b-2\sqrt{(a+b)b}}{ae^{(-2dx-2c)}+a+2b+2\sqrt{(a+b)b}}\right)}{4\sqrt{(a+b)b}(a+b)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] ((a + 2*b)*e^(-2*d*x - 2*c) + a)/((a^3 + a^2*b + 2*(a^3 + 3*a^2*b + 2*a*b^2
)*e^(-2*d*x - 2*c) + (a^3 + a^2*b)*e^(-4*d*x - 4*c))*d) - 1/4*log((a*e^(-2*
d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2
*sqrt((a + b)*b)))/(sqrt((a + b)*b)*(a + b)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh(c+d\,x)^2\left(a+\frac{b}{\cosh(c+d\,x)^2}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^2*(a + b/cosh(c + d*x)^2)^2),x)
```

```
[Out] int(1/(cosh(c + d*x)^2*(a + b/cosh(c + d*x)^2)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**2/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral(sech(c + d*x)**2/(a + b*sech(c + d*x)**2)**2, x)
```

3.88 $\int \frac{\text{sech}^3(c+dx)}{\left(a+b\,\text{sech}^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=73

$$\frac{\sinh(c+dx)}{2d(a+b)\left(a\sinh^2(c+dx)+a+b\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{2\sqrt{a}\,d(a+b)^{3/2}}$$

```
[Out] 1/2*sinh(d*x+c)/(a+b)/d/(a+b+a*sinh(d*x+c)^2)+1/2*arctan(sinh(d*x+c)*a^(1/2
)/(a+b)^(1/2))/(a+b)^(3/2)/d/a^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4147, 199, 205}

$$\frac{\sinh(c+dx)}{2d(a+b)\left(a\sinh^2(c+dx)+a+b\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{2\sqrt{a}\,d(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^3/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]]/(2*Sqrt[a]*(a + b)^(3/2)*d) + S
inh[c + d*x]/(2*(a + b)*d*(a + b + a*Sinh[c + d*x]^2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
```

```
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\text{sech}^3(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+b+ax^2\right)^2}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\sinh(c+dx)}{2(a+b)d\left(a+b+a\sinh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{a+b+ax^2}\,dx, x, \sinh(c+dx)\right)}{2(a+b)d}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{2\sqrt{a}\,(a+b)^{3/2}d} + \frac{\sinh(c+dx)}{2(a+b)d\left(a+b+a\sinh^2(c+dx)\right)}$$

Mathematica [A] time = 0.23, size = 108, normalized size = 1.48

$$\frac{\text{sech}^4(c+dx)(a\cosh(2c+2dx)+a+2b)^2\left(\frac{\sinh(c+dx)}{(a+b)\left(a\sinh^2(c+dx)+a+b\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{a}\,(a+b)^{3/2}}\right)}{8d\left(a+b\text{sech}^2(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^3/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*c + 2*d*x])^2*Sech[c + d*x]^4*(ArcTan[(Sqrt[a]*Sinh[c
+ d*x])/Sqrt[a + b]]/(Sqrt[a]*(a + b)^(3/2)) + Sinh[c + d*x]/((a + b)*(a +
b + a*Sinh[c + d*x]^2))))/(8*d*(a + b*Sech[c + d*x]^2)^2)
```

fricas [B] time = 0.48, size = 1570, normalized size = 21.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a^2 + a*b)*cosh(d*x + c)^3 + 12*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x
 + c)^2 + 4*(a^2 + a*b)*sinh(d*x + c)^3 - (a*cosh(d*x + c)^4 + 4*a*cosh(d*x
 + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2
```

```
*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (
a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)*sqrt(-a^2 - a*b)*log((a*cosh(d*x
 + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 - 2*(3*a +
2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 - 3*a - 2*b)*sinh(d*x + c)^2
+ 4*(a*cosh(d*x + c)^3 - (3*a + 2*b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh
(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d
*x + c)^2 - 1)*sinh(d*x + c) - cosh(d*x + c))*sqrt(-a^2 - a*b) + a)/(a*cosh
(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a
+ 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2
+ 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) - 4*(
a^2 + a*b)*cosh(d*x + c) + 4*(3*(a^2 + a*b)*cosh(d*x + c)^2 - a^2 - a*b)*si
nh(d*x + c))/((a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^4 + 4*(a^4 + 2*a^3*
b + a^2*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 + 2*a^3*b + a^2*b^2)*d*
sinh(d*x + c)^4 + 2*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d*cosh(d*x + c)^2
 + 2*(3*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^2 + (a^4 + 4*a^3*b + 5*a^
2*b^2 + 2*a*b^3)*d)*sinh(d*x + c)^2 + (a^4 + 2*a^3*b + a^2*b^2)*d + 4*((a^4
 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^3 + (a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*
b^3)*d*cosh(d*x + c))*sinh(d*x + c)), 1/2*(2*(a^2 + a*b)*cosh(d*x + c)^3 +
6*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c)^2 + 2*(a^2 + a*b)*sinh(d*x + c)^3
 + (a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)
^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d
*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) +
 a)*sqrt(a^2 + a*b)*arctan(1/2*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(
d*x + c)^2 + a*sinh(d*x + c)^3 + (3*a + 4*b)*cosh(d*x + c) + (3*a*cosh(d*x
+ c)^2 + 3*a + 4*b)*sinh(d*x + c))/sqrt(a^2 + a*b)) + (a*cosh(d*x + c)^4 +
4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*
x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*
x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)*sqrt(a^2 + a*b)*arct
an(1/2*sqrt(a^2 + a*b)*(cosh(d*x + c) + sinh(d*x + c))/(a + b)) - 2*(a^2 +
a*b)*cosh(d*x + c) + 2*(3*(a^2 + a*b)*cosh(d*x + c)^2 - a^2 - a*b)*sinh(d*x
 + c))/((a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^4 + 4*(a^4 + 2*a^3*b + a^
2*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 + 2*a^3*b + a^2*b^2)*d*sinh(d
*x + c)^4 + 2*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d*cosh(d*x + c)^2 + 2*(
3*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^2 + (a^4 + 4*a^3*b + 5*a^2*b^2
+ 2*a*b^3)*d)*sinh(d*x + c)^2 + (a^4 + 2*a^3*b + a^2*b^2)*d + 4*((a^4 + 2*a
^3*b + a^2*b^2)*d*cosh(d*x + c)^3 + (a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d
*cosh(d*x + c))*sinh(d*x + c))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[6,-20]Warning, need to choose a branch for the root of a po
lynomial with parameters. This might be wrong.The choice was done assuming
[a,b]=[89,-63]Warning, need to choose a branch for the root of a polynomial
 with parameters. This might be wrong.The choice was done assuming [a,b]=[1
2,-32]Warning, need to choose a branch for the root of a polynomial with pa
rameters. This might be wrong.The choice was done assuming [a,b]=[2,72]Unde
f/Unsigned Inf encountered in limitEvaluation time: 0.58Limit: Max order re
ached or unable to make series expansion Error: Bad Argument Value
```

maple [B] time = 0.27, size = 241, normalized size = 3.30

$$-\frac{\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\left(\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a+b\right)(a+b)}+\frac{}{d\left(\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] -1/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)
^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)/(a+b)*tanh(1/2*d*x+1/2*c)^3+1/d/(tanh(1
/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(
1/2*d*x+1/2*c)^2*b+a+b)/(a+b)*tanh(1/2*d*x+1/2*c)+1/2/d/(a+b)^(3/2)/a^(1/2)
*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)-2*b^(1/2))/a^(1/2))+1/2/d/(a
+b)^(3/2)/a^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)+2*b^(1/2))/
a^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{e^{(3\,dx+3\,c)}-e^{(dx+c)}}{a^2d+abd+\left(a^2de^{(4\,c)}+abde^{(4\,c)}\right)e^{(4\,dx)}+2\left(a^2de^{(2\,c)}+3\,abde^{(2\,c)}+2\,b^2de^{(2\,c)}\right)e^{(2\,dx)}}+8\int\frac{}{8\left(a^2+ab+\left(a^2e^{(4\,c)}+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] (e^(3*d*x + 3*c) - e^(d*x + c))/(a^2*d + a*b*d + (a^2*d*e^(4*c) + a*b*d*e^(
4*c))*e^(4*d*x) + 2*(a^2*d*e^(2*c) + 3*a*b*d*e^(2*c) + 2*b^2*d*e^(2*c))*e^(
2*d*x)) + 8*integrate(1/8*(e^(3*d*x + 3*c) + e^(d*x + c))/(a^2 + a*b + (a^2
*e^(4*c) + a*b*e^(4*c))*e^(4*d*x) + 2*(a^2*e^(2*c) + 3*a*b*e^(2*c) + 2*b^2*
e^(2*c))*e^(2*d*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh\left(c+d\,x\right)^3\,{\left(a+\frac{b}{{\cosh\left(c+d\,x\right)}^2}\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^3*(a + b/cosh(c + d*x)^2)^2),x)
```

```
[Out] int(1/(cosh(c + d*x)^3*(a + b/cosh(c + d*x)^2)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^{3}{\left(c+dx \right)}}{\left(a+b\operatorname{sech}^{2}{\left(c+dx \right)}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**3/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral(sech(c + d*x)**3/(a + b*sech(c + d*x)**2)**2, x)
```

3.89 $\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=83

$$\frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2b^{3/2}d(a+b)^{3/2}}-\frac{a\tanh(c+dx)}{2bd(a+b)\left(a-b\tanh^2(c+dx)+b\right)}$$

```
[Out] 1/2*(a+2*b)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))/b^(3/2)/(a+b)^(3/2)/d-
1/2*a*tanh(d*x+c)/b/(a+b)/d/(a+b-b*tanh(d*x+c)^2)
```

Rubi [A] time = 0.09, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4146, 385, 208}

$$\frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2b^{3/2}d(a+b)^{3/2}}-\frac{a\tanh(c+dx)}{2bd(a+b)\left(a-b\tanh^2(c+dx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^4/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + 2*b)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(2*b^(3/2)*(a + b)
^(3/2)*d) - (a*Tanh[c + d*x])/(2*b*(a + b)*d*(a + b - b*Tanh[c + d*x]^2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
```

```
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1-x^2}{\left(a+b-bx^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{a\tanh(c+dx)}{2b(a+b)d\left(a+b-b\tanh^2(c+dx)\right)} + \frac{(a+2b)\operatorname{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \tanh(c$$

$$= \frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2b^{3/2}(a+b)^{3/2}d} - \frac{a\tanh(c+dx)}{2b(a+b)d\left(a+b-b\tanh^2(c+dx)\right)}$$

Mathematica [A] time = 0.22, size = 88, normalized size = 1.06

$$4\left(\frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8b^{3/2}d(a+b)^{3/2}} - \frac{a\sinh(2(c+dx))}{8bd(a+b)(a\cosh(2(c+dx))+a+2b)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^4/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] 4*(((a + 2*b)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(8*b^(3/2)*(a +
 b)^(3/2)*d) - (a*Sinh[2*(c + d*x)])/(8*b*(a + b)*d*(a + 2*b + a*Cosh[2*(c
+ d*x)])))
```

fricas [B] time = 0.45, size = 1569, normalized size = 18.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*a^2*b + 4*a*b^2 + 4*(a^2*b + 3*a*b^2 + 2*b^3)*cosh(d*x + c)^2 + 8*(
a^2*b + 3*a*b^2 + 2*b^3)*cosh(d*x + c)*sinh(d*x + c) + 4*(a^2*b + 3*a*b^2 +
 2*b^3)*sinh(d*x + c)^2 + ((a^2 + 2*a*b)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b)*
```

```
cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b)*sinh(d*x + c)^4 + 2*(a^2 + 4*
a*b + 4*b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b)*cosh(d*x + c)^2 + a^2 + 4
*a*b + 4*b^2)*sinh(d*x + c)^2 + a^2 + 2*a*b + 4*((a^2 + 2*a*b)*cosh(d*x + c
)^3 + (a^2 + 4*a*b + 4*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a*b + b^2)*l
og((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*
x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2
 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 +
(a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) - 4*(a*cosh(d*x + c)^2 + 2*a*cos
h(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(a*b + b^2))/(a
*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 +
2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x +
c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)))
/((a^3*b^2 + 2*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^4 + 4*(a^3*b^2 + 2*a^2*b^3
+ a*b^4)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3*b^2 + 2*a^2*b^3 + a*b^4)*d*
sinh(d*x + c)^4 + 2*(a^3*b^2 + 4*a^2*b^3 + 5*a*b^4 + 2*b^5)*d*cosh(d*x + c)
^2 + 2*(3*(a^3*b^2 + 2*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^2 + (a^3*b^2 + 4*a^
2*b^3 + 5*a*b^4 + 2*b^5)*d)*sinh(d*x + c)^2 + (a^3*b^2 + 2*a^2*b^3 + a*b^4)
*d + 4*((a^3*b^2 + 2*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^3 + (a^3*b^2 + 4*a^2*
b^3 + 5*a*b^4 + 2*b^5)*d*cosh(d*x + c))*sinh(d*x + c)), 1/2*(2*a^2*b + 2*a*
b^2 + 2*(a^2*b + 3*a*b^2 + 2*b^3)*cosh(d*x + c)^2 + 4*(a^2*b + 3*a*b^2 + 2*
b^3)*cosh(d*x + c)*sinh(d*x + c) + 2*(a^2*b + 3*a*b^2 + 2*b^3)*sinh(d*x + c
)^2 + ((a^2 + 2*a*b)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b)*cosh(d*x + c)*sinh(d
*x + c)^3 + (a^2 + 2*a*b)*sinh(d*x + c)^4 + 2*(a^2 + 4*a*b + 4*b^2)*cosh(d*
x + c)^2 + 2*(3*(a^2 + 2*a*b)*cosh(d*x + c)^2 + a^2 + 4*a*b + 4*b^2)*sinh(d
*x + c)^2 + a^2 + 2*a*b + 4*((a^2 + 2*a*b)*cosh(d*x + c)^3 + (a^2 + 4*a*b +
 4*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a*b - b^2)*arctan(1/2*(a*cosh(d
*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*
sqrt(-a*b - b^2)/(a*b + b^2)))/((a^3*b^2 + 2*a^2*b^3 + a*b^4)*d*cosh(d*x +
c)^4 + 4*(a^3*b^2 + 2*a^2*b^3 + a*b^4)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a
^3*b^2 + 2*a^2*b^3 + a*b^4)*d*sinh(d*x + c)^4 + 2*(a^3*b^2 + 4*a^2*b^3 + 5*
a*b^4 + 2*b^5)*d*cosh(d*x + c)^2 + 2*(3*(a^3*b^2 + 2*a^2*b^3 + a*b^4)*d*cos
h(d*x + c)^2 + (a^3*b^2 + 4*a^2*b^3 + 5*a*b^4 + 2*b^5)*d)*sinh(d*x + c)^2 +
 (a^3*b^2 + 2*a^2*b^3 + a*b^4)*d + 4*((a^3*b^2 + 2*a^2*b^3 + a*b^4)*d*cosh(
d*x + c)^3 + (a^3*b^2 + 4*a^2*b^3 + 5*a*b^4 + 2*b^5)*d*cosh(d*x + c))*sinh(
d*x + c))]
```

giac [A] time = 0.77, size = 139, normalized size = 1.67

$$\frac{\frac{(a+2\,b)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{\left(ab+b^2\right)\sqrt{-ab-b^2}}+\frac{2\,\left(ae^{(2\,dx+2\,c)}+2\,be^{(2\,dx+2\,c)}+a\right)}{\left(ab+b^2\right)\left(ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+a\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((a + 2*b)*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/(
(a*b + b^2)*sqrt(-a*b - b^2)) + 2*(a*e^(2*d*x + 2*c) + 2*b*e^(2*d*x + 2*c)
+ a)/((a*b + b^2)*(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x +
 2*c) + a)))/d
```

maple [B] time = 0.29, size = 374, normalized size = 4.51

$$-\frac{a\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\left(\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a+b\right)b\,(a+b)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] -1/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)
^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)*a/b/(a+b)*tanh(1/2*d*x+1/2*c)^3-1/d/(ta
nh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*t
anh(1/2*d*x+1/2*c)^2*b+a+b)*a/b/(a+b)*tanh(1/2*d*x+1/2*c)-1/4/d/(a+b)^(3/2)
/b^(3/2)*a*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*b^(1/2)*tanh(1/2*d*x+1/2*
c)+(a+b)^(1/2))+1/4/d/(a+b)^(3/2)/b^(3/2)*a*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2
*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))-1/2/d/(a+b)^(3/2)/b^(1/2)*
ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1
/2))+1/2/d/(a+b)^(3/2)/b^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/
2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))
```

maxima [B] time = 0.48, size = 165, normalized size = 1.99

$$-\frac{(a+2\,b)\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{4\,\sqrt{(a+b)b}\left(ab+b^2\right)d}-\frac{(a+2\,b)e^{(-2\,dx-2\,c)}+a}{\left(a^2b+ab^2+2\left(a^2b+3\,ab^2+2\,b^3\right)e^{(-2\,dx-2\,c)}+\left(a^2b+ab^2\right)e^{(-4\,dx-4\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(a + 2*b)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^
(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*(a*b + b^2)
*d) - ((a + 2*b)*e^(-2*d*x - 2*c) + a)/((a^2*b + a*b^2 + 2*(a^2*b + 3*a*b^2
 + 2*b^3)*e^(-2*d*x - 2*c) + (a^2*b + a*b^2)*e^(-4*d*x - 4*c))*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh\left(c+d\,x\right)^4\left(a+\frac{b}{\cosh\left(c+d\,x\right)^2}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^4*(a + b/cosh(c + d*x)^2)^2),x)
```

```
[Out] int(1/(cosh(c + d*x)^4*(a + b/cosh(c + d*x)^2)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**4/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral(sech(c + d*x)**4/(a + b*sech(c + d*x)**2)**2, x)
```

3.90 $\int \frac{\text{sech}^5(c+dx)}{\left(a+b\,\text{sech}^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=101

$$-\frac{\sqrt{a}\,(2a+3b)\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{2b^2d(a+b)^{3/2}}-\frac{a\sinh(c+dx)}{2bd(a+b)\left(a\sinh^2(c+dx)+a+b\right)}+\frac{\tan^{-1}(\sinh(c+dx))}{b^2d}$$

```
[Out] arctan(sinh(d*x+c))/b^2/d-1/2*a*sinh(d*x+c)/b/(a+b)/d/(a+b+a*sinh(d*x+c)^2)
-1/2*(2*a+3*b)*arctan(sinh(d*x+c)*a^(1/2)/(a+b)^(1/2))*a^(1/2)/b^2/(a+b)^(3
/2)/d
```

Rubi [A] time = 0.12, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4147, 414, 522, 203, 205}

$$-\frac{\sqrt{a}\,(2a+3b)\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{2b^2d(a+b)^{3/2}}-\frac{a\sinh(c+dx)}{2bd(a+b)\left(a\sinh^2(c+dx)+a+b\right)}+\frac{\tan^{-1}(\sinh(c+dx))}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^5/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ArcTan[Sinh[c + d*x]]/(b^2*d) - (Sqrt[a]*(2*a + 3*b)*ArcTan[(Sqrt[a]*Sinh[c
 + d*x])/Sqrt[a + b]])/(2*b^2*(a + b)^(3/2)*d) - (a*Sinh[c + d*x])/(2*b*(a
+ b)*d*(a + b + a*Sinh[c + d*x]^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
```

```
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\text{sech}^5(c+dx)}{\left(a+b\,\text{sech}^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\left(a+b+ax^2\right)^2}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= -\frac{a\sinh(c+dx)}{2b(a+b)d\left(a+b+a\sinh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{a+2b-ax^2}{(1+x^2)\left(a+b+ax^2\right)}\,dx, x, \sinh(c+d\right.}{2b(a+b)d}$$

$$= -\frac{a\sinh(c+dx)}{2b(a+b)d\left(a+b+a\sinh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(c+dx)\right)}{b^2d} - \frac{(a(2}{}$$

$$= \frac{\tan^{-1}(\sinh(c+dx))}{b^2d} - \frac{\sqrt{a}\,(2a+3b)\tan^{-1}\left(\frac{\sqrt{a}\,\sinh(c+dx)}{\sqrt{a+b}}\right)}{2b^2(a+b)^{3/2}d} - \frac{a\sinh(c+}{2b(a+b)d\left(a+b+a\,s\right.}$$

Mathematica [B] time = 2.53, size = 282, normalized size = 2.79

$$\frac{\text{sech}^3(c+dx)(a\cosh(2(c+dx))+a+2b)\left(-2ab\sqrt{a+b}\,\sqrt{(\cosh(c)-\sinh(c))^2}\,\tanh(c+dx)+\sqrt{a}\,(2a+3b)\cosh\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^5/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^3*(Sqrt[a]*(2*a + 3*b)*ArcTa
n[(Sqrt[a + b]*Csch[c + d*x]*Sqrt[(Cosh[c] - Sinh[c])^2]*(Cosh[c] + Sinh[c]
))/Sqrt[a]]*Cosh[c]*(a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x] - (a + 2*
b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]*(-4*(a + b)^(3/2)*ArcTan[Tanh[(c + d
*x)/2]]*Sqrt[(Cosh[c] - Sinh[c])^2] + Sqrt[a]*(2*a + 3*b)*ArcTan[(Sqrt[a +
b]*Csch[c + d*x]*Sqrt[(Cosh[c] - Sinh[c])^2]*(Cosh[c] + Sinh[c]))/Sqrt[a]]*
Sinh[c]) - 2*a*b*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[c + d*x]))/(8
*b^2*(a + b)^(3/2)*d*(a + b*Sech[c + d*x]^2)^2*Sqrt[(Cosh[c] - Sinh[c])^2])
```

fricas [B] time = 0.50, size = 2069, normalized size = 20.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*a*b*cosh(d*x + c)^3 + 12*a*b*cosh(d*x + c)*sinh(d*x + c)^2 + 4*a*b
*sinh(d*x + c)^3 - 4*a*b*cosh(d*x + c) - ((2*a^2 + 3*a*b)*cosh(d*x + c)^4 +
 4*(2*a^2 + 3*a*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a^2 + 3*a*b)*sinh(d*x
 + c)^4 + 2*(2*a^2 + 7*a*b + 6*b^2)*cosh(d*x + c)^2 + 2*(3*(2*a^2 + 3*a*b)*
cosh(d*x + c)^2 + 2*a^2 + 7*a*b + 6*b^2)*sinh(d*x + c)^2 + 2*a^2 + 3*a*b +
4*((2*a^2 + 3*a*b)*cosh(d*x + c)^3 + (2*a^2 + 7*a*b + 6*b^2)*cosh(d*x + c))
*sinh(d*x + c))*sqrt(-a/(a + b))*log((a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)
*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 - 2*(3*a + 2*b)*cosh(d*x + c)^2 + 2*(3
*a*cosh(d*x + c)^2 - 3*a - 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 - (3
*a + 2*b)*cosh(d*x + c))*sinh(d*x + c) - 4*((a + b)*cosh(d*x + c)^3 + 3*(a
+ b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 - (a + b)*cosh
(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 - a - b)*sinh(d*x + c))*sqrt(-a/(a +
 b)) + a)/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d
*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)
*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x
 + c) + a)) - 8*((a^2 + a*b)*cosh(d*x + c)^4 + 4*(a^2 + a*b)*cosh(d*x + c)*
sinh(d*x + c)^3 + (a^2 + a*b)*sinh(d*x + c)^4 + 2*(a^2 + 3*a*b + 2*b^2)*cos
h(d*x + c)^2 + 2*(3*(a^2 + a*b)*cosh(d*x + c)^2 + a^2 + 3*a*b + 2*b^2)*sinh
(d*x + c)^2 + a^2 + a*b + 4*((a^2 + a*b)*cosh(d*x + c)^3 + (a^2 + 3*a*b + 2
*b^2)*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) +
 4*(3*a*b*cosh(d*x + c)^2 - a*b)*sinh(d*x + c))/((a^2*b^2 + a*b^3)*d*cosh(d
*x + c)^4 + 4*(a^2*b^2 + a*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2*b^2
+ a*b^3)*d*sinh(d*x + c)^4 + 2*(a^2*b^2 + 3*a*b^3 + 2*b^4)*d*cosh(d*x + c)^
2 + 2*(3*(a^2*b^2 + a*b^3)*d*cosh(d*x + c)^2 + (a^2*b^2 + 3*a*b^3 + 2*b^4)*
d)*sinh(d*x + c)^2 + (a^2*b^2 + a*b^3)*d + 4*((a^2*b^2 + a*b^3)*d*cosh(d*x
```

```
+ c)^3 + (a^2*b^2 + 3*a*b^3 + 2*b^4)*d*cosh(d*x + c))*sinh(d*x + c)), -1/2*
(2*a*b*cosh(d*x + c)^3 + 6*a*b*cosh(d*x + c)*sinh(d*x + c)^2 + 2*a*b*sinh(d
*x + c)^3 - 2*a*b*cosh(d*x + c) + ((2*a^2 + 3*a*b)*cosh(d*x + c)^4 + 4*(2*a
^2 + 3*a*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a^2 + 3*a*b)*sinh(d*x + c)^4
 + 2*(2*a^2 + 7*a*b + 6*b^2)*cosh(d*x + c)^2 + 2*(3*(2*a^2 + 3*a*b)*cosh(d*
x + c)^2 + 2*a^2 + 7*a*b + 6*b^2)*sinh(d*x + c)^2 + 2*a^2 + 3*a*b + 4*((2*a
^2 + 3*a*b)*cosh(d*x + c)^3 + (2*a^2 + 7*a*b + 6*b^2)*cosh(d*x + c))*sinh(d
*x + c))*sqrt(a/(a + b))*arctan(1/2*sqrt(a/(a + b))*(cosh(d*x + c) + sinh(d
*x + c))) + ((2*a^2 + 3*a*b)*cosh(d*x + c)^4 + 4*(2*a^2 + 3*a*b)*cosh(d*x +
 c)*sinh(d*x + c)^3 + (2*a^2 + 3*a*b)*sinh(d*x + c)^4 + 2*(2*a^2 + 7*a*b +
6*b^2)*cosh(d*x + c)^2 + 2*(3*(2*a^2 + 3*a*b)*cosh(d*x + c)^2 + 2*a^2 + 7*a
*b + 6*b^2)*sinh(d*x + c)^2 + 2*a^2 + 3*a*b + 4*((2*a^2 + 3*a*b)*cosh(d*x +
 c)^3 + (2*a^2 + 7*a*b + 6*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a/(a + b
))*arctan(1/2*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*si
nh(d*x + c)^3 + (3*a + 4*b)*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + 3*a + 4*
b)*sinh(d*x + c))*sqrt(a/(a + b))/a) - 4*((a^2 + a*b)*cosh(d*x + c)^4 + 4*(
a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + a*b)*sinh(d*x + c)^4 + 2*
(a^2 + 3*a*b + 2*b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + a*b)*cosh(d*x + c)^2 +
a^2 + 3*a*b + 2*b^2)*sinh(d*x + c)^2 + a^2 + a*b + 4*((a^2 + a*b)*cosh(d*x
+ c)^3 + (a^2 + 3*a*b + 2*b^2)*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*
x + c) + sinh(d*x + c)) + 2*(3*a*b*cosh(d*x + c)^2 - a*b)*sinh(d*x + c))/((
a^2*b^2 + a*b^3)*d*cosh(d*x + c)^4 + 4*(a^2*b^2 + a*b^3)*d*cosh(d*x + c)*si
nh(d*x + c)^3 + (a^2*b^2 + a*b^3)*d*sinh(d*x + c)^4 + 2*(a^2*b^2 + 3*a*b^3
+ 2*b^4)*d*cosh(d*x + c)^2 + 2*(3*(a^2*b^2 + a*b^3)*d*cosh(d*x + c)^2 + (a^
2*b^2 + 3*a*b^3 + 2*b^4)*d)*sinh(d*x + c)^2 + (a^2*b^2 + a*b^3)*d + 4*((a^2
*b^2 + a*b^3)*d*cosh(d*x + c)^3 + (a^2*b^2 + 3*a*b^3 + 2*b^4)*d*cosh(d*x +
c))*sinh(d*x + c))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[6,-20]Warning, need to choose a branch for the root of a po
lynomial with parameters. This might be wrong.The choice was done assuming
[a,b]=[89,-63]Warning, need to choose a branch for the root of a polynomial
 with parameters. This might be wrong.The choice was done assuming [a,b]=[1
2,-32]Warning, need to choose a branch for the root of a polynomial with pa
rameters. This might be wrong.The choice was done assuming [a,b]=[2,72]Unde
```

```
f/Unsigned Inf encountered in limitEvaluation time: 0.62Limit: Max order re
ached or unable to make series expansion Error: Bad Argument Value
```

maple [B] time = 0.27, size = 361, normalized size = 3.57

$$\frac{a\left(\tanh^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\left(\left(\tanh^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+b\left(\tanh^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a+b\right)b\left(a+b\right)}-\frac{}{d\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^5/(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] 1/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^
2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)*a/b/(a+b)*tanh(1/2*d*x+1/2*c)^3-1/d/(tan
h(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*ta
nh(1/2*d*x+1/2*c)^2*b+a+b)*a/b/(a+b)*tanh(1/2*d*x+1/2*c)-1/d*a^(3/2)/b^2/(a
+b)^(3/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)-2*b^(1/2))/a^(1/2))
-1/d*a^(3/2)/b^2/(a+b)^(3/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)+
2*b^(1/2))/a^(1/2))-3/2/d*a^(1/2)/b/(a+b)^(3/2)*arctan(1/2*(2*(a+b)^(1/2)*t
anh(1/2*d*x+1/2*c)-2*b^(1/2))/a^(1/2))-3/2/d*a^(1/2)/b/(a+b)^(3/2)*arctan(1
/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)+2*b^(1/2))/a^(1/2))+2/d/b^2*arctan(ta
nh(1/2*d*x+1/2*c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{ae^{(3\,dx+3\,c)}-ae^{(dx+c)}}{a^{2}bd+ab^{2}d+\left(a^{2}bde^{(4\,c)}+ab^{2}de^{(4\,c)}\right)e^{(4\,dx)}+2\left(a^{2}bde^{(2\,c)}+3\,ab^{2}de^{(2\,c)}+2\,b^{3}de^{(2\,c)}\right)e^{(2\,dx)}}+\frac{2\arctan\left(e^{(dx+c)}\right)}{b^{2}d}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -(a*e^(3*d*x + 3*c) - a*e^(d*x + c))/(a^2*b*d + a*b^2*d + (a^2*b*d*e^(4*c)
+ a*b^2*d*e^(4*c))*e^(4*d*x) + 2*(a^2*b*d*e^(2*c) + 3*a*b^2*d*e^(2*c) + 2*b
^3*d*e^(2*c))*e^(2*d*x)) + 2*arctan(e^(d*x + c))/(b^2*d) - 32*integrate(1/3
2*((2*a^2*e^(3*c) + 3*a*b*e^(3*c))*e^(3*d*x) + (2*a^2*e^c + 3*a*b*e^c)*e^(d
*x))/(a^2*b^2 + a*b^3 + (a^2*b^2*e^(4*c) + a*b^3*e^(4*c))*e^(4*d*x) + 2*(a^
2*b^2*e^(2*c) + 3*a*b^3*e^(2*c) + 2*b^4*e^(2*c))*e^(2*d*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh\left(c+d\,x\right)^{5}\left(a+\frac{b}{\cosh\left(c+d\,x\right)^{2}}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^5*(a + b/cosh(c + d*x)^2)^2),x)
[Out] int(1/(cosh(c + d*x)^5*(a + b/cosh(c + d*x)^2)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**5/(a+b*sech(d*x+c)**2)**2,x)
[Out] Integral(sech(c + d*x)**5/(a + b*sech(c + d*x)**2)**2, x)
```

3.91 $\int \frac{\text{sech}^6(c+dx)}{\left(a+b\,\text{sech}^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=101

$$\frac{a^2\tanh(c+dx)}{2b^2d(a+b)\left(a-b\tanh^2(c+dx)+b\right)}-\frac{a(3a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2b^{5/2}d(a+b)^{3/2}}+\frac{\tanh(c+dx)}{b^2d}$$

```
[Out] -1/2*a*(3*a+4*b)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))/b^(5/2)/(a+b)^(3/
2)/d+tanh(d*x+c)/b^2/d+1/2*a^2*tanh(d*x+c)/b^2/(a+b)/d/(a+b-b*tanh(d*x+c)^2
)
```

Rubi [A] time = 0.14, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4146, 390, 385, 208}

$$\frac{a^2\tanh(c+dx)}{2b^2d(a+b)\left(a-b\tanh^2(c+dx)+b\right)}-\frac{a(3a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2b^{5/2}d(a+b)^{3/2}}+\frac{\tanh(c+dx)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^6/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] -(a*(3*a + 4*b)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(2*b^(5/2)*(a
 + b)^(3/2)*d) + Tanh[c + d*x]/(b^2*d) + (a^2*Tanh[c + d*x])/(2*b^2*(a + b)
*d*(a + b - b*Tanh[c + d*x]^2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{\left(a+b-bx^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{b^2}-\frac{a(a+2b)-2abx^2}{b^2\left(a+b-bx^2\right)^2}\right)dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\tanh(c+dx)}{b^2d} - \frac{\text{Subst}\left(\int \frac{a(a+2b)-2abx^2}{\left(a+b-bx^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{b^2d}$$

$$= \frac{\tanh(c+dx)}{b^2d} + \frac{a^2\tanh(c+dx)}{2b^2(a+b)d\left(a+b-b\tanh^2(c+dx)\right)} - \frac{(a(3a+4b))\,\text{Subst}\left(\int \frac{\ldots}{a+\ldots}\right.}{2b^2(a\ldots}$$

$$= -\frac{a(3a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2b^{5/2}(a+b)^{3/2}d} + \frac{\tanh(c+dx)}{b^2d} + \frac{a^2\tanh(c+dx)}{2b^2(a+b)d\left(a+b-b\tanh^2\ldots\right.}$$

Mathematica [B] time = 3.81, size = 229, normalized size = 2.27

$$\frac{\operatorname{sech}^4(c+dx)(a\cosh(2(c+dx))+a+2b)\left(2\operatorname{sech}(c)\sinh(dx)\operatorname{sech}(c+dx)(a\cosh(2(c+dx))+a+2b)+\frac{a(a\operatorname{sech}(2\ldots}{\ldots}\right.}{8b^2d\left(a+b\operatorname{se}\ldots\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^6/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^4*(-((a*(3*a + 4*b)*ArcTanh[
(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]
))/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(a + 2*b + a*Cosh[2*(c +
d*x)])*(Cosh[2*c] - Sinh[2*c]))/((a + b)^(3/2)*Sqrt[b*(Cosh[c] - Sinh[c])^4
])) + 2*(a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c]*Sech[c + d*x]*Sinh[d*x] + (
a*(a*Sech[2*c]*Sinh[2*d*x] - (a + 2*b)*Tanh[2*c]))/(a + b)))/(8*b^2*d*(a +
b*Sech[c + d*x]^2)^2)
```

fricas [B] time = 0.48, size = 2958, normalized size = 29.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*(3*a^3*b + 7*a^2*b^2 + 4*a*b^3)*cosh(d*x + c)^4 + 16*(3*a^3*b + 7*
a^2*b^2 + 4*a*b^3)*cosh(d*x + c)*sinh(d*x + c)^3 + 4*(3*a^3*b + 7*a^2*b^2 +
 4*a*b^3)*sinh(d*x + c)^4 + 12*a^3*b + 20*a^2*b^2 + 8*a*b^3 + 8*(3*a^3*b +
10*a^2*b^2 + 11*a*b^3 + 4*b^4)*cosh(d*x + c)^2 + 8*(3*a^3*b + 10*a^2*b^2 +
11*a*b^3 + 4*b^4 + 3*(3*a^3*b + 7*a^2*b^2 + 4*a*b^3)*cosh(d*x + c)^2)*sinh(
d*x + c)^2 - ((3*a^3 + 4*a^2*b)*cosh(d*x + c)^6 + 6*(3*a^3 + 4*a^2*b)*cosh(
d*x + c)*sinh(d*x + c)^5 + (3*a^3 + 4*a^2*b)*sinh(d*x + c)^6 + (9*a^3 + 24*
a^2*b + 16*a*b^2)*cosh(d*x + c)^4 + (9*a^3 + 24*a^2*b + 16*a*b^2 + 15*(3*a^
3 + 4*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(3*a^3 + 4*a^2*b)*cosh
(d*x + c)^3 + (9*a^3 + 24*a^2*b + 16*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^3
+ 3*a^3 + 4*a^2*b + (9*a^3 + 24*a^2*b + 16*a*b^2)*cosh(d*x + c)^2 + (15*(3*
a^3 + 4*a^2*b)*cosh(d*x + c)^4 + 9*a^3 + 24*a^2*b + 16*a*b^2 + 6*(9*a^3 + 2
4*a^2*b + 16*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(3*(3*a^3 + 4*a^2*
b)*cosh(d*x + c)^5 + 2*(9*a^3 + 24*a^2*b + 16*a*b^2)*cosh(d*x + c)^3 + (9*a
^3 + 24*a^2*b + 16*a*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a*b + b^2)*log
((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x
+ c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 +
 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a
^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) + 4*(a*cosh(d*x + c)^2 + 2*a*cosh(
d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(a*b + b^2))/(a*c
osh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*
(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)
^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) +
16*((3*a^3*b + 7*a^2*b^2 + 4*a*b^3)*cosh(d*x + c)^3 + (3*a^3*b + 10*a^2*b^2
 + 11*a*b^3 + 4*b^4)*cosh(d*x + c))*sinh(d*x + c))/((a^3*b^3 + 2*a^2*b^4 +
a*b^5)*d*cosh(d*x + c)^6 + 6*(a^3*b^3 + 2*a^2*b^4 + a*b^5)*d*cosh(d*x + c)*
```

sinh(d*x + c)^5 + (a^3*b^3 + 2*a^2*b^4 + a*b^5)*d*sinh(d*x + c)^6 + (3*a^3*
b^3 + 10*a^2*b^4 + 11*a*b^5 + 4*b^6)*d*cosh(d*x + c)^4 + (15*(a^3*b^3 + 2*a
^2*b^4 + a*b^5)*d*cosh(d*x + c)^2 + (3*a^3*b^3 + 10*a^2*b^4 + 11*a*b^5 + 4*
b^6)*d)*sinh(d*x + c)^4 + (3*a^3*b^3 + 10*a^2*b^4 + 11*a*b^5 + 4*b^6)*d*cos
h(d*x + c)^2 + 4*(5*(a^3*b^3 + 2*a^2*b^4 + a*b^5)*d*cosh(d*x + c)^3 + (3*a^
3*b^3 + 10*a^2*b^4 + 11*a*b^5 + 4*b^6)*d*cosh(d*x + c))*sinh(d*x + c)^3 + (
15*(a^3*b^3 + 2*a^2*b^4 + a*b^5)*d*cosh(d*x + c)^4 + 6*(3*a^3*b^3 + 10*a^2*
b^4 + 11*a*b^5 + 4*b^6)*d*cosh(d*x + c)^2 + (3*a^3*b^3 + 10*a^2*b^4 + 11*a*
b^5 + 4*b^6)*d)*sinh(d*x + c)^2 + (a^3*b^3 + 2*a^2*b^4 + a*b^5)*d + 2*(3*(a
^3*b^3 + 2*a^2*b^4 + a*b^5)*d*cosh(d*x + c)^5 + 2*(3*a^3*b^3 + 10*a^2*b^4 +
 11*a*b^5 + 4*b^6)*d*cosh(d*x + c)^3 + (3*a^3*b^3 + 10*a^2*b^4 + 11*a*b^5 +
 4*b^6)*d*cosh(d*x + c))*sinh(d*x + c)), -1/2*(2*(3*a^3*b + 7*a^2*b^2 + 4*a
*b^3)*cosh(d*x + c)^4 + 8*(3*a^3*b + 7*a^2*b^2 + 4*a*b^3)*cosh(d*x + c)*sin
h(d*x + c)^3 + 2*(3*a^3*b + 7*a^2*b^2 + 4*a*b^3)*sinh(d*x + c)^4 + 6*a^3*b
+ 10*a^2*b^2 + 4*a*b^3 + 4*(3*a^3*b + 10*a^2*b^2 + 11*a*b^3 + 4*b^4)*cosh(d
x + c)^2 + 4(3*a^3*b + 10*a^2*b^2 + 11*a*b^3 + 4*b^4 + 3*(3*a^3*b + 7*a^2
*b^2 + 4*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((3*a^3 + 4*a^2*b)*cosh(
d*x + c)^6 + 6*(3*a^3 + 4*a^2*b)*cosh(d*x + c)*sinh(d*x + c)^5 + (3*a^3 + 4
*a^2*b)*sinh(d*x + c)^6 + (9*a^3 + 24*a^2*b + 16*a*b^2)*cosh(d*x + c)^4 + (
9*a^3 + 24*a^2*b + 16*a*b^2 + 15*(3*a^3 + 4*a^2*b)*cosh(d*x + c)^2)*sinh(d*
x + c)^4 + 4*(5*(3*a^3 + 4*a^2*b)*cosh(d*x + c)^3 + (9*a^3 + 24*a^2*b + 16*
a*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*a^3 + 4*a^2*b + (9*a^3 + 24*a^2*b
 + 16*a*b^2)*cosh(d*x + c)^2 + (15*(3*a^3 + 4*a^2*b)*cosh(d*x + c)^4 + 9*a^
3 + 24*a^2*b + 16*a*b^2 + 6*(9*a^3 + 24*a^2*b + 16*a*b^2)*cosh(d*x + c)^2)*
sinh(d*x + c)^2 + 2*(3*(3*a^3 + 4*a^2*b)*cosh(d*x + c)^5 + 2*(9*a^3 + 24*a^
2*b + 16*a*b^2)*cosh(d*x + c)^3 + (9*a^3 + 24*a^2*b + 16*a*b^2)*cosh(d*x +
c))*sinh(d*x + c))*sqrt(-a*b - b^2)*arctan(1/2*(a*cosh(d*x + c)^2 + 2*a*cos
h(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-a*b - b^2)/(a
b + b^2)) + 8((3*a^3*b + 7*a^2*b^2 + 4*a*b^3)*cosh(d*x + c)^3 + (3*a^3*b
+ 10*a^2*b^2 + 11*a*b^3 + 4*b^4)*cosh(d*x + c))*sinh(d*x + c))/((a^3*b^3 +
2*a^2*b^4 + a*b^5)*d*cosh(d*x + c)^6 + 6*(a^3*b^3 + 2*a^2*b^4 + a*b^5)*d*co
sh(d*x + c)*sinh(d*x + c)^5 + (a^3*b^3 + 2*a^2*b^4 + a*b^5)*d*sinh(d*x + c)
^6 + (3*a^3*b^3 + 10*a^2*b^4 + 11*a*b^5 + 4*b^6)*d*cosh(d*x + c)^4 + (15*(a
^3*b^3 + 2*a^2*b^4 + a*b^5)*d*cosh(d*x + c)^2 + (3*a^3*b^3 + 10*a^2*b^4 + 1
1*a*b^5 + 4*b^6)*d)*sinh(d*x + c)^4 + (3*a^3*b^3 + 10*a^2*b^4 + 11*a*b^5 +
4*b^6)*d*cosh(d*x + c)^2 + 4*(5*(a^3*b^3 + 2*a^2*b^4 + a*b^5)*d*cosh(d*x +
c)^3 + (3*a^3*b^3 + 10*a^2*b^4 + 11*a*b^5 + 4*b^6)*d*cosh(d*x + c))*sinh(d*
x + c)^3 + (15*(a^3*b^3 + 2*a^2*b^4 + a*b^5)*d*cosh(d*x + c)^4 + 6*(3*a^3*b
^3 + 10*a^2*b^4 + 11*a*b^5 + 4*b^6)*d*cosh(d*x + c)^2 + (3*a^3*b^3 + 10*a^2
*b^4 + 11*a*b^5 + 4*b^6)*d)*sinh(d*x + c)^2 + (a^3*b^3 + 2*a^2*b^4 + a*b^5)
d + 2(3*(a^3*b^3 + 2*a^2*b^4 + a*b^5)*d*cosh(d*x + c)^5 + 2*(3*a^3*b^3 +
10*a^2*b^4 + 11*a*b^5 + 4*b^6)*d*cosh(d*x + c)^3 + (3*a^3*b^3 + 10*a^2*b^4
+ 11*a*b^5 + 4*b^6)*d*cosh(d*x + c))*sinh(d*x + c))]

giac [B] time = 0.79, size = 225, normalized size = 2.23

$$-\frac{\frac{\left(3\,a^2+4\,ab\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{\left(ab^2+b^3\right)\sqrt{-ab-b^2}}+\frac{2\left(3\,a^2e^{(4\,dx+4\,c)}+4\,abe^{(4\,dx+4\,c)}+6\,a^2e^{(2\,dx+2\,c)}+14\,abe^{(2\,dx+2\,c)}+8\,b^2e^{(2\,dx+2\,c)}+3\,a^2+2\,ab\right)}{\left(ab^2+b^3\right)\left(ae^{(6\,dx+6\,c)}+3\,ae^{(4\,dx+4\,c)}+4\,be^{(4\,dx+4\,c)}+3\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+a\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*((3*a^2 + 4*a*b)*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b -
b^2))/((a*b^2 + b^3)*sqrt(-a*b - b^2)) + 2*(3*a^2*e^(4*d*x + 4*c) + 4*a*b*e
^(4*d*x + 4*c) + 6*a^2*e^(2*d*x + 2*c) + 14*a*b*e^(2*d*x + 2*c) + 8*b^2*e^(
2*d*x + 2*c) + 3*a^2 + 2*a*b)/((a*b^2 + b^3)*(a*e^(6*d*x + 6*c) + 3*a*e^(4*
d*x + 4*c) + 4*b*e^(4*d*x + 4*c) + 3*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c
) + a)))/d
```

maple [B] time = 0.28, size = 419, normalized size = 4.15

$$\frac{a^2\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,b^2\left(\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a+b\right)(a+b)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^6/(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] 1/d*a^2/b^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x
+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)/(a+b)*tanh(1/2*d*x+1/2*c)^3+1/d*
a^2/b^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2
*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)/(a+b)*tanh(1/2*d*x+1/2*c)+3/4/d*a^2/
b^(5/2)/(a+b)^(3/2)*ln(-(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/
2*d*x+1/2*c)-(a+b)^(1/2))-3/4/d*a^2/b^(5/2)/(a+b)^(3/2)*ln((a+b)^(1/2)*tanh
(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))+1/d*a/b^(3/2)/
(a+b)^(3/2)*ln(-(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/
2*c)-(a+b)^(1/2))-1/d/(a+b)^(3/2)/b^(3/2)*a*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2
*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))+2/d/b^2*tanh(1/2*d*x+1/2*c
)/(tanh(1/2*d*x+1/2*c)^2+1)
```

maxima [B] time = 0.55, size = 244, normalized size = 2.42

$$\frac{(3\,a+4\,b)a\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{4\left(ab^2+b^3\right)\sqrt{(a+b)b}\,d}+\frac{3\,a^2+2\,ab+2\left(3\,a^2+7\,ab+4\,b^2\right)e^{(-2\,dx-2\,c)}+(}{\left(a^2b^2+ab^3+\left(3\,a^2b^2+7\,ab^3+4\,b^4\right)e^{(-2\,dx-2\,c)}+\left(3\,a^2b^2+7\,ab^3+4\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(3*a + 4*b)*a*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a
*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a*b^2 + b^3)*sqrt((a +
b)*b)*d) + (3*a^2 + 2*a*b + 2*(3*a^2 + 7*a*b + 4*b^2)*e^(-2*d*x - 2*c) + (3
*a^2 + 4*a*b)*e^(-4*d*x - 4*c))/((a^2*b^2 + a*b^3 + (3*a^2*b^2 + 7*a*b^3 +
4*b^4)*e^(-2*d*x - 2*c) + (3*a^2*b^2 + 7*a*b^3 + 4*b^4)*e^(-4*d*x - 4*c) +
(a^2*b^2 + a*b^3)*e^(-6*d*x - 6*c))*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh\left(c+d\,x\right)^6\,{\left(a+\frac{b}{{\cosh\left(c+d\,x\right)}^2}\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^6*(a + b/cosh(c + d*x)^2)^2),x)
```

```
[Out] int(1/(cosh(c + d*x)^6*(a + b/cosh(c + d*x)^2)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^{6}{\left(c + d x \right)}}{\left(a + b \operatorname{sech}^{2}{\left(c + d x \right)}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**6/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral(sech(c + d*x)**6/(a + b*sech(c + d*x)**2)**2, x)
```

3.92 $\int \frac{\text{sech}^7(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=153

$$\frac{a^{3/2}(4a+5b)\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{2b^3d(a+b)^{3/2}}-\frac{(4a-b)\tan^{-1}(\sinh(c+dx))}{2b^3d}+\frac{a(2a+b)\sinh(c+dx)}{2b^2d(a+b)\left(a\sinh^2(c+dx)+a+b\right)}+\frac{\tanh}{2bd\,(a}$$

```
[Out] -1/2*(4*a-b)*arctan(sinh(d*x+c))/b^3/d+1/2*a^(3/2)*(4*a+5*b)*arctan(sinh(d*
x+c)*a^(1/2)/(a+b)^(1/2))/b^3/(a+b)^(3/2)/d+1/2*a*(2*a+b)*sinh(d*x+c)/b^2/(
a+b)/d/(a+b+a*sinh(d*x+c)^2)+1/2*sech(d*x+c)*tanh(d*x+c)/b/d/(a+b+a*sinh(d*
x+c)^2)
```

Rubi [A] time = 0.20, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4147, 414, 527, 522, 203, 205}

$$\frac{a^{3/2}(4a+5b)\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{2b^3d(a+b)^{3/2}}+\frac{a(2a+b)\sinh(c+dx)}{2b^2d(a+b)\left(a\sinh^2(c+dx)+a+b\right)}-\frac{(4a-b)\tan^{-1}(\sinh(c+dx))}{2b^3d}+\frac{\tanh}{2bd\,(a}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^7/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] -((4*a - b)*ArcTan[Sinh[c + d*x]])/(2*b^3*d) + (a^(3/2)*(4*a + 5*b)*ArcTan[
(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]])/(2*b^3*(a + b)^(3/2)*d) + (a*(2*a + b
)*Sinh[c + d*x])/(2*b^2*(a + b)*d*(a + b + a*Sinh[c + d*x]^2)) + (Sech[c +
d*x]*Tanh[c + d*x])/(2*b*d*(a + b + a*Sinh[c + d*x]^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
```

```
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\text{sech}^7(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^2\left(a+b+ax^2\right)^2}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\text{sech}(c+dx)\tanh(c+dx)}{2bd\left(a+b+a\sinh^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{a-b-3ax^2}{\left(1+x^2\right)\left(a+b+ax^2\right)^2}\,dx, x, \sinh(c+dx)\right)}{2bd}$$

$$= \frac{a(2a+b)\sinh(c+dx)}{2b^2(a+b)d\left(a+b+a\sinh^2(c+dx)\right)} + \frac{\text{sech}(c+dx)\tanh(c+dx)}{2bd\left(a+b+a\sinh^2(c+dx)\right)} - \frac{\text{Subst}\left(\right.}{}$$

$$= \frac{a(2a+b)\sinh(c+dx)}{2b^2(a+b)d\left(a+b+a\sinh^2(c+dx)\right)} + \frac{\text{sech}(c+dx)\tanh(c+dx)}{2bd\left(a+b+a\sinh^2(c+dx)\right)} - \frac{(4a-b}{}$$

$$= -\frac{(4a-b)\tan^{-1}(\sinh(c+dx))}{2b^3d} + \frac{a^{3/2}(4a+5b)\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{2b^3(a+b)^{3/2}d} + \frac{a(2}{2b^2(a+b)}$$

Mathematica [B] time = 5.19, size = 489, normalized size = 3.20

$$\frac{\text{sech}(c)\text{sech}^3(c+dx)(a\cosh(2(c+dx))+a+2b)\left(-a^{3/2}(4a+5b)\cosh^2(c)\text{sech}(c+dx)(a\cosh(2(c+dx))+a+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^7/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c]*Sech[c + d*x]^3*(-(a^(3/2)*(4*a +
5*b)*ArcTan[(Sqrt[a + b]*Csch[c + d*x]*Sqrt[(Cosh[c] - Sinh[c])^2]*(Cosh[c]
 + Sinh[c]))/Sqrt[a]]*Cosh[c]^2*(a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*
x]) + b*(a + b)^(3/2)*(a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*Sqrt[
(Cosh[c] - Sinh[c])^2]*Sinh[c] - Cosh[c]*Sech[c + d*x]*(2*Sqrt[a + b]*(4*a^
2 + 3*a*b - b^2)*ArcTan[Tanh[(c + d*x)/2]]*(a + 2*b + a*Cosh[2*(c + d*x)])*
Sqrt[(Cosh[c] - Sinh[c])^2] - a^(5/2)*b*ArcTan[(Sqrt[a + b]*Csch[c + d*x]*S
qrt[(Cosh[c] - Sinh[c])^2]*(Cosh[c] + Sinh[c]))/Sqrt[a]]*(13 + 5*Cosh[2*(c
+ d*x)])*Sinh[c]) + a^(3/2)*ArcTan[(Sqrt[a + b]*Csch[c + d*x]*Sqrt[(Cosh[c]
 - Sinh[c])^2]*(Cosh[c] + Sinh[c]))/Sqrt[a]]*(2*a^2 + 5*b^2 + 2*a^2*Cosh[2*
(c + d*x)])*Sech[c + d*x]*Sinh[2*c] + b*(a + b)^(3/2)*(a + 2*b + a*Cosh[2*(
c + d*x)])*Sech[c + d*x]^3*Sqrt[(Cosh[c] - Sinh[c])^2]*Sinh[d*x] + 2*a^2*b*
```

```
Sqrt[a + b]*Cosh[c]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[c + d*x]))/(8*b^3*(a +
 b)^(3/2)*d*(a + b*Sech[c + d*x]^2)^2*Sqrt[(Cosh[c] - Sinh[c])^2])
```

fricas [B] time = 0.58, size = 6499, normalized size = 42.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^7/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(2*a^2*b + a*b^2)*cosh(d*x + c)^7 + 28*(2*a^2*b + a*b^2)*cosh(d*x +
 c)*sinh(d*x + c)^6 + 4*(2*a^2*b + a*b^2)*sinh(d*x + c)^7 + 4*(2*a^2*b + 5*
a*b^2 + 4*b^3)*cosh(d*x + c)^5 + 4*(2*a^2*b + 5*a*b^2 + 4*b^3 + 21*(2*a^2*b
 + a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 20*(7*(2*a^2*b + a*b^2)*cosh(d
*x + c)^3 + (2*a^2*b + 5*a*b^2 + 4*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 - 4*
(2*a^2*b + 5*a*b^2 + 4*b^3)*cosh(d*x + c)^3 + 4*(35*(2*a^2*b + a*b^2)*cosh(
d*x + c)^4 - 2*a^2*b - 5*a*b^2 - 4*b^3 + 10*(2*a^2*b + 5*a*b^2 + 4*b^3)*cos
h(d*x + c)^2)*sinh(d*x + c)^3 + 4*(21*(2*a^2*b + a*b^2)*cosh(d*x + c)^5 + 1
0*(2*a^2*b + 5*a*b^2 + 4*b^3)*cosh(d*x + c)^3 - 3*(2*a^2*b + 5*a*b^2 + 4*b^
3)*cosh(d*x + c))*sinh(d*x + c)^2 + ((4*a^3 + 5*a^2*b)*cosh(d*x + c)^8 + 8*
(4*a^3 + 5*a^2*b)*cosh(d*x + c)*sinh(d*x + c)^7 + (4*a^3 + 5*a^2*b)*sinh(d*
x + c)^8 + 4*(4*a^3 + 9*a^2*b + 5*a*b^2)*cosh(d*x + c)^6 + 4*(4*a^3 + 9*a^2
*b + 5*a*b^2 + 7*(4*a^3 + 5*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*
(4*a^3 + 5*a^2*b)*cosh(d*x + c)^3 + 3*(4*a^3 + 9*a^2*b + 5*a*b^2)*cosh(d*x
+ c))*sinh(d*x + c)^5 + 2*(12*a^3 + 31*a^2*b + 20*a*b^2)*cosh(d*x + c)^4 +
2*(35*(4*a^3 + 5*a^2*b)*cosh(d*x + c)^4 + 12*a^3 + 31*a^2*b + 20*a*b^2 + 30
*(4*a^3 + 9*a^2*b + 5*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(4*a^3
 + 5*a^2*b)*cosh(d*x + c)^5 + 10*(4*a^3 + 9*a^2*b + 5*a*b^2)*cosh(d*x + c)^
3 + (12*a^3 + 31*a^2*b + 20*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*a^3 +
 5*a^2*b + 4*(4*a^3 + 9*a^2*b + 5*a*b^2)*cosh(d*x + c)^2 + 4*(7*(4*a^3 + 5*
a^2*b)*cosh(d*x + c)^6 + 15*(4*a^3 + 9*a^2*b + 5*a*b^2)*cosh(d*x + c)^4 + 4
*a^3 + 9*a^2*b + 5*a*b^2 + 3*(12*a^3 + 31*a^2*b + 20*a*b^2)*cosh(d*x + c)^2
)*sinh(d*x + c)^2 + 8*((4*a^3 + 5*a^2*b)*cosh(d*x + c)^7 + 3*(4*a^3 + 9*a^2
*b + 5*a*b^2)*cosh(d*x + c)^5 + (12*a^3 + 31*a^2*b + 20*a*b^2)*cosh(d*x + c
)^3 + (4*a^3 + 9*a^2*b + 5*a*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a/(a
+ b))*log((a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d
*x + c)^4 - 2*(3*a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 - 3*a -
2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 - (3*a + 2*b)*cosh(d*x + c))*si
nh(d*x + c) + 4*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)*sinh(d*x
 + c)^2 + (a + b)*sinh(d*x + c)^3 - (a + b)*cosh(d*x + c) + (3*(a + b)*cosh
(d*x + c)^2 - a - b)*sinh(d*x + c))*sqrt(-a/(a + b)) + a)/(a*cosh(d*x + c)^
4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cos
h(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cos
h(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) - 4*((4*a^3 + 3
*a^2*b - a*b^2)*cosh(d*x + c)^8 + 8*(4*a^3 + 3*a^2*b - a*b^2)*cosh(d*x + c)
```

```
*sinh(d*x + c)^7 + (4*a^3 + 3*a^2*b - a*b^2)*sinh(d*x + c)^8 + 4*(4*a^3 + 7
*a^2*b + 2*a*b^2 - b^3)*cosh(d*x + c)^6 + 4*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^
3 + 7*(4*a^3 + 3*a^2*b - a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(4*
a^3 + 3*a^2*b - a*b^2)*cosh(d*x + c)^3 + 3*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3
)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(12*a^3 + 25*a^2*b + 9*a*b^2 - 4*b^3)*
cosh(d*x + c)^4 + 2*(35*(4*a^3 + 3*a^2*b - a*b^2)*cosh(d*x + c)^4 + 12*a^3
+ 25*a^2*b + 9*a*b^2 - 4*b^3 + 30*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*cosh(d*
x + c)^2)*sinh(d*x + c)^4 + 8*(7*(4*a^3 + 3*a^2*b - a*b^2)*cosh(d*x + c)^5
+ 10*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*cosh(d*x + c)^3 + (12*a^3 + 25*a^2*b
 + 9*a*b^2 - 4*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*a^3 + 3*a^2*b - a*b^
2 + 4*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*cosh(d*x + c)^2 + 4*(7*(4*a^3 + 3*a
^2*b - a*b^2)*cosh(d*x + c)^6 + 15*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*cosh(d
*x + c)^4 + 4*a^3 + 7*a^2*b + 2*a*b^2 - b^3 + 3*(12*a^3 + 25*a^2*b + 9*a*b^
2 - 4*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((4*a^3 + 3*a^2*b - a*b^2)*
cosh(d*x + c)^7 + 3*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*cosh(d*x + c)^5 + (12
*a^3 + 25*a^2*b + 9*a*b^2 - 4*b^3)*cosh(d*x + c)^3 + (4*a^3 + 7*a^2*b + 2*a
*b^2 - b^3)*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x +
 c)) - 4*(2*a^2*b + a*b^2)*cosh(d*x + c) + 4*(7*(2*a^2*b + a*b^2)*cosh(d*x
+ c)^6 + 5*(2*a^2*b + 5*a*b^2 + 4*b^3)*cosh(d*x + c)^4 - 2*a^2*b - a*b^2 -
3*(2*a^2*b + 5*a*b^2 + 4*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/((a^2*b^3 + a
*b^4)*d*cosh(d*x + c)^8 + 8*(a^2*b^3 + a*b^4)*d*cosh(d*x + c)*sinh(d*x + c)
^7 + (a^2*b^3 + a*b^4)*d*sinh(d*x + c)^8 + 4*(a^2*b^3 + 2*a*b^4 + b^5)*d*co
sh(d*x + c)^6 + 4*(7*(a^2*b^3 + a*b^4)*d*cosh(d*x + c)^2 + (a^2*b^3 + 2*a*b
^4 + b^5)*d)*sinh(d*x + c)^6 + 2*(3*a^2*b^3 + 7*a*b^4 + 4*b^5)*d*cosh(d*x +
 c)^4 + 8*(7*(a^2*b^3 + a*b^4)*d*cosh(d*x + c)^3 + 3*(a^2*b^3 + 2*a*b^4 + b
^5)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^2*b^3 + a*b^4)*d*cosh(d*x +
 c)^4 + 30*(a^2*b^3 + 2*a*b^4 + b^5)*d*cosh(d*x + c)^2 + (3*a^2*b^3 + 7*a*b
^4 + 4*b^5)*d)*sinh(d*x + c)^4 + 4*(a^2*b^3 + 2*a*b^4 + b^5)*d*cosh(d*x + c
)^2 + 8*(7*(a^2*b^3 + a*b^4)*d*cosh(d*x + c)^5 + 10*(a^2*b^3 + 2*a*b^4 + b^
5)*d*cosh(d*x + c)^3 + (3*a^2*b^3 + 7*a*b^4 + 4*b^5)*d*cosh(d*x + c))*sinh(
d*x + c)^3 + 4*(7*(a^2*b^3 + a*b^4)*d*cosh(d*x + c)^6 + 15*(a^2*b^3 + 2*a*b
^4 + b^5)*d*cosh(d*x + c)^4 + 3*(3*a^2*b^3 + 7*a*b^4 + 4*b^5)*d*cosh(d*x +
c)^2 + (a^2*b^3 + 2*a*b^4 + b^5)*d)*sinh(d*x + c)^2 + (a^2*b^3 + a*b^4)*d +
 8*((a^2*b^3 + a*b^4)*d*cosh(d*x + c)^7 + 3*(a^2*b^3 + 2*a*b^4 + b^5)*d*cos
h(d*x + c)^5 + (3*a^2*b^3 + 7*a*b^4 + 4*b^5)*d*cosh(d*x + c)^3 + (a^2*b^3 +
 2*a*b^4 + b^5)*d*cosh(d*x + c))*sinh(d*x + c)), 1/2*(2*(2*a^2*b + a*b^2)*c
osh(d*x + c)^7 + 14*(2*a^2*b + a*b^2)*cosh(d*x + c)*sinh(d*x + c)^6 + 2*(2*
a^2*b + a*b^2)*sinh(d*x + c)^7 + 2*(2*a^2*b + 5*a*b^2 + 4*b^3)*cosh(d*x + c
)^5 + 2*(2*a^2*b + 5*a*b^2 + 4*b^3 + 21*(2*a^2*b + a*b^2)*cosh(d*x + c)^2)*
sinh(d*x + c)^5 + 10*(7*(2*a^2*b + a*b^2)*cosh(d*x + c)^3 + (2*a^2*b + 5*a*
b^2 + 4*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 - 2*(2*a^2*b + 5*a*b^2 + 4*b^3)
*cosh(d*x + c)^3 + 2*(35*(2*a^2*b + a*b^2)*cosh(d*x + c)^4 - 2*a^2*b - 5*a*
b^2 - 4*b^3 + 10*(2*a^2*b + 5*a*b^2 + 4*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)
^3 + 2*(21*(2*a^2*b + a*b^2)*cosh(d*x + c)^5 + 10*(2*a^2*b + 5*a*b^2 + 4*b^
3)*cosh(d*x + c)^3 - 3*(2*a^2*b + 5*a*b^2 + 4*b^3)*cosh(d*x + c))*sinh(d*x
```

+ c)^2 + ((4*a^3 + 5*a^2*b)*cosh(d*x + c)^8 + 8*(4*a^3 + 5*a^2*b)*cosh(d*x
+ c)*sinh(d*x + c)^7 + (4*a^3 + 5*a^2*b)*sinh(d*x + c)^8 + 4*(4*a^3 + 9*a^2
*b + 5*a*b^2)*cosh(d*x + c)^6 + 4*(4*a^3 + 9*a^2*b + 5*a*b^2 + 7*(4*a^3 + 5
*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(4*a^3 + 5*a^2*b)*cosh(d*x
+ c)^3 + 3*(4*a^3 + 9*a^2*b + 5*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(
12*a^3 + 31*a^2*b + 20*a*b^2)*cosh(d*x + c)^4 + 2*(35*(4*a^3 + 5*a^2*b)*cos
h(d*x + c)^4 + 12*a^3 + 31*a^2*b + 20*a*b^2 + 30*(4*a^3 + 9*a^2*b + 5*a*b^2
)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(4*a^3 + 5*a^2*b)*cosh(d*x + c)^5
 + 10*(4*a^3 + 9*a^2*b + 5*a*b^2)*cosh(d*x + c)^3 + (12*a^3 + 31*a^2*b + 20
*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*a^3 + 5*a^2*b + 4*(4*a^3 + 9*a^2
*b + 5*a*b^2)*cosh(d*x + c)^2 + 4*(7*(4*a^3 + 5*a^2*b)*cosh(d*x + c)^6 + 15
*(4*a^3 + 9*a^2*b + 5*a*b^2)*cosh(d*x + c)^4 + 4*a^3 + 9*a^2*b + 5*a*b^2 +
3*(12*a^3 + 31*a^2*b + 20*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((4*a
^3 + 5*a^2*b)*cosh(d*x + c)^7 + 3*(4*a^3 + 9*a^2*b + 5*a*b^2)*cosh(d*x + c)
^5 + (12*a^3 + 31*a^2*b + 20*a*b^2)*cosh(d*x + c)^3 + (4*a^3 + 9*a^2*b + 5*
a*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a/(a + b))*arctan(1/2*sqrt(a/(a +
 b))*(cosh(d*x + c) + sinh(d*x + c))) + ((4*a^3 + 5*a^2*b)*cosh(d*x + c)^8
+ 8*(4*a^3 + 5*a^2*b)*cosh(d*x + c)*sinh(d*x + c)^7 + (4*a^3 + 5*a^2*b)*sin
h(d*x + c)^8 + 4*(4*a^3 + 9*a^2*b + 5*a*b^2)*cosh(d*x + c)^6 + 4*(4*a^3 + 9
*a^2*b + 5*a*b^2 + 7*(4*a^3 + 5*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8
(7(4*a^3 + 5*a^2*b)*cosh(d*x + c)^3 + 3*(4*a^3 + 9*a^2*b + 5*a*b^2)*cosh(
d*x + c))*sinh(d*x + c)^5 + 2*(12*a^3 + 31*a^2*b + 20*a*b^2)*cosh(d*x + c)^
4 + 2*(35*(4*a^3 + 5*a^2*b)*cosh(d*x + c)^4 + 12*a^3 + 31*a^2*b + 20*a*b^2
+ 30*(4*a^3 + 9*a^2*b + 5*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(4
*a^3 + 5*a^2*b)*cosh(d*x + c)^5 + 10*(4*a^3 + 9*a^2*b + 5*a*b^2)*cosh(d*x +
 c)^3 + (12*a^3 + 31*a^2*b + 20*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*a
^3 + 5*a^2*b + 4*(4*a^3 + 9*a^2*b + 5*a*b^2)*cosh(d*x + c)^2 + 4*(7*(4*a^3
+ 5*a^2*b)*cosh(d*x + c)^6 + 15*(4*a^3 + 9*a^2*b + 5*a*b^2)*cosh(d*x + c)^4
 + 4*a^3 + 9*a^2*b + 5*a*b^2 + 3*(12*a^3 + 31*a^2*b + 20*a*b^2)*cosh(d*x +
c)^2)*sinh(d*x + c)^2 + 8*((4*a^3 + 5*a^2*b)*cosh(d*x + c)^7 + 3*(4*a^3 + 9
*a^2*b + 5*a*b^2)*cosh(d*x + c)^5 + (12*a^3 + 31*a^2*b + 20*a*b^2)*cosh(d*x
 + c)^3 + (4*a^3 + 9*a^2*b + 5*a*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a/
(a + b))*arctan(1/2*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2
+ a*sinh(d*x + c)^3 + (3*a + 4*b)*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + 3*
a + 4*b)*sinh(d*x + c))*sqrt(a/(a + b))/a) - 2*((4*a^3 + 3*a^2*b - a*b^2)*c
osh(d*x + c)^8 + 8*(4*a^3 + 3*a^2*b - a*b^2)*cosh(d*x + c)*sinh(d*x + c)^7
+ (4*a^3 + 3*a^2*b - a*b^2)*sinh(d*x + c)^8 + 4*(4*a^3 + 7*a^2*b + 2*a*b^2
- b^3)*cosh(d*x + c)^6 + 4*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3 + 7*(4*a^3 + 3*
a^2*b - a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(4*a^3 + 3*a^2*b - a
*b^2)*cosh(d*x + c)^3 + 3*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*cosh(d*x + c))*
sinh(d*x + c)^5 + 2*(12*a^3 + 25*a^2*b + 9*a*b^2 - 4*b^3)*cosh(d*x + c)^4 +
 2*(35*(4*a^3 + 3*a^2*b - a*b^2)*cosh(d*x + c)^4 + 12*a^3 + 25*a^2*b + 9*a*
b^2 - 4*b^3 + 30*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*cosh(d*x + c)^2)*sinh(d*
x + c)^4 + 8*(7*(4*a^3 + 3*a^2*b - a*b^2)*cosh(d*x + c)^5 + 10*(4*a^3 + 7*a
^2*b + 2*a*b^2 - b^3)*cosh(d*x + c)^3 + (12*a^3 + 25*a^2*b + 9*a*b^2 - 4*b^

```
3)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*a^3 + 3*a^2*b - a*b^2 + 4*(4*a^3 + 7*
a^2*b + 2*a*b^2 - b^3)*cosh(d*x + c)^2 + 4*(7*(4*a^3 + 3*a^2*b - a*b^2)*cos
h(d*x + c)^6 + 15*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*cosh(d*x + c)^4 + 4*a^3
 + 7*a^2*b + 2*a*b^2 - b^3 + 3*(12*a^3 + 25*a^2*b + 9*a*b^2 - 4*b^3)*cosh(d
*x + c)^2)*sinh(d*x + c)^2 + 8*((4*a^3 + 3*a^2*b - a*b^2)*cosh(d*x + c)^7 +
 3*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*cosh(d*x + c)^5 + (12*a^3 + 25*a^2*b +
 9*a*b^2 - 4*b^3)*cosh(d*x + c)^3 + (4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*cosh(
d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) - 2*(2*a^2*b
 + a*b^2)*cosh(d*x + c) + 2*(7*(2*a^2*b + a*b^2)*cosh(d*x + c)^6 + 5*(2*a^2
*b + 5*a*b^2 + 4*b^3)*cosh(d*x + c)^4 - 2*a^2*b - a*b^2 - 3*(2*a^2*b + 5*a*
b^2 + 4*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/((a^2*b^3 + a*b^4)*d*cosh(d*x
+ c)^8 + 8*(a^2*b^3 + a*b^4)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2*b^3 + a
*b^4)*d*sinh(d*x + c)^8 + 4*(a^2*b^3 + 2*a*b^4 + b^5)*d*cosh(d*x + c)^6 + 4
*(7*(a^2*b^3 + a*b^4)*d*cosh(d*x + c)^2 + (a^2*b^3 + 2*a*b^4 + b^5)*d)*sinh
(d*x + c)^6 + 2*(3*a^2*b^3 + 7*a*b^4 + 4*b^5)*d*cosh(d*x + c)^4 + 8*(7*(a^2
*b^3 + a*b^4)*d*cosh(d*x + c)^3 + 3*(a^2*b^3 + 2*a*b^4 + b^5)*d*cosh(d*x +
c))*sinh(d*x + c)^5 + 2*(35*(a^2*b^3 + a*b^4)*d*cosh(d*x + c)^4 + 30*(a^2*b
^3 + 2*a*b^4 + b^5)*d*cosh(d*x + c)^2 + (3*a^2*b^3 + 7*a*b^4 + 4*b^5)*d)*si
nh(d*x + c)^4 + 4*(a^2*b^3 + 2*a*b^4 + b^5)*d*cosh(d*x + c)^2 + 8*(7*(a^2*b
^3 + a*b^4)*d*cosh(d*x + c)^5 + 10*(a^2*b^3 + 2*a*b^4 + b^5)*d*cosh(d*x + c
)^3 + (3*a^2*b^3 + 7*a*b^4 + 4*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7
*(a^2*b^3 + a*b^4)*d*cosh(d*x + c)^6 + 15*(a^2*b^3 + 2*a*b^4 + b^5)*d*cosh(
d*x + c)^4 + 3*(3*a^2*b^3 + 7*a*b^4 + 4*b^5)*d*cosh(d*x + c)^2 + (a^2*b^3 +
 2*a*b^4 + b^5)*d)*sinh(d*x + c)^2 + (a^2*b^3 + a*b^4)*d + 8*((a^2*b^3 + a*
b^4)*d*cosh(d*x + c)^7 + 3*(a^2*b^3 + 2*a*b^4 + b^5)*d*cosh(d*x + c)^5 + (3
*a^2*b^3 + 7*a*b^4 + 4*b^5)*d*cosh(d*x + c)^3 + (a^2*b^3 + 2*a*b^4 + b^5)*d
*cosh(d*x + c))*sinh(d*x + c))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^7/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[6,-20]Warning, need to choose a branch for the root of a po
lynomial with parameters. This might be wrong.The choice was done assuming
[a,b]=[89,-63]Warning, need to choose a branch for the root of a polynomial
 with parameters. This might be wrong.The choice was done assuming [a,b]=[1
2,-32]Warning, need to choose a branch for the root of a polynomial with pa
rameters. This might be wrong.The choice was done assuming [a,b]=[2,72]Unde
```

```
f/Unsigned Inf encountered in limitEvaluation time: 0.71Limit: Max order re
ached or unable to make series expansion Error: Bad Argument Value
```

maple [B] time = 0.29, size = 448, normalized size = 2.93

$$-\frac{a^2\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,b^2\left(\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a+b\right)(a+b)}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^7/(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] -1/d*a^2/b^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*
x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)/(a+b)*tanh(1/2*d*x+1/2*c)^3+1/d
*a^2/b^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/
2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)/(a+b)*tanh(1/2*d*x+1/2*c)+2/d*a^(5/
2)/b^3/(a+b)^(3/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)-2*b^(1/2))
/a^(1/2))+2/d*a^(5/2)/b^3/(a+b)^(3/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*
x+1/2*c)+2*b^(1/2))/a^(1/2))+5/2/d*a^(3/2)/b^2/(a+b)^(3/2)*arctan(1/2*(2*(a
+b)^(1/2)*tanh(1/2*d*x+1/2*c)-2*b^(1/2))/a^(1/2))+5/2/d*a^(3/2)/b^2/(a+b)^(
3/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)+2*b^(1/2))/a^(1/2))-1/d/
b^2/(tanh(1/2*d*x+1/2*c)^2+1)^2*tanh(1/2*d*x+1/2*c)^3+1/d/b^2/(tanh(1/2*d*x
+1/2*c)^2+1)^2*tanh(1/2*d*x+1/2*c)+1/d/b^2*arctan(tanh(1/2*d*x+1/2*c))-4/d/
b^3*arctan(tanh(1/2*d*x+1/2*c))*a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(2\,a^2e^{(7\,c)}+abe^{(7\,c)}\right)e^{(7\,dx)}+\left(2\,a^2e^{(5\,c)}+5\,abe^{(5\,c)}+4\,b^2e^{(5\,c)}\right)e^{(5\,dx)}-\left(2\,a^2e^{(3\,c)}+5\,ab\cdots\right.}{a^2b^2d+ab^3d+\left(a^2b^2de^{(8\,c)}+ab^3de^{(8\,c)}\right)e^{(8\,dx)}+4\left(a^2b^2de^{(6\,c)}+2\,ab^3de^{(6\,c)}+b^4de^{(6\,c)}\right)e^{(6\,dx)}+2\left(3\,a^2b^2de^{(4\,c)}+7\,a\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^7/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] ((2*a^2*e^(7*c) + a*b*e^(7*c))*e^(7*d*x) + (2*a^2*e^(5*c) + 5*a*b*e^(5*c) +
 4*b^2*e^(5*c))*e^(5*d*x) - (2*a^2*e^(3*c) + 5*a*b*e^(3*c) + 4*b^2*e^(3*c))
*e^(3*d*x) - (2*a^2*e^c + a*b*e^c)*e^(d*x))/(a^2*b^2*d + a*b^3*d + (a^2*b^2
*d*e^(8*c) + a*b^3*d*e^(8*c))*e^(8*d*x) + 4*(a^2*b^2*d*e^(6*c) + 2*a*b^3*d*
e^(6*c) + b^4*d*e^(6*c))*e^(6*d*x) + 2*(3*a^2*b^2*d*e^(4*c) + 7*a*b^3*d*e^(
4*c) + 4*b^4*d*e^(4*c))*e^(4*d*x) + 4*(a^2*b^2*d*e^(2*c) + 2*a*b^3*d*e^(2*c
) + b^4*d*e^(2*c))*e^(2*d*x)) - (4*a*e^c - b*e^c)*arctan(e^(d*x + c))*e^(-c
)/(b^3*d) + 128*integrate(1/128*((4*a^3*e^(3*c) + 5*a^2*b*e^(3*c))*e^(3*d*x
) + (4*a^3*e^c + 5*a^2*b*e^c)*e^(d*x))/(a^2*b^3 + a*b^4 + (a^2*b^3*e^(4*c)
```

```
+ a*b^4*e^(4*c))*e^(4*d*x) + 2*(a^2*b^3*e^(2*c) + 3*a*b^4*e^(2*c) + 2*b^5*e
^(2*c))*e^(2*d*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh\left(c+d\,x\right)^7\,{\left(a+\frac{b}{{\cosh\left(c+d\,x\right)}^2}\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^7*(a + b/cosh(c + d*x)^2)^2),x)
[Out] int(1/(cosh(c + d*x)^7*(a + b/cosh(c + d*x)^2)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^{7}{\left(c + dx \right)}}{\left(a + b \operatorname{sech}^{2}{\left(c + dx \right)}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**7/(a+b*sech(d*x+c)**2)**2,x)
[Out] Integral(sech(c + d*x)**7/(a + b*sech(c + d*x)**2)**2, x)
```

3.93 $\int \frac{\cosh^2(c+dx)}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=204

$$\frac{x(a-6b)}{2a^4}+\frac{b(4a+3b)(a+4b)\tanh(c+dx)}{8a^3d(a+b)^2\left(a-b\tanh^2(c+dx)+b\right)}+\frac{b(2a+3b)\tanh(c+dx)}{4a^2d(a+b)\left(a-b\tanh^2(c+dx)+b\right)^2}+\frac{b^{3/2}\left(35a^2+56ab+2\right.}{8a^4}$$

```
[Out] 1/2*(a-6*b)*x/a^4+1/8*b^(3/2)*(35*a^2+56*a*b+24*b^2)*arctanh(b^(1/2)*tanh(d
*x+c)/(a+b)^(1/2))/a^4/(a+b)^(5/2)/d+1/2*cosh(d*x+c)*sinh(d*x+c)/a/d/(a+b-b
*tanh(d*x+c)^2)^2+1/4*b*(2*a+3*b)*tanh(d*x+c)/a^2/(a+b)/d/(a+b-b*tanh(d*x+c
)^2)^2+1/8*b*(4*a+3*b)*(a+4*b)*tanh(d*x+c)/a^3/(a+b)^2/d/(a+b-b*tanh(d*x+c)
^2)
```

Rubi [A] time = 0.38, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4146, 414, 527, 522, 206, 208}

$$\frac{b^{3/2}\left(35a^2+56ab+24b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^4d(a+b)^{5/2}}+\frac{b(4a+3b)(a+4b)\tanh(c+dx)}{8a^3d(a+b)^2\left(a-b\tanh^2(c+dx)+b\right)}+\frac{b(2a+3b)\tanh}{4a^2d(a+b)\left(a-b\tanh\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^2/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((a - 6*b)*x)/(2*a^4) + (b^(3/2)*(35*a^2 + 56*a*b + 24*b^2)*ArcTanh[(Sqrt[b
]*Tanh[c + d*x])/Sqrt[a + b]])/(8*a^4*(a + b)^(5/2)*d) + (Cosh[c + d*x]*Sin
h[c + d*x])/(2*a*d*(a + b - b*Tanh[c + d*x]^2)^2) + (b*(2*a + 3*b)*Tanh[c +
 d*x])/(4*a^2*(a + b)*d*(a + b - b*Tanh[c + d*x]^2)^2) + (b*(4*a + 3*b)*(a
+ 4*b)*Tanh[c + d*x])/(8*a^3*(a + b)^2*d*(a + b - b*Tanh[c + d*x]^2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^2(a+b-bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2ad\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{\text{Subst}\left(\int \frac{a-b-5bx^2}{(1-x^2)(a+b-bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{2ad}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2ad\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{b(2a+3b)\tanh(c+dx)}{4a^2(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2} - \frac{\text{Subst}}{}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2ad\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{b(2a+3b)\tanh(c+dx)}{4a^2(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{b(4}{8a^3(a}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2ad\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{b(2a+3b)\tanh(c+dx)}{4a^2(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{b(4}{8a^3(a}$$

$$= \frac{(a-6b)x}{2a^4} + \frac{b^{3/2}\left(35a^2+56ab+24b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^4(a+b)^{5/2}d} + \frac{\cosh(c+dx)\,\text{si}}{2ad\left(a+b-b\tan\right.}$$

Mathematica [A] time = 4.23, size = 156, normalized size = 0.76

$$\frac{\frac{b^{3/2}\left(35a^2+56ab+24b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}} + a\sinh(2(c+dx))\left(\frac{2b^3(5a\cosh(2(c+dx))+3a+8b)}{(a+b)^2(a\cosh(2(c+dx))+a+2b)^2} + \frac{13ab^2}{(a+b)^2(a\cosh(2(c+dx))+a+2b)} + 2\right)}{8a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^2/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (4*(a - 6*b)*(c + d*x) + (b^(3/2)*(35*a^2 + 56*a*b + 24*b^2)*ArcTanh[(Sqrt[
b]*Tanh[c + d*x])/Sqrt[a + b]])/(a + b)^(5/2) + a*(2 + (13*a*b^2)/((a + b)^
2*(a + 2*b + a*Cosh[2*(c + d*x)])) + (2*b^3*(3*a + 8*b + 5*a*Cosh[2*(c + d*
x)]))/((a + b)^2*(a + 2*b + a*Cosh[2*(c + d*x)])^2))*Sinh[2*(c + d*x)])/(8*
a^4*d)
```

fricas [B] time = 0.66, size = 11740, normalized size = 57.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(2*(a^5 + 2*a^4*b + a^3*b^2)*cosh(d*x + c)^12 + 24*(a^5 + 2*a^4*b + a
^3*b^2)*cosh(d*x + c)*sinh(d*x + c)^11 + 2*(a^5 + 2*a^4*b + a^3*b^2)*sinh(d
*x + c)^12 + 8*(a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3 + (a^5 - 4*a^4*b - 11
*a^3*b^2 - 6*a^2*b^3)*d*x)*cosh(d*x + c)^10 + 4*(2*a^5 + 8*a^4*b + 10*a^3*b
^2 + 4*a^2*b^3 + 2*(a^5 - 4*a^4*b - 11*a^3*b^2 - 6*a^2*b^3)*d*x + 33*(a^5 +
 2*a^4*b + a^3*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 40*(11*(a^5 + 2*a^4
*b + a^3*b^2)*cosh(d*x + c)^3 + 2*(a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3 +
(a^5 - 4*a^4*b - 11*a^3*b^2 - 6*a^2*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^
9 + 2*(5*a^5 + 26*a^4*b + 27*a^3*b^2 - 32*a^2*b^3 - 32*a*b^4 + 16*(a^5 - 2*
a^4*b - 19*a^3*b^2 - 28*a^2*b^3 - 12*a*b^4)*d*x)*cosh(d*x + c)^8 + 2*(5*a^5
 + 26*a^4*b + 27*a^3*b^2 - 32*a^2*b^3 - 32*a*b^4 + 495*(a^5 + 2*a^4*b + a^3
*b^2)*cosh(d*x + c)^4 + 16*(a^5 - 2*a^4*b - 19*a^3*b^2 - 28*a^2*b^3 - 12*a*
b^4)*d*x + 180*(a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3 + (a^5 - 4*a^4*b - 11
*a^3*b^2 - 6*a^2*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 16*(99*(a^5 +
 2*a^4*b + a^3*b^2)*cosh(d*x + c)^5 + 60*(a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2
*b^3 + (a^5 - 4*a^4*b - 11*a^3*b^2 - 6*a^2*b^3)*d*x)*cosh(d*x + c)^3 + (5*a
^5 + 26*a^4*b + 27*a^3*b^2 - 32*a^2*b^3 - 32*a*b^4 + 16*(a^5 - 2*a^4*b - 19
*a^3*b^2 - 28*a^2*b^3 - 12*a*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c)^7 - 4*(
39*a^3*b^2 + 134*a^2*b^3 + 184*a*b^4 + 80*b^5 - 4*(3*a^5 - 4*a^4*b - 57*a^3
*b^2 - 138*a^2*b^3 - 136*a*b^4 - 48*b^5)*d*x)*cosh(d*x + c)^6 + 4*(462*(a^5
 + 2*a^4*b + a^3*b^2)*cosh(d*x + c)^6 - 39*a^3*b^2 - 134*a^2*b^3 - 184*a*b^
4 - 80*b^5 + 420*(a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3 + (a^5 - 4*a^4*b -
11*a^3*b^2 - 6*a^2*b^3)*d*x)*cosh(d*x + c)^4 + 4*(3*a^5 - 4*a^4*b - 57*a^3*
b^2 - 138*a^2*b^3 - 136*a*b^4 - 48*b^5)*d*x + 14*(5*a^5 + 26*a^4*b + 27*a^3
*b^2 - 32*a^2*b^3 - 32*a*b^4 + 16*(a^5 - 2*a^4*b - 19*a^3*b^2 - 28*a^2*b^3
- 12*a*b^4)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(198*(a^5 + 2*a^4*b +
 a^3*b^2)*cosh(d*x + c)^7 + 252*(a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3 + (a
^5 - 4*a^4*b - 11*a^3*b^2 - 6*a^2*b^3)*d*x)*cosh(d*x + c)^5 + 14*(5*a^5 + 2
6*a^4*b + 27*a^3*b^2 - 32*a^2*b^3 - 32*a*b^4 + 16*(a^5 - 2*a^4*b - 19*a^3*b
^2 - 28*a^2*b^3 - 12*a*b^4)*d*x)*cosh(d*x + c)^3 - 3*(39*a^3*b^2 + 134*a^2*
b^3 + 184*a*b^4 + 80*b^5 - 4*(3*a^5 - 4*a^4*b - 57*a^3*b^2 - 138*a^2*b^3 -
136*a*b^4 - 48*b^5)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*a^5 - 4*a^4*b -
 2*a^3*b^2 - 2*(5*a^5 + 26*a^4*b + 131*a^3*b^2 + 256*a^2*b^3 + 128*a*b^4 -
16*(a^5 - 2*a^4*b - 19*a^3*b^2 - 28*a^2*b^3 - 12*a*b^4)*d*x)*cosh(d*x + c)^
4 + 2*(495*(a^5 + 2*a^4*b + a^3*b^2)*cosh(d*x + c)^8 + 840*(a^5 + 4*a^4*b +
 5*a^3*b^2 + 2*a^2*b^3 + (a^5 - 4*a^4*b - 11*a^3*b^2 - 6*a^2*b^3)*d*x)*cosh
(d*x + c)^6 - 5*a^5 - 26*a^4*b - 131*a^3*b^2 - 256*a^2*b^3 - 128*a*b^4 + 70
*(5*a^5 + 26*a^4*b + 27*a^3*b^2 - 32*a^2*b^3 - 32*a*b^4 + 16*(a^5 - 2*a^4*b
 - 19*a^3*b^2 - 28*a^2*b^3 - 12*a*b^4)*d*x)*cosh(d*x + c)^4 + 16*(a^5 - 2*a
^4*b - 19*a^3*b^2 - 28*a^2*b^3 - 12*a*b^4)*d*x - 30*(39*a^3*b^2 + 134*a^2*b
^3 + 184*a*b^4 + 80*b^5 - 4*(3*a^5 - 4*a^4*b - 57*a^3*b^2 - 138*a^2*b^3 - 1
```

36*a*b^4 - 48*b^5)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(55*(a^5 + 2*a
^4*b + a^3*b^2)*cosh(d*x + c)^9 + 120*(a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^
3 + (a^5 - 4*a^4*b - 11*a^3*b^2 - 6*a^2*b^3)*d*x)*cosh(d*x + c)^7 + 14*(5*a
^5 + 26*a^4*b + 27*a^3*b^2 - 32*a^2*b^3 - 32*a*b^4 + 16*(a^5 - 2*a^4*b - 19
*a^3*b^2 - 28*a^2*b^3 - 12*a*b^4)*d*x)*cosh(d*x + c)^5 - 10*(39*a^3*b^2 + 1
34*a^2*b^3 + 184*a*b^4 + 80*b^5 - 4*(3*a^5 - 4*a^4*b - 57*a^3*b^2 - 138*a^2
*b^3 - 136*a*b^4 - 48*b^5)*d*x)*cosh(d*x + c)^3 - (5*a^5 + 26*a^4*b + 131*a
^3*b^2 + 256*a^2*b^3 + 128*a*b^4 - 16*(a^5 - 2*a^4*b - 19*a^3*b^2 - 28*a^2*
b^3 - 12*a*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 - 4*(2*a^5 + 8*a^4*b +
23*a^3*b^2 + 14*a^2*b^3 - 2*(a^5 - 4*a^4*b - 11*a^3*b^2 - 6*a^2*b^3)*d*x)*c
osh(d*x + c)^2 + 4*(33*(a^5 + 2*a^4*b + a^3*b^2)*cosh(d*x + c)^10 + 90*(a^5
 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3 + (a^5 - 4*a^4*b - 11*a^3*b^2 - 6*a^2*b^
3)*d*x)*cosh(d*x + c)^8 + 14*(5*a^5 + 26*a^4*b + 27*a^3*b^2 - 32*a^2*b^3 -
32*a*b^4 + 16*(a^5 - 2*a^4*b - 19*a^3*b^2 - 28*a^2*b^3 - 12*a*b^4)*d*x)*cos
h(d*x + c)^6 - 2*a^5 - 8*a^4*b - 23*a^3*b^2 - 14*a^2*b^3 - 15*(39*a^3*b^2 +
 134*a^2*b^3 + 184*a*b^4 + 80*b^5 - 4*(3*a^5 - 4*a^4*b - 57*a^3*b^2 - 138*a
^2*b^3 - 136*a*b^4 - 48*b^5)*d*x)*cosh(d*x + c)^4 + 2*(a^5 - 4*a^4*b - 11*a
^3*b^2 - 6*a^2*b^3)*d*x - 3*(5*a^5 + 26*a^4*b + 131*a^3*b^2 + 256*a^2*b^3 +
 128*a*b^4 - 16*(a^5 - 2*a^4*b - 19*a^3*b^2 - 28*a^2*b^3 - 12*a*b^4)*d*x)*c
osh(d*x + c)^2)*sinh(d*x + c)^2 + ((35*a^4*b + 56*a^3*b^2 + 24*a^2*b^3)*cos
h(d*x + c)^10 + 10*(35*a^4*b + 56*a^3*b^2 + 24*a^2*b^3)*cosh(d*x + c)*sinh(
d*x + c)^9 + (35*a^4*b + 56*a^3*b^2 + 24*a^2*b^3)*sinh(d*x + c)^10 + 4*(35*
a^4*b + 126*a^3*b^2 + 136*a^2*b^3 + 48*a*b^4)*cosh(d*x + c)^8 + (140*a^4*b
+ 504*a^3*b^2 + 544*a^2*b^3 + 192*a*b^4 + 45*(35*a^4*b + 56*a^3*b^2 + 24*a^
2*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 8*(15*(35*a^4*b + 56*a^3*b^2 + 24
*a^2*b^3)*cosh(d*x + c)^3 + 4*(35*a^4*b + 126*a^3*b^2 + 136*a^2*b^3 + 48*a*
b^4)*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(105*a^4*b + 448*a^3*b^2 + 800*a^2*
b^3 + 640*a*b^4 + 192*b^5)*cosh(d*x + c)^6 + 2*(105*a^4*b + 448*a^3*b^2 + 8
00*a^2*b^3 + 640*a*b^4 + 192*b^5 + 105*(35*a^4*b + 56*a^3*b^2 + 24*a^2*b^3)
*cosh(d*x + c)^4 + 56*(35*a^4*b + 126*a^3*b^2 + 136*a^2*b^3 + 48*a*b^4)*cos
h(d*x + c)^2)*sinh(d*x + c)^6 + 4*(63*(35*a^4*b + 56*a^3*b^2 + 24*a^2*b^3)*
cosh(d*x + c)^5 + 56*(35*a^4*b + 126*a^3*b^2 + 136*a^2*b^3 + 48*a*b^4)*cosh
(d*x + c)^3 + 3*(105*a^4*b + 448*a^3*b^2 + 800*a^2*b^3 + 640*a*b^4 + 192*b^
5)*cosh(d*x + c))*sinh(d*x + c)^5 + 4*(35*a^4*b + 126*a^3*b^2 + 136*a^2*b^3
 + 48*a*b^4)*cosh(d*x + c)^4 + 2*(105*(35*a^4*b + 56*a^3*b^2 + 24*a^2*b^3)*
cosh(d*x + c)^6 + 70*a^4*b + 252*a^3*b^2 + 272*a^2*b^3 + 96*a*b^4 + 140*(35
*a^4*b + 126*a^3*b^2 + 136*a^2*b^3 + 48*a*b^4)*cosh(d*x + c)^4 + 15*(105*a^
4*b + 448*a^3*b^2 + 800*a^2*b^3 + 640*a*b^4 + 192*b^5)*cosh(d*x + c)^2)*sin
h(d*x + c)^4 + 8*(15*(35*a^4*b + 56*a^3*b^2 + 24*a^2*b^3)*cosh(d*x + c)^7 +
 28*(35*a^4*b + 126*a^3*b^2 + 136*a^2*b^3 + 48*a*b^4)*cosh(d*x + c)^5 + 5*(
105*a^4*b + 448*a^3*b^2 + 800*a^2*b^3 + 640*a*b^4 + 192*b^5)*cosh(d*x + c)^
3 + 2*(35*a^4*b + 126*a^3*b^2 + 136*a^2*b^3 + 48*a*b^4)*cosh(d*x + c))*sinh
(d*x + c)^3 + (35*a^4*b + 56*a^3*b^2 + 24*a^2*b^3)*cosh(d*x + c)^2 + (45*(3
5*a^4*b + 56*a^3*b^2 + 24*a^2*b^3)*cosh(d*x + c)^8 + 112*(35*a^4*b + 126*a^
3*b^2 + 136*a^2*b^3 + 48*a*b^4)*cosh(d*x + c)^6 + 35*a^4*b + 56*a^3*b^2 + 2

4*a^2*b^3 + 30*(105*a^4*b + 448*a^3*b^2 + 800*a^2*b^3 + 640*a*b^4 + 192*b^5
)*cosh(d*x + c)^4 + 24*(35*a^4*b + 126*a^3*b^2 + 136*a^2*b^3 + 48*a*b^4)*co
sh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(5*(35*a^4*b + 56*a^3*b^2 + 24*a^2*b^3)*
cosh(d*x + c)^9 + 16*(35*a^4*b + 126*a^3*b^2 + 136*a^2*b^3 + 48*a*b^4)*cosh
(d*x + c)^7 + 6*(105*a^4*b + 448*a^3*b^2 + 800*a^2*b^3 + 640*a*b^4 + 192*b^
5)*cosh(d*x + c)^5 + 8*(35*a^4*b + 126*a^3*b^2 + 136*a^2*b^3 + 48*a*b^4)*co
sh(d*x + c)^3 + (35*a^4*b + 56*a^3*b^2 + 24*a^2*b^3)*cosh(d*x + c))*sinh(d*
x + c))*sqrt(b/(a + b))*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh
(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*
a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 +
4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) - 4*((a
^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^
2 + a*b)*sinh(d*x + c)^2 + a^2 + 3*a*b + 2*b^2)*sqrt(b/(a + b)))/(a*cosh(d*
x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2
*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4
*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) + 8*(3*(
a^5 + 2*a^4*b + a^3*b^2)*cosh(d*x + c)^11 + 10*(a^5 + 4*a^4*b + 5*a^3*b^2 +
 2*a^2*b^3 + (a^5 - 4*a^4*b - 11*a^3*b^2 - 6*a^2*b^3)*d*x)*cosh(d*x + c)^9
+ 2*(5*a^5 + 26*a^4*b + 27*a^3*b^2 - 32*a^2*b^3 - 32*a*b^4 + 16*(a^5 - 2*a^
4*b - 19*a^3*b^2 - 28*a^2*b^3 - 12*a*b^4)*d*x)*cosh(d*x + c)^7 - 3*(39*a^3*
b^2 + 134*a^2*b^3 + 184*a*b^4 + 80*b^5 - 4*(3*a^5 - 4*a^4*b - 57*a^3*b^2 -
138*a^2*b^3 - 136*a*b^4 - 48*b^5)*d*x)*cosh(d*x + c)^5 - (5*a^5 + 26*a^4*b
+ 131*a^3*b^2 + 256*a^2*b^3 + 128*a*b^4 - 16*(a^5 - 2*a^4*b - 19*a^3*b^2 -
28*a^2*b^3 - 12*a*b^4)*d*x)*cosh(d*x + c)^3 - (2*a^5 + 8*a^4*b + 23*a^3*b^2
 + 14*a^2*b^3 - 2*(a^5 - 4*a^4*b - 11*a^3*b^2 - 6*a^2*b^3)*d*x)*cosh(d*x +
c))*sinh(d*x + c))/((a^8 + 2*a^7*b + a^6*b^2)*d*cosh(d*x + c)^10 + 10*(a^8
+ 2*a^7*b + a^6*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^9 + (a^8 + 2*a^7*b + a^6
*b^2)*d*sinh(d*x + c)^10 + 4*(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*b^3)*d*cosh
(d*x + c)^8 + (45*(a^8 + 2*a^7*b + a^6*b^2)*d*cosh(d*x + c)^2 + 4*(a^8 + 4*
a^7*b + 5*a^6*b^2 + 2*a^5*b^3)*d)*sinh(d*x + c)^8 + 2*(3*a^8 + 14*a^7*b + 2
7*a^6*b^2 + 24*a^5*b^3 + 8*a^4*b^4)*d*cosh(d*x + c)^6 + 8*(15*(a^8 + 2*a^7*
b + a^6*b^2)*d*cosh(d*x + c)^3 + 4*(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*b^3)*
d*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(105*(a^8 + 2*a^7*b + a^6*b^2)*d*cosh(
d*x + c)^4 + 56*(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*b^3)*d*cosh(d*x + c)^2 +
 (3*a^8 + 14*a^7*b + 27*a^6*b^2 + 24*a^5*b^3 + 8*a^4*b^4)*d)*sinh(d*x + c)^
6 + 4*(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*b^3)*d*cosh(d*x + c)^4 + 4*(63*(a^
8 + 2*a^7*b + a^6*b^2)*d*cosh(d*x + c)^5 + 56*(a^8 + 4*a^7*b + 5*a^6*b^2 +
2*a^5*b^3)*d*cosh(d*x + c)^3 + 3*(3*a^8 + 14*a^7*b + 27*a^6*b^2 + 24*a^5*b^
3 + 8*a^4*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(105*(a^8 + 2*a^7*b + a
^6*b^2)*d*cosh(d*x + c)^6 + 140*(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*b^3)*d*c
osh(d*x + c)^4 + 15*(3*a^8 + 14*a^7*b + 27*a^6*b^2 + 24*a^5*b^3 + 8*a^4*b^4
)*d*cosh(d*x + c)^2 + 2*(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*b^3)*d)*sinh(d*x
 + c)^4 + (a^8 + 2*a^7*b + a^6*b^2)*d*cosh(d*x + c)^2 + 8*(15*(a^8 + 2*a^7*
b + a^6*b^2)*d*cosh(d*x + c)^7 + 28*(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*b^3)
*d*cosh(d*x + c)^5 + 5*(3*a^8 + 14*a^7*b + 27*a^6*b^2 + 24*a^5*b^3 + 8*a^4*

b^4)*d*cosh(d*x + c)^3 + 2*(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*b^3)*d*cosh(d
*x + c))*sinh(d*x + c)^3 + (45*(a^8 + 2*a^7*b + a^6*b^2)*d*cosh(d*x + c)^8
+ 112*(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*b^3)*d*cosh(d*x + c)^6 + 30*(3*a^8
 + 14*a^7*b + 27*a^6*b^2 + 24*a^5*b^3 + 8*a^4*b^4)*d*cosh(d*x + c)^4 + 24*(
a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*b^3)*d*cosh(d*x + c)^2 + (a^8 + 2*a^7*b +
 a^6*b^2)*d)*sinh(d*x + c)^2 + 2*(5*(a^8 + 2*a^7*b + a^6*b^2)*d*cosh(d*x +
c)^9 + 16*(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*b^3)*d*cosh(d*x + c)^7 + 6*(3*
a^8 + 14*a^7*b + 27*a^6*b^2 + 24*a^5*b^3 + 8*a^4*b^4)*d*cosh(d*x + c)^5 + 8
*(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*b^3)*d*cosh(d*x + c)^3 + (a^8 + 2*a^7*b
 + a^6*b^2)*d*cosh(d*x + c))*sinh(d*x + c)), 1/8*((a^5 + 2*a^4*b + a^3*b^2)
*cosh(d*x + c)^12 + 12*(a^5 + 2*a^4*b + a^3*b^2)*cosh(d*x + c)*sinh(d*x + c
)^11 + (a^5 + 2*a^4*b + a^3*b^2)*sinh(d*x + c)^12 + 4*(a^5 + 4*a^4*b + 5*a^
3*b^2 + 2*a^2*b^3 + (a^5 - 4*a^4*b - 11*a^3*b^2 - 6*a^2*b^3)*d*x)*cosh(d*x
+ c)^10 + 2*(2*a^5 + 8*a^4*b + 10*a^3*b^2 + 4*a^2*b^3 + 2*(a^5 - 4*a^4*b -
11*a^3*b^2 - 6*a^2*b^3)*d*x + 33*(a^5 + 2*a^4*b + a^3*b^2)*cosh(d*x + c)^2)
*sinh(d*x + c)^10 + 20*(11*(a^5 + 2*a^4*b + a^3*b^2)*cosh(d*x + c)^3 + 2*(a
^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3 + (a^5 - 4*a^4*b - 11*a^3*b^2 - 6*a^2*
b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^9 + (5*a^5 + 26*a^4*b + 27*a^3*b^2 -
 32*a^2*b^3 - 32*a*b^4 + 16*(a^5 - 2*a^4*b - 19*a^3*b^2 - 28*a^2*b^3 - 12*a
*b^4)*d*x)*cosh(d*x + c)^8 + (5*a^5 + 26*a^4*b + 27*a^3*b^2 - 32*a^2*b^3 -
32*a*b^4 + 495*(a^5 + 2*a^4*b + a^3*b^2)*cosh(d*x + c)^4 + 16*(a^5 - 2*a^4*
b - 19*a^3*b^2 - 28*a^2*b^3 - 12*a*b^4)*d*x + 180*(a^5 + 4*a^4*b + 5*a^3*b^
2 + 2*a^2*b^3 + (a^5 - 4*a^4*b - 11*a^3*b^2 - 6*a^2*b^3)*d*x)*cosh(d*x + c)
^2)*sinh(d*x + c)^8 + 8*(99*(a^5 + 2*a^4*b + a^3*b^2)*cosh(d*x + c)^5 + 60*
(a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3 + (a^5 - 4*a^4*b - 11*a^3*b^2 - 6*a^
2*b^3)*d*x)*cosh(d*x + c)^3 + (5*a^5 + 26*a^4*b + 27*a^3*b^2 - 32*a^2*b^3 -
 32*a*b^4 + 16*(a^5 - 2*a^4*b - 19*a^3*b^2 - 28*a^2*b^3 - 12*a*b^4)*d*x)*co
sh(d*x + c))*sinh(d*x + c)^7 - 2*(39*a^3*b^2 + 134*a^2*b^3 + 184*a*b^4 + 80
b^5 - 4(3*a^5 - 4*a^4*b - 57*a^3*b^2 - 138*a^2*b^3 - 136*a*b^4 - 48*b^5)*
d*x)*cosh(d*x + c)^6 + 2*(462*(a^5 + 2*a^4*b + a^3*b^2)*cosh(d*x + c)^6 - 3
9*a^3*b^2 - 134*a^2*b^3 - 184*a*b^4 - 80*b^5 + 420*(a^5 + 4*a^4*b + 5*a^3*b
^2 + 2*a^2*b^3 + (a^5 - 4*a^4*b - 11*a^3*b^2 - 6*a^2*b^3)*d*x)*cosh(d*x + c
)^4 + 4*(3*a^5 - 4*a^4*b - 57*a^3*b^2 - 138*a^2*b^3 - 136*a*b^4 - 48*b^5)*d
x + 14(5*a^5 + 26*a^4*b + 27*a^3*b^2 - 32*a^2*b^3 - 32*a*b^4 + 16*(a^5 -
2*a^4*b - 19*a^3*b^2 - 28*a^2*b^3 - 12*a*b^4)*d*x)*cosh(d*x + c)^2)*sinh(d*
x + c)^6 + 4*(198*(a^5 + 2*a^4*b + a^3*b^2)*cosh(d*x + c)^7 + 252*(a^5 + 4*
a^4*b + 5*a^3*b^2 + 2*a^2*b^3 + (a^5 - 4*a^4*b - 11*a^3*b^2 - 6*a^2*b^3)*d*
x)*cosh(d*x + c)^5 + 14*(5*a^5 + 26*a^4*b + 27*a^3*b^2 - 32*a^2*b^3 - 32*a*
b^4 + 16*(a^5 - 2*a^4*b - 19*a^3*b^2 - 28*a^2*b^3 - 12*a*b^4)*d*x)*cosh(d*x
 + c)^3 - 3*(39*a^3*b^2 + 134*a^2*b^3 + 184*a*b^4 + 80*b^5 - 4*(3*a^5 - 4*a
^4*b - 57*a^3*b^2 - 138*a^2*b^3 - 136*a*b^4 - 48*b^5)*d*x)*cosh(d*x + c))*s
inh(d*x + c)^5 - a^5 - 2*a^4*b - a^3*b^2 - (5*a^5 + 26*a^4*b + 131*a^3*b^2
+ 256*a^2*b^3 + 128*a*b^4 - 16*(a^5 - 2*a^4*b - 19*a^3*b^2 - 28*a^2*b^3 - 1
2*a*b^4)*d*x)*cosh(d*x + c)^4 + (495*(a^5 + 2*a^4*b + a^3*b^2)*cosh(d*x + c
)^8 + 840*(a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3 + (a^5 - 4*a^4*b - 11*a^3*

b^2 - 6*a^2*b^3)*d*x)*cosh(d*x + c)^6 - 5*a^5 - 26*a^4*b - 131*a^3*b^2 - 25
6*a^2*b^3 - 128*a*b^4 + 70*(5*a^5 + 26*a^4*b + 27*a^3*b^2 - 32*a^2*b^3 - 32
*a*b^4 + 16*(a^5 - 2*a^4*b - 19*a^3*b^2 - 28*a^2*b^3 - 12*a*b^4)*d*x)*cosh(
d*x + c)^4 + 16*(a^5 - 2*a^4*b - 19*a^3*b^2 - 28*a^2*b^3 - 12*a*b^4)*d*x -
30*(39*a^3*b^2 + 134*a^2*b^3 + 184*a*b^4 + 80*b^5 - 4*(3*a^5 - 4*a^4*b - 57
*a^3*b^2 - 138*a^2*b^3 - 136*a*b^4 - 48*b^5)*d*x)*cosh(d*x + c)^2)*sinh(d*x
 + c)^4 + 4*(55*(a^5 + 2*a^4*b + a^3*b^2)*cosh(d*x + c)^9 + 120*(a^5 + 4*a^
4*b + 5*a^3*b^2 + 2*a^2*b^3 + (a^5 - 4*a^4*b - 11*a^3*b^2 - 6*a^2*b^3)*d*x)
*cosh(d*x + c)^7 + 14*(5*a^5 + 26*a^4*b + 27*a^3*b^2 - 32*a^2*b^3 - 32*a*b^
4 + 16*(a^5 - 2*a^4*b - 19*a^3*b^2 - 28*a^2*b^3 - 12*a*b^4)*d*x)*cosh(d*x +
 c)^5 - 10*(39*a^3*b^2 + 134*a^2*b^3 + 184*a*b^4 + 80*b^5 - 4*(3*a^5 - 4*a^
4*b - 57*a^3*b^2 - 138*a^2*b^3 - 136*a*b^4 - 48*b^5)*d*x)*cosh(d*x + c)^3 -
 (5*a^5 + 26*a^4*b + 131*a^3*b^2 + 256*a^2*b^3 + 128*a*b^4 - 16*(a^5 - 2*a^
4*b - 19*a^3*b^2 - 28*a^2*b^3 - 12*a*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c)
^3 - 2*(2*a^5 + 8*a^4*b + 23*a^3*b^2 + 14*a^2*b^3 - 2*(a^5 - 4*a^4*b - 11*a
^3*b^2 - 6*a^2*b^3)*d*x)*cosh(d*x + c)^2 + 2*(33*(a^5 + 2*a^4*b + a^3*b^2)*
cosh(d*x + c)^10 + 90*(a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3 + (a^5 - 4*a^4
*b - 11*a^3*b^2 - 6*a^2*b^3)*d*x)*cosh(d*x + c)^8 + 14*(5*a^5 + 26*a^4*b +
27*a^3*b^2 - 32*a^2*b^3 - 32*a*b^4 + 16*(a^5 - 2*a^4*b - 19*a^3*b^2 - 28*a^
2*b^3 - 12*a*b^4)*d*x)*cosh(d*x + c)^6 - 2*a^5 - 8*a^4*b - 23*a^3*b^2 - 14*
a^2*b^3 - 15*(39*a^3*b^2 + 134*a^2*b^3 + 184*a*b^4 + 80*b^5 - 4*(3*a^5 - 4*
a^4*b - 57*a^3*b^2 - 138*a^2*b^3 - 136*a*b^4 - 48*b^5)*d*x)*cosh(d*x + c)^4
 + 2*(a^5 - 4*a^4*b - 11*a^3*b^2 - 6*a^2*b^3)*d*x - 3*(5*a^5 + 26*a^4*b + 1
31*a^3*b^2 + 256*a^2*b^3 + 128*a*b^4 - 16*(a^5 - 2*a^4*b - 19*a^3*b^2 - 28*
a^2*b^3 - 12*a*b^4)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((35*a^4*b + 56
*a^3*b^2 + 24*a^2*b^3)*cosh(d*x + c)^10 + 10*(35*a^4*b + 56*a^3*b^2 + 24*a^
2*b^3)*cosh(d*x + c)*sinh(d*x + c)^9 + (35*a^4*b + 56*a^3*b^2 + 24*a^2*b^3)
*sinh(d*x + c)^10 + 4*(35*a^4*b + 126*a^3*b^2 + 136*a^2*b^3 + 48*a*b^4)*cos
h(d*x + c)^8 + (140*a^4*b + 504*a^3*b^2 + 544*a^2*b^3 + 192*a*b^4 + 45*(35*
a^4*b + 56*a^3*b^2 + 24*a^2*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 8*(15*(
35*a^4*b + 56*a^3*b^2 + 24*a^2*b^3)*cosh(d*x + c)^3 + 4*(35*a^4*b + 126*a^3
*b^2 + 136*a^2*b^3 + 48*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(105*a^4*
b + 448*a^3*b^2 + 800*a^2*b^3 + 640*a*b^4 + 192*b^5)*cosh(d*x + c)^6 + 2*(1
05*a^4*b + 448*a^3*b^2 + 800*a^2*b^3 + 640*a*b^4 + 192*b^5 + 105*(35*a^4*b
+ 56*a^3*b^2 + 24*a^2*b^3)*cosh(d*x + c)^4 + 56*(35*a^4*b + 126*a^3*b^2 + 1
36*a^2*b^3 + 48*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(63*(35*a^4*b +
 56*a^3*b^2 + 24*a^2*b^3)*cosh(d*x + c)^5 + 56*(35*a^4*b + 126*a^3*b^2 + 13
6*a^2*b^3 + 48*a*b^4)*cosh(d*x + c)^3 + 3*(105*a^4*b + 448*a^3*b^2 + 800*a^
2*b^3 + 640*a*b^4 + 192*b^5)*cosh(d*x + c))*sinh(d*x + c)^5 + 4*(35*a^4*b +
 126*a^3*b^2 + 136*a^2*b^3 + 48*a*b^4)*cosh(d*x + c)^4 + 2*(105*(35*a^4*b +
 56*a^3*b^2 + 24*a^2*b^3)*cosh(d*x + c)^6 + 70*a^4*b + 252*a^3*b^2 + 272*a^
2*b^3 + 96*a*b^4 + 140*(35*a^4*b + 126*a^3*b^2 + 136*a^2*b^3 + 48*a*b^4)*co
sh(d*x + c)^4 + 15*(105*a^4*b + 448*a^3*b^2 + 800*a^2*b^3 + 640*a*b^4 + 192
*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(15*(35*a^4*b + 56*a^3*b^2 + 24*
a^2*b^3)*cosh(d*x + c)^7 + 28*(35*a^4*b + 126*a^3*b^2 + 136*a^2*b^3 + 48*a*

b^4)*cosh(d*x + c)^5 + 5*(105*a^4*b + 448*a^3*b^2 + 800*a^2*b^3 + 640*a*b^4 + 192*b^5)*cosh(d*x + c)^3 + 2*(35*a^4*b + 126*a^3*b^2 + 136*a^2*b^3 + 48*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + (35*a^4*b + 56*a^3*b^2 + 24*a^2*b^3)*cosh(d*x + c)^2 + (45*(35*a^4*b + 56*a^3*b^2 + 24*a^2*b^3)*cosh(d*x + c)^8 + 112*(35*a^4*b + 126*a^3*b^2 + 136*a^2*b^3 + 48*a*b^4)*cosh(d*x + c)^6 + 35*a^4*b + 56*a^3*b^2 + 24*a^2*b^3 + 30*(105*a^4*b + 448*a^3*b^2 + 800*a^2*b^3 + 640*a*b^4 + 192*b^5)*cosh(d*x + c)^4 + 24*(35*a^4*b + 126*a^3*b^2 + 136*a^2*b^3 + 48*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(5*(35*a^4*b + 56*a^3*b^2 + 24*a^2*b^3)*cosh(d*x + c)^9 + 16*(35*a^4*b + 126*a^3*b^2 + 136*a^2*b^3 + 48*a*b^4)*cosh(d*x + c)^7 + 6*(105*a^4*b + 448*a^3*b^2 + 800*a^2*b^3 + 640*a*b^4 + 192*b^5)*cosh(d*x + c)^5 + 8*(35*a^4*b + 126*a^3*b^2 + 136*a^2*b^3 + 48*a*b^4)*cosh(d*x + c)^3 + (35*a^4*b + 56*a^3*b^2 + 24*a^2*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b/(a + b))*arctan(1/2*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-b/(a + b))/b) + 4*(3*(a^5 + 2*a^4*b + a^3*b^2)*cosh(d*x + c)^11 + 10*(a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3 + (a^5 - 4*a^4*b - 11*a^3*b^2 - 6*a^2*b^3)*d*x)*cosh(d*x + c)^9 + 2*(5*a^5 + 26*a^4*b + 27*a^3*b^2 - 32*a^2*b^3 - 32*a*b^4 + 16*(a^5 - 2*a^4*b - 19*a^3*b^2 - 28*a^2*b^3 - 12*a*b^4)*d*x)*cosh(d*x + c)^7 - 3*(39*a^3*b^2 + 134*a^2*b^3 + 184*a*b^4 + 80*b^5 - 4*(3*a^5 - 4*a^4*b - 57*a^3*b^2 - 138*a^2*b^3 - 136*a*b^4 - 48*b^5)*d*x)*cosh(d*x + c)^5 - (5*a^5 + 26*a^4*b + 131*a^3*b^2 + 256*a^2*b^3 + 128*a*b^4 - 16*(a^5 - 2*a^4*b - 19*a^3*b^2 - 28*a^2*b^3 - 12*a*b^4)*d*x)*cosh(d*x + c)^3 - (2*a^5 + 8*a^4*b + 23*a^3*b^2 + 14*a^2*b^3 - 2*(a^5 - 4*a^4*b - 11*a^3*b^2 - 6*a^2*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a^8 + 2*a^7*b + a^6*b^2)*d*cosh(d*x + c)^10 + 10*(a^8 + 2*a^7*b + a^6*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^9 + (a^8 + 2*a^7*b + a^6*b^2)*d*sinh(d*x + c)^10 + 4*(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*b^3)*d*cosh(d*x + c)^8 + (45*(a^8 + 2*a^7*b + a^6*b^2)*d*cosh(d*x + c)^2 + 4*(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*b^3)*d)*sinh(d*x + c)^8 + 2*(3*a^8 + 14*a^7*b + 27*a^6*b^2 + 24*a^5*b^3 + 8*a^4*b^4)*d*cosh(d*x + c)^6 + 8*(15*(a^8 + 2*a^7*b + a^6*b^2)*d*cosh(d*x + c)^3 + 4*(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(105*(a^8 + 2*a^7*b + a^6*b^2)*d*cosh(d*x + c)^4 + 56*(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*b^3)*d*cosh(d*x + c)^2 + (3*a^8 + 14*a^7*b + 27*a^6*b^2 + 24*a^5*b^3 + 8*a^4*b^4)*d)*sinh(d*x + c)^6 + 4*(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*b^3)*d*cosh(d*x + c)^4 + 4*(63*(a^8 + 2*a^7*b + a^6*b^2)*d*cosh(d*x + c)^5 + 56*(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*b^3)*d*cosh(d*x + c)^3 + 3*(3*a^8 + 14*a^7*b + 27*a^6*b^2 + 24*a^5*b^3 + 8*a^4*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(105*(a^8 + 2*a^7*b + a^6*b^2)*d*cosh(d*x + c)^6 + 140*(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*b^3)*d*cosh(d*x + c)^4 + 15*(3*a^8 + 14*a^7*b + 27*a^6*b^2 + 24*a^5*b^3 + 8*a^4*b^4)*d*cosh(d*x + c)^2 + 2*(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*b^3)*d)*sinh(d*x + c)^4 + (a^8 + 2*a^7*b + a^6*b^2)*d*cosh(d*x + c)^2 + 8*(15*(a^8 + 2*a^7*b + a^6*b^2)*d*cosh(d*x + c)^7 + 28*(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*b^3)*d*cosh(d*x + c)^5 + 5*(3*a^8 + 14*a^7*b + 27*a^6*b^2 + 24*a^5*b^3 + 8*a^4*b^4)*d*cosh(d*x + c)^3 + 2*(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^3 + (45*(a^8 + 2*a^7*b + a^6

```
*b^2)*d*cosh(d*x + c)^8 + 112*(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*b^3)*d*cos
h(d*x + c)^6 + 30*(3*a^8 + 14*a^7*b + 27*a^6*b^2 + 24*a^5*b^3 + 8*a^4*b^4)*
d*cosh(d*x + c)^4 + 24*(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*b^3)*d*cosh(d*x +
 c)^2 + (a^8 + 2*a^7*b + a^6*b^2)*d)*sinh(d*x + c)^2 + 2*(5*(a^8 + 2*a^7*b
+ a^6*b^2)*d*cosh(d*x + c)^9 + 16*(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*b^3)*d
*cosh(d*x + c)^7 + 6*(3*a^8 + 14*a^7*b + 27*a^6*b^2 + 24*a^5*b^3 + 8*a^4*b^
4)*d*cosh(d*x + c)^5 + 8*(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*b^3)*d*cosh(d*x
 + c)^3 + (a^8 + 2*a^7*b + a^6*b^2)*d*cosh(d*x + c))*sinh(d*x + c))]
```

giac [B] time = 3.67, size = 395, normalized size = 1.94

$$\frac{\left(35\,a^2b^2+56\,ab^3+24\,b^4\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{\left(a^6+2\,a^5b+a^4b^2\right)\sqrt{-ab-b^2}} - \frac{2\left(13\,a^3b^2e^{(6\,dx+6\,c)}+40\,a^2b^3e^{(6\,dx+6\,c)}+24\,ab^4e^{(6\,dx+6\,c)}+39\,a^3b^2e^{(4\,dx+4\,c)}+134\,a^2b^3e^{(4\,dx+4\,c)}+1\cdots\right.}{\left(a^6+2\,a^5b+a^4b^2\right)\left(ae^{(4\,dx+\cdots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*((35*a^2*b^2 + 56*a*b^3 + 24*b^4)*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2
*b)/sqrt(-a*b - b^2))/((a^6 + 2*a^5*b + a^4*b^2)*sqrt(-a*b - b^2)) - 2*(13*
a^3*b^2*e^(6*d*x + 6*c) + 40*a^2*b^3*e^(6*d*x + 6*c) + 24*a*b^4*e^(6*d*x +
6*c) + 39*a^3*b^2*e^(4*d*x + 4*c) + 134*a^2*b^3*e^(4*d*x + 4*c) + 184*a*b^4
*e^(4*d*x + 4*c) + 80*b^5*e^(4*d*x + 4*c) + 39*a^3*b^2*e^(2*d*x + 2*c) + 10
4*a^2*b^3*e^(2*d*x + 2*c) + 56*a*b^4*e^(2*d*x + 2*c) + 13*a^3*b^2 + 10*a^2*
b^3)/((a^6 + 2*a^5*b + a^4*b^2)*(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) +
4*b*e^(2*d*x + 2*c) + a)^2) + 4*(d*x + c)*(a - 6*b)/a^4 + e^(2*d*x + 2*c)/a
^3 - (2*a*e^(2*d*x + 2*c) - 12*b*e^(2*d*x + 2*c) + a)*e^(-2*d*x - 2*c)/a^4)
/d
```

maple [B] time = 0.59, size = 1435, normalized size = 7.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] 1/2/d/a^3/(tanh(1/2*d*x+1/2*c)-1)^2+1/2/d/a^3/(tanh(1/2*d*x+1/2*c)-1)-1/2/d
/a^3*ln(tanh(1/2*d*x+1/2*c)-1)+3/d/a^4*ln(tanh(1/2*d*x+1/2*c)-1)*b-1/2/d/a^
3/(tanh(1/2*d*x+1/2*c)+1)^2+1/2/d/a^3/(tanh(1/2*d*x+1/2*c)+1)+1/2/d/a^3*ln(
tanh(1/2*d*x+1/2*c)+1)-3/d/a^4*ln(tanh(1/2*d*x+1/2*c)+1)*b+13/4/d/a^2*b^2/(
tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2
*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)^7+2/d/a^3*b^3/(ta
nh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*t
anh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)^7+39/4/d/a*b^2/(tan
```

```
h(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*ta
nh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^5+19/4/d/a^2*b^3/(
tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2
*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^5-2/d/a^3*b^4/(
tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2
*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^5+39/4/d/a*b^2/
(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-
2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^3+19/4/d/a^2*b
^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2
*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^3-2/d/a^3*b
^4/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2
*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^3+13/4/d/a^
2*b^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c
)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)+2/d/a^3*b^
3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*
a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)-35/16/d/a^2*b^
(3/2)/(a^2+2*a*b+b^2)/(a+b)^(1/2)*ln(-(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b
^(1/2)*tanh(1/2*d*x+1/2*c)-(a+b)^(1/2))-7/2/d/a^3*b^(5/2)/(a^2+2*a*b+b^2)/(
a+b)^(1/2)*ln(-(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2
*c)-(a+b)^(1/2))-3/2/d/a^4*b^(7/2)/(a^2+2*a*b+b^2)/(a+b)^(1/2)*ln(-(a+b)^(1
/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)-(a+b)^(1/2))+35/16/
d/a^2*b^(3/2)/(a^2+2*a*b+b^2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c
)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))+7/2/d/a^3*b^(5/2)/(a^2+2*a*b
+b^2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d
*x+1/2*c)+(a+b)^(1/2))+3/2/d/a^4*b^(7/2)/(a^2+2*a*b+b^2)/(a+b)^(1/2)*ln((a+
b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))
```

maxima [B] time = 0.55, size = 1373, normalized size = 6.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 3/64*(5*a^3*b + 30*a^2*b^2 + 40*a*b^3 + 16*b^4)*log((a*e^(2*d*x + 2*c) + a
+ 2*b - 2*sqrt((a + b)*b))/(a*e^(2*d*x + 2*c) + a + 2*b + 2*sqrt((a + b)*b)
))/((a^6 + 2*a^5*b + a^4*b^2)*sqrt((a + b)*b)*d) - 3/64*(5*a^3*b + 30*a^2*b
^2 + 40*a*b^3 + 16*b^4)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*
b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^6 + 2*a^5*b + a
^4*b^2)*sqrt((a + b)*b)*d) + 1/32*(15*a^2*b + 20*a*b^2 + 8*b^3)*log((a*e^(-
2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b +
 2*sqrt((a + b)*b)))/((a^5 + 2*a^4*b + a^3*b^2)*sqrt((a + b)*b)*d) - 1/16*(
9*a^4*b + 32*a^3*b^2 + 20*a^2*b^3 + 3*(3*a^4*b + 34*a^3*b^2 + 64*a^2*b^3 +
32*a*b^4)*e^(6*d*x + 6*c) + (27*a^4*b + 264*a^3*b^2 + 740*a^2*b^3 + 832*a*b
^4 + 320*b^5)*e^(4*d*x + 4*c) + (27*a^4*b + 194*a^3*b^2 + 336*a^2*b^3 + 160
```

```
*a*b^4)*e^(2*d*x + 2*c))/((a^8 + 2*a^7*b + a^6*b^2 + (a^8 + 2*a^7*b + a^6*b
^2)*e^(8*d*x + 8*c) + 4*(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*b^3)*e^(6*d*x +
6*c) + 2*(3*a^8 + 14*a^7*b + 27*a^6*b^2 + 24*a^5*b^3 + 8*a^4*b^4)*e^(4*d*x
+ 4*c) + 4*(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*b^3)*e^(2*d*x + 2*c))*d) + 1/
16*(9*a^4*b + 32*a^3*b^2 + 20*a^2*b^3 + (27*a^4*b + 194*a^3*b^2 + 336*a^2*b
^3 + 160*a*b^4)*e^(-2*d*x - 2*c) + (27*a^4*b + 264*a^3*b^2 + 740*a^2*b^3 +
832*a*b^4 + 320*b^5)*e^(-4*d*x - 4*c) + 3*(3*a^4*b + 34*a^3*b^2 + 64*a^2*b^
3 + 32*a*b^4)*e^(-6*d*x - 6*c))/((a^8 + 2*a^7*b + a^6*b^2 + 4*(a^8 + 4*a^7*
b + 5*a^6*b^2 + 2*a^5*b^3)*e^(-2*d*x - 2*c) + 2*(3*a^8 + 14*a^7*b + 27*a^6*
b^2 + 24*a^5*b^3 + 8*a^4*b^4)*e^(-4*d*x - 4*c) + 4*(a^8 + 4*a^7*b + 5*a^6*b
^2 + 2*a^5*b^3)*e^(-6*d*x - 6*c) + (a^8 + 2*a^7*b + a^6*b^2)*e^(-8*d*x - 8*
c))*d) - 1/8*(9*a^3*b + 6*a^2*b^2 + (27*a^3*b + 68*a^2*b^2 + 32*a*b^3)*e^(-
2*d*x - 2*c) + 3*(9*a^3*b + 30*a^2*b^2 + 40*a*b^3 + 16*b^4)*e^(-4*d*x - 4*c
) + (9*a^3*b + 28*a^2*b^2 + 16*a*b^3)*e^(-6*d*x - 6*c))/((a^7 + 2*a^6*b + a
^5*b^2 + 4*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*e^(-2*d*x - 2*c) + 2*(3*
a^7 + 14*a^6*b + 27*a^5*b^2 + 24*a^4*b^3 + 8*a^3*b^4)*e^(-4*d*x - 4*c) + 4*
(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*e^(-6*d*x - 6*c) + (a^7 + 2*a^6*b +
 a^5*b^2)*e^(-8*d*x - 8*c))*d) + 1/2*(d*x + c)/(a^3*d) + 1/8*e^(2*d*x + 2*c
)/(a^3*d) - 1/8*e^(-2*d*x - 2*c)/(a^3*d) - 3/4*b*log(a*e^(4*d*x + 4*c) + 2*
(a + 2*b)*e^(2*d*x + 2*c) + a)/(a^4*d) + 3/4*b*log(2*(a + 2*b)*e^(-2*d*x -
2*c) + a*e^(-4*d*x - 4*c) + a)/(a^4*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh\left(c+d\,x\right)^2}{{\left(a+\frac{b}{\cosh\left(c+d\,x\right)^2}\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^2/(a + b/cosh(c + d*x)^2)^3,x)
[Out] int(cosh(c + d*x)^2/(a + b/cosh(c + d*x)^2)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2/(a+b*sech(d*x+c)**2)**3,x)
[Out] Timed out
```

3.94 $\int \frac{\cosh(c+dx)}{\left(a+b\,\mathbf{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=154

$$-\frac{3b\left(4(a+b)^2+(2a+b)^2\right)\tan^{-1}\left(\frac{\sqrt{a}\,\sinh(c+dx)}{\sqrt{a+b}}\right)}{8a^{7/2}d(a+b)^{5/2}}-\frac{b^3\sinh(c+dx)}{4a^3d(a+b)\left(a\sinh^2(c+dx)+a+b\right)^2}+\frac{3b^2(4a+3b)\sinh(c+dx)}{8a^3d(a+b)^2\left(a\sinh^2(c+dx)+a+b\right)}$$

```
[Out] -3/8*b*(4*(a+b)^2+(2*a+b)^2)*arctan(sinh(d*x+c)*a^(1/2)/(a+b)^(1/2))/a^(7/2
)/(a+b)^(5/2)/d+sinh(d*x+c)/a^3/d-1/4*b^3*sinh(d*x+c)/a^3/(a+b)/d/(a+b+a*si
nh(d*x+c)^2)^2+3/8*b^2*(4*a+3*b)*sinh(d*x+c)/a^3/(a+b)^2/d/(a+b+a*sinh(d*x+
c)^2)
```

Rubi [A] time = 0.18, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4147, 390, 1157, 385, 205}

$$-\frac{b^3\sinh(c+dx)}{4a^3d(a+b)\left(a\sinh^2(c+dx)+a+b\right)^2}+\frac{3b^2(4a+3b)\sinh(c+dx)}{8a^3d(a+b)^2\left(a\sinh^2(c+dx)+a+b\right)}-\frac{3b\left(4(a+b)^2+(2a+b)^2\right)\tan^{-1}\left(\frac{\sqrt{a}\,\sinh(c+dx)}{\sqrt{a+b}}\right)}{8a^{7/2}d(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (-3*b*(4*(a + b)^2 + (2*a + b)^2)*ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b
]])/(8*a^(7/2)*(a + b)^(5/2)*d) + Sinh[c + d*x]/(a^3*d) - (b^3*Sinh[c + d*x
])/(4*a^3*(a + b)*d*(a + b + a*Sinh[c + d*x]^2)^2) + (3*b^2*(4*a + 3*b)*Sin
h[c + d*x])/(8*a^3*(a + b)^2*d*(a + b + a*Sinh[c + d*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cosh(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^3}{\left(a+b+ax^2\right)^3}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{a^3} - \frac{b\left(3a^2+3ab+b^2\right)+3ab(2a+b)x^2+3a^2bx^4}{a^3\left(a+b+ax^2\right)^3}\right)dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\sinh(c+dx)}{a^3d} - \frac{\text{Subst}\left(\int \frac{b\left(3a^2+3ab+b^2\right)+3ab(2a+b)x^2+3a^2bx^4}{\left(a+b+ax^2\right)^3}\,dx, x, \sinh(c+dx)\right)}{a^3d}$$

$$= \frac{\sinh(c+dx)}{a^3d} - \frac{b^3\sinh(c+dx)}{4a^3(a+b)d\left(a+b+a\sinh^2(c+dx)\right)^2} + \frac{\text{Subst}\left(\int \frac{-3b(2a+b)^2-12ab(a}{\left(a+b+ax^2\right)^2}\right.}{4a^3(a}$$

$$= \frac{\sinh(c+dx)}{a^3d} - \frac{b^3\sinh(c+dx)}{4a^3(a+b)d\left(a+b+a\sinh^2(c+dx)\right)^2} + \frac{3b^2(4a+3b)\sinh}{8a^3(a+b)^2d\left(a+b+a\,s\right.}$$

$$= -\frac{3b\left(4(a+b)^2+(2a+b)^2\right)\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{8a^{7/2}(a+b)^{5/2}d} + \frac{\sinh(c+dx)}{a^3d} - \frac{b^3\,s}{4a^3(a+b)d\left(a\right.}$$

Mathematica [A] time = 3.65, size = 292, normalized size = 1.90

$$\frac{\operatorname{sech}^5(c+dx)(a\cosh(2(c+dx))+a+2b)\left(\frac{3b\left(8a^2+12ab+5b^2\right)(\cosh(c)-\sinh(c))\operatorname{sech}(c+dx)(a\cosh(2(c+dx))+a+2b)^2\tan^{-1}\left(\frac{\sqrt{a+b}\sqrt{(\cos}}{}\right.}{(a+b)^{5/2}\sqrt{(\cosh(c)-\sinh(c))^2}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^5*((3*b*(8*a^2 + 12*a*b + 5*
b^2)*ArcTan[(Sqrt[a + b]*Csch[c + d*x]*Sqrt[(Cosh[c] - Sinh[c])^2]*(Cosh[c]
 + Sinh[c]))/Sqrt[a]]*(a + 2*b + a*Cosh[2*(c + d*x)])^2*Sech[c + d*x]*(Cosh
[c] - Sinh[c]))/((a + b)^(5/2)*Sqrt[(Cosh[c] - Sinh[c])^2]) + 8*Sqrt[a]*Cos
h[d*x]*(a + 2*b + a*Cosh[2*(c + d*x)])^2*Sech[c + d*x]*Sinh[c] + 8*Sqrt[a]*
```

```
Cosh[c]*(a + 2*b + a*Cosh[2*(c + d*x)])^2*Sech[c + d*x]*Sinh[d*x] - (8*Sqrt
[a]*b^3*Tanh[c + d*x])/(a + b) + (6*Sqrt[a]*b^2*(4*a + 3*b)*(a + 2*b + a*Co
sh[2*(c + d*x)])*Tanh[c + d*x])/(a + b)^2))/(64*a^(7/2)*d*(a + b*Sech[c + d
*x]^2)^3)
```

fricas [B] time = 0.64, size = 9856, normalized size = 64.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(8*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cosh(d*x + c)^10 + 80*(a^6 +
 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cosh(d*x + c)*sinh(d*x + c)^9 + 8*(a^6 + 3*
a^5*b + 3*a^4*b^2 + a^3*b^3)*sinh(d*x + c)^10 + 4*(6*a^6 + 34*a^5*b + 78*a^
4*b^2 + 75*a^3*b^3 + 25*a^2*b^4)*cosh(d*x + c)^8 + 4*(6*a^6 + 34*a^5*b + 78
*a^4*b^2 + 75*a^3*b^3 + 25*a^2*b^4 + 90*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^
3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 32*(30*(a^6 + 3*a^5*b + 3*a^4*b^2 + a
^3*b^3)*cosh(d*x + c)^3 + (6*a^6 + 34*a^5*b + 78*a^4*b^2 + 75*a^3*b^3 + 25*
a^2*b^4)*cosh(d*x + c))*sinh(d*x + c)^7 + 4*(4*a^6 + 28*a^5*b + 104*a^4*b^2
 + 209*a^3*b^3 + 189*a^2*b^4 + 60*a*b^5)*cosh(d*x + c)^6 + 4*(4*a^6 + 28*a^
5*b + 104*a^4*b^2 + 209*a^3*b^3 + 189*a^2*b^4 + 60*a*b^5 + 420*(a^6 + 3*a^5
*b + 3*a^4*b^2 + a^3*b^3)*cosh(d*x + c)^4 + 28*(6*a^6 + 34*a^5*b + 78*a^4*b
^2 + 75*a^3*b^3 + 25*a^2*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 - 8*a^6 - 24
*a^5*b - 24*a^4*b^2 - 8*a^3*b^3 + 8*(252*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b
^3)*cosh(d*x + c)^5 + 28*(6*a^6 + 34*a^5*b + 78*a^4*b^2 + 75*a^3*b^3 + 25*a
^2*b^4)*cosh(d*x + c)^3 + 3*(4*a^6 + 28*a^5*b + 104*a^4*b^2 + 209*a^3*b^3 +
 189*a^2*b^4 + 60*a*b^5)*cosh(d*x + c))*sinh(d*x + c)^5 - 4*(4*a^6 + 28*a^5
*b + 104*a^4*b^2 + 209*a^3*b^3 + 189*a^2*b^4 + 60*a*b^5)*cosh(d*x + c)^4 +
4*(420*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cosh(d*x + c)^6 - 4*a^6 - 28*a
^5*b - 104*a^4*b^2 - 209*a^3*b^3 - 189*a^2*b^4 - 60*a*b^5 + 70*(6*a^6 + 34*
a^5*b + 78*a^4*b^2 + 75*a^3*b^3 + 25*a^2*b^4)*cosh(d*x + c)^4 + 15*(4*a^6 +
 28*a^5*b + 104*a^4*b^2 + 209*a^3*b^3 + 189*a^2*b^4 + 60*a*b^5)*cosh(d*x +
c)^2)*sinh(d*x + c)^4 + 16*(60*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cosh(d
*x + c)^7 + 14*(6*a^6 + 34*a^5*b + 78*a^4*b^2 + 75*a^3*b^3 + 25*a^2*b^4)*co
sh(d*x + c)^5 + 5*(4*a^6 + 28*a^5*b + 104*a^4*b^2 + 209*a^3*b^3 + 189*a^2*b
^4 + 60*a*b^5)*cosh(d*x + c)^3 - (4*a^6 + 28*a^5*b + 104*a^4*b^2 + 209*a^3*
b^3 + 189*a^2*b^4 + 60*a*b^5)*cosh(d*x + c))*sinh(d*x + c)^3 - 4*(6*a^6 + 3
4*a^5*b + 78*a^4*b^2 + 75*a^3*b^3 + 25*a^2*b^4)*cosh(d*x + c)^2 + 4*(90*(a^
6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cosh(d*x + c)^8 + 28*(6*a^6 + 34*a^5*b +
 78*a^4*b^2 + 75*a^3*b^3 + 25*a^2*b^4)*cosh(d*x + c)^6 - 6*a^6 - 34*a^5*b -
 78*a^4*b^2 - 75*a^3*b^3 - 25*a^2*b^4 + 15*(4*a^6 + 28*a^5*b + 104*a^4*b^2
+ 209*a^3*b^3 + 189*a^2*b^4 + 60*a*b^5)*cosh(d*x + c)^4 - 6*(4*a^6 + 28*a^5
*b + 104*a^4*b^2 + 209*a^3*b^3 + 189*a^2*b^4 + 60*a*b^5)*cosh(d*x + c)^2)*s
inh(d*x + c)^2 - 3*((8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)^9 + 9*
```

(8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)*sinh(d*x + c)^8 + (8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)*sinh(d*x + c)^9 + 4*(8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^4)*cosh(d*x + c)^7 + 4*(8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^4 + 9*(8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^7 + 28*(3*(8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)^3 + (8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^6 + 2*(24*a^4*b + 100*a^3*b^2 + 175*a^2*b^3 + 136*a*b^4 + 40*b^5)*cosh(d*x + c)^5 + 2*(24*a^4*b + 100*a^3*b^2 + 175*a^2*b^3 + 136*a*b^4 + 40*b^5 + 63*(8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)^4 + 42*(8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 2*(63*(8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)^5 + 70*(8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^4)*cosh(d*x + c)^3 + 5*(24*a^4*b + 100*a^3*b^2 + 175*a^2*b^3 + 136*a*b^4 + 40*b^5)*cosh(d*x + c))*sinh(d*x + c)^4 + 4*(8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^4)*cosh(d*x + c)^3 + 4*(21*(8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)^6 + 8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^4 + 35*(8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^4)*cosh(d*x + c)^4 + 5*(24*a^4*b + 100*a^3*b^2 + 175*a^2*b^3 + 136*a*b^4 + 40*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 4*(9*(8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)^7 + 21*(8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^4)*cosh(d*x + c)^5 + 5*(24*a^4*b + 100*a^3*b^2 + 175*a^2*b^3 + 136*a*b^4 + 40*b^5)*cosh(d*x + c)^3 + 3*(8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^2 + (8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c) + (9*(8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)^8 + 28*(8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^4)*cosh(d*x + c)^6 + 8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3 + 10*(24*a^4*b + 100*a^3*b^2 + 175*a^2*b^3 + 136*a*b^4 + 40*b^5)*cosh(d*x + c)^4 + 12*(8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c))*sqrt(-a^2 - a*b)*log((a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 - 2*(3*a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 - 3*a - 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 - (3*a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1)*sinh(d*x + c) - cosh(d*x + c))*sqrt(-a^2 - a*b) + a)/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) + 8*(10*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cosh(d*x + c)^9 + 4*(6*a^6 + 34*a^5*b + 78*a^4*b^2 + 75*a^3*b^3 + 25*a^2*b^4)*cosh(d*x + c)^7 + 3*(4*a^6 + 28*a^5*b + 104*a^4*b^2 + 209*a^3*b^3 + 189*a^2*b^4 + 60*a*b^5)*cosh(d*x + c)^5 - 2*(4*a^6 + 28*a^5*b + 104*a^4*b^2 + 209*a^3*b^3 + 189*a^2*b^4 + 60*a*b^5)*cosh(d*x + c)^3 - (6*a^6 + 34*a^5*b + 78*a^4*b^2 + 75*a^3*b^3 + 25*a^2*b^4)*cosh(d*x + c))*sinh(d*x + c))/((a^9 + 3*a^8*b + 3*a^7*b^2 + a^6*b^3)*d*cosh(d*x + c)^9 + 9*(a^9 + 3*a^8*b + 3*a^7*b^2 + a^6*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^8 + (a^9 + 3*a^8*b + 3*a^7*b^2 + a^6*b^3)*d*sinh(d*x + c)^9 + 4*(a^9 + 5*a^8*b + 9*a^7*b^2 + 7*a^6*b^3 + 2*a^5*b^4)*d*cosh(d*x + c)^7 + 4*(9*(a^9 + 3*a^8*b + 3*a^7*b^2 + a^6*b^3)*d*cosh(d*x + c)^2 + (a^9 + 5*a^8*b + 9*a^7*b^2 + 7*a^6*b^3 + 2*a^5*b^4)*d

)*sinh(d*x + c)^7 + 2*(3*a^9 + 17*a^8*b + 41*a^7*b^2 + 51*a^6*b^3 + 32*a^5*
b^4 + 8*a^4*b^5)*d*cosh(d*x + c)^5 + 28*(3*(a^9 + 3*a^8*b + 3*a^7*b^2 + a^6
*b^3)*d*cosh(d*x + c)^3 + (a^9 + 5*a^8*b + 9*a^7*b^2 + 7*a^6*b^3 + 2*a^5*b^
4)*d*cosh(d*x + c))*sinh(d*x + c)^6 + 2*(63*(a^9 + 3*a^8*b + 3*a^7*b^2 + a^
6*b^3)*d*cosh(d*x + c)^4 + 42*(a^9 + 5*a^8*b + 9*a^7*b^2 + 7*a^6*b^3 + 2*a^
5*b^4)*d*cosh(d*x + c)^2 + (3*a^9 + 17*a^8*b + 41*a^7*b^2 + 51*a^6*b^3 + 32
*a^5*b^4 + 8*a^4*b^5)*d)*sinh(d*x + c)^5 + 4*(a^9 + 5*a^8*b + 9*a^7*b^2 + 7
*a^6*b^3 + 2*a^5*b^4)*d*cosh(d*x + c)^3 + 2*(63*(a^9 + 3*a^8*b + 3*a^7*b^2
+ a^6*b^3)*d*cosh(d*x + c)^5 + 70*(a^9 + 5*a^8*b + 9*a^7*b^2 + 7*a^6*b^3 +
2*a^5*b^4)*d*cosh(d*x + c)^3 + 5*(3*a^9 + 17*a^8*b + 41*a^7*b^2 + 51*a^6*b^
3 + 32*a^5*b^4 + 8*a^4*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^4 + 4*(21*(a^9 +
 3*a^8*b + 3*a^7*b^2 + a^6*b^3)*d*cosh(d*x + c)^6 + 35*(a^9 + 5*a^8*b + 9*a
^7*b^2 + 7*a^6*b^3 + 2*a^5*b^4)*d*cosh(d*x + c)^4 + 5*(3*a^9 + 17*a^8*b + 4
1*a^7*b^2 + 51*a^6*b^3 + 32*a^5*b^4 + 8*a^4*b^5)*d*cosh(d*x + c)^2 + (a^9 +
 5*a^8*b + 9*a^7*b^2 + 7*a^6*b^3 + 2*a^5*b^4)*d)*sinh(d*x + c)^3 + (a^9 + 3
*a^8*b + 3*a^7*b^2 + a^6*b^3)*d*cosh(d*x + c) + 4*(9*(a^9 + 3*a^8*b + 3*a^7
*b^2 + a^6*b^3)*d*cosh(d*x + c)^7 + 21*(a^9 + 5*a^8*b + 9*a^7*b^2 + 7*a^6*b
^3 + 2*a^5*b^4)*d*cosh(d*x + c)^5 + 5*(3*a^9 + 17*a^8*b + 41*a^7*b^2 + 51*a
^6*b^3 + 32*a^5*b^4 + 8*a^4*b^5)*d*cosh(d*x + c)^3 + 3*(a^9 + 5*a^8*b + 9*a
^7*b^2 + 7*a^6*b^3 + 2*a^5*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^2 + (9*(a^9
+ 3*a^8*b + 3*a^7*b^2 + a^6*b^3)*d*cosh(d*x + c)^8 + 28*(a^9 + 5*a^8*b + 9*
a^7*b^2 + 7*a^6*b^3 + 2*a^5*b^4)*d*cosh(d*x + c)^6 + 10*(3*a^9 + 17*a^8*b +
 41*a^7*b^2 + 51*a^6*b^3 + 32*a^5*b^4 + 8*a^4*b^5)*d*cosh(d*x + c)^4 + 12*(
a^9 + 5*a^8*b + 9*a^7*b^2 + 7*a^6*b^3 + 2*a^5*b^4)*d*cosh(d*x + c)^2 + (a^9
 + 3*a^8*b + 3*a^7*b^2 + a^6*b^3)*d)*sinh(d*x + c)), 1/8*(4*(a^6 + 3*a^5*b
+ 3*a^4*b^2 + a^3*b^3)*cosh(d*x + c)^10 + 40*(a^6 + 3*a^5*b + 3*a^4*b^2 + a
^3*b^3)*cosh(d*x + c)*sinh(d*x + c)^9 + 4*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*
b^3)*sinh(d*x + c)^10 + 2*(6*a^6 + 34*a^5*b + 78*a^4*b^2 + 75*a^3*b^3 + 25*
a^2*b^4)*cosh(d*x + c)^8 + 2*(6*a^6 + 34*a^5*b + 78*a^4*b^2 + 75*a^3*b^3 +
25*a^2*b^4 + 90*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cosh(d*x + c)^2)*sinh
(d*x + c)^8 + 16*(30*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cosh(d*x + c)^3
+ (6*a^6 + 34*a^5*b + 78*a^4*b^2 + 75*a^3*b^3 + 25*a^2*b^4)*cosh(d*x + c))*
sinh(d*x + c)^7 + 2*(4*a^6 + 28*a^5*b + 104*a^4*b^2 + 209*a^3*b^3 + 189*a^2
*b^4 + 60*a*b^5)*cosh(d*x + c)^6 + 2*(4*a^6 + 28*a^5*b + 104*a^4*b^2 + 209*
a^3*b^3 + 189*a^2*b^4 + 60*a*b^5 + 420*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3
)*cosh(d*x + c)^4 + 28*(6*a^6 + 34*a^5*b + 78*a^4*b^2 + 75*a^3*b^3 + 25*a^2
*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 - 4*a^6 - 12*a^5*b - 12*a^4*b^2 - 4*
a^3*b^3 + 4*(252*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cosh(d*x + c)^5 + 28
*(6*a^6 + 34*a^5*b + 78*a^4*b^2 + 75*a^3*b^3 + 25*a^2*b^4)*cosh(d*x + c)^3
+ 3*(4*a^6 + 28*a^5*b + 104*a^4*b^2 + 209*a^3*b^3 + 189*a^2*b^4 + 60*a*b^5)
*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(4*a^6 + 28*a^5*b + 104*a^4*b^2 + 209*a
^3*b^3 + 189*a^2*b^4 + 60*a*b^5)*cosh(d*x + c)^4 + 2*(420*(a^6 + 3*a^5*b +
3*a^4*b^2 + a^3*b^3)*cosh(d*x + c)^6 - 4*a^6 - 28*a^5*b - 104*a^4*b^2 - 209
*a^3*b^3 - 189*a^2*b^4 - 60*a*b^5 + 70*(6*a^6 + 34*a^5*b + 78*a^4*b^2 + 75*
a^3*b^3 + 25*a^2*b^4)*cosh(d*x + c)^4 + 15*(4*a^6 + 28*a^5*b + 104*a^4*b^2

+ 209*a^3*b^3 + 189*a^2*b^4 + 60*a*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^4 +
8*(60*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cosh(d*x + c)^7 + 14*(6*a^6 + 3
4*a^5*b + 78*a^4*b^2 + 75*a^3*b^3 + 25*a^2*b^4)*cosh(d*x + c)^5 + 5*(4*a^6
+ 28*a^5*b + 104*a^4*b^2 + 209*a^3*b^3 + 189*a^2*b^4 + 60*a*b^5)*cosh(d*x +
c)^3 - (4*a^6 + 28*a^5*b + 104*a^4*b^2 + 209*a^3*b^3 + 189*a^2*b^4 + 60*a*
b^5)*cosh(d*x + c))*sinh(d*x + c)^3 - 2*(6*a^6 + 34*a^5*b + 78*a^4*b^2 + 75
*a^3*b^3 + 25*a^2*b^4)*cosh(d*x + c)^2 + 2*(90*(a^6 + 3*a^5*b + 3*a^4*b^2 +
a^3*b^3)*cosh(d*x + c)^8 + 28*(6*a^6 + 34*a^5*b + 78*a^4*b^2 + 75*a^3*b^3
+ 25*a^2*b^4)*cosh(d*x + c)^6 - 6*a^6 - 34*a^5*b - 78*a^4*b^2 - 75*a^3*b^3
- 25*a^2*b^4 + 15*(4*a^6 + 28*a^5*b + 104*a^4*b^2 + 209*a^3*b^3 + 189*a^2*b
^4 + 60*a*b^5)*cosh(d*x + c)^4 - 6*(4*a^6 + 28*a^5*b + 104*a^4*b^2 + 209*a^
3*b^3 + 189*a^2*b^4 + 60*a*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 3*((8*a^
4*b + 12*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)^9 + 9*(8*a^4*b + 12*a^3*b^2 + 5
*a^2*b^3)*cosh(d*x + c)*sinh(d*x + c)^8 + (8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3
)*sinh(d*x + c)^9 + 4*(8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^4)*cosh(d
x + c)^7 + 4(8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^4 + 9*(8*a^4*b +
12*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^7 + 28*(3*(8*a^4*b +
12*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)^3 + (8*a^4*b + 28*a^3*b^2 + 29*a^2*b
^3 + 10*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^6 + 2*(24*a^4*b + 100*a^3*b^2 +
175*a^2*b^3 + 136*a*b^4 + 40*b^5)*cosh(d*x + c)^5 + 2*(24*a^4*b + 100*a^3*
b^2 + 175*a^2*b^3 + 136*a*b^4 + 40*b^5 + 63*(8*a^4*b + 12*a^3*b^2 + 5*a^2*b
^3)*cosh(d*x + c)^4 + 42*(8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^4)*cos
h(d*x + c)^2)*sinh(d*x + c)^5 + 2*(63*(8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)*co
sh(d*x + c)^5 + 70*(8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^4)*cosh(d*x
+ c)^3 + 5*(24*a^4*b + 100*a^3*b^2 + 175*a^2*b^3 + 136*a*b^4 + 40*b^5)*cosh
(d*x + c))*sinh(d*x + c)^4 + 4*(8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^
4)*cosh(d*x + c)^3 + 4*(21*(8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)
^6 + 8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^4 + 35*(8*a^4*b + 28*a^3*b^
2 + 29*a^2*b^3 + 10*a*b^4)*cosh(d*x + c)^4 + 5*(24*a^4*b + 100*a^3*b^2 + 17
5*a^2*b^3 + 136*a*b^4 + 40*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 4*(9*(8*
a^4*b + 12*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)^7 + 21*(8*a^4*b + 28*a^3*b^2
+ 29*a^2*b^3 + 10*a*b^4)*cosh(d*x + c)^5 + 5*(24*a^4*b + 100*a^3*b^2 + 175*
a^2*b^3 + 136*a*b^4 + 40*b^5)*cosh(d*x + c)^3 + 3*(8*a^4*b + 28*a^3*b^2 + 2
9*a^2*b^3 + 10*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^2 + (8*a^4*b + 12*a^3*b^
2 + 5*a^2*b^3)*cosh(d*x + c) + (9*(8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)*cosh(d
x + c)^8 + 28(8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^4)*cosh(d*x + c)
^6 + 8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3 + 10*(24*a^4*b + 100*a^3*b^2 + 175*a^
2*b^3 + 136*a*b^4 + 40*b^5)*cosh(d*x + c)^4 + 12*(8*a^4*b + 28*a^3*b^2 + 29
*a^2*b^3 + 10*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c))*sqrt(a^2 + a*b)*arctan
(1/2*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x +
c)^3 + (3*a + 4*b)*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + 3*a + 4*b)*sinh(d
*x + c))/sqrt(a^2 + a*b)) - 3*((8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)*cosh(d*x
+ c)^9 + 9*(8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)*sinh(d*x + c)^8
+ (8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)*sinh(d*x + c)^9 + 4*(8*a^4*b + 28*a^3
*b^2 + 29*a^2*b^3 + 10*a*b^4)*cosh(d*x + c)^7 + 4*(8*a^4*b + 28*a^3*b^2 + 2

9*a^2*b^3 + 10*a*b^4 + 9*(8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)^2
)*sinh(d*x + c)^7 + 28*(3*(8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)^
3 + (8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^4)*cosh(d*x + c))*sinh(d*x
+ c)^6 + 2*(24*a^4*b + 100*a^3*b^2 + 175*a^2*b^3 + 136*a*b^4 + 40*b^5)*cosh
(d*x + c)^5 + 2*(24*a^4*b + 100*a^3*b^2 + 175*a^2*b^3 + 136*a*b^4 + 40*b^5
+ 63*(8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)^4 + 42*(8*a^4*b + 28*
a^3*b^2 + 29*a^2*b^3 + 10*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 2*(63*(
8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)^5 + 70*(8*a^4*b + 28*a^3*b^
2 + 29*a^2*b^3 + 10*a*b^4)*cosh(d*x + c)^3 + 5*(24*a^4*b + 100*a^3*b^2 + 17
5*a^2*b^3 + 136*a*b^4 + 40*b^5)*cosh(d*x + c))*sinh(d*x + c)^4 + 4*(8*a^4*b
 + 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^4)*cosh(d*x + c)^3 + 4*(21*(8*a^4*b + 1
2*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)^6 + 8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3
+ 10*a*b^4 + 35*(8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^4)*cosh(d*x + c
)^4 + 5*(24*a^4*b + 100*a^3*b^2 + 175*a^2*b^3 + 136*a*b^4 + 40*b^5)*cosh(d*
x + c)^2)*sinh(d*x + c)^3 + 4*(9*(8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)*cosh(d*
x + c)^7 + 21*(8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^4)*cosh(d*x + c)^
5 + 5*(24*a^4*b + 100*a^3*b^2 + 175*a^2*b^3 + 136*a*b^4 + 40*b^5)*cosh(d*x
+ c)^3 + 3*(8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^4)*cosh(d*x + c))*si
nh(d*x + c)^2 + (8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c) + (9*(8*a^
4*b + 12*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)^8 + 28*(8*a^4*b + 28*a^3*b^2 +
29*a^2*b^3 + 10*a*b^4)*cosh(d*x + c)^6 + 8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3 +
 10*(24*a^4*b + 100*a^3*b^2 + 175*a^2*b^3 + 136*a*b^4 + 40*b^5)*cosh(d*x +
c)^4 + 12*(8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^4)*cosh(d*x + c)^2)*s
inh(d*x + c))*sqrt(a^2 + a*b)*arctan(1/2*sqrt(a^2 + a*b)*(cosh(d*x + c) + s
inh(d*x + c))/(a + b)) + 4*(10*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cosh(d
x + c)^9 + 4(6*a^6 + 34*a^5*b + 78*a^4*b^2 + 75*a^3*b^3 + 25*a^2*b^4)*cos
h(d*x + c)^7 + 3*(4*a^6 + 28*a^5*b + 104*a^4*b^2 + 209*a^3*b^3 + 189*a^2*b^
4 + 60*a*b^5)*cosh(d*x + c)^5 - 2*(4*a^6 + 28*a^5*b + 104*a^4*b^2 + 209*a^3
*b^3 + 189*a^2*b^4 + 60*a*b^5)*cosh(d*x + c)^3 - (6*a^6 + 34*a^5*b + 78*a^4
*b^2 + 75*a^3*b^3 + 25*a^2*b^4)*cosh(d*x + c))*sinh(d*x + c))/((a^9 + 3*a^8
*b + 3*a^7*b^2 + a^6*b^3)*d*cosh(d*x + c)^9 + 9*(a^9 + 3*a^8*b + 3*a^7*b^2
+ a^6*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^8 + (a^9 + 3*a^8*b + 3*a^7*b^2 + a
^6*b^3)*d*sinh(d*x + c)^9 + 4*(a^9 + 5*a^8*b + 9*a^7*b^2 + 7*a^6*b^3 + 2*a^
5*b^4)*d*cosh(d*x + c)^7 + 4*(9*(a^9 + 3*a^8*b + 3*a^7*b^2 + a^6*b^3)*d*cos
h(d*x + c)^2 + (a^9 + 5*a^8*b + 9*a^7*b^2 + 7*a^6*b^3 + 2*a^5*b^4)*d)*sinh(
d*x + c)^7 + 2*(3*a^9 + 17*a^8*b + 41*a^7*b^2 + 51*a^6*b^3 + 32*a^5*b^4 + 8
*a^4*b^5)*d*cosh(d*x + c)^5 + 28*(3*(a^9 + 3*a^8*b + 3*a^7*b^2 + a^6*b^3)*d
*cosh(d*x + c)^3 + (a^9 + 5*a^8*b + 9*a^7*b^2 + 7*a^6*b^3 + 2*a^5*b^4)*d*co
sh(d*x + c))*sinh(d*x + c)^6 + 2*(63*(a^9 + 3*a^8*b + 3*a^7*b^2 + a^6*b^3)*
d*cosh(d*x + c)^4 + 42*(a^9 + 5*a^8*b + 9*a^7*b^2 + 7*a^6*b^3 + 2*a^5*b^4)*
d*cosh(d*x + c)^2 + (3*a^9 + 17*a^8*b + 41*a^7*b^2 + 51*a^6*b^3 + 32*a^5*b^
4 + 8*a^4*b^5)*d)*sinh(d*x + c)^5 + 4*(a^9 + 5*a^8*b + 9*a^7*b^2 + 7*a^6*b^
3 + 2*a^5*b^4)*d*cosh(d*x + c)^3 + 2*(63*(a^9 + 3*a^8*b + 3*a^7*b^2 + a^6*b
^3)*d*cosh(d*x + c)^5 + 70*(a^9 + 5*a^8*b + 9*a^7*b^2 + 7*a^6*b^3 + 2*a^5*b
^4)*d*cosh(d*x + c)^3 + 5*(3*a^9 + 17*a^8*b + 41*a^7*b^2 + 51*a^6*b^3 + 32*

```
a^5*b^4 + 8*a^4*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^4 + 4*(21*(a^9 + 3*a^8*
b + 3*a^7*b^2 + a^6*b^3)*d*cosh(d*x + c)^6 + 35*(a^9 + 5*a^8*b + 9*a^7*b^2
+ 7*a^6*b^3 + 2*a^5*b^4)*d*cosh(d*x + c)^4 + 5*(3*a^9 + 17*a^8*b + 41*a^7*b
^2 + 51*a^6*b^3 + 32*a^5*b^4 + 8*a^4*b^5)*d*cosh(d*x + c)^2 + (a^9 + 5*a^8*
b + 9*a^7*b^2 + 7*a^6*b^3 + 2*a^5*b^4)*d)*sinh(d*x + c)^3 + (a^9 + 3*a^8*b
+ 3*a^7*b^2 + a^6*b^3)*d*cosh(d*x + c) + 4*(9*(a^9 + 3*a^8*b + 3*a^7*b^2 +
a^6*b^3)*d*cosh(d*x + c)^7 + 21*(a^9 + 5*a^8*b + 9*a^7*b^2 + 7*a^6*b^3 + 2*
a^5*b^4)*d*cosh(d*x + c)^5 + 5*(3*a^9 + 17*a^8*b + 41*a^7*b^2 + 51*a^6*b^3
+ 32*a^5*b^4 + 8*a^4*b^5)*d*cosh(d*x + c)^3 + 3*(a^9 + 5*a^8*b + 9*a^7*b^2
+ 7*a^6*b^3 + 2*a^5*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^2 + (9*(a^9 + 3*a^8
*b + 3*a^7*b^2 + a^6*b^3)*d*cosh(d*x + c)^8 + 28*(a^9 + 5*a^8*b + 9*a^7*b^2
 + 7*a^6*b^3 + 2*a^5*b^4)*d*cosh(d*x + c)^6 + 10*(3*a^9 + 17*a^8*b + 41*a^7
*b^2 + 51*a^6*b^3 + 32*a^5*b^4 + 8*a^4*b^5)*d*cosh(d*x + c)^4 + 12*(a^9 + 5
*a^8*b + 9*a^7*b^2 + 7*a^6*b^3 + 2*a^5*b^4)*d*cosh(d*x + c)^2 + (a^9 + 3*a^
8*b + 3*a^7*b^2 + a^6*b^3)*d)*sinh(d*x + c))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[84,-86]Warning, need to choose a branch for the root of a p
olynomial with parameters. This might be wrong.The choice was done assuming
 [a,b]=[-42,-12]Warning, need to choose a branch for the root of a polynomi
al with parameters. This might be wrong.The choice was done assuming [a,b]=
[-43,-99]Warning, need to choose a branch for the root of a polynomial with
 parameters. This might be wrong.The choice was done assuming [a,b]=[-28,94
]Warning, need to choose a branch for the root of a polynomial with paramet
ers. This might be wrong.The choice was done assuming [a,b]=[-7,46]Warning,
 need to choose a branch for the root of a polynomial with parameters. This
 might be wrong.The choice was done assuming [a,b]=[-35,-99]Warning, need t
o choose a branch for the root of a polynomial with parameters. This might
be wrong.The choice was done assuming [a,b]=[7,50]Warning, need to choose a
 branch for the root of a polynomial with parameters. This might be wrong.T
he choice was done assuming [a,b]=[-63,-70]Warning, need to choose a branch
 for the root of a polynomial with parameters. This might be wrong.The choi
ce was done assuming [a,b]=[-82,81]Precision problem choosing root in commo
n_EXT, current precision 14Warning, need to choose a branch for the root of
 a polynomial with parameters. This might be wrong.The choice was done assu
ming [a,b]=[-60,-34]Undef/Unsigned Inf encountered in limitEvaluation time:
```

```
2.08Limit: Max order reached or unable to make series expansion Error: Bad
Argument Value
```

maple [B] time = 0.56, size = 1238, normalized size = 8.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] -1/d/a^3/(tanh(1/2*d*x+1/2*c)-1)-1/d/a^3/(tanh(1/2*d*x+1/2*c)+1)-3/d/a^2*b^
2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*
a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)^7-7/4/d/a^3*b^
3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*
a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)^7-3/d/a*b^2/(t
anh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*
tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^5+21/4/d/a^2*b^3
/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a
-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^5+21/4/d/a^3*
b^4/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^
2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^5+3/d/a*b^
2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*
a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^3-21/4/d/a^2
*b^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)
^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^3-21/4/d/
a^3*b^4/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2
*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^3+3/d/
a^2*b^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2
*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)+7/4/d/a^
3*b^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c
)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)-3/d/a^(3/2
)*b/(a^2+2*a*b+b^2)/(a+b)^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*
c)-2*b^(1/2))/a^(1/2))-9/2/d/a^(5/2)*b^2/(a^2+2*a*b+b^2)/(a+b)^(1/2)*arctan
(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)-2*b^(1/2))/a^(1/2))-15/8/d/a^(7/2)*
b^3/(a^2+2*a*b+b^2)/(a+b)^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*
c)-2*b^(1/2))/a^(1/2))-3/d/a^(3/2)*b/(a^2+2*a*b+b^2)/(a+b)^(1/2)*arctan(1/2
*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)+2*b^(1/2))/a^(1/2))-9/2/d/a^(5/2)*b^2/(
a^2+2*a*b+b^2)/(a+b)^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)+2*
b^(1/2))/a^(1/2))-15/8/d/a^(7/2)*b^3/(a^2+2*a*b+b^2)/(a+b)^(1/2)*arctan(1/2
*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)+2*b^(1/2))/a^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\,a^4 + 4\,a^3b + 2\,a^2b^2 - 2\left(a^4e^{(10\,c)} + 2\,a^3be^{(10\,c)} + a^2b^2e^{(10\,c)}\right)e^{(10\,dx)} - \left(6\,a^4e^{(8\,c)} + 28\,a^3be^{(8\,c)} + 50\,a^2b^2e^{(8\,c)} + 25}{4\left(\left(a^7de^{(9\,c)} + 2\,a^6bde^{(9\,c)} + a^5b^2de^{(9\,c)}\right)e^{(9\,dx)} + 4\left(a^7de^{(7\,c)} + 4\,a^6bde^{(7\,c)} + 5\,a^5b^2de^{(7\,c)} + \right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*a^4 + 4*a^3*b + 2*a^2*b^2 - 2*(a^4*e^(10*c) + 2*a^3*b*e^(10*c) + a^
2*b^2*e^(10*c))*e^(10*d*x) - (6*a^4*e^(8*c) + 28*a^3*b*e^(8*c) + 50*a^2*b^2
*e^(8*c) + 25*a*b^3*e^(8*c))*e^(8*d*x) - (4*a^4*e^(6*c) + 24*a^3*b*e^(6*c)
+ 80*a^2*b^2*e^(6*c) + 129*a*b^3*e^(6*c) + 60*b^4*e^(6*c))*e^(6*d*x) + (4*a
^4*e^(4*c) + 24*a^3*b*e^(4*c) + 80*a^2*b^2*e^(4*c) + 129*a*b^3*e^(4*c) + 60
*b^4*e^(4*c))*e^(4*d*x) + (6*a^4*e^(2*c) + 28*a^3*b*e^(2*c) + 50*a^2*b^2*e^
(2*c) + 25*a*b^3*e^(2*c))*e^(2*d*x))/((a^7*d*e^(9*c) + 2*a^6*b*d*e^(9*c) +
a^5*b^2*d*e^(9*c))*e^(9*d*x) + 4*(a^7*d*e^(7*c) + 4*a^6*b*d*e^(7*c) + 5*a^5
*b^2*d*e^(7*c) + 2*a^4*b^3*d*e^(7*c))*e^(7*d*x) + 2*(3*a^7*d*e^(5*c) + 14*a
^6*b*d*e^(5*c) + 27*a^5*b^2*d*e^(5*c) + 24*a^4*b^3*d*e^(5*c) + 8*a^3*b^4*d*
e^(5*c))*e^(5*d*x) + 4*(a^7*d*e^(3*c) + 4*a^6*b*d*e^(3*c) + 5*a^5*b^2*d*e^(
3*c) + 2*a^4*b^3*d*e^(3*c))*e^(3*d*x) + (a^7*d*e^c + 2*a^6*b*d*e^c + a^5*b^
2*d*e^c)*e^(d*x)) - 1/2*integrate(3/2*((8*a^2*b*e^(3*c) + 12*a*b^2*e^(3*c)
+ 5*b^3*e^(3*c))*e^(3*d*x) + (8*a^2*b*e^c + 12*a*b^2*e^c + 5*b^3*e^c)*e^(d*
x))/(a^6 + 2*a^5*b + a^4*b^2 + (a^6*e^(4*c) + 2*a^5*b*e^(4*c) + a^4*b^2*e^(
4*c))*e^(4*d*x) + 2*(a^6*e^(2*c) + 4*a^5*b*e^(2*c) + 5*a^4*b^2*e^(2*c) + 2*
a^3*b^3*e^(2*c))*e^(2*d*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh (c+d\,x)}{{\left(a+\frac{b}{{\cosh (c+d\,x)}^2}\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)/(a + b/cosh(c + d*x)^2)^3,x)
```

```
[Out] int(cosh(c + d*x)/(a + b/cosh(c + d*x)^2)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

3.95 $\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=142

$$-\frac{3b(2a+b)\sinh(c+dx)}{8a^2d(a+b)^2\left(a\sinh^2(c+dx)+a+b\right)}+\frac{\left(8a^2+8ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{8a^{5/2}d(a+b)^{5/2}}-\frac{b\sinh(c+dx)\cosh^2(c+}{4ad(a+b)\left(a\sinh^2(c+dx)+\right.}$$

```
[Out] 1/8*(8*a^2+8*a*b+3*b^2)*arctan(sinh(d*x+c)*a^(1/2)/(a+b)^(1/2))/a^(5/2)/(a+
b)^(5/2)/d-1/4*b*cosh(d*x+c)^2*sinh(d*x+c)/a/(a+b)/d/(a+b+a*sinh(d*x+c)^2)^
2-3/8*b*(2*a+b)*sinh(d*x+c)/a^2/(a+b)^2/d/(a+b+a*sinh(d*x+c)^2)
```

Rubi [A] time = 0.14, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {4147, 413, 385, 205}

$$\frac{\left(8a^2+8ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{8a^{5/2}d(a+b)^{5/2}}-\frac{3b(2a+b)\sinh(c+dx)}{8a^2d(a+b)^2\left(a\sinh^2(c+dx)+a+b\right)}-\frac{b\sinh(c+dx)\cosh^2(c+d}{4ad(a+b)\left(a\sinh^2(c+dx)+a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((8*a^2 + 8*a*b + 3*b^2)*ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]])/(8*a^
(5/2)*(a + b)^(5/2)*d) - (b*Cosh[c + d*x]^2*Sinh[c + d*x])/(4*a*(a + b)*d*(
a + b + a*Sinh[c + d*x]^2)^2) - (3*b*(2*a + b)*Sinh[c + d*x])/(8*a^2*(a + b
)^2*d*(a + b + a*Sinh[c + d*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\text{sech}(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{\left(a+b+ax^2\right)^3}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= -\frac{b\cosh^2(c+dx)\sinh(c+dx)}{4a(a+b)d\left(a+b+a\sinh^2(c+dx)\right)^2} + \frac{\text{Subst}\left(\int \frac{4a+b+(4a+3b)x^2}{\left(a+b+ax^2\right)^2}\,dx, x, \sinh(c+d\right.}{4a(a+b)d}$$

$$= -\frac{b\cosh^2(c+dx)\sinh(c+dx)}{4a(a+b)d\left(a+b+a\sinh^2(c+dx)\right)^2} - \frac{3b(2a+b)\sinh(c+dx)}{8a^2(a+b)^2d\left(a+b+a\sinh^2(c+dx)\right)}$$

$$= \frac{\left(8a^2+8ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{8a^{5/2}(a+b)^{5/2}d} - \frac{b\cosh^2(c+dx)\sinh(c+dx)}{4a(a+b)d\left(a+b+a\sinh^2(c+dx)\right)^2}$$

Mathematica [A] time = 2.94, size = 214, normalized size = 1.51

$$\frac{\text{sech}^5(c+dx)(a\cosh(2(c+dx))+a+2b)\left(\frac{\left(8a^2+8ab+3b^2\right)(\sinh(c)-\cosh(c))\text{sech}(c+dx)(a\cosh(2(c+dx))+a+2b)^2\tan^{-1}\left(\frac{\sqrt{a+b}\sqrt{(\cosh(c)\cdots}}{\cdots}\right)}{\sqrt{a+b}\sqrt{(\cosh(c)-\sinh(c))^2}}\right.}{64a^{5/2}d(a+b)^2\left(\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^5*(((8*a^2 + 8*a*b + 3*b^2)*
ArcTan[(Sqrt[a + b]*Csch[c + d*x]*Sqrt[(Cosh[c] - Sinh[c])^2]*(Cosh[c] + Si
nh[c]))/Sqrt[a]]*(a + 2*b + a*Cosh[2*(c + d*x)])^2*Sech[c + d*x]*(-Cosh[c]
+ Sinh[c]))/(Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]) + 8*Sqrt[a]*b^2*(a +
b)*Tanh[c + d*x] - 2*Sqrt[a]*b*(8*a + 5*b)*(a + 2*b + a*Cosh[2*(c + d*x)])*
Tanh[c + d*x]))/(64*a^(5/2)*(a + b)^2*d*(a + b*Sech[c + d*x]^2)^3)
```

fricas [B] time = 0.54, size = 6806, normalized size = 47.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*(8*a^4*b + 13*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)^7 + 28*(8*a^4*b
+ 13*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)*sinh(d*x + c)^6 + 4*(8*a^4*b + 13*a
^3*b^2 + 5*a^2*b^3)*sinh(d*x + c)^7 + 4*(8*a^4*b + 37*a^3*b^2 + 41*a^2*b^3
+ 12*a*b^4)*cosh(d*x + c)^5 + 4*(8*a^4*b + 37*a^3*b^2 + 41*a^2*b^3 + 12*a*b
^4 + 21*(8*a^4*b + 13*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5
 + 20*(7*(8*a^4*b + 13*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)^3 + (8*a^4*b + 37
*a^3*b^2 + 41*a^2*b^3 + 12*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^4 - 4*(8*a^4
*b + 37*a^3*b^2 + 41*a^2*b^3 + 12*a*b^4)*cosh(d*x + c)^3 - 4*(8*a^4*b + 37*
a^3*b^2 + 41*a^2*b^3 + 12*a*b^4 - 35*(8*a^4*b + 13*a^3*b^2 + 5*a^2*b^3)*cos
h(d*x + c)^4 - 10*(8*a^4*b + 37*a^3*b^2 + 41*a^2*b^3 + 12*a*b^4)*cosh(d*x +
 c)^2)*sinh(d*x + c)^3 + 4*(21*(8*a^4*b + 13*a^3*b^2 + 5*a^2*b^3)*cosh(d*x
+ c)^5 + 10*(8*a^4*b + 37*a^3*b^2 + 41*a^2*b^3 + 12*a*b^4)*cosh(d*x + c)^3
- 3*(8*a^4*b + 37*a^3*b^2 + 41*a^2*b^3 + 12*a*b^4)*cosh(d*x + c))*sinh(d*x
+ c)^2 + ((8*a^4 + 8*a^3*b + 3*a^2*b^2)*cosh(d*x + c)^8 + 8*(8*a^4 + 8*a^3*
b + 3*a^2*b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (8*a^4 + 8*a^3*b + 3*a^2*b^2
)*sinh(d*x + c)^8 + 4*(8*a^4 + 24*a^3*b + 19*a^2*b^2 + 6*a*b^3)*cosh(d*x +
c)^6 + 4*(8*a^4 + 24*a^3*b + 19*a^2*b^2 + 6*a*b^3 + 7*(8*a^4 + 8*a^3*b + 3*
a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(8*a^4 + 8*a^3*b + 3*a^2*b
^2)*cosh(d*x + c)^3 + 3*(8*a^4 + 24*a^3*b + 19*a^2*b^2 + 6*a*b^3)*cosh(d*x
+ c))*sinh(d*x + c)^5 + 2*(24*a^4 + 88*a^3*b + 137*a^2*b^2 + 88*a*b^3 + 24*
b^4)*cosh(d*x + c)^4 + 2*(35*(8*a^4 + 8*a^3*b + 3*a^2*b^2)*cosh(d*x + c)^4
+ 24*a^4 + 88*a^3*b + 137*a^2*b^2 + 88*a*b^3 + 24*b^4 + 30*(8*a^4 + 24*a^3*
b + 19*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*a^4 + 8*a^3*
b + 3*a^2*b^2 + 8*(7*(8*a^4 + 8*a^3*b + 3*a^2*b^2)*cosh(d*x + c)^5 + 10*(8*
a^4 + 24*a^3*b + 19*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^3 + (24*a^4 + 88*a^3*b
 + 137*a^2*b^2 + 88*a*b^3 + 24*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(8*a
^4 + 24*a^3*b + 19*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^2 + 4*(7*(8*a^4 + 8*a^3
```

```
*b + 3*a^2*b^2)*cosh(d*x + c)^6 + 15*(8*a^4 + 24*a^3*b + 19*a^2*b^2 + 6*a*b
^3)*cosh(d*x + c)^4 + 8*a^4 + 24*a^3*b + 19*a^2*b^2 + 6*a*b^3 + 3*(24*a^4 +
 88*a^3*b + 137*a^2*b^2 + 88*a*b^3 + 24*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)
^2 + 8*((8*a^4 + 8*a^3*b + 3*a^2*b^2)*cosh(d*x + c)^7 + 3*(8*a^4 + 24*a^3*b
 + 19*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^5 + (24*a^4 + 88*a^3*b + 137*a^2*b^2
 + 88*a*b^3 + 24*b^4)*cosh(d*x + c)^3 + (8*a^4 + 24*a^3*b + 19*a^2*b^2 + 6*
a*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a^2 - a*b)*log((a*cosh(d*x + c)^
4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 - 2*(3*a + 2*b)*c
osh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 - 3*a - 2*b)*sinh(d*x + c)^2 + 4*(a
*cosh(d*x + c)^3 - (3*a + 2*b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x +
 c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c
)^2 - 1)*sinh(d*x + c) - cosh(d*x + c))*sqrt(-a^2 - a*b) + a)/(a*cosh(d*x +
 c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)
*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a
*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) - 4*(8*a^4*
b + 13*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c) + 4*(7*(8*a^4*b + 13*a^3*b^2 + 5*
a^2*b^3)*cosh(d*x + c)^6 - 8*a^4*b - 13*a^3*b^2 - 5*a^2*b^3 + 5*(8*a^4*b +
37*a^3*b^2 + 41*a^2*b^3 + 12*a*b^4)*cosh(d*x + c)^4 - 3*(8*a^4*b + 37*a^3*b
^2 + 41*a^2*b^3 + 12*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c))/((a^8 + 3*a^7*b
 + 3*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^8 + 8*(a^8 + 3*a^7*b + 3*a^6*b^2 +
a^5*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^8 + 3*a^7*b + 3*a^6*b^2 + a^5
*b^3)*d*sinh(d*x + c)^8 + 4*(a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a^5*b^3 + 2*a^4*
b^4)*d*cosh(d*x + c)^6 + 4*(7*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*cosh(
d*x + c)^2 + (a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a^5*b^3 + 2*a^4*b^4)*d)*sinh(d*
x + c)^6 + 2*(3*a^8 + 17*a^7*b + 41*a^6*b^2 + 51*a^5*b^3 + 32*a^4*b^4 + 8*a
^3*b^5)*d*cosh(d*x + c)^4 + 8*(7*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*co
sh(d*x + c)^3 + 3*(a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a^5*b^3 + 2*a^4*b^4)*d*cos
h(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d
*cosh(d*x + c)^4 + 30*(a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a^5*b^3 + 2*a^4*b^4)*d
*cosh(d*x + c)^2 + (3*a^8 + 17*a^7*b + 41*a^6*b^2 + 51*a^5*b^3 + 32*a^4*b^4
 + 8*a^3*b^5)*d)*sinh(d*x + c)^4 + 4*(a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a^5*b^3
 + 2*a^4*b^4)*d*cosh(d*x + c)^2 + 8*(7*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3
)*d*cosh(d*x + c)^5 + 10*(a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a^5*b^3 + 2*a^4*b^4
)*d*cosh(d*x + c)^3 + (3*a^8 + 17*a^7*b + 41*a^6*b^2 + 51*a^5*b^3 + 32*a^4*
b^4 + 8*a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^8 + 3*a^7*b + 3
*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^6 + 15*(a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a
^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c)^4 + 3*(3*a^8 + 17*a^7*b + 41*a^6*b^2 +
51*a^5*b^3 + 32*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c)^2 + (a^8 + 5*a^7*b + 9
*a^6*b^2 + 7*a^5*b^3 + 2*a^4*b^4)*d)*sinh(d*x + c)^2 + (a^8 + 3*a^7*b + 3*a
^6*b^2 + a^5*b^3)*d + 8*((a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*cosh(d*x +
 c)^7 + 3*(a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a^5*b^3 + 2*a^4*b^4)*d*cosh(d*x +
c)^5 + (3*a^8 + 17*a^7*b + 41*a^6*b^2 + 51*a^5*b^3 + 32*a^4*b^4 + 8*a^3*b^5
)*d*cosh(d*x + c)^3 + (a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a^5*b^3 + 2*a^4*b^4)*d
*cosh(d*x + c))*sinh(d*x + c)), -1/8*(2*(8*a^4*b + 13*a^3*b^2 + 5*a^2*b^3)*
cosh(d*x + c)^7 + 14*(8*a^4*b + 13*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)*sinh(
```

d*x + c)^6 + 2*(8*a^4*b + 13*a^3*b^2 + 5*a^2*b^3)*sinh(d*x + c)^7 + 2*(8*a^
4*b + 37*a^3*b^2 + 41*a^2*b^3 + 12*a*b^4)*cosh(d*x + c)^5 + 2*(8*a^4*b + 37
*a^3*b^2 + 41*a^2*b^3 + 12*a*b^4 + 21*(8*a^4*b + 13*a^3*b^2 + 5*a^2*b^3)*co
sh(d*x + c)^2)*sinh(d*x + c)^5 + 10*(7*(8*a^4*b + 13*a^3*b^2 + 5*a^2*b^3)*c
osh(d*x + c)^3 + (8*a^4*b + 37*a^3*b^2 + 41*a^2*b^3 + 12*a*b^4)*cosh(d*x +
c))*sinh(d*x + c)^4 - 2*(8*a^4*b + 37*a^3*b^2 + 41*a^2*b^3 + 12*a*b^4)*cosh
(d*x + c)^3 - 2*(8*a^4*b + 37*a^3*b^2 + 41*a^2*b^3 + 12*a*b^4 - 35*(8*a^4*b
 + 13*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)^4 - 10*(8*a^4*b + 37*a^3*b^2 + 41*
a^2*b^3 + 12*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 2*(21*(8*a^4*b + 13*
a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)^5 + 10*(8*a^4*b + 37*a^3*b^2 + 41*a^2*b^
3 + 12*a*b^4)*cosh(d*x + c)^3 - 3*(8*a^4*b + 37*a^3*b^2 + 41*a^2*b^3 + 12*a
*b^4)*cosh(d*x + c))*sinh(d*x + c)^2 - ((8*a^4 + 8*a^3*b + 3*a^2*b^2)*cosh(
d*x + c)^8 + 8*(8*a^4 + 8*a^3*b + 3*a^2*b^2)*cosh(d*x + c)*sinh(d*x + c)^7
+ (8*a^4 + 8*a^3*b + 3*a^2*b^2)*sinh(d*x + c)^8 + 4*(8*a^4 + 24*a^3*b + 19*
a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^6 + 4*(8*a^4 + 24*a^3*b + 19*a^2*b^2 + 6*a
b^3 + 7(8*a^4 + 8*a^3*b + 3*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8
(7(8*a^4 + 8*a^3*b + 3*a^2*b^2)*cosh(d*x + c)^3 + 3*(8*a^4 + 24*a^3*b + 1
9*a^2*b^2 + 6*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(24*a^4 + 88*a^3*b
+ 137*a^2*b^2 + 88*a*b^3 + 24*b^4)*cosh(d*x + c)^4 + 2*(35*(8*a^4 + 8*a^3*b
 + 3*a^2*b^2)*cosh(d*x + c)^4 + 24*a^4 + 88*a^3*b + 137*a^2*b^2 + 88*a*b^3
+ 24*b^4 + 30*(8*a^4 + 24*a^3*b + 19*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^2)*si
nh(d*x + c)^4 + 8*a^4 + 8*a^3*b + 3*a^2*b^2 + 8*(7*(8*a^4 + 8*a^3*b + 3*a^2
*b^2)*cosh(d*x + c)^5 + 10*(8*a^4 + 24*a^3*b + 19*a^2*b^2 + 6*a*b^3)*cosh(d
*x + c)^3 + (24*a^4 + 88*a^3*b + 137*a^2*b^2 + 88*a*b^3 + 24*b^4)*cosh(d*x
+ c))*sinh(d*x + c)^3 + 4*(8*a^4 + 24*a^3*b + 19*a^2*b^2 + 6*a*b^3)*cosh(d*
x + c)^2 + 4*(7*(8*a^4 + 8*a^3*b + 3*a^2*b^2)*cosh(d*x + c)^6 + 15*(8*a^4 +
 24*a^3*b + 19*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^4 + 8*a^4 + 24*a^3*b + 19*a
^2*b^2 + 6*a*b^3 + 3*(24*a^4 + 88*a^3*b + 137*a^2*b^2 + 88*a*b^3 + 24*b^4)*
cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((8*a^4 + 8*a^3*b + 3*a^2*b^2)*cosh(d*
x + c)^7 + 3*(8*a^4 + 24*a^3*b + 19*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^5 + (2
4*a^4 + 88*a^3*b + 137*a^2*b^2 + 88*a*b^3 + 24*b^4)*cosh(d*x + c)^3 + (8*a^
4 + 24*a^3*b + 19*a^2*b^2 + 6*a*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2
 + a*b)*arctan(1/2*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 +
 a*sinh(d*x + c)^3 + (3*a + 4*b)*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + 3*a
 + 4*b)*sinh(d*x + c))/sqrt(a^2 + a*b)) - ((8*a^4 + 8*a^3*b + 3*a^2*b^2)*co
sh(d*x + c)^8 + 8*(8*a^4 + 8*a^3*b + 3*a^2*b^2)*cosh(d*x + c)*sinh(d*x + c)
^7 + (8*a^4 + 8*a^3*b + 3*a^2*b^2)*sinh(d*x + c)^8 + 4*(8*a^4 + 24*a^3*b +
19*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^6 + 4*(8*a^4 + 24*a^3*b + 19*a^2*b^2 +
6*a*b^3 + 7*(8*a^4 + 8*a^3*b + 3*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^6
+ 8*(7*(8*a^4 + 8*a^3*b + 3*a^2*b^2)*cosh(d*x + c)^3 + 3*(8*a^4 + 24*a^3*b
+ 19*a^2*b^2 + 6*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(24*a^4 + 88*a^3
*b + 137*a^2*b^2 + 88*a*b^3 + 24*b^4)*cosh(d*x + c)^4 + 2*(35*(8*a^4 + 8*a^
3*b + 3*a^2*b^2)*cosh(d*x + c)^4 + 24*a^4 + 88*a^3*b + 137*a^2*b^2 + 88*a*b
^3 + 24*b^4 + 30*(8*a^4 + 24*a^3*b + 19*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^2)
*sinh(d*x + c)^4 + 8*a^4 + 8*a^3*b + 3*a^2*b^2 + 8*(7*(8*a^4 + 8*a^3*b + 3*

```
a^2*b^2)*cosh(d*x + c)^5 + 10*(8*a^4 + 24*a^3*b + 19*a^2*b^2 + 6*a*b^3)*cos
h(d*x + c)^3 + (24*a^4 + 88*a^3*b + 137*a^2*b^2 + 88*a*b^3 + 24*b^4)*cosh(d
*x + c))*sinh(d*x + c)^3 + 4*(8*a^4 + 24*a^3*b + 19*a^2*b^2 + 6*a*b^3)*cosh
(d*x + c)^2 + 4*(7*(8*a^4 + 8*a^3*b + 3*a^2*b^2)*cosh(d*x + c)^6 + 15*(8*a^
4 + 24*a^3*b + 19*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^4 + 8*a^4 + 24*a^3*b + 1
9*a^2*b^2 + 6*a*b^3 + 3*(24*a^4 + 88*a^3*b + 137*a^2*b^2 + 88*a*b^3 + 24*b^
4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((8*a^4 + 8*a^3*b + 3*a^2*b^2)*cosh
(d*x + c)^7 + 3*(8*a^4 + 24*a^3*b + 19*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^5 +
 (24*a^4 + 88*a^3*b + 137*a^2*b^2 + 88*a*b^3 + 24*b^4)*cosh(d*x + c)^3 + (8
*a^4 + 24*a^3*b + 19*a^2*b^2 + 6*a*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(
a^2 + a*b)*arctan(1/2*sqrt(a^2 + a*b)*(cosh(d*x + c) + sinh(d*x + c))/(a +
b)) - 2*(8*a^4*b + 13*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c) + 2*(7*(8*a^4*b +
13*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)^6 - 8*a^4*b - 13*a^3*b^2 - 5*a^2*b^3
+ 5*(8*a^4*b + 37*a^3*b^2 + 41*a^2*b^3 + 12*a*b^4)*cosh(d*x + c)^4 - 3*(8*a
^4*b + 37*a^3*b^2 + 41*a^2*b^3 + 12*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c))/
((a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^8 + 8*(a^8 + 3*a^7*b
 + 3*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^8 + 3*a^7*b +
3*a^6*b^2 + a^5*b^3)*d*sinh(d*x + c)^8 + 4*(a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a
^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c)^6 + 4*(7*(a^8 + 3*a^7*b + 3*a^6*b^2 + a
^5*b^3)*d*cosh(d*x + c)^2 + (a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a^5*b^3 + 2*a^4*
b^4)*d)*sinh(d*x + c)^6 + 2*(3*a^8 + 17*a^7*b + 41*a^6*b^2 + 51*a^5*b^3 + 3
2*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c)^4 + 8*(7*(a^8 + 3*a^7*b + 3*a^6*b^2
+ a^5*b^3)*d*cosh(d*x + c)^3 + 3*(a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a^5*b^3 + 2
*a^4*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^8 + 3*a^7*b + 3*a^6*b
^2 + a^5*b^3)*d*cosh(d*x + c)^4 + 30*(a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a^5*b^3
 + 2*a^4*b^4)*d*cosh(d*x + c)^2 + (3*a^8 + 17*a^7*b + 41*a^6*b^2 + 51*a^5*b
^3 + 32*a^4*b^4 + 8*a^3*b^5)*d)*sinh(d*x + c)^4 + 4*(a^8 + 5*a^7*b + 9*a^6*
b^2 + 7*a^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c)^2 + 8*(7*(a^8 + 3*a^7*b + 3*a^
6*b^2 + a^5*b^3)*d*cosh(d*x + c)^5 + 10*(a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a^5*
b^3 + 2*a^4*b^4)*d*cosh(d*x + c)^3 + (3*a^8 + 17*a^7*b + 41*a^6*b^2 + 51*a^
5*b^3 + 32*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^
8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^6 + 15*(a^8 + 5*a^7*b +
9*a^6*b^2 + 7*a^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c)^4 + 3*(3*a^8 + 17*a^7*b
+ 41*a^6*b^2 + 51*a^5*b^3 + 32*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c)^2 + (a^
8 + 5*a^7*b + 9*a^6*b^2 + 7*a^5*b^3 + 2*a^4*b^4)*d)*sinh(d*x + c)^2 + (a^8
+ 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d + 8*((a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^
3)*d*cosh(d*x + c)^7 + 3*(a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a^5*b^3 + 2*a^4*b^4
)*d*cosh(d*x + c)^5 + (3*a^8 + 17*a^7*b + 41*a^6*b^2 + 51*a^5*b^3 + 32*a^4*
b^4 + 8*a^3*b^5)*d*cosh(d*x + c)^3 + (a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a^5*b^3
 + 2*a^4*b^4)*d*cosh(d*x + c))*sinh(d*x + c))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[84,-86]Warning, need to choose a branch for the root of a p
olynomial with parameters. This might be wrong.The choice was done assuming
 [a,b]=[-42,-12]Warning, need to choose a branch for the root of a polynomi
al with parameters. This might be wrong.The choice was done assuming [a,b]=
[-43,-99]Warning, need to choose a branch for the root of a polynomial with
 parameters. This might be wrong.The choice was done assuming [a,b]=[-28,94
]Warning, need to choose a branch for the root of a polynomial with paramet
ers. This might be wrong.The choice was done assuming [a,b]=[-7,46]Warning,
 need to choose a branch for the root of a polynomial with parameters. This
 might be wrong.The choice was done assuming [a,b]=[-35,-99]Undef/Unsigned
Inf encountered in limitEvaluation time: 1.18Limit: Max order reached or un
able to make series expansion Error: Bad Argument Value
```

maple [B] time = 0.40, size = 1172, normalized size = 8.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)/(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] 2/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^
2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*b/(a+b)/a*tanh(1/2*d*x+1/2*c)^7+3/4/d/
a^2*b^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2
*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)^7+2/d/(t
anh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*
tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*b/(a+b)^2*tanh(1/2*d*x+1/2*c)^5-13/4/d/a*b^2
/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a
-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^5-9/4/d/a^2*b
^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2
*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^5-2/d/(tanh
(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tan
h(1/2*d*x+1/2*c)^2*b+a+b)^2*b/(a+b)^2*tanh(1/2*d*x+1/2*c)^3+13/4/d/a*b^2/(t
anh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*
tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^3+9/4/d/a^2*b^3/
(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-
2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^3-2/d/(tanh(1/
2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1
/2*d*x+1/2*c)^2*b+a+b)^2*b/(a+b)/a*tanh(1/2*d*x+1/2*c)-3/4/d/a^2*b^2/(tanh(
1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh
(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)+1/d/(a^2+2*a*b+b^2)/(a
```

```
+b)^(1/2)/a^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)-2*b^(1/2))/
a^(1/2))+1/d/a^(3/2)*b/(a^2+2*a*b+b^2)/(a+b)^(1/2)*arctan(1/2*(2*(a+b)^(1/2
)*tanh(1/2*d*x+1/2*c)-2*b^(1/2))/a^(1/2))+3/8/d/a^(5/2)*b^2/(a^2+2*a*b+b^2)
/(a+b)^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)-2*b^(1/2))/a^(1/
2))+1/d/(a^2+2*a*b+b^2)/(a+b)^(1/2)/a^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(
1/2*d*x+1/2*c)+2*b^(1/2))/a^(1/2))+1/d/a^(3/2)*b/(a^2+2*a*b+b^2)/(a+b)^(1/2
)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)+2*b^(1/2))/a^(1/2))+3/8/d/a
^(5/2)*b^2/(a^2+2*a*b+b^2)/(a+b)^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d
*x+1/2*c)+2*b^(1/2))/a^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(8\,a^2be^{(7\,c)}+5\,ab^2e^{(7\,c)}\right)e^{(7\,dx)}+\left(8\,a^2be^{(5\,c)}+29\right.}{4\left(a^6d+2\,a^5bd+a^4b^2d+\left(a^6de^{(8\,c)}+2\,a^5bde^{(8\,c)}+a^4b^2de^{(8\,c)}\right)e^{(8\,dx)}+4\left(a^6de^{(6\,c)}+4\,a^5bde^{(6\,c)}+5\,a^4b^2de^{(6\,c)}+2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*((8*a^2*b*e^(7*c) + 5*a*b^2*e^(7*c))*e^(7*d*x) + (8*a^2*b*e^(5*c) + 29
*a*b^2*e^(5*c) + 12*b^3*e^(5*c))*e^(5*d*x) - (8*a^2*b*e^(3*c) + 29*a*b^2*e^
(3*c) + 12*b^3*e^(3*c))*e^(3*d*x) - (8*a^2*b*e^c + 5*a*b^2*e^c)*e^(d*x))/(a
^6*d + 2*a^5*b*d + a^4*b^2*d + (a^6*d*e^(8*c) + 2*a^5*b*d*e^(8*c) + a^4*b^2
*d*e^(8*c))*e^(8*d*x) + 4*(a^6*d*e^(6*c) + 4*a^5*b*d*e^(6*c) + 5*a^4*b^2*d*
e^(6*c) + 2*a^3*b^3*d*e^(6*c))*e^(6*d*x) + 2*(3*a^6*d*e^(4*c) + 14*a^5*b*d*
e^(4*c) + 27*a^4*b^2*d*e^(4*c) + 24*a^3*b^3*d*e^(4*c) + 8*a^2*b^4*d*e^(4*c)
)*e^(4*d*x) + 4*(a^6*d*e^(2*c) + 4*a^5*b*d*e^(2*c) + 5*a^4*b^2*d*e^(2*c) +
2*a^3*b^3*d*e^(2*c))*e^(2*d*x)) + 2*integrate(1/8*((8*a^2*e^(3*c) + 8*a*b*e
^(3*c) + 3*b^2*e^(3*c))*e^(3*d*x) + (8*a^2*e^c + 8*a*b*e^c + 3*b^2*e^c)*e^(
d*x))/(a^5 + 2*a^4*b + a^3*b^2 + (a^5*e^(4*c) + 2*a^4*b*e^(4*c) + a^3*b^2*e
^(4*c))*e^(4*d*x) + 2*(a^5*e^(2*c) + 4*a^4*b*e^(2*c) + 5*a^3*b^2*e^(2*c) +
2*a^2*b^3*e^(2*c))*e^(2*d*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh\left(c+d\,x\right)\,{\left(a+\frac{b}{{\cosh\left(c+d\,x\right)}^2}\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)*(a + b/cosh(c + d*x)^2)^3),x)
```

```
[Out] int(1/(cosh(c + d*x)*(a + b/cosh(c + d*x)^2)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Integral(sech(c + d*x)/(a + b*sech(c + d*x)**2)**3, x)
```

3.96 $\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=108

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8\sqrt{b}\,d(a+b)^{5/2}}+\frac{3\tanh(c+dx)}{8d(a+b)^2\left(a-b\tanh^2(c+dx)+b\right)}+\frac{\tanh(c+dx)}{4d(a+b)\left(a-b\tanh^2(c+dx)+b\right)^2}$$

```
[Out] 3/8*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))/(a+b)^(5/2)/d/b^(1/2)+1/4*tanh
(d*x+c)/(a+b)/d/(a+b-b*tanh(d*x+c)^2)^2+3/8*tanh(d*x+c)/(a+b)^2/d/(a+b-b*ta
nh(d*x+c)^2)
```

Rubi [A] time = 0.09, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4146, 199, 208}

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8\sqrt{b}\,d(a+b)^{5/2}}+\frac{3\tanh(c+dx)}{8d(a+b)^2\left(a-b\tanh^2(c+dx)+b\right)}+\frac{\tanh(c+dx)}{4d(a+b)\left(a-b\tanh^2(c+dx)+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^2/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (3*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(8*Sqrt[b]*(a + b)^(5/2)*d
) + Tanh[c + d*x]/(4*(a + b)*d*(a + b - b*Tanh[c + d*x]^2)^2) + (3*Tanh[c +
 d*x])/(8*(a + b)^2*d*(a + b - b*Tanh[c + d*x]^2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{\left(a+b-bx^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\tanh(c+dx)}{4(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{3\operatorname{Subst}\left(\int \frac{1}{\left(a+b-bx^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{4(a+b)d}$$

$$= \frac{\tanh(c+dx)}{4(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{3\tanh(c+dx)}{8(a+b)^2d\left(a+b-b\tanh^2(c+dx)\right)} + -$$

$$= \frac{3\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8\sqrt{b}\,(a+b)^{5/2}d} + \frac{\tanh(c+dx)}{4(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{3\,\text{ta}}{8(a+b)^2d\left(a\right.}$$

Mathematica [B] time = 2.17, size = 258, normalized size = 2.39

$$\frac{\operatorname{sech}^6(c+dx)(a\cosh(2(c+dx))+a+2b)\left(-\frac{\operatorname{sech}(2c)\left(\left(5a^2+16ab+8b^2\right)\sinh(2c)-a(5a+2b)\sinh(2dx)\right)(a\cosh(2(c+dx))+a+2b)}{a^2}+\frac{4b}{}\right.}{64d(a+b)^2\left(\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^2/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^6*((3*ArcTanh[(Sech[d*x]*(Co
sh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a +
 b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(a + 2*b + a*Cosh[2*(c + d*x)])^2*(Cosh
[2*c] - Sinh[2*c]))/(Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4]) + (4*b*(a +
```

```
 b)*Sech[2*c]*((a + 2*b)*Sinh[2*c] - a*Sinh[2*d*x]))/a^2 - ((a + 2*b + a*Co
sh[2*(c + d*x)])*Sech[2*c]*((5*a^2 + 16*a*b + 8*b^2)*Sinh[2*c] - a*(5*a + 2
*b)*Sinh[2*d*x]))/a^2))/(64*(a + b)^2*d*(a + b*Sech[c + d*x]^2)^3)
```

fricas [B] time = 0.51, size = 5109, normalized size = 47.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*(5*a^4*b + 21*a^3*b^2 + 24*a^2*b^3 + 8*a*b^4)*cosh(d*x + c)^6 + 2
4*(5*a^4*b + 21*a^3*b^2 + 24*a^2*b^3 + 8*a*b^4)*cosh(d*x + c)*sinh(d*x + c)
^5 + 4*(5*a^4*b + 21*a^3*b^2 + 24*a^2*b^3 + 8*a*b^4)*sinh(d*x + c)^6 + 20*a
^4*b + 28*a^3*b^2 + 8*a^2*b^3 + 4*(15*a^4*b + 61*a^3*b^2 + 102*a^2*b^3 + 72
*a*b^4 + 16*b^5)*cosh(d*x + c)^4 + 4*(15*a^4*b + 61*a^3*b^2 + 102*a^2*b^3 +
 72*a*b^4 + 16*b^5 + 15*(5*a^4*b + 21*a^3*b^2 + 24*a^2*b^3 + 8*a*b^4)*cosh(
d*x + c)^2)*sinh(d*x + c)^4 + 16*(5*(5*a^4*b + 21*a^3*b^2 + 24*a^2*b^3 + 8*
a*b^4)*cosh(d*x + c)^3 + (15*a^4*b + 61*a^3*b^2 + 102*a^2*b^3 + 72*a*b^4 +
16*b^5)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(15*a^4*b + 47*a^3*b^2 + 40*a^2*
b^3 + 8*a*b^4)*cosh(d*x + c)^2 + 4*(15*a^4*b + 47*a^3*b^2 + 40*a^2*b^3 + 8*
a*b^4 + 15*(5*a^4*b + 21*a^3*b^2 + 24*a^2*b^3 + 8*a*b^4)*cosh(d*x + c)^4 +
6*(15*a^4*b + 61*a^3*b^2 + 102*a^2*b^3 + 72*a*b^4 + 16*b^5)*cosh(d*x + c)^2
)*sinh(d*x + c)^2 - 3*(a^4*cosh(d*x + c)^8 + 8*a^4*cosh(d*x + c)*sinh(d*x +
 c)^7 + a^4*sinh(d*x + c)^8 + 4*(a^4 + 2*a^3*b)*cosh(d*x + c)^6 + 4*(7*a^4*
cosh(d*x + c)^2 + a^4 + 2*a^3*b)*sinh(d*x + c)^6 + 8*(7*a^4*cosh(d*x + c)^3
 + 3*(a^4 + 2*a^3*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*a^4 + 8*a^3*b +
8*a^2*b^2)*cosh(d*x + c)^4 + 2*(35*a^4*cosh(d*x + c)^4 + 3*a^4 + 8*a^3*b +
8*a^2*b^2 + 30*(a^4 + 2*a^3*b)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + a^4 + 8*(
7*a^4*cosh(d*x + c)^5 + 10*(a^4 + 2*a^3*b)*cosh(d*x + c)^3 + (3*a^4 + 8*a^3
*b + 8*a^2*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a^4 + 2*a^3*b)*cosh(d*x
 + c)^2 + 4*(7*a^4*cosh(d*x + c)^6 + 15*(a^4 + 2*a^3*b)*cosh(d*x + c)^4 + a
^4 + 2*a^3*b + 3*(3*a^4 + 8*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x +
c)^2 + 8*(a^4*cosh(d*x + c)^7 + 3*(a^4 + 2*a^3*b)*cosh(d*x + c)^5 + (3*a^4
+ 8*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^3 + (a^4 + 2*a^3*b)*cosh(d*x + c))*sin
h(d*x + c))*sqrt(a*b + b^2)*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*
sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2
*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^
2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) - 4
*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 +
 a + 2*b)*sqrt(a*b + b^2))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x
+ c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x
+ c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d
*x + c))*sinh(d*x + c) + a)) + 8*(3*(5*a^4*b + 21*a^3*b^2 + 24*a^2*b^3 + 8*
a*b^4)*cosh(d*x + c)^5 + 2*(15*a^4*b + 61*a^3*b^2 + 102*a^2*b^3 + 72*a*b^4
```

+ 16*b^5)*cosh(d*x + c)^3 + (15*a^4*b + 47*a^3*b^2 + 40*a^2*b^3 + 8*a*b^4)*
cosh(d*x + c))*sinh(d*x + c))/((a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*d*
cosh(d*x + c)^8 + 8*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*d*cosh(d*x +
c)*sinh(d*x + c)^7 + (a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*d*sinh(d*x +
c)^8 + 4*(a^7*b + 5*a^6*b^2 + 9*a^5*b^3 + 7*a^4*b^4 + 2*a^3*b^5)*d*cosh(d*
x + c)^6 + 4*(7*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*d*cosh(d*x + c)^2
+ (a^7*b + 5*a^6*b^2 + 9*a^5*b^3 + 7*a^4*b^4 + 2*a^3*b^5)*d)*sinh(d*x + c)
^6 + 2*(3*a^7*b + 17*a^6*b^2 + 41*a^5*b^3 + 51*a^4*b^4 + 32*a^3*b^5 + 8*a^2
*b^6)*d*cosh(d*x + c)^4 + 8*(7*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*d*
cosh(d*x + c)^3 + 3*(a^7*b + 5*a^6*b^2 + 9*a^5*b^3 + 7*a^4*b^4 + 2*a^3*b^5)
*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3 +
a^4*b^4)*d*cosh(d*x + c)^4 + 30*(a^7*b + 5*a^6*b^2 + 9*a^5*b^3 + 7*a^4*b^4
+ 2*a^3*b^5)*d*cosh(d*x + c)^2 + (3*a^7*b + 17*a^6*b^2 + 41*a^5*b^3 + 51*a^
4*b^4 + 32*a^3*b^5 + 8*a^2*b^6)*d)*sinh(d*x + c)^4 + 4*(a^7*b + 5*a^6*b^2 +
9*a^5*b^3 + 7*a^4*b^4 + 2*a^3*b^5)*d*cosh(d*x + c)^2 + 8*(7*(a^7*b + 3*a^6
*b^2 + 3*a^5*b^3 + a^4*b^4)*d*cosh(d*x + c)^5 + 10*(a^7*b + 5*a^6*b^2 + 9*a
^5*b^3 + 7*a^4*b^4 + 2*a^3*b^5)*d*cosh(d*x + c)^3 + (3*a^7*b + 17*a^6*b^2 +
41*a^5*b^3 + 51*a^4*b^4 + 32*a^3*b^5 + 8*a^2*b^6)*d*cosh(d*x + c))*sinh(d*
x + c)^3 + 4*(7*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*d*cosh(d*x + c)^6
+ 15*(a^7*b + 5*a^6*b^2 + 9*a^5*b^3 + 7*a^4*b^4 + 2*a^3*b^5)*d*cosh(d*x +
c)^4 + 3*(3*a^7*b + 17*a^6*b^2 + 41*a^5*b^3 + 51*a^4*b^4 + 32*a^3*b^5 + 8*a
^2*b^6)*d*cosh(d*x + c)^2 + (a^7*b + 5*a^6*b^2 + 9*a^5*b^3 + 7*a^4*b^4 + 2*
a^3*b^5)*d)*sinh(d*x + c)^2 + (a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*d +
8*((a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*d*cosh(d*x + c)^7 + 3*(a^7*b
+ 5*a^6*b^2 + 9*a^5*b^3 + 7*a^4*b^4 + 2*a^3*b^5)*d*cosh(d*x + c)^5 + (3*a^7
*b + 17*a^6*b^2 + 41*a^5*b^3 + 51*a^4*b^4 + 32*a^3*b^5 + 8*a^2*b^6)*d*cosh(
d*x + c)^3 + (a^7*b + 5*a^6*b^2 + 9*a^5*b^3 + 7*a^4*b^4 + 2*a^3*b^5)*d*cosh
(d*x + c))*sinh(d*x + c)), -1/8*(2*(5*a^4*b + 21*a^3*b^2 + 24*a^2*b^3 + 8*a
*b^4)*cosh(d*x + c)^6 + 12*(5*a^4*b + 21*a^3*b^2 + 24*a^2*b^3 + 8*a*b^4)*co
sh(d*x + c)*sinh(d*x + c)^5 + 2*(5*a^4*b + 21*a^3*b^2 + 24*a^2*b^3 + 8*a*b^
4)*sinh(d*x + c)^6 + 10*a^4*b + 14*a^3*b^2 + 4*a^2*b^3 + 2*(15*a^4*b + 61*a
^3*b^2 + 102*a^2*b^3 + 72*a*b^4 + 16*b^5)*cosh(d*x + c)^4 + 2*(15*a^4*b + 6
1*a^3*b^2 + 102*a^2*b^3 + 72*a*b^4 + 16*b^5 + 15*(5*a^4*b + 21*a^3*b^2 + 24
*a^2*b^3 + 8*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(5*(5*a^4*b + 21*a
^3*b^2 + 24*a^2*b^3 + 8*a*b^4)*cosh(d*x + c)^3 + (15*a^4*b + 61*a^3*b^2 + 1
02*a^2*b^3 + 72*a*b^4 + 16*b^5)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(15*a^4*
b + 47*a^3*b^2 + 40*a^2*b^3 + 8*a*b^4)*cosh(d*x + c)^2 + 2*(15*a^4*b + 47*a
^3*b^2 + 40*a^2*b^3 + 8*a*b^4 + 15*(5*a^4*b + 21*a^3*b^2 + 24*a^2*b^3 + 8*a
*b^4)*cosh(d*x + c)^4 + 6*(15*a^4*b + 61*a^3*b^2 + 102*a^2*b^3 + 72*a*b^4 +
16*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 3*(a^4*cosh(d*x + c)^8 + 8*a^4*
cosh(d*x + c)*sinh(d*x + c)^7 + a^4*sinh(d*x + c)^8 + 4*(a^4 + 2*a^3*b)*cos
h(d*x + c)^6 + 4*(7*a^4*cosh(d*x + c)^2 + a^4 + 2*a^3*b)*sinh(d*x + c)^6 +
8*(7*a^4*cosh(d*x + c)^3 + 3*(a^4 + 2*a^3*b)*cosh(d*x + c))*sinh(d*x + c)^5
+ 2*(3*a^4 + 8*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^4 + 2*(35*a^4*cosh(d*x + c
)^4 + 3*a^4 + 8*a^3*b + 8*a^2*b^2 + 30*(a^4 + 2*a^3*b)*cosh(d*x + c)^2)*sin

```
h(d*x + c)^4 + a^4 + 8*(7*a^4*cosh(d*x + c)^5 + 10*(a^4 + 2*a^3*b)*cosh(d*x
 + c)^3 + (3*a^4 + 8*a^3*b + 8*a^2*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*
(a^4 + 2*a^3*b)*cosh(d*x + c)^2 + 4*(7*a^4*cosh(d*x + c)^6 + 15*(a^4 + 2*a^
3*b)*cosh(d*x + c)^4 + a^4 + 2*a^3*b + 3*(3*a^4 + 8*a^3*b + 8*a^2*b^2)*cosh
(d*x + c)^2)*sinh(d*x + c)^2 + 8*(a^4*cosh(d*x + c)^7 + 3*(a^4 + 2*a^3*b)*c
osh(d*x + c)^5 + (3*a^4 + 8*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^3 + (a^4 + 2*a
^3*b)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a*b - b^2)*arctan(1/2*(a*cosh(d*x
 + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sq
rt(-a*b - b^2)/(a*b + b^2)) + 4*(3*(5*a^4*b + 21*a^3*b^2 + 24*a^2*b^3 + 8*a
*b^4)*cosh(d*x + c)^5 + 2*(15*a^4*b + 61*a^3*b^2 + 102*a^2*b^3 + 72*a*b^4 +
 16*b^5)*cosh(d*x + c)^3 + (15*a^4*b + 47*a^3*b^2 + 40*a^2*b^3 + 8*a*b^4)*c
osh(d*x + c))*sinh(d*x + c))/((a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*d*c
osh(d*x + c)^8 + 8*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*d*cosh(d*x + c
)*sinh(d*x + c)^7 + (a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*d*sinh(d*x +
c)^8 + 4*(a^7*b + 5*a^6*b^2 + 9*a^5*b^3 + 7*a^4*b^4 + 2*a^3*b^5)*d*cosh(d*x
 + c)^6 + 4*(7*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*d*cosh(d*x + c)^2
+ (a^7*b + 5*a^6*b^2 + 9*a^5*b^3 + 7*a^4*b^4 + 2*a^3*b^5)*d)*sinh(d*x + c)^
6 + 2*(3*a^7*b + 17*a^6*b^2 + 41*a^5*b^3 + 51*a^4*b^4 + 32*a^3*b^5 + 8*a^2*
b^6)*d*cosh(d*x + c)^4 + 8*(7*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*d*c
osh(d*x + c)^3 + 3*(a^7*b + 5*a^6*b^2 + 9*a^5*b^3 + 7*a^4*b^4 + 2*a^3*b^5)*
d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a
^4*b^4)*d*cosh(d*x + c)^4 + 30*(a^7*b + 5*a^6*b^2 + 9*a^5*b^3 + 7*a^4*b^4 +
 2*a^3*b^5)*d*cosh(d*x + c)^2 + (3*a^7*b + 17*a^6*b^2 + 41*a^5*b^3 + 51*a^4
*b^4 + 32*a^3*b^5 + 8*a^2*b^6)*d)*sinh(d*x + c)^4 + 4*(a^7*b + 5*a^6*b^2 +
9*a^5*b^3 + 7*a^4*b^4 + 2*a^3*b^5)*d*cosh(d*x + c)^2 + 8*(7*(a^7*b + 3*a^6*
b^2 + 3*a^5*b^3 + a^4*b^4)*d*cosh(d*x + c)^5 + 10*(a^7*b + 5*a^6*b^2 + 9*a^
5*b^3 + 7*a^4*b^4 + 2*a^3*b^5)*d*cosh(d*x + c)^3 + (3*a^7*b + 17*a^6*b^2 +
41*a^5*b^3 + 51*a^4*b^4 + 32*a^3*b^5 + 8*a^2*b^6)*d*cosh(d*x + c))*sinh(d*x
 + c)^3 + 4*(7*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*d*cosh(d*x + c)^6
+ 15*(a^7*b + 5*a^6*b^2 + 9*a^5*b^3 + 7*a^4*b^4 + 2*a^3*b^5)*d*cosh(d*x + c
)^4 + 3*(3*a^7*b + 17*a^6*b^2 + 41*a^5*b^3 + 51*a^4*b^4 + 32*a^3*b^5 + 8*a^
2*b^6)*d*cosh(d*x + c)^2 + (a^7*b + 5*a^6*b^2 + 9*a^5*b^3 + 7*a^4*b^4 + 2*a
^3*b^5)*d)*sinh(d*x + c)^2 + (a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*d +
8*((a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*d*cosh(d*x + c)^7 + 3*(a^7*b +
 5*a^6*b^2 + 9*a^5*b^3 + 7*a^4*b^4 + 2*a^3*b^5)*d*cosh(d*x + c)^5 + (3*a^7*
b + 17*a^6*b^2 + 41*a^5*b^3 + 51*a^4*b^4 + 32*a^3*b^5 + 8*a^2*b^6)*d*cosh(d
*x + c)^3 + (a^7*b + 5*a^6*b^2 + 9*a^5*b^3 + 7*a^4*b^4 + 2*a^3*b^5)*d*cosh(
d*x + c))*sinh(d*x + c))]
```

giac [B] time = 1.75, size = 282, normalized size = 2.61

$$\frac{\frac{3\arctan\left(\frac{ae^{(2dx+2c)}+a+2b}{2\sqrt{-ab-b^2}}\right)}{(a^2+2ab+b^2)\sqrt{-ab-b^2}} - \frac{2\left(5a^3e^{(6dx+6c)}+16a^2be^{(6dx+6c)}+8ab^2e^{(6dx+6c)}+15a^3e^{(4dx+4c)}+46a^2be^{(4dx+4c)}+56ab^2e^{(4dx+4c)}+16b^3e^{(4dx+4c)}+15\right.}{(a^4+2a^3b+a^2b^2)\left(ae^{(4dx+4c)}+2ae^{(2dx+2c)}+4be^{(2dx+2c)}+a\right)^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/((a^2 + 2
*a*b + b^2)*sqrt(-a*b - b^2)) - 2*(5*a^3*e^(6*d*x + 6*c) + 16*a^2*b*e^(6*d*
x + 6*c) + 8*a*b^2*e^(6*d*x + 6*c) + 15*a^3*e^(4*d*x + 4*c) + 46*a^2*b*e^(4
*d*x + 4*c) + 56*a*b^2*e^(4*d*x + 4*c) + 16*b^3*e^(4*d*x + 4*c) + 15*a^3*e^
(2*d*x + 2*c) + 32*a^2*b*e^(2*d*x + 2*c) + 8*a*b^2*e^(2*d*x + 2*c) + 5*a^3
+ 2*a^2*b)/((a^4 + 2*a^3*b + a^2*b^2)*(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2
*c) + 4*b*e^(2*d*x + 2*c) + a)^2))/d
```

maple [B] time = 0.34, size = 615, normalized size = 5.69

$$\frac{5\left(\tanh^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d\left(\left(\tanh^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+b\left(\tanh^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a+b\right)^{2}(a+b)}+\frac{}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] 5/4/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c
)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)^7+15/4/d/(
tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2
*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^5*a+3/4/d/(tanh
(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tan
h(1/2*d*x+1/2*c)^2*b+a+b)^2*b/(a+b)^2*tanh(1/2*d*x+1/2*c)^5+15/4/d/(tanh(1/
2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1
/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^3*a+3/4/d/(tanh(1/2*d*
x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d
*x+1/2*c)^2*b+a+b)^2*b/(a+b)^2*tanh(1/2*d*x+1/2*c)^3+5/4/d/(tanh(1/2*d*x+1/
2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1
/2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)-3/16/d/(a^2+2*a*b+b^2)/b^(1/2)/(
a+b)^(1/2)*ln(-(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2
*c)-(a+b)^(1/2))+3/16/d/(a^2+2*a*b+b^2)/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*
tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))
```

maxima [B] time = 0.51, size = 353, normalized size = 3.27

$$\frac{5a^{3}+2a^{2}b+\left(15a^{3}+32a^{2}b+8ab^{2}\right)e^{(-2dx-2c)}+\left(15a^{3}+46a^{2}b+56ab^{2}\right.}{4\left(a^{6}+2a^{5}b+a^{4}b^{2}+4\left(a^{6}+4a^{5}b+5a^{4}b^{2}+2a^{3}b^{3}\right)e^{(-2dx-2c)}+2\left(3a^{6}+14a^{5}b+27a^{4}b^{2}+24a^{3}b^{3}+8a^{2}b^{4}\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(5*a^3 + 2*a^2*b + (15*a^3 + 32*a^2*b + 8*a*b^2)*e^(-2*d*x - 2*c) + (15
*a^3 + 46*a^2*b + 56*a*b^2 + 16*b^3)*e^(-4*d*x - 4*c) + (5*a^3 + 16*a^2*b +
 8*a*b^2)*e^(-6*d*x - 6*c))/((a^6 + 2*a^5*b + a^4*b^2 + 4*(a^6 + 4*a^5*b +
5*a^4*b^2 + 2*a^3*b^3)*e^(-2*d*x - 2*c) + 2*(3*a^6 + 14*a^5*b + 27*a^4*b^2
+ 24*a^3*b^3 + 8*a^2*b^4)*e^(-4*d*x - 4*c) + 4*(a^6 + 4*a^5*b + 5*a^4*b^2 +
 2*a^3*b^3)*e^(-6*d*x - 6*c) + (a^6 + 2*a^5*b + a^4*b^2)*e^(-8*d*x - 8*c))*
d) - 3/16*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d
*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^2 + 2*a*b + b^2)*sqrt((a + b)
*b)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh\left(c+d\,x\right)^2\,{\left(a+\frac{b}{{\cosh\left(c+d\,x\right)}^2}\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^2*(a + b/cosh(c + d*x)^2)^3),x)
```

```
[Out] int(1/(cosh(c + d*x)^2*(a + b/cosh(c + d*x)^2)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^{2}{\left(c + dx \right)}}{\left(a + b \operatorname{sech}^{2}{\left(c + dx \right)}\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**2/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Integral(sech(c + d*x)**2/(a + b*sech(c + d*x)**2)**3, x)
```

3.97 $\int \frac{\text{sech}^3(c+dx)}{\left(a+b\,\text{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=123

$$\frac{(4a+b)\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{8a^{3/2}d(a+b)^{5/2}}+\frac{(4a+b)\sinh(c+dx)}{8ad(a+b)^2\left(a\sinh^2(c+dx)+a+b\right)}-\frac{b\sinh(c+dx)}{4ad(a+b)\left(a\sinh^2(c+dx)+a+b\right)^2}$$

```
[Out] 1/8*(4*a+b)*arctan(sinh(d*x+c)*a^(1/2)/(a+b)^(1/2))/a^(3/2)/(a+b)^(5/2)/d-1
/4*b*sinh(d*x+c)/a/(a+b)/d/(a+b+a*sinh(d*x+c)^2)^2+1/8*(4*a+b)*sinh(d*x+c)/
a/(a+b)^2/d/(a+b+a*sinh(d*x+c)^2)
```

Rubi [A] time = 0.11, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4147, 385, 199, 205}

$$\frac{(4a+b)\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{8a^{3/2}d(a+b)^{5/2}}+\frac{(4a+b)\sinh(c+dx)}{8ad(a+b)^2\left(a\sinh^2(c+dx)+a+b\right)}-\frac{b\sinh(c+dx)}{4ad(a+b)\left(a\sinh^2(c+dx)+a+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^3/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((4*a + b)*ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]])/(8*a^(3/2)*(a + b)^
(5/2)*d) - (b*Sinh[c + d*x])/(4*a*(a + b)*d*(a + b + a*Sinh[c + d*x]^2)^2)
+ ((4*a + b)*Sinh[c + d*x])/(8*a*(a + b)^2*d*(a + b + a*Sinh[c + d*x]^2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1+x^2}{\left(a+b+ax^2\right)^3}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= -\frac{b\sinh(c+dx)}{4a(a+b)d\left(a+b+a\sinh^2(c+dx)\right)^2} + \frac{(4a+b)\operatorname{Subst}\left(\int \frac{1}{\left(a+b+ax^2\right)^2}\,dx, x, \sinh(\right.}{4a(a+b)d}$$

$$= -\frac{b\sinh(c+dx)}{4a(a+b)d\left(a+b+a\sinh^2(c+dx)\right)^2} + \frac{(4a+b)\sinh(c+dx)}{8a(a+b)^2d\left(a+b+a\sinh^2(c+dx)\right)} +$$

$$= \frac{(4a+b)\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{8a^{3/2}(a+b)^{5/2}d} - \frac{b\sinh(c+dx)}{4a(a+b)d\left(a+b+a\sinh^2(c+dx)\right)^2} + \frac{(4}{8a(a+b)}$$

Mathematica [A] time = 0.79, size = 159, normalized size = 1.29

$$-\frac{\operatorname{sech}^6(c+dx)(a\cosh(2(c+dx))+a+2b)^3\left(\frac{8\sinh(c+dx)}{\left(a\sinh^2(c+dx)+a+b\right)^2} - (4a+b)\left(\frac{5(a+b)\sinh(c+dx)+3a\sinh^3(c+dx)}{(a+b)^2\left(a\sinh^2(c+dx)+a+b\right)^2} + \frac{3\tan^{-1}\left(\right.}{\sqrt{a}\,(}\right.\right.}{192ad\left(a+b\operatorname{sech}^2(c+dx)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^3/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] -1/192*((a + 2*b + a*Cosh[2*(c + d*x)])^3*Sech[c + d*x]^6*((8*Sinh[c + d*x]
)/(a + b + a*Sinh[c + d*x]^2)^2 - (4*a + b)*((3*ArcTan[(Sqrt[a]*Sinh[c + d*
x])/Sqrt[a + b]])/(Sqrt[a]*(a + b)^(5/2)) + (5*(a + b)*Sinh[c + d*x] + 3*a*
Sinh[c + d*x]^3)/((a + b)^2*(a + b + a*Sinh[c + d*x]^2)^2))))/(a*d*(a + b*S
ech[c + d*x]^2)^3)
```

fricas [B] time = 0.53, size = 6037, normalized size = 49.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(4*(4*a^4 + 5*a^3*b + a^2*b^2)*cosh(d*x + c)^7 + 28*(4*a^4 + 5*a^3*b
+ a^2*b^2)*cosh(d*x + c)*sinh(d*x + c)^6 + 4*(4*a^4 + 5*a^3*b + a^2*b^2)*si
nh(d*x + c)^7 + 4*(4*a^4 + 13*a^3*b + 5*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^5
+ 4*(4*a^4 + 13*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 21*(4*a^4 + 5*a^3*b + a^2*b^2
)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 20*(7*(4*a^4 + 5*a^3*b + a^2*b^2)*cosh
(d*x + c)^3 + (4*a^4 + 13*a^3*b + 5*a^2*b^2 - 4*a*b^3)*cosh(d*x + c))*sinh(
d*x + c)^4 - 4*(4*a^4 + 13*a^3*b + 5*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^3 + 4
*(35*(4*a^4 + 5*a^3*b + a^2*b^2)*cosh(d*x + c)^4 - 4*a^4 - 13*a^3*b - 5*a^2
*b^2 + 4*a*b^3 + 10*(4*a^4 + 13*a^3*b + 5*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^
2)*sinh(d*x + c)^3 + 4*(21*(4*a^4 + 5*a^3*b + a^2*b^2)*cosh(d*x + c)^5 + 10
*(4*a^4 + 13*a^3*b + 5*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^3 - 3*(4*a^4 + 13*a
^3*b + 5*a^2*b^2 - 4*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 - ((4*a^3 + a^2*
b)*cosh(d*x + c)^8 + 8*(4*a^3 + a^2*b)*cosh(d*x + c)*sinh(d*x + c)^7 + (4*a
^3 + a^2*b)*sinh(d*x + c)^8 + 4*(4*a^3 + 9*a^2*b + 2*a*b^2)*cosh(d*x + c)^6
 + 4*(4*a^3 + 9*a^2*b + 2*a*b^2 + 7*(4*a^3 + a^2*b)*cosh(d*x + c)^2)*sinh(d
*x + c)^6 + 8*(7*(4*a^3 + a^2*b)*cosh(d*x + c)^3 + 3*(4*a^3 + 9*a^2*b + 2*a
*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(12*a^3 + 35*a^2*b + 40*a*b^2 + 8*
b^3)*cosh(d*x + c)^4 + 2*(35*(4*a^3 + a^2*b)*cosh(d*x + c)^4 + 12*a^3 + 35*
a^2*b + 40*a*b^2 + 8*b^3 + 30*(4*a^3 + 9*a^2*b + 2*a*b^2)*cosh(d*x + c)^2)*
sinh(d*x + c)^4 + 8*(7*(4*a^3 + a^2*b)*cosh(d*x + c)^5 + 10*(4*a^3 + 9*a^2*
b + 2*a*b^2)*cosh(d*x + c)^3 + (12*a^3 + 35*a^2*b + 40*a*b^2 + 8*b^3)*cosh(
d*x + c))*sinh(d*x + c)^3 + 4*a^3 + a^2*b + 4*(4*a^3 + 9*a^2*b + 2*a*b^2)*c
osh(d*x + c)^2 + 4*(7*(4*a^3 + a^2*b)*cosh(d*x + c)^6 + 15*(4*a^3 + 9*a^2*b
 + 2*a*b^2)*cosh(d*x + c)^4 + 4*a^3 + 9*a^2*b + 2*a*b^2 + 3*(12*a^3 + 35*a^
2*b + 40*a*b^2 + 8*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((4*a^3 + a^2*
b)*cosh(d*x + c)^7 + 3*(4*a^3 + 9*a^2*b + 2*a*b^2)*cosh(d*x + c)^5 + (12*a^
3 + 35*a^2*b + 40*a*b^2 + 8*b^3)*cosh(d*x + c)^3 + (4*a^3 + 9*a^2*b + 2*a*b
^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a^2 - a*b)*log((a*cosh(d*x + c)^4 +
 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 - 2*(3*a + 2*b)*cosh
(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 - 3*a - 2*b)*sinh(d*x + c)^2 + 4*(a*co
```

sh(d*x + c)^3 - (3*a + 2*b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)
^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2
 - 1)*sinh(d*x + c) - cosh(d*x + c))*sqrt(-a^2 - a*b) + a)/(a*cosh(d*x + c)
^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*co
sh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*co
sh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) - 4*(4*a^4 + 5
*a^3*b + a^2*b^2)*cosh(d*x + c) + 4*(7*(4*a^4 + 5*a^3*b + a^2*b^2)*cosh(d*x
 + c)^6 + 5*(4*a^4 + 13*a^3*b + 5*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^4 - 4*a^
4 - 5*a^3*b - a^2*b^2 - 3*(4*a^4 + 13*a^3*b + 5*a^2*b^2 - 4*a*b^3)*cosh(d*x
 + c)^2)*sinh(d*x + c))/((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x +
 c)^8 + 8*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)*sinh(d*x +
c)^7 + (a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*sinh(d*x + c)^8 + 4*(a^7 + 5
*a^6*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d*cosh(d*x + c)^6 + 4*(7*(a^7 +
 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^2 + (a^7 + 5*a^6*b + 9*a^5*
b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d)*sinh(d*x + c)^6 + 2*(3*a^7 + 17*a^6*b + 41*
a^5*b^2 + 51*a^4*b^3 + 32*a^3*b^4 + 8*a^2*b^5)*d*cosh(d*x + c)^4 + 8*(7*(a^
7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^3 + 3*(a^7 + 5*a^6*b + 9
*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*
(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^4 + 30*(a^7 + 5*a^6*b
 + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d*cosh(d*x + c)^2 + (3*a^7 + 17*a^6*b
 + 41*a^5*b^2 + 51*a^4*b^3 + 32*a^3*b^4 + 8*a^2*b^5)*d)*sinh(d*x + c)^4 + 4
*(a^7 + 5*a^6*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d*cosh(d*x + c)^2 + 8*
(7*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^5 + 10*(a^7 + 5*a^
6*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d*cosh(d*x + c)^3 + (3*a^7 + 17*a^
6*b + 41*a^5*b^2 + 51*a^4*b^3 + 32*a^3*b^4 + 8*a^2*b^5)*d*cosh(d*x + c))*si
nh(d*x + c)^3 + 4*(7*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^
6 + 15*(a^7 + 5*a^6*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d*cosh(d*x + c)^
4 + 3*(3*a^7 + 17*a^6*b + 41*a^5*b^2 + 51*a^4*b^3 + 32*a^3*b^4 + 8*a^2*b^5)
*d*cosh(d*x + c)^2 + (a^7 + 5*a^6*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d)
*sinh(d*x + c)^2 + (a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d + 8*((a^7 + 3*a^
6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^7 + 3*(a^7 + 5*a^6*b + 9*a^5*b^2
 + 7*a^4*b^3 + 2*a^3*b^4)*d*cosh(d*x + c)^5 + (3*a^7 + 17*a^6*b + 41*a^5*b^
2 + 51*a^4*b^3 + 32*a^3*b^4 + 8*a^2*b^5)*d*cosh(d*x + c)^3 + (a^7 + 5*a^6*b
 + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d*cosh(d*x + c))*sinh(d*x + c)), 1/8*
(2*(4*a^4 + 5*a^3*b + a^2*b^2)*cosh(d*x + c)^7 + 14*(4*a^4 + 5*a^3*b + a^2*
b^2)*cosh(d*x + c)*sinh(d*x + c)^6 + 2*(4*a^4 + 5*a^3*b + a^2*b^2)*sinh(d*x
 + c)^7 + 2*(4*a^4 + 13*a^3*b + 5*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^5 + 2*(4
*a^4 + 13*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 21*(4*a^4 + 5*a^3*b + a^2*b^2)*cosh
(d*x + c)^2)*sinh(d*x + c)^5 + 10*(7*(4*a^4 + 5*a^3*b + a^2*b^2)*cosh(d*x +
 c)^3 + (4*a^4 + 13*a^3*b + 5*a^2*b^2 - 4*a*b^3)*cosh(d*x + c))*sinh(d*x +
c)^4 - 2*(4*a^4 + 13*a^3*b + 5*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^3 + 2*(35*(
4*a^4 + 5*a^3*b + a^2*b^2)*cosh(d*x + c)^4 - 4*a^4 - 13*a^3*b - 5*a^2*b^2 +
 4*a*b^3 + 10*(4*a^4 + 13*a^3*b + 5*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^2)*sin
h(d*x + c)^3 + 2*(21*(4*a^4 + 5*a^3*b + a^2*b^2)*cosh(d*x + c)^5 + 10*(4*a^
4 + 13*a^3*b + 5*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^3 - 3*(4*a^4 + 13*a^3*b +

5*a^2*b^2 - 4*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + ((4*a^3 + a^2*b)*cos
h(d*x + c)^8 + 8*(4*a^3 + a^2*b)*cosh(d*x + c)*sinh(d*x + c)^7 + (4*a^3 + a
^2*b)*sinh(d*x + c)^8 + 4*(4*a^3 + 9*a^2*b + 2*a*b^2)*cosh(d*x + c)^6 + 4*(
4*a^3 + 9*a^2*b + 2*a*b^2 + 7*(4*a^3 + a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c
)^6 + 8*(7*(4*a^3 + a^2*b)*cosh(d*x + c)^3 + 3*(4*a^3 + 9*a^2*b + 2*a*b^2)*
cosh(d*x + c))*sinh(d*x + c)^5 + 2*(12*a^3 + 35*a^2*b + 40*a*b^2 + 8*b^3)*c
osh(d*x + c)^4 + 2*(35*(4*a^3 + a^2*b)*cosh(d*x + c)^4 + 12*a^3 + 35*a^2*b
+ 40*a*b^2 + 8*b^3 + 30*(4*a^3 + 9*a^2*b + 2*a*b^2)*cosh(d*x + c)^2)*sinh(d
x + c)^4 + 8(7*(4*a^3 + a^2*b)*cosh(d*x + c)^5 + 10*(4*a^3 + 9*a^2*b + 2*
a*b^2)*cosh(d*x + c)^3 + (12*a^3 + 35*a^2*b + 40*a*b^2 + 8*b^3)*cosh(d*x +
c))*sinh(d*x + c)^3 + 4*a^3 + a^2*b + 4*(4*a^3 + 9*a^2*b + 2*a*b^2)*cosh(d*
x + c)^2 + 4*(7*(4*a^3 + a^2*b)*cosh(d*x + c)^6 + 15*(4*a^3 + 9*a^2*b + 2*a
*b^2)*cosh(d*x + c)^4 + 4*a^3 + 9*a^2*b + 2*a*b^2 + 3*(12*a^3 + 35*a^2*b +
40*a*b^2 + 8*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((4*a^3 + a^2*b)*cos
h(d*x + c)^7 + 3*(4*a^3 + 9*a^2*b + 2*a*b^2)*cosh(d*x + c)^5 + (12*a^3 + 35
*a^2*b + 40*a*b^2 + 8*b^3)*cosh(d*x + c)^3 + (4*a^3 + 9*a^2*b + 2*a*b^2)*co
sh(d*x + c))*sinh(d*x + c))*sqrt(a^2 + a*b)*arctan(1/2*(a*cosh(d*x + c)^3 +
3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 + (3*a + 4*b)*cosh(d
*x + c) + (3*a*cosh(d*x + c)^2 + 3*a + 4*b)*sinh(d*x + c))/sqrt(a^2 + a*b))
+ ((4*a^3 + a^2*b)*cosh(d*x + c)^8 + 8*(4*a^3 + a^2*b)*cosh(d*x + c)*sinh(
d*x + c)^7 + (4*a^3 + a^2*b)*sinh(d*x + c)^8 + 4*(4*a^3 + 9*a^2*b + 2*a*b^2
)*cosh(d*x + c)^6 + 4*(4*a^3 + 9*a^2*b + 2*a*b^2 + 7*(4*a^3 + a^2*b)*cosh(d
*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(4*a^3 + a^2*b)*cosh(d*x + c)^3 + 3*(4*a^
3 + 9*a^2*b + 2*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(12*a^3 + 35*a^2*
b + 40*a*b^2 + 8*b^3)*cosh(d*x + c)^4 + 2*(35*(4*a^3 + a^2*b)*cosh(d*x + c)
^4 + 12*a^3 + 35*a^2*b + 40*a*b^2 + 8*b^3 + 30*(4*a^3 + 9*a^2*b + 2*a*b^2)*
cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(4*a^3 + a^2*b)*cosh(d*x + c)^5 + 1
0*(4*a^3 + 9*a^2*b + 2*a*b^2)*cosh(d*x + c)^3 + (12*a^3 + 35*a^2*b + 40*a*b
^2 + 8*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*a^3 + a^2*b + 4*(4*a^3 + 9*a
^2*b + 2*a*b^2)*cosh(d*x + c)^2 + 4*(7*(4*a^3 + a^2*b)*cosh(d*x + c)^6 + 15
*(4*a^3 + 9*a^2*b + 2*a*b^2)*cosh(d*x + c)^4 + 4*a^3 + 9*a^2*b + 2*a*b^2 +
3*(12*a^3 + 35*a^2*b + 40*a*b^2 + 8*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 +
8*((4*a^3 + a^2*b)*cosh(d*x + c)^7 + 3*(4*a^3 + 9*a^2*b + 2*a*b^2)*cosh(d*
x + c)^5 + (12*a^3 + 35*a^2*b + 40*a*b^2 + 8*b^3)*cosh(d*x + c)^3 + (4*a^3
+ 9*a^2*b + 2*a*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 + a*b)*arctan(1
/2*sqrt(a^2 + a*b)*(cosh(d*x + c) + sinh(d*x + c))/(a + b)) - 2*(4*a^4 + 5*
a^3*b + a^2*b^2)*cosh(d*x + c) + 2*(7*(4*a^4 + 5*a^3*b + a^2*b^2)*cosh(d*x
+ c)^6 + 5*(4*a^4 + 13*a^3*b + 5*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^4 - 4*a^4
- 5*a^3*b - a^2*b^2 - 3*(4*a^4 + 13*a^3*b + 5*a^2*b^2 - 4*a*b^3)*cosh(d*x
+ c)^2)*sinh(d*x + c))/((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x +
c)^8 + 8*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)*sinh(d*x + c
)^7 + (a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*sinh(d*x + c)^8 + 4*(a^7 + 5*
a^6*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d*cosh(d*x + c)^6 + 4*(7*(a^7 +
3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^2 + (a^7 + 5*a^6*b + 9*a^5*b
^2 + 7*a^4*b^3 + 2*a^3*b^4)*d)*sinh(d*x + c)^6 + 2*(3*a^7 + 17*a^6*b + 41*a

```
^5*b^2 + 51*a^4*b^3 + 32*a^3*b^4 + 8*a^2*b^5)*d*cosh(d*x + c)^4 + 8*(7*(a^7
 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^3 + 3*(a^7 + 5*a^6*b + 9*
a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(
a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^4 + 30*(a^7 + 5*a^6*b
+ 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d*cosh(d*x + c)^2 + (3*a^7 + 17*a^6*b
+ 41*a^5*b^2 + 51*a^4*b^3 + 32*a^3*b^4 + 8*a^2*b^5)*d)*sinh(d*x + c)^4 + 4*
(a^7 + 5*a^6*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d*cosh(d*x + c)^2 + 8*(
7*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^5 + 10*(a^7 + 5*a^6
*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d*cosh(d*x + c)^3 + (3*a^7 + 17*a^6
*b + 41*a^5*b^2 + 51*a^4*b^3 + 32*a^3*b^4 + 8*a^2*b^5)*d*cosh(d*x + c))*sin
h(d*x + c)^3 + 4*(7*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^6
 + 15*(a^7 + 5*a^6*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d*cosh(d*x + c)^4
 + 3*(3*a^7 + 17*a^6*b + 41*a^5*b^2 + 51*a^4*b^3 + 32*a^3*b^4 + 8*a^2*b^5)*
d*cosh(d*x + c)^2 + (a^7 + 5*a^6*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d)*
sinh(d*x + c)^2 + (a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d + 8*((a^7 + 3*a^6
*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^7 + 3*(a^7 + 5*a^6*b + 9*a^5*b^2
+ 7*a^4*b^3 + 2*a^3*b^4)*d*cosh(d*x + c)^5 + (3*a^7 + 17*a^6*b + 41*a^5*b^2
 + 51*a^4*b^3 + 32*a^3*b^4 + 8*a^2*b^5)*d*cosh(d*x + c)^3 + (a^7 + 5*a^6*b
+ 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d*cosh(d*x + c))*sinh(d*x + c))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[84,-86]Warning, need to choose a branch for the root of a p
olynomial with parameters. This might be wrong.The choice was done assuming
 [a,b]=[-42,-12]Warning, need to choose a branch for the root of a polynomi
al with parameters. This might be wrong.The choice was done assuming [a,b]=
[-43,-99]Warning, need to choose a branch for the root of a polynomial with
 parameters. This might be wrong.The choice was done assuming [a,b]=[-28,94
]Warning, need to choose a branch for the root of a polynomial with paramet
ers. This might be wrong.The choice was done assuming [a,b]=[-7,46]Warning,
 need to choose a branch for the root of a polynomial with parameters. This
 might be wrong.The choice was done assuming [a,b]=[-35,-99]Warning, need t
o choose a branch for the root of a polynomial with parameters. This might
be wrong.The choice was done assuming [a,b]=[7,50]Warning, need to choose a
 branch for the root of a polynomial with parameters. This might be wrong.T
he choice was done assuming [a,b]=[-63,-70]Undef/Unsigned Inf encountered i
```

```
n limitEvaluation time: 1.54Limit: Max order reached or unable to make seri
es expansion Error: Bad Argument Value
```

maple [B] time = 0.34, size = 1038, normalized size = 8.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] -1/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)
^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)^7+1/4/d/(ta
nh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*t
anh(1/2*d*x+1/2*c)^2*b+a+b)^2*b/(a+b)/a*tanh(1/2*d*x+1/2*c)^7-1/d/(tanh(1/2
*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/
2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^5*a+5/4/d/(tanh(1/2*d*x
+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*
x+1/2*c)^2*b+a+b)^2*b/(a+b)^2*tanh(1/2*d*x+1/2*c)^5-3/4/d/a*b^2/(tanh(1/2*d
*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*
d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^5+1/d/(tanh(1/2*d*x+1/2*c
)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*
c)^2*b+a+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^3*a-5/4/d/(tanh(1/2*d*x+1/2*c)^4*
a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2
*b+a+b)^2*b/(a+b)^2*tanh(1/2*d*x+1/2*c)^3+3/4/d/a*b^2/(tanh(1/2*d*x+1/2*c)^
4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)
^2*b+a+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^3+1/d/(tanh(1/2*d*x+1/2*c)^4*a+b*ta
nh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b
)^2/(a+b)*tanh(1/2*d*x+1/2*c)-1/4/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x
+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*b/(a+b
)/a*tanh(1/2*d*x+1/2*c)+1/2/d/(a^2+2*a*b+b^2)/(a+b)^(1/2)/a^(1/2)*arctan(1/
2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)-2*b^(1/2))/a^(1/2))+1/2/d/(a^2+2*a*b+b
^2)/(a+b)^(1/2)/a^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)+2*b^(
1/2))/a^(1/2))+1/8/d/a^(3/2)*b/(a^2+2*a*b+b^2)/(a+b)^(1/2)*arctan(1/2*(2*(a
+b)^(1/2)*tanh(1/2*d*x+1/2*c)-2*b^(1/2))/a^(1/2))+1/8/d/a^(3/2)*b/(a^2+2*a*
b+b^2)/(a+b)^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)+2*b^(1/2))
/a^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(4\,a^2e^{(7\,c)} + abe^{(7\,c)}\right)e^{(7\,dx)} + \left(4\,a^2e^{(5\,c)} + \cdots\right.}{4\left(a^5d + 2\,a^4bd + a^3b^2d + \left(a^5de^{(8\,c)} + 2\,a^4bde^{(8\,c)} + a^3b^2de^{(8\,c)}\right)e^{(8\,dx)} + 4\left(a^5de^{(6\,c)} + 4\,a^4bde^{(6\,c)} + 5\,a^3b^2de^{(6\,c)} + \cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*((4*a^2*e^(7*c) + a*b*e^(7*c))*e^(7*d*x) + (4*a^2*e^(5*c) + 9*a*b*e^(5*
c) - 4*b^2*e^(5*c))*e^(5*d*x) - (4*a^2*e^(3*c) + 9*a*b*e^(3*c) - 4*b^2*e^(3
*c))*e^(3*d*x) - (4*a^2*e^c + a*b*e^c)*e^(d*x))/(a^5*d + 2*a^4*b*d + a^3*b^
2*d + (a^5*d*e^(8*c) + 2*a^4*b*d*e^(8*c) + a^3*b^2*d*e^(8*c))*e^(8*d*x) + 4
*(a^5*d*e^(6*c) + 4*a^4*b*d*e^(6*c) + 5*a^3*b^2*d*e^(6*c) + 2*a^2*b^3*d*e^(
6*c))*e^(6*d*x) + 2*(3*a^5*d*e^(4*c) + 14*a^4*b*d*e^(4*c) + 27*a^3*b^2*d*e^
(4*c) + 24*a^2*b^3*d*e^(4*c) + 8*a*b^4*d*e^(4*c))*e^(4*d*x) + 4*(a^5*d*e^(2
*c) + 4*a^4*b*d*e^(2*c) + 5*a^3*b^2*d*e^(2*c) + 2*a^2*b^3*d*e^(2*c))*e^(2*d
*x)) + 8*integrate(1/32*((4*a*e^(3*c) + b*e^(3*c))*e^(3*d*x) + (4*a*e^c + b
*e^c)*e^(d*x))/(a^4 + 2*a^3*b + a^2*b^2 + (a^4*e^(4*c) + 2*a^3*b*e^(4*c) +
a^2*b^2*e^(4*c))*e^(4*d*x) + 2*(a^4*e^(2*c) + 4*a^3*b*e^(2*c) + 5*a^2*b^2*e
^(2*c) + 2*a*b^3*e^(2*c))*e^(2*d*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh\left(c+d\,x\right)^3\,{\left(a+\frac{b}{{\cosh\left(c+d\,x\right)}^2}\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^3*(a + b/cosh(c + d*x)^2)^3),x)
```

```
[Out] int(1/(cosh(c + d*x)^3*(a + b/cosh(c + d*x)^2)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^{3}{\left(c + d x \right)}}{\left(a + b \operatorname{sech}^{2}{\left(c + d x \right)}\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**3/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Integral(sech(c + d*x)**3/(a + b*sech(c + d*x)**2)**3, x)
```

3.98 $\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3} dx$

Optimal. Leaf size=125

$$\frac{(a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8b^{3/2}d(a+b)^{5/2}}+\frac{(a+4b)\tanh(c+dx)}{8bd(a+b)^2\left(a-b\tanh^2(c+dx)+b\right)}-\frac{a\tanh(c+dx)}{4bd(a+b)\left(a-b\tanh^2(c+dx)+b\right)^2}$$

```
[Out] 1/8*(a+4*b)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))/b^(3/2)/(a+b)^(5/2)/d-
1/4*a*tanh(d*x+c)/b/(a+b)/d/(a+b-b*tanh(d*x+c)^2)^2+1/8*(a+4*b)*tanh(d*x+c)
/b/(a+b)^2/d/(a+b-b*tanh(d*x+c)^2)
```

Rubi [A] time = 0.11, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4146, 385, 199, 208}

$$\frac{(a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8b^{3/2}d(a+b)^{5/2}}+\frac{(a+4b)\tanh(c+dx)}{8bd(a+b)^2\left(a-b\tanh^2(c+dx)+b\right)}-\frac{a\tanh(c+dx)}{4bd(a+b)\left(a-b\tanh^2(c+dx)+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^4/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((a + 4*b)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(8*b^(3/2)*(a + b)
^(5/2)*d) - (a*Tanh[c + d*x])/(4*b*(a + b)*d*(a + b - b*Tanh[c + d*x]^2)^2)
 + ((a + 4*b)*Tanh[c + d*x])/(8*b*(a + b)^2*d*(a + b - b*Tanh[c + d*x]^2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1-x^2}{\left(a+b-bx^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{a\tanh(c+dx)}{4b(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{(a+4b)\operatorname{Subst}\left(\int \frac{1}{\left(a+b-bx^2\right)^2}\,dx, x, \tanh(\right.}{4b(a+b)d}$$

$$= -\frac{a\tanh(c+dx)}{4b(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{(a+4b)\tanh(c+dx)}{8b(a+b)^2d\left(a+b-b\tanh^2(c+dx)\right)} +$$

$$= \frac{(a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8b^{3/2}(a+b)^{5/2}d} - \frac{a\tanh(c+dx)}{4b(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{}{8b(a+}$$

Mathematica [A] time = 3.39, size = 250, normalized size = 2.00

$$\frac{\operatorname{sech}^6(c+dx)(a\cosh(2(c+dx))+a+2b)\left(-\frac{4(a+b)\operatorname{sech}(2c)((a+2b)\sinh(2c)-a\sinh(2dx))}{a} + \frac{\operatorname{sech}(2c)((a+4b)\sinh(2c)-(a-2b)\sinh(2\right.}{b}}{64d(a+b)^2\left(a+b\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^4/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^6*(((a + 4*b)*ArcTanh[(Sech[
d*x]*(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*
Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(a + 2*b + a*Cosh[2*(c + d*x)])
^2*(Cosh[2*c] - Sinh[2*c]))/(b*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4]) -
 (4*(a + b)*Sech[2*c]*((a + 2*b)*Sinh[2*c] - a*Sinh[2*d*x]))/a + ((a + 2*b
+ a*Cosh[2*(c + d*x)])*Sech[2*c]*((a + 4*b)*Sinh[2*c] - (a - 2*b)*Sinh[2*d*
x]))/b))/(64*(a + b)^2*d*(a + b*Sech[c + d*x]^2)^3)
```

fricas [B] time = 0.52, size = 5447, normalized size = 43.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(4*(a^4*b + 5*a^3*b^2 + 4*a^2*b^3)*cosh(d*x + c)^6 + 24*(a^4*b + 5*a^
3*b^2 + 4*a^2*b^3)*cosh(d*x + c)*sinh(d*x + c)^5 + 4*(a^4*b + 5*a^3*b^2 + 4
*a^2*b^3)*sinh(d*x + c)^6 + 4*a^4*b - 4*a^3*b^2 - 8*a^2*b^3 + 4*(3*a^4*b +
5*a^3*b^2 - 6*a^2*b^3 - 24*a*b^4 - 16*b^5)*cosh(d*x + c)^4 + 4*(3*a^4*b + 5
*a^3*b^2 - 6*a^2*b^3 - 24*a*b^4 - 16*b^5 + 15*(a^4*b + 5*a^3*b^2 + 4*a^2*b^
3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 16*(5*(a^4*b + 5*a^3*b^2 + 4*a^2*b^3)
*cosh(d*x + c)^3 + (3*a^4*b + 5*a^3*b^2 - 6*a^2*b^3 - 24*a*b^4 - 16*b^5)*co
sh(d*x + c))*sinh(d*x + c)^3 + 4*(3*a^4*b - a^3*b^2 - 20*a^2*b^3 - 16*a*b^4
)*cosh(d*x + c)^2 + 4*(3*a^4*b - a^3*b^2 - 20*a^2*b^3 - 16*a*b^4 + 15*(a^4*
b + 5*a^3*b^2 + 4*a^2*b^3)*cosh(d*x + c)^4 + 6*(3*a^4*b + 5*a^3*b^2 - 6*a^2
*b^3 - 24*a*b^4 - 16*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((a^4 + 4*a^3*
b)*cosh(d*x + c)^8 + 8*(a^4 + 4*a^3*b)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^4
 + 4*a^3*b)*sinh(d*x + c)^8 + 4*(a^4 + 6*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^6
 + 4*(a^4 + 6*a^3*b + 8*a^2*b^2 + 7*(a^4 + 4*a^3*b)*cosh(d*x + c)^2)*sinh(d
*x + c)^6 + 8*(7*(a^4 + 4*a^3*b)*cosh(d*x + c)^3 + 3*(a^4 + 6*a^3*b + 8*a^2
*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*a^4 + 20*a^3*b + 40*a^2*b^2 + 3
2*a*b^3)*cosh(d*x + c)^4 + 2*(35*(a^4 + 4*a^3*b)*cosh(d*x + c)^4 + 3*a^4 +
20*a^3*b + 40*a^2*b^2 + 32*a*b^3 + 30*(a^4 + 6*a^3*b + 8*a^2*b^2)*cosh(d*x
+ c)^2)*sinh(d*x + c)^4 + a^4 + 4*a^3*b + 8*(7*(a^4 + 4*a^3*b)*cosh(d*x + c
)^5 + 10*(a^4 + 6*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^3 + (3*a^4 + 20*a^3*b +
40*a^2*b^2 + 32*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a^4 + 6*a^3*b +
8*a^2*b^2)*cosh(d*x + c)^2 + 4*(7*(a^4 + 4*a^3*b)*cosh(d*x + c)^6 + 15*(a^4
 + 6*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^4 + a^4 + 6*a^3*b + 8*a^2*b^2 + 3*(3*
a^4 + 20*a^3*b + 40*a^2*b^2 + 32*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 +
8*((a^4 + 4*a^3*b)*cosh(d*x + c)^7 + 3*(a^4 + 6*a^3*b + 8*a^2*b^2)*cosh(d*x
 + c)^5 + (3*a^4 + 20*a^3*b + 40*a^2*b^2 + 32*a*b^3)*cosh(d*x + c)^3 + (a^4
 + 6*a^3*b + 8*a^2*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a*b + b^2)*log((
a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x +
```

```
c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2
*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2
 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) - 4*(a*cosh(d*x + c)^2 + 2*a*cosh(d*
x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(a*b + b^2))/(a*cos
h(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a
 + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2
 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) + 8*
(3*(a^4*b + 5*a^3*b^2 + 4*a^2*b^3)*cosh(d*x + c)^5 + 2*(3*a^4*b + 5*a^3*b^2
 - 6*a^2*b^3 - 24*a*b^4 - 16*b^5)*cosh(d*x + c)^3 + (3*a^4*b - a^3*b^2 - 20
*a^2*b^3 - 16*a*b^4)*cosh(d*x + c))*sinh(d*x + c))/((a^6*b^2 + 3*a^5*b^3 +
3*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^8 + 8*(a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4
 + a^3*b^5)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^6*b^2 + 3*a^5*b^3 + 3*a^4*
b^4 + a^3*b^5)*d*sinh(d*x + c)^8 + 4*(a^6*b^2 + 5*a^5*b^3 + 9*a^4*b^4 + 7*a
^3*b^5 + 2*a^2*b^6)*d*cosh(d*x + c)^6 + 4*(7*(a^6*b^2 + 3*a^5*b^3 + 3*a^4*b
^4 + a^3*b^5)*d*cosh(d*x + c)^2 + (a^6*b^2 + 5*a^5*b^3 + 9*a^4*b^4 + 7*a^3*
b^5 + 2*a^2*b^6)*d)*sinh(d*x + c)^6 + 2*(3*a^6*b^2 + 17*a^5*b^3 + 41*a^4*b^
4 + 51*a^3*b^5 + 32*a^2*b^6 + 8*a*b^7)*d*cosh(d*x + c)^4 + 8*(7*(a^6*b^2 +
3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^3 + 3*(a^6*b^2 + 5*a^5*b^3
 + 9*a^4*b^4 + 7*a^3*b^5 + 2*a^2*b^6)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*
(35*(a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^4 + 30*(a^6
*b^2 + 5*a^5*b^3 + 9*a^4*b^4 + 7*a^3*b^5 + 2*a^2*b^6)*d*cosh(d*x + c)^2 + (
3*a^6*b^2 + 17*a^5*b^3 + 41*a^4*b^4 + 51*a^3*b^5 + 32*a^2*b^6 + 8*a*b^7)*d)
*sinh(d*x + c)^4 + 4*(a^6*b^2 + 5*a^5*b^3 + 9*a^4*b^4 + 7*a^3*b^5 + 2*a^2*b
^6)*d*cosh(d*x + c)^2 + 8*(7*(a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*d*
cosh(d*x + c)^5 + 10*(a^6*b^2 + 5*a^5*b^3 + 9*a^4*b^4 + 7*a^3*b^5 + 2*a^2*b
^6)*d*cosh(d*x + c)^3 + (3*a^6*b^2 + 17*a^5*b^3 + 41*a^4*b^4 + 51*a^3*b^5 +
 32*a^2*b^6 + 8*a*b^7)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^6*b^2 + 3
*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^6 + 15*(a^6*b^2 + 5*a^5*b^3
 + 9*a^4*b^4 + 7*a^3*b^5 + 2*a^2*b^6)*d*cosh(d*x + c)^4 + 3*(3*a^6*b^2 + 17
*a^5*b^3 + 41*a^4*b^4 + 51*a^3*b^5 + 32*a^2*b^6 + 8*a*b^7)*d*cosh(d*x + c)^
2 + (a^6*b^2 + 5*a^5*b^3 + 9*a^4*b^4 + 7*a^3*b^5 + 2*a^2*b^6)*d)*sinh(d*x +
 c)^2 + (a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*d + 8*((a^6*b^2 + 3*a^5
*b^3 + 3*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^7 + 3*(a^6*b^2 + 5*a^5*b^3 + 9*
a^4*b^4 + 7*a^3*b^5 + 2*a^2*b^6)*d*cosh(d*x + c)^5 + (3*a^6*b^2 + 17*a^5*b^
3 + 41*a^4*b^4 + 51*a^3*b^5 + 32*a^2*b^6 + 8*a*b^7)*d*cosh(d*x + c)^3 + (a^
6*b^2 + 5*a^5*b^3 + 9*a^4*b^4 + 7*a^3*b^5 + 2*a^2*b^6)*d*cosh(d*x + c))*sin
h(d*x + c)), 1/8*(2*(a^4*b + 5*a^3*b^2 + 4*a^2*b^3)*cosh(d*x + c)^6 + 12*(a
^4*b + 5*a^3*b^2 + 4*a^2*b^3)*cosh(d*x + c)*sinh(d*x + c)^5 + 2*(a^4*b + 5*
a^3*b^2 + 4*a^2*b^3)*sinh(d*x + c)^6 + 2*a^4*b - 2*a^3*b^2 - 4*a^2*b^3 + 2*
(3*a^4*b + 5*a^3*b^2 - 6*a^2*b^3 - 24*a*b^4 - 16*b^5)*cosh(d*x + c)^4 + 2*(
3*a^4*b + 5*a^3*b^2 - 6*a^2*b^3 - 24*a*b^4 - 16*b^5 + 15*(a^4*b + 5*a^3*b^2
 + 4*a^2*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(5*(a^4*b + 5*a^3*b^2 +
4*a^2*b^3)*cosh(d*x + c)^3 + (3*a^4*b + 5*a^3*b^2 - 6*a^2*b^3 - 24*a*b^4 -
16*b^5)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(3*a^4*b - a^3*b^2 - 20*a^2*b^3
- 16*a*b^4)*cosh(d*x + c)^2 + 2*(3*a^4*b - a^3*b^2 - 20*a^2*b^3 - 16*a*b^4
```

+ 15*(a^4*b + 5*a^3*b^2 + 4*a^2*b^3)*cosh(d*x + c)^4 + 6*(3*a^4*b + 5*a^3*b
^2 - 6*a^2*b^3 - 24*a*b^4 - 16*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((a^
4 + 4*a^3*b)*cosh(d*x + c)^8 + 8*(a^4 + 4*a^3*b)*cosh(d*x + c)*sinh(d*x + c
)^7 + (a^4 + 4*a^3*b)*sinh(d*x + c)^8 + 4*(a^4 + 6*a^3*b + 8*a^2*b^2)*cosh(
d*x + c)^6 + 4*(a^4 + 6*a^3*b + 8*a^2*b^2 + 7*(a^4 + 4*a^3*b)*cosh(d*x + c)
^2)*sinh(d*x + c)^6 + 8*(7*(a^4 + 4*a^3*b)*cosh(d*x + c)^3 + 3*(a^4 + 6*a^3
*b + 8*a^2*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*a^4 + 20*a^3*b + 40*a
^2*b^2 + 32*a*b^3)*cosh(d*x + c)^4 + 2*(35*(a^4 + 4*a^3*b)*cosh(d*x + c)^4
+ 3*a^4 + 20*a^3*b + 40*a^2*b^2 + 32*a*b^3 + 30*(a^4 + 6*a^3*b + 8*a^2*b^2)
*cosh(d*x + c)^2)*sinh(d*x + c)^4 + a^4 + 4*a^3*b + 8*(7*(a^4 + 4*a^3*b)*co
sh(d*x + c)^5 + 10*(a^4 + 6*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^3 + (3*a^4 + 2
0*a^3*b + 40*a^2*b^2 + 32*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a^4 +
6*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^2 + 4*(7*(a^4 + 4*a^3*b)*cosh(d*x + c)^6
 + 15*(a^4 + 6*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^4 + a^4 + 6*a^3*b + 8*a^2*b
^2 + 3*(3*a^4 + 20*a^3*b + 40*a^2*b^2 + 32*a*b^3)*cosh(d*x + c)^2)*sinh(d*x
 + c)^2 + 8*((a^4 + 4*a^3*b)*cosh(d*x + c)^7 + 3*(a^4 + 6*a^3*b + 8*a^2*b^2
)*cosh(d*x + c)^5 + (3*a^4 + 20*a^3*b + 40*a^2*b^2 + 32*a*b^3)*cosh(d*x + c
)^3 + (a^4 + 6*a^3*b + 8*a^2*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a*b -
 b^2)*arctan(1/2*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*s
inh(d*x + c)^2 + a + 2*b)*sqrt(-a*b - b^2)/(a*b + b^2)) + 4*(3*(a^4*b + 5*a
^3*b^2 + 4*a^2*b^3)*cosh(d*x + c)^5 + 2*(3*a^4*b + 5*a^3*b^2 - 6*a^2*b^3 -
24*a*b^4 - 16*b^5)*cosh(d*x + c)^3 + (3*a^4*b - a^3*b^2 - 20*a^2*b^3 - 16*a
*b^4)*cosh(d*x + c))*sinh(d*x + c))/((a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3
*b^5)*d*cosh(d*x + c)^8 + 8*(a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*d*c
osh(d*x + c)*sinh(d*x + c)^7 + (a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*
d*sinh(d*x + c)^8 + 4*(a^6*b^2 + 5*a^5*b^3 + 9*a^4*b^4 + 7*a^3*b^5 + 2*a^2*
b^6)*d*cosh(d*x + c)^6 + 4*(7*(a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*d
*cosh(d*x + c)^2 + (a^6*b^2 + 5*a^5*b^3 + 9*a^4*b^4 + 7*a^3*b^5 + 2*a^2*b^6
)*d)*sinh(d*x + c)^6 + 2*(3*a^6*b^2 + 17*a^5*b^3 + 41*a^4*b^4 + 51*a^3*b^5
+ 32*a^2*b^6 + 8*a*b^7)*d*cosh(d*x + c)^4 + 8*(7*(a^6*b^2 + 3*a^5*b^3 + 3*a
^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^3 + 3*(a^6*b^2 + 5*a^5*b^3 + 9*a^4*b^4 +
7*a^3*b^5 + 2*a^2*b^6)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^6*b^2 +
3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^4 + 30*(a^6*b^2 + 5*a^5*b^
3 + 9*a^4*b^4 + 7*a^3*b^5 + 2*a^2*b^6)*d*cosh(d*x + c)^2 + (3*a^6*b^2 + 17*
a^5*b^3 + 41*a^4*b^4 + 51*a^3*b^5 + 32*a^2*b^6 + 8*a*b^7)*d)*sinh(d*x + c)^
4 + 4*(a^6*b^2 + 5*a^5*b^3 + 9*a^4*b^4 + 7*a^3*b^5 + 2*a^2*b^6)*d*cosh(d*x
+ c)^2 + 8*(7*(a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^5
 + 10*(a^6*b^2 + 5*a^5*b^3 + 9*a^4*b^4 + 7*a^3*b^5 + 2*a^2*b^6)*d*cosh(d*x
+ c)^3 + (3*a^6*b^2 + 17*a^5*b^3 + 41*a^4*b^4 + 51*a^3*b^5 + 32*a^2*b^6 + 8
*a*b^7)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^6*b^2 + 3*a^5*b^3 + 3*a^
4*b^4 + a^3*b^5)*d*cosh(d*x + c)^6 + 15*(a^6*b^2 + 5*a^5*b^3 + 9*a^4*b^4 +
7*a^3*b^5 + 2*a^2*b^6)*d*cosh(d*x + c)^4 + 3*(3*a^6*b^2 + 17*a^5*b^3 + 41*a
^4*b^4 + 51*a^3*b^5 + 32*a^2*b^6 + 8*a*b^7)*d*cosh(d*x + c)^2 + (a^6*b^2 +
5*a^5*b^3 + 9*a^4*b^4 + 7*a^3*b^5 + 2*a^2*b^6)*d)*sinh(d*x + c)^2 + (a^6*b^
2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*d + 8*((a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^

```
4 + a^3*b^5)*d*cosh(d*x + c)^7 + 3*(a^6*b^2 + 5*a^5*b^3 + 9*a^4*b^4 + 7*a^3
*b^5 + 2*a^2*b^6)*d*cosh(d*x + c)^5 + (3*a^6*b^2 + 17*a^5*b^3 + 41*a^4*b^4
+ 51*a^3*b^5 + 32*a^2*b^6 + 8*a*b^7)*d*cosh(d*x + c)^3 + (a^6*b^2 + 5*a^5*b
^3 + 9*a^4*b^4 + 7*a^3*b^5 + 2*a^2*b^6)*d*cosh(d*x + c))*sinh(d*x + c))]
```

giac [B] time = 1.73, size = 274, normalized size = 2.19

$$\frac{\frac{(a+4\,b)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{\left(a^2b+2\,ab^2+b^3\right)\sqrt{-ab-b^2}}+\frac{2\left(a^3e^{(6\,dx+6\,c)}+4\,a^2be^{(6\,dx+6\,c)}+3\,a^3e^{(4\,dx+4\,c)}+2\,a^2be^{(4\,dx+4\,c)}-8\,ab^2e^{(4\,dx+4\,c)}-16\,b^3e^{(4\,dx+4\,c)}+3\,a^3e^{(2\,dx+2\,c)}-4\,a^2b}{\left(a^3b+2\,a^2b^2+ab^3\right)\left(ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+a\right)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*((a + 4*b)*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/(
(a^2*b + 2*a*b^2 + b^3)*sqrt(-a*b - b^2)) + 2*(a^3*e^(6*d*x + 6*c) + 4*a^2*
b*e^(6*d*x + 6*c) + 3*a^3*e^(4*d*x + 4*c) + 2*a^2*b*e^(4*d*x + 4*c) - 8*a*b
^2*e^(4*d*x + 4*c) - 16*b^3*e^(4*d*x + 4*c) + 3*a^3*e^(2*d*x + 2*c) - 4*a^2
*b*e^(2*d*x + 2*c) - 16*a*b^2*e^(2*d*x + 2*c) + a^3 - 2*a^2*b)/((a^3*b + 2*
a^2*b^2 + a*b^3)*(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x +
2*c) + a)^2))/d
```

maple [B] time = 0.28, size = 1084, normalized size = 8.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] -1/4/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*
c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/b/(a+b)*tanh(1/2*d*x+1/2*c)^7*a+1/d
/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a
-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)^7-3/4/d/(tanh(1
/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(
1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2/b*tanh(1/2*d*x+1/2*c)^5*a^2+5/4/d/(tanh(1
/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(
1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^5*a-1/d/(tanh(1/2*d*x
+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*
x+1/2*c)^2*b+a+b)^2*b/(a+b)^2*tanh(1/2*d*x+1/2*c)^5-3/4/d/(tanh(1/2*d*x+1/2
*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/
2*c)^2*b+a+b)^2/(a+b)^2/b*tanh(1/2*d*x+1/2*c)^3*a^2+5/4/d/(tanh(1/2*d*x+1/2
*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/
2*c)^2*b+a+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^3*a-1/d/(tanh(1/2*d*x+1/2*c)^4*
a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2
```

```
*b+a+b)^2*b/(a+b)^2*tanh(1/2*d*x+1/2*c)^3-1/4/d/(tanh(1/2*d*x+1/2*c)^4*a+b*
tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a
+b)^2/b/(a+b)*tanh(1/2*d*x+1/2*c)*a+1/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2
*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a
+b)*tanh(1/2*d*x+1/2*c)-1/16/d/b^(3/2)/(a^2+2*a*b+b^2)*a/(a+b)^(1/2)*ln(-(a
+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)-(a+b)^(1/2))+
1/16/d/b^(3/2)/(a^2+2*a*b+b^2)*a/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/
2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))-1/4/d/(a^2+2*a*b+b^2)/b^(
1/2)/(a+b)^(1/2)*ln(-(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d
*x+1/2*c)-(a+b)^(1/2))+1/4/d/(a^2+2*a*b+b^2)/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(
1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))
```

maxima [B] time = 0.53, size = 369, normalized size = 2.95

$$-\frac{(a+4\,b)\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{16\left(a^2b+2\,ab^2+b^3\right)\sqrt{(a+b)b}\,d}-\frac{a^3-2\,a^2b+\left(3\,a^3-\right.}{4\left(a^5b+2\,a^4b^2+a^3b^3+4\left(a^5b+4\,a^4b^2+5\,a^3b^3+2\,a^2b^4\right)e^{(-2\,dx-2\,c)}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(a + 4*b)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e
^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^2*b + 2*a*b^2 + b^3)*sq
rt((a + b)*b)*d) - 1/4*(a^3 - 2*a^2*b + (3*a^3 - 4*a^2*b - 16*a*b^2)*e^(-2*
d*x - 2*c) + (3*a^3 + 2*a^2*b - 8*a*b^2 - 16*b^3)*e^(-4*d*x - 4*c) + (a^3 +
 4*a^2*b)*e^(-6*d*x - 6*c))/((a^5*b + 2*a^4*b^2 + a^3*b^3 + 4*(a^5*b + 4*a^
4*b^2 + 5*a^3*b^3 + 2*a^2*b^4)*e^(-2*d*x - 2*c) + 2*(3*a^5*b + 14*a^4*b^2 +
 27*a^3*b^3 + 24*a^2*b^4 + 8*a*b^5)*e^(-4*d*x - 4*c) + 4*(a^5*b + 4*a^4*b^2
 + 5*a^3*b^3 + 2*a^2*b^4)*e^(-6*d*x - 6*c) + (a^5*b + 2*a^4*b^2 + a^3*b^3)*
e^(-8*d*x - 8*c))*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh\left(c+d\,x\right)^4\,\left(a+\frac{b}{\cosh\left(c+d\,x\right)^2}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^4*(a + b/cosh(c + d*x)^2)^3),x)
```

```
[Out] int(1/(cosh(c + d*x)^4*(a + b/cosh(c + d*x)^2)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^{4}{\left(c+dx \right)}}{\left(a+b\operatorname{sech}^{2}{\left(c+dx \right)}\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**4/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Integral(sech(c + d*x)**4/(a + b*sech(c + d*x)**2)**3, x)
```

3.99 $\int \frac{\mathbf{sech}^5(c+dx)}{\left(a+b\mathbf{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=106

$$\frac{3\sinh(c+dx)}{8d(a+b)^2\left(a\sinh^2(c+dx)+a+b\right)}+\frac{\sinh(c+dx)}{4d(a+b)\left(a\sinh^2(c+dx)+a+b\right)^2}+\frac{3\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{8\sqrt{a}\,d(a+b)^{5/2}}$$

```
[Out] 1/4*sinh(d*x+c)/(a+b)/d/(a+b+a*sinh(d*x+c)^2)^2+3/8*sinh(d*x+c)/(a+b)^2/d/(
a+b+a*sinh(d*x+c)^2)+3/8*arctan(sinh(d*x+c)*a^(1/2)/(a+b)^(1/2))/(a+b)^(5/2
)/d/a^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4147, 199, 205}

$$\frac{3\sinh(c+dx)}{8d(a+b)^2\left(a\sinh^2(c+dx)+a+b\right)}+\frac{\sinh(c+dx)}{4d(a+b)\left(a\sinh^2(c+dx)+a+b\right)^2}+\frac{3\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{8\sqrt{a}\,d(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^5/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (3*ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]])/(8*Sqrt[a]*(a + b)^(5/2)*d)
 + Sinh[c + d*x]/(4*(a + b)*d*(a + b + a*Sinh[c + d*x]^2)^2) + (3*Sinh[c +
d*x])/(8*(a + b)^2*d*(a + b + a*Sinh[c + d*x]^2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+b+ax^2\right)^3}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\sinh(c+dx)}{4(a+b)d\left(a+b+a\sinh^2(c+dx)\right)^2} + \frac{3\,\text{Subst}\left(\int \frac{1}{\left(a+b+ax^2\right)^2}\,dx, x, \sinh(c+dx)\right)}{4(a+b)d}$$

$$= \frac{\sinh(c+dx)}{4(a+b)d\left(a+b+a\sinh^2(c+dx)\right)^2} + \frac{3\sinh(c+dx)}{8(a+b)^2d\left(a+b+a\sinh^2(c+dx)\right)} + \frac{3\,\text{S}}{}$$

$$= \frac{3\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{8\sqrt{a}\,(a+b)^{5/2}d} + \frac{\sinh(c+dx)}{4(a+b)d\left(a+b+a\sinh^2(c+dx)\right)^2} + \frac{3\sinh}{8(a+b)^2d\left(a+b\right.}$$

Mathematica [A] time = 0.29, size = 125, normalized size = 1.18

$$\frac{\operatorname{sech}^6(c+dx)(a\cosh(2(c+dx))+a+2b)^3\left(\frac{5(a+b)\sinh(c+dx)+3a\sinh^3(c+dx)}{(a+b)^2\left(a\sinh^2(c+dx)+a+b\right)^2}+\frac{3\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{a}\,(a+b)^{5/2}}\right)}{64d\left(a+b\operatorname{sech}^2(c+dx)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^5/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])^3*Sech[c + d*x]^6*((3*ArcTan[(Sqrt[a]*Sinh
[c + d*x])/Sqrt[a + b]])/(Sqrt[a]*(a + b)^(5/2)) + (5*(a + b)*Sinh[c + d*x]
 + 3*a*Sinh[c + d*x]^3)/((a + b)^2*(a + b + a*Sinh[c + d*x]^2)^2)))/(64*d*(
a + b*Sech[c + d*x]^2)^3)
```

fricas [B] time = 0.51, size = 5006, normalized size = 47.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^5/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")

[Out] [1/16*(12*(a^3 + a^2*b)*cosh(d*x + c)^7 + 84*(a^3 + a^2*b)*cosh(d*x + c)*sinh(d*x + c)^6 + 12*(a^3 + a^2*b)*sinh(d*x + c)^7 + 4*(11*a^3 + 31*a^2*b + 20*a*b^2)*cosh(d*x + c)^5 + 4*(11*a^3 + 31*a^2*b + 20*a*b^2 + 63*(a^3 + a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 20*(21*(a^3 + a^2*b)*cosh(d*x + c)^3 + (11*a^3 + 31*a^2*b + 20*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^4 - 4*(11*a^3 + 31*a^2*b + 20*a*b^2)*cosh(d*x + c)^3 + 4*(105*(a^3 + a^2*b)*cosh(d*x + c)^4 - 11*a^3 - 31*a^2*b - 20*a*b^2 + 10*(11*a^3 + 31*a^2*b + 20*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 4*(63*(a^3 + a^2*b)*cosh(d*x + c)^5 + 10*(11*a^3 + 31*a^2*b + 20*a*b^2)*cosh(d*x + c)^3 - 3*(11*a^3 + 31*a^2*b + 20*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 - 3*(a^2*cosh(d*x + c)^8 + 8*a^2*cosh(d*x + c)*sinh(d*x + c)^7 + a^2*sinh(d*x + c)^8 + 4*(a^2 + 2*a*b)*cosh(d*x + c)^6 + 4*(7*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^6 + 8*(7*a^2*cosh(d*x + c)^3 + 3*(a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*a^2 + 8*a*b + 8*b^2)*cosh(d*x + c)^4 + 2*(35*a^2*cosh(d*x + c)^4 + 30*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 3*a^2 + 8*a*b + 8*b^2)*sinh(d*x + c)^4 + 8*(7*a^2*cosh(d*x + c)^5 + 10*(a^2 + 2*a*b)*cosh(d*x + c)^3 + (3*a^2 + 8*a*b + 8*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 4*(7*a^2*cosh(d*x + c)^6 + 15*(a^2 + 2*a*b)*cosh(d*x + c)^4 + 3*(3*a^2 + 8*a*b + 8*b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*(a^2*cosh(d*x + c)^7 + 3*(a^2 + 2*a*b)*cosh(d*x + c)^5 + (3*a^2 + 8*a*b + 8*b^2)*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a^2 - a*b)*log((a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 - 2*(3*a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 - 3*a - 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 - (3*a + 2*b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1)*sinh(d*x + c) - cosh(d*x + c))*sqrt(-a^2 - a*b) + a)/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) - 12*(a^3 + a^2*b)*cosh(d*x + c) + 4*(21*(a^3 + a^2*b)*cosh(d*x + c)^6 + 5*(11*a^3 + 31*a^2*b + 20*a*b^2)*cosh(d*x + c)^4 - 3*a^3 - 3*a^2*b - 3*(11*a^3 + 31*a^2*b + 20*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^8 + 8*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*sinh(d*x + c)^8 + 4*(a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 + 2*a^2*b^4)*d*cosh(d*x + c)^6 + 4*(7*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^2 + (a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 + 2*a^2*b^4)*d)*sinh(d*x + c)^6 + 2*(3*a^6 + 17*a^5*b + 41*a^4*b^2 + 51*a^3*b^3 + 32*a^2*b^4 + 8*a*b^5)*d*cosh(d*x + c)^4 + 8*(7*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^3 + 3*(a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 + 2*a^2*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*

d*cosh(d*x + c)^4 + 30*(a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 + 2*a^2*b^4)*
d*cosh(d*x + c)^2 + (3*a^6 + 17*a^5*b + 41*a^4*b^2 + 51*a^3*b^3 + 32*a^2*b^
4 + 8*a*b^5)*d)*sinh(d*x + c)^4 + 4*(a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3
+ 2*a^2*b^4)*d*cosh(d*x + c)^2 + 8*(7*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)
*d*cosh(d*x + c)^5 + 10*(a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 + 2*a^2*b^4)
*d*cosh(d*x + c)^3 + (3*a^6 + 17*a^5*b + 41*a^4*b^2 + 51*a^3*b^3 + 32*a^2*b
^4 + 8*a*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^6 + 3*a^5*b + 3*a^
4*b^2 + a^3*b^3)*d*cosh(d*x + c)^6 + 15*(a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*
b^3 + 2*a^2*b^4)*d*cosh(d*x + c)^4 + 3*(3*a^6 + 17*a^5*b + 41*a^4*b^2 + 51*
a^3*b^3 + 32*a^2*b^4 + 8*a*b^5)*d*cosh(d*x + c)^2 + (a^6 + 5*a^5*b + 9*a^4*
b^2 + 7*a^3*b^3 + 2*a^2*b^4)*d)*sinh(d*x + c)^2 + (a^6 + 3*a^5*b + 3*a^4*b^
2 + a^3*b^3)*d + 8*((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^7
 + 3*(a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 + 2*a^2*b^4)*d*cosh(d*x + c)^5
+ (3*a^6 + 17*a^5*b + 41*a^4*b^2 + 51*a^3*b^3 + 32*a^2*b^4 + 8*a*b^5)*d*cos
h(d*x + c)^3 + (a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 + 2*a^2*b^4)*d*cosh(d
*x + c))*sinh(d*x + c)), 1/8*(6*(a^3 + a^2*b)*cosh(d*x + c)^7 + 42*(a^3 + a
^2*b)*cosh(d*x + c)*sinh(d*x + c)^6 + 6*(a^3 + a^2*b)*sinh(d*x + c)^7 + 2*(
11*a^3 + 31*a^2*b + 20*a*b^2)*cosh(d*x + c)^5 + 2*(11*a^3 + 31*a^2*b + 20*a
b^2 + 63(a^3 + a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 10*(21*(a^3 + a^
2*b)*cosh(d*x + c)^3 + (11*a^3 + 31*a^2*b + 20*a*b^2)*cosh(d*x + c))*sinh(d
x + c)^4 - 2(11*a^3 + 31*a^2*b + 20*a*b^2)*cosh(d*x + c)^3 + 2*(105*(a^3
+ a^2*b)*cosh(d*x + c)^4 - 11*a^3 - 31*a^2*b - 20*a*b^2 + 10*(11*a^3 + 31*a
^2*b + 20*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 2*(63*(a^3 + a^2*b)*cos
h(d*x + c)^5 + 10*(11*a^3 + 31*a^2*b + 20*a*b^2)*cosh(d*x + c)^3 - 3*(11*a^
3 + 31*a^2*b + 20*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 3*(a^2*cosh(d*x +
 c)^8 + 8*a^2*cosh(d*x + c)*sinh(d*x + c)^7 + a^2*sinh(d*x + c)^8 + 4*(a^2
+ 2*a*b)*cosh(d*x + c)^6 + 4*(7*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x
 + c)^6 + 8*(7*a^2*cosh(d*x + c)^3 + 3*(a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*
x + c)^5 + 2*(3*a^2 + 8*a*b + 8*b^2)*cosh(d*x + c)^4 + 2*(35*a^2*cosh(d*x +
 c)^4 + 30*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 3*a^2 + 8*a*b + 8*b^2)*sinh(d*x
+ c)^4 + 8*(7*a^2*cosh(d*x + c)^5 + 10*(a^2 + 2*a*b)*cosh(d*x + c)^3 + (3*a
^2 + 8*a*b + 8*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a^2 + 2*a*b)*cosh(d
x + c)^2 + 4(7*a^2*cosh(d*x + c)^6 + 15*(a^2 + 2*a*b)*cosh(d*x + c)^4 + 3
*(3*a^2 + 8*a*b + 8*b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a
^2 + 8*(a^2*cosh(d*x + c)^7 + 3*(a^2 + 2*a*b)*cosh(d*x + c)^5 + (3*a^2 + 8*
a*b + 8*b^2)*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c))*
sqrt(a^2 + a*b)*arctan(1/2*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x
+ c)^2 + a*sinh(d*x + c)^3 + (3*a + 4*b)*cosh(d*x + c) + (3*a*cosh(d*x + c)
^2 + 3*a + 4*b)*sinh(d*x + c))/sqrt(a^2 + a*b)) + 3*(a^2*cosh(d*x + c)^8 +
8*a^2*cosh(d*x + c)*sinh(d*x + c)^7 + a^2*sinh(d*x + c)^8 + 4*(a^2 + 2*a*b)
*cosh(d*x + c)^6 + 4*(7*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^6
+ 8*(7*a^2*cosh(d*x + c)^3 + 3*(a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c)^5
 + 2*(3*a^2 + 8*a*b + 8*b^2)*cosh(d*x + c)^4 + 2*(35*a^2*cosh(d*x + c)^4 +
30*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 3*a^2 + 8*a*b + 8*b^2)*sinh(d*x + c)^4 +
 8*(7*a^2*cosh(d*x + c)^5 + 10*(a^2 + 2*a*b)*cosh(d*x + c)^3 + (3*a^2 + 8*a

```
*b + 8*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a^2 + 2*a*b)*cosh(d*x + c)^
2 + 4*(7*a^2*cosh(d*x + c)^6 + 15*(a^2 + 2*a*b)*cosh(d*x + c)^4 + 3*(3*a^2
+ 8*a*b + 8*b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*(
a^2*cosh(d*x + c)^7 + 3*(a^2 + 2*a*b)*cosh(d*x + c)^5 + (3*a^2 + 8*a*b + 8*
b^2)*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2
 + a*b)*arctan(1/2*sqrt(a^2 + a*b)*(cosh(d*x + c) + sinh(d*x + c))/(a + b))
 - 6*(a^3 + a^2*b)*cosh(d*x + c) + 2*(21*(a^3 + a^2*b)*cosh(d*x + c)^6 + 5*
(11*a^3 + 31*a^2*b + 20*a*b^2)*cosh(d*x + c)^4 - 3*a^3 - 3*a^2*b - 3*(11*a^
3 + 31*a^2*b + 20*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c))/((a^6 + 3*a^5*b +
3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^8 + 8*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3
*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^
3)*d*sinh(d*x + c)^8 + 4*(a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 + 2*a^2*b^4
)*d*cosh(d*x + c)^6 + 4*(7*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x
 + c)^2 + (a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 + 2*a^2*b^4)*d)*sinh(d*x +
 c)^6 + 2*(3*a^6 + 17*a^5*b + 41*a^4*b^2 + 51*a^3*b^3 + 32*a^2*b^4 + 8*a*b^
5)*d*cosh(d*x + c)^4 + 8*(7*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*
x + c)^3 + 3*(a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 + 2*a^2*b^4)*d*cosh(d*x
 + c))*sinh(d*x + c)^5 + 2*(35*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh
(d*x + c)^4 + 30*(a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 + 2*a^2*b^4)*d*cosh
(d*x + c)^2 + (3*a^6 + 17*a^5*b + 41*a^4*b^2 + 51*a^3*b^3 + 32*a^2*b^4 + 8*
a*b^5)*d)*sinh(d*x + c)^4 + 4*(a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 + 2*a^
2*b^4)*d*cosh(d*x + c)^2 + 8*(7*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cos
h(d*x + c)^5 + 10*(a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 + 2*a^2*b^4)*d*cos
h(d*x + c)^3 + (3*a^6 + 17*a^5*b + 41*a^4*b^2 + 51*a^3*b^3 + 32*a^2*b^4 + 8
*a*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^6 + 3*a^5*b + 3*a^4*b^2
+ a^3*b^3)*d*cosh(d*x + c)^6 + 15*(a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 +
2*a^2*b^4)*d*cosh(d*x + c)^4 + 3*(3*a^6 + 17*a^5*b + 41*a^4*b^2 + 51*a^3*b^
3 + 32*a^2*b^4 + 8*a*b^5)*d*cosh(d*x + c)^2 + (a^6 + 5*a^5*b + 9*a^4*b^2 +
7*a^3*b^3 + 2*a^2*b^4)*d)*sinh(d*x + c)^2 + (a^6 + 3*a^5*b + 3*a^4*b^2 + a^
3*b^3)*d + 8*((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^7 + 3*(
a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 + 2*a^2*b^4)*d*cosh(d*x + c)^5 + (3*a
^6 + 17*a^5*b + 41*a^4*b^2 + 51*a^3*b^3 + 32*a^2*b^4 + 8*a*b^5)*d*cosh(d*x
+ c)^3 + (a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 + 2*a^2*b^4)*d*cosh(d*x + c
))*sinh(d*x + c))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
```

```
assuming [a,b]=[84,-86]Warning, need to choose a branch for the root of a p
olynomial with parameters. This might be wrong.The choice was done assuming
 [a,b]=[-42,-12]Warning, need to choose a branch for the root of a polynomi
al with parameters. This might be wrong.The choice was done assuming [a,b]=
[-43,-99]Warning, need to choose a branch for the root of a polynomial with
 parameters. This might be wrong.The choice was done assuming [a,b]=[-28,94
]Warning, need to choose a branch for the root of a polynomial with paramet
ers. This might be wrong.The choice was done assuming [a,b]=[-7,46]Warning,
 need to choose a branch for the root of a polynomial with parameters. This
 might be wrong.The choice was done assuming [a,b]=[-35,-99]Warning, need t
o choose a branch for the root of a polynomial with parameters. This might
be wrong.The choice was done assuming [a,b]=[7,50]Warning, need to choose a
 branch for the root of a polynomial with parameters. This might be wrong.T
he choice was done assuming [a,b]=[-63,-70]Undef/Unsigned Inf encountered i
n limitEvaluation time: 1.6Limit: Max order reached or unable to make serie
s expansion Error: Bad Argument Value
```

maple [B] time = 0.32, size = 592, normalized size = 5.58

$$-\frac{5\left(\tanh^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d\left(\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a+b\right)^2(a+b)}+\cdots 4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^5/(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] -5/4/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*
c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)^7+3/4/d/(
tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2
*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^5*a+15/4/d/(tan
h(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*ta
nh(1/2*d*x+1/2*c)^2*b+a+b)^2*b/(a+b)^2*tanh(1/2*d*x+1/2*c)^5-3/4/d/(tanh(1/
2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1
/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^3*a-15/4/d/(tanh(1/2*d
*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*
d*x+1/2*c)^2*b+a+b)^2*b/(a+b)^2*tanh(1/2*d*x+1/2*c)^3+5/4/d/(tanh(1/2*d*x+1
/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+
1/2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)+3/8/d/(a^2+2*a*b+b^2)/(a+b)^(1/
2)/a^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)-2*b^(1/2))/a^(1/2)
)+3/8/d/(a^2+2*a*b+b^2)/(a+b)^(1/2)/a^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(
1/2*d*x+1/2*c)+2*b^(1/2))/a^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(11\,ae^{(5\,c)}+20\,be^{(5}\right.}{4\left(a^4d+2\,a^3bd+a^2b^2d+\left(a^4de^{(8\,c)}+2\,a^3bde^{(8\,c)}+a^2b^2de^{(8\,c)}\right)e^{(8\,dx)}+4\left(a^4de^{(6\,c)}+4\,a^3bde^{(6\,c)}+5\,a^2b^2de^{(6\,c)}+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*((11*a*e^(5*c) + 20*b*e^(5*c))*e^(5*d*x) - (11*a*e^(3*c) + 20*b*e^(3*c)
)*e^(3*d*x) + 3*a*e^(7*d*x + 7*c) - 3*a*e^(d*x + c))/(a^4*d + 2*a^3*b*d + a
^2*b^2*d + (a^4*d*e^(8*c) + 2*a^3*b*d*e^(8*c) + a^2*b^2*d*e^(8*c))*e^(8*d*x
) + 4*(a^4*d*e^(6*c) + 4*a^3*b*d*e^(6*c) + 5*a^2*b^2*d*e^(6*c) + 2*a*b^3*d*
e^(6*c))*e^(6*d*x) + 2*(3*a^4*d*e^(4*c) + 14*a^3*b*d*e^(4*c) + 27*a^2*b^2*d
*e^(4*c) + 24*a*b^3*d*e^(4*c) + 8*b^4*d*e^(4*c))*e^(4*d*x) + 4*(a^4*d*e^(2*
c) + 4*a^3*b*d*e^(2*c) + 5*a^2*b^2*d*e^(2*c) + 2*a*b^3*d*e^(2*c))*e^(2*d*x)
) + 32*integrate(3/128*(e^(3*d*x + 3*c) + e^(d*x + c))/(a^3 + 2*a^2*b + a*b
^2 + (a^3*e^(4*c) + 2*a^2*b*e^(4*c) + a*b^2*e^(4*c))*e^(4*d*x) + 2*(a^3*e^(
2*c) + 4*a^2*b*e^(2*c) + 5*a*b^2*e^(2*c) + 2*b^3*e^(2*c))*e^(2*d*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh\left(c+d\,x\right)^5\,\left(a+\frac{b}{\cosh\left(c+d\,x\right)^2}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^5*(a + b/cosh(c + d*x)^2)^3),x)
```

```
[Out] int(1/(cosh(c + d*x)^5*(a + b/cosh(c + d*x)^2)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^{5}{\left(c + dx \right)}}{\left(a + b \operatorname{sech}^{2}{\left(c + dx \right)}\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**5/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Integral(sech(c + d*x)**5/(a + b*sech(c + d*x)**2)**3, x)
```

3.100 $\int \frac{\text{sech}^6(c+dx)}{\left(a+b\,\text{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=144

$$\frac{\left(3a^2+8ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8b^{5/2}d(a+b)^{5/2}}-\frac{3a(a+2b)\tanh(c+dx)}{8b^2d(a+b)^2\left(a-b\tanh^2(c+dx)+b\right)}-\frac{a\tanh(c+dx)\text{sech}^2(c+d}{4bd(a+b)\left(a-b\tanh^2(c+dx\right.}$$

```
[Out] 1/8*(3*a^2+8*a*b+8*b^2)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))/b^(5/2)/(a
+b)^(5/2)/d-1/4*a*sech(d*x+c)^2*tanh(d*x+c)/b/(a+b)/d/(a+b-b*tanh(d*x+c)^2)
^2-3/8*a*(a+2*b)*tanh(d*x+c)/b^2/(a+b)^2/d/(a+b-b*tanh(d*x+c)^2)
```

Rubi [A] time = 0.14, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4146, 413, 385, 208}

$$\frac{\left(3a^2+8ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8b^{5/2}d(a+b)^{5/2}}-\frac{3a(a+2b)\tanh(c+dx)}{8b^2d(a+b)^2\left(a-b\tanh^2(c+dx)+b\right)}-\frac{a\tanh(c+dx)\text{sech}^2(c+d}{4bd(a+b)\left(a-b\tanh^2(c+dx\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^6/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((3*a^2 + 8*a*b + 8*b^2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(8*b
^(5/2)*(a + b)^(5/2)*d) - (a*Sech[c + d*x]^2*Tanh[c + d*x])/(4*b*(a + b)*d*
(a + b - b*Tanh[c + d*x]^2)^2) - (3*a*(a + 2*b)*Tanh[c + d*x])/(8*b^2*(a +
b)^2*d*(a + b - b*Tanh[c + d*x]^2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = \frac{\operatorname{Subst}\left(\int \frac{\left(1-x^2\right)^2}{\left(a+b-bx^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{a\operatorname{sech}^2(c+dx)\tanh(c+dx)}{4b(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2} - \frac{\operatorname{Subst}\left(\int \frac{-a-4b+(3a+4b)x^2}{\left(a+b-bx^2\right)^2}\,dx, x, \tanh(c+\right.}{4b(a+b)d}$$

$$= -\frac{a\operatorname{sech}^2(c+dx)\tanh(c+dx)}{4b(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2} - \frac{3a(a+2b)\tanh(c+dx)}{8b^2(a+b)^2d\left(a+b-b\tanh^2(c+dx)\right)}$$

$$= \frac{\left(3a^2+8ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8b^{5/2}(a+b)^{5/2}d} - \frac{a\operatorname{sech}^2(c+dx)\tanh(c+dx)}{4b(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2}$$

Mathematica [A] time = 1.01, size = 125, normalized size = 0.87

$$\frac{\frac{\left(3a^2+8ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}} - \frac{a\sqrt{b}\sinh(2(c+dx))\left(3a^2+3a(a+2b)\cosh(2(c+dx))+16ab+16b^2\right)}{(a+b)^2(a\cosh(2(c+dx))+a+2b)^2}}{8b^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^6/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (((3*a^2 + 8*a*b + 8*b^2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a
+ b)^(5/2) - (a*Sqrt[b]*(3*a^2 + 16*a*b + 16*b^2 + 3*a*(a + 2*b)*Cosh[2*(c
+ d*x)])*Sinh[2*(c + d*x)])/((a + b)^2*(a + 2*b + a*Cosh[2*(c + d*x)])^2))/
(8*b^(5/2)*d)
```

fricas [B] time = 0.53, size = 5887, normalized size = 40.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(4*(3*a^4*b + 11*a^3*b^2 + 16*a^2*b^3 + 8*a*b^4)*cosh(d*x + c)^6 + 24
*(3*a^4*b + 11*a^3*b^2 + 16*a^2*b^3 + 8*a*b^4)*cosh(d*x + c)*sinh(d*x + c)^
5 + 4*(3*a^4*b + 11*a^3*b^2 + 16*a^2*b^3 + 8*a*b^4)*sinh(d*x + c)^6 + 12*a^
4*b + 36*a^3*b^2 + 24*a^2*b^3 + 12*(3*a^4*b + 17*a^3*b^2 + 38*a^2*b^3 + 40*
a*b^4 + 16*b^5)*cosh(d*x + c)^4 + 12*(3*a^4*b + 17*a^3*b^2 + 38*a^2*b^3 + 4
0*a*b^4 + 16*b^5 + 5*(3*a^4*b + 11*a^3*b^2 + 16*a^2*b^3 + 8*a*b^4)*cosh(d*x
 + c)^2)*sinh(d*x + c)^4 + 16*(5*(3*a^4*b + 11*a^3*b^2 + 16*a^2*b^3 + 8*a*b
^4)*cosh(d*x + c)^3 + 3*(3*a^4*b + 17*a^3*b^2 + 38*a^2*b^3 + 40*a*b^4 + 16*
b^5)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(9*a^4*b + 49*a^3*b^2 + 80*a^2*b^3
+ 40*a*b^4)*cosh(d*x + c)^2 + 4*(9*a^4*b + 49*a^3*b^2 + 80*a^2*b^3 + 40*a*b
^4 + 15*(3*a^4*b + 11*a^3*b^2 + 16*a^2*b^3 + 8*a*b^4)*cosh(d*x + c)^4 + 18*
(3*a^4*b + 17*a^3*b^2 + 38*a^2*b^3 + 40*a*b^4 + 16*b^5)*cosh(d*x + c)^2)*si
nh(d*x + c)^2 + ((3*a^4 + 8*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^8 + 8*(3*a^4 +
 8*a^3*b + 8*a^2*b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (3*a^4 + 8*a^3*b + 8*
a^2*b^2)*sinh(d*x + c)^8 + 4*(3*a^4 + 14*a^3*b + 24*a^2*b^2 + 16*a*b^3)*cos
h(d*x + c)^6 + 4*(3*a^4 + 14*a^3*b + 24*a^2*b^2 + 16*a*b^3 + 7*(3*a^4 + 8*a
^3*b + 8*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(3*a^4 + 8*a^3*b
+ 8*a^2*b^2)*cosh(d*x + c)^3 + 3*(3*a^4 + 14*a^3*b + 24*a^2*b^2 + 16*a*b^3)
*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(9*a^4 + 48*a^3*b + 112*a^2*b^2 + 128*a
*b^3 + 64*b^4)*cosh(d*x + c)^4 + 2*(35*(3*a^4 + 8*a^3*b + 8*a^2*b^2)*cosh(d
*x + c)^4 + 9*a^4 + 48*a^3*b + 112*a^2*b^2 + 128*a*b^3 + 64*b^4 + 30*(3*a^4
 + 14*a^3*b + 24*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 3*a
^4 + 8*a^3*b + 8*a^2*b^2 + 8*(7*(3*a^4 + 8*a^3*b + 8*a^2*b^2)*cosh(d*x + c)
^5 + 10*(3*a^4 + 14*a^3*b + 24*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^3 + (9*a^4
 + 48*a^3*b + 112*a^2*b^2 + 128*a*b^3 + 64*b^4)*cosh(d*x + c))*sinh(d*x + c
)^3 + 4*(3*a^4 + 14*a^3*b + 24*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^2 + 4*(7*(
3*a^4 + 8*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^6 + 15*(3*a^4 + 14*a^3*b + 24*a^
2*b^2 + 16*a*b^3)*cosh(d*x + c)^4 + 3*a^4 + 14*a^3*b + 24*a^2*b^2 + 16*a*b^
3 + 3*(9*a^4 + 48*a^3*b + 112*a^2*b^2 + 128*a*b^3 + 64*b^4)*cosh(d*x + c)^2
)*sinh(d*x + c)^2 + 8*((3*a^4 + 8*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^7 + 3*(3
*a^4 + 14*a^3*b + 24*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^5 + (9*a^4 + 48*a^3*
```

```
b + 112*a^2*b^2 + 128*a*b^3 + 64*b^4)*cosh(d*x + c)^3 + (3*a^4 + 14*a^3*b +
 24*a^2*b^2 + 16*a*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a*b + b^2)*log((
a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x +
c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2
*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2
 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) - 4*(a*cosh(d*x + c)^2 + 2*a*cosh(d*
x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(a*b + b^2))/(a*cos
h(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a
 + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2
 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) + 8*
(3*(3*a^4*b + 11*a^3*b^2 + 16*a^2*b^3 + 8*a*b^4)*cosh(d*x + c)^5 + 6*(3*a^4
*b + 17*a^3*b^2 + 38*a^2*b^3 + 40*a*b^4 + 16*b^5)*cosh(d*x + c)^3 + (9*a^4*
b + 49*a^3*b^2 + 80*a^2*b^3 + 40*a*b^4)*cosh(d*x + c))*sinh(d*x + c))/((a^5
*b^3 + 3*a^4*b^4 + 3*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c)^8 + 8*(a^5*b^3 + 3*
a^4*b^4 + 3*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^5*b^3 +
 3*a^4*b^4 + 3*a^3*b^5 + a^2*b^6)*d*sinh(d*x + c)^8 + 4*(a^5*b^3 + 5*a^4*b^
4 + 9*a^3*b^5 + 7*a^2*b^6 + 2*a*b^7)*d*cosh(d*x + c)^6 + 4*(7*(a^5*b^3 + 3*
a^4*b^4 + 3*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c)^2 + (a^5*b^3 + 5*a^4*b^4 + 9
*a^3*b^5 + 7*a^2*b^6 + 2*a*b^7)*d)*sinh(d*x + c)^6 + 2*(3*a^5*b^3 + 17*a^4*
b^4 + 41*a^3*b^5 + 51*a^2*b^6 + 32*a*b^7 + 8*b^8)*d*cosh(d*x + c)^4 + 8*(7*
(a^5*b^3 + 3*a^4*b^4 + 3*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c)^3 + 3*(a^5*b^3
+ 5*a^4*b^4 + 9*a^3*b^5 + 7*a^2*b^6 + 2*a*b^7)*d*cosh(d*x + c))*sinh(d*x +
c)^5 + 2*(35*(a^5*b^3 + 3*a^4*b^4 + 3*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c)^4
+ 30*(a^5*b^3 + 5*a^4*b^4 + 9*a^3*b^5 + 7*a^2*b^6 + 2*a*b^7)*d*cosh(d*x + c
)^2 + (3*a^5*b^3 + 17*a^4*b^4 + 41*a^3*b^5 + 51*a^2*b^6 + 32*a*b^7 + 8*b^8)
*d)*sinh(d*x + c)^4 + 4*(a^5*b^3 + 5*a^4*b^4 + 9*a^3*b^5 + 7*a^2*b^6 + 2*a*
b^7)*d*cosh(d*x + c)^2 + 8*(7*(a^5*b^3 + 3*a^4*b^4 + 3*a^3*b^5 + a^2*b^6)*d
*cosh(d*x + c)^5 + 10*(a^5*b^3 + 5*a^4*b^4 + 9*a^3*b^5 + 7*a^2*b^6 + 2*a*b^
7)*d*cosh(d*x + c)^3 + (3*a^5*b^3 + 17*a^4*b^4 + 41*a^3*b^5 + 51*a^2*b^6 +
32*a*b^7 + 8*b^8)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^5*b^3 + 3*a^4*
b^4 + 3*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c)^6 + 15*(a^5*b^3 + 5*a^4*b^4 + 9*
a^3*b^5 + 7*a^2*b^6 + 2*a*b^7)*d*cosh(d*x + c)^4 + 3*(3*a^5*b^3 + 17*a^4*b^
4 + 41*a^3*b^5 + 51*a^2*b^6 + 32*a*b^7 + 8*b^8)*d*cosh(d*x + c)^2 + (a^5*b^
3 + 5*a^4*b^4 + 9*a^3*b^5 + 7*a^2*b^6 + 2*a*b^7)*d)*sinh(d*x + c)^2 + (a^5*
b^3 + 3*a^4*b^4 + 3*a^3*b^5 + a^2*b^6)*d + 8*((a^5*b^3 + 3*a^4*b^4 + 3*a^3*
b^5 + a^2*b^6)*d*cosh(d*x + c)^7 + 3*(a^5*b^3 + 5*a^4*b^4 + 9*a^3*b^5 + 7*a
^2*b^6 + 2*a*b^7)*d*cosh(d*x + c)^5 + (3*a^5*b^3 + 17*a^4*b^4 + 41*a^3*b^5
+ 51*a^2*b^6 + 32*a*b^7 + 8*b^8)*d*cosh(d*x + c)^3 + (a^5*b^3 + 5*a^4*b^4 +
 9*a^3*b^5 + 7*a^2*b^6 + 2*a*b^7)*d*cosh(d*x + c))*sinh(d*x + c)), 1/8*(2*(
3*a^4*b + 11*a^3*b^2 + 16*a^2*b^3 + 8*a*b^4)*cosh(d*x + c)^6 + 12*(3*a^4*b
+ 11*a^3*b^2 + 16*a^2*b^3 + 8*a*b^4)*cosh(d*x + c)*sinh(d*x + c)^5 + 2*(3*a
^4*b + 11*a^3*b^2 + 16*a^2*b^3 + 8*a*b^4)*sinh(d*x + c)^6 + 6*a^4*b + 18*a^
3*b^2 + 12*a^2*b^3 + 6*(3*a^4*b + 17*a^3*b^2 + 38*a^2*b^3 + 40*a*b^4 + 16*b
^5)*cosh(d*x + c)^4 + 6*(3*a^4*b + 17*a^3*b^2 + 38*a^2*b^3 + 40*a*b^4 + 16*
b^5 + 5*(3*a^4*b + 11*a^3*b^2 + 16*a^2*b^3 + 8*a*b^4)*cosh(d*x + c)^2)*sinh
```

(d*x + c)^4 + 8*(5*(3*a^4*b + 11*a^3*b^2 + 16*a^2*b^3 + 8*a*b^4)*cosh(d*x +
c)^3 + 3*(3*a^4*b + 17*a^3*b^2 + 38*a^2*b^3 + 40*a*b^4 + 16*b^5)*cosh(d*x
+ c))*sinh(d*x + c)^3 + 2*(9*a^4*b + 49*a^3*b^2 + 80*a^2*b^3 + 40*a*b^4)*co
sh(d*x + c)^2 + 2*(9*a^4*b + 49*a^3*b^2 + 80*a^2*b^3 + 40*a*b^4 + 15*(3*a^4
*b + 11*a^3*b^2 + 16*a^2*b^3 + 8*a*b^4)*cosh(d*x + c)^4 + 18*(3*a^4*b + 17*
a^3*b^2 + 38*a^2*b^3 + 40*a*b^4 + 16*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^2
+ ((3*a^4 + 8*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^8 + 8*(3*a^4 + 8*a^3*b + 8*a
^2*b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (3*a^4 + 8*a^3*b + 8*a^2*b^2)*sinh(
d*x + c)^8 + 4*(3*a^4 + 14*a^3*b + 24*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^6 +
4*(3*a^4 + 14*a^3*b + 24*a^2*b^2 + 16*a*b^3 + 7*(3*a^4 + 8*a^3*b + 8*a^2*b
^2)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(3*a^4 + 8*a^3*b + 8*a^2*b^2)*c
osh(d*x + c)^3 + 3*(3*a^4 + 14*a^3*b + 24*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)
)*sinh(d*x + c)^5 + 2*(9*a^4 + 48*a^3*b + 112*a^2*b^2 + 128*a*b^3 + 64*b^4)
*cosh(d*x + c)^4 + 2*(35*(3*a^4 + 8*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^4 + 9*
a^4 + 48*a^3*b + 112*a^2*b^2 + 128*a*b^3 + 64*b^4 + 30*(3*a^4 + 14*a^3*b +
24*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 3*a^4 + 8*a^3*b +
8*a^2*b^2 + 8*(7*(3*a^4 + 8*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^5 + 10*(3*a^4
+ 14*a^3*b + 24*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^3 + (9*a^4 + 48*a^3*b +
112*a^2*b^2 + 128*a*b^3 + 64*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(3*a^4
+ 14*a^3*b + 24*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^2 + 4*(7*(3*a^4 + 8*a^3*
b + 8*a^2*b^2)*cosh(d*x + c)^6 + 15*(3*a^4 + 14*a^3*b + 24*a^2*b^2 + 16*a*b
^3)*cosh(d*x + c)^4 + 3*a^4 + 14*a^3*b + 24*a^2*b^2 + 16*a*b^3 + 3*(9*a^4 +
48*a^3*b + 112*a^2*b^2 + 128*a*b^3 + 64*b^4)*cosh(d*x + c)^2)*sinh(d*x + c
)^2 + 8*((3*a^4 + 8*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^7 + 3*(3*a^4 + 14*a^3*
b + 24*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^5 + (9*a^4 + 48*a^3*b + 112*a^2*b^
2 + 128*a*b^3 + 64*b^4)*cosh(d*x + c)^3 + (3*a^4 + 14*a^3*b + 24*a^2*b^2 +
16*a*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a*b - b^2)*arctan(1/2*(a*cosh
(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b
)*sqrt(-a*b - b^2)/(a*b + b^2)) + 4*(3*(3*a^4*b + 11*a^3*b^2 + 16*a^2*b^3 +
8*a*b^4)*cosh(d*x + c)^5 + 6*(3*a^4*b + 17*a^3*b^2 + 38*a^2*b^3 + 40*a*b^4
+ 16*b^5)*cosh(d*x + c)^3 + (9*a^4*b + 49*a^3*b^2 + 80*a^2*b^3 + 40*a*b^4)
*cosh(d*x + c))*sinh(d*x + c))/((a^5*b^3 + 3*a^4*b^4 + 3*a^3*b^5 + a^2*b^6)
*d*cosh(d*x + c)^8 + 8*(a^5*b^3 + 3*a^4*b^4 + 3*a^3*b^5 + a^2*b^6)*d*cosh(d
*x + c)*sinh(d*x + c)^7 + (a^5*b^3 + 3*a^4*b^4 + 3*a^3*b^5 + a^2*b^6)*d*sin
h(d*x + c)^8 + 4*(a^5*b^3 + 5*a^4*b^4 + 9*a^3*b^5 + 7*a^2*b^6 + 2*a*b^7)*d*
cosh(d*x + c)^6 + 4*(7*(a^5*b^3 + 3*a^4*b^4 + 3*a^3*b^5 + a^2*b^6)*d*cosh(d
*x + c)^2 + (a^5*b^3 + 5*a^4*b^4 + 9*a^3*b^5 + 7*a^2*b^6 + 2*a*b^7)*d)*sinh
(d*x + c)^6 + 2*(3*a^5*b^3 + 17*a^4*b^4 + 41*a^3*b^5 + 51*a^2*b^6 + 32*a*b^
7 + 8*b^8)*d*cosh(d*x + c)^4 + 8*(7*(a^5*b^3 + 3*a^4*b^4 + 3*a^3*b^5 + a^2*
b^6)*d*cosh(d*x + c)^3 + 3*(a^5*b^3 + 5*a^4*b^4 + 9*a^3*b^5 + 7*a^2*b^6 + 2
*a*b^7)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^5*b^3 + 3*a^4*b^4 + 3*a
^3*b^5 + a^2*b^6)*d*cosh(d*x + c)^4 + 30*(a^5*b^3 + 5*a^4*b^4 + 9*a^3*b^5 +
7*a^2*b^6 + 2*a*b^7)*d*cosh(d*x + c)^2 + (3*a^5*b^3 + 17*a^4*b^4 + 41*a^3*
b^5 + 51*a^2*b^6 + 32*a*b^7 + 8*b^8)*d)*sinh(d*x + c)^4 + 4*(a^5*b^3 + 5*a^
4*b^4 + 9*a^3*b^5 + 7*a^2*b^6 + 2*a*b^7)*d*cosh(d*x + c)^2 + 8*(7*(a^5*b^3

```
+ 3*a^4*b^4 + 3*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c)^5 + 10*(a^5*b^3 + 5*a^4*
b^4 + 9*a^3*b^5 + 7*a^2*b^6 + 2*a*b^7)*d*cosh(d*x + c)^3 + (3*a^5*b^3 + 17*
a^4*b^4 + 41*a^3*b^5 + 51*a^2*b^6 + 32*a*b^7 + 8*b^8)*d*cosh(d*x + c))*sinh
(d*x + c)^3 + 4*(7*(a^5*b^3 + 3*a^4*b^4 + 3*a^3*b^5 + a^2*b^6)*d*cosh(d*x +
 c)^6 + 15*(a^5*b^3 + 5*a^4*b^4 + 9*a^3*b^5 + 7*a^2*b^6 + 2*a*b^7)*d*cosh(d
*x + c)^4 + 3*(3*a^5*b^3 + 17*a^4*b^4 + 41*a^3*b^5 + 51*a^2*b^6 + 32*a*b^7
+ 8*b^8)*d*cosh(d*x + c)^2 + (a^5*b^3 + 5*a^4*b^4 + 9*a^3*b^5 + 7*a^2*b^6 +
 2*a*b^7)*d)*sinh(d*x + c)^2 + (a^5*b^3 + 3*a^4*b^4 + 3*a^3*b^5 + a^2*b^6)*
d + 8*((a^5*b^3 + 3*a^4*b^4 + 3*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c)^7 + 3*(a
^5*b^3 + 5*a^4*b^4 + 9*a^3*b^5 + 7*a^2*b^6 + 2*a*b^7)*d*cosh(d*x + c)^5 + (
3*a^5*b^3 + 17*a^4*b^4 + 41*a^3*b^5 + 51*a^2*b^6 + 32*a*b^7 + 8*b^8)*d*cosh
(d*x + c)^3 + (a^5*b^3 + 5*a^4*b^4 + 9*a^3*b^5 + 7*a^2*b^6 + 2*a*b^7)*d*cos
h(d*x + c))*sinh(d*x + c))]
```

giac [B] time = 1.75, size = 302, normalized size = 2.10

$$\frac{\frac{\left(3\,a^{2}+8\,ab+8\,b^{2}\right)\arctan\left(\frac{ae^{\left(2\,dx+2\,c\right)}+a+2\,b}{2\,\sqrt{-ab-b^{2}}}\right)}{\left(a^{2}b^{2}+2\,ab^{3}+b^{4}\right)\sqrt{-ab-b^{2}}}+\frac{2\left(3\,a^{3}e^{\left(6\,dx+6\,c\right)}+8\,a^{2}be^{\left(6\,dx+6\,c\right)}+8\,ab^{2}e^{\left(6\,dx+6\,c\right)}+9\,a^{3}e^{\left(4\,dx+4\,c\right)}+42\,a^{2}be^{\left(4\,dx+4\,c\right)}+72\,ab^{2}e^{\left(4\,dx+4\,c\right)}+48\,b}{\left(a^{2}b^{2}+2\,ab^{3}+b^{4}\right)\left(ae^{\left(4\,dx+4\,c\right)}+2\,ae^{\left(2\,dx+2\,c\right)}+4}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*((3*a^2 + 8*a*b + 8*b^2)*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(
-a*b - b^2))/((a^2*b^2 + 2*a*b^3 + b^4)*sqrt(-a*b - b^2)) + 2*(3*a^3*e^(6*d
*x + 6*c) + 8*a^2*b*e^(6*d*x + 6*c) + 8*a*b^2*e^(6*d*x + 6*c) + 9*a^3*e^(4*
d*x + 4*c) + 42*a^2*b*e^(4*d*x + 4*c) + 72*a*b^2*e^(4*d*x + 4*c) + 48*b^3*e
^(4*d*x + 4*c) + 9*a^3*e^(2*d*x + 2*c) + 40*a^2*b*e^(2*d*x + 2*c) + 40*a*b^
2*e^(2*d*x + 2*c) + 3*a^3 + 6*a^2*b)/((a^2*b^2 + 2*a*b^3 + b^4)*(a*e^(4*d*x
 + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) + a)^2))/d
```

maple [B] time = 0.34, size = 1245, normalized size = 8.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^6/(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] -3/4/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*
c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*a^2/(a+b)/b^2*tanh(1/2*d*x+1/2*c)^7
-2/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)
^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/b/(a+b)*tanh(1/2*d*x+1/2*c)^7*a-9/4/d
/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a
-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*a^3/(a+b)^2/b^2*tanh(1/2*d*x+1/2*c)^5-13/
```

```
4/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^
2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2/b*tanh(1/2*d*x+1/2*c)^5*a^2+2/
d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*
a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^5*a-9/4/d/(t
anh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*
tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*a^3/(a+b)^2/b^2*tanh(1/2*d*x+1/2*c)^3-13/4/d
/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a
-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2/b*tanh(1/2*d*x+1/2*c)^3*a^2+2/d/(
tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2
*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^3*a-3/4/d/(tanh
(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tan
h(1/2*d*x+1/2*c)^2*b+a+b)^2*a^2/(a+b)/b^2*tanh(1/2*d*x+1/2*c)-2/d/(tanh(1/2
*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/
2*d*x+1/2*c)^2*b+a+b)^2/b/(a+b)*tanh(1/2*d*x+1/2*c)*a-3/16/d/b^(5/2)/(a^2+2
*a*b+b^2)/(a+b)^(1/2)*ln(-(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(
1/2*d*x+1/2*c)-(a+b)^(1/2))*a^2-1/2/d/b^(3/2)/(a^2+2*a*b+b^2)*a/(a+b)^(1/2)
*ln(-(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)-(a+b)^
(1/2))-1/2/d/(a^2+2*a*b+b^2)/b^(1/2)/(a+b)^(1/2)*ln(-(a+b)^(1/2)*tanh(1/2*d
*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)-(a+b)^(1/2))+3/16/d/b^(5/2)/(a^2+
2*a*b+b^2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(
1/2*d*x+1/2*c)+(a+b)^(1/2))*a^2+1/2/d/b^(3/2)/(a^2+2*a*b+b^2)*a/(a+b)^(1/2)
*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(
1/2))+1/2/d/(a^2+2*a*b+b^2)/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x
+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))
```

maxima [B] time = 1.37, size = 395, normalized size = 2.74

$$-\frac{\left(3\,a^2+8\,ab+8\,b^2\right)\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{16\left(a^2b^2+2\,ab^3+b^4\right)\sqrt{(a+b)b}\,d}-\frac{3\,a^3+6\,a^2b+\left(9\,a^3\right.}{4\left(a^4b^2+2\,a^3b^3+a^2b^4+4\left(a^4b^2+4\,a^3b^3+5\,a^2b^4+2\,ab^5\right)e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(3*a^2 + 8*a*b + 8*b^2)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a
 + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^2*b^2 + 2
*a*b^3 + b^4)*sqrt((a + b)*b)*d) - 1/4*(3*a^3 + 6*a^2*b + (9*a^3 + 40*a^2*b
 + 40*a*b^2)*e^(-2*d*x - 2*c) + 3*(3*a^3 + 14*a^2*b + 24*a*b^2 + 16*b^3)*e^
(-4*d*x - 4*c) + (3*a^3 + 8*a^2*b + 8*a*b^2)*e^(-6*d*x - 6*c))/((a^4*b^2 +
2*a^3*b^3 + a^2*b^4 + 4*(a^4*b^2 + 4*a^3*b^3 + 5*a^2*b^4 + 2*a*b^5)*e^(-2*d
*x - 2*c) + 2*(3*a^4*b^2 + 14*a^3*b^3 + 27*a^2*b^4 + 24*a*b^5 + 8*b^6)*e^(-
4*d*x - 4*c) + 4*(a^4*b^2 + 4*a^3*b^3 + 5*a^2*b^4 + 2*a*b^5)*e^(-6*d*x - 6*
c) + (a^4*b^2 + 2*a^3*b^3 + a^2*b^4)*e^(-8*d*x - 8*c))*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh(c+dx)^6 \left(a+\frac{b}{\cosh(c+dx)^2}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^6*(a + b/cosh(c + d*x)^2)^3),x)
[Out] int(1/(cosh(c + d*x)^6*(a + b/cosh(c + d*x)^2)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**6/(a+b*sech(d*x+c)**2)**3,x)
[Out] Integral(sech(c + d*x)**6/(a + b*sech(c + d*x)**2)**3, x)
```

3.101 $\int \frac{\operatorname{sech}^7(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=153

$$-\frac{\sqrt{a}\left(8a^2+20ab+15b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{8b^3d(a+b)^{5/2}}-\frac{a(4a+7b)\sinh(c+dx)}{8b^2d(a+b)^2\left(a\sinh^2(c+dx)+a+b\right)}-\frac{a\sinh(c+dx}{4bd(a+b)\left(a\sinh^2(c+\right.}$$

[Out] arctan(sinh(d*x+c))/b^3/d-1/4*a*sinh(d*x+c)/b/(a+b)/d/(a+b+a*sinh(d*x+c)^2)^2-1/8*a*(4*a+7*b)*sinh(d*x+c)/b^2/(a+b)^2/d/(a+b+a*sinh(d*x+c)^2)-1/8*(8*a^2+20*a*b+15*b^2)*arctan(sinh(d*x+c)*a^(1/2)/(a+b)^(1/2))*a^(1/2)/b^3/(a+b)^(5/2)/d

Rubi [A] time = 0.20, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4147, 414, 527, 522, 203, 205}

$$-\frac{\sqrt{a}\left(8a^2+20ab+15b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{8b^3d(a+b)^{5/2}}-\frac{a(4a+7b)\sinh(c+dx)}{8b^2d(a+b)^2\left(a\sinh^2(c+dx)+a+b\right)}-\frac{a\sinh(c+dx}{4bd(a+b)\left(a\sinh^2(c+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^7/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ArcTan[Sinh[c + d*x]]/(b^3*d) - (Sqrt[a]*(8*a^2 + 20*a*b + 15*b^2)*ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]])/(8*b^3*(a + b)^(5/2)*d) - (a*Sinh[c + d*x])/(4*b*(a + b)*d*(a + b + a*Sinh[c + d*x]^2)^2) - (a*(4*a + 7*b)*Sinh[c + d*x])/(8*b^2*(a + b)^2*d*(a + b + a*Sinh[c + d*x]^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{sech}^7(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{(1+x^2)(a+b+ax^2)^3}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= -\frac{a\sinh(c+dx)}{4b(a+b)d\left(a+b+a\sinh^2(c+dx)\right)^2} + \frac{\operatorname{Subst}\left(\int \frac{a+4b-3ax^2}{(1+x^2)(a+b+ax^2)^2}\,dx, x, \sinh(c+\right.}{4b(a+b)d}$$

$$= -\frac{a\sinh(c+dx)}{4b(a+b)d\left(a+b+a\sinh^2(c+dx)\right)^2} - \frac{a(4a+7b)\sinh(c+dx)}{8b^2(a+b)^2d\left(a+b+a\sinh^2(c+dx)\right)} +$$

$$= -\frac{a\sinh(c+dx)}{4b(a+b)d\left(a+b+a\sinh^2(c+dx)\right)^2} - \frac{a(4a+7b)\sinh(c+dx)}{8b^2(a+b)^2d\left(a+b+a\sinh^2(c+dx)\right)} +$$

$$= \frac{\tan^{-1}(\sinh(c+dx))}{b^3d} - \frac{\sqrt{a}\left(8a^2+20ab+15b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{8b^3(a+b)^{5/2}d} - \frac{}{4b(a+b)d}$$

Mathematica [A] time = 4.72, size = 247, normalized size = 1.61

$$\frac{\operatorname{sech}^5(c+dx)(a\cosh(2(c+dx))+a+2b)\left(\frac{\sqrt{a}\left(8a^2+20ab+15b^2\right)(\cosh(c)-\sinh(c))\operatorname{sech}(c+dx)(a\cosh(2(c+dx))+a+2b)^2\tan^{-1}\left(\frac{\sqrt{a+b}\sqrt{(}}{}\right.}{(a+b)^{5/2}\sqrt{(\cosh(c)-\sinh(c))^2}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^7/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^5*(16*ArcTan[Tanh[(c + d*x)/
2]]*(a + 2*b + a*Cosh[2*(c + d*x)])^2*Sech[c + d*x] + (Sqrt[a]*(8*a^2 + 20*
a*b + 15*b^2)*ArcTan[(Sqrt[a + b]*Csch[c + d*x]*Sqrt[(Cosh[c] - Sinh[c])^2]
*(Cosh[c] + Sinh[c]))/Sqrt[a]]*(a + 2*b + a*Cosh[2*(c + d*x)])^2*Sech[c + d
*x]*(Cosh[c] - Sinh[c]))/((a + b)^(5/2)*Sqrt[(Cosh[c] - Sinh[c])^2]) - (8*a
*b^2*Tanh[c + d*x])/(a + b) - (2*a*b*(4*a + 7*b)*(a + 2*b + a*Cosh[2*(c + d
*x)])*Tanh[c + d*x])/(a + b)^2))/(64*b^3*d*(a + b*Sech[c + d*x]^2)^3)
```

fricas [B] time = 0.62, size = 7993, normalized size = 52.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^7/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*(4*a^3*b + 7*a^2*b^2)*cosh(d*x + c)^7 + 28*(4*a^3*b + 7*a^2*b^2)*
cosh(d*x + c)*sinh(d*x + c)^6 + 4*(4*a^3*b + 7*a^2*b^2)*sinh(d*x + c)^7 + 4
*(4*a^3*b + 31*a^2*b^2 + 36*a*b^3)*cosh(d*x + c)^5 + 4*(4*a^3*b + 31*a^2*b^
2 + 36*a*b^3 + 21*(4*a^3*b + 7*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^5 +
20*(7*(4*a^3*b + 7*a^2*b^2)*cosh(d*x + c)^3 + (4*a^3*b + 31*a^2*b^2 + 36*a*
b^3)*cosh(d*x + c))*sinh(d*x + c)^4 - 4*(4*a^3*b + 31*a^2*b^2 + 36*a*b^3)*c
osh(d*x + c)^3 + 4*(35*(4*a^3*b + 7*a^2*b^2)*cosh(d*x + c)^4 - 4*a^3*b - 31
*a^2*b^2 - 36*a*b^3 + 10*(4*a^3*b + 31*a^2*b^2 + 36*a*b^3)*cosh(d*x + c)^2)
*sinh(d*x + c)^3 + 4*(21*(4*a^3*b + 7*a^2*b^2)*cosh(d*x + c)^5 + 10*(4*a^3*
b + 31*a^2*b^2 + 36*a*b^3)*cosh(d*x + c)^3 - 3*(4*a^3*b + 31*a^2*b^2 + 36*a
*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 - ((8*a^4 + 20*a^3*b + 15*a^2*b^2)*cos
h(d*x + c)^8 + 8*(8*a^4 + 20*a^3*b + 15*a^2*b^2)*cosh(d*x + c)*sinh(d*x + c
)^7 + (8*a^4 + 20*a^3*b + 15*a^2*b^2)*sinh(d*x + c)^8 + 4*(8*a^4 + 36*a^3*b
 + 55*a^2*b^2 + 30*a*b^3)*cosh(d*x + c)^6 + 4*(8*a^4 + 36*a^3*b + 55*a^2*b^
2 + 30*a*b^3 + 7*(8*a^4 + 20*a^3*b + 15*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x
+ c)^6 + 8*(7*(8*a^4 + 20*a^3*b + 15*a^2*b^2)*cosh(d*x + c)^3 + 3*(8*a^4 +
36*a^3*b + 55*a^2*b^2 + 30*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(24*a^
4 + 124*a^3*b + 269*a^2*b^2 + 280*a*b^3 + 120*b^4)*cosh(d*x + c)^4 + 2*(35*
(8*a^4 + 20*a^3*b + 15*a^2*b^2)*cosh(d*x + c)^4 + 24*a^4 + 124*a^3*b + 269*
a^2*b^2 + 280*a*b^3 + 120*b^4 + 30*(8*a^4 + 36*a^3*b + 55*a^2*b^2 + 30*a*b^
3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*a^4 + 20*a^3*b + 15*a^2*b^2 + 8*(7*
(8*a^4 + 20*a^3*b + 15*a^2*b^2)*cosh(d*x + c)^5 + 10*(8*a^4 + 36*a^3*b + 55
*a^2*b^2 + 30*a*b^3)*cosh(d*x + c)^3 + (24*a^4 + 124*a^3*b + 269*a^2*b^2 +
280*a*b^3 + 120*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(8*a^4 + 36*a^3*b +
 55*a^2*b^2 + 30*a*b^3)*cosh(d*x + c)^2 + 4*(7*(8*a^4 + 20*a^3*b + 15*a^2*b
^2)*cosh(d*x + c)^6 + 15*(8*a^4 + 36*a^3*b + 55*a^2*b^2 + 30*a*b^3)*cosh(d*
x + c)^4 + 8*a^4 + 36*a^3*b + 55*a^2*b^2 + 30*a*b^3 + 3*(24*a^4 + 124*a^3*b
 + 269*a^2*b^2 + 280*a*b^3 + 120*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*
((8*a^4 + 20*a^3*b + 15*a^2*b^2)*cosh(d*x + c)^7 + 3*(8*a^4 + 36*a^3*b + 55
*a^2*b^2 + 30*a*b^3)*cosh(d*x + c)^5 + (24*a^4 + 124*a^3*b + 269*a^2*b^2 +
280*a*b^3 + 120*b^4)*cosh(d*x + c)^3 + (8*a^4 + 36*a^3*b + 55*a^2*b^2 + 30*
a*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a/(a + b))*log((a*cosh(d*x + c)^
4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 - 2*(3*a + 2*b)*c
osh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 - 3*a - 2*b)*sinh(d*x + c)^2 + 4*(a
*cosh(d*x + c)^3 - (3*a + 2*b)*cosh(d*x + c))*sinh(d*x + c) - 4*((a + b)*co
sh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x
+ c)^3 - (a + b)*cosh(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 - a - b)*sinh(d
*x + c))*sqrt(-a/(a + b)) + a)/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(
d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(
d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*co
sh(d*x + c))*sinh(d*x + c) + a)) - 32*((a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x +
```

c)^8 + 8*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^4 +
2*a^3*b + a^2*b^2)*sinh(d*x + c)^8 + 4*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3
)*cosh(d*x + c)^6 + 4*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3 + 7*(a^4 + 2*a^3
*b + a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(a^4 + 2*a^3*b + a^2*
b^2)*cosh(d*x + c)^3 + 3*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*cosh(d*x + c
))*sinh(d*x + c)^5 + 2*(3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4)*c
osh(d*x + c)^4 + 2*(35*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)^4 + 3*a^4 +
14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4 + 30*(a^4 + 4*a^3*b + 5*a^2*b^2 +
2*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + a^4 + 2*a^3*b + a^2*b^2 + 8*(7*
(a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)^5 + 10*(a^4 + 4*a^3*b + 5*a^2*b^2 +
 2*a*b^3)*cosh(d*x + c)^3 + (3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b
^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3
)*cosh(d*x + c)^2 + 4*(7*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)^6 + 15*(a^
4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*cosh(d*x + c)^4 + a^4 + 4*a^3*b + 5*a^2*
b^2 + 2*a*b^3 + 3*(3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4)*cosh(d
*x + c)^2)*sinh(d*x + c)^2 + 8*((a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)^7 +
 3*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*cosh(d*x + c)^5 + (3*a^4 + 14*a^3*
b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4)*cosh(d*x + c)^3 + (a^4 + 4*a^3*b + 5*a^2
*b^2 + 2*a*b^3)*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d
x + c)) - 4(4*a^3*b + 7*a^2*b^2)*cosh(d*x + c) + 4*(7*(4*a^3*b + 7*a^2*b^
2)*cosh(d*x + c)^6 + 5*(4*a^3*b + 31*a^2*b^2 + 36*a*b^3)*cosh(d*x + c)^4 -
4*a^3*b - 7*a^2*b^2 - 3*(4*a^3*b + 31*a^2*b^2 + 36*a*b^3)*cosh(d*x + c)^2)*
sinh(d*x + c))/((a^4*b^3 + 2*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^8 + 8*(a^4*
b^3 + 2*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^4*b^3 + 2*a
^3*b^4 + a^2*b^5)*d*sinh(d*x + c)^8 + 4*(a^4*b^3 + 4*a^3*b^4 + 5*a^2*b^5 +
2*a*b^6)*d*cosh(d*x + c)^6 + 4*(7*(a^4*b^3 + 2*a^3*b^4 + a^2*b^5)*d*cosh(d*
x + c)^2 + (a^4*b^3 + 4*a^3*b^4 + 5*a^2*b^5 + 2*a*b^6)*d)*sinh(d*x + c)^6 +
 2*(3*a^4*b^3 + 14*a^3*b^4 + 27*a^2*b^5 + 24*a*b^6 + 8*b^7)*d*cosh(d*x + c)
^4 + 8*(7*(a^4*b^3 + 2*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^3 + 3*(a^4*b^3 +
4*a^3*b^4 + 5*a^2*b^5 + 2*a*b^6)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(
a^4*b^3 + 2*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^4 + 30*(a^4*b^3 + 4*a^3*b^4
+ 5*a^2*b^5 + 2*a*b^6)*d*cosh(d*x + c)^2 + (3*a^4*b^3 + 14*a^3*b^4 + 27*a^2
*b^5 + 24*a*b^6 + 8*b^7)*d)*sinh(d*x + c)^4 + 4*(a^4*b^3 + 4*a^3*b^4 + 5*a^
2*b^5 + 2*a*b^6)*d*cosh(d*x + c)^2 + 8*(7*(a^4*b^3 + 2*a^3*b^4 + a^2*b^5)*d
*cosh(d*x + c)^5 + 10*(a^4*b^3 + 4*a^3*b^4 + 5*a^2*b^5 + 2*a*b^6)*d*cosh(d*
x + c)^3 + (3*a^4*b^3 + 14*a^3*b^4 + 27*a^2*b^5 + 24*a*b^6 + 8*b^7)*d*cosh(
d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^4*b^3 + 2*a^3*b^4 + a^2*b^5)*d*cosh(d*x
 + c)^6 + 15*(a^4*b^3 + 4*a^3*b^4 + 5*a^2*b^5 + 2*a*b^6)*d*cosh(d*x + c)^4
+ 3*(3*a^4*b^3 + 14*a^3*b^4 + 27*a^2*b^5 + 24*a*b^6 + 8*b^7)*d*cosh(d*x + c
)^2 + (a^4*b^3 + 4*a^3*b^4 + 5*a^2*b^5 + 2*a*b^6)*d)*sinh(d*x + c)^2 + (a^4
*b^3 + 2*a^3*b^4 + a^2*b^5)*d + 8*((a^4*b^3 + 2*a^3*b^4 + a^2*b^5)*d*cosh(d
x + c)^7 + 3(a^4*b^3 + 4*a^3*b^4 + 5*a^2*b^5 + 2*a*b^6)*d*cosh(d*x + c)^5
 + (3*a^4*b^3 + 14*a^3*b^4 + 27*a^2*b^5 + 24*a*b^6 + 8*b^7)*d*cosh(d*x + c)
^3 + (a^4*b^3 + 4*a^3*b^4 + 5*a^2*b^5 + 2*a*b^6)*d*cosh(d*x + c))*sinh(d*x
+ c)), -1/8*(2*(4*a^3*b + 7*a^2*b^2)*cosh(d*x + c)^7 + 14*(4*a^3*b + 7*a^2*

```
b^2)*cosh(d*x + c)*sinh(d*x + c)^6 + 2*(4*a^3*b + 7*a^2*b^2)*sinh(d*x + c)^
7 + 2*(4*a^3*b + 31*a^2*b^2 + 36*a*b^3)*cosh(d*x + c)^5 + 2*(4*a^3*b + 31*a
^2*b^2 + 36*a*b^3 + 21*(4*a^3*b + 7*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)
^5 + 10*(7*(4*a^3*b + 7*a^2*b^2)*cosh(d*x + c)^3 + (4*a^3*b + 31*a^2*b^2 +
36*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 - 2*(4*a^3*b + 31*a^2*b^2 + 36*a*b
^3)*cosh(d*x + c)^3 + 2*(35*(4*a^3*b + 7*a^2*b^2)*cosh(d*x + c)^4 - 4*a^3*b
 - 31*a^2*b^2 - 36*a*b^3 + 10*(4*a^3*b + 31*a^2*b^2 + 36*a*b^3)*cosh(d*x +
c)^2)*sinh(d*x + c)^3 + 2*(21*(4*a^3*b + 7*a^2*b^2)*cosh(d*x + c)^5 + 10*(4
*a^3*b + 31*a^2*b^2 + 36*a*b^3)*cosh(d*x + c)^3 - 3*(4*a^3*b + 31*a^2*b^2 +
 36*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + ((8*a^4 + 20*a^3*b + 15*a^2*b^2
)*cosh(d*x + c)^8 + 8*(8*a^4 + 20*a^3*b + 15*a^2*b^2)*cosh(d*x + c)*sinh(d*
x + c)^7 + (8*a^4 + 20*a^3*b + 15*a^2*b^2)*sinh(d*x + c)^8 + 4*(8*a^4 + 36*
a^3*b + 55*a^2*b^2 + 30*a*b^3)*cosh(d*x + c)^6 + 4*(8*a^4 + 36*a^3*b + 55*a
^2*b^2 + 30*a*b^3 + 7*(8*a^4 + 20*a^3*b + 15*a^2*b^2)*cosh(d*x + c)^2)*sinh
(d*x + c)^6 + 8*(7*(8*a^4 + 20*a^3*b + 15*a^2*b^2)*cosh(d*x + c)^3 + 3*(8*a
^4 + 36*a^3*b + 55*a^2*b^2 + 30*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(
24*a^4 + 124*a^3*b + 269*a^2*b^2 + 280*a*b^3 + 120*b^4)*cosh(d*x + c)^4 + 2
*(35*(8*a^4 + 20*a^3*b + 15*a^2*b^2)*cosh(d*x + c)^4 + 24*a^4 + 124*a^3*b +
 269*a^2*b^2 + 280*a*b^3 + 120*b^4 + 30*(8*a^4 + 36*a^3*b + 55*a^2*b^2 + 30
*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*a^4 + 20*a^3*b + 15*a^2*b^2 +
8*(7*(8*a^4 + 20*a^3*b + 15*a^2*b^2)*cosh(d*x + c)^5 + 10*(8*a^4 + 36*a^3*b
 + 55*a^2*b^2 + 30*a*b^3)*cosh(d*x + c)^3 + (24*a^4 + 124*a^3*b + 269*a^2*b
^2 + 280*a*b^3 + 120*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(8*a^4 + 36*a^
3*b + 55*a^2*b^2 + 30*a*b^3)*cosh(d*x + c)^2 + 4*(7*(8*a^4 + 20*a^3*b + 15*
a^2*b^2)*cosh(d*x + c)^6 + 15*(8*a^4 + 36*a^3*b + 55*a^2*b^2 + 30*a*b^3)*co
sh(d*x + c)^4 + 8*a^4 + 36*a^3*b + 55*a^2*b^2 + 30*a*b^3 + 3*(24*a^4 + 124*
a^3*b + 269*a^2*b^2 + 280*a*b^3 + 120*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2
 + 8*((8*a^4 + 20*a^3*b + 15*a^2*b^2)*cosh(d*x + c)^7 + 3*(8*a^4 + 36*a^3*b
 + 55*a^2*b^2 + 30*a*b^3)*cosh(d*x + c)^5 + (24*a^4 + 124*a^3*b + 269*a^2*b
^2 + 280*a*b^3 + 120*b^4)*cosh(d*x + c)^3 + (8*a^4 + 36*a^3*b + 55*a^2*b^2
+ 30*a*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a/(a + b))*arctan(1/2*sqrt(a
/(a + b))*(cosh(d*x + c) + sinh(d*x + c))) + ((8*a^4 + 20*a^3*b + 15*a^2*b^
2)*cosh(d*x + c)^8 + 8*(8*a^4 + 20*a^3*b + 15*a^2*b^2)*cosh(d*x + c)*sinh(d
*x + c)^7 + (8*a^4 + 20*a^3*b + 15*a^2*b^2)*sinh(d*x + c)^8 + 4*(8*a^4 + 36
*a^3*b + 55*a^2*b^2 + 30*a*b^3)*cosh(d*x + c)^6 + 4*(8*a^4 + 36*a^3*b + 55*
a^2*b^2 + 30*a*b^3 + 7*(8*a^4 + 20*a^3*b + 15*a^2*b^2)*cosh(d*x + c)^2)*sin
h(d*x + c)^6 + 8*(7*(8*a^4 + 20*a^3*b + 15*a^2*b^2)*cosh(d*x + c)^3 + 3*(8*
a^4 + 36*a^3*b + 55*a^2*b^2 + 30*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*
(24*a^4 + 124*a^3*b + 269*a^2*b^2 + 280*a*b^3 + 120*b^4)*cosh(d*x + c)^4 +
2*(35*(8*a^4 + 20*a^3*b + 15*a^2*b^2)*cosh(d*x + c)^4 + 24*a^4 + 124*a^3*b
+ 269*a^2*b^2 + 280*a*b^3 + 120*b^4 + 30*(8*a^4 + 36*a^3*b + 55*a^2*b^2 + 3
0*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*a^4 + 20*a^3*b + 15*a^2*b^2 +
 8*(7*(8*a^4 + 20*a^3*b + 15*a^2*b^2)*cosh(d*x + c)^5 + 10*(8*a^4 + 36*a^3*
b + 55*a^2*b^2 + 30*a*b^3)*cosh(d*x + c)^3 + (24*a^4 + 124*a^3*b + 269*a^2*
b^2 + 280*a*b^3 + 120*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(8*a^4 + 36*a
```

^3*b + 55*a^2*b^2 + 30*a*b^3)*cosh(d*x + c)^2 + 4*(7*(8*a^4 + 20*a^3*b + 15
*a^2*b^2)*cosh(d*x + c)^6 + 15*(8*a^4 + 36*a^3*b + 55*a^2*b^2 + 30*a*b^3)*c
osh(d*x + c)^4 + 8*a^4 + 36*a^3*b + 55*a^2*b^2 + 30*a*b^3 + 3*(24*a^4 + 124
*a^3*b + 269*a^2*b^2 + 280*a*b^3 + 120*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^
2 + 8*((8*a^4 + 20*a^3*b + 15*a^2*b^2)*cosh(d*x + c)^7 + 3*(8*a^4 + 36*a^3*
b + 55*a^2*b^2 + 30*a*b^3)*cosh(d*x + c)^5 + (24*a^4 + 124*a^3*b + 269*a^2*
b^2 + 280*a*b^3 + 120*b^4)*cosh(d*x + c)^3 + (8*a^4 + 36*a^3*b + 55*a^2*b^2
 + 30*a*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a/(a + b))*arctan(1/2*(a*co
sh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 + (3*
a + 4*b)*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + 3*a + 4*b)*sinh(d*x + c))*s
qrt(a/(a + b))/a) - 16*((a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)^8 + 8*(a^4
+ 2*a^3*b + a^2*b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^4 + 2*a^3*b + a^2*b
^2)*sinh(d*x + c)^8 + 4*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*cosh(d*x + c)
^6 + 4*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3 + 7*(a^4 + 2*a^3*b + a^2*b^2)*c
osh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x +
 c)^3 + 3*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*cosh(d*x + c))*sinh(d*x + c
)^5 + 2*(3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4)*cosh(d*x + c)^4
+ 2*(35*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)^4 + 3*a^4 + 14*a^3*b + 27*a
^2*b^2 + 24*a*b^3 + 8*b^4 + 30*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*cosh(d
*x + c)^2)*sinh(d*x + c)^4 + a^4 + 2*a^3*b + a^2*b^2 + 8*(7*(a^4 + 2*a^3*b
+ a^2*b^2)*cosh(d*x + c)^5 + 10*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*cosh(
d*x + c)^3 + (3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4)*cosh(d*x +
c))*sinh(d*x + c)^3 + 4*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*cosh(d*x + c)
^2 + 4*(7*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)^6 + 15*(a^4 + 4*a^3*b + 5
*a^2*b^2 + 2*a*b^3)*cosh(d*x + c)^4 + a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3 +
 3*(3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4)*cosh(d*x + c)^2)*sinh
(d*x + c)^2 + 8*((a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)^7 + 3*(a^4 + 4*a^3
*b + 5*a^2*b^2 + 2*a*b^3)*cosh(d*x + c)^5 + (3*a^4 + 14*a^3*b + 27*a^2*b^2
+ 24*a*b^3 + 8*b^4)*cosh(d*x + c)^3 + (a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)
*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) - 2*(4
*a^3*b + 7*a^2*b^2)*cosh(d*x + c) + 2*(7*(4*a^3*b + 7*a^2*b^2)*cosh(d*x + c
)^6 + 5*(4*a^3*b + 31*a^2*b^2 + 36*a*b^3)*cosh(d*x + c)^4 - 4*a^3*b - 7*a^2
b^2 - 3(4*a^3*b + 31*a^2*b^2 + 36*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/
((a^4*b^3 + 2*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^8 + 8*(a^4*b^3 + 2*a^3*b^4
 + a^2*b^5)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^4*b^3 + 2*a^3*b^4 + a^2*b^
5)*d*sinh(d*x + c)^8 + 4*(a^4*b^3 + 4*a^3*b^4 + 5*a^2*b^5 + 2*a*b^6)*d*cosh
(d*x + c)^6 + 4*(7*(a^4*b^3 + 2*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^2 + (a^4
*b^3 + 4*a^3*b^4 + 5*a^2*b^5 + 2*a*b^6)*d)*sinh(d*x + c)^6 + 2*(3*a^4*b^3 +
 14*a^3*b^4 + 27*a^2*b^5 + 24*a*b^6 + 8*b^7)*d*cosh(d*x + c)^4 + 8*(7*(a^4*
b^3 + 2*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^3 + 3*(a^4*b^3 + 4*a^3*b^4 + 5*a
^2*b^5 + 2*a*b^6)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^4*b^3 + 2*a^3
*b^4 + a^2*b^5)*d*cosh(d*x + c)^4 + 30*(a^4*b^3 + 4*a^3*b^4 + 5*a^2*b^5 + 2
*a*b^6)*d*cosh(d*x + c)^2 + (3*a^4*b^3 + 14*a^3*b^4 + 27*a^2*b^5 + 24*a*b^6
 + 8*b^7)*d)*sinh(d*x + c)^4 + 4*(a^4*b^3 + 4*a^3*b^4 + 5*a^2*b^5 + 2*a*b^6
)*d*cosh(d*x + c)^2 + 8*(7*(a^4*b^3 + 2*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^

```
5 + 10*(a^4*b^3 + 4*a^3*b^4 + 5*a^2*b^5 + 2*a*b^6)*d*cosh(d*x + c)^3 + (3*a
^4*b^3 + 14*a^3*b^4 + 27*a^2*b^5 + 24*a*b^6 + 8*b^7)*d*cosh(d*x + c))*sinh(
d*x + c)^3 + 4*(7*(a^4*b^3 + 2*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^6 + 15*(a
^4*b^3 + 4*a^3*b^4 + 5*a^2*b^5 + 2*a*b^6)*d*cosh(d*x + c)^4 + 3*(3*a^4*b^3
+ 14*a^3*b^4 + 27*a^2*b^5 + 24*a*b^6 + 8*b^7)*d*cosh(d*x + c)^2 + (a^4*b^3
+ 4*a^3*b^4 + 5*a^2*b^5 + 2*a*b^6)*d)*sinh(d*x + c)^2 + (a^4*b^3 + 2*a^3*b^
4 + a^2*b^5)*d + 8*((a^4*b^3 + 2*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^7 + 3*(
a^4*b^3 + 4*a^3*b^4 + 5*a^2*b^5 + 2*a*b^6)*d*cosh(d*x + c)^5 + (3*a^4*b^3 +
 14*a^3*b^4 + 27*a^2*b^5 + 24*a*b^6 + 8*b^7)*d*cosh(d*x + c)^3 + (a^4*b^3 +
 4*a^3*b^4 + 5*a^2*b^5 + 2*a*b^6)*d*cosh(d*x + c))*sinh(d*x + c))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^7/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[84,-86]Warning, need to choose a branch for the root of a p
olynomial with parameters. This might be wrong.The choice was done assuming
 [a,b]=[-42,-12]Warning, need to choose a branch for the root of a polynomi
al with parameters. This might be wrong.The choice was done assuming [a,b]=
[-43,-99]Warning, need to choose a branch for the root of a polynomial with
 parameters. This might be wrong.The choice was done assuming [a,b]=[-28,94
]Warning, need to choose a branch for the root of a polynomial with paramet
ers. This might be wrong.The choice was done assuming [a,b]=[-7,46]Warning,
 need to choose a branch for the root of a polynomial with parameters. This
 might be wrong.The choice was done assuming [a,b]=[-35,-99]Warning, need t
o choose a branch for the root of a polynomial with parameters. This might
be wrong.The choice was done assuming [a,b]=[7,50]Warning, need to choose a
 branch for the root of a polynomial with parameters. This might be wrong.T
he choice was done assuming [a,b]=[-63,-70]Undef/Unsigned Inf encountered i
n limitEvaluation time: 1.51Limit: Max order reached or unable to make seri
es expansion Error: Bad Argument Value
```

maple [B] time = 0.30, size = 1201, normalized size = 7.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^7/(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] 1/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^
2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*a^2/(a+b)/b^2*tanh(1/2*d*x+1/2*c)^7+9/
4/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^
2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/b/(a+b)*tanh(1/2*d*x+1/2*c)^7*a+1/d/(t
anh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*
tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*a^3/(a+b)^2/b^2*tanh(1/2*d*x+1/2*c)^5-11/4/d
/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a
-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2/b*tanh(1/2*d*x+1/2*c)^5*a^2-27/4/
d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*
a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^5*a-1/d/(tan
h(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*ta
nh(1/2*d*x+1/2*c)^2*b+a+b)^2*a^3/(a+b)^2/b^2*tanh(1/2*d*x+1/2*c)^3+11/4/d/(
tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2
*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2/b*tanh(1/2*d*x+1/2*c)^3*a^2+27/4/d/
(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-
2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^3*a-1/d/(tanh(
1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh
(1/2*d*x+1/2*c)^2*b+a+b)^2*a^2/(a+b)/b^2*tanh(1/2*d*x+1/2*c)-9/4/d/(tanh(1/
2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1
/2*d*x+1/2*c)^2*b+a+b)^2/b/(a+b)*tanh(1/2*d*x+1/2*c)*a+1/d*a^(5/2)/b^3/(a^2
+2*a*b+b^2)/(a+b)^(1/2)*arctan(1/2*(-2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)+2*b^
(1/2))/a^(1/2))+5/2/d*a^(3/2)/b^2/(a^2+2*a*b+b^2)/(a+b)^(1/2)*arctan(1/2*(-
2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)+2*b^(1/2))/a^(1/2))+15/8/d*a^(1/2)/b/(a^2
+2*a*b+b^2)/(a+b)^(1/2)*arctan(1/2*(-2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)+2*b^
(1/2))/a^(1/2))-1/d*a^(5/2)/b^3/(a^2+2*a*b+b^2)/(a+b)^(1/2)*arctan(1/2*(2*(
a+b)^(1/2)*tanh(1/2*d*x+1/2*c)+2*b^(1/2))/a^(1/2))-5/2/d*a^(3/2)/b^2/(a^2+2
*a*b+b^2)/(a+b)^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)+2*b^(1/
2))/a^(1/2))-15/8/d*a^(1/2)/b/(a^2+2*a*b+b^2)/(a+b)^(1/2)*arctan(1/2*(2*(a+
b)^(1/2)*tanh(1/2*d*x+1/2*c)+2*b^(1/2))/a^(1/2))+2/d/b^3*arctan(tanh(1/2*d*
x+1/2*c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(4\,a^3e^{(7\,c)}+7\,a^2be^{(7\,c)}\right)e^{(7\,dx)}+\left(4\,a^3e^{(5\,c)}\right.}{4\left(a^4b^2d+2\,a^3b^3d+a^2b^4d+\left(a^4b^2de^{(8\,c)}+2\,a^3b^3de^{(8\,c)}+a^2b^4de^{(8\,c)}\right)e^{(8\,dx)}+4\left(a^4b^2de^{(6\,c)}+4\,a^3b^3de^{(6\,c)}+5\,a^2b\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^7/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*((4*a^3*e^(7*c) + 7*a^2*b*e^(7*c))*e^(7*d*x) + (4*a^3*e^(5*c) + 31*a^2
*b*e^(5*c) + 36*a*b^2*e^(5*c))*e^(5*d*x) - (4*a^3*e^(3*c) + 31*a^2*b*e^(3*c
) + 36*a*b^2*e^(3*c))*e^(3*d*x) - (4*a^3*e^c + 7*a^2*b*e^c)*e^(d*x))/(a^4*b
^2*d + 2*a^3*b^3*d + a^2*b^4*d + (a^4*b^2*d*e^(8*c) + 2*a^3*b^3*d*e^(8*c) +
 a^2*b^4*d*e^(8*c))*e^(8*d*x) + 4*(a^4*b^2*d*e^(6*c) + 4*a^3*b^3*d*e^(6*c)
```

```
+ 5*a^2*b^4*d*e^(6*c) + 2*a*b^5*d*e^(6*c))*e^(6*d*x) + 2*(3*a^4*b^2*d*e^(4*
c) + 14*a^3*b^3*d*e^(4*c) + 27*a^2*b^4*d*e^(4*c) + 24*a*b^5*d*e^(4*c) + 8*b
^6*d*e^(4*c))*e^(4*d*x) + 4*(a^4*b^2*d*e^(2*c) + 4*a^3*b^3*d*e^(2*c) + 5*a^
2*b^4*d*e^(2*c) + 2*a*b^5*d*e^(2*c))*e^(2*d*x)) + 2*arctan(e^(d*x + c))/(b^
3*d) - 128*integrate(1/512*((8*a^3*e^(3*c) + 20*a^2*b*e^(3*c) + 15*a*b^2*e^
(3*c))*e^(3*d*x) + (8*a^3*e^c + 20*a^2*b*e^c + 15*a*b^2*e^c)*e^(d*x))/(a^3*
b^3 + 2*a^2*b^4 + a*b^5 + (a^3*b^3*e^(4*c) + 2*a^2*b^4*e^(4*c) + a*b^5*e^(4
*c))*e^(4*d*x) + 2*(a^3*b^3*e^(2*c) + 4*a^2*b^4*e^(2*c) + 5*a*b^5*e^(2*c) +
 2*b^6*e^(2*c))*e^(2*d*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh\left(c+d\,x\right)^7\,{\left(a+\frac{b}{{\cosh\left(c+d\,x\right)}^2}\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^7*(a + b/cosh(c + d*x)^2)^3),x)
```

```
[Out] int(1/(cosh(c + d*x)^7*(a + b/cosh(c + d*x)^2)^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**7/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

3.102 $\int \left(a+b\,\text{sech}^2(c+dx)\right)\tanh^4(c+dx)\,dx$

Optimal. Leaf size=48

$$-\frac{a\tanh^3(c+dx)}{3d}-\frac{a\tanh(c+dx)}{d}+ax+\frac{b\tanh^5(c+dx)}{5d}$$

```
[Out] a*x-a*tanh(d*x+c)/d-1/3*a*tanh(d*x+c)^3/d+1/5*b*tanh(d*x+c)^5/d
```

Rubi [A] time = 0.06, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4141, 1802, 206}

$$-\frac{a\tanh^3(c+dx)}{3d}-\frac{a\tanh(c+dx)}{d}+ax+\frac{b\tanh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)*Tanh[c + d*x]^4,x]
```

```
[Out] a*x - (a*Tanh[c + d*x])/d - (a*Tanh[c + d*x]^3)/(3*d) + (b*Tanh[c + d*x]^5)
/(5*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \left(a+b\text{sech}^2(c+dx)\right)\tanh^4(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x^4\left(a+b\left(1-x^2\right)\right)}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-a-ax^2+bx^4+\frac{a}{1-x^2}\right)\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= -\frac{a\tanh(c+dx)}{d} - \frac{a\tanh^3(c+dx)}{3d} + \frac{b\tanh^5(c+dx)}{5d} + \frac{a\,\text{Subst}\left(\int\right.}{}$$
$$= ax - \frac{a\tanh(c+dx)}{d} - \frac{a\tanh^3(c+dx)}{3d} + \frac{b\tanh^5(c+dx)}{5d}$$

Mathematica [A] time = 0.03, size = 57, normalized size = 1.19

$$\frac{a\tanh^{-1}(\tanh(c+dx))}{d} - \frac{a\tanh^3(c+dx)}{3d} - \frac{a\tanh(c+dx)}{d} + \frac{b\tanh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)*Tanh[c + d*x]^4,x]
```

```
[Out] (a*ArcTanh[Tanh[c + d*x]])/d - (a*Tanh[c + d*x])/d - (a*Tanh[c + d*x]^3)/(3
*d) + (b*Tanh[c + d*x]^5)/(5*d)
```

fricas [B] time = 0.42, size = 327, normalized size = 6.81

$$\frac{(15\,adx + 20\,a - 3\,b)\cosh(dx+c)^5 + 5\,(15\,adx + 20\,a - 3\,b)\cosh(dx+c)\sinh(dx+c)^4 - (20\,a - 3\,b)\sinh(dx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)*tanh(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/15*((15*a*d*x + 20*a - 3*b)*cosh(d*x + c)^5 + 5*(15*a*d*x + 20*a - 3*b)*c
osh(d*x + c)*sinh(d*x + c)^4 - (20*a - 3*b)*sinh(d*x + c)^5 + 5*(15*a*d*x +
 20*a - 3*b)*cosh(d*x + c)^3 - 5*(2*(20*a - 3*b)*cosh(d*x + c)^2 + 8*a + 3*
b)*sinh(d*x + c)^3 + 5*(2*(15*a*d*x + 20*a - 3*b)*cosh(d*x + c)^3 + 3*(15*a
*d*x + 20*a - 3*b)*cosh(d*x + c))*sinh(d*x + c)^2 + 10*(15*a*d*x + 20*a - 3
*b)*cosh(d*x + c) - 5*((20*a - 3*b)*cosh(d*x + c)^4 + 3*(8*a + 3*b)*cosh(d*
x + c)^2 + 4*a - 6*b)*sinh(d*x + c))/(d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)
*sinh(d*x + c)^4 + 5*d*cosh(d*x + c)^3 + 5*(2*d*cosh(d*x + c)^3 + 3*d*cosh(
d*x + c))*sinh(d*x + c)^2 + 10*d*cosh(d*x + c))
```

giac [B] time = 0.20, size = 105, normalized size = 2.19

$$\frac{15\,adx + \frac{2\left(30\,ae^{(8\,dx+8\,c)} - 15\,be^{(8\,dx+8\,c)} + 90\,ae^{(6\,dx+6\,c)} + 110\,ae^{(4\,dx+4\,c)} - 30\,be^{(4\,dx+4\,c)} + 70\,ae^{(2\,dx+2\,c)} + 20\,a - 3\,b\right)}{\left(e^{(2\,dx+2\,c)}+1\right)^5}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)*tanh(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/15*(15*a*d*x + 2*(30*a*e^(8*d*x + 8*c) - 15*b*e^(8*d*x + 8*c) + 90*a*e^(6
*d*x + 6*c) + 110*a*e^(4*d*x + 4*c) - 30*b*e^(4*d*x + 4*c) + 70*a*e^(2*d*x
+ 2*c) + 20*a - 3*b)/(e^(2*d*x + 2*c) + 1)^5)/d
```

maple [B] time = 0.33, size = 98, normalized size = 2.04

$$\frac{a\left(dx + c - \tanh\left(dx + c\right) - \frac{\left(\tanh^3(dx+c)\right)}{3}\right) + b\left(-\frac{\sinh^3(dx+c)}{2\cosh(dx+c)^5} - \frac{3\sinh(dx+c)}{8\cosh(dx+c)^5} + \frac{3\left(\frac{8}{15} + \frac{\mathrm{sech}(dx+c)^4}{5} + \frac{4\mathrm{sech}(dx+c)^2}{15}\right)\tanh(dx+c)}{8}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)*tanh(d*x+c)^4,x)
```

```
[Out] 1/d*(a*(d*x+c-tanh(d*x+c)-1/3*tanh(d*x+c)^3)+b*(-1/2*sinh(d*x+c)^3/cosh(d*x
+c)^5-3/8*sinh(d*x+c)/cosh(d*x+c)^5+3/8*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d
*x+c)^2)*tanh(d*x+c)))
```

maxima [B] time = 0.61, size = 92, normalized size = 1.92

$$\frac{b\tanh\left(dx + c\right)^5}{5\,d} + \frac{1}{3}\,a\left(3\,x + \frac{3\,c}{d} - \frac{4\left(3\,e^{(-2\,dx-2\,c)} + 3\,e^{(-4\,dx-4\,c)} + 2\right)}{d\left(3\,e^{(-2\,dx-2\,c)} + 3\,e^{(-4\,dx-4\,c)} + e^{(-6\,dx-6\,c)} + 1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)*tanh(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/5*b*tanh(d*x + c)^5/d + 1/3*a*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^
(-4*d*x - 4*c) + 2)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x
 - 6*c) + 1)))
```

mupad [B] time = 1.54, size = 433, normalized size = 9.02

$$a\,x+\frac{\frac{2\,(2\,a-3\,b)}{15\,d}+\frac{4\,e^{2\,c+2\,d\,x}\,(a+b)}{5\,d}+\frac{2\,e^{4\,c+4\,d\,x}\,(2\,a-b)}{5\,d}}{3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1}+\frac{\frac{2\,(2\,a-b)}{5\,d}+\frac{8\,e^{2\,c+2\,d\,x}\,(a+b)}{5\,d}+\frac{8\,e^{6\,c+6\,d\,x}\,(a+b)}{5\,d}+\frac{4\,e^{4\,c+4\,d\,x}\,(2\,a-3\,b)}{5\,d}+\frac{2\,e^{8\,c+8\,d\,x}\,\cdots}{5\,d}}{5\,e^{2\,c+2\,d\,x}+10\,e^{4\,c+4\,d\,x}+10\,e^{6\,c+6\,d\,x}+5\,e^{8\,c+8\,d\,x}+e^{10\,c+10\,d\,x}+\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^4*(a + b/cosh(c + d*x)^2),x)
```

```
[Out] a*x + ((2*(2*a - 3*b))/(15*d) + (4*exp(2*c + 2*d*x)*(a + b))/(5*d) + (2*exp
(4*c + 4*d*x)*(2*a - b))/(5*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) +
exp(6*c + 6*d*x) + 1) + ((2*(2*a - b))/(5*d) + (8*exp(2*c + 2*d*x)*(a + b))
/(5*d) + (8*exp(6*c + 6*d*x)*(a + b))/(5*d) + (4*exp(4*c + 4*d*x)*(2*a - 3*
b))/(5*d) + (2*exp(8*c + 8*d*x)*(2*a - b))/(5*d))/(5*exp(2*c + 2*d*x) + 10*
exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10
*d*x) + 1) + ((2*(a + b))/(5*d) + (2*exp(2*c + 2*d*x)*(2*a - b))/(5*d))/(2*
exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) + ((2*(a + b))/(5*d) + (6*exp(4*c
+ 4*d*x)*(a + b))/(5*d) + (2*exp(2*c + 2*d*x)*(2*a - 3*b))/(5*d) + (2*exp(6
*c + 6*d*x)*(2*a - b))/(5*d))/(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*
exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) + (2*(2*a - b))/(5*d*(exp(2*c + 2*
d*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\operatorname{sech}^2(c + dx)\right)\tanh^4(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)*tanh(d*x+c)**4,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)*tanh(c + d*x)**4, x)
```

3.103 $\int \left(a+b\,\mathbf{sech}^2(c+dx)\right)\tanh^3(c+dx)\,dx$

Optimal. Leaf size=49

$$\frac{(a-b)\text{sech}^2(c+dx)}{2d}+\frac{a\log(\cosh(c+dx))}{d}+\frac{b\text{sech}^4(c+dx)}{4d}$$

```
[Out] a*ln(cosh(d*x+c))/d+1/2*(a-b)*sech(d*x+c)^2/d+1/4*b*sech(d*x+c)^4/d
```

Rubi [A] time = 0.06, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4138, 446, 76}

$$\frac{(a-b)\text{sech}^2(c+dx)}{2d}+\frac{a\log(\cosh(c+dx))}{d}+\frac{b\text{sech}^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)*Tanh[c + d*x]^3,x]
```

```
[Out] (a*Log[Cosh[c + d*x]])/d + ((a - b)*Sech[c + d*x]^2)/(2*d) + (b*Sech[c + d*
x]^4)/(4*d)
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
```

```
x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \left(a+b\operatorname{sech}^2(c+dx)\right)\tanh^3(c+dx)\,dx &= -\frac{\operatorname{Subst}\left(\int \frac{(1-x^2)(b+ax^2)}{x^5}\,dx, x, \cosh(c+dx)\right)}{d}\\
&= -\frac{\operatorname{Subst}\left(\int \frac{(1-x)(b+ax)}{x^3}\,dx, x, \cosh^2(c+dx)\right)}{2d}\\
&= -\frac{\operatorname{Subst}\left(\int \left(\frac{b}{x^3}+\frac{a-b}{x^2}-\frac{a}{x}\right)dx, x, \cosh^2(c+dx)\right)}{2d}\\
&= \frac{a\log(\cosh(c+dx))}{d}+\frac{(a-b)\operatorname{sech}^2(c+dx)}{2d}+\frac{b\operatorname{sech}^4(c+dx)}{4d}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 45, normalized size = 0.92

$$
-\frac{a\tanh^2(c+dx)}{2d}+\frac{a\log(\cosh(c+dx))}{d}+\frac{b\tanh^4(c+dx)}{4d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)*Tanh[c + d*x]^3,x]
```

```
[Out] (a*Log[Cosh[c + d*x]])/d - (a*Tanh[c + d*x]^2)/(2*d) + (b*Tanh[c + d*x]^4)/
(4*d)
```

fricas [B] time = 0.43, size = 1072, normalized size = 21.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)*tanh(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -(a*d*x*cosh(d*x + c)^8 + 8*a*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + a*d*x*sin
h(d*x + c)^8 + 2*(2*a*d*x - a + b)*cosh(d*x + c)^6 + 2*(14*a*d*x*cosh(d*x +
 c)^2 + 2*a*d*x - a + b)*sinh(d*x + c)^6 + 4*(14*a*d*x*cosh(d*x + c)^3 + 3*
(2*a*d*x - a + b)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*a*d*x - 2*a)*cosh(d
*x + c)^4 + 2*(35*a*d*x*cosh(d*x + c)^4 + 3*a*d*x + 15*(2*a*d*x - a + b)*co
sh(d*x + c)^2 - 2*a)*sinh(d*x + c)^4 + 8*(7*a*d*x*cosh(d*x + c)^5 + 5*(2*a*
d*x - a + b)*cosh(d*x + c)^3 + (3*a*d*x - 2*a)*cosh(d*x + c))*sinh(d*x + c)
```

```
^3 + a*d*x + 2*(2*a*d*x - a + b)*cosh(d*x + c)^2 + 2*(14*a*d*x*cosh(d*x + c
)^6 + 15*(2*a*d*x - a + b)*cosh(d*x + c)^4 + 2*a*d*x + 6*(3*a*d*x - 2*a)*co
sh(d*x + c)^2 - a + b)*sinh(d*x + c)^2 - (a*cosh(d*x + c)^8 + 8*a*cosh(d*x
+ c)*sinh(d*x + c)^7 + a*sinh(d*x + c)^8 + 4*a*cosh(d*x + c)^6 + 4*(7*a*cos
h(d*x + c)^2 + a)*sinh(d*x + c)^6 + 8*(7*a*cosh(d*x + c)^3 + 3*a*cosh(d*x +
 c))*sinh(d*x + c)^5 + 6*a*cosh(d*x + c)^4 + 2*(35*a*cosh(d*x + c)^4 + 30*a
*cosh(d*x + c)^2 + 3*a)*sinh(d*x + c)^4 + 8*(7*a*cosh(d*x + c)^5 + 10*a*cos
h(d*x + c)^3 + 3*a*cosh(d*x + c))*sinh(d*x + c)^3 + 4*a*cosh(d*x + c)^2 + 4
*(7*a*cosh(d*x + c)^6 + 15*a*cosh(d*x + c)^4 + 9*a*cosh(d*x + c)^2 + a)*sin
h(d*x + c)^2 + 8*(a*cosh(d*x + c)^7 + 3*a*cosh(d*x + c)^5 + 3*a*cosh(d*x +
c)^3 + a*cosh(d*x + c))*sinh(d*x + c) + a)*log(2*cosh(d*x + c)/(cosh(d*x +
c) - sinh(d*x + c))) + 4*(2*a*d*x*cosh(d*x + c)^7 + 3*(2*a*d*x - a + b)*cos
h(d*x + c)^5 + 2*(3*a*d*x - 2*a)*cosh(d*x + c)^3 + (2*a*d*x - a + b)*cosh(d
*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^8 + 8*d*cosh(d*x + c)*sinh(d*x + c
)^7 + d*sinh(d*x + c)^8 + 4*d*cosh(d*x + c)^6 + 4*(7*d*cosh(d*x + c)^2 + d)
*sinh(d*x + c)^6 + 8*(7*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c
)^5 + 6*d*cosh(d*x + c)^4 + 2*(35*d*cosh(d*x + c)^4 + 30*d*cosh(d*x + c)^2
+ 3*d)*sinh(d*x + c)^4 + 8*(7*d*cosh(d*x + c)^5 + 10*d*cosh(d*x + c)^3 + 3*
d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*d*cosh(d*x + c)^2 + 4*(7*d*cosh(d*x +
c)^6 + 15*d*cosh(d*x + c)^4 + 9*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 8*
(d*cosh(d*x + c)^7 + 3*d*cosh(d*x + c)^5 + 3*d*cosh(d*x + c)^3 + d*cosh(d*x
 + c))*sinh(d*x + c) + d)
```

giac [B] time = 0.18, size = 116, normalized size = 2.37

$$-\frac{12\,adx - 12\,a\log\left(e^{(2\,dx+2\,c)} + 1\right) + \frac{25\,ae^{(8\,dx+8\,c)} + 76\,ae^{(6\,dx+6\,c)} + 24\,be^{(6\,dx+6\,c)} + 102\,ae^{(4\,dx+4\,c)} + 76\,ae^{(2\,dx+2\,c)} + 24\,be^{(2\,dx+2\,c)} + 25\,a}{\left(e^{(2\,dx+2\,c)} + 1\right)^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)*tanh(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/12*(12*a*d*x - 12*a*log(e^(2*d*x + 2*c) + 1) + (25*a*e^(8*d*x + 8*c) + 7
6*a*e^(6*d*x + 6*c) + 24*b*e^(6*d*x + 6*c) + 102*a*e^(4*d*x + 4*c) + 76*a*e
^(2*d*x + 2*c) + 24*b*e^(2*d*x + 2*c) + 25*a)/(e^(2*d*x + 2*c) + 1)^4)/d
```

maple [A] time = 0.20, size = 64, normalized size = 1.31

$$\frac{a\ln\left(\cosh\left(dx+c\right)\right)}{d} - \frac{\left(\tanh^2\left(dx+c\right)\right)a}{2d} - \frac{b\left(\sinh^2\left(dx+c\right)\right)}{2d\cosh\left(dx+c\right)^4} - \frac{b}{4d\cosh\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)*tanh(d*x+c)^3,x)
```

[Out] a*ln(cosh(d*x+c))/d-1/2/d*tanh(d*x+c)^2*a-1/2/d*b*sinh(d*x+c)^2/cosh(d*x+c)^4-1/4/d*b/cosh(d*x+c)^4

maxima [A] time = 0.75, size = 78, normalized size = 1.59

$$\frac{b\tanh(dx+c)^4}{4\,d}+a\left(x+\frac{c}{d}+\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d\left(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)*tanh(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/4*b*tanh(d*x + c)^4/d + a*(x + c/d + log(e^(-2*d*x - 2*c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1)))
```

mupad [B] time = 0.12, size = 173, normalized size = 3.53

$$\frac{2\,(a-b)}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)}-a\,x-\frac{2\,(a-3\,b)}{d\,\left(2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1\right)}-\frac{8\,b}{d\,\left(3\,\mathrm{e}^{2\,c+2\,d\,x}+3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}+1\right)}+\frac{}{d\,\left(4\,\mathrm{e}^{2\,c+2\,d\,x}+6\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^3*(a + b/cosh(c + d*x)^2),x)
```

```
[Out] (2*(a - b))/(d*(exp(2*c + 2*d*x) + 1)) - a*x - (2*(a - 3*b))/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (8*b)/(d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) + (4*b)/(d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) + (a*log(exp(2*c)*exp(2*d*x) + 1))/d
```

sympy [A] time = 1.66, size = 80, normalized size = 1.63

$$\begin{cases} ax-\frac{a\log(\tanh(c+dx)+1)}{d}-\frac{a\tanh^2(c+dx)}{2d}-\frac{b\tanh^2(c+dx)\operatorname{sech}^2(c+dx)}{4d}-\frac{b\operatorname{sech}^2(c+dx)}{4d} & \text{for } d\neq 0\\ x\left(a+b\operatorname{sech}^2(c)\right)\tanh^3(c) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)*tanh(d*x+c)**3,x)
```

```
[Out] Piecewise((a*x - a*log(tanh(c + d*x) + 1)/d - a*tanh(c + d*x)**2/(2*d) - b*tanh(c + d*x)**2*sech(c + d*x)**2/(4*d) - b*sech(c + d*x)**2/(4*d), Ne(d, 0)), (x*(a + b*sech(c)**2)*tanh(c)**3, True))
```

3.104 $\int \left(a+b\,\mathbf{sech}^2(c+dx)\right)\tanh^2(c+dx)\,dx$

Optimal. Leaf size=32

$$-\frac{a\tanh(c+dx)}{d}+ax+\frac{b\tanh^3(c+dx)}{3d}$$

[Out] a*x-a*tanh(d*x+c)/d+1/3*b*tanh(d*x+c)^3/d

Rubi [A] time = 0.06, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4141, 1802, 206}

$$-\frac{a\tanh(c+dx)}{d}+ax+\frac{b\tanh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sech[c + d*x]^2)*Tanh[c + d*x]^2,x]

[Out] a*x - (a*Tanh[c + d*x])/d + (b*Tanh[c + d*x]^3)/(3*d)

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \left(a+b\text{sech}^2(c+dx)\right)\tanh^2(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x^2\left(a+b\left(1-x^2\right)\right)}{1-x^2}\,dx,x,\tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-a+bx^2+\frac{a}{1-x^2}\right)\,dx,x,\tanh(c+dx)\right)}{d}$$
$$= -\frac{a\tanh(c+dx)}{d}+\frac{b\tanh^3(c+dx)}{3d}+\frac{a\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\tanh(c$$
$$= ax-\frac{a\tanh(c+dx)}{d}+\frac{b\tanh^3(c+dx)}{3d}$$

Mathematica [A] time = 0.02, size = 41, normalized size = 1.28

$$\frac{a\tanh^{-1}(\tanh(c+dx))}{d}-\frac{a\tanh(c+dx)}{d}+\frac{b\tanh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)*Tanh[c + d*x]^2,x]
```

```
[Out] (a*ArcTanh[Tanh[c + d*x]])/d - (a*Tanh[c + d*x])/d + (b*Tanh[c + d*x]^3)/(3
*d)
```

fricas [B] time = 0.40, size = 155, normalized size = 4.84

$$\frac{(3\,adx+3\,a-b)\cosh(dx+c)^3+3\,(3\,adx+3\,a-b)\cosh(dx+c)\sinh(dx+c)^2-(3\,a-b)\sinh(dx+c)^3+3\,(}{3\left(d\cosh(dx+c)^3+3\,d\cosh(dx+c)\sinh(dx+c)^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)*tanh(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/3*((3*a*d*x + 3*a - b)*cosh(d*x + c)^3 + 3*(3*a*d*x + 3*a - b)*cosh(d*x +
 c)*sinh(d*x + c)^2 - (3*a - b)*sinh(d*x + c)^3 + 3*(3*a*d*x + 3*a - b)*cos
h(d*x + c) - 3*((3*a - b)*cosh(d*x + c)^2 + a + b)*sinh(d*x + c))/(d*cosh(d
*x + c)^3 + 3*d*cosh(d*x + c)*sinh(d*x + c)^2 + 3*d*cosh(d*x + c))
```

giac [B] time = 0.16, size = 69, normalized size = 2.16

$$\frac{3\,adx+\frac{2\left(3\,ae^{(4\,dx+4\,c)}-3\,be^{(4\,dx+4\,c)}+6\,ae^{(2\,dx+2\,c)}+3\,a-b\right)}{\left(e^{(2\,dx+2\,c)}+1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)*tanh(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*a*d*x + 2*(3*a*e^(4*d*x + 4*c) - 3*b*e^(4*d*x + 4*c) + 6*a*e^(2*d*x
+ 2*c) + 3*a - b)/(e^(2*d*x + 2*c) + 1)^3)/d
```

maple [A] time = 0.33, size = 60, normalized size = 1.88

$$\frac{a\left(dx+c-\tanh\left(dx+c\right)\right)+b\left(-\frac{\sinh(dx+c)}{2\cosh(dx+c)^{3}}+\frac{\left(\frac{2}{3}+\frac{\mathrm{sech}(dx+c)^{2}}{3}\right)\tanh(dx+c)}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)*tanh(d*x+c)^2,x)
```

```
[Out] 1/d*(a*(d*x+c-tanh(d*x+c))+b*(-1/2*sinh(d*x+c)/cosh(d*x+c)^3+1/2*(2/3+1/3*s
ech(d*x+c)^2)*tanh(d*x+c)))
```

maxima [A] time = 0.40, size = 42, normalized size = 1.31

$$\frac{b\tanh\left(dx+c\right)^{3}}{3\,d}+a\left(x+\frac{c}{d}-\frac{2}{d\left(e^{(-2\,dx-2\,c)}+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)*tanh(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/3*b*tanh(d*x + c)^3/d + a*(x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1)))
```

mupad [B] time = 1.50, size = 163, normalized size = 5.09

$$\frac{\frac{2\,(a+b)}{3\,d}+\frac{2\,\mathrm{e}^{2\,c+2\,d\,x}\,(a-b)}{3\,d}}{2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1}+a\,x+\frac{\frac{2\,(a-b)}{3\,d}+\frac{4\,\mathrm{e}^{2\,c+2\,d\,x}\,(a+b)}{3\,d}+\frac{2\,\mathrm{e}^{4\,c+4\,d\,x}\,(a-b)}{3\,d}}{3\,\mathrm{e}^{2\,c+2\,d\,x}+3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}+1}+\frac{2\,\left(a-b\right)}{3\,d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^2*(a + b/cosh(c + d*x)^2),x)
```

```
[Out] ((2*(a + b))/(3*d) + (2*exp(2*c + 2*d*x)*(a - b))/(3*d))/(2*exp(2*c + 2*d*x
) + exp(4*c + 4*d*x) + 1) + a*x + ((2*(a - b))/(3*d) + (4*exp(2*c + 2*d*x)*
(a + b))/(3*d) + (2*exp(4*c + 4*d*x)*(a - b))/(3*d))/(3*exp(2*c + 2*d*x) +
3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) + (2*(a - b))/(3*d*(exp(2*c + 2*
d*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\operatorname{sech}^2\left(c + dx\right)\right)\tanh^2\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)*tanh(d*x+c)**2,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)*tanh(c + d*x)**2, x)
```

3.105 $\int \left(a+b\,\mathbf{sech}^2(c+dx)\right)\tanh(c+dx)\,dx$

Optimal. Leaf size=29

$$\frac{a\log(\cosh(c+dx))}{d}-\frac{b\mathrm{sech}^2(c+dx)}{2d}$$

```
[Out] a*ln(cosh(d*x+c))/d-1/2*b*sech(d*x+c)^2/d
```

Rubi [A] time = 0.03, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {4138, 14}

$$\frac{a\log(\cosh(c+dx))}{d}-\frac{b\mathrm{sech}^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)*Tanh[c + d*x],x]
```

```
[Out] (a*Log[Cosh[c + d*x]])/d - (b*Sech[c + d*x]^2)/(2*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \left(a+b\operatorname{sech}^2(c+dx)\right)\tanh(c+dx)\,dx = \frac{\operatorname{Subst}\left(\int \frac{b+ax^2}{x^3}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= \frac{\operatorname{Subst}\left(\int \left(\frac{b}{x^3}+\frac{a}{x}\right)dx, x, \cosh(c+dx)\right)}{d}$$
$$= \frac{a\log(\cosh(c+dx))}{d} - \frac{b\operatorname{sech}^2(c+dx)}{2d}$$

Mathematica [A] time = 0.02, size = 29, normalized size = 1.00

$$\frac{a\log(\cosh(c+dx))}{d} - \frac{b\operatorname{sech}^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)*Tanh[c + d*x],x]
```

```
[Out] (a*Log[Cosh[c + d*x]])/d - (b*Sech[c + d*x]^2)/(2*d)
```

fricas [B] time = 0.42, size = 359, normalized size = 12.38

$$-\frac{adx\cosh(dx+c)^4 + 4\,adx\cosh(dx+c)\sinh(dx+c)^3 + adx\sinh(dx+c)^4 + adx + 2\,(adx+b)\cosh(dx+c)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)*tanh(d*x+c),x, algorithm="fricas")
```

```
[Out] -(a*d*x*cosh(d*x + c)^4 + 4*a*d*x*cosh(d*x + c)*sinh(d*x + c)^3 + a*d*x*sin
h(d*x + c)^4 + a*d*x + 2*(a*d*x + b)*cosh(d*x + c)^2 + 2*(3*a*d*x*cosh(d*x
+ c)^2 + a*d*x + b)*sinh(d*x + c)^2 - (a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c
)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*a*cosh(d*x + c)^2 + 2*(3*a*cosh(d
*x + c)^2 + a)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + a*cosh(d*x + c))*si
nh(d*x + c) + a)*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 4*(
a*d*x*cosh(d*x + c)^3 + (a*d*x + b)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d
*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x + c)^3 + d*sinh(d*x + c)^4 + 2*d*cos
h(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 4*(d*cosh(d*x
+ c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

giac [B] time = 0.13, size = 80, normalized size = 2.76

$$-\frac{2\,adx - 2\,a\log\left(e^{(2\,dx+2\,c)}+1\right) + \frac{3\,ae^{(4\,dx+4\,c)}+6\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+3\,a}{\left(e^{(2\,dx+2\,c)}+1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)*tanh(d*x+c),x, algorithm="giac")
```

```
[Out] -1/2*(2*a*d*x - 2*a*log(e^(2*d*x + 2*c) + 1) + (3*a*e^(4*d*x + 4*c) + 6*a*e
^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) + 3*a)/(e^(2*d*x + 2*c) + 1)^2)/d
```

maple [A] time = 0.14, size = 29, normalized size = 1.00

$$-\frac{b\operatorname{sech}\left(dx+c\right)^{2}}{2d}-\frac{a\ln\left(\operatorname{sech}\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)*tanh(d*x+c),x)
```

```
[Out] -1/2*b*sech(d*x+c)^2/d-1/d*a*ln(sech(d*x+c))
```

maxima [A] time = 0.33, size = 27, normalized size = 0.93

$$\frac{b\tanh\left(dx+c\right)^{2}}{2\,d}+\frac{a\log\left(\cosh\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)*tanh(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*b*tanh(d*x + c)^2/d + a*log(cosh(d*x + c))/d
```

mupad [B] time = 1.47, size = 72, normalized size = 2.48

$$\frac{2\,b}{d\,\left(2\,{\mathrm{e}}^{2\,c+2\,d\,x}+{\mathrm{e}}^{4\,c+4\,d\,x}+1\right)}-\frac{2\,b}{d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}-a\,x+\frac{a\,\ln\left({\mathrm{e}}^{2\,c}\,{\mathrm{e}}^{2\,d\,x}+1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)*(a + b/cosh(c + d*x)^2),x)
```

```
[Out] (2*b)/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (2*b)/(d*(exp(2*c +
 2*d*x) + 1)) - a*x + (a*log(exp(2*c)*exp(2*d*x) + 1))/d
```

sympy [A] time = 0.49, size = 42, normalized size = 1.45

$$\begin{cases} ax-\frac{a\log\left(\tanh\left(c+dx\right)+1\right)}{d}-\frac{b\operatorname{sech}^{2}\left(c+dx\right)}{2d} & \text{for}\ d\neq 0\\ x\left(a+b\operatorname{sech}^{2}\left(c\right)\right)\tanh\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)*tanh(d*x+c),x)
```

```
[Out] Piecewise((a*x - a*log(tanh(c + d*x) + 1)/d - b*sech(c + d*x)**2/(2*d), Ne(
d, 0)), (x*(a + b*sech(c)**2)*tanh(c), True))
```

3.106 $\int \left(a+b\,\mathbf{sech}^2(c+dx)\right)\,dx$

Optimal. Leaf size=15

$$ax+\frac{b\tanh(c+dx)}{d}$$

```
[Out] a*x+b*tanh(d*x+c)/d
```

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3767, 8}

$$ax+\frac{b\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Sech[c + d*x]^2,x]
```

```
[Out] a*x + (b*Tanh[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\operatorname{sech}^2(c+dx)\right)\,dx &= ax+b\int \operatorname{sech}^2(c+dx)\,dx\\ &= ax+\frac{(ib)\,\text{Subst}\left(\int 1\,dx,x,-i\tanh(c+dx)\right)}{d}\\ &= ax+\frac{b\tanh(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$ax+\frac{b\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Sech[c + d*x]^2,x]
```

```
[Out] a*x + (b*Tanh[c + d*x])/d
```

fricas [B] time = 0.40, size = 36, normalized size = 2.40

$$\frac{(adx - b)\cosh(dx + c) + b\sinh(dx + c)}{d\cosh(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sech(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] ((a*d*x - b)*cosh(d*x + c) + b*sinh(d*x + c))/(d*cosh(d*x + c))
```

giac [A] time = 0.12, size = 23, normalized size = 1.53

$$ax - \frac{2b}{d\left(e^{(2dx+2c)} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sech(d*x+c)^2,x, algorithm="giac")
```

```
[Out] a*x - 2*b/(d*(e^(2*d*x + 2*c) + 1))
```

maple [A] time = 0.27, size = 16, normalized size = 1.07

$$ax + \frac{b\tanh(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*sech(d*x+c)^2,x)
```

```
[Out] a*x+b*tanh(d*x+c)/d
```

maxima [A] time = 0.37, size = 23, normalized size = 1.53

$$ax + \frac{2b}{d\left(e^{(-2dx-2c)} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sech(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] a*x + 2*b/(d*(e^(-2*d*x - 2*c) + 1))
```

mupad [B] time = 1.39, size = 23, normalized size = 1.53

$$ax - \frac{2b}{d\left(e^{2c+2dx} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b/cosh(c + d*x)^2,x)
[Out] a*x - (2*b)/(d*(exp(2*c + 2*d*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\operatorname{sech}^{2}(c + dx)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sech(d*x+c)**2,x)
[Out] Integral(a + b*sech(c + d*x)**2, x)
```

3.107 $\int \coth(c+dx)\left(a+b\,\mathbf{sech}^2(c+dx)\right)\,dx$

Optimal. Leaf size=28

$$\frac{(a+b)\log(\sinh(c+dx))}{d}-\frac{b\log(\cosh(c+dx))}{d}$$

```
[Out] -b*ln(cosh(d*x+c))/d+(a+b)*ln(sinh(d*x+c))/d
```

Rubi [A] time = 0.05, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {4138, 446, 72}

$$\frac{(a+b)\log(\sinh(c+dx))}{d}-\frac{b\log(\cosh(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -((b*Log[Cosh[c + d*x]])/d) + ((a + b)*Log[Sinh[c + d*x]])/d
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \coth(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx = -\frac{\text{Subst}\left(\int \frac{b+ax^2}{x(1-x^2)}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \frac{b+ax}{(1-x)x}\,dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{-a-b}{-1+x}+\frac{b}{x}\right)\,dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= -\frac{b\log(\cosh(c+dx))}{d}+\frac{(a+b)\log(\sinh(c+dx))}{d}$$

Mathematica [**A**] time = 0.04, size = 44, normalized size = 1.57

$$\frac{a(\log(\tanh(c+dx))+\log(\cosh(c+dx)))}{d}-\frac{b(\log(\cosh(c+dx))-\log(\sinh(c+dx)))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -((b*(Log[Cosh[c + d*x]] - Log[Sinh[c + d*x]]))/d) + (a*(Log[Cosh[c + d*x]]
 + Log[Tanh[c + d*x]]))/d
```

fricas [**B**] time = 0.42, size = 69, normalized size = 2.46

$$-\frac{adx+b\log\left(\frac{2\cosh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)-(a+b)\log\left(\frac{2\sinh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -(a*d*x + b*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) - (a + b)*
log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))))/d
```

giac [**A**] time = 0.15, size = 56, normalized size = 2.00

$$-\frac{adx-\left(ae^{(2c)}+be^{(2c)}\right)e^{(-2c)}\log\left(\left|e^{(2dx+2c)}-1\right|\right)+b\log\left(e^{(2dx+2c)}+1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -(a*d*x - (a*e^(2*c) + b*e^(2*c))*e^(-2*c)*log(abs(e^(2*d*x + 2*c) - 1)) +
b*log(e^(2*d*x + 2*c) + 1))/d
```

maple [A] time = 0.26, size = 26, normalized size = 0.93

$$\frac{b\ln\left(\tanh\left(dx+c\right)\right)}{d}+\frac{a\ln\left(\sinh\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)*(a+b*sech(d*x+c)^2),x)
```

```
[Out] b*ln(tanh(d*x+c))/d+a*ln(sinh(d*x+c))/d
```

maxima [B] time = 0.53, size = 65, normalized size = 2.32

$$b\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}-\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}\right)+\frac{a\log\left(\sinh\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] b*(log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d - log(e^(-2*d*x - 2*c)
 + 1)/d) + a*log(sinh(d*x + c))/d
```

mupad [B] time = 0.17, size = 167, normalized size = 5.96

$$\frac{a\,\ln\left(4\,a^2\,e^{4\,c}\,e^{4\,d\,x}-4\,a^2-16\,b^2-16\,a\,b+16\,b^2\,e^{4\,c}\,e^{4\,d\,x}+16\,a\,b\,e^{4\,c}\,e^{4\,d\,x}\right)}{2\,d}-a\,x-\frac{\mathrm{atan}\left(\frac{a\,e^{2\,c}\,e^{2\,d\,x}\,\sqrt{-d^2}}{d\,\sqrt{a^2+4\,a\,b+4\,b^2}}+\frac{2\,b\,e^{2\,c}\,e^{2}}{d\,\sqrt{a^2+4}}\right.}{\sqrt{-d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)*(a + b/cosh(c + d*x)^2),x)
```

```
[Out] (a*log(4*a^2*exp(4*c)*exp(4*d*x) - 4*a^2 - 16*b^2 - 16*a*b + 16*b^2*exp(4*c
)*exp(4*d*x) + 16*a*b*exp(4*c)*exp(4*d*x)))/(2*d) - a*x - (atan((a*exp(2*c)
*exp(2*d*x)*(-d^2)^(1/2))/(d*(4*a*b + a^2 + 4*b^2)^(1/2)) + (2*b*exp(2*c)*e
xp(2*d*x)*(-d^2)^(1/2))/(d*(4*a*b + a^2 + 4*b^2)^(1/2)))*(4*a*b + a^2 + 4*b
^2)^(1/2))/(-d^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\,\mathrm{sech}^2\left(c+dx\right)\right)\coth\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)*coth(c + d*x), x)
```

3.108 $\int \coth^2(c+dx)\left(a+b\,\mathbf{sech}^2(c+dx)\right)\,dx$

Optimal. Leaf size=18

$$ax-\frac{(a+b)\coth(c+dx)}{d}$$

```
[Out] a*x-(a+b)*coth(d*x+c)/d
```

Rubi [A] time = 0.06, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4141, 1802, 207}

$$ax-\frac{(a+b)\coth(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^2*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] a*x - ((a + b)*Coth[c + d*x])/d
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \coth^2(c+dx)\left(a+b\text{sech}^2(c+dx)\right)dx = \frac{\text{Subst}\left(\int \frac{a+b\left(1-x^2\right)}{x^2\left(1-x^2\right)}\,dx,x,\tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a+b}{x^2}-\frac{a}{-1+x^2}\right)dx,x,\tanh(c+dx)\right)}{d}$$
$$= -\frac{(a+b)\coth(c+dx)}{d}-\frac{a\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\tanh(c+dx)\right)}{d}$$
$$= ax-\frac{(a+b)\coth(c+dx)}{d}$$

Mathematica [C] time = 0.03, size = 41, normalized size = 2.28

$$-\frac{a\coth(c+dx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\tanh^2(c+dx)\right)}{d}-\frac{b\coth(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^2*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -((b*Coth[c + d*x])/d) - (a*Coth[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, T
anh[c + d*x]^2])/d
```

fricas [B] time = 0.41, size = 39, normalized size = 2.17

$$-\frac{(a+b)\cosh(dx+c)-(adx+a+b)\sinh(dx+c)}{d\sinh(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -((a + b)*cosh(d*x + c) - (a*d*x + a + b)*sinh(d*x + c))/(d*sinh(d*x + c))
```

giac [A] time = 0.15, size = 27, normalized size = 1.50

$$\frac{adx-\frac{2\,(a+b)}{e^{(2\,dx+2\,c)}-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] (a*d*x - 2*(a + b)/(e^(2*d*x + 2*c) - 1))/d
```

maple [A] time = 0.33, size = 30, normalized size = 1.67

$$\frac{a\left(dx+c-\coth\left(dx+c\right)\right)-b\coth\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^2*(a+b*sech(d*x+c)^2),x)
[Out] 1/d*(a*(d*x+c-coth(d*x+c))-b*coth(d*x+c))
```

maxima [B] time = 0.34, size = 47, normalized size = 2.61

$$a\left(x+\frac{c}{d}+\frac{2}{d\left(e^{(-2\,dx-2\,c)}-1\right)}\right)+\frac{2\,b}{d\left(e^{(-2\,dx-2\,c)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*(a+b*sech(d*x+c)^2),x, algorithm="maxima")
[Out] a*(x + c/d + 2/(d*(e^(-2*d*x - 2*c) - 1))) + 2*b/(d*(e^(-2*d*x - 2*c) - 1))
```

mupad [B] time = 0.11, size = 25, normalized size = 1.39

$$a\,x-\frac{2\,\left(a+b\right)}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^2*(a + b/cosh(c + d*x)^2),x)
[Out] a*x - (2*(a + b))/(d*(exp(2*c + 2*d*x) - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\operatorname{sech}^{2}\left(c+dx\right)\right)\coth^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**2*(a+b*sech(d*x+c)**2),x)
[Out] Integral((a + b*sech(c + d*x)**2)*coth(c + d*x)**2, x)
```

3.109 $\int \coth^3(c+dx)\left(a+b\,\mathbf{sech}^2(c+dx)\right)\,dx$

Optimal. Leaf size=31

$$\frac{a\log(\sinh(c+dx))}{d}-\frac{(a+b)\operatorname{csch}^2(c+dx)}{2d}$$

```
[Out] -1/2*(a+b)*csch(d*x+c)^2/d+a*ln(sinh(d*x+c))/d
```

Rubi [A] time = 0.06, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4138, 444, 43}

$$\frac{a\log(\sinh(c+dx))}{d}-\frac{(a+b)\operatorname{csch}^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^3*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -((a + b)*Csch[c + d*x]^2)/(2*d) + (a*Log[Sinh[c + d*x]])/d
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \coth^3(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx = \frac{\text{Subst}\left(\int \frac{x\left(b+ax^2\right)}{\left(1-x^2\right)^2}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{b+ax}{(1-x)^2}\,dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a+b}{(-1+x)^2}+\frac{a}{-1+x}\right)dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= -\frac{(a+b)\text{csch}^2(c+dx)}{2d}+\frac{a\log(\sinh(c+dx))}{d}$$

Mathematica [A] time = 0.18, size = 52, normalized size = 1.68

$$-\frac{a\left(\coth^2(c+dx)-2\log(\tanh(c+dx))-2\log(\cosh(c+dx))\right)}{2d}-\frac{b\text{csch}^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^3*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -1/2*(b*Csch[c + d*x]^2)/d - (a*(Coth[c + d*x]^2 - 2*Log[Cosh[c + d*x]] - 2
*Log[Tanh[c + d*x]]))/(2*d)
```

fricas [B] time = 0.42, size = 378, normalized size = 12.19

$$-\frac{adx\cosh(dx+c)^4+4\,adx\cosh(dx+c)\sinh(dx+c)^3+adx\sinh(dx+c)^4+adx-2\,(adx-a-b)\cosh(dx+\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3*(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -(a*d*x*cosh(d*x + c)^4 + 4*a*d*x*cosh(d*x + c)*sinh(d*x + c)^3 + a*d*x*sin
h(d*x + c)^4 + a*d*x - 2*(a*d*x - a - b)*cosh(d*x + c)^2 + 2*(3*a*d*x*cosh(
d*x + c)^2 - a*d*x + a + b)*sinh(d*x + c)^2 - (a*cosh(d*x + c)^4 + 4*a*cosh
(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 - 2*a*cosh(d*x + c)^2 + 2*(3*
a*cosh(d*x + c)^2 - a)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 - a*cosh(d*x
+ c))*sinh(d*x + c) + a)*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c)
)) + 4*(a*d*x*cosh(d*x + c)^3 - (a*d*x - a - b)*cosh(d*x + c))*sinh(d*x + c
))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x + c)^3 + d*sinh(d*x + c)
```

```
^4 - 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^2 + 4*
(d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c) + d)
```

giac [B] time = 0.21, size = 81, normalized size = 2.61

$$-\frac{2\,adx - 2\,a\log\left(\left|e^{(2\,dx+2\,c)} - 1\right|\right) + \frac{3\,ae^{(4\,dx+4\,c)} - 2\,ae^{(2\,dx+2\,c)} + 4\,be^{(2\,dx+2\,c)} + 3\,a}{\left(e^{(2\,dx+2\,c)} - 1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3*(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/2*(2*a*d*x - 2*a*log(abs(e^(2*d*x + 2*c) - 1)) + (3*a*e^(4*d*x + 4*c) -
2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) + 3*a)/(e^(2*d*x + 2*c) - 1)^2)/d
```

maple [A] time = 0.33, size = 42, normalized size = 1.35

$$\frac{a\ln\left(\sinh\left(dx+c\right)\right)}{d} - \frac{a\left(\coth^2\left(dx+c\right)\right)}{2d} - \frac{b}{2d\sinh\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^3*(a+b*sech(d*x+c)^2),x)
```

```
[Out] a*ln(sinh(d*x+c))/d-1/2*a*coth(d*x+c)^2/d-1/2/d/sinh(d*x+c)^2*b
```

maxima [B] time = 0.33, size = 108, normalized size = 3.48

$$a\left(x + \frac{c}{d} + \frac{\log\left(e^{(-dx-c)} + 1\right)}{d} + \frac{\log\left(e^{(-dx-c)} - 1\right)}{d} + \frac{2\,e^{(-2\,dx-2\,c)}}{d\left(2\,e^{(-2\,dx-2\,c)} - e^{(-4\,dx-4\,c)} - 1\right)}\right) - \frac{2\,b}{d\left(e^{(dx+c)} - e^{(-dx-c)}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3*(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] a*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 2*e^(-2*d*
x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) - 2*b/(d*(e^(d*x
+ c) - e^(-d*x - c))^2)
```

mupad [B] time = 1.41, size = 76, normalized size = 2.45

$$\frac{a\,\ln\left(\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x} - 1\right)}{d} - a\,x - \frac{2\,\left(a+b\right)}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x} - 1\right)} - \frac{2\,\left(a+b\right)}{d\,\left(\mathrm{e}^{4\,c+4\,d\,x} - 2\,\mathrm{e}^{2\,c+2\,d\,x} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^3*(a + b/cosh(c + d*x)^2),x)
```

```
[Out] (a*log(exp(2*c)*exp(2*d*x) - 1))/d - a*x - (2*(a + b))/(d*(exp(2*c + 2*d*x)
 - 1)) - (2*(a + b))/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\operatorname{sech}^2\left(c + dx\right)\right)\coth^3\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**3*(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)*coth(c + d*x)**3, x)
```

3.110 $\int \coth^4(c+dx) \left(a+b\,\mathbf{sech}^2(c+dx)\right) dx$

Optimal. Leaf size=34

$$-\frac{(a+b)\coth^3(c+dx)}{3d}-\frac{a\coth(c+dx)}{d}+ax$$

```
[Out] a*x-a*coth(d*x+c)/d-1/3*(a+b)*coth(d*x+c)^3/d
```

Rubi [A] time = 0.06, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4141, 1802, 207}

$$-\frac{(a+b)\coth^3(c+dx)}{3d}-\frac{a\coth(c+dx)}{d}+ax$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^4*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] a*x - (a*Coth[c + d*x])/d - ((a + b)*Coth[c + d*x]^3)/(3*d)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \coth^4(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx = \frac{\text{Subst}\left(\int \frac{a+b\left(1-x^2\right)}{x^4\left(1-x^2\right)}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a+b}{x^4}+\frac{a}{x^2}-\frac{a}{-1+x^2}\right)dx, x, \tanh(c+dx)\right)}{d}$$
$$= -\frac{a\coth(c+dx)}{d}-\frac{(a+b)\coth^3(c+dx)}{3d}-\frac{a\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \text{t}\right.}{d}$$
$$= ax-\frac{a\coth(c+dx)}{d}-\frac{(a+b)\coth^3(c+dx)}{3d}$$

Mathematica [C] time = 0.02, size = 49, normalized size = 1.44

$$-\frac{a\coth^3(c+dx)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\tanh^2(c+dx)\right)}{3d}-\frac{b\coth^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^4*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -1/3*(b*Coth[c + d*x]^3)/d - (a*Coth[c + d*x]^3*Hypergeometric2F1[-3/2, 1,
-1/2, Tanh[c + d*x]^2])/(3*d)
```

fricas [B] time = 0.41, size = 140, normalized size = 4.12

$$-\frac{(4a+b)\cosh(dx+c)^3+3(4a+b)\cosh(dx+c)\sinh(dx+c)^2-(3adx+4a+b)\sinh(dx+c)^3+3b\cosh}{3\left(d\sinh(dx+c)^3+3\left(d\cosh(dx+c)^2-d\right)\text{s}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4*(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/3*((4*a + b)*cosh(d*x + c)^3 + 3*(4*a + b)*cosh(d*x + c)*sinh(d*x + c)^2
 - (3*a*d*x + 4*a + b)*sinh(d*x + c)^3 + 3*b*cosh(d*x + c) + 3*(3*a*d*x - (
3*a*d*x + 4*a + b)*cosh(d*x + c)^2 + 4*a + b)*sinh(d*x + c))/(d*sinh(d*x +
c)^3 + 3*(d*cosh(d*x + c)^2 - d)*sinh(d*x + c))
```

giac [B] time = 0.21, size = 67, normalized size = 1.97

$$\frac{3\,adx-\frac{2\left(6\,ae^{(4\,dx+4\,c)}+3\,be^{(4\,dx+4\,c)}-6\,ae^{(2\,dx+2\,c)}+4\,a+b\right)}{\left(e^{(2\,dx+2\,c)}-1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4*(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/3*(3*a*d*x - 2*(6*a*e^(4*d*x + 4*c) + 3*b*e^(4*d*x + 4*c) - 6*a*e^(2*d*x
+ 2*c) + 4*a + b)/(e^(2*d*x + 2*c) - 1)^3)/d
```

maple [B] time = 0.39, size = 70, normalized size = 2.06

$$\frac{a\left(dx+c-\coth\left(dx+c\right)-\frac{\left(\coth^{3}(dx+c)\right)}{3}\right)+b\left(-\frac{\cosh(dx+c)}{2\sinh(dx+c)^{3}}-\frac{\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^{2}}{3}\right)\coth(dx+c)}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^4*(a+b*sech(d*x+c)^2),x)
```

```
[Out] 1/d*(a*(d*x+c-coth(d*x+c)-1/3*coth(d*x+c)^3)+b*(-1/2/sinh(d*x+c)^3*cosh(d*x
+c)-1/2*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)))
```

maxima [B] time = 0.34, size = 170, normalized size = 5.00

$$\frac{1}{3}\,a\left(3\,x+\frac{3\,c}{d}-\frac{4\left(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}-2\right)}{d\left(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1\right)}\right)+\frac{2}{3}\,b\left(\frac{3\,e^{(-4\,dx-4\,c)}}{d\left(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4*(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/3*a*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) - 2)/(d*(3*
e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1))) + 2/3*b*(3*
e^(-4*d*x - 4*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x -
6*c) - 1)) + 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*
c) - 1)))
```

mupad [B] time = 1.49, size = 161, normalized size = 4.74

$$a\,x-\frac{\frac{2\,b}{3\,d}+\frac{2\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(2\,a+b\right)}{3\,d}}{{\mathrm{e}}^{4\,c+4\,d\,x}-2\,{\mathrm{e}}^{2\,c+2\,d\,x}+1}-\frac{\frac{2\,\left(2\,a+b\right)}{3\,d}+\frac{4\,b\,{\mathrm{e}}^{2\,c+2\,d\,x}}{3\,d}+\frac{2\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(2\,a+b\right)}{3\,d}}{3\,{\mathrm{e}}^{2\,c+2\,d\,x}-3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}-1}-\frac{2\,\left(2\,a+b\right)}{3\,d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^4*(a + b/cosh(c + d*x)^2),x)
```

```
[Out] a*x - ((2*b)/(3*d) + (2*exp(2*c + 2*d*x)*(2*a + b))/(3*d))/(exp(4*c + 4*d*x
) - 2*exp(2*c + 2*d*x) + 1) - ((2*(2*a + b))/(3*d) + (4*b*exp(2*c + 2*d*x))
/(3*d) + (2*exp(4*c + 4*d*x)*(2*a + b))/(3*d))/(3*exp(2*c + 2*d*x) - 3*exp(
4*c + 4*d*x) + exp(6*c + 6*d*x) - 1) - (2*(2*a + b))/(3*d*(exp(2*c + 2*d*x)
 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\operatorname{sech}^2(c + dx)\right)\coth^4(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**4*(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)*coth(c + d*x)**4, x)
```

3.111 $\int \coth^5(c+dx)\left(a+b\,\mathbf{sech}^2(c+dx)\right)\,dx$

Optimal. Leaf size=51

$$-\frac{(a+b)\operatorname{csch}^4(c+dx)}{4d}-\frac{(2a+b)\operatorname{csch}^2(c+dx)}{2d}+\frac{a\log(\sinh(c+dx))}{d}$$

```
[Out] -1/2*(2*a+b)*csch(d*x+c)^2/d-1/4*(a+b)*csch(d*x+c)^4/d+a*ln(sinh(d*x+c))/d
```

Rubi [A] time = 0.08, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4138, 446, 77}

$$-\frac{(a+b)\operatorname{csch}^4(c+dx)}{4d}-\frac{(2a+b)\operatorname{csch}^2(c+dx)}{2d}+\frac{a\log(\sinh(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^5*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -((2*a + b)*Csch[c + d*x]^2)/(2*d) - ((a + b)*Csch[c + d*x]^4)/(4*d) + (a*L
og[Sinh[c + d*x]])/d
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
```

```
x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \coth^5(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx = -\frac{\text{Subst}\left(\int \frac{x^3\left(b+ax^2\right)}{\left(1-x^2\right)^3}\,dx,x,\cosh(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \frac{x(b+ax)}{(1-x)^3}\,dx,x,\cosh^2(c+dx)\right)}{2d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{-a-b}{(-1+x)^3}+\frac{-2a-b}{(-1+x)^2}-\frac{a}{-1+x}\right)dx,x,\cosh^2(c+dx)\right)}{2d}$$
$$= -\frac{(2a+b)\text{csch}^2(c+dx)}{2d}-\frac{(a+b)\text{csch}^4(c+dx)}{4d}+\frac{a\log(\sinh(c+d}{d}$$

Mathematica [A] time = 0.26, size = 62, normalized size = 1.22

$$-\frac{a\left(\coth^4(c+dx)+2\coth^2(c+dx)-4\log(\tanh(c+dx))-4\log(\cosh(c+dx))\right)}{4d}-\frac{b\coth^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^5*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -1/4*(b*Coth[c + d*x]^4)/d - (a*(2*Coth[c + d*x]^2 + Coth[c + d*x]^4 - 4*Lo
g[Cosh[c + d*x]] - 4*Log[Tanh[c + d*x]]))/(4*d)
```

fricas [B] time = 0.43, size = 1099, normalized size = 21.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^5*(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -(a*d*x*cosh(d*x + c)^8 + 8*a*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + a*d*x*sin
h(d*x + c)^8 - 2*(2*a*d*x - 2*a - b)*cosh(d*x + c)^6 + 2*(14*a*d*x*cosh(d*x
 + c)^2 - 2*a*d*x + 2*a + b)*sinh(d*x + c)^6 + 4*(14*a*d*x*cosh(d*x + c)^3
- 3*(2*a*d*x - 2*a - b)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*a*d*x - 2*a)*
cosh(d*x + c)^4 + 2*(35*a*d*x*cosh(d*x + c)^4 + 3*a*d*x - 15*(2*a*d*x - 2*a
 - b)*cosh(d*x + c)^2 - 2*a)*sinh(d*x + c)^4 + 8*(7*a*d*x*cosh(d*x + c)^5 -
```

```
5*(2*a*d*x - 2*a - b)*cosh(d*x + c)^3 + (3*a*d*x - 2*a)*cosh(d*x + c))*sin
h(d*x + c)^3 + a*d*x - 2*(2*a*d*x - 2*a - b)*cosh(d*x + c)^2 + 2*(14*a*d*x*
cosh(d*x + c)^6 - 15*(2*a*d*x - 2*a - b)*cosh(d*x + c)^4 - 2*a*d*x + 6*(3*a
*d*x - 2*a)*cosh(d*x + c)^2 + 2*a + b)*sinh(d*x + c)^2 - (a*cosh(d*x + c)^8
 + 8*a*cosh(d*x + c)*sinh(d*x + c)^7 + a*sinh(d*x + c)^8 - 4*a*cosh(d*x + c
)^6 + 4*(7*a*cosh(d*x + c)^2 - a)*sinh(d*x + c)^6 + 8*(7*a*cosh(d*x + c)^3
- 3*a*cosh(d*x + c))*sinh(d*x + c)^5 + 6*a*cosh(d*x + c)^4 + 2*(35*a*cosh(d
*x + c)^4 - 30*a*cosh(d*x + c)^2 + 3*a)*sinh(d*x + c)^4 + 8*(7*a*cosh(d*x +
 c)^5 - 10*a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c))*sinh(d*x + c)^3 - 4*a*cos
h(d*x + c)^2 + 4*(7*a*cosh(d*x + c)^6 - 15*a*cosh(d*x + c)^4 + 9*a*cosh(d*x
 + c)^2 - a)*sinh(d*x + c)^2 + 8*(a*cosh(d*x + c)^7 - 3*a*cosh(d*x + c)^5 +
 3*a*cosh(d*x + c)^3 - a*cosh(d*x + c))*sinh(d*x + c) + a)*log(2*sinh(d*x +
 c)/(cosh(d*x + c) - sinh(d*x + c))) + 4*(2*a*d*x*cosh(d*x + c)^7 - 3*(2*a*
d*x - 2*a - b)*cosh(d*x + c)^5 + 2*(3*a*d*x - 2*a)*cosh(d*x + c)^3 - (2*a*d
*x - 2*a - b)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^8 + 8*d*cosh(d
*x + c)*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 - 4*d*cosh(d*x + c)^6 + 4*(7*d*
cosh(d*x + c)^2 - d)*sinh(d*x + c)^6 + 8*(7*d*cosh(d*x + c)^3 - 3*d*cosh(d*
x + c))*sinh(d*x + c)^5 + 6*d*cosh(d*x + c)^4 + 2*(35*d*cosh(d*x + c)^4 - 3
0*d*cosh(d*x + c)^2 + 3*d)*sinh(d*x + c)^4 + 8*(7*d*cosh(d*x + c)^5 - 10*d*
cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^3 - 4*d*cosh(d*x + c)^2
+ 4*(7*d*cosh(d*x + c)^6 - 15*d*cosh(d*x + c)^4 + 9*d*cosh(d*x + c)^2 - d)*
sinh(d*x + c)^2 + 8*(d*cosh(d*x + c)^7 - 3*d*cosh(d*x + c)^5 + 3*d*cosh(d*x
 + c)^3 - d*cosh(d*x + c))*sinh(d*x + c) + d)
```

giac [B] time = 0.23, size = 117, normalized size = 2.29

$$-\frac{12\,adx - 12\,a\log\left(\left|e^{(2\,dx+2\,c)} - 1\right|\right) + \frac{25\,ae^{(8\,dx+8\,c)} - 52\,ae^{(6\,dx+6\,c)} + 24\,be^{(6\,dx+6\,c)} + 102\,ae^{(4\,dx+4\,c)} - 52\,ae^{(2\,dx+2\,c)} + 24\,be^{(2\,dx+2\,c)} + 25\,a}{\left(e^{(2\,dx+2\,c)} - 1\right)^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^5*(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/12*(12*a*d*x - 12*a*log(abs(e^(2*d*x + 2*c) - 1)) + (25*a*e^(8*d*x + 8*c
) - 52*a*e^(6*d*x + 6*c) + 24*b*e^(6*d*x + 6*c) + 102*a*e^(4*d*x + 4*c) - 5
2*a*e^(2*d*x + 2*c) + 24*b*e^(2*d*x + 2*c) + 25*a)/(e^(2*d*x + 2*c) - 1)^4)
/d
```

maple [A] time = 0.31, size = 78, normalized size = 1.53

$$\frac{a\ln\left(\sinh\left(dx+c\right)\right)}{d} - \frac{a\left(\coth^2\left(dx+c\right)\right)}{2d} - \frac{a\left(\coth^4\left(dx+c\right)\right)}{4d} - \frac{b\left(\cosh^2\left(dx+c\right)\right)}{2d\sinh\left(dx+c\right)^4} + \frac{b}{4d\sinh\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^5*(a+b*sech(d*x+c)^2),x)
```

```
[Out] a*ln(sinh(d*x+c))/d-1/2*a*coth(d*x+c)^2/d-1/4*a*coth(d*x+c)^4/d-1/2/d*b/sin
h(d*x+c)^4*cosh(d*x+c)^2+1/4/d*b/sinh(d*x+c)^4
```

maxima [B] time = 0.51, size = 251, normalized size = 4.92

$$a\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{4\left(e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}\right)}{d\left(4\,e^{(-2\,dx-2\,c)}-6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}-e^{(-8\,dx-8\,c)}-1\right)}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^5*(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] a*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 4*(e^(-2*d
*x - 2*c) - e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/(d*(4*e^(-2*d*x - 2*c) - 6
*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) - 1))) + 2*b*(e^(
-2*d*x - 2*c)/(d*(4*e^(-2*d*x - 2*c) - 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6
*c) - e^(-8*d*x - 8*c) - 1)) + e^(-6*d*x - 6*c)/(d*(4*e^(-2*d*x - 2*c) - 6*
e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) - 1)))
```

mupad [B] time = 0.11, size = 179, normalized size = 3.51

$$\frac{a\,\ln\left(\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}-1\right)}{d}-\frac{8\,\left(a+b\right)}{d\,\left(3\,\mathrm{e}^{2\,c+2\,d\,x}-3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}-1\right)}-\frac{2\,\left(2\,a+b\right)}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}-\frac{4\,\left(a+b\right)}{d\,\left(6\,\mathrm{e}^{4\,c+4\,d\,x}-4\,\mathrm{e}^{2\,c+2\,d\,x}-4\,\mathrm{e}^{6}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^5*(a + b/cosh(c + d*x)^2),x)
```

```
[Out] (a*log(exp(2*c)*exp(2*d*x) - 1))/d - (8*(a + b))/(d*(3*exp(2*c + 2*d*x) - 3
*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1)) - (2*(2*a + b))/(d*(exp(2*c + 2*
d*x) - 1)) - (4*(a + b))/(d*(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*ex
p(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) - a*x - (2*(4*a + 3*b))/(d*(exp(4*c
 + 4*d*x) - 2*exp(2*c + 2*d*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\,\mathrm{sech}^{2}\,(c+dx)\right)\coth^{5}(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**5*(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)*coth(c + d*x)**5, x)
```

3.112 $\int (a+b\,\mathrm{sech}^2(c+dx))^2 \tanh^4(c+dx)\,dx$

Optimal. Leaf size=77

$$-\frac{a^2\tanh^3(c+dx)}{3d}-\frac{a^2\tanh(c+dx)}{d}+a^2x+\frac{b(2a+b)\tanh^5(c+dx)}{5d}-\frac{b^2\tanh^7(c+dx)}{7d}$$

```
[Out] a^2*x-a^2*tanh(d*x+c)/d-1/3*a^2*tanh(d*x+c)^3/d+1/5*b*(2*a+b)*tanh(d*x+c)^5
/d-1/7*b^2*tanh(d*x+c)^7/d
```

Rubi [A] time = 0.10, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4141, 1802, 206}

$$-\frac{a^2\tanh^3(c+dx)}{3d}-\frac{a^2\tanh(c+dx)}{d}+a^2x+\frac{b(2a+b)\tanh^5(c+dx)}{5d}-\frac{b^2\tanh^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^2*Tanh[c + d*x]^4,x]
```

```
[Out] a^2*x - (a^2*Tanh[c + d*x])/d - (a^2*Tanh[c + d*x]^3)/(3*d) + (b*(2*a + b)*
Tanh[c + d*x]^5)/(5*d) - (b^2*Tanh[c + d*x]^7)/(7*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \left(a+b\text{sech}^2(c+dx)\right)^2 \tanh^4(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x^4\left(a+b\left(1-x^2\right)\right)^2}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-a^2-a^2x^2+b(2a+b)x^4-b^2x^6+\frac{a^2}{1-x^2}\right)dx, x, \tanh(c+\right.}{d}$$
$$= -\frac{a^2\tanh(c+dx)}{d} - \frac{a^2\tanh^3(c+dx)}{3d} + \frac{b(2a+b)\tanh^5(c+dx)}{5d} -$$
$$= a^2x - \frac{a^2\tanh(c+dx)}{d} - \frac{a^2\tanh^3(c+dx)}{3d} + \frac{b(2a+b)\tanh^5(c+d}{5d}$$

Mathematica [B] time = 1.18, size = 395, normalized size = 5.13

$$\frac{\text{sech}(c)\text{sech}^7(c+dx)\left(4480a^2\sinh(2c+dx) - 3780a^2\sinh(2c+3dx) + 2100a^2\sinh(4c+3dx) - 1540a^2\sinh(4\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^2*Tanh[c + d*x]^4,x]
```

```
[Out] (Sech[c]*Sech[c + d*x]^7*(3675*a^2*d*x*Cosh[d*x] + 3675*a^2*d*x*Cosh[2*c +
d*x] + 2205*a^2*d*x*Cosh[2*c + 3*d*x] + 2205*a^2*d*x*Cosh[4*c + 3*d*x] + 73
5*a^2*d*x*Cosh[4*c + 5*d*x] + 735*a^2*d*x*Cosh[6*c + 5*d*x] + 105*a^2*d*x*C
osh[6*c + 7*d*x] + 105*a^2*d*x*Cosh[8*c + 7*d*x] - 5320*a^2*Sinh[d*x] + 168
0*a*b*Sinh[d*x] + 840*b^2*Sinh[d*x] + 4480*a^2*Sinh[2*c + d*x] - 1260*a*b*S
inh[2*c + d*x] + 420*b^2*Sinh[2*c + d*x] - 3780*a^2*Sinh[2*c + 3*d*x] + 924
*a*b*Sinh[2*c + 3*d*x] - 168*b^2*Sinh[2*c + 3*d*x] + 2100*a^2*Sinh[4*c + 3*
d*x] - 840*a*b*Sinh[4*c + 3*d*x] - 420*b^2*Sinh[4*c + 3*d*x] - 1540*a^2*Sin
h[4*c + 5*d*x] + 168*a*b*Sinh[4*c + 5*d*x] + 84*b^2*Sinh[4*c + 5*d*x] + 420
*a^2*Sinh[6*c + 5*d*x] - 420*a*b*Sinh[6*c + 5*d*x] - 280*a^2*Sinh[6*c + 7*d
*x] + 84*a*b*Sinh[6*c + 7*d*x] + 12*b^2*Sinh[6*c + 7*d*x]))/(13440*d)
```

fricas [B] time = 0.42, size = 721, normalized size = 9.36

$$\frac{\left(105\,a^2dx + 140\,a^2 - 42\,ab - 6\,b^2\right)\cosh\left(dx+c\right)^7 + 7\left(105\,a^2dx + 140\,a^2 - 42\,ab - 6\,b^2\right)\cosh\left(dx+c\right)\sinh\left(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2*tanh(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/105*((105*a^2*d*x + 140*a^2 - 42*a*b - 6*b^2)*cosh(d*x + c)^7 + 7*(105*a^
2*d*x + 140*a^2 - 42*a*b - 6*b^2)*cosh(d*x + c)*sinh(d*x + c)^6 - 2*(70*a^2
 - 21*a*b - 3*b^2)*sinh(d*x + c)^7 + 7*(105*a^2*d*x + 140*a^2 - 42*a*b - 6*
b^2)*cosh(d*x + c)^5 - 14*(3*(70*a^2 - 21*a*b - 3*b^2)*cosh(d*x + c)^2 + 40
*a^2 + 9*a*b - 3*b^2)*sinh(d*x + c)^5 + 35*((105*a^2*d*x + 140*a^2 - 42*a*b
 - 6*b^2)*cosh(d*x + c)^3 + (105*a^2*d*x + 140*a^2 - 42*a*b - 6*b^2)*cosh(d
*x + c))*sinh(d*x + c)^4 + 21*(105*a^2*d*x + 140*a^2 - 42*a*b - 6*b^2)*cosh
(d*x + c)^3 - 14*(5*(70*a^2 - 21*a*b - 3*b^2)*cosh(d*x + c)^4 + 10*(40*a^2
+ 9*a*b - 3*b^2)*cosh(d*x + c)^2 + 60*a^2 - 3*a*b + 21*b^2)*sinh(d*x + c)^3
 + 7*(3*(105*a^2*d*x + 140*a^2 - 42*a*b - 6*b^2)*cosh(d*x + c)^5 + 10*(105*
a^2*d*x + 140*a^2 - 42*a*b - 6*b^2)*cosh(d*x + c)^3 + 9*(105*a^2*d*x + 140*
a^2 - 42*a*b - 6*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 35*(105*a^2*d*x + 14
0*a^2 - 42*a*b - 6*b^2)*cosh(d*x + c) - 14*((70*a^2 - 21*a*b - 3*b^2)*cosh(
d*x + c)^6 + 5*(40*a^2 + 9*a*b - 3*b^2)*cosh(d*x + c)^4 + 9*(20*a^2 - a*b +
 7*b^2)*cosh(d*x + c)^2 + 30*a^2 - 15*a*b - 45*b^2)*sinh(d*x + c))/(d*cosh(
d*x + c)^7 + 7*d*cosh(d*x + c)*sinh(d*x + c)^6 + 7*d*cosh(d*x + c)^5 + 35*(
d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^4 + 21*d*cosh(d*x + c)^3
 + 7*(3*d*cosh(d*x + c)^5 + 10*d*cosh(d*x + c)^3 + 9*d*cosh(d*x + c))*sinh(
d*x + c)^2 + 35*d*cosh(d*x + c))
```

giac [B] time = 0.25, size = 275, normalized size = 3.57

$$105\,a^2dx + \frac{4\left(105\,a^2e^{(12\,dx+12\,c)} - 105\,abe^{(12\,dx+12\,c)} + 525\,a^2e^{(10\,dx+10\,c)} - 210\,abe^{(10\,dx+10\,c)} - 105\,b^2e^{(10\,dx+10\,c)} + 1120\,a^2e^{(8\,dx+8\,c)} - 315\,abe^{(8\,dx+8\,c)} + 10\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2*tanh(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/105*(105*a^2*d*x + 4*(105*a^2*e^(12*d*x + 12*c) - 105*a*b*e^(12*d*x + 12*
c) + 525*a^2*e^(10*d*x + 10*c) - 210*a*b*e^(10*d*x + 10*c) - 105*b^2*e^(10*
d*x + 10*c) + 1120*a^2*e^(8*d*x + 8*c) - 315*a*b*e^(8*d*x + 8*c) + 105*b^2*
e^(8*d*x + 8*c) + 1330*a^2*e^(6*d*x + 6*c) - 420*a*b*e^(6*d*x + 6*c) - 210*
b^2*e^(6*d*x + 6*c) + 945*a^2*e^(4*d*x + 4*c) - 231*a*b*e^(4*d*x + 4*c) + 4
2*b^2*e^(4*d*x + 4*c) + 385*a^2*e^(2*d*x + 2*c) - 42*a*b*e^(2*d*x + 2*c) -
21*b^2*e^(2*d*x + 2*c) + 70*a^2 - 21*a*b - 3*b^2)/(e^(2*d*x + 2*c) + 1)^7)/
d
```

maple [B] time = 0.44, size = 181, normalized size = 2.35

$$\frac{a^2\left(dx+c-\tanh\left(dx+c\right)-\frac{\left(\tanh^3(dx+c)\right)}{3}\right)+2ab\left(-\frac{\sinh^3(dx+c)}{2\cosh(dx+c)^5}-\frac{3\sinh(dx+c)}{8\cosh(dx+c)^5}+\frac{3\left(\frac{8}{15}+\frac{\operatorname{sech}(dx+c)^4}{5}+\frac{4\operatorname{sech}(dx+c)^2}{15}\right)\tanh(dx+c)}{8}\right)\cdots}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)^2*tanh(d*x+c)^4,x)
```

```
[Out] 1/d*(a^2*(d*x+c-tanh(d*x+c)-1/3*tanh(d*x+c)^3)+2*a*b*(-1/2*sinh(d*x+c)^3/co
sh(d*x+c)^5-3/8*sinh(d*x+c)/cosh(d*x+c)^5+3/8*(8/15+1/5*sech(d*x+c)^4+4/15*
sech(d*x+c)^2)*tanh(d*x+c))+b^2*(-1/4*sinh(d*x+c)^3/cosh(d*x+c)^7-1/8*sinh(
d*x+c)/cosh(d*x+c)^7+1/8*(16/35+1/7*sech(d*x+c)^6+6/35*sech(d*x+c)^4+8/35*s
ech(d*x+c)^2)*tanh(d*x+c)))
```

maxima [B] time = 0.55, size = 649, normalized size = 8.43

$$\frac{2\,ab\tanh\left(dx+c\right)^{5}}{5\,d}+\frac{1}{3}\,a^{2}\left(3\,x+\frac{3\,c}{d}-\frac{4\left(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+2\right)}{d\left(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1\right)}\right)+\frac{4}{35}\,b^{2}\left(\frac{}{d\left(7\,e^{(-2\,dx-2\,c)}+21\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2*tanh(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 2/5*a*b*tanh(d*x + c)^5/d + 1/3*a^2*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) +
3*e^(-4*d*x - 4*c) + 2)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6
*d*x - 6*c) + 1))) + 4/35*b^2*(7*e^(-2*d*x - 2*c)/(d*(7*e^(-2*d*x - 2*c) +
21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10
*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) - 14*e^(-4*d
*x - 4*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c
) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^
(-14*d*x - 14*c) + 1)) + 70*e^(-6*d*x - 6*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^
(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x
- 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) - 35*e^(-8*d*x -
8*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 3
5*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*
d*x - 14*c) + 1)) + 35*e^(-10*d*x - 10*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4
*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 1
0*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 1/(d*(7*e^(-2*d*x
- 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) +
21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)))
```

mupad [B] time = 0.18, size = 1022, normalized size = 13.27

$$\frac{\frac{4\left(7\,a^{2}+a\,b+8\,b^{2}\right)}{105\,d}-\frac{4\,e^{8\,c+8\,d\,x}\left(a\,b-a^{2}\right)}{7\,d}-\frac{16\,e^{2\,c+2\,d\,x}\left(-2\,a^{2}+a\,b+3\,b^{2}\right)}{35\,d}+\frac{16\,e^{6\,c+6\,d\,x}\left(2\,a^{2}+a\,b-b^{2}\right)}{21\,d}+\frac{8\,e^{4\,c+4\,d\,x}\left(7\,a^{2}+a\,b+8\,b^{2}\right)}{35\,d}}{5\,e^{2\,c+2\,d\,x}+10\,e^{4\,c+4\,d\,x}+10\,e^{6\,c+6\,d\,x}+5\,e^{8\,c+8\,d\,x}+e^{10\,c+10\,d\,x}+1}-\frac{4\left(-2\,a^{2}+a\right.}{35\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^4*(a + b/cosh(c + d*x)^2)^2,x)
```

```
[Out] ((4*(a*b + 7*a^2 + 8*b^2))/(105*d) - (4*exp(8*c + 8*d*x)*(a*b - a^2))/(7*d)
 - (16*exp(2*c + 2*d*x)*(a*b - 2*a^2 + 3*b^2))/(35*d) + (16*exp(6*c + 6*d*x
)*(a*b + 2*a^2 - b^2))/(21*d) + (8*exp(4*c + 4*d*x)*(a*b + 7*a^2 + 8*b^2))/
(35*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5
*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1) - ((4*(a*b - 2*a^2 + 3*b^2))/(3
5*d) + (4*exp(6*c + 6*d*x)*(a*b - a^2))/(7*d) - (4*exp(4*c + 4*d*x)*(a*b +
2*a^2 - b^2))/(7*d) - (4*exp(2*c + 2*d*x)*(a*b + 7*a^2 + 8*b^2))/(35*d))/(4
*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d
*x) + 1) + a^2*x + ((4*(a*b + 7*a^2 + 8*b^2))/(105*d) - (4*exp(4*c + 4*d*x)
*(a*b - a^2))/(7*d) + (8*exp(2*c + 2*d*x)*(a*b + 2*a^2 - b^2))/(21*d))/(3*e
xp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) + ((8*exp(2*c
+ 2*d*x)*(a*b + 2*a^2 - b^2))/(7*d) - (4*exp(12*c + 12*d*x)*(a*b - a^2))/(7
*d) - (4*(a*b - a^2))/(7*d) - (16*exp(6*c + 6*d*x)*(a*b - 2*a^2 + 3*b^2))/(
7*d) + (4*exp(4*c + 4*d*x)*(a*b + 7*a^2 + 8*b^2))/(7*d) + (8*exp(10*c + 10*
d*x)*(a*b + 2*a^2 - b^2))/(7*d) + (4*exp(8*c + 8*d*x)*(a*b + 7*a^2 + 8*b^2)
)/(7*d))/(7*exp(2*c + 2*d*x) + 21*exp(4*c + 4*d*x) + 35*exp(6*c + 6*d*x) +
35*exp(8*c + 8*d*x) + 21*exp(10*c + 10*d*x) + 7*exp(12*c + 12*d*x) + exp(14
*c + 14*d*x) + 1) + ((4*(a*b + 2*a^2 - b^2))/(21*d) - (4*exp(2*c + 2*d*x)*(
a*b - a^2))/(7*d))/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) + ((4*(a*b +
 2*a^2 - b^2))/(21*d) - (4*exp(10*c + 10*d*x)*(a*b - a^2))/(7*d) - (8*exp(4
*c + 4*d*x)*(a*b - 2*a^2 + 3*b^2))/(7*d) + (4*exp(2*c + 2*d*x)*(a*b + 7*a^2
 + 8*b^2))/(21*d) + (20*exp(8*c + 8*d*x)*(a*b + 2*a^2 - b^2))/(21*d) + (8*e
xp(6*c + 6*d*x)*(a*b + 7*a^2 + 8*b^2))/(21*d))/(6*exp(2*c + 2*d*x) + 15*exp
(4*c + 4*d*x) + 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*exp(10*c + 10
*d*x) + exp(12*c + 12*d*x) + 1) - (4*(a*b - a^2))/(7*d*(exp(2*c + 2*d*x) +
1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\operatorname{sech}^2(c + dx)\right)^2 \tanh^4(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)**2*tanh(d*x+c)**4,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**2*tanh(c + d*x)**4, x)
```

3.113 $\int \left(a+b\,\text{sech}^2(c+dx)\right)^2 \tanh^3(c+dx)\,dx$

Optimal. Leaf size=77

$$\frac{a^2\log(\cosh(c+dx))}{d}+\frac{b(2a-b)\text{sech}^4(c+dx)}{4d}+\frac{a(a-2b)\text{sech}^2(c+dx)}{2d}+\frac{b^2\text{sech}^6(c+dx)}{6d}$$

```
[Out] a^2*ln(cosh(d*x+c))/d+1/2*a*(a-2*b)*sech(d*x+c)^2/d+1/4*(2*a-b)*b*sech(d*x+
c)^4/d+1/6*b^2*sech(d*x+c)^6/d
```

Rubi [A] time = 0.09, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4138, 446, 76}

$$\frac{a^2\log(\cosh(c+dx))}{d}+\frac{b(2a-b)\text{sech}^4(c+dx)}{4d}+\frac{a(a-2b)\text{sech}^2(c+dx)}{2d}+\frac{b^2\text{sech}^6(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^2*Tanh[c + d*x]^3,x]
```

```
[Out] (a^2*Log[Cosh[c + d*x]])/d + (a*(a - 2*b)*Sech[c + d*x]^2)/(2*d) + ((2*a -
b)*b*Sech[c + d*x]^4)/(4*d) + (b^2*Sech[c + d*x]^6)/(6*d)
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
```

```
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \left(a+b\operatorname{sech}^2(c+dx)\right)^2\tanh^3(c+dx)\,dx=-\frac{\operatorname{Subst}\left(\int \frac{(1-x^2)(b+ax^2)^2}{x^7}\,dx,x,\cosh(c+dx)\right)}{d}$$
$$=-\frac{\operatorname{Subst}\left(\int \frac{(1-x)(b+ax)^2}{x^4}\,dx,x,\cosh^2(c+dx)\right)}{2d}$$
$$=-\frac{\operatorname{Subst}\left(\int \left(\frac{b^2}{x^4}+\frac{(2a-b)b}{x^3}+\frac{a(a-2b)}{x^2}-\frac{a^2}{x}\right)dx,x,\cosh^2(c+dx)\right)}{2d}$$
$$=\frac{a^2\log(\cosh(c+dx))}{d}+\frac{a(a-2b)\operatorname{sech}^2(c+dx)}{2d}+\frac{(2a-b)b\operatorname{sech}^4(c$$

Mathematica [A] time = 0.29, size = 107, normalized size = 1.39

$$\frac{\cosh^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\left(12a^2\log(\cosh(c+dx))+3b(2a-b)\operatorname{sech}^4(c+dx)+6a(a-2b)\operatorname{sech}^2(c+d\right.}{3d(a\cosh(2c+2dx)+a+2b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^2*Tanh[c + d*x]^3,x]
```

```
[Out] (Cosh[c + d*x]^4*(a + b*Sech[c + d*x]^2)^2*(12*a^2*Log[Cosh[c + d*x]] + 6*a
*(a - 2*b)*Sech[c + d*x]^2 + 3*(2*a - b)*b*Sech[c + d*x]^4 + 2*b^2*Sech[c +
 d*x]^6))/(3*d*(a + 2*b + a*Cosh[2*c + 2*d*x])^2)
```

fricas [B] time = 0.45, size = 2591, normalized size = 33.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2*tanh(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/3*(3*a^2*d*x*cosh(d*x + c)^12 + 36*a^2*d*x*cosh(d*x + c)*sinh(d*x + c)^1
1 + 3*a^2*d*x*sinh(d*x + c)^12 + 6*(3*a^2*d*x - a^2 + 2*a*b)*cosh(d*x + c)^
10 + 6*(33*a^2*d*x*cosh(d*x + c)^2 + 3*a^2*d*x - a^2 + 2*a*b)*sinh(d*x + c)
^10 + 60*(11*a^2*d*x*cosh(d*x + c)^3 + (3*a^2*d*x - a^2 + 2*a*b)*cosh(d*x +
```

c))*sinh(d*x + c)^9 + 3*(15*a^2*d*x - 8*a^2 + 8*a*b + 4*b^2)*cosh(d*x + c)
^8 + 3*(495*a^2*d*x*cosh(d*x + c)^4 + 15*a^2*d*x + 90*(3*a^2*d*x - a^2 + 2*
a*b)*cosh(d*x + c)^2 - 8*a^2 + 8*a*b + 4*b^2)*sinh(d*x + c)^8 + 24*(99*a^2*
d*x*cosh(d*x + c)^5 + 30*(3*a^2*d*x - a^2 + 2*a*b)*cosh(d*x + c)^3 + (15*a^
2*d*x - 8*a^2 + 8*a*b + 4*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 + 4*(15*a^2*d
*x - 9*a^2 + 6*a*b - 2*b^2)*cosh(d*x + c)^6 + 4*(693*a^2*d*x*cosh(d*x + c)^
6 + 315*(3*a^2*d*x - a^2 + 2*a*b)*cosh(d*x + c)^4 + 15*a^2*d*x + 21*(15*a^2
*d*x - 8*a^2 + 8*a*b + 4*b^2)*cosh(d*x + c)^2 - 9*a^2 + 6*a*b - 2*b^2)*sinh
(d*x + c)^6 + 24*(99*a^2*d*x*cosh(d*x + c)^7 + 63*(3*a^2*d*x - a^2 + 2*a*b)
*cosh(d*x + c)^5 + 7*(15*a^2*d*x - 8*a^2 + 8*a*b + 4*b^2)*cosh(d*x + c)^3 +
(15*a^2*d*x - 9*a^2 + 6*a*b - 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 3*(1
5*a^2*d*x - 8*a^2 + 8*a*b + 4*b^2)*cosh(d*x + c)^4 + 3*(495*a^2*d*x*cosh(d*
x + c)^8 + 420*(3*a^2*d*x - a^2 + 2*a*b)*cosh(d*x + c)^6 + 70*(15*a^2*d*x -
8*a^2 + 8*a*b + 4*b^2)*cosh(d*x + c)^4 + 15*a^2*d*x + 20*(15*a^2*d*x - 9*a
^2 + 6*a*b - 2*b^2)*cosh(d*x + c)^2 - 8*a^2 + 8*a*b + 4*b^2)*sinh(d*x + c)^
4 + 3*a^2*d*x + 4*(165*a^2*d*x*cosh(d*x + c)^9 + 180*(3*a^2*d*x - a^2 + 2*a
*b)*cosh(d*x + c)^7 + 42*(15*a^2*d*x - 8*a^2 + 8*a*b + 4*b^2)*cosh(d*x + c)
^5 + 20*(15*a^2*d*x - 9*a^2 + 6*a*b - 2*b^2)*cosh(d*x + c)^3 + 3*(15*a^2*d*
x - 8*a^2 + 8*a*b + 4*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 6*(3*a^2*d*x -
a^2 + 2*a*b)*cosh(d*x + c)^2 + 6*(33*a^2*d*x*cosh(d*x + c)^10 + 45*(3*a^2*d
*x - a^2 + 2*a*b)*cosh(d*x + c)^8 + 14*(15*a^2*d*x - 8*a^2 + 8*a*b + 4*b^2)
*cosh(d*x + c)^6 + 10*(15*a^2*d*x - 9*a^2 + 6*a*b - 2*b^2)*cosh(d*x + c)^4
+ 3*a^2*d*x + 3*(15*a^2*d*x - 8*a^2 + 8*a*b + 4*b^2)*cosh(d*x + c)^2 - a^2
+ 2*a*b)*sinh(d*x + c)^2 - 3*(a^2*cosh(d*x + c)^12 + 12*a^2*cosh(d*x + c)*s
inh(d*x + c)^11 + a^2*sinh(d*x + c)^12 + 6*a^2*cosh(d*x + c)^10 + 6*(11*a^2
*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^10 + 15*a^2*cosh(d*x + c)^8 + 20*(11*
a^2*cosh(d*x + c)^3 + 3*a^2*cosh(d*x + c))*sinh(d*x + c)^9 + 15*(33*a^2*cos
h(d*x + c)^4 + 18*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^8 + 20*a^2*cosh(
d*x + c)^6 + 24*(33*a^2*cosh(d*x + c)^5 + 30*a^2*cosh(d*x + c)^3 + 5*a^2*co
sh(d*x + c))*sinh(d*x + c)^7 + 4*(231*a^2*cosh(d*x + c)^6 + 315*a^2*cosh(d*
x + c)^4 + 105*a^2*cosh(d*x + c)^2 + 5*a^2)*sinh(d*x + c)^6 + 15*a^2*cosh(d
x + c)^4 + 24(33*a^2*cosh(d*x + c)^7 + 63*a^2*cosh(d*x + c)^5 + 35*a^2*co
sh(d*x + c)^3 + 5*a^2*cosh(d*x + c))*sinh(d*x + c)^5 + 15*(33*a^2*cosh(d*x
+ c)^8 + 84*a^2*cosh(d*x + c)^6 + 70*a^2*cosh(d*x + c)^4 + 20*a^2*cosh(d*x
+ c)^2 + a^2)*sinh(d*x + c)^4 + 6*a^2*cosh(d*x + c)^2 + 20*(11*a^2*cosh(d*x
+ c)^9 + 36*a^2*cosh(d*x + c)^7 + 42*a^2*cosh(d*x + c)^5 + 20*a^2*cosh(d*x
+ c)^3 + 3*a^2*cosh(d*x + c))*sinh(d*x + c)^3 + 6*(11*a^2*cosh(d*x + c)^10
+ 45*a^2*cosh(d*x + c)^8 + 70*a^2*cosh(d*x + c)^6 + 50*a^2*cosh(d*x + c)^4
+ 15*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^2 + a^2 + 12*(a^2*cosh(d*x +
c)^11 + 5*a^2*cosh(d*x + c)^9 + 10*a^2*cosh(d*x + c)^7 + 10*a^2*cosh(d*x +
c)^5 + 5*a^2*cosh(d*x + c)^3 + a^2*cosh(d*x + c))*sinh(d*x + c))*log(2*cos
h(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 12*(3*a^2*d*x*cosh(d*x + c)^1
1 + 5*(3*a^2*d*x - a^2 + 2*a*b)*cosh(d*x + c)^9 + 2*(15*a^2*d*x - 8*a^2 + 8
*a*b + 4*b^2)*cosh(d*x + c)^7 + 2*(15*a^2*d*x - 9*a^2 + 6*a*b - 2*b^2)*cosh
(d*x + c)^5 + (15*a^2*d*x - 8*a^2 + 8*a*b + 4*b^2)*cosh(d*x + c)^3 + (3*a^2

```
*d*x - a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^12 + 12*
d*cosh(d*x + c)*sinh(d*x + c)^11 + d*sinh(d*x + c)^12 + 6*d*cosh(d*x + c)^1
0 + 6*(11*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^10 + 20*(11*d*cosh(d*x + c)^
3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^9 + 15*d*cosh(d*x + c)^8 + 15*(33*d*co
sh(d*x + c)^4 + 18*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^8 + 24*(33*d*cosh(d
*x + c)^5 + 30*d*cosh(d*x + c)^3 + 5*d*cosh(d*x + c))*sinh(d*x + c)^7 + 20*
d*cosh(d*x + c)^6 + 4*(231*d*cosh(d*x + c)^6 + 315*d*cosh(d*x + c)^4 + 105*
d*cosh(d*x + c)^2 + 5*d)*sinh(d*x + c)^6 + 24*(33*d*cosh(d*x + c)^7 + 63*d*
cosh(d*x + c)^5 + 35*d*cosh(d*x + c)^3 + 5*d*cosh(d*x + c))*sinh(d*x + c)^5
 + 15*d*cosh(d*x + c)^4 + 15*(33*d*cosh(d*x + c)^8 + 84*d*cosh(d*x + c)^6 +
 70*d*cosh(d*x + c)^4 + 20*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^4 + 20*(11*
d*cosh(d*x + c)^9 + 36*d*cosh(d*x + c)^7 + 42*d*cosh(d*x + c)^5 + 20*d*cosh
(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^3 + 6*d*cosh(d*x + c)^2 + 6*
(11*d*cosh(d*x + c)^10 + 45*d*cosh(d*x + c)^8 + 70*d*cosh(d*x + c)^6 + 50*d
*cosh(d*x + c)^4 + 15*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 12*(d*cosh(d
*x + c)^11 + 5*d*cosh(d*x + c)^9 + 10*d*cosh(d*x + c)^7 + 10*d*cosh(d*x + c
)^5 + 5*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

giac [B] time = 0.20, size = 241, normalized size = 3.13

$$-\frac{60\,a^2dx - 60\,a^2\log\left(e^{(2\,dx+2\,c)}+1\right) + \frac{147\,a^2e^{(12\,dx+12\,c)}+762\,a^2e^{(10\,dx+10\,c)}+240\,abe^{(10\,dx+10\,c)}+1725\,a^2e^{(8\,dx+8\,c)}+480\,abe^{(8\,dx+8\,c)}+240\,b^2\cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2*tanh(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/60*(60*a^2*d*x - 60*a^2*log(e^(2*d*x + 2*c) + 1) + (147*a^2*e^(12*d*x +
12*c) + 762*a^2*e^(10*d*x + 10*c) + 240*a*b*e^(10*d*x + 10*c) + 1725*a^2*e^
(8*d*x + 8*c) + 480*a*b*e^(8*d*x + 8*c) + 240*b^2*e^(8*d*x + 8*c) + 2220*a^
2*e^(6*d*x + 6*c) + 480*a*b*e^(6*d*x + 6*c) - 160*b^2*e^(6*d*x + 6*c) + 172
5*a^2*e^(4*d*x + 4*c) + 480*a*b*e^(4*d*x + 4*c) + 240*b^2*e^(4*d*x + 4*c) +
 762*a^2*e^(2*d*x + 2*c) + 240*a*b*e^(2*d*x + 2*c) + 147*a^2)/(e^(2*d*x + 2
*c) + 1)^6)/d
```

maple [A] time = 0.24, size = 110, normalized size = 1.43

$$\frac{a^2\ln\left(\cosh\left(dx+c\right)\right)}{d}-\frac{\left(\tanh^2\left(dx+c\right)\right)a^2}{2d}-\frac{ab\left(\sinh^2\left(dx+c\right)\right)}{d\cosh\left(dx+c\right)^4}-\frac{ab}{2d\cosh\left(dx+c\right)^4}-\frac{b^2\left(\sinh^2\left(dx+c\right)\right)}{4d\cosh\left(dx+c\right)^6}-\frac{b\cdots}{12d\cosh\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)^2*tanh(d*x+c)^3,x)
```

```
[Out] a^2*ln(cosh(d*x+c))/d-1/2/d*tanh(d*x+c)^2*a^2-1/d*a*b*sinh(d*x+c)^2/cosh(d*
x+c)^4-1/2/d*a*b/cosh(d*x+c)^4-1/4/d*b^2*sinh(d*x+c)^2/cosh(d*x+c)^6-1/12/d
*b^2/cosh(d*x+c)^6
```

maxima [B] time = 0.44, size = 333, normalized size = 4.32

$$\frac{ab\tanh(dx+c)^4}{2d}+a^2\left(x+\frac{c}{d}+\frac{\log\left(e^{(-2dx-2c)}+1\right)}{d}+\frac{2e^{(-2dx-2c)}}{d\left(2e^{(-2dx-2c)}+e^{(-4dx-4c)}+1\right)}\right)-\frac{4}{3}b^2\left(\frac{}{d\left(6e^{(-2dx-2c)}+15\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2*tanh(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/2*a*b*tanh(d*x + c)^4/d + a^2*(x + c/d + log(e^(-2*d*x - 2*c) + 1)/d + 2*
e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) - 4/3*b^2
*(3*e^(-4*d*x - 4*c)/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x - 4*c) + 20*e^(-
6*d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) + e^(-12*d*x - 12
*c) + 1)) - 2*e^(-6*d*x - 6*c)/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x - 4*c)
 + 20*e^(-6*d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) + e^(-1
2*d*x - 12*c) + 1)) + 3*e^(-8*d*x - 8*c)/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*
d*x - 4*c) + 20*e^(-6*d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*
c) + e^(-12*d*x - 12*c) + 1)))
```

mupad [B] time = 1.52, size = 349, normalized size = 4.53

$$\frac{4\left(2ab-9b^2\right)}{d\left(4e^{2c+2dx}+6e^{4c+4dx}+4e^{6c+6dx}+e^{8c+8dx}+1\right)}-\frac{32b^2}{3d\left(6e^{2c+2dx}+15e^{4c+4dx}+20e^{6c+6dx}+15e^{8c+8dx}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^3*(a + b/cosh(c + d*x)^2)^2,x)
```

```
[Out] (4*(2*a*b - 9*b^2))/(d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c
 + 6*d*x) + exp(8*c + 8*d*x) + 1)) - (32*b^2)/(3*d*(6*exp(2*c + 2*d*x) + 15
*exp(4*c + 4*d*x) + 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*exp(10*c
+ 10*d*x) + exp(12*c + 12*d*x) + 1)) - (2*(a^2 - 6*a*b + 2*b^2))/(d*(2*exp(
2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (2*(2*a*b - a^2))/(d*(exp(2*c + 2*d
*x) + 1)) - (8*(6*a*b - 7*b^2))/(3*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*
x) + exp(6*c + 6*d*x) + 1)) - a^2*x + (a^2*log(exp(2*c)*exp(2*d*x) + 1))/d
+ (32*b^2)/(d*(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*
x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1))
```

sympy [A] time = 5.09, size = 129, normalized size = 1.68

$$\begin{cases} a^2x - \frac{a^2 \log (\tanh (c+dx)+1)}{d} - \frac{a^2 \tanh^2 (c+dx)}{2d} - \frac{ab \tanh^2 (c+dx) \operatorname{sech}^2 (c+dx)}{2d} - \frac{ab \operatorname{sech}^2 (c+dx)}{2d} - \frac{b^2 \tanh^2 (c+dx) \operatorname{sech}^4 (c+dx)}{6d} - \frac{b^2 \operatorname{sech}}{12} \\ x \left(a + b \operatorname{sech}^2 (c)\right)^2 \tanh^3 (c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)**2*tanh(d*x+c)**3,x)
```

```
[Out] Piecewise((a**2*x - a**2*log(tanh(c + d*x) + 1)/d - a**2*tanh(c + d*x)**2/(
2*d) - a*b*tanh(c + d*x)**2*sech(c + d*x)**2/(2*d) - a*b*sech(c + d*x)**2/(
2*d) - b**2*tanh(c + d*x)**2*sech(c + d*x)**4/(6*d) - b**2*sech(c + d*x)**4
/(12*d), Ne(d, 0)), (x*(a + b*sech(c)**2)**2*tanh(c)**3, True))
```

3.114 $\int \left(a+b\,\mathbf{sech}^2(c+dx)\right)^2 \tanh^2(c+dx)\,dx$

Optimal. Leaf size=59

$$-\frac{a^2\tanh(c+dx)}{d}+a^2x+\frac{b(2a+b)\tanh^3(c+dx)}{3d}-\frac{b^2\tanh^5(c+dx)}{5d}$$

```
[Out] a^2*x-a^2*tanh(d*x+c)/d+1/3*b*(2*a+b)*tanh(d*x+c)^3/d-1/5*b^2*tanh(d*x+c)^5
/d
```

Rubi [A] time = 0.10, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4141, 1802, 206}

$$-\frac{a^2\tanh(c+dx)}{d}+a^2x+\frac{b(2a+b)\tanh^3(c+dx)}{3d}-\frac{b^2\tanh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^2*Tanh[c + d*x]^2,x]
```

```
[Out] a^2*x - (a^2*Tanh[c + d*x])/d + (b*(2*a + b)*Tanh[c + d*x]^3)/(3*d) - (b^2*
Tanh[c + d*x]^5)/(5*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \left(a+b\text{sech}^2(c+dx)\right)^2 \tanh^2(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x^2\left(a+b\left(1-x^2\right)\right)^2}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-a^2+b(2a+b)x^2-b^2x^4+\frac{a^2}{1-x^2}\right)dx, x, \tanh(c+dx)\right)}{d}$$
$$= -\frac{a^2\tanh(c+dx)}{d}+\frac{b(2a+b)\tanh^3(c+dx)}{3d}-\frac{b^2\tanh^5(c+dx)}{5d}+$$
$$= a^2x-\frac{a^2\tanh(c+dx)}{d}+\frac{b(2a+b)\tanh^3(c+dx)}{3d}-\frac{b^2\tanh^5(c+dx)}{5d}$$

Mathematica [B] time = 0.86, size = 281, normalized size = 4.76

$$\frac{\text{sech}(c)\text{sech}^5(c+dx)\left(120a^2\sinh(2c+dx)-120a^2\sinh(2c+3dx)+30a^2\sinh(4c+3dx)-30a^2\sinh(4c+5dx)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^2*Tanh[c + d*x]^2,x]
```

```
[Out] (Sech[c]*Sech[c + d*x]^5*(150*a^2*d*x*Cosh[d*x] + 150*a^2*d*x*Cosh[2*c + d*
x] + 75*a^2*d*x*Cosh[2*c + 3*d*x] + 75*a^2*d*x*Cosh[4*c + 3*d*x] + 15*a^2*d
*x*Cosh[4*c + 5*d*x] + 15*a^2*d*x*Cosh[6*c + 5*d*x] - 180*a^2*Sinh[d*x] + 8
0*a*b*Sinh[d*x] - 20*b^2*Sinh[d*x] + 120*a^2*Sinh[2*c + d*x] - 120*a*b*Sinh
[2*c + d*x] - 60*b^2*Sinh[2*c + d*x] - 120*a^2*Sinh[2*c + 3*d*x] + 40*a*b*S
inh[2*c + 3*d*x] + 20*b^2*Sinh[2*c + 3*d*x] + 30*a^2*Sinh[4*c + 3*d*x] - 60
*a*b*Sinh[4*c + 3*d*x] - 30*a^2*Sinh[4*c + 5*d*x] + 20*a*b*Sinh[4*c + 5*d*x
] + 4*b^2*Sinh[4*c + 5*d*x]))/(480*d)
```

fricas [B] time = 0.42, size = 435, normalized size = 7.37

$$\frac{\left(15\,a^2dx+15\,a^2-10\,ab-2\,b^2\right)\cosh\left(dx+c\right)^5+5\left(15\,a^2dx+15\,a^2-10\,ab-2\,b^2\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^4-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2*tanh(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/15*((15*a^2*d*x + 15*a^2 - 10*a*b - 2*b^2)*cosh(d*x + c)^5 + 5*(15*a^2*d*
x + 15*a^2 - 10*a*b - 2*b^2)*cosh(d*x + c)*sinh(d*x + c)^4 - (15*a^2 - 10*a
*b - 2*b^2)*sinh(d*x + c)^5 + 5*(15*a^2*d*x + 15*a^2 - 10*a*b - 2*b^2)*cosh
```

```
(d*x + c)^3 - 5*(2*(15*a^2 - 10*a*b - 2*b^2)*cosh(d*x + c)^2 + 9*a^2 + 2*a*
b - 2*b^2)*sinh(d*x + c)^3 + 5*(2*(15*a^2*d*x + 15*a^2 - 10*a*b - 2*b^2)*co
sh(d*x + c)^3 + 3*(15*a^2*d*x + 15*a^2 - 10*a*b - 2*b^2)*cosh(d*x + c))*sin
h(d*x + c)^2 + 10*(15*a^2*d*x + 15*a^2 - 10*a*b - 2*b^2)*cosh(d*x + c) - 5*
((15*a^2 - 10*a*b - 2*b^2)*cosh(d*x + c)^4 + 3*(9*a^2 + 2*a*b - 2*b^2)*cosh
(d*x + c)^2 + 6*a^2 + 4*a*b + 8*b^2)*sinh(d*x + c))/(d*cosh(d*x + c)^5 + 5*
d*cosh(d*x + c)*sinh(d*x + c)^4 + 5*d*cosh(d*x + c)^3 + 5*(2*d*cosh(d*x + c
)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 + 10*d*cosh(d*x + c))
```

giac [B] time = 0.17, size = 193, normalized size = 3.27

$$\frac{15\,a^2dx + \frac{2\left(15\,a^2e^{(8\,dx+8\,c)}-30\,abe^{(8\,dx+8\,c)}+60\,a^2e^{(6\,dx+6\,c)}-60\,abe^{(6\,dx+6\,c)}-30\,b^2e^{(6\,dx+6\,c)}+90\,a^2e^{(4\,dx+4\,c)}-40\,abe^{(4\,dx+4\,c)}+10\,b^2e^{(4\,dx+4\,c)}+60\,a^2\cdots\right.}{\left(e^{(2\,dx+2\,c)}+1\right)^5}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2*tanh(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/15*(15*a^2*d*x + 2*(15*a^2*e^(8*d*x + 8*c) - 30*a*b*e^(8*d*x + 8*c) + 60*
a^2*e^(6*d*x + 6*c) - 60*a*b*e^(6*d*x + 6*c) - 30*b^2*e^(6*d*x + 6*c) + 90*
a^2*e^(4*d*x + 4*c) - 40*a*b*e^(4*d*x + 4*c) + 10*b^2*e^(4*d*x + 4*c) + 60*
a^2*e^(2*d*x + 2*c) - 20*a*b*e^(2*d*x + 2*c) - 10*b^2*e^(2*d*x + 2*c) + 15*
a^2 - 10*a*b - 2*b^2)/(e^(2*d*x + 2*c) + 1)^5)/d
```

maple [B] time = 0.40, size = 115, normalized size = 1.95

$$\frac{a^2\left(dx+c-\tanh\left(dx+c\right)\right)+2ab\left(-\frac{\sinh(dx+c)}{2\cosh(dx+c)^3}+\frac{\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)}{2}\right)+b^2\left(-\frac{\sinh(dx+c)}{4\cosh(dx+c)^5}+\frac{\left(\frac{8}{15}+\frac{\operatorname{sech}(dx+c)^4}{5}+\frac{4\operatorname{sec}\cdots}{}\right.}{4\cdots}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)^2*tanh(d*x+c)^2,x)
```

```
[Out] 1/d*(a^2*(d*x+c-tanh(d*x+c))+2*a*b*(-1/2*sinh(d*x+c)/cosh(d*x+c)^3+1/2*(2/3
+1/3*sech(d*x+c)^2)*tanh(d*x+c))+b^2*(-1/4*sinh(d*x+c)/cosh(d*x+c)^5+1/4*(8
/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)*tanh(d*x+c)))
```

maxima [B] time = 0.52, size = 325, normalized size = 5.51

$$\frac{2\,ab\tanh\left(dx+c\right)^3}{3\,d}+a^2\left(x+\frac{c}{d}-\frac{2}{d\left(e^{(-2\,dx-2\,c)}+1\right)}\right)+\frac{4}{15}\,b^2\left(\frac{5\,e^{(-2\,dx-2\,c)}}{d\left(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}+5\,e^{(-8\,\cdots}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2*tanh(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 2/3*a*b*tanh(d*x + c)^3/d + a^2*(x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1))) +
4/15*b^2*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) +
 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) - 5*e^
(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x
- 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 15*e^(-6*d*x - 6*c
)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^
(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*d*x - 2*c) + 10*e
^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 1
0*c) + 1)))
```

mupad [B] time = 0.14, size = 513, normalized size = 8.69

$$\frac{\frac{8\,e^{2c+2dx}\left(a^2-b^2\right)}{5d}-\frac{2\left(2ab-a^2\right)}{5d}+\frac{8\,e^{6c+6dx}\left(a^2-b^2\right)}{5d}-\frac{2\,e^{8c+8dx}\left(2ab-a^2\right)}{5d}+\frac{4\,e^{4c+4dx}\left(3a^2+2ab+4b^2\right)}{5d}}{5\,e^{2c+2dx}+10\,e^{4c+4dx}+10\,e^{6c+6dx}+5\,e^{8c+8dx}+e^{10c+10dx}+1}+\frac{\frac{2\left(3a^2+2ab+4b^2\right)}{15d}+\frac{4\,e^{2c+2dx}\cdots}{5\cdots}}{3\,e^{2c+2dx}+3\,e^{4c+\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^2*(a + b/cosh(c + d*x)^2)^2,x)
```

```
[Out] ((8*exp(2*c + 2*d*x)*(a^2 - b^2))/(5*d) - (2*(2*a*b - a^2))/(5*d) + (8*exp(
6*c + 6*d*x)*(a^2 - b^2))/(5*d) - (2*exp(8*c + 8*d*x)*(2*a*b - a^2))/(5*d)
+ (4*exp(4*c + 4*d*x)*(2*a*b + 3*a^2 + 4*b^2))/(5*d))/(5*exp(2*c + 2*d*x) +
 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c
+ 10*d*x) + 1) + ((2*(2*a*b + 3*a^2 + 4*b^2))/(15*d) + (4*exp(2*c + 2*d*x)*
(a^2 - b^2))/(5*d) - (2*exp(4*c + 4*d*x)*(2*a*b - a^2))/(5*d))/(3*exp(2*c +
 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) + ((2*(a^2 - b^2))/(5*
d) - (2*exp(2*c + 2*d*x)*(2*a*b - a^2))/(5*d))/(2*exp(2*c + 2*d*x) + exp(4*
c + 4*d*x) + 1) + ((2*(a^2 - b^2))/(5*d) + (6*exp(4*c + 4*d*x)*(a^2 - b^2))
/(5*d) - (2*exp(6*c + 6*d*x)*(2*a*b - a^2))/(5*d) + (2*exp(2*c + 2*d*x)*(2*
a*b + 3*a^2 + 4*b^2))/(5*d))/(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*e
xp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) + a^2*x - (2*(2*a*b - a^2))/(5*d*(e
xp(2*c + 2*d*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\operatorname{sech}^{2}\left(c + dx\right)\right)^{2}\tanh^{2}\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)**2*tanh(d*x+c)**2,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**2*tanh(c + d*x)**2, x)
```

3.115 $\int \left(a+b\,\mathbf{sech}^2(c+dx)\right)^2 \tanh(c+dx)\,dx$

Optimal. Leaf size=48

$$\frac{a^2\log(\cosh(c+dx))}{d}-\frac{ab\operatorname{sech}^2(c+dx)}{d}-\frac{b^2\operatorname{sech}^4(c+dx)}{4d}$$

```
[Out] a^2*ln(cosh(d*x+c))/d-a*b*sech(d*x+c)^2/d-1/4*b^2*sech(d*x+c)^4/d
```

Rubi [A] time = 0.05, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4138, 266, 43}

$$\frac{a^2\log(\cosh(c+dx))}{d}-\frac{ab\operatorname{sech}^2(c+dx)}{d}-\frac{b^2\operatorname{sech}^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^2*Tanh[c + d*x],x]
```

```
[Out] (a^2*Log[Cosh[c + d*x]])/d - (a*b*Sech[c + d*x]^2)/d - (b^2*Sech[c + d*x]^4
)/(4*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \left(a+b\text{sech}^2(c+dx)\right)^2 \tanh(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{\left(b+ax^2\right)^2}{x^5}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{(b+ax)^2}{x^3}\,dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{b^2}{x^3}+\frac{2ab}{x^2}+\frac{a^2}{x}\right) dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= \frac{a^2\log(\cosh(c+dx))}{d} - \frac{ab\text{sech}^2(c+dx)}{d} - \frac{b^2\text{sech}^4(c+dx)}{4d}$$

Mathematica [A] time = 0.14, size = 81, normalized size = 1.69

$$\frac{\text{sech}^4(c+dx)\left(a\cosh^2(c+dx)+b\right)^2\left(4a^2\cosh^4(c+dx)\log(\cosh(c+dx))-4ab\cosh^2(c+dx)-b^2\right)}{d(a\cosh(2(c+dx))+a+2b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^2*Tanh[c + d*x],x]
```

```
[Out] ((b + a*Cosh[c + d*x]^2)^2*(-b^2 - 4*a*b*Cosh[c + d*x]^2 + 4*a^2*Cosh[c + d
*x]^4*Log[Cosh[c + d*x]])*Sech[c + d*x]^4)/(d*(a + 2*b + a*Cosh[2*(c + d*x)
])^2)
```

fricas [B] time = 0.42, size = 1180, normalized size = 24.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2*tanh(d*x+c),x, algorithm="fricas")
```

```
[Out] -(a^2*d*x*cosh(d*x + c)^8 + 8*a^2*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + a^2*d
*x*sinh(d*x + c)^8 + 4*(a^2*d*x + a*b)*cosh(d*x + c)^6 + 4*(7*a^2*d*x*cosh(
d*x + c)^2 + a^2*d*x + a*b)*sinh(d*x + c)^6 + 8*(7*a^2*d*x*cosh(d*x + c)^3
+ 3*(a^2*d*x + a*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*a^2*d*x + 4*a*b +
 2*b^2)*cosh(d*x + c)^4 + 2*(35*a^2*d*x*cosh(d*x + c)^4 + 3*a^2*d*x + 30*(a
^2*d*x + a*b)*cosh(d*x + c)^2 + 4*a*b + 2*b^2)*sinh(d*x + c)^4 + a^2*d*x +
8*(7*a^2*d*x*cosh(d*x + c)^5 + 10*(a^2*d*x + a*b)*cosh(d*x + c)^3 + (3*a^2*
d*x + 4*a*b + 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a^2*d*x + a*b)*cos
h(d*x + c)^2 + 4*(7*a^2*d*x*cosh(d*x + c)^6 + 15*(a^2*d*x + a*b)*cosh(d*x +
```

```
 c)^4 + a^2*d*x + 3*(3*a^2*d*x + 4*a*b + 2*b^2)*cosh(d*x + c)^2 + a*b)*sinh
(d*x + c)^2 - (a^2*cosh(d*x + c)^8 + 8*a^2*cosh(d*x + c)*sinh(d*x + c)^7 +
a^2*sinh(d*x + c)^8 + 4*a^2*cosh(d*x + c)^6 + 4*(7*a^2*cosh(d*x + c)^2 + a^
2)*sinh(d*x + c)^6 + 6*a^2*cosh(d*x + c)^4 + 8*(7*a^2*cosh(d*x + c)^3 + 3*a
^2*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*a^2*cosh(d*x + c)^4 + 30*a^2*cosh
(d*x + c)^2 + 3*a^2)*sinh(d*x + c)^4 + 4*a^2*cosh(d*x + c)^2 + 8*(7*a^2*cos
h(d*x + c)^5 + 10*a^2*cosh(d*x + c)^3 + 3*a^2*cosh(d*x + c))*sinh(d*x + c)^
3 + 4*(7*a^2*cosh(d*x + c)^6 + 15*a^2*cosh(d*x + c)^4 + 9*a^2*cosh(d*x + c)
^2 + a^2)*sinh(d*x + c)^2 + a^2 + 8*(a^2*cosh(d*x + c)^7 + 3*a^2*cosh(d*x +
 c)^5 + 3*a^2*cosh(d*x + c)^3 + a^2*cosh(d*x + c))*sinh(d*x + c))*log(2*cos
h(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 8*(a^2*d*x*cosh(d*x + c)^7 +
3*(a^2*d*x + a*b)*cosh(d*x + c)^5 + (3*a^2*d*x + 4*a*b + 2*b^2)*cosh(d*x +
c)^3 + (a^2*d*x + a*b)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^8 + 8
*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 + 4*d*cosh(d*x + c)^6
+ 4*(7*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^6 + 8*(7*d*cosh(d*x + c)^3 + 3*
d*cosh(d*x + c))*sinh(d*x + c)^5 + 6*d*cosh(d*x + c)^4 + 2*(35*d*cosh(d*x +
 c)^4 + 30*d*cosh(d*x + c)^2 + 3*d)*sinh(d*x + c)^4 + 8*(7*d*cosh(d*x + c)^
5 + 10*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*d*cosh(d*
x + c)^2 + 4*(7*d*cosh(d*x + c)^6 + 15*d*cosh(d*x + c)^4 + 9*d*cosh(d*x + c
)^2 + d)*sinh(d*x + c)^2 + 8*(d*cosh(d*x + c)^7 + 3*d*cosh(d*x + c)^5 + 3*d
*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

giac [B] time = 0.14, size = 159, normalized size = 3.31

$$-\frac{12\,a^2dx - 12\,a^2\log\left(e^{(2\,dx+2\,c)}+1\right) + \frac{25\,a^2e^{(8\,dx+8\,c)}+100\,a^2e^{(6\,dx+6\,c)}+48\,abe^{(6\,dx+6\,c)}+150\,a^2e^{(4\,dx+4\,c)}+96\,abe^{(4\,dx+4\,c)}+48\,b^2e^{(4\,dx+4\,c)}+}{\left(e^{(2\,dx+2\,c)}+1\right)^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2*tanh(d*x+c),x, algorithm="giac")
```

```
[Out] -1/12*(12*a^2*d*x - 12*a^2*log(e^(2*d*x + 2*c) + 1) + (25*a^2*e^(8*d*x + 8*
c) + 100*a^2*e^(6*d*x + 6*c) + 48*a*b*e^(6*d*x + 6*c) + 150*a^2*e^(4*d*x +
4*c) + 96*a*b*e^(4*d*x + 4*c) + 48*b^2*e^(4*d*x + 4*c) + 100*a^2*e^(2*d*x +
 2*c) + 48*a*b*e^(2*d*x + 2*c) + 25*a^2)/(e^(2*d*x + 2*c) + 1)^4)/d
```

maple [A] time = 0.16, size = 48, normalized size = 1.00

$$-\frac{b^2\operatorname{sech}(dx+c)^4}{4d} - \frac{ab\operatorname{sech}(dx+c)^2}{d} - \frac{a^2\ln(\operatorname{sech}(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)^2*tanh(d*x+c),x)
```

```
[Out] -1/4*b^2*sech(d*x+c)^4/d-a*b*sech(d*x+c)^2/d-1/d*a^2*ln(sech(d*x+c))
```

maxima [A] time = 0.48, size = 55, normalized size = 1.15

$$\frac{ab\tanh\left(dx+c\right)^2}{d}+\frac{a^2\log\left(\cosh\left(dx+c\right)\right)}{d}-\frac{4\,b^2}{d\left(e^{(dx+c)}+e^{(-dx-c)}\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2*tanh(d*x+c),x, algorithm="maxima")
```

```
[Out] a*b*tanh(d*x + c)^2/d + a^2*log(cosh(d*x + c))/d - 4*b^2/(d*(e^(d*x + c) +
e^(-d*x - c))^4)
```

mupad [B] time = 1.69, size = 182, normalized size = 3.79

$$\frac{4\left(a\,b-b^2\right)}{d\left(2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1\right)}-a^2\,x+\frac{8\,b^2}{d\left(3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1\right)}-\frac{4\,b^2}{d\left(4\,e^{2\,c+2\,d\,x}+6\,e^{4\,c+4\,d\,x}+4\,e^{6\,c+6}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)*(a + b/cosh(c + d*x)^2)^2,x)
```

```
[Out] (4*(a*b - b^2))/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - a^2*x + (
8*b^2)/(d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1))
 - (4*b^2)/(d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x)
 + exp(8*c + 8*d*x) + 1)) + (a^2*log(exp(2*c)*exp(2*d*x) + 1))/d - (4*a*b)/
(d*(exp(2*c + 2*d*x) + 1))
```

sympy [A] time = 1.67, size = 63, normalized size = 1.31

$$\begin{cases} a^2x-\frac{a^2\log\left(\tanh\left(c+dx\right)+1\right)}{d}-\frac{ab\,\mathrm{sech}^2\left(c+dx\right)}{d}-\frac{b^2\,\mathrm{sech}^4\left(c+dx\right)}{4d} & \text{for } d\neq 0\\ x\left(a+b\,\mathrm{sech}^2\left(c\right)\right)^2\tanh\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)**2*tanh(d*x+c),x)
```

```
[Out] Piecewise((a**2*x - a**2*log(tanh(c + d*x) + 1)/d - a*b*sech(c + d*x)**2/d
- b**2*sech(c + d*x)**4/(4*d), Ne(d, 0)), (x*(a + b*sech(c)**2)**2*tanh(c),
 True))
```

3.116 $\int \left(a+b\,\mathbf{sech}^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=40

$$a^2x+\frac{b(2a+b)\tanh(c+dx)}{d}-\frac{b^2\tanh^3(c+dx)}{3d}$$

```
[Out] a^2*x+b*(2*a+b)*tanh(d*x+c)/d-1/3*b^2*tanh(d*x+c)^3/d
```

Rubi [A] time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4128, 390, 206}

$$a^2x+\frac{b(2a+b)\tanh(c+dx)}{d}-\frac{b^2\tanh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] a^2*x + (b*(2*a + b)*Tanh[c + d*x])/d - (b^2*Tanh[c + d*x]^3)/(3*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \left(a+b\text{sech}^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b-bx^2\right)^2}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(b(2a+b)-b^2x^2+\frac{a^2}{1-x^2}\right)dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{b(2a+b)\tanh(c+dx)}{d} - \frac{b^2\tanh^3(c+dx)}{3d} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= a^2x + \frac{b(2a+b)\tanh(c+dx)}{d} - \frac{b^2\tanh^3(c+dx)}{3d}$$

Mathematica [B] time = 0.38, size = 106, normalized size = 2.65

$$\frac{4\text{sech}^3(c+dx)\left(a\cosh^2(c+dx)+b\right)^2\left(3a^2dx\cosh^3(c+dx)+2b(3a+b)\text{sech}(c)\sinh(dx)\cosh^2(c+dx)+b^2\tan\ldots\right.}{3d(a\cosh(2(c+dx))+a+2b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (4*(b + a*Cosh[c + d*x]^2)^2*Sech[c + d*x]^3*(3*a^2*d*x*Cosh[c + d*x]^3 + b
^2*Sech[c]*Sinh[d*x] + 2*b*(3*a + b)*Cosh[c + d*x]^2*Sech[c]*Sinh[d*x] + b^
2*Cosh[c + d*x]*Tanh[c]))/(3*d*(a + 2*b + a*Cosh[2*(c + d*x)])^2)
```

fricas [B] time = 0.40, size = 176, normalized size = 4.40

$$\frac{\left(3\,a^2dx-6\,ab-2\,b^2\right)\cosh\left(dx+c\right)^3+3\left(3\,a^2dx-6\,ab-2\,b^2\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^2+2\left(3\,ab+b^2\right)\sinh\left(\ldots\right.}{3\left(d\cosh\left(dx+c\right)^3+3\,d\cosh\left(dx+c\right)\sin\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*((3*a^2*d*x - 6*a*b - 2*b^2)*cosh(d*x + c)^3 + 3*(3*a^2*d*x - 6*a*b - 2
*b^2)*cosh(d*x + c)*sinh(d*x + c)^2 + 2*(3*a*b + b^2)*sinh(d*x + c)^3 + 3*(
3*a^2*d*x - 6*a*b - 2*b^2)*cosh(d*x + c) + 6*((3*a*b + b^2)*cosh(d*x + c)^2
 + a*b + b^2)*sinh(d*x + c))/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)*sinh(d*
x + c)^2 + 3*d*cosh(d*x + c))
```

giac [B] time = 0.14, size = 79, normalized size = 1.98

$$\frac{3\,(dx+c)a^2 - \frac{4\left(3\,abe^{(4\,dx+4\,c)}+6\,abe^{(2\,dx+2\,c)}+3\,b^2e^{(2\,dx+2\,c)}+3\,ab+b^2\right)}{\left(e^{(2\,dx+2\,c)}+1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*(d*x + c)*a^2 - 4*(3*a*b*e^(4*d*x + 4*c) + 6*a*b*e^(2*d*x + 2*c) + 3
*b^2*e^(2*d*x + 2*c) + 3*a*b + b^2)/(e^(2*d*x + 2*c) + 1)^3)/d
```

maple [A] time = 0.36, size = 47, normalized size = 1.18

$$\frac{a^2\,(dx+c) + 2ab\tanh(dx+c) + b^2\left(\frac{2}{3} + \frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(a^2*(d*x+c)+2*a*b*tanh(d*x+c)+b^2*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c))
```

maxima [B] time = 0.46, size = 120, normalized size = 3.00

$$a^2x+\frac{4}{3}\,b^2\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d\left(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1\right)}+\frac{1}{d\left(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1\right)}\right)+\frac{\cdots}{d\left(e^{(-\cdots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] a^2*x + 4/3*b^2*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x -
4*c) + e^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*
c) + e^(-6*d*x - 6*c) + 1))) + 4*a*b/(d*(e^(-2*d*x - 2*c) + 1))
```

mupad [B] time = 1.46, size = 163, normalized size = 4.08

$$a^2\,x-\frac{\frac{4\left(b^2+a\,b\right)}{3\,d}+\frac{4\,a\,b\,\mathrm{e}^{2\,c+2\,d\,x}}{3\,d}}{2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1}-\frac{\frac{8\,\mathrm{e}^{2\,c+2\,d\,x}\left(b^2+a\,b\right)}{3\,d}+\frac{4\,a\,b}{3\,d}+\frac{4\,a\,b\,\mathrm{e}^{4\,c+4\,d\,x}}{3\,d}}{3\,\mathrm{e}^{2\,c+2\,d\,x}+3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}+1}-\frac{4\,a\,b}{3\,d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cosh(c + d*x)^2)^2,x)
```

```
[Out] a^2*x - ((4*(a*b + b^2))/(3*d) + (4*a*b*exp(2*c + 2*d*x))/(3*d))/(2*exp(2*c
 + 2*d*x) + exp(4*c + 4*d*x) + 1) - ((8*exp(2*c + 2*d*x)*(a*b + b^2))/(3*d)
 + (4*a*b)/(3*d) + (4*a*b*exp(4*c + 4*d*x))/(3*d))/(3*exp(2*c + 2*d*x) + 3*
exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) - (4*a*b)/(3*d*(exp(2*c + 2*d*x) +
 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\,\mathrm{sech}^2\left(c + dx\right)\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**2, x)
```

3.117 $\int \coth(c+dx)\left(a+b\,\mathbf{sech}^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=53

$$\frac{(a+b)^2\log(\sinh(c+dx))}{d}-\frac{b(2a+b)\log(\cosh(c+dx))}{d}+\frac{b^2\text{sech}^2(c+dx)}{2d}$$

```
[Out] -b*(2*a+b)*ln(cosh(d*x+c))/d+(a+b)^2*ln(sinh(d*x+c))/d+1/2*b^2*sech(d*x+c)^
2/d
```

Rubi [A] time = 0.08, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4138, 446, 88}

$$\frac{(a+b)^2\log(\sinh(c+dx))}{d}-\frac{b(2a+b)\log(\cosh(c+dx))}{d}+\frac{b^2\text{sech}^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] -((b*(2*a + b)*Log[Cosh[c + d*x]])/d) + ((a + b)^2*Log[Sinh[c + d*x]])/d +
(b^2*Sech[c + d*x]^2)/(2*d)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \coth(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2 dx = -\frac{\text{Subst}\left(\int \frac{\left(b+ax^2\right)^2}{x^3\left(1-x^2\right)} dx, x, \cosh(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \frac{(b+ax)^2}{(1-x)x^2} dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= -\frac{\text{Subst}\left(\int \left(-\frac{(a+b)^2}{-1+x}+\frac{b^2}{x^2}+\frac{b(2a+b)}{x}\right) dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= -\frac{b(2a+b)\log(\cosh(c+dx))}{d}+\frac{(a+b)^2\log(\sinh(c+dx))}{d}+\frac{b^2\text{sech}}{}$$

Mathematica [A] time = 0.25, size = 84, normalized size = 1.58

$$\frac{2(a\cosh(c+dx)+b\text{sech}(c+dx))^2\left(2\cosh^2(c+dx)\left((a+b)^2\log(\sinh(c+dx))-b(2a+b)\log(\cosh(c+dx))\right)+\right.}{d(a\cosh(2(c+dx))+a+2b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (2*(b^2 + 2*Cosh[c + d*x]^2*(-(b*(2*a + b)*Log[Cosh[c + d*x]]) + (a + b)^2*
Log[Sinh[c + d*x]]))*(a*Cosh[c + d*x] + b*Sech[c + d*x])^2)/(d*(a + 2*b + a
*Cosh[2*(c + d*x)])^2)
```

fricas [B] time = 0.46, size = 665, normalized size = 12.55

$$-\frac{a^2dx\cosh(dx+c)^4+4a^2dx\cosh(dx+c)\sinh(dx+c)^3+a^2dx\sinh(dx+c)^4+a^2dx+2\left(a^2dx-b^2\right)\cosh(dx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -(a^2*d*x*cosh(d*x + c)^4 + 4*a^2*d*x*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*d
*x*sinh(d*x + c)^4 + a^2*d*x + 2*(a^2*d*x - b^2)*cosh(d*x + c)^2 + 2*(3*a^2
*d*x*cosh(d*x + c)^2 + a^2*d*x - b^2)*sinh(d*x + c)^2 + ((2*a*b + b^2)*cosh
(d*x + c)^4 + 4*(2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a*b + b^2)
*sinh(d*x + c)^4 + 2*(2*a*b + b^2)*cosh(d*x + c)^2 + 2*(3*(2*a*b + b^2)*cos
h(d*x + c)^2 + 2*a*b + b^2)*sinh(d*x + c)^2 + 2*a*b + b^2 + 4*((2*a*b + b^2
```

```
)*cosh(d*x + c)^3 + (2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(
d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) - ((a^2 + 2*a*b + b^2)*cosh(d*x +
 c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b
+ b^2)*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 2*(3*(a^2
+ 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b + b^2)*sinh(d*x + c)^2 + a^2 +
 2*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 + 2*a*b + b^2)
*cosh(d*x + c))*sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*
x + c))) + 4*(a^2*d*x*cosh(d*x + c)^3 + (a^2*d*x - b^2)*cosh(d*x + c))*sinh
(d*x + c))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x + c)^3 + d*sinh(
d*x + c)^4 + 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 + d)*sinh(d*x + c
)^2 + 4*(d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

giac [B] time = 0.17, size = 171, normalized size = 3.23

$$-\frac{2\,a^2dx+2\left(2\,abe^{(2\,c)}+b^2e^{(2\,c)}\right)e^{(-2\,c)}\log\left(e^{(2\,dx+2\,c)}+1\right)-2\left(a^2e^{(2\,c)}+2\,abe^{(2\,c)}+b^2e^{(2\,c)}\right)e^{(-2\,c)}\log\left(\left|e^{(2\,dx+2\,c)}-\right.\right.}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*a^2*d*x + 2*(2*a*b*e^(2*c) + b^2*e^(2*c))*e^(-2*c)*log(e^(2*d*x + 2
*c) + 1) - 2*(a^2*e^(2*c) + 2*a*b*e^(2*c) + b^2*e^(2*c))*e^(-2*c)*log(abs(e
^(2*d*x + 2*c) - 1)) - (6*a*b*e^(4*d*x + 4*c) + 3*b^2*e^(4*d*x + 4*c) + 12*
a*b*e^(2*d*x + 2*c) + 10*b^2*e^(2*d*x + 2*c) + 6*a*b + 3*b^2)/(e^(2*d*x + 2
*c) + 1)^2)/d
```

maple [A] time = 0.26, size = 60, normalized size = 1.13

$$\frac{a^2\ln\left(\sinh\left(dx+c\right)\right)}{d}+\frac{2ab\ln\left(\tanh\left(dx+c\right)\right)}{d}+\frac{b^2}{2d\cosh\left(dx+c\right)^2}+\frac{b^2\ln\left(\tanh\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)*(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] a^2*ln(sinh(d*x+c))/d+2*a*b*ln(tanh(d*x+c))/d+1/2/d*b^2/cosh(d*x+c)^2+1/d*b
^2*ln(tanh(d*x+c))
```

maxima [B] time = 0.70, size = 161, normalized size = 3.04

$$b^2\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}-\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d\left(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1\right)}\right)+2\,ab\left(\frac{\log\left(e^{(\cdot}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] b^2*(log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d - log(e^(-2*d*x - 2*
c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) +
1))) + 2*a*b*(log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d - log(e^(-2
*d*x - 2*c) + 1)/d) + a^2*log(sinh(d*x + c))/d
```

mupad [B] time = 0.30, size = 308, normalized size = 5.81

$$\frac{2\,b^2}{d\,\left(e^{2\,c+2\,d\,x}+1\right)}-a^2\,x+\frac{a^2\,\ln\left(e^{4\,c+4\,d\,x}-1\right)}{2\,d}-\frac{2\,b^2}{d\,\left(2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1\right)}-\frac{\mathrm{atan}\left(\frac{e^{2\,c}\,e^{2\,d\,x}\,\left(a^4\,\sqrt{-d^2}\right.}{a^2\,d\,\sqrt{a^4+8\,a^3\,b+20\,a^2\,b^2+16\,a\,b^3+4\,b^4}+2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)*(a + b/cosh(c + d*x)^2)^2,x)
```

```
[Out] (2*b^2)/(d*(exp(2*c + 2*d*x) + 1)) - a^2*x + (a^2*log(exp(4*c + 4*d*x) - 1)
)/(2*d) - (2*b^2)/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (atan((
exp(2*c)*exp(2*d*x)*(a^4*(-d^2)^(1/2) + 4*b^4*(-d^2)^(1/2) + 16*a*b^3*(-d^2
)^(1/2) + 8*a^3*b*(-d^2)^(1/2) + 20*a^2*b^2*(-d^2)^(1/2)))/(a^2*d*(16*a*b^3
 + 8*a^3*b + a^4 + 4*b^4 + 20*a^2*b^2)^(1/2) + 2*b^2*d*(16*a*b^3 + 8*a^3*b
+ a^4 + 4*b^4 + 20*a^2*b^2)^(1/2) + 4*a*b*d*(16*a*b^3 + 8*a^3*b + a^4 + 4*b
^4 + 20*a^2*b^2)^(1/2)))*(16*a*b^3 + 8*a^3*b + a^4 + 4*b^4 + 20*a^2*b^2)^(1
/2))/(-d^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\,\mathrm{sech}^2\left(c + dx\right)\right)^2 \coth\left(c + dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**2*coth(c + d*x), x)
```

3.118 $\int \coth^2(c+dx)\left(a+b\,\mathbf{sech}^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=36

$$a^2x-\frac{(a+b)^2\coth(c+dx)}{d}-\frac{b^2\tanh(c+dx)}{d}$$

```
[Out] a^2*x-(a+b)^2*coth(d*x+c)/d-b^2*tanh(d*x+c)/d
```

Rubi [A] time = 0.09, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4141, 1802, 207}

$$a^2x-\frac{(a+b)^2\coth(c+dx)}{d}-\frac{b^2\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^2*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] a^2*x - ((a + b)^2*Coth[c + d*x])/d - (b^2*Tanh[c + d*x])/d
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \coth^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2 dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\left(1-x^2\right)\right)^2}{x^2\left(1-x^2\right)}dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-b^2+\frac{(a+b)^2}{x^2}-\frac{a^2}{-1+x^2}\right)dx, x, \tanh(c+dx)\right)}{d}$$
$$= -\frac{(a+b)^2\coth(c+dx)}{d}-\frac{b^2\tanh(c+dx)}{d}-\frac{a^2\,\text{Subst}\left(\int \frac{1}{-1+x^2}dx, x\right.}{d}$$
$$= a^2x-\frac{(a+b)^2\coth(c+dx)}{d}-\frac{b^2\tanh(c+dx)}{d}$$

Mathematica [B] time = 0.74, size = 82, normalized size = 2.28

$$\frac{4\operatorname{sech}(c+dx)\left(a\cosh^2(c+dx)+b\right)^2\left(a^2dx\cosh(c+dx)+\sinh(dx)\left((a+b)^2\operatorname{csch}(c)\coth(c+dx)-b^2\operatorname{sech}(c)\right)\right)}{d(a\cosh(2(c+dx))+a+2b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^2*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (4*(b + a*Cosh[c + d*x]^2)^2*Sech[c + d*x]*(a^2*d*x*Cosh[c + d*x] + ((a + b
)^2*Coth[c + d*x]*Csch[c] - b^2*Sech[c])*Sinh[d*x]))/(d*(a + 2*b + a*Cosh[2
*(c + d*x)])^2)
```

fricas [B] time = 0.44, size = 106, normalized size = 2.94

$$-\frac{\left(a^2+2\,ab+2\,b^2\right)\cosh\left(dx+c\right)^2-2\left(a^2dx+a^2+2\,ab+2\,b^2\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right)+\left(a^2+2\,ab+2\,b^2\right)\text{si}}{2\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*((a^2 + 2*a*b + 2*b^2)*cosh(d*x + c)^2 - 2*(a^2*d*x + a^2 + 2*a*b + 2*
b^2)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + 2*a*b + 2*b^2)*sinh(d*x + c)^2 +
a^2 + 2*a*b)/(d*cosh(d*x + c)*sinh(d*x + c))
```

giac [A] time = 0.20, size = 65, normalized size = 1.81

$$\frac{a^2dx-\frac{2\left(a^2e^{(2\,dx+2\,c)}+2\,abe^{(2\,dx+2\,c)}+a^2+2\,ab+2\,b^2\right)}{e^{(4\,dx+4\,c)}-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] (a^2*d*x - 2*(a^2*e^(2*d*x + 2*c) + 2*a*b*e^(2*d*x + 2*c) + a^2 + 2*a*b + 2
*b^2)/(e^(4*d*x + 4*c) - 1))/d
```

maple [A] time = 0.39, size = 64, normalized size = 1.78

$$\frac{a^2(dx+c-\coth(dx+c)) - 2ab\coth(dx+c) + b^2\left(-\frac{1}{\sinh(dx+c)\cosh(dx+c)} - 2\tanh(dx+c)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(a^2*(d*x+c-coth(d*x+c))-2*a*b*coth(d*x+c)+b^2*(-1/sinh(d*x+c)/cosh(d*x
+c)-2*tanh(d*x+c)))
```

maxima [A] time = 0.55, size = 71, normalized size = 1.97

$$a^2\left(x+\frac{c}{d}+\frac{2}{d\left(e^{(-2dx-2c)}-1\right)}\right)+\frac{4ab}{d\left(e^{(-2dx-2c)}-1\right)}+\frac{4b^2}{d\left(e^{(-4dx-4c)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] a^2*(x + c/d + 2/(d*(e^(-2*d*x - 2*c) - 1))) + 4*a*b/(d*(e^(-2*d*x - 2*c) -
 1)) + 4*b^2/(d*(e^(-4*d*x - 4*c) - 1))
```

mupad [B] time = 1.42, size = 60, normalized size = 1.67

$$a^2\,x-\frac{\frac{2\left(a^2+2\,a\,b+2\,b^2\right)}{d}+\frac{2\,a\,\mathrm{e}^{2\,c+2\,d\,x}\,\left(a+2\,b\right)}{d}}{\mathrm{e}^{4\,c+4\,d\,x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^2*(a + b/cosh(c + d*x)^2)^2,x)
```

```
[Out] a^2*x - ((2*(2*a*b + a^2 + 2*b^2))/d + (2*a*exp(2*c + 2*d*x)*(a + 2*b))/d)/
(exp(4*c + 4*d*x) - 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\operatorname{sech}^2(c+dx)\right)^2\coth^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**2*(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**2*coth(c + d*x)**2, x)
```

3.119 $\int \coth^3(c+dx)\left(a+b\,\mathbf{sech}^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=55

$$\frac{(a^2-b^2)\log(\sinh(c+dx))}{d}-\frac{(a+b)^2\operatorname{csch}^2(c+dx)}{2d}+\frac{b^2\log(\cosh(c+dx))}{d}$$

```
[Out] -1/2*(a+b)^2*csch(d*x+c)^2/d+b^2*ln(cosh(d*x+c))/d+(a^2-b^2)*ln(sinh(d*x+c)
)/d
```

Rubi [A] time = 0.09, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4138, 446, 88}

$$\frac{(a^2-b^2)\log(\sinh(c+dx))}{d}-\frac{(a+b)^2\operatorname{csch}^2(c+dx)}{2d}+\frac{b^2\log(\cosh(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^3*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] -((a + b)^2*Csch[c + d*x]^2)/(2*d) + (b^2*Log[Cosh[c + d*x]])/d + ((a^2 - b
^2)*Log[Sinh[c + d*x]])/d
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
```

```
x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \coth^3(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2\,dx &= \frac{\text{Subst}\left(\int \frac{\left(b+ax^2\right)^2}{x\left(1-x^2\right)^2}\,dx,x,\cosh(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{(b+ax)^2}{(1-x)^2x}\,dx,x,\cosh^2(c+dx)\right)}{2d}\\
&= \frac{\text{Subst}\left(\int \left(\frac{(a+b)^2}{(-1+x)^2}+\frac{a^2-b^2}{-1+x}+\frac{b^2}{x}\right)dx,x,\cosh^2(c+dx)\right)}{2d}\\
&= -\frac{(a+b)^2\text{csch}^2(c+dx)}{2d}+\frac{b^2\log(\cosh(c+dx))}{d}+\frac{\left(a^2-b^2\right)\log(\sin}{d}
\end{aligned}$$

Mathematica [A] time = 0.20, size = 82, normalized size = 1.49

$$-\frac{2\left(a\cosh^2(c+dx)+b\right)^2\left((a+b)^2\text{csch}^2(c+dx)-2\left(\left(a^2-b^2\right)\log(\sinh(c+dx))+b^2\log(\cosh(c+dx))\right)\right)}{d(a\cosh(2(c+dx))+a+2b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^3*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (-2*(b + a*Cosh[c + d*x]^2)^2*((a + b)^2*Csch[c + d*x]^2 - 2*(b^2*Log[Cosh[
c + d*x]] + (a^2 - b^2)*Log[Sinh[c + d*x]])))/(d*(a + 2*b + a*Cosh[2*(c + d
*x)])^2)
```

fricas [B] time = 0.45, size = 637, normalized size = 11.58

$$-\frac{a^2dx\cosh(dx+c)^4+4\,a^2dx\cosh(dx+c)\sinh(dx+c)^3+a^2dx\sinh(dx+c)^4+a^2dx-2\left(a^2dx-a^2-2\,ab-b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -(a^2*d*x*cosh(d*x + c)^4 + 4*a^2*d*x*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*d
*x*sinh(d*x + c)^4 + a^2*d*x - 2*(a^2*d*x - a^2 - 2*a*b - b^2)*cosh(d*x + c
)^2 + 2*(3*a^2*d*x*cosh(d*x + c)^2 - a^2*d*x + a^2 + 2*a*b + b^2)*sinh(d*x
```

```
+ c)^2 - (b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*s
inh(d*x + c)^4 - 2*b^2*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 - b^2)*si
nh(d*x + c)^2 + b^2 + 4*(b^2*cosh(d*x + c)^3 - b^2*cosh(d*x + c))*sinh(d*x
+ c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) - ((a^2 - b^2)*c
osh(d*x + c)^4 + 4*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 - b^2)*
sinh(d*x + c)^4 - 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 - b^2)*cosh(d*x
 + c)^2 - a^2 + b^2)*sinh(d*x + c)^2 + a^2 - b^2 + 4*((a^2 - b^2)*cosh(d*x
+ c)^3 - (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c))*log(2*sinh(d*x + c)/(cos
h(d*x + c) - sinh(d*x + c))) + 4*(a^2*d*x*cosh(d*x + c)^3 - (a^2*d*x - a^2
- 2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^4 + 4*d*cosh(
d*x + c)*sinh(d*x + c)^3 + d*sinh(d*x + c)^4 - 2*d*cosh(d*x + c)^2 + 2*(3*d
*cosh(d*x + c)^2 - d)*sinh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 - d*cosh(d*x +
 c))*sinh(d*x + c) + d)
```

giac [B] time = 0.25, size = 161, normalized size = 2.93

$$-\frac{2\,a^2dx - 2\,b^2\log\left(e^{(2\,dx+2\,c)}+1\right) - 2\left(a^2e^{(2\,c)} - b^2e^{(2\,c)}\right)e^{(-2\,c)}\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right) + \frac{3\,a^2e^{(4\,dx+4\,c)} - 3\,b^2e^{(4\,dx+4\,c)} - 2\,a^2e^{(2\,dx}}{(e^{(2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*a^2*d*x - 2*b^2*log(e^(2*d*x + 2*c) + 1) - 2*(a^2*e^(2*c) - b^2*e^(
2*c))*e^(-2*c)*log(abs(e^(2*d*x + 2*c) - 1)) + (3*a^2*e^(4*d*x + 4*c) - 3*b
^2*e^(4*d*x + 4*c) - 2*a^2*e^(2*d*x + 2*c) + 8*a*b*e^(2*d*x + 2*c) + 10*b^2
*e^(2*d*x + 2*c) + 3*a^2 - 3*b^2)/(e^(2*d*x + 2*c) - 1)^2)/d
```

maple [A] time = 0.31, size = 78, normalized size = 1.42

$$\frac{a^2\ln(\sinh(dx+c))}{d} - \frac{a^2\left(\coth^2(dx+c)\right)}{2d} - \frac{ab}{d\sinh(dx+c)^2} - \frac{b^2}{2d\sinh(dx+c)^2} - \frac{b^2\ln(\tanh(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] a^2*ln(sinh(d*x+c))/d-1/2*a^2*coth(d*x+c)^2/d-1/d/sinh(d*x+c)^2*a*b-1/2/d*b
^2/sinh(d*x+c)^2-1/d*b^2*ln(tanh(d*x+c))
```

maxima [B] time = 0.48, size = 206, normalized size = 3.75

$$a^2\left(x + \frac{c}{d} + \frac{\log\left(e^{(-dx-c)}+1\right)}{d} + \frac{\log\left(e^{(-dx-c)}-1\right)}{d} + \frac{2\,e^{(-2\,dx-2\,c)}}{d\left(2\,e^{(-2\,dx-2\,c)} - e^{(-4\,dx-4\,c)} - 1\right)}\right) - b^2\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d} + \frac{\log}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] a^2*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 2*e^(-2*
d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) - b^2*(log(e^(-
d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d - log(e^(-2*d*x - 2*c) + 1)/d - 2
*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) - 4*a*b/
(d*(e^(d*x + c) - e^(-d*x - c))^2)
```

mupad [B] time = 0.22, size = 240, normalized size = 4.36

$$\frac{\ln\left(e^{4c+4dx}-1\right)\left(d\left(a^2-b^2\right)+b^2 d\right)}{2d^2}-a^2 x-\frac{2\left(a^2+2ab+b^2\right)}{d\left(e^{2c+2dx}-1\right)}-\frac{\operatorname{atan}\left(\frac{e^{2c}\,e^{2dx}\left(a^4\sqrt{-d^2}+4b^4\sqrt{-d^2}-4a^2b^2\sqrt{-d^2}\right)}{a^2 d\sqrt{a^4-4a^2b^2+4b^4}-2b^2 d\sqrt{a^4-4a^2b^2+4b^4}}\right)\sqrt{a^4}}{\sqrt{-d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^3*(a + b/cosh(c + d*x)^2)^2,x)
```

```
[Out] (log(exp(4*c + 4*d*x) - 1)*(d*(a^2 - b^2) + b^2*d))/(2*d^2) - a^2*x - (2*(2
*a*b + a^2 + b^2))/(d*(exp(2*c + 2*d*x) - 1)) - (atan((exp(2*c)*exp(2*d*x)*
(a^4*(-d^2)^(1/2) + 4*b^4*(-d^2)^(1/2) - 4*a^2*b^2*(-d^2)^(1/2)))/(a^2*d*(a
^4 + 4*b^4 - 4*a^2*b^2)^(1/2) - 2*b^2*d*(a^4 + 4*b^4 - 4*a^2*b^2)^(1/2)))*(
a^4 + 4*b^4 - 4*a^2*b^2)^(1/2))/(-d^2)^(1/2) - (2*(2*a*b + a^2 + b^2))/(d*(
exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\operatorname{sech}^2(c+dx)\right)^2 \coth^3(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**3*(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**2*coth(c + d*x)**3, x)
```

3.120 $\int \coth^4(c+dx)\left(a+b\,\mathbf{sech}^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=46

$$-\frac{\left(a^2-b^2\right)\coth(c+dx)}{d}+a^2x-\frac{(a+b)^2\coth^3(c+dx)}{3d}$$

```
[Out] a^2*x-(a^2-b^2)*coth(d*x+c)/d-1/3*(a+b)^2*coth(d*x+c)^3/d
```

Rubi [A] time = 0.09, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4141, 1802, 207}

$$-\frac{\left(a^2-b^2\right)\coth(c+dx)}{d}+a^2x-\frac{(a+b)^2\coth^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^4*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] a^2*x - ((a^2 - b^2)*Coth[c + d*x])/d - ((a + b)^2*Coth[c + d*x]^3)/(3*d)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \coth^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx = \frac{\operatorname{Subst}\left(\int \frac{\left(a+b\left(1-x^2\right)\right)^2}{x^4\left(1-x^2\right)}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\operatorname{Subst}\left(\int \left(\frac{(a+b)^2}{x^4}+\frac{a^2-b^2}{x^2}-\frac{a^2}{-1+x^2}\right)dx, x, \tanh(c+dx)\right)}{d}$$
$$= -\frac{\left(a^2-b^2\right)\coth(c+dx)}{d}-\frac{(a+b)^2\coth^3(c+dx)}{3d}-\frac{a^2\operatorname{Subst}\left(\int \frac{1}{-1+\dots}\right.}{}$$
$$= a^2x-\frac{\left(a^2-b^2\right)\coth(c+dx)}{d}-\frac{(a+b)^2\coth^3(c+dx)}{3d}$$

Mathematica [B] time = 0.81, size = 160, normalized size = 3.48

$$\frac{\operatorname{csch}(c)\operatorname{csch}^3(c+dx)\left(-12a^2\sinh(2c+dx)+8a^2\sinh(2c+3dx)-9a^2dx\cosh(2c+dx)-3a^2dx\cosh(2c+3dx)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^4*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (Csch[c]*Csch[c + d*x]^3*(9*a^2*d*x*Cosh[d*x] - 9*a^2*d*x*Cosh[2*c + d*x] -
 3*a^2*d*x*Cosh[2*c + 3*d*x] + 3*a^2*d*x*Cosh[4*c + 3*d*x] - 12*a^2*Sinh[d*
x] + 12*b^2*Sinh[d*x] - 12*a^2*Sinh[2*c + d*x] - 12*a*b*Sinh[2*c + d*x] + 8
*a^2*Sinh[2*c + 3*d*x] + 4*a*b*Sinh[2*c + 3*d*x] - 4*b^2*Sinh[2*c + 3*d*x])
)/(24*d)
```

fricas [B] time = 0.43, size = 201, normalized size = 4.37

$$-\frac{2\left(2a^2+ab-b^2\right)\cosh(dx+c)^3+6\left(2a^2+ab-b^2\right)\cosh(dx+c)\sinh(dx+c)^2-\left(3a^2dx+4a^2+2ab-2b^2\right)}{3\left(d\sinh(dx+c)^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(2*(2*a^2 + a*b - b^2)*cosh(d*x + c)^3 + 6*(2*a^2 + a*b - b^2)*cosh(d*
x + c)*sinh(d*x + c)^2 - (3*a^2*d*x + 4*a^2 + 2*a*b - 2*b^2)*sinh(d*x + c)^
3 + 6*(a*b + b^2)*cosh(d*x + c) + 3*(3*a^2*d*x - (3*a^2*d*x + 4*a^2 + 2*a*b
 - 2*b^2)*cosh(d*x + c)^2 + 4*a^2 + 2*a*b - 2*b^2)*sinh(d*x + c))/(d*sinh(d
*x + c)^3 + 3*(d*cosh(d*x + c)^2 - d)*sinh(d*x + c))
```

giac [B] time = 0.27, size = 97, normalized size = 2.11

$$\frac{3\,a^2dx-\frac{4\left(3\,a^2e^{(4\,dx+4\,c)}+3\,abe^{(4\,dx+4\,c)}-3\,a^2e^{(2\,dx+2\,c)}+3\,b^2e^{(2\,dx+2\,c)}+2\,a^2+ab-b^2\right)}{\left(e^{(2\,dx+2\,c)}-1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*a^2*d*x - 4*(3*a^2*e^(4*d*x + 4*c) + 3*a*b*e^(4*d*x + 4*c) - 3*a^2*e
^(2*d*x + 2*c) + 3*b^2*e^(2*d*x + 2*c) + 2*a^2 + a*b - b^2)/(e^(2*d*x + 2*c
) - 1)^3)/d
```

maple [B] time = 0.36, size = 96, normalized size = 2.09

$$\frac{a^2\left(dx+c-\coth\left(dx+c\right)-\frac{\left(\coth^3(dx+c)\right)}{3}\right)+2ab\left(-\frac{\cosh(dx+c)}{2\sinh(dx+c)^3}-\frac{\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^2}{3}\right)\coth(dx+c)}{2}\right)+b^2\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^2}{3}\right)\mathrm{cot}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(a^2*(d*x+c-coth(d*x+c)-1/3*coth(d*x+c)^3)+2*a*b*(-1/2/sinh(d*x+c)^3*co
sh(d*x+c)-1/2*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c))+b^2*(2/3-1/3*csch(d*x+c)
^2)*coth(d*x+c))
```

maxima [B] time = 0.43, size = 268, normalized size = 5.83

$$\frac{1}{3}\,a^2\left(3\,x+\frac{3\,c}{d}-\frac{4\left(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}-2\right)}{d\left(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1\right)}\right)+\frac{4}{3}\,b^2\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d\left(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) - 2)/(d*(
3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1))) + 4/3*b^2
*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*
x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x
- 6*c) - 1))) + 4/3*a*b*(3*e^(-4*d*x - 4*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-
4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)) + 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*
d*x - 4*c) + e^(-6*d*x - 6*c) - 1)))
```

mupad [B] time = 1.42, size = 183, normalized size = 3.98

$$a^2\,x-\frac{\frac{4\,\left(a^2+b\,a\right)}{3\,d}+\frac{4\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(a^2+b\,a\right)}{3\,d}+\frac{8\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(b^2+a\,b\right)}{3\,d}}{3\,{\mathrm{e}}^{2\,c+2\,d\,x}-3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}-1}-\frac{\frac{4\,\left(b^2+a\,b\right)}{3\,d}+\frac{4\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(a^2+b\,a\right)}{3\,d}}{{\mathrm{e}}^{4\,c+4\,d\,x}-2\,{\mathrm{e}}^{2\,c+2\,d\,x}+1}-\frac{4\,\left(a^2+b\,a\right)}{3\,d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^4*(a + b/cosh(c + d*x)^2)^2,x)
```

```
[Out] a^2*x - ((4*(a*b + a^2))/(3*d) + (4*exp(4*c + 4*d*x)*(a*b + a^2))/(3*d) + (
8*exp(2*c + 2*d*x)*(a*b + b^2))/(3*d))/(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*
d*x) + exp(6*c + 6*d*x) - 1) - ((4*(a*b + b^2))/(3*d) + (4*exp(2*c + 2*d*x)
*(a*b + a^2))/(3*d))/(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1) - (4*(a*b
+ a^2))/(3*d*(exp(2*c + 2*d*x) - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\operatorname{sech}^{2}{\left(c + dx \right)}\right)^{2}\coth^{4}{\left(c + dx \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**4*(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**2*coth(c + d*x)**4, x)
```

3.121 $\int \coth^5(c+dx)\left(a+b\,\mathbf{sech}^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=52

$$\frac{a^2\log(\sinh(c+dx))}{d}-\frac{(a+b)^2\text{csch}^4(c+dx)}{4d}-\frac{a(a+b)\text{csch}^2(c+dx)}{d}$$

```
[Out] -a*(a+b)*csch(d*x+c)^2/d-1/4*(a+b)^2*csch(d*x+c)^4/d+a^2*ln(sinh(d*x+c))/d
```

Rubi [A] time = 0.09, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4138, 444, 43}

$$\frac{a^2\log(\sinh(c+dx))}{d}-\frac{(a+b)^2\text{csch}^4(c+dx)}{4d}-\frac{a(a+b)\text{csch}^2(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^5*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] -((a*(a + b)*Csch[c + d*x]^2)/d) - ((a + b)^2*Csch[c + d*x]^4)/(4*d) + (a^2
*Log[Sinh[c + d*x]])/d
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \coth^5(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx = -\frac{\operatorname{Subst}\left(\int \frac{x\left(b+ax^2\right)^2}{\left(1-x^2\right)^3}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= -\frac{\operatorname{Subst}\left(\int \frac{(b+ax)^2}{(1-x)^3}\,dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= -\frac{\operatorname{Subst}\left(\int \left(-\frac{(a+b)^2}{(-1+x)^3}-\frac{2a(a+b)}{(-1+x)^2}-\frac{a^2}{-1+x}\right)dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= -\frac{a(a+b)\operatorname{csch}^2(c+dx)}{d}-\frac{(a+b)^2\operatorname{csch}^4(c+dx)}{4d}+\frac{a^2\log(\sinh(c+}{d}$$

Mathematica [A] time = 0.23, size = 77, normalized size = 1.48

$$-\frac{\left(a\cosh^2(c+dx)+b\right)^2\left(-4a^2\log(\sinh(c+dx))+(a+b)^2\operatorname{csch}^4(c+dx)+4a(a+b)\operatorname{csch}^2(c+dx)\right)}{d(a\cosh(2(c+dx))+a+2b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^5*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] -(((b + a*Cosh[c + d*x]^2)^2*(4*a*(a + b)*Csch[c + d*x]^2 + (a + b)^2*Csch[
c + d*x]^4 - 4*a^2*Log[Sinh[c + d*x]]))/(d*(a + 2*b + a*Cosh[2*(c + d*x)])^
2))
```

fricas [B] time = 0.44, size = 1252, normalized size = 24.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^5*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -(a^2*d*x*cosh(d*x + c)^8 + 8*a^2*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + a^2*d
*x*sinh(d*x + c)^8 - 4*(a^2*d*x - a^2 - a*b)*cosh(d*x + c)^6 + 4*(7*a^2*d*x
*cosh(d*x + c)^2 - a^2*d*x + a^2 + a*b)*sinh(d*x + c)^6 + 8*(7*a^2*d*x*cosh
(d*x + c)^3 - 3*(a^2*d*x - a^2 - a*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3
*a^2*d*x - 2*a^2 + 2*b^2)*cosh(d*x + c)^4 + 2*(35*a^2*d*x*cosh(d*x + c)^4 +
 3*a^2*d*x - 30*(a^2*d*x - a^2 - a*b)*cosh(d*x + c)^2 - 2*a^2 + 2*b^2)*sinh
(d*x + c)^4 + a^2*d*x + 8*(7*a^2*d*x*cosh(d*x + c)^5 - 10*(a^2*d*x - a^2 -
a*b)*cosh(d*x + c)^3 + (3*a^2*d*x - 2*a^2 + 2*b^2)*cosh(d*x + c))*sinh(d*x
```

```
+ c)^3 - 4*(a^2*d*x - a^2 - a*b)*cosh(d*x + c)^2 + 4*(7*a^2*d*x*cosh(d*x +
c)^6 - 15*(a^2*d*x - a^2 - a*b)*cosh(d*x + c)^4 - a^2*d*x + 3*(3*a^2*d*x -
2*a^2 + 2*b^2)*cosh(d*x + c)^2 + a^2 + a*b)*sinh(d*x + c)^2 - (a^2*cosh(d*x
 + c)^8 + 8*a^2*cosh(d*x + c)*sinh(d*x + c)^7 + a^2*sinh(d*x + c)^8 - 4*a^2
*cosh(d*x + c)^6 + 4*(7*a^2*cosh(d*x + c)^2 - a^2)*sinh(d*x + c)^6 + 6*a^2*
cosh(d*x + c)^4 + 8*(7*a^2*cosh(d*x + c)^3 - 3*a^2*cosh(d*x + c))*sinh(d*x
+ c)^5 + 2*(35*a^2*cosh(d*x + c)^4 - 30*a^2*cosh(d*x + c)^2 + 3*a^2)*sinh(d
*x + c)^4 - 4*a^2*cosh(d*x + c)^2 + 8*(7*a^2*cosh(d*x + c)^5 - 10*a^2*cosh(
d*x + c)^3 + 3*a^2*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*a^2*cosh(d*x + c)^
6 - 15*a^2*cosh(d*x + c)^4 + 9*a^2*cosh(d*x + c)^2 - a^2)*sinh(d*x + c)^2 +
 a^2 + 8*(a^2*cosh(d*x + c)^7 - 3*a^2*cosh(d*x + c)^5 + 3*a^2*cosh(d*x + c)
^3 - a^2*cosh(d*x + c))*sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) -
 sinh(d*x + c))) + 8*(a^2*d*x*cosh(d*x + c)^7 - 3*(a^2*d*x - a^2 - a*b)*cos
h(d*x + c)^5 + (3*a^2*d*x - 2*a^2 + 2*b^2)*cosh(d*x + c)^3 - (a^2*d*x - a^2
 - a*b)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^8 + 8*d*cosh(d*x + c
)*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 - 4*d*cosh(d*x + c)^6 + 4*(7*d*cosh(d
*x + c)^2 - d)*sinh(d*x + c)^6 + 8*(7*d*cosh(d*x + c)^3 - 3*d*cosh(d*x + c)
)*sinh(d*x + c)^5 + 6*d*cosh(d*x + c)^4 + 2*(35*d*cosh(d*x + c)^4 - 30*d*co
sh(d*x + c)^2 + 3*d)*sinh(d*x + c)^4 + 8*(7*d*cosh(d*x + c)^5 - 10*d*cosh(d
*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^3 - 4*d*cosh(d*x + c)^2 + 4*(7
*d*cosh(d*x + c)^6 - 15*d*cosh(d*x + c)^4 + 9*d*cosh(d*x + c)^2 - d)*sinh(d
*x + c)^2 + 8*(d*cosh(d*x + c)^7 - 3*d*cosh(d*x + c)^5 + 3*d*cosh(d*x + c)^
3 - d*cosh(d*x + c))*sinh(d*x + c) + d)
```

giac [B] time = 0.34, size = 147, normalized size = 2.83

$$-\frac{12\,a^2dx - 12\,a^2\log\left(\left|e^{(2\,dx+2\,c)} - 1\right|\right) + \frac{25\,a^2e^{(8\,dx+8\,c)} - 52\,a^2e^{(6\,dx+6\,c)} + 48\,abe^{(6\,dx+6\,c)} + 102\,a^2e^{(4\,dx+4\,c)} + 48\,b^2e^{(4\,dx+4\,c)} - 52\,a^2e^{(2\,dx+2\,c)} +}{\left(e^{(2\,dx+2\,c)} - 1\right)^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^5*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/12*(12*a^2*d*x - 12*a^2*log(abs(e^(2*d*x + 2*c) - 1)) + (25*a^2*e^(8*d*x
 + 8*c) - 52*a^2*e^(6*d*x + 6*c) + 48*a*b*e^(6*d*x + 6*c) + 102*a^2*e^(4*d*
x + 4*c) + 48*b^2*e^(4*d*x + 4*c) - 52*a^2*e^(2*d*x + 2*c) + 48*a*b*e^(2*d*
x + 2*c) + 25*a^2)/(e^(2*d*x + 2*c) - 1)^4)/d
```

maple [B] time = 0.32, size = 102, normalized size = 1.96

$$\frac{a^2\ln\left(\sinh\left(dx+c\right)\right)}{d} - \frac{a^2\left(\coth^2\left(dx+c\right)\right)}{2d} - \frac{a^2\left(\coth^4\left(dx+c\right)\right)}{4d} - \frac{ab\left(\cosh^2\left(dx+c\right)\right)}{d\sinh\left(dx+c\right)^4} + \frac{ab}{2d\sinh\left(dx+c\right)^4} - \frac{}{4d\sinh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^5*(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] a^2*ln(sinh(d*x+c))/d-1/2*a^2*coth(d*x+c)^2/d-1/4*a^2*coth(d*x+c)^4/d-1/d*a
*b/sinh(d*x+c)^4*cosh(d*x+c)^2+1/2/d*a*b/sinh(d*x+c)^4-1/4/d/sinh(d*x+c)^4*
b^2
```

maxima [B] time = 0.45, size = 282, normalized size = 5.42

$$a^2\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{4\left(e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}\right)}{d\left(4\,e^{(-2\,dx-2\,c)}-6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}-e^{(-8\,dx-8\,c)}-1\right)}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^5*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] a^2*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 4*(e^(-2
*d*x - 2*c) - e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/(d*(4*e^(-2*d*x - 2*c) -
 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) - 1))) + 4*a*b*
(e^(-2*d*x - 2*c)/(d*(4*e^(-2*d*x - 2*c) - 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x
 - 6*c) - e^(-8*d*x - 8*c) - 1)) + e^(-6*d*x - 6*c)/(d*(4*e^(-2*d*x - 2*c)
- 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) - 1))) - 4*b^2
/(d*(e^(d*x + c) - e^(-d*x - c))^4)
```

mupad [B] time = 1.47, size = 207, normalized size = 3.98

$$\frac{a^2\,\ln\left(\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}-1\right)}{d}-\frac{4\,\left(a^2+2\,a\,b+b^2\right)}{d\,\left(6\,\mathrm{e}^{4\,c+4\,d\,x}-4\,\mathrm{e}^{2\,c+2\,d\,x}-4\,\mathrm{e}^{6\,c+6\,d\,x}+\mathrm{e}^{8\,c+8\,d\,x}+1\right)}-\frac{4\,\left(2\,a^2+3\,a\,b+b^2\right)}{d\,\left(\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1\right)}-a^2\,x-\frac{}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^5*(a + b/cosh(c + d*x)^2)^2,x)
```

```
[Out] (a^2*log(exp(2*c)*exp(2*d*x) - 1))/d - (4*(2*a*b + a^2 + b^2))/(d*(6*exp(4*
c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1
)) - (4*(3*a*b + 2*a^2 + b^2))/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) +
1)) - a^2*x - (4*(a*b + a^2))/(d*(exp(2*c + 2*d*x) - 1)) - (8*(2*a*b + a^2
+ b^2))/(d*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1)
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**5*(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

3.122 $\int \coth^6(c+dx)\left(a+b\,\mathbf{sech}^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=64

$$-\frac{\left(a^2-b^2\right)\coth^3(c+dx)}{3d}-\frac{a^2\coth(c+dx)}{d}+a^2x-\frac{(a+b)^2\coth^5(c+dx)}{5d}$$

```
[Out] a^2*x-a^2*coth(d*x+c)/d-1/3*(a^2-b^2)*coth(d*x+c)^3/d-1/5*(a+b)^2*coth(d*x+
c)^5/d
```

Rubi [A] time = 0.10, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4141, 1802, 207}

$$-\frac{\left(a^2-b^2\right)\coth^3(c+dx)}{3d}-\frac{a^2\coth(c+dx)}{d}+a^2x-\frac{(a+b)^2\coth^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^6*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] a^2*x - (a^2*Coth[c + d*x])/d - ((a^2 - b^2)*Coth[c + d*x]^3)/(3*d) - ((a +
 b)^2*Coth[c + d*x]^5)/(5*d)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \coth^6(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\left(1-x^2\right)\right)^2}{x^6\left(1-x^2\right)}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{(a+b)^2}{x^6}+\frac{a^2-b^2}{x^4}+\frac{a^2}{x^2}-\frac{a^2}{-1+x^2}\right)dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{a^2\coth(c+dx)}{d}-\frac{\left(a^2-b^2\right)\coth^3(c+dx)}{3d}-\frac{(a+b)^2\coth^5(c+dx}{5d}$$

$$= a^2x-\frac{a^2\coth(c+dx)}{d}-\frac{\left(a^2-b^2\right)\coth^3(c+dx)}{3d}-\frac{(a+b)^2\coth^5(c}{5d}$$

Mathematica [B] time = 1.10, size = 256, normalized size = 4.00

$$\frac{\text{csch}(c)\text{csch}^5(c+dx)\left(180a^2\sinh(2c+dx)-140a^2\sinh(2c+3dx)-90a^2\sinh(4c+3dx)+46a^2\sinh(4c+5dx)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^6*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (Csch[c]*Csch[c + d*x]^5*(-150*a^2*d*x*Cosh[d*x] + 150*a^2*d*x*Cosh[2*c + d
*x] + 75*a^2*d*x*Cosh[2*c + 3*d*x] - 75*a^2*d*x*Cosh[4*c + 3*d*x] - 15*a^2*
d*x*Cosh[4*c + 5*d*x] + 15*a^2*d*x*Cosh[6*c + 5*d*x] + 280*a^2*Sinh[d*x] +
120*a*b*Sinh[d*x] + 20*b^2*Sinh[d*x] + 180*a^2*Sinh[2*c + d*x] - 60*b^2*Sin
h[2*c + d*x] - 140*a^2*Sinh[2*c + 3*d*x] + 20*b^2*Sinh[2*c + 3*d*x] - 90*a^
2*Sinh[4*c + 3*d*x] - 60*a*b*Sinh[4*c + 3*d*x] + 46*a^2*Sinh[4*c + 5*d*x] +
 12*a*b*Sinh[4*c + 5*d*x] - 4*b^2*Sinh[4*c + 5*d*x]))/(480*d)
```

fricas [B] time = 0.42, size = 425, normalized size = 6.64

$$-\frac{\left(23\,a^2+6\,ab-2\,b^2\right)\cosh\left(dx+c\right)^5+5\left(23\,a^2+6\,ab-2\,b^2\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^4-\left(15\,a^2dx+23\,a^2+6\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^6*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/15*((23*a^2 + 6*a*b - 2*b^2)*cosh(d*x + c)^5 + 5*(23*a^2 + 6*a*b - 2*b^2
)*cosh(d*x + c)*sinh(d*x + c)^4 - (15*a^2*d*x + 23*a^2 + 6*a*b - 2*b^2)*sin
h(d*x + c)^5 - 5*(5*a^2 - 6*a*b - 2*b^2)*cosh(d*x + c)^3 + 5*(15*a^2*d*x -
2*(15*a^2*d*x + 23*a^2 + 6*a*b - 2*b^2)*cosh(d*x + c)^2 + 23*a^2 + 6*a*b -
```

```
2*b^2)*sinh(d*x + c)^3 + 5*(2*(23*a^2 + 6*a*b - 2*b^2)*cosh(d*x + c)^3 - 3*
(5*a^2 - 6*a*b - 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 10*(5*a^2 + 6*a*b
+ 4*b^2)*cosh(d*x + c) - 5*((15*a^2*d*x + 23*a^2 + 6*a*b - 2*b^2)*cosh(d*x
+ c)^4 + 30*a^2*d*x - 3*(15*a^2*d*x + 23*a^2 + 6*a*b - 2*b^2)*cosh(d*x + c)
^2 + 46*a^2 + 12*a*b - 4*b^2)*sinh(d*x + c))/(d*sinh(d*x + c)^5 + 5*(2*d*co
sh(d*x + c)^2 - d)*sinh(d*x + c)^3 + 5*(d*cosh(d*x + c)^4 - 3*d*cosh(d*x +
c)^2 + 2*d)*sinh(d*x + c))
```

giac [B] time = 0.37, size = 167, normalized size = 2.61

$$\frac{15\,a^2dx-\frac{2\left(45\,a^2e^{(8\,dx+8\,c)}+30\,abe^{(8\,dx+8\,c)}-90\,a^2e^{(6\,dx+6\,c)}+30\,b^2e^{(6\,dx+6\,c)}+140\,a^2e^{(4\,dx+4\,c)}+60\,abe^{(4\,dx+4\,c)}+10\,b^2e^{(4\,dx+4\,c)}-70\,a^2e^{(2\,dx+2\,c)}+10\,b}{\left(e^{(2\,dx+2\,c)}-1\right)^5}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^6*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/15*(15*a^2*d*x - 2*(45*a^2*e^(8*d*x + 8*c) + 30*a*b*e^(8*d*x + 8*c) - 90*
a^2*e^(6*d*x + 6*c) + 30*b^2*e^(6*d*x + 6*c) + 140*a^2*e^(4*d*x + 4*c) + 60
*a*b*e^(4*d*x + 4*c) + 10*b^2*e^(4*d*x + 4*c) - 70*a^2*e^(2*d*x + 2*c) + 10
*b^2*e^(2*d*x + 2*c) + 23*a^2 + 6*a*b - 2*b^2)/(e^(2*d*x + 2*c) - 1)^5)/d
```

maple [B] time = 0.44, size = 163, normalized size = 2.55

$$\frac{a^2\left(dx+c-\coth\left(dx+c\right)-\frac{\left(\coth^3(dx+c)\right)}{3}-\frac{\left(\coth^5(dx+c)\right)}{5}\right)+2ab\left(-\frac{\cosh^3(dx+c)}{2\sinh(dx+c)^5}+\frac{3\cosh(dx+c)}{8\sinh(dx+c)^5}+\frac{3\left(-\frac{8}{15}-\frac{\operatorname{csch}(dx+c)^4}{5}+\frac{4\operatorname{csch}(}{1}\right.}{8}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^6*(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(a^2*(d*x+c-coth(d*x+c)-1/3*coth(d*x+c)^3-1/5*coth(d*x+c)^5)+2*a*b*(-1/
2/sinh(d*x+c)^5*cosh(d*x+c)^3+3/8/sinh(d*x+c)^5*cosh(d*x+c)+3/8*(-8/15-1/5*
csch(d*x+c)^4+4/15*csch(d*x+c)^2)*coth(d*x+c))+b^2*(-1/4/sinh(d*x+c)^5*cosh
(d*x+c)-1/4*(-8/15-1/5*csch(d*x+c)^4+4/15*csch(d*x+c)^2)*coth(d*x+c)))
```

maxima [B] time = 0.54, size = 613, normalized size = 9.58

$$\frac{1}{15}\,a^2\left(15\,x+\frac{15\,c}{d}-\frac{2\left(70\,e^{(-2\,dx-2\,c)}-140\,e^{(-4\,dx-4\,c)}+90\,e^{(-6\,dx-6\,c)}-45\,e^{(-8\,dx-8\,c)}-23\right)}{d\left(5\,e^{(-2\,dx-2\,c)}-10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}-5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}-1\right)}\right)+\frac{4}{15}\,b^2\left(\frac{}{d\left(5\,e^{(}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^6*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/15*a^2*(15*x + 15*c/d - 2*(70*e^(-2*d*x - 2*c) - 140*e^(-4*d*x - 4*c) + 9
0*e^(-6*d*x - 6*c) - 45*e^(-8*d*x - 8*c) - 23)/(d*(5*e^(-2*d*x - 2*c) - 10*
e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x -
10*c) - 1))) + 4/15*b^2*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) - 10*e^(
-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*
c) - 1)) + 5*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c)
+ 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)) + 15*
e^(-6*d*x - 6*c)/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*
x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)) - 1/(d*(5*e^(-2*d*
x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) +
 e^(-10*d*x - 10*c) - 1))) + 4/5*a*b*(10*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x -
 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^
(-10*d*x - 10*c) - 1)) + 5*e^(-8*d*x - 8*c)/(d*(5*e^(-2*d*x - 2*c) - 10*e^(
-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*
c) - 1)) + 1/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x -
6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)))
```

mupad [B] time = 1.44, size = 511, normalized size = 7.98

$$a^2\,x-\frac{\frac{2\,\left(5\,a^2+6\,a\,b+4\,b^2\right)}{15\,d}+\frac{4\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(b^2+2\,a\,b\right)}{5\,d}+\frac{2\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(3\,a^2+2\,b\,a\right)}{5\,d}}{3\,{\mathrm{e}}^{2\,c+2\,d\,x}-3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}-1}-\frac{\frac{2\,\left(b^2+2\,a\,b\right)}{5\,d}+\frac{2\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(3\,a^2+2\,b\,a\right)}{5\,d}}{{\mathrm{e}}^{4\,c+4\,d\,x}-2\,{\mathrm{e}}^{2\,c+2\,d\,x}+1}-\frac{\frac{2\,\left(b^2+2\,a\,b\right)}{5\,d}+\frac{6\,{\mathrm{e}}^{4\,c+4\,d\,x}\,(}{5\,a}}{6\,{\mathrm{e}}^{4\,c+4\,d\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^6*(a + b/cosh(c + d*x)^2)^2,x)
```

```
[Out] a^2*x - ((2*(6*a*b + 5*a^2 + 4*b^2))/(15*d) + (4*exp(2*c + 2*d*x)*(2*a*b +
b^2))/(5*d) + (2*exp(4*c + 4*d*x)*(2*a*b + 3*a^2))/(5*d))/(3*exp(2*c + 2*d*
x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1) - ((2*(2*a*b + b^2))/(5*d)
+ (2*exp(2*c + 2*d*x)*(2*a*b + 3*a^2))/(5*d))/(exp(4*c + 4*d*x) - 2*exp(2*c
 + 2*d*x) + 1) - ((2*(2*a*b + b^2))/(5*d) + (6*exp(4*c + 4*d*x)*(2*a*b + b^
2))/(5*d) + (2*exp(6*c + 6*d*x)*(2*a*b + 3*a^2))/(5*d) + (2*exp(2*c + 2*d*x
)*(6*a*b + 5*a^2 + 4*b^2))/(5*d))/(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x)
- 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) - ((2*(2*a*b + 3*a^2))/(5*d) +
 (8*exp(2*c + 2*d*x)*(2*a*b + b^2))/(5*d) + (8*exp(6*c + 6*d*x)*(2*a*b + b^
2))/(5*d) + (2*exp(8*c + 8*d*x)*(2*a*b + 3*a^2))/(5*d) + (4*exp(4*c + 4*d*x
)*(6*a*b + 5*a^2 + 4*b^2))/(5*d))/(5*exp(2*c + 2*d*x) - 10*exp(4*c + 4*d*x)
 + 10*exp(6*c + 6*d*x) - 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) - 1) - (2*
(2*a*b + 3*a^2))/(5*d*(exp(2*c + 2*d*x) - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**6*(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

3.123 $\int \coth^7(c+dx)\left(a+b\,\mathbf{sech}^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=86

$$\frac{a^2\log(\sinh(c+dx))}{d}-\frac{(a+b)^2\operatorname{csch}^4(c+dx)}{4d}-\frac{a(a+b)\operatorname{csch}^2(c+dx)}{d}-\frac{\operatorname{csch}^6(c+dx)\left(a\cosh^2(c+dx)+b\right)^3}{6d(a+b)}$$

```
[Out] -a*(a+b)*csch(d*x+c)^2/d-1/4*(a+b)^2*csch(d*x+c)^4/d-1/6*(b+a*cosh(d*x+c)^2
)^3*csch(d*x+c)^6/(a+b)/d+a^2*ln(sinh(d*x+c))/d
```

Rubi [A] time = 0.12, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4138, 446, 78, 43}

$$\frac{a^2\log(\sinh(c+dx))}{d}-\frac{(a+b)^2\operatorname{csch}^4(c+dx)}{4d}-\frac{a(a+b)\operatorname{csch}^2(c+dx)}{d}-\frac{\operatorname{csch}^6(c+dx)\left(a\cosh^2(c+dx)+b\right)^3}{6d(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^7*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] -((a*(a + b)*Csch[c + d*x]^2)/d) - ((a + b)^2*Csch[c + d*x]^4)/(4*d) - ((b
+ a*Cosh[c + d*x]^2)^3*Csch[c + d*x]^6)/(6*(a + b)*d) + (a^2*Log[Sinh[c + d
*x]])/d
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \coth^7(c+dx)\left(a+b\mathrm{sech}^2(c+dx)\right)^2\,dx = \frac{\mathrm{Subst}\left(\int \frac{x^3\left(b+ax^2\right)^2}{\left(1-x^2\right)^4}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= \frac{\mathrm{Subst}\left(\int \frac{x(b+ax)^2}{(1-x)^4}\,dx, x, \cosh^2(c+dx)\right)}{2d}$$

$$= -\frac{\left(b+a\cosh^2(c+dx)\right)^3\mathrm{csch}^6(c+dx)}{6(a+b)d} - \frac{\mathrm{Subst}\left(\int \frac{(b+ax)^2}{(1-x)^3}\,dx, x, \mathrm{co}\right.}{2d}$$

$$= -\frac{\left(b+a\cosh^2(c+dx)\right)^3\mathrm{csch}^6(c+dx)}{6(a+b)d} - \frac{\mathrm{Subst}\left(\int\left(-\frac{(a+b)^2}{(-1+x)^3} - \frac{2a(}{(-1}\right.\right.}{}$$

$$= -\frac{a(a+b)\mathrm{csch}^2(c+dx)}{d} - \frac{(a+b)^2\mathrm{csch}^4(c+dx)}{4d} - \frac{\left(b+a\cosh^2(c\right.}{6}$$

Mathematica [A] time = 0.52, size = 107, normalized size = 1.24

$$-\frac{\left(a\cosh^2(c+dx)+b\right)^2\left(3\left(3a^2+4ab+b^2\right)\mathrm{csch}^4(c+dx)-12a^2\log(\sinh(c+dx))+2(a+b)^2\mathrm{csch}^6(c+dx)+\right.}{3d(a\cosh(2(c+dx))+a+2b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^7*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] -1/3*((b + a*Cosh[c + d*x]^2)^2*(6*a*(3*a + 2*b)*Csch[c + d*x]^2 + 3*(3*a^2
 + 4*a*b + b^2)*Csch[c + d*x]^4 + 2*(a + b)^2*Csch[c + d*x]^6 - 12*a^2*Log[
Sinh[c + d*x]]))/(d*(a + 2*b + a*Cosh[2*(c + d*x)])^2)
```

fricas [B] time = 0.48, size = 2548, normalized size = 29.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^7*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(3*a^2*d*x*cosh(d*x + c)^12 + 36*a^2*d*x*cosh(d*x + c)*sinh(d*x + c)^1
1 + 3*a^2*d*x*sinh(d*x + c)^12 - 6*(3*a^2*d*x - 3*a^2 - 2*a*b)*cosh(d*x + c
)^10 + 6*(33*a^2*d*x*cosh(d*x + c)^2 - 3*a^2*d*x + 3*a^2 + 2*a*b)*sinh(d*x
+ c)^10 + 60*(11*a^2*d*x*cosh(d*x + c)^3 - (3*a^2*d*x - 3*a^2 - 2*a*b)*cosh
(d*x + c))*sinh(d*x + c)^9 + 3*(15*a^2*d*x - 12*a^2 + 4*b^2)*cosh(d*x + c)^
8 + 3*(495*a^2*d*x*cosh(d*x + c)^4 + 15*a^2*d*x - 90*(3*a^2*d*x - 3*a^2 - 2
*a*b)*cosh(d*x + c)^2 - 12*a^2 + 4*b^2)*sinh(d*x + c)^8 + 24*(99*a^2*d*x*co
sh(d*x + c)^5 - 30*(3*a^2*d*x - 3*a^2 - 2*a*b)*cosh(d*x + c)^3 + (15*a^2*d*
x - 12*a^2 + 4*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 - 4*(15*a^2*d*x - 17*a^2
 - 10*a*b - 2*b^2)*cosh(d*x + c)^6 + 4*(693*a^2*d*x*cosh(d*x + c)^6 - 315*(
3*a^2*d*x - 3*a^2 - 2*a*b)*cosh(d*x + c)^4 - 15*a^2*d*x + 21*(15*a^2*d*x -
12*a^2 + 4*b^2)*cosh(d*x + c)^2 + 17*a^2 + 10*a*b + 2*b^2)*sinh(d*x + c)^6
+ 24*(99*a^2*d*x*cosh(d*x + c)^7 - 63*(3*a^2*d*x - 3*a^2 - 2*a*b)*cosh(d*x
+ c)^5 + 7*(15*a^2*d*x - 12*a^2 + 4*b^2)*cosh(d*x + c)^3 - (15*a^2*d*x - 17
*a^2 - 10*a*b - 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 3*(15*a^2*d*x - 12*
a^2 + 4*b^2)*cosh(d*x + c)^4 + 3*(495*a^2*d*x*cosh(d*x + c)^8 - 420*(3*a^2*
d*x - 3*a^2 - 2*a*b)*cosh(d*x + c)^6 + 70*(15*a^2*d*x - 12*a^2 + 4*b^2)*cos
h(d*x + c)^4 + 15*a^2*d*x - 20*(15*a^2*d*x - 17*a^2 - 10*a*b - 2*b^2)*cosh(
d*x + c)^2 - 12*a^2 + 4*b^2)*sinh(d*x + c)^4 + 3*a^2*d*x + 4*(165*a^2*d*x*c
osh(d*x + c)^9 - 180*(3*a^2*d*x - 3*a^2 - 2*a*b)*cosh(d*x + c)^7 + 42*(15*a
^2*d*x - 12*a^2 + 4*b^2)*cosh(d*x + c)^5 - 20*(15*a^2*d*x - 17*a^2 - 10*a*b
 - 2*b^2)*cosh(d*x + c)^3 + 3*(15*a^2*d*x - 12*a^2 + 4*b^2)*cosh(d*x + c))*
sinh(d*x + c)^3 - 6*(3*a^2*d*x - 3*a^2 - 2*a*b)*cosh(d*x + c)^2 + 6*(33*a^2
*d*x*cosh(d*x + c)^10 - 45*(3*a^2*d*x - 3*a^2 - 2*a*b)*cosh(d*x + c)^8 + 14
*(15*a^2*d*x - 12*a^2 + 4*b^2)*cosh(d*x + c)^6 - 10*(15*a^2*d*x - 17*a^2 -
10*a*b - 2*b^2)*cosh(d*x + c)^4 - 3*a^2*d*x + 3*(15*a^2*d*x - 12*a^2 + 4*b^
2)*cosh(d*x + c)^2 + 3*a^2 + 2*a*b)*sinh(d*x + c)^2 - 3*(a^2*cosh(d*x + c)^
12 + 12*a^2*cosh(d*x + c)*sinh(d*x + c)^11 + a^2*sinh(d*x + c)^12 - 6*a^2*c
osh(d*x + c)^10 + 6*(11*a^2*cosh(d*x + c)^2 - a^2)*sinh(d*x + c)^10 + 15*a^
2*cosh(d*x + c)^8 + 20*(11*a^2*cosh(d*x + c)^3 - 3*a^2*cosh(d*x + c))*sinh(
d*x + c)^9 + 15*(33*a^2*cosh(d*x + c)^4 - 18*a^2*cosh(d*x + c)^2 + a^2)*sin
h(d*x + c)^8 - 20*a^2*cosh(d*x + c)^6 + 24*(33*a^2*cosh(d*x + c)^5 - 30*a^2
*cosh(d*x + c)^3 + 5*a^2*cosh(d*x + c))*sinh(d*x + c)^7 + 4*(231*a^2*cosh(d
*x + c)^6 - 315*a^2*cosh(d*x + c)^4 + 105*a^2*cosh(d*x + c)^2 - 5*a^2)*sinh
(d*x + c)^6 + 15*a^2*cosh(d*x + c)^4 + 24*(33*a^2*cosh(d*x + c)^7 - 63*a^2*
cosh(d*x + c)^5 + 35*a^2*cosh(d*x + c)^3 - 5*a^2*cosh(d*x + c))*sinh(d*x +
c)^5 + 15*(33*a^2*cosh(d*x + c)^8 - 84*a^2*cosh(d*x + c)^6 + 70*a^2*cosh(d*
```

x + c)^4 - 20*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^4 - 6*a^2*cosh(d*x + c)^2 + 20*(11*a^2*cosh(d*x + c)^9 - 36*a^2*cosh(d*x + c)^7 + 42*a^2*cosh(d*x + c)^5 - 20*a^2*cosh(d*x + c)^3 + 3*a^2*cosh(d*x + c))*sinh(d*x + c)^3 + 6*(11*a^2*cosh(d*x + c)^10 - 45*a^2*cosh(d*x + c)^8 + 70*a^2*cosh(d*x + c)^6 - 50*a^2*cosh(d*x + c)^4 + 15*a^2*cosh(d*x + c)^2 - a^2)*sinh(d*x + c)^2 + a^2 + 12*(a^2*cosh(d*x + c)^11 - 5*a^2*cosh(d*x + c)^9 + 10*a^2*cosh(d*x + c)^7 - 10*a^2*cosh(d*x + c)^5 + 5*a^2*cosh(d*x + c)^3 - a^2*cosh(d*x + c))*sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 12*(3*a^2*d*x*cosh(d*x + c)^11 - 5*(3*a^2*d*x - 3*a^2 - 2*a*b)*cosh(d*x + c)^9 + 2*(15*a^2*d*x - 12*a^2 + 4*b^2)*cosh(d*x + c)^7 - 2*(15*a^2*d*x - 17*a^2 - 10*a*b - 2*b^2)*cosh(d*x + c)^5 + (15*a^2*d*x - 12*a^2 + 4*b^2)*cosh(d*x + c)^3 - (3*a^2*d*x - 3*a^2 - 2*a*b)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^12 + 12*d*cosh(d*x + c)*sinh(d*x + c)^11 + d*sinh(d*x + c)^12 - 6*d*cosh(d*x + c)^10 + 6*(11*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^10 + 20*(11*d*cosh(d*x + c)^3 - 3*d*cosh(d*x + c))*sinh(d*x + c)^9 + 15*d*cosh(d*x + c)^8 + 15*(33*d*cosh(d*x + c)^4 - 18*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^8 + 24*(33*d*cosh(d*x + c)^5 - 30*d*cosh(d*x + c)^3 + 5*d*cosh(d*x + c))*sinh(d*x + c)^7 - 20*d*cosh(d*x + c)^6 + 4*(231*d*cosh(d*x + c)^6 - 315*d*cosh(d*x + c)^4 + 105*d*cosh(d*x + c)^2 - 5*d)*sinh(d*x + c)^6 + 24*(33*d*cosh(d*x + c)^7 - 63*d*cosh(d*x + c)^5 + 35*d*cosh(d*x + c)^3 - 5*d*cosh(d*x + c))*sinh(d*x + c)^5 + 15*d*cosh(d*x + c)^4 + 15*(33*d*cosh(d*x + c)^8 - 84*d*cosh(d*x + c)^6 + 70*d*cosh(d*x + c)^4 - 20*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^4 + 20*(11*d*cosh(d*x + c)^9 - 36*d*cosh(d*x + c)^7 + 42*d*cosh(d*x + c)^5 - 20*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^3 - 6*d*cosh(d*x + c)^2 + 6*(11*d*cosh(d*x + c)^10 - 45*d*cosh(d*x + c)^8 + 70*d*cosh(d*x + c)^6 - 50*d*cosh(d*x + c)^4 + 15*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^2 + 12*(d*cosh(d*x + c)^11 - 5*d*cosh(d*x + c)^9 + 10*d*cosh(d*x + c)^7 - 10*d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c) + d)

giac [B] time = 0.46, size = 216, normalized size = 2.51

$$-\frac{60\,a^2dx - 60\,a^2\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right) + \frac{147\,a^2e^{(12\,dx+12\,c)} - 522\,a^2e^{(10\,dx+10\,c)} + 240\,abe^{(10\,dx+10\,c)} + 1485\,a^2e^{(8\,dx+8\,c)} + 240\,b^2e^{(8\,dx+8\,c)} - 158\cdots}{\cdots}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^7*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")

[Out] -1/60*(60*a^2*d*x - 60*a^2*log(abs(e^(2*d*x + 2*c) - 1)) + (147*a^2*e^(12*d*x + 12*c) - 522*a^2*e^(10*d*x + 10*c) + 240*a*b*e^(10*d*x + 10*c) + 1485*a^2*e^(8*d*x + 8*c) + 240*b^2*e^(8*d*x + 8*c) - 1580*a^2*e^(6*d*x + 6*c) + 800*a*b*e^(6*d*x + 6*c) + 160*b^2*e^(6*d*x + 6*c) + 1485*a^2*e^(4*d*x + 4*c) + 240*b^2*e^(4*d*x + 4*c) - 522*a^2*e^(2*d*x + 2*c) + 240*a*b*e^(2*d*x + 2*c) + 147*a^2)/(e^(2*d*x + 2*c) - 1)^6)/d

maple [A] time = 0.34, size = 164, normalized size = 1.91

$$\frac{a^2\ln(\sinh(dx+c))}{d}-\frac{a^2\left(\coth^2(dx+c)\right)}{2d}-\frac{a^2\left(\coth^4(dx+c)\right)}{4d}-\frac{a^2\left(\coth^6(dx+c)\right)}{6d}-\frac{ab\left(\cosh^4(dx+c)\right)}{d\sinh(dx+c)^6}+\frac{ab\left(\cos\right.}{d\sinh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^7*(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] a^2*ln(sinh(d*x+c))/d-1/2*a^2*coth(d*x+c)^2/d-1/4*a^2*coth(d*x+c)^4/d-1/6*a
^2*coth(d*x+c)^6/d-1/d*a*b/sinh(d*x+c)^6*cosh(d*x+c)^4+1/d*a*b/sinh(d*x+c)^
6*cosh(d*x+c)^2-1/3/d*a*b/sinh(d*x+c)^6-1/4/d*b^2/sinh(d*x+c)^6*cosh(d*x+c)
^2+1/12/d*b^2/sinh(d*x+c)^6
```

maxima [B] time = 0.41, size = 696, normalized size = 8.09

$$\frac{1}{3}a^2\left(3x+\frac{3c}{d}+\frac{3\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{3\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(9e^{(-2dx-2c)}-18e^{(-4dx-4c)}+34e^{(-6dx-6c}\right.}{d\left(6e^{(-2dx-2c)}-15e^{(-4dx-4c)}+20e^{(-6dx-6c)}-15e^{(-}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^7*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*(3*x + 3*c/d + 3*log(e^(-d*x - c) + 1)/d + 3*log(e^(-d*x - c) - 1)/
d + 2*(9*e^(-2*d*x - 2*c) - 18*e^(-4*d*x - 4*c) + 34*e^(-6*d*x - 6*c) - 18*
e^(-8*d*x - 8*c) + 9*e^(-10*d*x - 10*c))/(d*(6*e^(-2*d*x - 2*c) - 15*e^(-4*
d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*
c) - e^(-12*d*x - 12*c) - 1))) + 4/3*a*b*(3*e^(-2*d*x - 2*c)/(d*(6*e^(-2*d*
x - 2*c) - 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c)
+ 6*e^(-10*d*x - 10*c) - e^(-12*d*x - 12*c) - 1)) + 10*e^(-6*d*x - 6*c)/(d*
(6*e^(-2*d*x - 2*c) - 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*
d*x - 8*c) + 6*e^(-10*d*x - 10*c) - e^(-12*d*x - 12*c) - 1)) + 3*e^(-10*d*x
 - 10*c)/(d*(6*e^(-2*d*x - 2*c) - 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c)
 - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) - e^(-12*d*x - 12*c) - 1))) +
 4/3*b^2*(3*e^(-4*d*x - 4*c)/(d*(6*e^(-2*d*x - 2*c) - 15*e^(-4*d*x - 4*c) +
 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) - e^(-12*
d*x - 12*c) - 1)) + 2*e^(-6*d*x - 6*c)/(d*(6*e^(-2*d*x - 2*c) - 15*e^(-4*d*
x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c)
 - e^(-12*d*x - 12*c) - 1)) + 3*e^(-8*d*x - 8*c)/(d*(6*e^(-2*d*x - 2*c) - 1
5*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d
*x - 10*c) - e^(-12*d*x - 12*c) - 1)))
```

mupad [B] time = 1.53, size = 377, normalized size = 4.38

$$\frac{a^2\ln\left(e^{2c}e^{2dx}-1\right)}{d}-\frac{32\left(a^2+2ab+b^2\right)}{d\left(5e^{2c+2dx}-10e^{4c+4dx}+10e^{6c+6dx}-5e^{8c+8dx}+e^{10c+10dx}-1\right)}-\frac{2\left(3a^2+2ba\right)}{d\left(e^{2c+2dx}-1\right)}-\frac{}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^7*(a + b/cosh(c + d*x)^2)^2,x)
```

```
[Out] (a^2*log(exp(2*c)*exp(2*d*x) - 1))/d - (32*(2*a*b + a^2 + b^2))/(d*(5*exp(2
*c + 2*d*x) - 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) - 5*exp(8*c + 8*d*x
) + exp(10*c + 10*d*x) - 1)) - (2*(2*a*b + 3*a^2))/(d*(exp(2*c + 2*d*x) - 1
)) - (32*(2*a*b + a^2 + b^2))/(3*d*(15*exp(4*c + 4*d*x) - 6*exp(2*c + 2*d*x
) - 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) - 6*exp(10*c + 10*d*x) + exp(
12*c + 12*d*x) + 1)) - (2*(10*a*b + 9*a^2 + 2*b^2))/(d*(exp(4*c + 4*d*x) -
2*exp(2*c + 2*d*x) + 1)) - (8*(20*a*b + 13*a^2 + 7*b^2))/(3*d*(3*exp(2*c +
2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1)) - (4*(20*a*b + 11*a^2
+ 9*b^2))/(d*(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c + 6*d*x)
+ exp(8*c + 8*d*x) + 1)) - a^2*x
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**7*(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

3.124 $\int \left(a+b\,\mathbf{sech}^2(c+dx)\right)^3 \tanh^4(c+dx)\,dx$

Optimal. Leaf size=110

$$-\frac{a^3\tanh^3(c+dx)}{3d}-\frac{a^3\tanh(c+dx)}{d}+a^3x+\frac{b\left(3a^2+3ab+b^2\right)\tanh^5(c+dx)}{5d}-\frac{b^2(3a+2b)\tanh^7(c+dx)}{7d}+\frac{b^3\tanh}{}$$

```
[Out] a^3*x-a^3*tanh(d*x+c)/d-1/3*a^3*tanh(d*x+c)^3/d+1/5*b*(3*a^2+3*a*b+b^2)*tan
h(d*x+c)^5/d-1/7*b^2*(3*a+2*b)*tanh(d*x+c)^7/d+1/9*b^3*tanh(d*x+c)^9/d
```

Rubi [A] time = 0.12, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4141, 1802, 206}

$$\frac{b\left(3a^2+3ab+b^2\right)\tanh^5(c+dx)}{5d}-\frac{a^3\tanh^3(c+dx)}{3d}-\frac{a^3\tanh(c+dx)}{d}+a^3x-\frac{b^2(3a+2b)\tanh^7(c+dx)}{7d}+\frac{b^3\tanh^9}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^3*Tanh[c + d*x]^4,x]
```

```
[Out] a^3*x - (a^3*Tanh[c + d*x])/d - (a^3*Tanh[c + d*x]^3)/(3*d) + (b*(3*a^2 + 3
*a*b + b^2)*Tanh[c + d*x]^5)/(5*d) - (b^2*(3*a + 2*b)*Tanh[c + d*x]^7)/(7*d
) + (b^3*Tanh[c + d*x]^9)/(9*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \left(a+b\text{sech}^2(c+dx)\right)^3 \tanh^4(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x^4\left(a+b\left(1-x^2\right)\right)^3}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-a^3-a^3x^2+b\left(3a^2+3ab+b^2\right)x^4-b^2(3a+2b)x^6+b^3\right.\right.}{d}$$
$$= -\frac{a^3\tanh(c+dx)}{d}-\frac{a^3\tanh^3(c+dx)}{3d}+\frac{b\left(3a^2+3ab+b^2\right)\tanh^5(c}{5d}$$
$$= a^3x-\frac{a^3\tanh(c+dx)}{d}-\frac{a^3\tanh^3(c+dx)}{3d}+\frac{b\left(3a^2+3ab+b^2\right)\tan}{5d}$$

Mathematica [B] time = 5.94, size = 301, normalized size = 2.74

$$\frac{8\text{sech}^9(c+dx)\left(a\cosh^2(c+dx)+b\right)^3\left(315a^3dx\cosh^9(c+dx)+3b\left(63a^2-72ab+b^2\right)\tanh(c)\cosh^5(c+dx)+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^3*Tanh[c + d*x]^4,x]
```

```
[Out] (8*(b + a*Cosh[c + d*x]^2)^3*Sech[c + d*x]^9*(315*a^3*d*x*Cosh[c + d*x]^9 +
 35*b^3*Sech[c]*Sinh[d*x] + 5*(27*a - 10*b)*b^2*Cosh[c + d*x]^2*Sech[c]*Sin
h[d*x] + 3*b*(63*a^2 - 72*a*b + b^2)*Cosh[c + d*x]^4*Sech[c]*Sinh[d*x] + (1
05*a^3 - 378*a^2*b + 27*a*b^2 + 4*b^3)*Cosh[c + d*x]^6*Sech[c]*Sinh[d*x] -
(420*a^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)*Cosh[c + d*x]^8*Sech[c]*Sinh[d*x]
+ 35*b^3*Cosh[c + d*x]*Tanh[c] + 5*(27*a - 10*b)*b^2*Cosh[c + d*x]^3*Tanh[c
] + 3*b*(63*a^2 - 72*a*b + b^2)*Cosh[c + d*x]^5*Tanh[c] + (105*a^3 - 378*a^
2*b + 27*a*b^2 + 4*b^3)*Cosh[c + d*x]^7*Tanh[c]))/(315*d*(a + 2*b + a*Cosh[
2*(c + d*x)])^3)
```

fricas [B] time = 0.43, size = 1323, normalized size = 12.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3*tanh(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/315*((315*a^3*d*x + 420*a^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x + c)
^9 + 9*(315*a^3*d*x + 420*a^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x + c)
```

```
*sinh(d*x + c)^8 - (420*a^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)*sinh(d*x + c)^9
 + 9*(315*a^3*d*x + 420*a^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x + c)^7
 - 9*(280*a^3 + 21*a^2*b - 54*a*b^2 - 8*b^3 + 4*(420*a^3 - 189*a^2*b - 54*a
*b^2 - 8*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^7 + 21*(4*(315*a^3*d*x + 420*a
^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x + c)^3 + 3*(315*a^3*d*x + 420*a
^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x + c))*sinh(d*x + c)^6 + 36*(315
*a^3*d*x + 420*a^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x + c)^5 - 9*(14*
(420*a^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x + c)^4 + 700*a^3 + 84*a^2
*b + 204*a*b^2 - 32*b^3 + 21*(280*a^3 + 21*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d
*x + c)^2)*sinh(d*x + c)^5 + 9*(14*(315*a^3*d*x + 420*a^3 - 189*a^2*b - 54*
a*b^2 - 8*b^3)*cosh(d*x + c)^5 + 35*(315*a^3*d*x + 420*a^3 - 189*a^2*b - 54
*a*b^2 - 8*b^3)*cosh(d*x + c)^3 + 20*(315*a^3*d*x + 420*a^3 - 189*a^2*b - 5
4*a*b^2 - 8*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 84*(315*a^3*d*x + 420*a^3
 - 189*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x + c)^3 - 3*(28*(420*a^3 - 189*a^2
*b - 54*a*b^2 - 8*b^3)*cosh(d*x + c)^6 + 105*(280*a^3 + 21*a^2*b - 54*a*b^2
 - 8*b^3)*cosh(d*x + c)^4 + 2660*a^3 - 252*a^2*b - 252*a*b^2 + 896*b^3 + 12
0*(175*a^3 + 21*a^2*b + 51*a*b^2 - 8*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3
+ 9*(4*(315*a^3*d*x + 420*a^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x + c)
^7 + 21*(315*a^3*d*x + 420*a^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x + c
)^5 + 40*(315*a^3*d*x + 420*a^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x +
c)^3 + 28*(315*a^3*d*x + 420*a^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x +
 c))*sinh(d*x + c)^2 + 126*(315*a^3*d*x + 420*a^3 - 189*a^2*b - 54*a*b^2 -
8*b^3)*cosh(d*x + c) - 9*((420*a^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x
 + c)^8 + 7*(280*a^3 + 21*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x + c)^6 + 20*(1
75*a^3 + 21*a^2*b + 51*a*b^2 - 8*b^3)*cosh(d*x + c)^4 + 420*a^3 - 126*a^2*b
 - 336*a*b^2 - 672*b^3 + 28*(95*a^3 - 9*a^2*b - 9*a*b^2 + 32*b^3)*cosh(d*x
+ c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^9 + 9*d*cosh(d*x + c)*sinh(d*x + c)
^8 + 9*d*cosh(d*x + c)^7 + 21*(4*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sin
h(d*x + c)^6 + 36*d*cosh(d*x + c)^5 + 9*(14*d*cosh(d*x + c)^5 + 35*d*cosh(d
*x + c)^3 + 20*d*cosh(d*x + c))*sinh(d*x + c)^4 + 84*d*cosh(d*x + c)^3 + 9*
(4*d*cosh(d*x + c)^7 + 21*d*cosh(d*x + c)^5 + 40*d*cosh(d*x + c)^3 + 28*d*c
osh(d*x + c))*sinh(d*x + c)^2 + 126*d*cosh(d*x + c))
```

giac [B] time = 0.30, size = 472, normalized size = 4.29

$$315\,a^3dx + \frac{2\left(630\,a^3e^{(16\,dx+16\,c)} - 945\,a^2be^{(16\,dx+16\,c)} + 4410\,a^3e^{(14\,dx+14\,c)} - 3780\,a^2be^{(14\,dx+14\,c)} - 1890\,ab^2e^{(14\,dx+14\,c)} + 13650\,a^3e^{(12\,dx+12\,c)} - 7560\,a^2b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3*tanh(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/315*(315*a^3*d*x + 2*(630*a^3*e^(16*d*x + 16*c) - 945*a^2*b*e^(16*d*x + 1
6*c) + 4410*a^3*e^(14*d*x + 14*c) - 3780*a^2*b*e^(14*d*x + 14*c) - 1890*a*b
```

```
^2*e^(14*d*x + 14*c) + 13650*a^3*e^(12*d*x + 12*c) - 7560*a^2*b*e^(12*d*x +
 12*c) - 1890*a*b^2*e^(12*d*x + 12*c) - 1680*b^3*e^(12*d*x + 12*c) + 24570*
a^3*e^(10*d*x + 10*c) - 11340*a^2*b*e^(10*d*x + 10*c) - 1890*a*b^2*e^(10*d*
x + 10*c) + 2520*b^3*e^(10*d*x + 10*c) + 28350*a^3*e^(8*d*x + 8*c) - 12474*
a^2*b*e^(8*d*x + 8*c) - 4914*a*b^2*e^(8*d*x + 8*c) - 3528*b^3*e^(8*d*x + 8*
c) + 21630*a^3*e^(6*d*x + 6*c) - 8316*a^2*b*e^(6*d*x + 6*c) - 2646*a*b^2*e^
(6*d*x + 6*c) + 1008*b^3*e^(6*d*x + 6*c) + 10710*a^3*e^(4*d*x + 4*c) - 3024
*a^2*b*e^(4*d*x + 4*c) - 54*a*b^2*e^(4*d*x + 4*c) - 288*b^3*e^(4*d*x + 4*c)
 + 3150*a^3*e^(2*d*x + 2*c) - 756*a^2*b*e^(2*d*x + 2*c) - 486*a*b^2*e^(2*d*
x + 2*c) - 72*b^3*e^(2*d*x + 2*c) + 420*a^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)
/(e^(2*d*x + 2*c) + 1)^9)/d
```

maple [B] time = 0.55, size = 274, normalized size = 2.49

$$a^3\left(dx+c-\tanh(dx+c)-\frac{\left(\tanh^3(dx+c)\right)}{3}\right)+3a^2b\left(-\frac{\sinh^3(dx+c)}{2\cosh(dx+c)^5}-\frac{3\sinh(dx+c)}{8\cosh(dx+c)^5}+\frac{3\left(\frac{8}{15}+\frac{\operatorname{sech}(dx+c)^4}{5}+\frac{4\operatorname{sech}(dx+c)^2}{15}\right)\tanh(dx+}{8}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)^3*tanh(d*x+c)^4,x)
```

```
[Out] 1/d*(a^3*(d*x+c-tanh(d*x+c)-1/3*tanh(d*x+c)^3)+3*a^2*b*(-1/2*sinh(d*x+c)^3/
cosh(d*x+c)^5-3/8*sinh(d*x+c)/cosh(d*x+c)^5+3/8*(8/15+1/5*sech(d*x+c)^4+4/1
5*sech(d*x+c)^2)*tanh(d*x+c))+3*a*b^2*(-1/4*sinh(d*x+c)^3/cosh(d*x+c)^7-1/8
*sinh(d*x+c)/cosh(d*x+c)^7+1/8*(16/35+1/7*sech(d*x+c)^6+6/35*sech(d*x+c)^4+
8/35*sech(d*x+c)^2)*tanh(d*x+c))+b^3*(-1/6*sinh(d*x+c)^3/cosh(d*x+c)^9-1/16
*sinh(d*x+c)/cosh(d*x+c)^9+1/16*(128/315+1/9*sech(d*x+c)^8+8/63*sech(d*x+c)
^6+16/105*sech(d*x+c)^4+64/315*sech(d*x+c)^2)*tanh(d*x+c)))
```

maxima [B] time = 0.39, size = 1453, normalized size = 13.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3*tanh(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 3/5*a^2*b*tanh(d*x + c)^5/d + 1/3*a^3*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c)
+ 3*e^(-4*d*x - 4*c) + 2)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(
-6*d*x - 6*c) + 1))) + 16/315*b^3*(9*e^(-2*d*x - 2*c)/(d*(9*e^(-2*d*x - 2*c
) + 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*c) + 126*
e^(-10*d*x - 10*c) + 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) + 9*e^(-
16*d*x - 16*c) + e^(-18*d*x - 18*c) + 1)) + 36*e^(-4*d*x - 4*c)/(d*(9*e^(-2
*d*x - 2*c) + 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8
```

```
*c) + 126*e^(-10*d*x - 10*c) + 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c
) + 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) + 1)) - 126*e^(-6*d*x - 6*c)/
(d*(9*e^(-2*d*x - 2*c) + 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) + 126*e^
(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) + 84*e^(-12*d*x - 12*c) + 36*e^(-14
*d*x - 14*c) + 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) + 1)) + 441*e^(-8*
d*x - 8*c)/(d*(9*e^(-2*d*x - 2*c) + 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*
c) + 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) + 84*e^(-12*d*x - 12*c)
+ 36*e^(-14*d*x - 14*c) + 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) + 1)) -
 315*e^(-10*d*x - 10*c)/(d*(9*e^(-2*d*x - 2*c) + 36*e^(-4*d*x - 4*c) + 84*e
^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) + 84*e^(-12
*d*x - 12*c) + 36*e^(-14*d*x - 14*c) + 9*e^(-16*d*x - 16*c) + e^(-18*d*x -
18*c) + 1)) + 210*e^(-12*d*x - 12*c)/(d*(9*e^(-2*d*x - 2*c) + 36*e^(-4*d*x
- 4*c) + 84*e^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c
) + 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) + 9*e^(-16*d*x - 16*c) +
e^(-18*d*x - 18*c) + 1)) + 1/(d*(9*e^(-2*d*x - 2*c) + 36*e^(-4*d*x - 4*c) +
 84*e^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) + 84*e
^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) + 9*e^(-16*d*x - 16*c) + e^(-18*d
*x - 18*c) + 1))) + 12/35*a*b^2*(7*e^(-2*d*x - 2*c)/(d*(7*e^(-2*d*x - 2*c)
+ 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-
10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) - 14*e^(-4
*d*x - 4*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6
*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) +
e^(-14*d*x - 14*c) + 1)) + 70*e^(-6*d*x - 6*c)/(d*(7*e^(-2*d*x - 2*c) + 21*
e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*
x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) - 35*e^(-8*d*x
- 8*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) +
 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-1
4*d*x - 14*c) + 1)) + 35*e^(-10*d*x - 10*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(
-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x -
 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 1/(d*(7*e^(-2*d*
x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c)
+ 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)))
```

mupad [B] time = 1.62, size = 1834, normalized size = 16.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^4*(a + b/cosh(c + d*x)^2)^3,x)
```

```
[Out] ((3*a*b^2 + 13*a^3 + 16*b^3)/(63*d) + (10*exp(4*c + 4*d*x)*(3*a*b^2 + 13*a^
3 + 16*b^3))/(63*d) + (20*exp(6*c + 6*d*x)*(6*a*b^2 + 3*a^2*b + 8*a^3 - 4*b
^3))/(63*d) - (2*exp(2*c + 2*d*x)*(8*a*b^2 + 3*a^2*b - 10*a^3 + 16*b^3))/(2
1*d) - (5*exp(8*c + 8*d*x)*(a*b^2 - a^3))/(3*d) - (2*exp(10*c + 10*d*x)*(3*
a^2*b - 2*a^3))/(9*d))/(6*exp(2*c + 2*d*x) + 15*exp(4*c + 4*d*x) + 20*exp(6
```

```
*c + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*exp(10*c + 10*d*x) + exp(12*c + 12*d*
x) + 1) - ((2*exp(2*c + 2*d*x)*(a*b^2 - a^3))/(3*d) - (2*(6*a*b^2 + 3*a^2*b
 + 8*a^3 - 4*b^3))/(63*d) + (2*exp(4*c + 4*d*x)*(3*a^2*b - 2*a^3))/(9*d))/(
3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) + ((3*a*b^2
 + 13*a^3 + 16*b^3)/(63*d) + (2*exp(2*c + 2*d*x)*(6*a*b^2 + 3*a^2*b + 8*a^3
 - 4*b^3))/(21*d) - (exp(4*c + 4*d*x)*(a*b^2 - a^3))/d - (2*exp(6*c + 6*d*x
)*(3*a^2*b - 2*a^3))/(9*d))/(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*ex
p(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) - ((a*b^2 - a^3)/(3*d) + (2*exp(2*c
+ 2*d*x)*(3*a^2*b - 2*a^3))/(9*d))/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) +
 1) + a^3*x + ((2*(6*a*b^2 + 3*a^2*b + 8*a^3 - 4*b^3))/(63*d) + (2*exp(2*c
+ 2*d*x)*(3*a*b^2 + 13*a^3 + 16*b^3))/(21*d) + (20*exp(6*c + 6*d*x)*(3*a*b^
2 + 13*a^3 + 16*b^3))/(63*d) + (10*exp(8*c + 8*d*x)*(6*a*b^2 + 3*a^2*b + 8*
a^3 - 4*b^3))/(21*d) - (2*exp(4*c + 4*d*x)*(8*a*b^2 + 3*a^2*b - 10*a^3 + 16
*b^3))/(7*d) - (2*exp(10*c + 10*d*x)*(a*b^2 - a^3))/d - (2*exp(12*c + 12*d*
x)*(3*a^2*b - 2*a^3))/(9*d))/(7*exp(2*c + 2*d*x) + 21*exp(4*c + 4*d*x) + 35
*exp(6*c + 6*d*x) + 35*exp(8*c + 8*d*x) + 21*exp(10*c + 10*d*x) + 7*exp(12*
c + 12*d*x) + exp(14*c + 14*d*x) + 1) - ((a*b^2 - a^3)/(3*d) - (exp(4*c + 4
*d*x)*(3*a*b^2 + 13*a^3 + 16*b^3))/(3*d) - (5*exp(8*c + 8*d*x)*(3*a*b^2 + 1
3*a^3 + 16*b^3))/(9*d) - (2*exp(2*c + 2*d*x)*(6*a*b^2 + 3*a^2*b + 8*a^3 - 4
*b^3))/(9*d) - (2*exp(10*c + 10*d*x)*(6*a*b^2 + 3*a^2*b + 8*a^3 - 4*b^3))/(
3*d) + (2*exp(6*c + 6*d*x)*(8*a*b^2 + 3*a^2*b - 10*a^3 + 16*b^3))/(3*d) + (
7*exp(12*c + 12*d*x)*(a*b^2 - a^3))/(3*d) + (2*exp(14*c + 14*d*x)*(3*a^2*b
 - 2*a^3))/(9*d))/(8*exp(2*c + 2*d*x) + 28*exp(4*c + 4*d*x) + 56*exp(6*c + 6
*d*x) + 70*exp(8*c + 8*d*x) + 56*exp(10*c + 10*d*x) + 28*exp(12*c + 12*d*x)
 + 8*exp(14*c + 14*d*x) + exp(16*c + 16*d*x) + 1) - ((2*(3*a^2*b - 2*a^3))/
(9*d) - (8*exp(6*c + 6*d*x)*(3*a*b^2 + 13*a^3 + 16*b^3))/(9*d) - (8*exp(10*
c + 10*d*x)*(3*a*b^2 + 13*a^3 + 16*b^3))/(9*d) - (8*exp(4*c + 4*d*x)*(6*a*b
^2 + 3*a^2*b + 8*a^3 - 4*b^3))/(9*d) - (8*exp(12*c + 12*d*x)*(6*a*b^2 + 3*a
^2*b + 8*a^3 - 4*b^3))/(9*d) + (4*exp(8*c + 8*d*x)*(8*a*b^2 + 3*a^2*b - 10*
a^3 + 16*b^3))/(3*d) + (8*exp(2*c + 2*d*x)*(a*b^2 - a^3))/(3*d) + (8*exp(14
*c + 14*d*x)*(a*b^2 - a^3))/(3*d) + (2*exp(16*c + 16*d*x)*(3*a^2*b - 2*a^3)
)/(9*d))/(9*exp(2*c + 2*d*x) + 36*exp(4*c + 4*d*x) + 84*exp(6*c + 6*d*x) +
126*exp(8*c + 8*d*x) + 126*exp(10*c + 10*d*x) + 84*exp(12*c + 12*d*x) + 36*
exp(14*c + 14*d*x) + 9*exp(16*c + 16*d*x) + exp(18*c + 18*d*x) + 1) - ((2*(
8*a*b^2 + 3*a^2*b - 10*a^3 + 16*b^3))/(105*d) - (4*exp(2*c + 2*d*x)*(3*a*b^
2 + 13*a^3 + 16*b^3))/(63*d) - (4*exp(4*c + 4*d*x)*(6*a*b^2 + 3*a^2*b + 8*a
^3 - 4*b^3))/(21*d) + (4*exp(6*c + 6*d*x)*(a*b^2 - a^3))/(3*d) + (2*exp(8*c
 + 8*d*x)*(3*a^2*b - 2*a^3))/(9*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*
x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1) - (
2*(3*a^2*b - 2*a^3))/(9*d*(exp(2*c + 2*d*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\operatorname{sech}^2(c + dx)\right)^3 \tanh^4(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)**3*tanh(d*x+c)**4,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**3*tanh(c + d*x)**4, x)
```

3.125 $\int \left(a+b\,\mathrm{sech}^2(c+dx)\right)^3 \tanh^3(c+dx)\,dx$

Optimal. Leaf size=103

$$\frac{a^3\log(\cosh(c+dx))}{d}-\frac{3a^2b\,\mathrm{sech}^2(c+dx)}{2d}-\frac{3ab^2\mathrm{sech}^4(c+dx)}{4d}+\frac{\mathrm{sech}^8(c+dx)\left(a\cosh^2(c+dx)+b\right)^4}{8bd}-\frac{b^3\mathrm{sech}^6(\ldots)}{6d}$$

```
[Out] a^3*ln(cosh(d*x+c))/d-3/2*a^2*b*sech(d*x+c)^2/d-3/4*a*b^2*sech(d*x+c)^4/d-1
/6*b^3*sech(d*x+c)^6/d+1/8*(b+a*cosh(d*x+c)^2)^4*sech(d*x+c)^8/b/d
```

Rubi [A] time = 0.10, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4138, 446, 78, 43}

$$-\frac{3a^2b\,\mathrm{sech}^2(c+dx)}{2d}+\frac{a^3\log(\cosh(c+dx))}{d}-\frac{3ab^2\mathrm{sech}^4(c+dx)}{4d}+\frac{\mathrm{sech}^8(c+dx)\left(a\cosh^2(c+dx)+b\right)^4}{8bd}-\frac{b^3\mathrm{sech}^6(\ldots)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^3*Tanh[c + d*x]^3,x]
```

```
[Out] (a^3*Log[Cosh[c + d*x]])/d - (3*a^2*b*Sech[c + d*x]^2)/(2*d) - (3*a*b^2*Sec
h[c + d*x]^4)/(4*d) - (b^3*Sech[c + d*x]^6)/(6*d) + ((b + a*Cosh[c + d*x]^2
)^4*Sech[c + d*x]^8)/(8*b*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \left(a+b\text{sech}^2(c+dx)\right)^3 \tanh^3(c+dx)\,dx = -\frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)\left(b+ax^2\right)^3}{x^9}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \frac{(1-x)(b+ax)^3}{x^5}\,dx, x, \cosh^2(c+dx)\right)}{2d}$$

$$= \frac{\left(b+a\cosh^2(c+dx)\right)^4 \text{sech}^8(c+dx)}{8bd} + \frac{\text{Subst}\left(\int \frac{(b+ax)^3}{x^4}\,dx, x, \cosh^2\right.}{2d}$$

$$= \frac{\left(b+a\cosh^2(c+dx)\right)^4 \text{sech}^8(c+dx)}{8bd} + \frac{\text{Subst}\left(\int \left(\frac{b^3}{x^4}+\frac{3ab^2}{x^3}+\frac{3a^2b}{x^2}\right.\right.}{2d}$$

$$= \frac{a^3\log(\cosh(c+dx))}{d} - \frac{3a^2b\text{sech}^2(c+dx)}{2d} - \frac{3ab^2\text{sech}^4(c+dx)}{4d} - \frac{b}{}$$

Mathematica [A] time = 0.75, size = 128, normalized size = 1.24

$$\frac{\cosh^6(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\left(24a^3\log(\cosh(c+dx))+12a^2(a-3b)\text{sech}^2(c+dx)+4b^2(3a-b)\text{sech}^6(c\right.}{3d(a\cosh(2c+2dx)+a+2b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^3*Tanh[c + d*x]^3,x]
```

```
[Out] (Cosh[c + d*x]^6*(a + b*Sech[c + d*x]^2)^3*(24*a^3*Log[Cosh[c + d*x]] + 12*
a^2*(a - 3*b)*Sech[c + d*x]^2 + 18*a*(a - b)*b*Sech[c + d*x]^4 + 4*(3*a - b
)*b^2*Sech[c + d*x]^6 + 3*b^3*Sech[c + d*x]^8))/(3*d*(a + 2*b + a*Cosh[2*c
+ 2*d*x])^3)
```

fricas [B] time = 0.51, size = 4658, normalized size = 45.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3*tanh(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/3*(3*a^3*d*x*cosh(d*x + c)^16 + 48*a^3*d*x*cosh(d*x + c)*sinh(d*x + c)^1
5 + 3*a^3*d*x*sinh(d*x + c)^16 + 6*(4*a^3*d*x - a^3 + 3*a^2*b)*cosh(d*x + c
)^14 + 6*(60*a^3*d*x*cosh(d*x + c)^2 + 4*a^3*d*x - a^3 + 3*a^2*b)*sinh(d*x
+ c)^14 + 84*(20*a^3*d*x*cosh(d*x + c)^3 + (4*a^3*d*x - a^3 + 3*a^2*b)*cosh
(d*x + c))*sinh(d*x + c)^13 + 12*(7*a^3*d*x - 3*a^3 + 6*a^2*b + 3*a*b^2)*co
sh(d*x + c)^12 + 6*(910*a^3*d*x*cosh(d*x + c)^4 + 14*a^3*d*x - 6*a^3 + 12*a
^2*b + 6*a*b^2 + 91*(4*a^3*d*x - a^3 + 3*a^2*b)*cosh(d*x + c)^2)*sinh(d*x +
 c)^12 + 24*(546*a^3*d*x*cosh(d*x + c)^5 + 91*(4*a^3*d*x - a^3 + 3*a^2*b)*c
osh(d*x + c)^3 + 6*(7*a^3*d*x - 3*a^3 + 6*a^2*b + 3*a*b^2)*cosh(d*x + c))*s
inh(d*x + c)^11 + 2*(84*a^3*d*x - 45*a^3 + 63*a^2*b + 24*a*b^2 + 16*b^3)*co
sh(d*x + c)^10 + 2*(12012*a^3*d*x*cosh(d*x + c)^6 + 84*a^3*d*x + 3003*(4*a^
3*d*x - a^3 + 3*a^2*b)*cosh(d*x + c)^4 - 45*a^3 + 63*a^2*b + 24*a*b^2 + 16*
b^3 + 396*(7*a^3*d*x - 3*a^3 + 6*a^2*b + 3*a*b^2)*cosh(d*x + c)^2)*sinh(d*x
 + c)^10 + 4*(8580*a^3*d*x*cosh(d*x + c)^7 + 3003*(4*a^3*d*x - a^3 + 3*a^2*
b)*cosh(d*x + c)^5 + 660*(7*a^3*d*x - 3*a^3 + 6*a^2*b + 3*a*b^2)*cosh(d*x +
 c)^3 + 5*(84*a^3*d*x - 45*a^3 + 63*a^2*b + 24*a*b^2 + 16*b^3)*cosh(d*x + c
))*sinh(d*x + c)^9 + 2*(105*a^3*d*x - 60*a^3 + 72*a^2*b + 12*a*b^2 - 16*b^3
)*cosh(d*x + c)^8 + 2*(19305*a^3*d*x*cosh(d*x + c)^8 + 9009*(4*a^3*d*x - a^
3 + 3*a^2*b)*cosh(d*x + c)^6 + 105*a^3*d*x + 2970*(7*a^3*d*x - 3*a^3 + 6*a^
2*b + 3*a*b^2)*cosh(d*x + c)^4 - 60*a^3 + 72*a^2*b + 12*a*b^2 - 16*b^3 + 45
*(84*a^3*d*x - 45*a^3 + 63*a^2*b + 24*a*b^2 + 16*b^3)*cosh(d*x + c)^2)*sinh
(d*x + c)^8 + 16*(2145*a^3*d*x*cosh(d*x + c)^9 + 1287*(4*a^3*d*x - a^3 + 3*
a^2*b)*cosh(d*x + c)^7 + 594*(7*a^3*d*x - 3*a^3 + 6*a^2*b + 3*a*b^2)*cosh(d
*x + c)^5 + 15*(84*a^3*d*x - 45*a^3 + 63*a^2*b + 24*a*b^2 + 16*b^3)*cosh(d*
x + c)^3 + (105*a^3*d*x - 60*a^3 + 72*a^2*b + 12*a*b^2 - 16*b^3)*cosh(d*x +
 c))*sinh(d*x + c)^7 + 2*(84*a^3*d*x - 45*a^3 + 63*a^2*b + 24*a*b^2 + 16*b^
3)*cosh(d*x + c)^6 + 2*(12012*a^3*d*x*cosh(d*x + c)^10 + 9009*(4*a^3*d*x -
a^3 + 3*a^2*b)*cosh(d*x + c)^8 + 5544*(7*a^3*d*x - 3*a^3 + 6*a^2*b + 3*a*b^
2)*cosh(d*x + c)^6 + 84*a^3*d*x + 210*(84*a^3*d*x - 45*a^3 + 63*a^2*b + 24*
a*b^2 + 16*b^3)*cosh(d*x + c)^4 - 45*a^3 + 63*a^2*b + 24*a*b^2 + 16*b^3 + 2
8*(105*a^3*d*x - 60*a^3 + 72*a^2*b + 12*a*b^2 - 16*b^3)*cosh(d*x + c)^2)*si
nh(d*x + c)^6 + 4*(3276*a^3*d*x*cosh(d*x + c)^11 + 3003*(4*a^3*d*x - a^3 +
3*a^2*b)*cosh(d*x + c)^9 + 2376*(7*a^3*d*x - 3*a^3 + 6*a^2*b + 3*a*b^2)*cos
h(d*x + c)^7 + 126*(84*a^3*d*x - 45*a^3 + 63*a^2*b + 24*a*b^2 + 16*b^3)*cos
h(d*x + c)^5 + 28*(105*a^3*d*x - 60*a^3 + 72*a^2*b + 12*a*b^2 - 16*b^3)*cos
h(d*x + c)^3 + 3*(84*a^3*d*x - 45*a^3 + 63*a^2*b + 24*a*b^2 + 16*b^3)*cosh(
d*x + c))*sinh(d*x + c)^5 + 3*a^3*d*x + 12*(7*a^3*d*x - 3*a^3 + 6*a^2*b + 3
```

*a*b^2)*cosh(d*x + c)^4 + 2*(2730*a^3*d*x*cosh(d*x + c)^12 + 3003*(4*a^3*d*
x - a^3 + 3*a^2*b)*cosh(d*x + c)^10 + 2970*(7*a^3*d*x - 3*a^3 + 6*a^2*b + 3
*a*b^2)*cosh(d*x + c)^8 + 210*(84*a^3*d*x - 45*a^3 + 63*a^2*b + 24*a*b^2 +
16*b^3)*cosh(d*x + c)^6 + 42*a^3*d*x + 70*(105*a^3*d*x - 60*a^3 + 72*a^2*b
+ 12*a*b^2 - 16*b^3)*cosh(d*x + c)^4 - 18*a^3 + 36*a^2*b + 18*a*b^2 + 15*(8
4*a^3*d*x - 45*a^3 + 63*a^2*b + 24*a*b^2 + 16*b^3)*cosh(d*x + c)^2)*sinh(d*
x + c)^4 + 8*(210*a^3*d*x*cosh(d*x + c)^13 + 273*(4*a^3*d*x - a^3 + 3*a^2*b
)*cosh(d*x + c)^11 + 330*(7*a^3*d*x - 3*a^3 + 6*a^2*b + 3*a*b^2)*cosh(d*x +
c)^9 + 30*(84*a^3*d*x - 45*a^3 + 63*a^2*b + 24*a*b^2 + 16*b^3)*cosh(d*x +
c)^7 + 14*(105*a^3*d*x - 60*a^3 + 72*a^2*b + 12*a*b^2 - 16*b^3)*cosh(d*x +
c)^5 + 5*(84*a^3*d*x - 45*a^3 + 63*a^2*b + 24*a*b^2 + 16*b^3)*cosh(d*x + c)
^3 + 6*(7*a^3*d*x - 3*a^3 + 6*a^2*b + 3*a*b^2)*cosh(d*x + c))*sinh(d*x + c)
^3 + 6*(4*a^3*d*x - a^3 + 3*a^2*b)*cosh(d*x + c)^2 + 2*(180*a^3*d*x*cosh(d*
x + c)^14 + 273*(4*a^3*d*x - a^3 + 3*a^2*b)*cosh(d*x + c)^12 + 396*(7*a^3*d
*x - 3*a^3 + 6*a^2*b + 3*a*b^2)*cosh(d*x + c)^10 + 45*(84*a^3*d*x - 45*a^3
+ 63*a^2*b + 24*a*b^2 + 16*b^3)*cosh(d*x + c)^8 + 28*(105*a^3*d*x - 60*a^3
+ 72*a^2*b + 12*a*b^2 - 16*b^3)*cosh(d*x + c)^6 + 12*a^3*d*x + 15*(84*a^3*d
*x - 45*a^3 + 63*a^2*b + 24*a*b^2 + 16*b^3)*cosh(d*x + c)^4 - 3*a^3 + 9*a^2
b + 36(7*a^3*d*x - 3*a^3 + 6*a^2*b + 3*a*b^2)*cosh(d*x + c)^2)*sinh(d*x +
c)^2 - 3*(a^3*cosh(d*x + c)^16 + 16*a^3*cosh(d*x + c)*sinh(d*x + c)^15 + a
^3*sinh(d*x + c)^16 + 8*a^3*cosh(d*x + c)^14 + 28*a^3*cosh(d*x + c)^12 + 8*
(15*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^14 + 112*(5*a^3*cosh(d*x + c)^
3 + a^3*cosh(d*x + c))*sinh(d*x + c)^13 + 56*a^3*cosh(d*x + c)^10 + 28*(65*
a^3*cosh(d*x + c)^4 + 26*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^12 + 112*
(39*a^3*cosh(d*x + c)^5 + 26*a^3*cosh(d*x + c)^3 + 3*a^3*cosh(d*x + c))*sin
h(d*x + c)^11 + 70*a^3*cosh(d*x + c)^8 + 56*(143*a^3*cosh(d*x + c)^6 + 143*
a^3*cosh(d*x + c)^4 + 33*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^10 + 16*(
715*a^3*cosh(d*x + c)^7 + 1001*a^3*cosh(d*x + c)^5 + 385*a^3*cosh(d*x + c)^
3 + 35*a^3*cosh(d*x + c))*sinh(d*x + c)^9 + 56*a^3*cosh(d*x + c)^6 + 2*(643
5*a^3*cosh(d*x + c)^8 + 12012*a^3*cosh(d*x + c)^6 + 6930*a^3*cosh(d*x + c)^
4 + 1260*a^3*cosh(d*x + c)^2 + 35*a^3)*sinh(d*x + c)^8 + 16*(715*a^3*cosh(d
*x + c)^9 + 1716*a^3*cosh(d*x + c)^7 + 1386*a^3*cosh(d*x + c)^5 + 420*a^3*c
osh(d*x + c)^3 + 35*a^3*cosh(d*x + c))*sinh(d*x + c)^7 + 28*a^3*cosh(d*x +
c)^4 + 56*(143*a^3*cosh(d*x + c)^10 + 429*a^3*cosh(d*x + c)^8 + 462*a^3*cos
h(d*x + c)^6 + 210*a^3*cosh(d*x + c)^4 + 35*a^3*cosh(d*x + c)^2 + a^3)*sinh
(d*x + c)^6 + 112*(39*a^3*cosh(d*x + c)^11 + 143*a^3*cosh(d*x + c)^9 + 198*
a^3*cosh(d*x + c)^7 + 126*a^3*cosh(d*x + c)^5 + 35*a^3*cosh(d*x + c)^3 + 3*
a^3*cosh(d*x + c))*sinh(d*x + c)^5 + 8*a^3*cosh(d*x + c)^2 + 28*(65*a^3*cos
h(d*x + c)^12 + 286*a^3*cosh(d*x + c)^10 + 495*a^3*cosh(d*x + c)^8 + 420*a^
3*cosh(d*x + c)^6 + 175*a^3*cosh(d*x + c)^4 + 30*a^3*cosh(d*x + c)^2 + a^3)
*sinh(d*x + c)^4 + 112*(5*a^3*cosh(d*x + c)^13 + 26*a^3*cosh(d*x + c)^11 +
55*a^3*cosh(d*x + c)^9 + 60*a^3*cosh(d*x + c)^7 + 35*a^3*cosh(d*x + c)^5 +
10*a^3*cosh(d*x + c)^3 + a^3*cosh(d*x + c))*sinh(d*x + c)^3 + a^3 + 8*(15*a
^3*cosh(d*x + c)^14 + 91*a^3*cosh(d*x + c)^12 + 231*a^3*cosh(d*x + c)^10 +
315*a^3*cosh(d*x + c)^8 + 245*a^3*cosh(d*x + c)^6 + 105*a^3*cosh(d*x + c)^4

```
 + 21*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^2 + 16*(a^3*cosh(d*x + c)^15
 + 7*a^3*cosh(d*x + c)^13 + 21*a^3*cosh(d*x + c)^11 + 35*a^3*cosh(d*x + c)^
9 + 35*a^3*cosh(d*x + c)^7 + 21*a^3*cosh(d*x + c)^5 + 7*a^3*cosh(d*x + c)^3
 + a^3*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - s
inh(d*x + c))) + 4*(12*a^3*d*x*cosh(d*x + c)^15 + 21*(4*a^3*d*x - a^3 + 3*a
^2*b)*cosh(d*x + c)^13 + 36*(7*a^3*d*x - 3*a^3 + 6*a^2*b + 3*a*b^2)*cosh(d*
x + c)^11 + 5*(84*a^3*d*x - 45*a^3 + 63*a^2*b + 24*a*b^2 + 16*b^3)*cosh(d*x
 + c)^9 + 4*(105*a^3*d*x - 60*a^3 + 72*a^2*b + 12*a*b^2 - 16*b^3)*cosh(d*x
+ c)^7 + 3*(84*a^3*d*x - 45*a^3 + 63*a^2*b + 24*a*b^2 + 16*b^3)*cosh(d*x +
c)^5 + 12*(7*a^3*d*x - 3*a^3 + 6*a^2*b + 3*a*b^2)*cosh(d*x + c)^3 + 3*(4*a^
3*d*x - a^3 + 3*a^2*b)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^16 +
16*d*cosh(d*x + c)*sinh(d*x + c)^15 + d*sinh(d*x + c)^16 + 8*d*cosh(d*x + c
)^14 + 8*(15*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^14 + 112*(5*d*cosh(d*x +
c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^13 + 28*d*cosh(d*x + c)^12 + 28*(65*d
*cosh(d*x + c)^4 + 26*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^12 + 112*(39*d*c
osh(d*x + c)^5 + 26*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^11
 + 56*d*cosh(d*x + c)^10 + 56*(143*d*cosh(d*x + c)^6 + 143*d*cosh(d*x + c)^
4 + 33*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^10 + 16*(715*d*cosh(d*x + c)^7
+ 1001*d*cosh(d*x + c)^5 + 385*d*cosh(d*x + c)^3 + 35*d*cosh(d*x + c))*sinh
(d*x + c)^9 + 70*d*cosh(d*x + c)^8 + 2*(6435*d*cosh(d*x + c)^8 + 12012*d*co
sh(d*x + c)^6 + 6930*d*cosh(d*x + c)^4 + 1260*d*cosh(d*x + c)^2 + 35*d)*sin
h(d*x + c)^8 + 16*(715*d*cosh(d*x + c)^9 + 1716*d*cosh(d*x + c)^7 + 1386*d*
cosh(d*x + c)^5 + 420*d*cosh(d*x + c)^3 + 35*d*cosh(d*x + c))*sinh(d*x + c)
^7 + 56*d*cosh(d*x + c)^6 + 56*(143*d*cosh(d*x + c)^10 + 429*d*cosh(d*x + c
)^8 + 462*d*cosh(d*x + c)^6 + 210*d*cosh(d*x + c)^4 + 35*d*cosh(d*x + c)^2
+ d)*sinh(d*x + c)^6 + 112*(39*d*cosh(d*x + c)^11 + 143*d*cosh(d*x + c)^9 +
 198*d*cosh(d*x + c)^7 + 126*d*cosh(d*x + c)^5 + 35*d*cosh(d*x + c)^3 + 3*d
*cosh(d*x + c))*sinh(d*x + c)^5 + 28*d*cosh(d*x + c)^4 + 28*(65*d*cosh(d*x
+ c)^12 + 286*d*cosh(d*x + c)^10 + 495*d*cosh(d*x + c)^8 + 420*d*cosh(d*x +
 c)^6 + 175*d*cosh(d*x + c)^4 + 30*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^4 +
 112*(5*d*cosh(d*x + c)^13 + 26*d*cosh(d*x + c)^11 + 55*d*cosh(d*x + c)^9 +
 60*d*cosh(d*x + c)^7 + 35*d*cosh(d*x + c)^5 + 10*d*cosh(d*x + c)^3 + d*cos
h(d*x + c))*sinh(d*x + c)^3 + 8*d*cosh(d*x + c)^2 + 8*(15*d*cosh(d*x + c)^1
4 + 91*d*cosh(d*x + c)^12 + 231*d*cosh(d*x + c)^10 + 315*d*cosh(d*x + c)^8
+ 245*d*cosh(d*x + c)^6 + 105*d*cosh(d*x + c)^4 + 21*d*cosh(d*x + c)^2 + d)
*sinh(d*x + c)^2 + 16*(d*cosh(d*x + c)^15 + 7*d*cosh(d*x + c)^13 + 21*d*cos
h(d*x + c)^11 + 35*d*cosh(d*x + c)^9 + 35*d*cosh(d*x + c)^7 + 21*d*cosh(d*x
 + c)^5 + 7*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

giac [B] time = 0.27, size = 384, normalized size = 3.73

$$-\frac{840\,a^3dx - 840\,a^3\log\left(e^{(2\,dx+2\,c)}+1\right) + \frac{2283\,a^3e^{(16\,dx+16\,c)}+16584\,a^3e^{(14\,dx+14\,c)}+5040\,a^2be^{(14\,dx+14\,c)}+53844\,a^3e^{(12\,dx+12\,c)}+20160\,a^2}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3*tanh(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/840*(840*a^3*d*x - 840*a^3*log(e^(2*d*x + 2*c) + 1) + (2283*a^3*e^(16*d*
x + 16*c) + 16584*a^3*e^(14*d*x + 14*c) + 5040*a^2*b*e^(14*d*x + 14*c) + 53
844*a^3*e^(12*d*x + 12*c) + 20160*a^2*b*e^(12*d*x + 12*c) + 10080*a*b^2*e^(
12*d*x + 12*c) + 102648*a^3*e^(10*d*x + 10*c) + 35280*a^2*b*e^(10*d*x + 10*
c) + 13440*a*b^2*e^(10*d*x + 10*c) + 8960*b^3*e^(10*d*x + 10*c) + 126210*a^
3*e^(8*d*x + 8*c) + 40320*a^2*b*e^(8*d*x + 8*c) + 6720*a*b^2*e^(8*d*x + 8*c
) - 8960*b^3*e^(8*d*x + 8*c) + 102648*a^3*e^(6*d*x + 6*c) + 35280*a^2*b*e^(
6*d*x + 6*c) + 13440*a*b^2*e^(6*d*x + 6*c) + 8960*b^3*e^(6*d*x + 6*c) + 538
44*a^3*e^(4*d*x + 4*c) + 20160*a^2*b*e^(4*d*x + 4*c) + 10080*a*b^2*e^(4*d*x
 + 4*c) + 16584*a^3*e^(2*d*x + 2*c) + 5040*a^2*b*e^(2*d*x + 2*c) + 2283*a^3
)/(e^(2*d*x + 2*c) + 1)^8)/d
```

maple [A] time = 0.30, size = 156, normalized size = 1.51

$$\frac{a^3 \ln(\cosh(dx+c))}{d} - \frac{(\tanh^2(dx+c))\,a^3}{2d} - \frac{3a^2b\,(\sinh^2(dx+c))}{2d\cosh(dx+c)^4} - \frac{3a^2b}{4d\cosh(dx+c)^4} - \frac{3a\,b^2\,(\sinh^2(dx+c))}{4d\cosh(dx+c)^6} - \frac{}{4d\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)^3*tanh(d*x+c)^3,x)
```

```
[Out] a^3*ln(cosh(d*x+c))/d-1/2/d*tanh(d*x+c)^2*a^3-3/2/d*a^2*b*sinh(d*x+c)^2/cos
h(d*x+c)^4-3/4/d*a^2*b/cosh(d*x+c)^4-3/4/d*a*b^2*sinh(d*x+c)^2/cosh(d*x+c)^
6-1/4/d*a*b^2/cosh(d*x+c)^6-1/6/d*b^3*sinh(d*x+c)^2/cosh(d*x+c)^8-1/24/d*b^
3/cosh(d*x+c)^8
```

maxima [B] time = 0.46, size = 652, normalized size = 6.33

$$\frac{3\,a^2b\tanh(dx+c)^4}{4\,d} + a^3\left(x + \frac{c}{d} + \frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d} + \frac{2\,e^{(-2\,dx-2\,c)}}{d\left(2\,e^{(-2\,dx-2\,c)} + e^{(-4\,dx-4\,c)} + 1\right)}\right) - 4\,ab^2\left(\frac{}{d\left(6\,e^{(-2\,dx-2\,c)} + 1\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3*tanh(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 3/4*a^2*b*tanh(d*x + c)^4/d + a^3*(x + c/d + log(e^(-2*d*x - 2*c) + 1)/d +
2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) - 4*a*b
^2*(3*e^(-4*d*x - 4*c)/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x - 4*c) + 20*e^
(-6*d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) + e^(-12*d*x -
12*c) + 1)) - 2*e^(-6*d*x - 6*c)/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x - 4*
c) + 20*e^(-6*d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) + e^(
-12*d*x - 12*c) + 1)) + 3*e^(-8*d*x - 8*c)/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-
4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 1
0*c) + e^(-12*d*x - 12*c) + 1))) - 32/3*b^3*(e^(-6*d*x - 6*c)/(d*(8*e^(-2*d
```

```
*x - 2*c) + 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x - 6*c) + 70*e^(-8*d*x - 8*c)
 + 56*e^(-10*d*x - 10*c) + 28*e^(-12*d*x - 12*c) + 8*e^(-14*d*x - 14*c) + e
^(-16*d*x - 16*c) + 1)) - e^(-8*d*x - 8*c)/(d*(8*e^(-2*d*x - 2*c) + 28*e^(-
4*d*x - 4*c) + 56*e^(-6*d*x - 6*c) + 70*e^(-8*d*x - 8*c) + 56*e^(-10*d*x -
10*c) + 28*e^(-12*d*x - 12*c) + 8*e^(-14*d*x - 14*c) + e^(-16*d*x - 16*c) +
 1)) + e^(-10*d*x - 10*c)/(d*(8*e^(-2*d*x - 2*c) + 28*e^(-4*d*x - 4*c) + 56
*e^(-6*d*x - 6*c) + 70*e^(-8*d*x - 8*c) + 56*e^(-10*d*x - 10*c) + 28*e^(-12
*d*x - 12*c) + 8*e^(-14*d*x - 14*c) + e^(-16*d*x - 16*c) + 1)))
```

mupad [B] time = 1.64, size = 573, normalized size = 5.56

$$\frac{32\left(3\,a\,b^2-5\,b^3\right)}{d\,\left(5\,{\mathrm{e}}^{2\,c+2\,d\,x}+10\,{\mathrm{e}}^{4\,c+4\,d\,x}+10\,{\mathrm{e}}^{6\,c+6\,d\,x}+5\,{\mathrm{e}}^{8\,c+8\,d\,x}+{\mathrm{e}}^{10\,c+10\,d\,x}+1\right)}-a^3\,x-\frac{}{d\,\left(7\,{\mathrm{e}}^{2\,c+2\,d\,x}+21\,{\mathrm{e}}^{4\,c+4\,d\,x}+35\,{\mathrm{e}}^{6\,c}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^3*(a + b/cosh(c + d*x)^2)^3,x)
```

```
[Out] (32*(3*a*b^2 - 5*b^3))/(d*(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*ex
p(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1)) - a^3*x - (1
28*b^3)/(d*(7*exp(2*c + 2*d*x) + 21*exp(4*c + 4*d*x) + 35*exp(6*c + 6*d*x)
+ 35*exp(8*c + 8*d*x) + 21*exp(10*c + 10*d*x) + 7*exp(12*c + 12*d*x) + exp(
14*c + 14*d*x) + 1)) - (32*(3*a*b^2 - 19*b^3))/(3*d*(6*exp(2*c + 2*d*x) + 1
5*exp(4*c + 4*d*x) + 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*exp(10*c
 + 10*d*x) + exp(12*c + 12*d*x) + 1)) + (32*b^3)/(d*(8*exp(2*c + 2*d*x) + 2
8*exp(4*c + 4*d*x) + 56*exp(6*c + 6*d*x) + 70*exp(8*c + 8*d*x) + 56*exp(10*
c + 10*d*x) + 28*exp(12*c + 12*d*x) + 8*exp(14*c + 14*d*x) + exp(16*c + 16*
d*x) + 1)) - (8*(9*a^2*b - 21*a*b^2 + 4*b^3))/(3*d*(3*exp(2*c + 2*d*x) + 3*
exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) + (4*(3*a^2*b - 27*a*b^2 + 16*b^3
))/(d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8
*c + 8*d*x) + 1)) - (2*(3*a^2*b - a^3))/(d*(exp(2*c + 2*d*x) + 1)) + (a^3*l
og(exp(2*c)*exp(2*d*x) + 1))/d - (2*(6*a*b^2 - 9*a^2*b + a^3))/(d*(2*exp(2*
c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

sympy [A] time = 13.79, size = 178, normalized size = 1.73

$$\begin{cases} a^3x-\frac{a^3\log\left(\tanh\left(c+dx\right)+1\right)}{d}-\frac{a^3\tanh^2\left(c+dx\right)}{2d}-\frac{3a^2b\tanh^2\left(c+dx\right)\operatorname{sech}^2\left(c+dx\right)}{4d}-\frac{3a^2b\operatorname{sech}^2\left(c+dx\right)}{4d}-\frac{ab^2\tanh^2\left(c+dx\right)\operatorname{sech}^4\left(c+dx\right)}{2d}- \\ x\left(a+b\operatorname{sech}^2\left(c\right)\right)^3\tanh^3\left(c\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)**3*tanh(d*x+c)**3,x)
```

```
[Out] Piecewise((a**3*x - a**3*log(tanh(c + d*x) + 1)/d - a**3*tanh(c + d*x)**2/(
2*d) - 3*a**2*b*tanh(c + d*x)**2*sech(c + d*x)**2/(4*d) - 3*a**2*b*sech(c +
 d*x)**2/(4*d) - a*b**2*tanh(c + d*x)**2*sech(c + d*x)**4/(2*d) - a*b**2*se
ch(c + d*x)**4/(4*d) - b**3*tanh(c + d*x)**2*sech(c + d*x)**6/(8*d) - b**3*
sech(c + d*x)**6/(24*d), Ne(d, 0)), (x*(a + b*sech(c)**2)**3*tanh(c)**3, Tr
ue))
```

3.126 $\int \left(a+b\,\mathbf{sech}^2(c+dx)\right)^3 \tanh^2(c+dx)\,dx$

Optimal. Leaf size=92

$$-\frac{a^3\tanh(c+dx)}{d}+a^3x+\frac{b\left(3a^2+3ab+b^2\right)\tanh^3(c+dx)}{3d}-\frac{b^2(3a+2b)\tanh^5(c+dx)}{5d}+\frac{b^3\tanh^7(c+dx)}{7d}$$

```
[Out] a^3*x-a^3*tanh(d*x+c)/d+1/3*b*(3*a^2+3*a*b+b^2)*tanh(d*x+c)^3/d-1/5*b^2*(3*
a+2*b)*tanh(d*x+c)^5/d+1/7*b^3*tanh(d*x+c)^7/d
```

Rubi [A] time = 0.11, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4141, 1802, 206}

$$\frac{b\left(3a^2+3ab+b^2\right)\tanh^3(c+dx)}{3d}-\frac{a^3\tanh(c+dx)}{d}+a^3x-\frac{b^2(3a+2b)\tanh^5(c+dx)}{5d}+\frac{b^3\tanh^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^3*Tanh[c + d*x]^2,x]
```

```
[Out] a^3*x - (a^3*Tanh[c + d*x])/d + (b*(3*a^2 + 3*a*b + b^2)*Tanh[c + d*x]^3)/(
3*d) - (b^2*(3*a + 2*b)*Tanh[c + d*x]^5)/(5*d) + (b^3*Tanh[c + d*x]^7)/(7*d
)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \left(a+b\text{sech}^2(c+dx)\right)^3\tanh^2(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x^2\left(a+b\left(1-x^2\right)\right)^3}{1-x^2}\,dx,x,\tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-a^3+b\left(3a^2+3ab+b^2\right)x^2-b^2(3a+2b)x^4+b^3x^6+\frac{a^3}{1-x}\right.\right.}{d}$$

$$= -\frac{a^3\tanh(c+dx)}{d}+\frac{b\left(3a^2+3ab+b^2\right)\tanh^3(c+dx)}{3d}-\frac{b^2(3a+2b)}{}$$

$$= a^3x-\frac{a^3\tanh(c+dx)}{d}+\frac{b\left(3a^2+3ab+b^2\right)\tanh^3(c+dx)}{3d}-\frac{b^2(3a+}{}$$

Mathematica [B] time = 1.79, size = 479, normalized size = 5.21

$$\frac{\text{sech}(c)\text{sech}^7(c+dx)\left(3150a^3\sinh(2c+dx)-3150a^3\sinh(2c+3dx)+1260a^3\sinh(4c+3dx)-1260a^3\sinh(4c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^3*Tanh[c + d*x]^2,x]
```

```
[Out] (Sech[c]*Sech[c + d*x]^7*(3675*a^3*d*x*Cosh[d*x] + 3675*a^3*d*x*Cosh[2*c +
d*x] + 2205*a^3*d*x*Cosh[2*c + 3*d*x] + 2205*a^3*d*x*Cosh[4*c + 3*d*x] + 73
5*a^3*d*x*Cosh[4*c + 5*d*x] + 735*a^3*d*x*Cosh[6*c + 5*d*x] + 105*a^3*d*x*C
osh[6*c + 7*d*x] + 105*a^3*d*x*Cosh[8*c + 7*d*x] - 4200*a^3*Sinh[d*x] + 336
0*a^2*b*Sinh[d*x] + 840*a*b^2*Sinh[d*x] - 560*b^3*Sinh[d*x] + 3150*a^3*Sinh
[2*c + d*x] - 3990*a^2*b*Sinh[2*c + d*x] - 2100*a*b^2*Sinh[2*c + d*x] - 112
0*b^3*Sinh[2*c + d*x] - 3150*a^3*Sinh[2*c + 3*d*x] + 1890*a^2*b*Sinh[2*c +
3*d*x] + 504*a*b^2*Sinh[2*c + 3*d*x] + 336*b^3*Sinh[2*c + 3*d*x] + 1260*a^3
*Sinh[4*c + 3*d*x] - 2520*a^2*b*Sinh[4*c + 3*d*x] - 1260*a*b^2*Sinh[4*c + 3
*d*x] - 1260*a^3*Sinh[4*c + 5*d*x] + 840*a^2*b*Sinh[4*c + 5*d*x] + 588*a*b^
2*Sinh[4*c + 5*d*x] + 112*b^3*Sinh[4*c + 5*d*x] + 210*a^3*Sinh[6*c + 5*d*x]
 - 630*a^2*b*Sinh[6*c + 5*d*x] - 210*a^3*Sinh[6*c + 7*d*x] + 210*a^2*b*Sinh
[6*c + 7*d*x] + 84*a*b^2*Sinh[6*c + 7*d*x] + 16*b^3*Sinh[6*c + 7*d*x]))/(13
440*d)
```

fricas [B] time = 0.42, size = 881, normalized size = 9.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3*tanh(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/105*((105*a^3*d*x + 105*a^3 - 105*a^2*b - 42*a*b^2 - 8*b^3)*cosh(d*x + c)
^7 + 7*(105*a^3*d*x + 105*a^3 - 105*a^2*b - 42*a*b^2 - 8*b^3)*cosh(d*x + c)
*sinh(d*x + c)^6 - (105*a^3 - 105*a^2*b - 42*a*b^2 - 8*b^3)*sinh(d*x + c)^7
 + 7*(105*a^3*d*x + 105*a^3 - 105*a^2*b - 42*a*b^2 - 8*b^3)*cosh(d*x + c)^5
 - 7*(75*a^3 - 15*a^2*b - 42*a*b^2 - 8*b^3 + 3*(105*a^3 - 105*a^2*b - 42*a*
b^2 - 8*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 35*((105*a^3*d*x + 105*a^3
- 105*a^2*b - 42*a*b^2 - 8*b^3)*cosh(d*x + c)^3 + (105*a^3*d*x + 105*a^3 -
105*a^2*b - 42*a*b^2 - 8*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 21*(105*a^3*
d*x + 105*a^3 - 105*a^2*b - 42*a*b^2 - 8*b^3)*cosh(d*x + c)^3 - 7*(5*(105*a
^3 - 105*a^2*b - 42*a*b^2 - 8*b^3)*cosh(d*x + c)^4 + 135*a^3 + 45*a^2*b + 5
4*a*b^2 - 24*b^3 + 10*(75*a^3 - 15*a^2*b - 42*a*b^2 - 8*b^3)*cosh(d*x + c)^
2)*sinh(d*x + c)^3 + 7*(3*(105*a^3*d*x + 105*a^3 - 105*a^2*b - 42*a*b^2 - 8
*b^3)*cosh(d*x + c)^5 + 10*(105*a^3*d*x + 105*a^3 - 105*a^2*b - 42*a*b^2 -
8*b^3)*cosh(d*x + c)^3 + 9*(105*a^3*d*x + 105*a^3 - 105*a^2*b - 42*a*b^2 -
8*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 35*(105*a^3*d*x + 105*a^3 - 105*a^2
*b - 42*a*b^2 - 8*b^3)*cosh(d*x + c) - 7*((105*a^3 - 105*a^2*b - 42*a*b^2 -
 8*b^3)*cosh(d*x + c)^6 + 5*(75*a^3 - 15*a^2*b - 42*a*b^2 - 8*b^3)*cosh(d*x
 + c)^4 + 75*a^3 + 45*a^2*b + 90*a*b^2 + 120*b^3 + 9*(45*a^3 + 15*a^2*b + 1
8*a*b^2 - 8*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^7 + 7*d*c
osh(d*x + c)*sinh(d*x + c)^6 + 7*d*cosh(d*x + c)^5 + 35*(d*cosh(d*x + c)^3
+ d*cosh(d*x + c))*sinh(d*x + c)^4 + 21*d*cosh(d*x + c)^3 + 7*(3*d*cosh(d*x
 + c)^5 + 10*d*cosh(d*x + c)^3 + 9*d*cosh(d*x + c))*sinh(d*x + c)^2 + 35*d*
cosh(d*x + c))
```

giac [B] time = 0.20, size = 356, normalized size = 3.87

$$105\,a^3dx + \frac{2\left(105\,a^3e^{(12\,dx+12\,c)} - 315\,a^2be^{(12\,dx+12\,c)} + 630\,a^3e^{(10\,dx+10\,c)} - 1260\,a^2be^{(10\,dx+10\,c)} - 630\,ab^2e^{(10\,dx+10\,c)} + 1575\,a^3e^{(8\,dx+8\,c)} - 1995\,a^2be^{(8\,c}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3*tanh(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/105*(105*a^3*d*x + 2*(105*a^3*e^(12*d*x + 12*c) - 315*a^2*b*e^(12*d*x + 1
2*c) + 630*a^3*e^(10*d*x + 10*c) - 1260*a^2*b*e^(10*d*x + 10*c) - 630*a*b^2
*e^(10*d*x + 10*c) + 1575*a^3*e^(8*d*x + 8*c) - 1995*a^2*b*e^(8*d*x + 8*c)
- 1050*a*b^2*e^(8*d*x + 8*c) - 560*b^3*e^(8*d*x + 8*c) + 2100*a^3*e^(6*d*x
+ 6*c) - 1680*a^2*b*e^(6*d*x + 6*c) - 420*a*b^2*e^(6*d*x + 6*c) + 280*b^3*e
^(6*d*x + 6*c) + 1575*a^3*e^(4*d*x + 4*c) - 945*a^2*b*e^(4*d*x + 4*c) - 252
*a*b^2*e^(4*d*x + 4*c) - 168*b^3*e^(4*d*x + 4*c) + 630*a^3*e^(2*d*x + 2*c)
- 420*a^2*b*e^(2*d*x + 2*c) - 294*a*b^2*e^(2*d*x + 2*c) - 56*b^3*e^(2*d*x +
 2*c) + 105*a^3 - 105*a^2*b - 42*a*b^2 - 8*b^3)/(e^(2*d*x + 2*c) + 1)^7)/d
```

maple [B] time = 0.52, size = 180, normalized size = 1.96

$$\frac{a^3\left(dx+c-\tanh\left(dx+c\right)\right)+3a^2b\left(-\frac{\sinh(dx+c)}{2\cosh(dx+c)^3}+\frac{\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)}{2}\right)+3a\,b^2\left(-\frac{\sinh(dx+c)}{4\cosh(dx+c)^5}+\frac{\left(\frac{8}{15}+\frac{\operatorname{sech}(dx+c)^4}{5}+\cdots\right.}{}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)^3*tanh(d*x+c)^2,x)
```

```
[Out] 1/d*(a^3*(d*x+c-tanh(d*x+c))+3*a^2*b*(-1/2*sinh(d*x+c)/cosh(d*x+c)^3+1/2*(2
/3+1/3*sech(d*x+c)^2)*tanh(d*x+c))+3*a*b^2*(-1/4*sinh(d*x+c)/cosh(d*x+c)^5+
1/4*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)*tanh(d*x+c))+b^3*(-1/6*sinh
(d*x+c)/cosh(d*x+c)^7+1/6*(16/35+1/7*sech(d*x+c)^6+6/35*sech(d*x+c)^4+8/35*
sech(d*x+c)^2)*tanh(d*x+c)))
```

maxima [B] time = 0.44, size = 788, normalized size = 8.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3*tanh(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] a^2*b*tanh(d*x + c)^3/d + a^3*(x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1))) + 16
/105*b^3*(7*e^(-2*d*x - 2*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) +
 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-
12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 21*e^(-4*d*x - 4*c)/(d*(7*e^(-2
*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*
c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)
) - 35*e^(-6*d*x - 6*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e
^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*
x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 70*e^(-8*d*x - 8*c)/(d*(7*e^(-2*d*x
- 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) +
21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 1
/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^
(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x
- 14*c) + 1))) + 4/5*a*b^2*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 10*
e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x -
10*c) + 1)) - 5*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*
c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) +
15*e^(-6*d*x - 6*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6
*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2
*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c
) + e^(-10*d*x - 10*c) + 1)))
```

mupad [B] time = 0.21, size = 1133, normalized size = 12.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^2*(a + b/cosh(c + d*x)^2)^3,x)
```

```
[Out] a^3*x - ((2*(2*a*b^2 + a^2*b - a^3))/(7*d) - (2*exp(2*c + 2*d*x)*(3*a^2*b +
 15*a^3 - 16*b^3))/(21*d) - (4*exp(6*c + 6*d*x)*(3*a^2*b + 15*a^3 - 16*b^3)
)/(21*d) + (10*exp(8*c + 8*d*x)*(2*a*b^2 + a^2*b - a^3))/(7*d) - (4*exp(4*c
 + 4*d*x)*(6*a*b^2 + 3*a^2*b + 5*a^3 + 8*b^3))/(7*d) + (2*exp(10*c + 10*d*x
)*(3*a^2*b - a^3))/(7*d))/(6*exp(2*c + 2*d*x) + 15*exp(4*c + 4*d*x) + 20*ex
p(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*exp(10*c + 10*d*x) + exp(12*c + 12
*d*x) + 1) - ((2*(3*a^2*b - a^3))/(7*d) - (2*exp(4*c + 4*d*x)*(3*a^2*b + 15
*a^3 - 16*b^3))/(7*d) - (2*exp(8*c + 8*d*x)*(3*a^2*b + 15*a^3 - 16*b^3))/(7
*d) + (12*exp(2*c + 2*d*x)*(2*a*b^2 + a^2*b - a^3))/(7*d) + (12*exp(10*c +
10*d*x)*(2*a*b^2 + a^2*b - a^3))/(7*d) - (8*exp(6*c + 6*d*x)*(6*a*b^2 + 3*a
^2*b + 5*a^3 + 8*b^3))/(7*d) + (2*exp(12*c + 12*d*x)*(3*a^2*b - a^3))/(7*d)
)/(7*exp(2*c + 2*d*x) + 21*exp(4*c + 4*d*x) + 35*exp(6*c + 6*d*x) + 35*exp(
8*c + 8*d*x) + 21*exp(10*c + 10*d*x) + 7*exp(12*c + 12*d*x) + exp(14*c + 14
*d*x) + 1) - ((4*exp(2*c + 2*d*x)*(2*a*b^2 + a^2*b - a^3))/(7*d) - (2*(3*a^
2*b + 15*a^3 - 16*b^3))/(105*d) + (2*exp(4*c + 4*d*x)*(3*a^2*b - a^3))/(7*d
))/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) + ((2*(
6*a*b^2 + 3*a^2*b + 5*a^3 + 8*b^3))/(35*d) + (2*exp(2*c + 2*d*x)*(3*a^2*b +
 15*a^3 - 16*b^3))/(35*d) - (6*exp(4*c + 4*d*x)*(2*a*b^2 + a^2*b - a^3))/(7
*d) - (2*exp(6*c + 6*d*x)*(3*a^2*b - a^3))/(7*d))/(4*exp(2*c + 2*d*x) + 6*e
xp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) + ((2*(3*a^2*b
 + 15*a^3 - 16*b^3))/(105*d) + (4*exp(4*c + 4*d*x)*(3*a^2*b + 15*a^3 - 16*b
^3))/(35*d) - (8*exp(6*c + 6*d*x)*(2*a*b^2 + a^2*b - a^3))/(7*d) + (8*exp(2
*c + 2*d*x)*(6*a*b^2 + 3*a^2*b + 5*a^3 + 8*b^3))/(35*d) - (2*exp(8*c + 8*d*
x)*(3*a^2*b - a^3))/(7*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*e
xp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1) - ((2*(2*a*b
^2 + a^2*b - a^3))/(7*d) + (2*exp(2*c + 2*d*x)*(3*a^2*b - a^3))/(7*d))/(2*e
xp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) - (2*(3*a^2*b - a^3))/(7*d*(exp(2*c
 + 2*d*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\operatorname{sech}^2(c + dx)\right)^3 \tanh^2(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)**3*tanh(d*x+c)**2,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**3*tanh(c + d*x)**2, x)
```

3.127 $\int \left(a+b\,\mathbf{sech}^2(c+dx)\right)^3 \tanh(c+dx)\,dx$

Optimal. Leaf size=71

$$\frac{a^3\log(\cosh(c+dx))}{d}-\frac{3a^2b\,\mathrm{sech}^2(c+dx)}{2d}-\frac{3ab^2\,\mathrm{sech}^4(c+dx)}{4d}-\frac{b^3\,\mathrm{sech}^6(c+dx)}{6d}$$

```
[Out] a^3*ln(cosh(d*x+c))/d-3/2*a^2*b*sech(d*x+c)^2/d-3/4*a*b^2*sech(d*x+c)^4/d-1
/6*b^3*sech(d*x+c)^6/d
```

Rubi [A] time = 0.06, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4138, 266, 43}

$$-\frac{3a^2b\,\mathrm{sech}^2(c+dx)}{2d}+\frac{a^3\log(\cosh(c+dx))}{d}-\frac{3ab^2\,\mathrm{sech}^4(c+dx)}{4d}-\frac{b^3\,\mathrm{sech}^6(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^3*Tanh[c + d*x],x]
```

```
[Out] (a^3*Log[Cosh[c + d*x]])/d - (3*a^2*b*Sech[c + d*x]^2)/(2*d) - (3*a*b^2*Sec
h[c + d*x]^4)/(4*d) - (b^3*Sech[c + d*x]^6)/(6*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \left(a+b\operatorname{sech}^2(c+dx)\right)^3 \tanh(c+dx)\,dx = \frac{\operatorname{Subst}\left(\int \frac{\left(b+ax^2\right)^3}{x^7}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= \frac{\operatorname{Subst}\left(\int \frac{(b+ax)^3}{x^4}\,dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= \frac{\operatorname{Subst}\left(\int \left(\frac{b^3}{x^4}+\frac{3ab^2}{x^3}+\frac{3a^2b}{x^2}+\frac{a^3}{x}\right)dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= \frac{a^3\log(\cosh(c+dx))}{d} - \frac{3a^2b\operatorname{sech}^2(c+dx)}{2d} - \frac{3ab^2\operatorname{sech}^4(c+dx)}{4d} -$$

Mathematica [A] time = 0.30, size = 100, normalized size = 1.41

$$\frac{2\operatorname{sech}^6(c+dx)\left(a\cosh^2(c+dx)+b\right)^3\left(12a^3\cosh^6(c+dx)\log(\cosh(c+dx))-18a^2b\cosh^4(c+dx)-9ab^2\cosh\right.}{3d(a\cosh(2(c+dx))+a+2b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^3*Tanh[c + d*x],x]
```

```
[Out] (2*(b + a*Cosh[c + d*x]^2)^3*(-2*b^3 - 9*a*b^2*Cosh[c + d*x]^2 - 18*a^2*b*C
osh[c + d*x]^4 + 12*a^3*Cosh[c + d*x]^6*Log[Cosh[c + d*x]])*Sech[c + d*x]^6
)/(3*d*(a + 2*b + a*Cosh[2*(c + d*x)])^3)
```

fricas [B] time = 0.44, size = 2519, normalized size = 35.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3*tanh(d*x+c),x, algorithm="fricas")
```

```
[Out] -1/3*(3*a^3*d*x*cosh(d*x + c)^12 + 36*a^3*d*x*cosh(d*x + c)*sinh(d*x + c)^1
1 + 3*a^3*d*x*sinh(d*x + c)^12 + 18*(a^3*d*x + a^2*b)*cosh(d*x + c)^10 + 18
*(11*a^3*d*x*cosh(d*x + c)^2 + a^3*d*x + a^2*b)*sinh(d*x + c)^10 + 60*(11*a
^3*d*x*cosh(d*x + c)^3 + 3*(a^3*d*x + a^2*b)*cosh(d*x + c))*sinh(d*x + c)^9
 + 9*(5*a^3*d*x + 8*a^2*b + 4*a*b^2)*cosh(d*x + c)^8 + 9*(165*a^3*d*x*cosh(
d*x + c)^4 + 5*a^3*d*x + 8*a^2*b + 4*a*b^2 + 90*(a^3*d*x + a^2*b)*cosh(d*x
+ c)^2)*sinh(d*x + c)^8 + 72*(33*a^3*d*x*cosh(d*x + c)^5 + 30*(a^3*d*x + a^
2*b)*cosh(d*x + c)^3 + (5*a^3*d*x + 8*a^2*b + 4*a*b^2)*cosh(d*x + c))*sinh(
```

```
d*x + c)^7 + 4*(15*a^3*d*x + 27*a^2*b + 18*a*b^2 + 8*b^3)*cosh(d*x + c)^6 +
 4*(693*a^3*d*x*cosh(d*x + c)^6 + 15*a^3*d*x + 945*(a^3*d*x + a^2*b)*cosh(d
*x + c)^4 + 27*a^2*b + 18*a*b^2 + 8*b^3 + 63*(5*a^3*d*x + 8*a^2*b + 4*a*b^2
)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 24*(99*a^3*d*x*cosh(d*x + c)^7 + 189*(
a^3*d*x + a^2*b)*cosh(d*x + c)^5 + 21*(5*a^3*d*x + 8*a^2*b + 4*a*b^2)*cosh(
d*x + c)^3 + (15*a^3*d*x + 27*a^2*b + 18*a*b^2 + 8*b^3)*cosh(d*x + c))*sinh
(d*x + c)^5 + 3*a^3*d*x + 9*(5*a^3*d*x + 8*a^2*b + 4*a*b^2)*cosh(d*x + c)^4
 + 3*(495*a^3*d*x*cosh(d*x + c)^8 + 1260*(a^3*d*x + a^2*b)*cosh(d*x + c)^6
+ 15*a^3*d*x + 210*(5*a^3*d*x + 8*a^2*b + 4*a*b^2)*cosh(d*x + c)^4 + 24*a^2
*b + 12*a*b^2 + 20*(15*a^3*d*x + 27*a^2*b + 18*a*b^2 + 8*b^3)*cosh(d*x + c)
^2)*sinh(d*x + c)^4 + 4*(165*a^3*d*x*cosh(d*x + c)^9 + 540*(a^3*d*x + a^2*b
)*cosh(d*x + c)^7 + 126*(5*a^3*d*x + 8*a^2*b + 4*a*b^2)*cosh(d*x + c)^5 + 2
0*(15*a^3*d*x + 27*a^2*b + 18*a*b^2 + 8*b^3)*cosh(d*x + c)^3 + 9*(5*a^3*d*x
 + 8*a^2*b + 4*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 18*(a^3*d*x + a^2*b)
*cosh(d*x + c)^2 + 6*(33*a^3*d*x*cosh(d*x + c)^10 + 135*(a^3*d*x + a^2*b)*c
osh(d*x + c)^8 + 42*(5*a^3*d*x + 8*a^2*b + 4*a*b^2)*cosh(d*x + c)^6 + 3*a^3
*d*x + 10*(15*a^3*d*x + 27*a^2*b + 18*a*b^2 + 8*b^3)*cosh(d*x + c)^4 + 3*a^
2*b + 9*(5*a^3*d*x + 8*a^2*b + 4*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 -
3*(a^3*cosh(d*x + c)^12 + 12*a^3*cosh(d*x + c)*sinh(d*x + c)^11 + a^3*sinh(
d*x + c)^12 + 6*a^3*cosh(d*x + c)^10 + 15*a^3*cosh(d*x + c)^8 + 6*(11*a^3*c
osh(d*x + c)^2 + a^3)*sinh(d*x + c)^10 + 20*(11*a^3*cosh(d*x + c)^3 + 3*a^3
*cosh(d*x + c))*sinh(d*x + c)^9 + 20*a^3*cosh(d*x + c)^6 + 15*(33*a^3*cosh(
d*x + c)^4 + 18*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^8 + 24*(33*a^3*cos
h(d*x + c)^5 + 30*a^3*cosh(d*x + c)^3 + 5*a^3*cosh(d*x + c))*sinh(d*x + c)^
7 + 15*a^3*cosh(d*x + c)^4 + 4*(231*a^3*cosh(d*x + c)^6 + 315*a^3*cosh(d*x
+ c)^4 + 105*a^3*cosh(d*x + c)^2 + 5*a^3)*sinh(d*x + c)^6 + 24*(33*a^3*cosh
(d*x + c)^7 + 63*a^3*cosh(d*x + c)^5 + 35*a^3*cosh(d*x + c)^3 + 5*a^3*cosh(
d*x + c))*sinh(d*x + c)^5 + 6*a^3*cosh(d*x + c)^2 + 15*(33*a^3*cosh(d*x + c
)^8 + 84*a^3*cosh(d*x + c)^6 + 70*a^3*cosh(d*x + c)^4 + 20*a^3*cosh(d*x + c
)^2 + a^3)*sinh(d*x + c)^4 + 20*(11*a^3*cosh(d*x + c)^9 + 36*a^3*cosh(d*x +
 c)^7 + 42*a^3*cosh(d*x + c)^5 + 20*a^3*cosh(d*x + c)^3 + 3*a^3*cosh(d*x +
c))*sinh(d*x + c)^3 + a^3 + 6*(11*a^3*cosh(d*x + c)^10 + 45*a^3*cosh(d*x +
c)^8 + 70*a^3*cosh(d*x + c)^6 + 50*a^3*cosh(d*x + c)^4 + 15*a^3*cosh(d*x +
c)^2 + a^3)*sinh(d*x + c)^2 + 12*(a^3*cosh(d*x + c)^11 + 5*a^3*cosh(d*x + c
)^9 + 10*a^3*cosh(d*x + c)^7 + 10*a^3*cosh(d*x + c)^5 + 5*a^3*cosh(d*x + c)
^3 + a^3*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) -
 sinh(d*x + c))) + 12*(3*a^3*d*x*cosh(d*x + c)^11 + 15*(a^3*d*x + a^2*b)*co
sh(d*x + c)^9 + 6*(5*a^3*d*x + 8*a^2*b + 4*a*b^2)*cosh(d*x + c)^7 + 2*(15*a
^3*d*x + 27*a^2*b + 18*a*b^2 + 8*b^3)*cosh(d*x + c)^5 + 3*(5*a^3*d*x + 8*a^
2*b + 4*a*b^2)*cosh(d*x + c)^3 + 3*(a^3*d*x + a^2*b)*cosh(d*x + c))*sinh(d*
x + c))/(d*cosh(d*x + c)^12 + 12*d*cosh(d*x + c)*sinh(d*x + c)^11 + d*sinh(
d*x + c)^12 + 6*d*cosh(d*x + c)^10 + 6*(11*d*cosh(d*x + c)^2 + d)*sinh(d*x
+ c)^10 + 20*(11*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^9 + 1
5*d*cosh(d*x + c)^8 + 15*(33*d*cosh(d*x + c)^4 + 18*d*cosh(d*x + c)^2 + d)*
sinh(d*x + c)^8 + 24*(33*d*cosh(d*x + c)^5 + 30*d*cosh(d*x + c)^3 + 5*d*cos
```

```
h(d*x + c))*sinh(d*x + c)^7 + 20*d*cosh(d*x + c)^6 + 4*(231*d*cosh(d*x + c)
^6 + 315*d*cosh(d*x + c)^4 + 105*d*cosh(d*x + c)^2 + 5*d)*sinh(d*x + c)^6 +
 24*(33*d*cosh(d*x + c)^7 + 63*d*cosh(d*x + c)^5 + 35*d*cosh(d*x + c)^3 + 5
*d*cosh(d*x + c))*sinh(d*x + c)^5 + 15*d*cosh(d*x + c)^4 + 15*(33*d*cosh(d*
x + c)^8 + 84*d*cosh(d*x + c)^6 + 70*d*cosh(d*x + c)^4 + 20*d*cosh(d*x + c)
^2 + d)*sinh(d*x + c)^4 + 20*(11*d*cosh(d*x + c)^9 + 36*d*cosh(d*x + c)^7 +
 42*d*cosh(d*x + c)^5 + 20*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x
+ c)^3 + 6*d*cosh(d*x + c)^2 + 6*(11*d*cosh(d*x + c)^10 + 45*d*cosh(d*x + c
)^8 + 70*d*cosh(d*x + c)^6 + 50*d*cosh(d*x + c)^4 + 15*d*cosh(d*x + c)^2 +
d)*sinh(d*x + c)^2 + 12*(d*cosh(d*x + c)^11 + 5*d*cosh(d*x + c)^9 + 10*d*co
sh(d*x + c)^7 + 10*d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)^3 + d*cosh(d*x + c
))*sinh(d*x + c) + d)
```

giac [B] time = 0.19, size = 268, normalized size = 3.77

$$-\frac{60\,a^3dx - 60\,a^3\log\left(e^{(2\,dx+2\,c)}+1\right) + \frac{147\,a^3e^{(12\,dx+12\,c)}+882\,a^3e^{(10\,dx+10\,c)}+360\,a^2be^{(10\,dx+10\,c)}+2205\,a^3e^{(8\,dx+8\,c)}+1440\,a^2be^{(8\,dx+8\,c)}+\cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3*tanh(d*x+c),x, algorithm="giac")
```

```
[Out] -1/60*(60*a^3*d*x - 60*a^3*log(e^(2*d*x + 2*c) + 1) + (147*a^3*e^(12*d*x +
12*c) + 882*a^3*e^(10*d*x + 10*c) + 360*a^2*b*e^(10*d*x + 10*c) + 2205*a^3*
e^(8*d*x + 8*c) + 1440*a^2*b*e^(8*d*x + 8*c) + 720*a*b^2*e^(8*d*x + 8*c) +
2940*a^3*e^(6*d*x + 6*c) + 2160*a^2*b*e^(6*d*x + 6*c) + 1440*a*b^2*e^(6*d*x
 + 6*c) + 640*b^3*e^(6*d*x + 6*c) + 2205*a^3*e^(4*d*x + 4*c) + 1440*a^2*b*e
^(4*d*x + 4*c) + 720*a*b^2*e^(4*d*x + 4*c) + 882*a^3*e^(2*d*x + 2*c) + 360*
a^2*b*e^(2*d*x + 2*c) + 147*a^3)/(e^(2*d*x + 2*c) + 1)^6)/d
```

maple [A] time = 0.16, size = 67, normalized size = 0.94

$$-\frac{b^3\mathrm{sech}\,(dx+c)^6}{6d}-\frac{3a\,b^2\mathrm{sech}\,(dx+c)^4}{4d}-\frac{3a^2b\mathrm{sech}\,(dx+c)^2}{2d}-\frac{a^3\ln\left(\mathrm{sech}\,(dx+c)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)^3*tanh(d*x+c),x)
```

```
[Out] -1/6*b^3*sech(d*x+c)^6/d-3/4*a*b^2*sech(d*x+c)^4/d-3/2*a^2*b*sech(d*x+c)^2/
d-1/d*a^3*ln(sech(d*x+c))
```

maxima [A] time = 0.43, size = 85, normalized size = 1.20

$$\frac{3\,a^2b\tanh\left(dx+c\right)^2}{2\,d}+\frac{a^3\log\left(\cosh\left(dx+c\right)\right)}{d}-\frac{12\,ab^2}{d\left(e^{(dx+c)}+e^{(-dx-c)}\right)^4}-\frac{32\,b^3}{3\,d\left(e^{(dx+c)}+e^{(-dx-c)}\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3*tanh(d*x+c),x, algorithm="maxima")
```

```
[Out] 3/2*a^2*b*tanh(d*x + c)^2/d + a^3*log(cosh(d*x + c))/d - 12*a*b^2/(d*(e^(d*
x + c) + e^(-d*x - c))^4) - 32/3*b^3/(d*(e^(d*x + c) + e^(-d*x - c))^6)
```

mupad [B] time = 0.21, size = 347, normalized size = 4.89

$$\frac{32\,b^3}{3\,d\,\left(6\,{\mathrm{e}}^{2\,c+2\,d\,x}+15\,{\mathrm{e}}^{4\,c+4\,d\,x}+20\,{\mathrm{e}}^{6\,c+6\,d\,x}+15\,{\mathrm{e}}^{8\,c+8\,d\,x}+6\,{\mathrm{e}}^{10\,c+10\,d\,x}+{\mathrm{e}}^{12\,c+12\,d\,x}+1\right)}-\frac{4\,\left(3\,a\right.}{d\,\left(4\,{\mathrm{e}}^{2\,c+2\,d\,x}+6\,{\mathrm{e}}^{4\,c+4\,d\,x}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)*(a + b/cosh(c + d*x)^2)^3,x)
```

```
[Out] (32*b^3)/(3*d*(6*exp(2*c + 2*d*x) + 15*exp(4*c + 4*d*x) + 20*exp(6*c + 6*d*
x) + 15*exp(8*c + 8*d*x) + 6*exp(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1))
- (4*(3*a*b^2 - 8*b^3))/(d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp
(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) - a^3*x - (6*(2*a*b^2 - a^2*b))/(d*(
2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) + (a^3*log(exp(2*c)*exp(2*d*x)
+ 1))/d + (8*(9*a*b^2 - 4*b^3))/(3*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*
x) + exp(6*c + 6*d*x) + 1)) - (32*b^3)/(d*(5*exp(2*c + 2*d*x) + 10*exp(4*c
+ 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) +
1)) - (6*a^2*b)/(d*(exp(2*c + 2*d*x) + 1))
```

sympy [A] time = 4.91, size = 87, normalized size = 1.23

$$\begin{cases} a^{3} x - \frac{a^{3} \log{\left(\tanh{\left(c + d x \right)} + 1 \right)}}{d} - \frac{3 a^{2} b \operatorname{sech}^{2}{\left(c + d x \right)}}{2 d} - \frac{3 a b^{2} \operatorname{sech}^{4}{\left(c + d x \right)}}{4 d} - \frac{b^{3} \operatorname{sech}^{6}{\left(c + d x \right)}}{6 d} & \text{for}\: d \neq 0 \\ x \left(a + b \operatorname{sech}^{2}{\left(c \right)}\right)^{3} \tanh{\left(c \right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)**3*tanh(d*x+c),x)
```

```
[Out] Piecewise((a**3*x - a**3*log(tanh(c + d*x) + 1)/d - 3*a**2*b*sech(c + d*x)*
*2/(2*d) - 3*a*b**2*sech(c + d*x)**4/(4*d) - b**3*sech(c + d*x)**6/(6*d), N
e(d, 0)), (x*(a + b*sech(c)**2)**3*tanh(c), True))
```

3.128 $\int \left(a + b\,\mathrm{sech}^2(c + dx)\right)^3 \,dx$

Optimal. Leaf size=73

$$a^3x + \frac{b\left(3a^2 + 3ab + b^2\right)\tanh(c + dx)}{d} - \frac{b^2(3a + 2b)\tanh^3(c + dx)}{3d} + \frac{b^3\tanh^5(c + dx)}{5d}$$

```
[Out] a^3*x+b*(3*a^2+3*a*b+b^2)*tanh(d*x+c)/d-1/3*b^2*(3*a+2*b)*tanh(d*x+c)^3/d+1
/5*b^3*tanh(d*x+c)^5/d
```

Rubi [A] time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4128, 390, 206}

$$\frac{b\left(3a^2 + 3ab + b^2\right)\tanh(c + dx)}{d} + a^3x - \frac{b^2(3a + 2b)\tanh^3(c + dx)}{3d} + \frac{b^3\tanh^5(c + dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] a^3*x + (b*(3*a^2 + 3*a*b + b^2)*Tanh[c + d*x])/d - (b^2*(3*a + 2*b)*Tanh[c
 + d*x]^3)/(3*d) + (b^3*Tanh[c + d*x]^5)/(5*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \left(a+b\text{sech}^2(c+dx)\right)^3\,dx=\frac{\text{Subst}\left(\int \frac{\left(a+b-bx^2\right)^3}{1-x^2}\,dx,x,\tanh(c+dx)\right)}{d}$$
$$=\frac{\text{Subst}\left(\int \left(b\left(3a^2+3ab+b^2\right)-b^2(3a+2b)x^2+b^3x^4+\frac{a^3}{1-x^2}\right)dx,x,\tanh(c+dx)\right)}{d}$$
$$=\frac{b\left(3a^2+3ab+b^2\right)\tanh(c+dx)}{d}-\frac{b^2(3a+2b)\tanh^3(c+dx)}{3d}+\frac{b^3\tanh^5(c+dx)}{5d}$$
$$=a^3x+\frac{b\left(3a^2+3ab+b^2\right)\tanh(c+dx)}{d}-\frac{b^2(3a+2b)\tanh^3(c+dx)}{3d}+\frac{b^3\tanh^5(c+dx)}{5d}$$

Mathematica [B] time = 0.93, size = 268, normalized size = 3.67

$$\frac{\text{sech}(c)\text{sech}^5(c+dx)\left(150a^3dx\cosh(2c+dx)+75a^3dx\cosh(2c+3dx)+75a^3dx\cosh(4c+3dx)+15a^3dx\cosh\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (Sech[c]*Sech[c + d*x]^5*(150*a^3*d*x*Cosh[d*x] + 150*a^3*d*x*Cosh[2*c + d*
x] + 75*a^3*d*x*Cosh[2*c + 3*d*x] + 75*a^3*d*x*Cosh[4*c + 3*d*x] + 15*a^3*d
*x*Cosh[4*c + 5*d*x] + 15*a^3*d*x*Cosh[6*c + 5*d*x] + 540*a^2*b*Sinh[d*x] +
 420*a*b^2*Sinh[d*x] + 160*b^3*Sinh[d*x] - 360*a^2*b*Sinh[2*c + d*x] - 180*
a*b^2*Sinh[2*c + d*x] + 360*a^2*b*Sinh[2*c + 3*d*x] + 300*a*b^2*Sinh[2*c +
3*d*x] + 80*b^3*Sinh[2*c + 3*d*x] - 90*a^2*b*Sinh[4*c + 3*d*x] + 90*a^2*b*S
inh[4*c + 5*d*x] + 60*a*b^2*Sinh[4*c + 5*d*x] + 16*b^3*Sinh[4*c + 5*d*x]))/
(480*d)
```

fricas [B] time = 0.42, size = 470, normalized size = 6.44

$$\frac{\left(15\,a^3dx-45\,a^2b-30\,ab^2-8\,b^3\right)\cosh\left(dx+c\right)^5+5\left(15\,a^3dx-45\,a^2b-30\,ab^2-8\,b^3\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/15*((15*a^3*d*x - 45*a^2*b - 30*a*b^2 - 8*b^3)*cosh(d*x + c)^5 + 5*(15*a^
3*d*x - 45*a^2*b - 30*a*b^2 - 8*b^3)*cosh(d*x + c)*sinh(d*x + c)^4 + (45*a^
2*b + 30*a*b^2 + 8*b^3)*sinh(d*x + c)^5 + 5*(15*a^3*d*x - 45*a^2*b - 30*a*b
```

```
^2 - 8*b^3)*cosh(d*x + c)^3 + 5*(27*a^2*b + 30*a*b^2 + 8*b^3 + 2*(45*a^2*b
+ 30*a*b^2 + 8*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 5*(2*(15*a^3*d*x - 4
5*a^2*b - 30*a*b^2 - 8*b^3)*cosh(d*x + c)^3 + 3*(15*a^3*d*x - 45*a^2*b - 30
*a*b^2 - 8*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 10*(15*a^3*d*x - 45*a^2*b
- 30*a*b^2 - 8*b^3)*cosh(d*x + c) + 5*((45*a^2*b + 30*a*b^2 + 8*b^3)*cosh(d
*x + c)^4 + 18*a^2*b + 24*a*b^2 + 16*b^3 + 3*(27*a^2*b + 30*a*b^2 + 8*b^3)*
cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)*sinh
(d*x + c)^4 + 5*d*cosh(d*x + c)^3 + 5*(2*d*cosh(d*x + c)^3 + 3*d*cosh(d*x +
 c))*sinh(d*x + c)^2 + 10*d*cosh(d*x + c))
```

giac [B] time = 0.15, size = 182, normalized size = 2.49

$$\frac{15\,(dx+c)a^3-\frac{2\left(45\,a^2be^{(8\,dx+8\,c)}+180\,a^2be^{(6\,dx+6\,c)}+90\,ab^2e^{(6\,dx+6\,c)}+270\,a^2be^{(4\,dx+4\,c)}+210\,ab^2e^{(4\,dx+4\,c)}+80\,b^3e^{(4\,dx+4\,c)}+180\,a^2be^{(2\,dx+2\,c)}+15}{\left(e^{(2\,dx+2\,c)}+1\right)^5}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/15*(15*(d*x + c)*a^3 - 2*(45*a^2*b*e^(8*d*x + 8*c) + 180*a^2*b*e^(6*d*x +
 6*c) + 90*a*b^2*e^(6*d*x + 6*c) + 270*a^2*b*e^(4*d*x + 4*c) + 210*a*b^2*e^
(4*d*x + 4*c) + 80*b^3*e^(4*d*x + 4*c) + 180*a^2*b*e^(2*d*x + 2*c) + 150*a*
b^2*e^(2*d*x + 2*c) + 40*b^3*e^(2*d*x + 2*c) + 45*a^2*b + 30*a*b^2 + 8*b^3)
/(e^(2*d*x + 2*c) + 1)^5)/d
```

maple [A] time = 0.42, size = 83, normalized size = 1.14

$$\frac{a^3\,(dx+c)+3a^2b\tanh\,(dx+c)+3a\,b^2\left(\frac{2}{3}+\frac{\mathrm{sech}(dx+c)^2}{3}\right)\tanh\,(dx+c)+b^3\left(\frac{8}{15}+\frac{\mathrm{sech}(dx+c)^4}{5}+\frac{4\mathrm{sech}(dx+c)^2}{15}\right)\mathrm{tanh}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(a^3*(d*x+c)+3*a^2*b*tanh(d*x+c)+3*a*b^2*(2/3+1/3*sech(d*x+c)^2)*tanh(d
*x+c)+b^3*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)*tanh(d*x+c))
```

maxima [B] time = 0.41, size = 332, normalized size = 4.55

$$a^3x+\frac{16}{15}\,b^3\left(\frac{5\,e^{(-2\,dx-2\,c)}}{d\left(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}+5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}+1\right)}+\frac{}{d\left(5\,e^{(-2\,dx-2\,c)}+10\,e\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] a^3*x + 16/15*b^3*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x
 - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1
)) + 10*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*
e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*
e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x
- 8*c) + e^(-10*d*x - 10*c) + 1))) + 4*a*b^2*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-
2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*
d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + 6*a^2*b/(d*(e^(
-2*d*x - 2*c) + 1))
```

mupad [B] time = 1.44, size = 502, normalized size = 6.88

$$a^3\,x-\frac{\frac{2\left(9\,a^2\,b+12\,a\,b^2+8\,b^3\right)}{15\,d}+\frac{12\,e^{2\,c+2\,d\,x}\left(a^2\,b+a\,b^2\right)}{5\,d}+\frac{6\,a^2\,b\,e^{4\,c+4\,d\,x}}{5\,d}}{3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1}-\frac{\frac{6\,a^2\,b}{5\,d}+\frac{24\,e^{2\,c+2\,d\,x}\left(a^2\,b+a\,b^2\right)}{5\,d}+\frac{24\,e^{6\,c+6\,d\,x}\left(a^2\,b+a\,b^2\right)}{5\,d}+\frac{4\,e^{4\,c+4\cdots}}{\cdots}}{5\,e^{2\,c+2\,d\,x}+10\,e^{4\,c+4\,d\,x}+10\,e^{6\,c+6\,d\,x}+5\,e^{8\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cosh(c + d*x)^2)^3,x)
```

```
[Out] a^3*x - ((2*(12*a*b^2 + 9*a^2*b + 8*b^3))/(15*d) + (12*exp(2*c + 2*d*x)*(a*
b^2 + a^2*b))/(5*d) + (6*a^2*b*exp(4*c + 4*d*x))/(5*d))/(3*exp(2*c + 2*d*x)
 + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) - ((6*a^2*b)/(5*d) + (24*exp(
2*c + 2*d*x)*(a*b^2 + a^2*b))/(5*d) + (24*exp(6*c + 6*d*x)*(a*b^2 + a^2*b))
/(5*d) + (4*exp(4*c + 4*d*x)*(12*a*b^2 + 9*a^2*b + 8*b^3))/(5*d) + (6*a^2*b
*exp(8*c + 8*d*x))/(5*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*ex
p(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1) - ((6*(a*b^2
+ a^2*b))/(5*d) + (18*exp(4*c + 4*d*x)*(a*b^2 + a^2*b))/(5*d) + (2*exp(2*c
+ 2*d*x)*(12*a*b^2 + 9*a^2*b + 8*b^3))/(5*d) + (6*a^2*b*exp(6*c + 6*d*x))/(
5*d))/(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8
*c + 8*d*x) + 1) - ((6*(a*b^2 + a^2*b))/(5*d) + (6*a^2*b*exp(2*c + 2*d*x))/
(5*d))/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) - (6*a^2*b)/(5*d*(exp(2*
c + 2*d*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\operatorname{sech}^2\left(c+dx\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**3, x)
```

3.129 $\int \coth(c+dx)\left(a+b\,\mathbf{sech}^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=84

$$-\frac{b\left(3a^2+3ab+b^2\right)\log(\cosh(c+dx))}{d}+\frac{b^2(3a+b)\operatorname{sech}^2(c+dx)}{2d}+\frac{(a+b)^3\log(\sinh(c+dx))}{d}+\frac{b^3\operatorname{sech}^4(c+dx)}{4d}$$

```
[Out] -b*(3*a^2+3*a*b+b^2)*ln(cosh(d*x+c))/d+(a+b)^3*ln(sinh(d*x+c))/d+1/2*b^2*(3
*a+b)*sech(d*x+c)^2/d+1/4*b^3*sech(d*x+c)^4/d
```

Rubi [A] time = 0.11, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4138, 446, 88}

$$-\frac{b\left(3a^2+3ab+b^2\right)\log(\cosh(c+dx))}{d}+\frac{b^2(3a+b)\operatorname{sech}^2(c+dx)}{2d}+\frac{(a+b)^3\log(\sinh(c+dx))}{d}+\frac{b^3\operatorname{sech}^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] -((b*(3*a^2 + 3*a*b + b^2)*Log[Cosh[c + d*x]])/d) + ((a + b)^3*Log[Sinh[c +
 d*x]])/d + (b^2*(3*a + b)*Sech[c + d*x]^2)/(2*d) + (b^3*Sech[c + d*x]^4)/(
4*d)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
```

```
x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \coth(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx &= -\frac{\text{Subst}\left(\int \frac{(b+ax^2)^3}{x^5(1-x^2)}\,dx,x,\cosh(c+dx)\right)}{d}\\ &= -\frac{\text{Subst}\left(\int \frac{(b+ax)^3}{(1-x)x^3}\,dx,x,\cosh^2(c+dx)\right)}{2d}\\ &= -\frac{\text{Subst}\left(\int \left(-\frac{(a+b)^3}{-1+x}+\frac{b^3}{x^3}+\frac{b^2(3a+b)}{x^2}+\frac{b(3a^2+3ab+b^2)}{x}\right)dx,x,\cosh^2(c+dx)\right)}{2d}\\ &= -\frac{b\left(3a^2+3ab+b^2\right)\log(\cosh(c+dx))}{d}+\frac{(a+b)^3\log(\sinh(c+dx))}{d}\end{aligned}$$

Mathematica [A] time = 0.66, size = 114, normalized size = 1.36

$$-\frac{2\cosh^6(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\left(4b\left(3a^2+3ab+b^2\right)\log(\cosh(c+dx))-2b^2(3a+b)\text{sech}^2(c+dx)-4(a\right.}{d(a\cosh(2c+2dx)+a+2b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (-2*Cosh[c + d*x]^6*(a + b*Sech[c + d*x]^2)^3*(4*b*(3*a^2 + 3*a*b + b^2)*Lo
g[Cosh[c + d*x]] - 4*(a + b)^3*Log[Sinh[c + d*x]] - 2*b^2*(3*a + b)*Sech[c
+ d*x]^2 - b^3*Sech[c + d*x]^4))/(d*(a + 2*b + a*Cosh[2*c + 2*d*x])^3)
```

fricas [B] time = 0.46, size = 2376, normalized size = 28.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -(a^3*d*x*cosh(d*x + c)^8 + 8*a^3*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + a^3*d
*x*sinh(d*x + c)^8 + 2*(2*a^3*d*x - 3*a*b^2 - b^3)*cosh(d*x + c)^6 + 2*(14*
a^3*d*x*cosh(d*x + c)^2 + 2*a^3*d*x - 3*a*b^2 - b^3)*sinh(d*x + c)^6 + 4*(1
4*a^3*d*x*cosh(d*x + c)^3 + 3*(2*a^3*d*x - 3*a*b^2 - b^3)*cosh(d*x + c))*si
nh(d*x + c)^5 + a^3*d*x + 2*(3*a^3*d*x - 6*a*b^2 - 4*b^3)*cosh(d*x + c)^4 +
```

```
2*(35*a^3*d*x*cosh(d*x + c)^4 + 3*a^3*d*x - 6*a*b^2 - 4*b^3 + 15*(2*a^3*d*
x - 3*a*b^2 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*a^3*d*x*cosh(d*x
 + c)^5 + 5*(2*a^3*d*x - 3*a*b^2 - b^3)*cosh(d*x + c)^3 + (3*a^3*d*x - 6*a*
b^2 - 4*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(2*a^3*d*x - 3*a*b^2 - b^3)
*cosh(d*x + c)^2 + 2*(14*a^3*d*x*cosh(d*x + c)^6 + 2*a^3*d*x + 15*(2*a^3*d*
x - 3*a*b^2 - b^3)*cosh(d*x + c)^4 - 3*a*b^2 - b^3 + 6*(3*a^3*d*x - 6*a*b^2
 - 4*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((3*a^2*b + 3*a*b^2 + b^3)*cos
h(d*x + c)^8 + 8*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^7 +
(3*a^2*b + 3*a*b^2 + b^3)*sinh(d*x + c)^8 + 4*(3*a^2*b + 3*a*b^2 + b^3)*cos
h(d*x + c)^6 + 4*(3*a^2*b + 3*a*b^2 + b^3 + 7*(3*a^2*b + 3*a*b^2 + b^3)*cos
h(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c
)^3 + 3*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 6*(3*a^2
*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 2*(35*(3*a^2*b + 3*a*b^2 + b^3)*cosh(
d*x + c)^4 + 9*a^2*b + 9*a*b^2 + 3*b^3 + 30*(3*a^2*b + 3*a*b^2 + b^3)*cosh(
d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^
5 + 10*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + 3*(3*a^2*b + 3*a*b^2 + b
^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*a^2*b + 3*a*b^2 + b^3 + 4*(3*a^2*b +
 3*a*b^2 + b^3)*cosh(d*x + c)^2 + 4*(7*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x +
 c)^6 + 15*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 3*a^2*b + 3*a*b^2 +
b^3 + 9*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((3*
a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^7 + 3*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d
*x + c)^5 + 3*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + (3*a^2*b + 3*a*b^
2 + b^3)*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) -
 sinh(d*x + c))) - ((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^8 + 8*(a^
3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^3 + 3*a^2*b
 + 3*a*b^2 + b^3)*sinh(d*x + c)^8 + 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(
d*x + c)^6 + 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 7*(a^3 + 3*a^2*b + 3*a*b^2
+ b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(a^3 + 3*a^2*b + 3*a*b^2 + b
^3)*cosh(d*x + c)^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c))*sinh
(d*x + c)^5 + 6*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 2*(35*(a^
3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 3*a^3 + 9*a^2*b + 9*a*b^2 +
3*b^3 + 30*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4
 + 8*(7*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^5 + 10*(a^3 + 3*a^2*b
 + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(
d*x + c))*sinh(d*x + c)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 4*(a^3 + 3*a^2*
b + 3*a*b^2 + b^3)*cosh(d*x + c)^2 + 4*(7*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*c
osh(d*x + c)^6 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + a^3 +
 3*a^2*b + 3*a*b^2 + b^3 + 9*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^
2)*sinh(d*x + c)^2 + 8*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^7 + 3
*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^5 + 3*(a^3 + 3*a^2*b + 3*a*b
^2 + b^3)*cosh(d*x + c)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c))*
sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 4*(2*
a^3*d*x*cosh(d*x + c)^7 + 3*(2*a^3*d*x - 3*a*b^2 - b^3)*cosh(d*x + c)^5 + 2
*(3*a^3*d*x - 6*a*b^2 - 4*b^3)*cosh(d*x + c)^3 + (2*a^3*d*x - 3*a*b^2 - b^3
)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^8 + 8*d*cosh(d*x + c)*sinh
```

```
(d*x + c)^7 + d*sinh(d*x + c)^8 + 4*d*cosh(d*x + c)^6 + 4*(7*d*cosh(d*x + c
)^2 + d)*sinh(d*x + c)^6 + 8*(7*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh
(d*x + c)^5 + 6*d*cosh(d*x + c)^4 + 2*(35*d*cosh(d*x + c)^4 + 30*d*cosh(d*x
 + c)^2 + 3*d)*sinh(d*x + c)^4 + 8*(7*d*cosh(d*x + c)^5 + 10*d*cosh(d*x + c
)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*d*cosh(d*x + c)^2 + 4*(7*d*cos
h(d*x + c)^6 + 15*d*cosh(d*x + c)^4 + 9*d*cosh(d*x + c)^2 + d)*sinh(d*x + c
)^2 + 8*(d*cosh(d*x + c)^7 + 3*d*cosh(d*x + c)^5 + 3*d*cosh(d*x + c)^3 + d*
cosh(d*x + c))*sinh(d*x + c) + d)
```

giac [B] time = 0.18, size = 325, normalized size = 3.87

$$-\frac{12\,a^{3}dx+12\left(3\,a^{2}be^{(2\,c)}+3\,ab^{2}e^{(2\,c)}+b^{3}e^{(2\,c)}\right)e^{(-2\,c)}\log\left(e^{(2\,dx+2\,c)}+1\right)-12\left(a^{3}e^{(2\,c)}+3\,a^{2}be^{(2\,c)}+3\,ab^{2}e^{(2\,c)}+b^{3}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/12*(12*a^3*d*x + 12*(3*a^2*b*e^(2*c) + 3*a*b^2*e^(2*c) + b^3*e^(2*c))*e^
(-2*c)*log(e^(2*d*x + 2*c) + 1) - 12*(a^3*e^(2*c) + 3*a^2*b*e^(2*c) + 3*a*b
^2*e^(2*c) + b^3*e^(2*c))*e^(-2*c)*log(abs(e^(2*d*x + 2*c) - 1)) - (75*a^2*
b*e^(8*d*x + 8*c) + 75*a*b^2*e^(8*d*x + 8*c) + 25*b^3*e^(8*d*x + 8*c) + 300
*a^2*b*e^(6*d*x + 6*c) + 372*a*b^2*e^(6*d*x + 6*c) + 124*b^3*e^(6*d*x + 6*c
) + 450*a^2*b*e^(4*d*x + 4*c) + 594*a*b^2*e^(4*d*x + 4*c) + 246*b^3*e^(4*d*
x + 4*c) + 300*a^2*b*e^(2*d*x + 2*c) + 372*a*b^2*e^(2*d*x + 2*c) + 124*b^3*
e^(2*d*x + 2*c) + 75*a^2*b + 75*a*b^2 + 25*b^3)/(e^(2*d*x + 2*c) + 1)^4)/d
```

maple [A] time = 0.31, size = 111, normalized size = 1.32

$$\frac{a^{3}\ln\left(\sinh\left(dx+c\right)\right)}{d}+\frac{3a^{2}b\ln\left(\tanh\left(dx+c\right)\right)}{d}+\frac{3a\,b^{2}}{2d\cosh\left(dx+c\right)^{2}}+\frac{3a\,b^{2}\ln\left(\tanh\left(dx+c\right)\right)}{d}+\frac{b^{3}}{4d\cosh\left(dx+c\right)^{4}}+\frac{}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)*(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] a^3*ln(sinh(d*x+c))/d+3*a^2*b*ln(tanh(d*x+c))/d+3/2/d*a*b^2/cosh(d*x+c)^2+3
/d*a*b^2*ln(tanh(d*x+c))+1/4/d*b^3/cosh(d*x+c)^4+1/2/d*b^3/cosh(d*x+c)^2+1/
d*b^3*ln(tanh(d*x+c))
```

maxima [B] time = 0.47, size = 300, normalized size = 3.57

$$b^{3}\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}-\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}+\frac{2\left(e^{(-2\,dx-2\,c)}+4\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}\right.}{d\left(4\,e^{(-2\,dx-2\,c)}+6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}+e^{(-8}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] b^3*(log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d - log(e^(-2*d*x - 2*
c) + 1)/d + 2*(e^(-2*d*x - 2*c) + 4*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/(d
*(4*e^(-2*d*x - 2*c) + 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x
- 8*c) + 1))) + 3*a*b^2*(log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d
- log(e^(-2*d*x - 2*c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) +
 e^(-4*d*x - 4*c) + 1))) + 3*a^2*b*(log(e^(-d*x - c) + 1)/d + log(e^(-d*x -
 c) - 1)/d - log(e^(-2*d*x - 2*c) + 1)/d) + a^3*log(sinh(d*x + c))/d
```

mupad [B] time = 1.64, size = 360, normalized size = 4.29

$$\frac{2\left(b^3+3\,a\,b^2\right)}{d\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)}-a^3\,x-\frac{\mathrm{atan}\left(\frac{\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}\left(a^3\,\sqrt{-d^2}+2\,b^3\,\sqrt{-d^2}+6\,a\,b^2\,\sqrt{-d^2}+6\,a^2\,b\,\sqrt{-d^2}\right)}{d\,\sqrt{a^6+12\,a^5\,b+48\,a^4\,b^2+76\,a^3\,b^3+60\,a^2\,b^4+24\,a\,b^5+4\,b^6}}\right)\sqrt{a^6+12\,a^5\,b+48\,a^4\,b^2+76\,a^3\,b^3}}{\sqrt{-d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)*(a + b/cosh(c + d*x)^2)^3,x)
```

```
[Out] (2*(3*a*b^2 + b^3))/(d*(exp(2*c + 2*d*x) + 1)) - a^3*x - (atan((exp(2*c)*ex
p(2*d*x)*(a^3*(-d^2)^(1/2) + 2*b^3*(-d^2)^(1/2) + 6*a*b^2*(-d^2)^(1/2) + 6*
a^2*b*(-d^2)^(1/2)))/(d*(24*a*b^5 + 12*a^5*b + a^6 + 4*b^6 + 60*a^2*b^4 + 7
6*a^3*b^3 + 48*a^4*b^2)^(1/2)))*(24*a*b^5 + 12*a^5*b + a^6 + 4*b^6 + 60*a^2
*b^4 + 76*a^3*b^3 + 48*a^4*b^2)^(1/2))/(-d^2)^(1/2) + (a^3*log(exp(4*c + 4*
d*x) - 1))/(2*d) - (8*b^3)/(d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + ex
p(6*c + 6*d*x) + 1)) - (2*(3*a*b^2 - b^3))/(d*(2*exp(2*c + 2*d*x) + exp(4*c
 + 4*d*x) + 1)) + (4*b^3)/(d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*e
xp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\,\mathrm{sech}^2\left(c+dx\right)\right)^3\coth\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**3*coth(c + d*x), x)
```

3.130 $\int \coth^2(c+dx)\left(a+b\,\mathbf{sech}^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=61

$$a^3x-\frac{b^2(3a+2b)\tanh(c+dx)}{d}-\frac{(a+b)^3\coth(c+dx)}{d}+\frac{b^3\tanh^3(c+dx)}{3d}$$

```
[Out] a^3*x-(a+b)^3*coth(d*x+c)/d-b^2*(3*a+2*b)*tanh(d*x+c)/d+1/3*b^3*tanh(d*x+c)
^3/d
```

Rubi [A] time = 0.10, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4141, 1802, 207}

$$a^3x-\frac{b^2(3a+2b)\tanh(c+dx)}{d}-\frac{(a+b)^3\coth(c+dx)}{d}+\frac{b^3\tanh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^2*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] a^3*x - ((a + b)^3*Coth[c + d*x])/d - (b^2*(3*a + 2*b)*Tanh[c + d*x])/d + (
b^3*Tanh[c + d*x]^3)/(3*d)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \coth^2(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\left(1-x^2\right)\right)^3}{x^2\left(1-x^2\right)}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-b^2(3a+2b)+\frac{(a+b)^3}{x^2}+b^3x^2-\frac{a^3}{-1+x^2}\right)dx, x, \tanh(c+d$$
$$= -\frac{(a+b)^3\coth(c+dx)}{d}-\frac{b^2(3a+2b)\tanh(c+dx)}{d}+\frac{b^3\tanh^3(c+}{3d}$$
$$= a^3x-\frac{(a+b)^3\coth(c+dx)}{d}-\frac{b^2(3a+2b)\tanh(c+dx)}{d}+\frac{b^3\tanh}{3}$$

Mathematica [B] time = 1.80, size = 126, normalized size = 2.07

$$\frac{8(a\cosh(c+dx)+b\text{sech}(c+dx))^3\left(3a^3dx\cosh^3(c+dx)+\sinh(dx)\cosh^2(c+dx)\left(3(a+b)^3\text{csch}(c)\coth(c+\right.\right.}{3d(a\cosh(2(c+dx))+a+2b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^2*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (8*(a*Cosh[c + d*x] + b*Sech[c + d*x])^3*(3*a^3*d*x*Cosh[c + d*x]^3 - b^3*S
ech[c]*Sinh[d*x] + Cosh[c + d*x]^2*(3*(a + b)^3*Coth[c + d*x]*Csch[c] - b^2
*(9*a + 5*b)*Sech[c])*Sinh[d*x] - b^3*Cosh[c + d*x]*Tanh[c]))/(3*d*(a + 2*b
 + a*Cosh[2*(c + d*x)])^3)
```

fricas [B] time = 0.41, size = 359, normalized size = 5.89

$$-\frac{\left(3\,a^3+9\,a^2b+18\,ab^2+8\,b^3\right)\cosh\left(dx+c\right)^4-4\left(3\,a^3dx+3\,a^3+9\,a^2b+18\,ab^2+8\,b^3\right)\cosh\left(dx+c\right)\sinh\left(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/12*((3*a^3 + 9*a^2*b + 18*a*b^2 + 8*b^3)*cosh(d*x + c)^4 - 4*(3*a^3*d*x
+ 3*a^3 + 9*a^2*b + 18*a*b^2 + 8*b^3)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a^
3 + 9*a^2*b + 18*a*b^2 + 8*b^3)*sinh(d*x + c)^4 + 9*a^3 + 27*a^2*b + 18*a*b
^2 + 4*(3*a^3 + 9*a^2*b + 9*a*b^2 + 4*b^3)*cosh(d*x + c)^2 + 2*(6*a^3 + 18*
a^2*b + 18*a*b^2 + 8*b^3 + 3*(3*a^3 + 9*a^2*b + 18*a*b^2 + 8*b^3)*cosh(d*x
+ c)^2)*sinh(d*x + c)^2 - 4*((3*a^3*d*x + 3*a^3 + 9*a^2*b + 18*a*b^2 + 8*b^
3)*cosh(d*x + c)^3 + (3*a^3*d*x + 3*a^3 + 9*a^2*b + 18*a*b^2 + 8*b^3)*cosh(
```

```
d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)*sinh(d*x + c)^3 + (d*cosh(d*x + c
)^3 + d*cosh(d*x + c))*sinh(d*x + c))
```

giac [B] time = 0.21, size = 132, normalized size = 2.16

$$\frac{3\,a^3dx - \frac{6\left(a^3+3\,a^2b+3\,ab^2+b^3\right)}{e^{(2\,dx+2\,c)}-1} + \frac{2\left(9\,ab^2e^{(4\,dx+4\,c)}+3\,b^3e^{(4\,dx+4\,c)}+18\,ab^2e^{(2\,dx+2\,c)}+12\,b^3e^{(2\,dx+2\,c)}+9\,ab^2+5\,b^3\right)}{\left(e^{(2\,dx+2\,c)}+1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/3*(3*a^3*d*x - 6*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)/(e^(2*d*x + 2*c) - 1) +
2*(9*a*b^2*e^(4*d*x + 4*c) + 3*b^3*e^(4*d*x + 4*c) + 18*a*b^2*e^(2*d*x + 2*
c) + 12*b^3*e^(2*d*x + 2*c) + 9*a*b^2 + 5*b^3)/(e^(2*d*x + 2*c) + 1)^3)/d
```

maple [A] time = 0.57, size = 111, normalized size = 1.82

$$\frac{a^3\left(dx+c-\coth\left(dx+c\right)\right)-3a^2b\coth\left(dx+c\right)+3a\,b^2\left(-\frac{1}{\sinh(dx+c)\cosh(dx+c)}-2\tanh\left(dx+c\right)\right)+b^3\left(-\frac{1}{\sinh(dx+c)\,\mathrm{co}}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(a^3*(d*x+c-coth(d*x+c))-3*a^2*b*coth(d*x+c)+3*a*b^2*(-1/sinh(d*x+c)/co
sh(d*x+c)-2*tanh(d*x+c))+b^3*(-1/sinh(d*x+c)/cosh(d*x+c)^3-4*(2/3+1/3*sech(
d*x+c)^2)*tanh(d*x+c)))
```

maxima [B] time = 0.41, size = 172, normalized size = 2.82

$$a^3\left(x+\frac{c}{d}+\frac{2}{d\left(e^{(-2\,dx-2\,c)}-1\right)}\right)-\frac{16}{3}\,b^3\left(\frac{2\,e^{(-2\,dx-2\,c)}}{d\left(2\,e^{(-2\,dx-2\,c)}-2\,e^{(-6\,dx-6\,c)}-e^{(-8\,dx-8\,c)}+1\right)}+\frac{1}{d\left(2\,e^{(-2\,dx-2\,c)}-2\,e^{(-6\,dx-6\,c)}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] a^3*(x + c/d + 2/(d*(e^(-2*d*x - 2*c) - 1))) - 16/3*b^3*(2*e^(-2*d*x - 2*c)
/(d*(2*e^(-2*d*x - 2*c) - 2*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) + 1)) + 1/(
d*(2*e^(-2*d*x - 2*c) - 2*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) + 1))) + 6*a^
2*b/(d*(e^(-2*d*x - 2*c) - 1)) + 12*a*b^2/(d*(e^(-4*d*x - 4*c) - 1))
```

mupad [B] time = 0.14, size = 234, normalized size = 3.84

$$\frac{\frac{2\left(b^3+3\,a\,b^2\right)}{3\,d}+\frac{4\,\mathrm{e}^{2\,c+2\,d\,x}\left(b^3+a\,b^2\right)}{d}+\frac{2\,\mathrm{e}^{4\,c+4\,d\,x}\left(b^3+3\,a\,b^2\right)}{3\,d}}{3\,\mathrm{e}^{2\,c+2\,d\,x}+3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}+1}+a^3\,x+\frac{\frac{2\left(b^3+a\,b^2\right)}{d}+\frac{2\,\mathrm{e}^{2\,c+2\,d\,x}\left(b^3+3\,a\,b^2\right)}{3\,d}}{2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1}+\frac{2\left(b^3+3\,a\,b^2\right)}{3\,d\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)}-\frac{2\left(a^3+\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^2*(a + b/cosh(c + d*x)^2)^3,x)
```

```
[Out] ((2*(3*a*b^2 + b^3))/(3*d) + (4*exp(2*c + 2*d*x)*(a*b^2 + b^3))/d + (2*exp(
4*c + 4*d*x)*(3*a*b^2 + b^3))/(3*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*
x) + exp(6*c + 6*d*x) + 1) + a^3*x + ((2*(a*b^2 + b^3))/d + (2*exp(2*c + 2*
d*x)*(3*a*b^2 + b^3))/(3*d))/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) +
(2*(3*a*b^2 + b^3))/(3*d*(exp(2*c + 2*d*x) + 1)) - (2*(3*a*b^2 + 3*a^2*b +
a^3 + b^3))/(d*(exp(2*c + 2*d*x) - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\,\mathrm{sech}^2\,(c + dx)\right)^3 \coth^2\,(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**2*(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**3*coth(c + d*x)**2, x)
```

3.131 $\int \coth^3(c+dx)\left(a+b\,\mathbf{sech}^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=81

$$\frac{b^2(3a+2b)\log(\cosh(c+dx))}{d}-\frac{(a+b)^3\operatorname{csch}^2(c+dx)}{2d}+\frac{(a-2b)(a+b)^2\log(\sinh(c+dx))}{d}-\frac{b^3\operatorname{sech}^2(c+dx)}{2d}$$

```
[Out] -1/2*(a+b)^3*csch(d*x+c)^2/d+b^2*(3*a+2*b)*ln(cosh(d*x+c))/d+(a-2*b)*(a+b)^
2*ln(sinh(d*x+c))/d-1/2*b^3*sech(d*x+c)^2/d
```

Rubi [A] time = 0.12, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4138, 446, 88}

$$\frac{b^2(3a+2b)\log(\cosh(c+dx))}{d}-\frac{(a+b)^3\operatorname{csch}^2(c+dx)}{2d}+\frac{(a-2b)(a+b)^2\log(\sinh(c+dx))}{d}-\frac{b^3\operatorname{sech}^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^3*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] -((a + b)^3*Csch[c + d*x]^2)/(2*d) + (b^2*(3*a + 2*b)*Log[Cosh[c + d*x]])/d
 + ((a - 2*b)*(a + b)^2*Log[Sinh[c + d*x]])/d - (b^3*Sech[c + d*x]^2)/(2*d)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \coth^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{\left(b+ax^2\right)^3}{x^3\left(1-x^2\right)^2}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{(b+ax)^3}{(1-x)^2x^2}\,dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{(a+b)^3}{(-1+x)^2}+\frac{(a-2b)(a+b)^2}{-1+x}+\frac{b^3}{x^2}+\frac{b^2(3a+2b)}{x}\right)dx, x, \cosh^2(c+a\right.}{2d}$$
$$= -\frac{(a+b)^3\operatorname{csch}^2(c+dx)}{2d}+\frac{b^2(3a+2b)\log(\cosh(c+dx))}{d}+\frac{(a-2l}{}$$

Mathematica [A] time = 1.24, size = 110, normalized size = 1.36

$$-\frac{4\cosh^6(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\left(-2b^2(3a+2b)\log(\cosh(c+dx))+(a+b)^3\operatorname{csch}^2(c+dx)-2(a-2b)(a\right.}{d(a\cosh(2c+2dx)+a+2b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^3*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (-4*Cosh[c + d*x]^6*(a + b*Sech[c + d*x]^2)^3*((a + b)^3*Csch[c + d*x]^2 -
2*b^2*(3*a + 2*b)*Log[Cosh[c + d*x]] - 2*(a - 2*b)*(a + b)^2*Log[Sinh[c + d
*x]] + b^3*Sech[c + d*x]^2))/(d*(a + 2*b + a*Cosh[2*c + 2*d*x])^3)
```

fricas [B] time = 0.44, size = 1701, normalized size = 21.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -(a^3*d*x*cosh(d*x + c)^8 + 8*a^3*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + a^3*d
*x*sinh(d*x + c)^8 + 2*(a^3 + 3*a^2*b + 3*a*b^2 + 2*b^3)*cosh(d*x + c)^6 +
2*(14*a^3*d*x*cosh(d*x + c)^2 + a^3 + 3*a^2*b + 3*a*b^2 + 2*b^3)*sinh(d*x +
 c)^6 + 4*(14*a^3*d*x*cosh(d*x + c)^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + 2*b^3)
*cosh(d*x + c))*sinh(d*x + c)^5 + a^3*d*x - 2*(a^3*d*x - 2*a^3 - 6*a^2*b -
6*a*b^2)*cosh(d*x + c)^4 + 2*(35*a^3*d*x*cosh(d*x + c)^4 - a^3*d*x + 2*a^3
+ 6*a^2*b + 6*a*b^2 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + 2*b^3)*cosh(d*x + c)^2)
*sinh(d*x + c)^4 + 8*(7*a^3*d*x*cosh(d*x + c)^5 + 5*(a^3 + 3*a^2*b + 3*a*b^
```

```
2 + 2*b^3)*cosh(d*x + c)^3 - (a^3*d*x - 2*a^3 - 6*a^2*b - 6*a*b^2)*cosh(d*x
 + c))*sinh(d*x + c)^3 + 2*(a^3 + 3*a^2*b + 3*a*b^2 + 2*b^3)*cosh(d*x + c)^
2 + 2*(14*a^3*d*x*cosh(d*x + c)^6 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + 2*b^3)*co
sh(d*x + c)^4 + a^3 + 3*a^2*b + 3*a*b^2 + 2*b^3 - 6*(a^3*d*x - 2*a^3 - 6*a^
2*b - 6*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - ((3*a*b^2 + 2*b^3)*cosh(d
*x + c)^8 + 56*(3*a*b^2 + 2*b^3)*cosh(d*x + c)^3*sinh(d*x + c)^5 + 28*(3*a*
b^2 + 2*b^3)*cosh(d*x + c)^2*sinh(d*x + c)^6 + 8*(3*a*b^2 + 2*b^3)*cosh(d*x
 + c)*sinh(d*x + c)^7 + (3*a*b^2 + 2*b^3)*sinh(d*x + c)^8 - 2*(3*a*b^2 + 2*
b^3)*cosh(d*x + c)^4 + 2*(35*(3*a*b^2 + 2*b^3)*cosh(d*x + c)^4 - 3*a*b^2 -
2*b^3)*sinh(d*x + c)^4 + 8*(7*(3*a*b^2 + 2*b^3)*cosh(d*x + c)^5 - (3*a*b^2
+ 2*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*a*b^2 + 2*b^3 + 4*(7*(3*a*b^2 +
 2*b^3)*cosh(d*x + c)^6 - 3*(3*a*b^2 + 2*b^3)*cosh(d*x + c)^2)*sinh(d*x + c
)^2 + 8*((3*a*b^2 + 2*b^3)*cosh(d*x + c)^7 - (3*a*b^2 + 2*b^3)*cosh(d*x + c
)^3)*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) -
((a^3 - 3*a*b^2 - 2*b^3)*cosh(d*x + c)^8 + 56*(a^3 - 3*a*b^2 - 2*b^3)*cosh(
d*x + c)^3*sinh(d*x + c)^5 + 28*(a^3 - 3*a*b^2 - 2*b^3)*cosh(d*x + c)^2*sin
h(d*x + c)^6 + 8*(a^3 - 3*a*b^2 - 2*b^3)*cosh(d*x + c)*sinh(d*x + c)^7 + (a
^3 - 3*a*b^2 - 2*b^3)*sinh(d*x + c)^8 - 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(d*x
+ c)^4 + 2*(35*(a^3 - 3*a*b^2 - 2*b^3)*cosh(d*x + c)^4 - a^3 + 3*a*b^2 + 2*
b^3)*sinh(d*x + c)^4 + 8*(7*(a^3 - 3*a*b^2 - 2*b^3)*cosh(d*x + c)^5 - (a^3
- 3*a*b^2 - 2*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + a^3 - 3*a*b^2 - 2*b^3 +
 4*(7*(a^3 - 3*a*b^2 - 2*b^3)*cosh(d*x + c)^6 - 3*(a^3 - 3*a*b^2 - 2*b^3)*c
osh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((a^3 - 3*a*b^2 - 2*b^3)*cosh(d*x + c)^
7 - (a^3 - 3*a*b^2 - 2*b^3)*cosh(d*x + c)^3)*sinh(d*x + c))*log(2*sinh(d*x
+ c)/(cosh(d*x + c) - sinh(d*x + c))) + 4*(2*a^3*d*x*cosh(d*x + c)^7 + 3*(a
^3 + 3*a^2*b + 3*a*b^2 + 2*b^3)*cosh(d*x + c)^5 - 2*(a^3*d*x - 2*a^3 - 6*a^
2*b - 6*a*b^2)*cosh(d*x + c)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + 2*b^3)*cosh(d*x
 + c))*sinh(d*x + c))/(d*cosh(d*x + c)^8 + 56*d*cosh(d*x + c)^3*sinh(d*x +
c)^5 + 28*d*cosh(d*x + c)^2*sinh(d*x + c)^6 + 8*d*cosh(d*x + c)*sinh(d*x +
c)^7 + d*sinh(d*x + c)^8 - 2*d*cosh(d*x + c)^4 + 2*(35*d*cosh(d*x + c)^4 -
d)*sinh(d*x + c)^4 + 8*(7*d*cosh(d*x + c)^5 - d*cosh(d*x + c))*sinh(d*x + c
)^3 + 4*(7*d*cosh(d*x + c)^6 - 3*d*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*(d*
cosh(d*x + c)^7 - d*cosh(d*x + c)^3)*sinh(d*x + c) + d)
```

giac [B] time = 0.29, size = 292, normalized size = 3.60

$$-\frac{4\,a^{3}dx-4\left(3\,ab^{2}e^{(2\,c)}+2\,b^{3}e^{(2\,c)}\right)e^{(-2\,c)}\log\left(e^{(2\,dx+2\,c)}+1\right)-4\left(a^{3}e^{(2\,c)}-3\,ab^{2}e^{(2\,c)}-2\,b^{3}e^{(2\,c)}\right)e^{(-2\,c)}\log\left(\left|e^{(2\,dx+2\,c}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/4*(4*a^3*d*x - 4*(3*a*b^2*e^(2*c) + 2*b^3*e^(2*c))*e^(-2*c)*log(e^(2*d*x
 + 2*c) + 1) - 4*(a^3*e^(2*c) - 3*a*b^2*e^(2*c) - 2*b^3*e^(2*c))*e^(-2*c)*l
```

```
og(abs(e^(2*d*x + 2*c) - 1)) + (3*a^3*e^(8*d*x + 8*c) + 8*a^3*e^(6*d*x + 6*
c) + 24*a^2*b*e^(6*d*x + 6*c) + 24*a*b^2*e^(6*d*x + 6*c) + 16*b^3*e^(6*d*x
+ 6*c) + 10*a^3*e^(4*d*x + 4*c) + 48*a^2*b*e^(4*d*x + 4*c) + 48*a*b^2*e^(4*
d*x + 4*c) + 8*a^3*e^(2*d*x + 2*c) + 24*a^2*b*e^(2*d*x + 2*c) + 24*a*b^2*e^
(2*d*x + 2*c) + 16*b^3*e^(2*d*x + 2*c) + 3*a^3)/(e^(4*d*x + 4*c) - 1)^2)/d
```

maple [A] time = 0.36, size = 137, normalized size = 1.69

$$\frac{a^3 \ln(\sinh(dx+c))}{d} - \frac{a^3\left(\coth^2(dx+c)\right)}{2d} - \frac{3a^2b}{2d\sinh(dx+c)^2} - \frac{3ab^2}{2d\sinh(dx+c)^2} - \frac{3ab^2\ln(\tanh(dx+c))}{d} - \frac{}{2d\sin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] a^3*ln(sinh(d*x+c))/d-1/2*a^3*coth(d*x+c)^2/d-3/2/d/sinh(d*x+c)^2*a^2*b-3/2
/d*a*b^2/sinh(d*x+c)^2-3/d*a*b^2*ln(tanh(d*x+c))-1/2/d*b^3/sinh(d*x+c)^2/co
sh(d*x+c)^2-1/d*b^3/cosh(d*x+c)^2-2/d*b^3*ln(tanh(d*x+c))
```

maxima [B] time = 0.47, size = 314, normalized size = 3.88

$$a^3\left(x + \frac{c}{d} + \frac{\log\left(e^{(-dx-c)}+1\right)}{d} + \frac{\log\left(e^{(-dx-c)}-1\right)}{d} + \frac{2\,e^{(-2\,dx-2\,c)}}{d\left(2\,e^{(-2\,dx-2\,c)} - e^{(-4\,dx-4\,c)} - 1\right)}\right) - 2\,b^3\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d} + \cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] a^3*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 2*e^(-2*
d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) - 2*b^3*(log(e^
(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d - log(e^(-2*d*x - 2*c) + 1)/d -
 2*(e^(-2*d*x - 2*c) + e^(-6*d*x - 6*c))/(d*(2*e^(-4*d*x - 4*c) - e^(-8*d*x
 - 8*c) - 1))) - 3*a*b^2*(log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d
 - log(e^(-2*d*x - 2*c) + 1)/d - 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c)
- e^(-4*d*x - 4*c) - 1))) - 6*a^2*b/(d*(e^(d*x + c) - e^(-d*x - c))^2)
```

mupad [B] time = 1.72, size = 324, normalized size = 4.00

$$\frac{\operatorname{atan}\left(\frac{e^{2c}\,e^{2dx}\left(4\,b^3\,\sqrt{-d^2} - a^3\,\sqrt{-d^2} + 6\,a\,b^2\,\sqrt{-d^2}\right)}{d\,\sqrt{a^6-12\,a^4\,b^2-8\,a^3\,b^3+36\,a^2\,b^4+48\,a\,b^5+16\,b^6}}\right)\sqrt{a^6 - 12\,a^4\,b^2 - 8\,a^3\,b^3 + 36\,a^2\,b^4 + 48\,a\,b^5 + 16\,b^6}}{\sqrt{-d^2}} - \frac{4\left(a^3+3\,a^2\,b+3\,a\,b\cdots\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^3*(a + b/cosh(c + d*x)^2)^3,x)
```

```
[Out] (atan((exp(2*c)*exp(2*d*x)*(4*b^3*(-d^2)^(1/2) - a^3*(-d^2)^(1/2) + 6*a*b^2
*(-d^2)^(1/2)))/(d*(48*a*b^5 + a^6 + 16*b^6 + 36*a^2*b^4 - 8*a^3*b^3 - 12*a
^4*b^2)^(1/2)))*(48*a*b^5 + a^6 + 16*b^6 + 36*a^2*b^4 - 8*a^3*b^3 - 12*a^4*
b^2)^(1/2))/(-d^2)^(1/2) - ((4*(3*a*b^2 + 3*a^2*b + a^3))/d + (2*exp(2*c +
2*d*x)*(3*a*b^2 + 3*a^2*b + a^3 + 2*b^3))/d)/(exp(4*c + 4*d*x) - 1) - ((4*(
3*a*b^2 + 3*a^2*b + a^3))/d + (4*exp(2*c + 2*d*x)*(3*a*b^2 + 3*a^2*b + a^3
+ 2*b^3))/d)/(exp(8*c + 8*d*x) - 2*exp(4*c + 4*d*x) + 1) - a^3*x + (a^3*log
(exp(4*c + 4*d*x) - 1))/(2*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**3*(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

3.132 $\int \coth^4(c+dx)\left(a+b\,\mathbf{sech}^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=60

$$a^3x-\frac{(a+b)^3\coth^3(c+dx)}{3d}-\frac{(a-2b)(a+b)^2\coth(c+dx)}{d}+\frac{b^3\tanh(c+dx)}{d}$$

```
[Out] a^3*x-(a-2*b)*(a+b)^2*coth(d*x+c)/d-1/3*(a+b)^3*coth(d*x+c)^3/d+b^3*tanh(d*
x+c)/d
```

Rubi [A] time = 0.10, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4141, 1802, 207}

$$a^3x-\frac{(a+b)^3\coth^3(c+dx)}{3d}-\frac{(a-2b)(a+b)^2\coth(c+dx)}{d}+\frac{b^3\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^4*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] a^3*x - ((a - 2*b)*(a + b)^2*Coth[c + d*x])/d - ((a + b)^3*Coth[c + d*x]^3)
/(3*d) + (b^3*Tanh[c + d*x])/d
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \coth^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\left(1-x^2\right)\right)^3}{x^4\left(1-x^2\right)}\,dx,x,\tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(b^3+\frac{(a+b)^3}{x^4}+\frac{(a-2b)(a+b)^2}{x^2}-\frac{a^3}{-1+x^2}\right)dx,x,\tanh(c+dx)\right)}{d}$$

$$= -\frac{(a-2b)(a+b)^2\coth(c+dx)}{d}-\frac{(a+b)^3\coth^3(c+dx)}{3d}+\frac{b^3\tanh(}{d}$$

$$= a^3x-\frac{(a-2b)(a+b)^2\coth(c+dx)}{d}-\frac{(a+b)^3\coth^3(c+dx)}{3d}+\frac{b^3\text{ta}}{}$$

Mathematica [B] time = 1.73, size = 343, normalized size = 5.72

$$\frac{\operatorname{csch}(c)\operatorname{sech}(c)\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)\left(-16a^3\sinh(2(c+dx))+8a^3\sinh(4(c+dx))+8a^3\sinh(2(c+2dx))-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^4*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (Csch[c]*Csch[c + d*x]^3*Sech[c]*Sech[c + d*x]*(6*a^3*d*x*Cosh[2*d*x] - 3*a
^3*d*x*Cosh[2*(c + 2*d*x)] - 6*a^3*d*x*Cosh[4*c + 2*d*x] + 3*a^3*d*x*Cosh[6
*c + 4*d*x] - 18*a^2*b*Sinh[2*c] - 36*a*b^2*Sinh[2*c] - 4*a^3*Sinh[2*d*x] +
 6*a^2*b*Sinh[2*d*x] + 24*a*b^2*Sinh[2*d*x] + 32*b^3*Sinh[2*d*x] - 16*a^3*S
inh[2*(c + d*x)] - 12*a^2*b*Sinh[2*(c + d*x)] + 24*a*b^2*Sinh[2*(c + d*x)]
+ 8*b^3*Sinh[2*(c + d*x)] + 8*a^3*Sinh[4*(c + d*x)] + 6*a^2*b*Sinh[4*(c + d
*x)] - 12*a*b^2*Sinh[4*(c + d*x)] - 4*b^3*Sinh[4*(c + d*x)] + 8*a^3*Sinh[2*
(c + 2*d*x)] + 6*a^2*b*Sinh[2*(c + 2*d*x)] - 12*a*b^2*Sinh[2*(c + 2*d*x)] -
 16*b^3*Sinh[2*(c + 2*d*x)] - 12*a^3*Sinh[4*c + 2*d*x] - 18*a^2*b*Sinh[4*c
+ 2*d*x]))/(96*d)
```

fricas [B] time = 0.40, size = 354, normalized size = 5.90

$$-\frac{\left(4a^3+3a^2b-6ab^2-8b^3\right)\cosh(dx+c)^4-4\left(3a^3dx+4a^3+3a^2b-6ab^2-8b^3\right)\cosh(dx+c)\sinh(dx+c)^{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/12*((4*a^3 + 3*a^2*b - 6*a*b^2 - 8*b^3)*cosh(d*x + c)^4 - 4*(3*a^3*d*x +
 4*a^3 + 3*a^2*b - 6*a*b^2 - 8*b^3)*cosh(d*x + c)*sinh(d*x + c)^3 + (4*a^3
```

```
+ 3*a^2*b - 6*a*b^2 - 8*b^3)*sinh(d*x + c)^4 + 9*a^2*b + 18*a*b^2 + 4*(a^3
+ 3*a^2*b + 3*a*b^2 + 4*b^3)*cosh(d*x + c)^2 + 2*(2*a^3 + 6*a^2*b + 6*a*b^2
 + 8*b^3 + 3*(4*a^3 + 3*a^2*b - 6*a*b^2 - 8*b^3)*cosh(d*x + c)^2)*sinh(d*x
+ c)^2 - 4*((3*a^3*d*x + 4*a^3 + 3*a^2*b - 6*a*b^2 - 8*b^3)*cosh(d*x + c)^3
 - (3*a^3*d*x + 4*a^3 + 3*a^2*b - 6*a*b^2 - 8*b^3)*cosh(d*x + c))*sinh(d*x
+ c))/(d*cosh(d*x + c)*sinh(d*x + c)^3 + (d*cosh(d*x + c)^3 - d*cosh(d*x +
c))*sinh(d*x + c))
```

giac [B] time = 0.34, size = 155, normalized size = 2.58

$$\frac{3\,a^3dx-\frac{6\,b^3}{e^{(2\,dx+2\,c)}+1}-\frac{2\left(6\,a^3e^{(4\,dx+4\,c)}+9\,a^2be^{(4\,dx+4\,c)}-3\,b^3e^{(4\,dx+4\,c)}-6\,a^3e^{(2\,dx+2\,c)}+18\,ab^2e^{(2\,dx+2\,c)}+12\,b^3e^{(2\,dx+2\,c)}+4\,a^3+3\,a^2b-6\,ab^2-5\,b^3\right)}{\left(e^{(2\,dx+2\,c)}-1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/3*(3*a^3*d*x - 6*b^3/(e^(2*d*x + 2*c) + 1) - 2*(6*a^3*e^(4*d*x + 4*c) + 9
*a^2*b*e^(4*d*x + 4*c) - 3*b^3*e^(4*d*x + 4*c) - 6*a^3*e^(2*d*x + 2*c) + 18
*a*b^2*e^(2*d*x + 2*c) + 12*b^3*e^(2*d*x + 2*c) + 4*a^3 + 3*a^2*b - 6*a*b^2
 - 5*b^3)/(e^(2*d*x + 2*c) - 1)^3)/d
```

maple [B] time = 0.48, size = 149, normalized size = 2.48

$$\frac{a^3\left(dx+c-\coth(dx+c)-\frac{\left(\coth^3(dx+c)\right)}{3}\right)+3a^2b\left(-\frac{\cosh(dx+c)}{2\sinh(dx+c)^3}-\frac{\left(\frac{2}{3}-\frac{\mathrm{csch}(dx+c)^2}{3}\right)\coth(dx+c)}{2}\right)+3a\,b^2\left(\frac{2}{3}-\frac{\mathrm{csch}(dx+c)^2}{3}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(a^3*(d*x+c-coth(d*x+c)-1/3*coth(d*x+c)^3)+3*a^2*b*(-1/2/sinh(d*x+c)^3*
cosh(d*x+c)-1/2*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c))+3*a*b^2*(2/3-1/3*csch(
d*x+c)^2)*coth(d*x+c)+b^3*(-1/3/sinh(d*x+c)^3/cosh(d*x+c)+4/3/sinh(d*x+c)/c
osh(d*x+c)+8/3*tanh(d*x+c)))
```

maxima [B] time = 0.52, size = 366, normalized size = 6.10

$$\frac{1}{3}\,a^3\left(3\,x+\frac{3\,c}{d}-\frac{4\left(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}-2\right)}{d\left(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1\right)}\right)+4\,ab^2\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d\left(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/3*a^3*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) - 2)/(d*(
3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1))) + 4*a*b^2
*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*
x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x
- 6*c) - 1))) + 16/3*b^3*(2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - 2*e^(
-6*d*x - 6*c) + e^(-8*d*x - 8*c) - 1)) - 1/(d*(2*e^(-2*d*x - 2*c) - 2*e^(-6
*d*x - 6*c) + e^(-8*d*x - 8*c) - 1))) + 2*a^2*b*(3*e^(-4*d*x - 4*c)/(d*(3*e
^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)) + 1/(d*(3*e^(
-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)))
```

mupad [B] time = 1.48, size = 260, normalized size = 4.33

$$a^3\,x-\frac{\frac{2\left(a^2\,b+2\,a\,b^2+b^3\right)}{d}+\frac{2\,\mathrm{e}^{2\,c+2\,d\,x}\left(2\,a^3+3\,a^2\,b-b^3\right)}{3\,d}}{\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1}-\frac{\frac{2\left(2\,a^3+3\,a^2\,b-b^3\right)}{3\,d}+\frac{2\,\mathrm{e}^{4\,c+4\,d\,x}\left(2\,a^3+3\,a^2\,b-b^3\right)}{3\,d}+\frac{4\,\mathrm{e}^{2\,c+2\,d\,x}\left(a^2\,b+2\,a\,b^2+b^3\right)}{d}}{3\,\mathrm{e}^{2\,c+2\,d\,x}-3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}-1}-\frac{}{d\,\left(\mathrm{e}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^4*(a + b/cosh(c + d*x)^2)^3,x)
```

```
[Out] a^3*x - ((2*(2*a*b^2 + a^2*b + b^3))/d + (2*exp(2*c + 2*d*x)*(3*a^2*b + 2*a
^3 - b^3))/(3*d))/(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1) - ((2*(3*a^2*
b + 2*a^3 - b^3))/(3*d) + (2*exp(4*c + 4*d*x)*(3*a^2*b + 2*a^3 - b^3))/(3*d
) + (4*exp(2*c + 2*d*x)*(2*a*b^2 + a^2*b + b^3))/d)/(3*exp(2*c + 2*d*x) - 3
*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1) - (2*b^3)/(d*(exp(2*c + 2*d*x) +
1)) - (2*(3*a^2*b + 2*a^3 - b^3))/(3*d*(exp(2*c + 2*d*x) - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**4*(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

3.133 $\int \coth^5(c+dx)\left(a+b\,\mathbf{sech}^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=81

$$\frac{\left(a^3+b^3\right)\log(\sinh(c+dx))}{d}-\frac{(a+b)^3\text{csch}^4(c+dx)}{4d}-\frac{(2a-b)(a+b)^2\text{csch}^2(c+dx)}{2d}-\frac{b^3\log(\cosh(c+dx))}{d}$$

```
[Out] -1/2*(2*a-b)*(a+b)^2*csch(d*x+c)^2/d-1/4*(a+b)^3*csch(d*x+c)^4/d-b^3*ln(cos
h(d*x+c))/d+(a^3+b^3)*ln(sinh(d*x+c))/d
```

Rubi [A] time = 0.12, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4138, 446, 88}

$$\frac{\left(a^3+b^3\right)\log(\sinh(c+dx))}{d}-\frac{(a+b)^3\text{csch}^4(c+dx)}{4d}-\frac{(2a-b)(a+b)^2\text{csch}^2(c+dx)}{2d}-\frac{b^3\log(\cosh(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^5*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] -((2*a - b)*(a + b)^2*Csch[c + d*x]^2)/(2*d) - ((a + b)^3*Csch[c + d*x]^4)/
(4*d) - (b^3*Log[Cosh[c + d*x]])/d + ((a^3 + b^3)*Log[Sinh[c + d*x]])/d
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
```

```
x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \coth^5(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx=-\frac{\text{Subst}\left(\int \frac{\left(b+ax^2\right)^3}{x\left(1-x^2\right)^3}\,dx,x,\cosh(c+dx)\right)}{d}$$
$$=-\frac{\text{Subst}\left(\int \frac{(b+ax)^3}{(1-x)^3x}\,dx,x,\cosh^2(c+dx)\right)}{2d}$$
$$=-\frac{\text{Subst}\left(\int \left(-\frac{(a+b)^3}{(-1+x)^3}-\frac{(2a-b)(a+b)^2}{(-1+x)^2}+\frac{-a^3-b^3}{-1+x}+\frac{b^3}{x}\right)dx,x,\cosh^2(c+d\right.}{2d}$$
$$=-\frac{(2a-b)(a+b)^2\text{csch}^2(c+dx)}{2d}-\frac{(a+b)^3\text{csch}^4(c+dx)}{4d}-\frac{b^3\log(c}{}$$

Mathematica [A] time = 0.91, size = 101, normalized size = 1.25

$$-\frac{2\left(a\cosh^2(c+dx)+b\right)^3\left(-4\left(a^3+b^3\right)\log(\sinh(c+dx))+(a+b)^3\text{csch}^4(c+dx)+2(2a-b)(a+b)^2\text{csch}^2(c+a\right.}{d(a\cosh(2(c+dx))+a+2b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^5*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (-2*(b + a*Cosh[c + d*x]^2)^3*(2*(2*a - b)*(a + b)^2*Csch[c + d*x]^2 + (a +
 b)^3*Csch[c + d*x]^4 + 4*b^3*Log[Cosh[c + d*x]] - 4*(a^3 + b^3)*Log[Sinh[c
 + d*x]]))/(d*(a + 2*b + a*Cosh[2*(c + d*x)])^3)
```

fricas [B] time = 0.45, size = 1830, normalized size = 22.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^5*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -(a^3*d*x*cosh(d*x + c)^8 + 8*a^3*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + a^3*d
*x*sinh(d*x + c)^8 - 2*(2*a^3*d*x - 2*a^3 - 3*a^2*b + b^3)*cosh(d*x + c)^6
+ 2*(14*a^3*d*x*cosh(d*x + c)^2 - 2*a^3*d*x + 2*a^3 + 3*a^2*b - b^3)*sinh(d
*x + c)^6 + 4*(14*a^3*d*x*cosh(d*x + c)^3 - 3*(2*a^3*d*x - 2*a^3 - 3*a^2*b
+ b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + a^3*d*x + 2*(3*a^3*d*x - 2*a^3 + 6*
```

```
a*b^2 + 4*b^3)*cosh(d*x + c)^4 + 2*(35*a^3*d*x*cosh(d*x + c)^4 + 3*a^3*d*x
- 2*a^3 + 6*a*b^2 + 4*b^3 - 15*(2*a^3*d*x - 2*a^3 - 3*a^2*b + b^3)*cosh(d*x
 + c)^2)*sinh(d*x + c)^4 + 8*(7*a^3*d*x*cosh(d*x + c)^5 - 5*(2*a^3*d*x - 2*
a^3 - 3*a^2*b + b^3)*cosh(d*x + c)^3 + (3*a^3*d*x - 2*a^3 + 6*a*b^2 + 4*b^3
)*cosh(d*x + c))*sinh(d*x + c)^3 - 2*(2*a^3*d*x - 2*a^3 - 3*a^2*b + b^3)*co
sh(d*x + c)^2 + 2*(14*a^3*d*x*cosh(d*x + c)^6 - 2*a^3*d*x - 15*(2*a^3*d*x -
 2*a^3 - 3*a^2*b + b^3)*cosh(d*x + c)^4 + 2*a^3 + 3*a^2*b - b^3 + 6*(3*a^3*
d*x - 2*a^3 + 6*a*b^2 + 4*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + (b^3*cosh
(d*x + c)^8 + 8*b^3*cosh(d*x + c)*sinh(d*x + c)^7 + b^3*sinh(d*x + c)^8 - 4
*b^3*cosh(d*x + c)^6 + 6*b^3*cosh(d*x + c)^4 + 4*(7*b^3*cosh(d*x + c)^2 - b
^3)*sinh(d*x + c)^6 + 8*(7*b^3*cosh(d*x + c)^3 - 3*b^3*cosh(d*x + c))*sinh(
d*x + c)^5 - 4*b^3*cosh(d*x + c)^2 + 2*(35*b^3*cosh(d*x + c)^4 - 30*b^3*cos
h(d*x + c)^2 + 3*b^3)*sinh(d*x + c)^4 + 8*(7*b^3*cosh(d*x + c)^5 - 10*b^3*c
osh(d*x + c)^3 + 3*b^3*cosh(d*x + c))*sinh(d*x + c)^3 + b^3 + 4*(7*b^3*cosh
(d*x + c)^6 - 15*b^3*cosh(d*x + c)^4 + 9*b^3*cosh(d*x + c)^2 - b^3)*sinh(d*
x + c)^2 + 8*(b^3*cosh(d*x + c)^7 - 3*b^3*cosh(d*x + c)^5 + 3*b^3*cosh(d*x
+ c)^3 - b^3*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x +
c) - sinh(d*x + c))) - ((a^3 + b^3)*cosh(d*x + c)^8 + 8*(a^3 + b^3)*cosh(d*
x + c)*sinh(d*x + c)^7 + (a^3 + b^3)*sinh(d*x + c)^8 - 4*(a^3 + b^3)*cosh(d
*x + c)^6 - 4*(a^3 + b^3 - 7*(a^3 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 +
 8*(7*(a^3 + b^3)*cosh(d*x + c)^3 - 3*(a^3 + b^3)*cosh(d*x + c))*sinh(d*x +
 c)^5 + 6*(a^3 + b^3)*cosh(d*x + c)^4 + 2*(35*(a^3 + b^3)*cosh(d*x + c)^4 +
 3*a^3 + 3*b^3 - 30*(a^3 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(a^
3 + b^3)*cosh(d*x + c)^5 - 10*(a^3 + b^3)*cosh(d*x + c)^3 + 3*(a^3 + b^3)*c
osh(d*x + c))*sinh(d*x + c)^3 + a^3 + b^3 - 4*(a^3 + b^3)*cosh(d*x + c)^2 +
 4*(7*(a^3 + b^3)*cosh(d*x + c)^6 - 15*(a^3 + b^3)*cosh(d*x + c)^4 - a^3 -
b^3 + 9*(a^3 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((a^3 + b^3)*cosh(
d*x + c)^7 - 3*(a^3 + b^3)*cosh(d*x + c)^5 + 3*(a^3 + b^3)*cosh(d*x + c)^3
- (a^3 + b^3)*cosh(d*x + c))*sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x +
 c) - sinh(d*x + c))) + 4*(2*a^3*d*x*cosh(d*x + c)^7 - 3*(2*a^3*d*x - 2*a^3
 - 3*a^2*b + b^3)*cosh(d*x + c)^5 + 2*(3*a^3*d*x - 2*a^3 + 6*a*b^2 + 4*b^3)
*cosh(d*x + c)^3 - (2*a^3*d*x - 2*a^3 - 3*a^2*b + b^3)*cosh(d*x + c))*sinh(
d*x + c))/(d*cosh(d*x + c)^8 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d
*x + c)^8 - 4*d*cosh(d*x + c)^6 + 4*(7*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)
^6 + 8*(7*d*cosh(d*x + c)^3 - 3*d*cosh(d*x + c))*sinh(d*x + c)^5 + 6*d*cosh
(d*x + c)^4 + 2*(35*d*cosh(d*x + c)^4 - 30*d*cosh(d*x + c)^2 + 3*d)*sinh(d*
x + c)^4 + 8*(7*d*cosh(d*x + c)^5 - 10*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c
))*sinh(d*x + c)^3 - 4*d*cosh(d*x + c)^2 + 4*(7*d*cosh(d*x + c)^6 - 15*d*co
sh(d*x + c)^4 + 9*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^2 + 8*(d*cosh(d*x +
c)^7 - 3*d*cosh(d*x + c)^5 + 3*d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*
x + c) + d)
```

giac [B] time = 0.41, size = 248, normalized size = 3.06

$$-\frac{12\,a^3dx + 12\,b^3\log\left(e^{(2\,dx+2\,c)}+1\right) - 12\left(a^3e^{(2\,c)} + b^3e^{(2\,c)}\right)e^{(-2\,c)}\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right) + \frac{25\,a^3e^{(8\,dx+8\,c)}+25\,b^3e^{(8\,dx+8\,c)}-52\,\cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^5*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/12*(12*a^3*d*x + 12*b^3*log(e^(2*d*x + 2*c) + 1) - 12*(a^3*e^(2*c) + b^3
*e^(2*c))*e^(-2*c)*log(abs(e^(2*d*x + 2*c) - 1)) + (25*a^3*e^(8*d*x + 8*c)
+ 25*b^3*e^(8*d*x + 8*c) - 52*a^3*e^(6*d*x + 6*c) + 72*a^2*b*e^(6*d*x + 6*c
) - 124*b^3*e^(6*d*x + 6*c) + 102*a^3*e^(4*d*x + 4*c) + 144*a*b^2*e^(4*d*x
+ 4*c) + 246*b^3*e^(4*d*x + 4*c) - 52*a^3*e^(2*d*x + 2*c) + 72*a^2*b*e^(2*d
*x + 2*c) - 124*b^3*e^(2*d*x + 2*c) + 25*a^3 + 25*b^3)/(e^(2*d*x + 2*c) - 1
)^4)/d
```

maple [A] time = 0.32, size = 153, normalized size = 1.89

$$\frac{a^3\ln(\sinh(dx+c))}{d} - \frac{a^3\left(\coth^2(dx+c)\right)}{2d} - \frac{a^3\left(\coth^4(dx+c)\right)}{4d} - \frac{3a^2b\left(\cosh^2(dx+c)\right)}{2d\sinh(dx+c)^4} + \frac{3a^2b}{4d\sinh(dx+c)^4} - \frac{3\cdots}{4d\sinh\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^5*(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] a^3*ln(sinh(d*x+c))/d-1/2*a^3*coth(d*x+c)^2/d-1/4*a^3*coth(d*x+c)^4/d-3/2/d
*a^2*b/sinh(d*x+c)^4*cosh(d*x+c)^2+3/4/d*a^2*b/sinh(d*x+c)^4-3/4/d/sinh(d*x
+c)^4*a*b^2-1/4/d*b^3/sinh(d*x+c)^4+1/2/d*b^3/sinh(d*x+c)^2+1/d*b^3*ln(tanh
(d*x+c))
```

maxima [B] time = 0.50, size = 422, normalized size = 5.21

$$a^3\left(x + \frac{c}{d} + \frac{\log\left(e^{(-dx-c)}+1\right)}{d} + \frac{\log\left(e^{(-dx-c)}-1\right)}{d} + \frac{4\left(e^{(-2\,dx-2\,c)} - e^{(-4\,dx-4\,c)} + e^{(-6\,dx-6\,c)}\right)}{d\left(4\,e^{(-2\,dx-2\,c)} - 6\,e^{(-4\,dx-4\,c)} + 4\,e^{(-6\,dx-6\,c)} - e^{(-8\,dx-8\,c)} - 1\right)}\right) + \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^5*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] a^3*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 4*(e^(-2
*d*x - 2*c) - e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/(d*(4*e^(-2*d*x - 2*c) -
 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) - 1))) + b^3*(l
og(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d - log(e^(-2*d*x - 2*c) + 1
```

```
)/d - 2*(e^(-2*d*x - 2*c) - 4*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/(d*(4*e^
(-2*d*x - 2*c) - 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c)
 - 1))) + 6*a^2*b*(e^(-2*d*x - 2*c)/(d*(4*e^(-2*d*x - 2*c) - 6*e^(-4*d*x -
4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) - 1)) + e^(-6*d*x - 6*c)/(d*(4
*e^(-2*d*x - 2*c) - 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x - 8
*c) - 1))) - 12*a*b^2/(d*(e^(d*x + c) - e^(-d*x - c))^4)
```

mupad [B] time = 1.64, size = 384, normalized size = 4.74

$$-a^3\,x-\frac{2\,\left(4\,a^3+9\,a^2\,b+6\,a\,b^2+b^3\right)}{d\,\left({\mathrm{e}}^{4\,c+4\,d\,x}-2\,{\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}-\frac{\ln\left({\mathrm{e}}^{4\,c+4\,d\,x}-1\right)\,\left(b^3\,d-d\,\left(a^3+b^3\right)\right)}{2\,d^2}-\frac{\mathrm{atan}\left(\frac{{\mathrm{e}}^{2\,c}\,{\mathrm{e}}^{2\,d\,x}\,\left(a^6\,\sqrt{-d^2}+4\,b^6\,\sqrt{-d^2}+4\,a^3}{a^3\,d\,\sqrt{a^6+4\,a^3\,b^3+4\,b^6}+2\,b^3\,d\,\sqrt{a^6+4}}\right.}{\sqrt{-d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^5*(a + b/cosh(c + d*x)^2)^3,x)
```

```
[Out] - a^3*x - (2*(6*a*b^2 + 9*a^2*b + 4*a^3 + b^3))/(d*(exp(4*c + 4*d*x) - 2*ex
p(2*c + 2*d*x) + 1)) - (log(exp(4*c + 4*d*x) - 1)*(b^3*d - d*(a^3 + b^3)))/
(2*d^2) - (atan((exp(2*c)*exp(2*d*x)*(a^6*(-d^2)^(1/2) + 4*b^6*(-d^2)^(1/2)
 + 4*a^3*b^3*(-d^2)^(1/2)))/(a^3*d*(a^6 + 4*b^6 + 4*a^3*b^3)^(1/2) + 2*b^3*
d*(a^6 + 4*b^6 + 4*a^3*b^3)^(1/2)))*(a^6 + 4*b^6 + 4*a^3*b^3)^(1/2))/(-d^2)
^(1/2) - (2*(3*a^2*b + 2*a^3 - b^3))/(d*(exp(2*c + 2*d*x) - 1)) - (8*(3*a*b
^2 + 3*a^2*b + a^3 + b^3))/(d*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + ex
p(6*c + 6*d*x) - 1)) - (4*(3*a*b^2 + 3*a^2*b + a^3 + b^3))/(d*(6*exp(4*c +
4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**5*(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

3.134 $\int \coth^6(c+dx)\left(a+b\,\mathrm{sech}^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=69

$$-\frac{\left(a^3+b^3\right)\coth(c+dx)}{d}+a^3x-\frac{(a+b)^3\coth^5(c+dx)}{5d}-\frac{(a-2b)(a+b)^2\coth^3(c+dx)}{3d}$$

```
[Out] a^3*x-(a^3+b^3)*coth(d*x+c)/d-1/3*(a-2*b)*(a+b)^2*coth(d*x+c)^3/d-1/5*(a+b)
^3*coth(d*x+c)^5/d
```

Rubi [A] time = 0.11, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4141, 1802, 207}

$$-\frac{\left(a^3+b^3\right)\coth(c+dx)}{d}+a^3x-\frac{(a+b)^3\coth^5(c+dx)}{5d}-\frac{(a-2b)(a+b)^2\coth^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^6*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] a^3*x - ((a^3 + b^3)*Coth[c + d*x])/d - ((a - 2*b)*(a + b)^2*Coth[c + d*x]^
3)/(3*d) - ((a + b)^3*Coth[c + d*x]^5)/(5*d)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \coth^6(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\left(1-x^2\right)\right)^3}{x^6\left(1-x^2\right)}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{(a+b)^3}{x^6}+\frac{(a-2b)(a+b)^2}{x^4}+\frac{a^3+b^3}{x^2}-\frac{a^3}{-1+x^2}\right)dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{\left(a^3+b^3\right)\coth(c+dx)}{d}-\frac{(a-2b)(a+b)^2\coth^3(c+dx)}{3d}-\frac{(a+b}{}$$

$$= a^3x-\frac{\left(a^3+b^3\right)\coth(c+dx)}{d}-\frac{(a-2b)(a+b)^2\coth^3(c+dx)}{3d}-\frac{(a}{}$$

Mathematica [B] time = 1.09, size = 303, normalized size = 4.39

$$\frac{\operatorname{csch}(c)\operatorname{csch}^5(c+dx)\left(180a^3\sinh(2c+dx)-140a^3\sinh(2c+3dx)-90a^3\sinh(4c+3dx)+46a^3\sinh(4c+5dx\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^6*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (Csch[c]*Csch[c + d*x]^5*(-150*a^3*d*x*Cosh[d*x] + 150*a^3*d*x*Cosh[2*c + d
*x] + 75*a^3*d*x*Cosh[2*c + 3*d*x] - 75*a^3*d*x*Cosh[4*c + 3*d*x] - 15*a^3*
d*x*Cosh[4*c + 5*d*x] + 15*a^3*d*x*Cosh[6*c + 5*d*x] + 280*a^3*Sinh[d*x] +
180*a^2*b*Sinh[d*x] + 60*a*b^2*Sinh[d*x] + 160*b^3*Sinh[d*x] + 180*a^3*Sinh
[2*c + d*x] - 180*a*b^2*Sinh[2*c + d*x] - 140*a^3*Sinh[2*c + 3*d*x] + 60*a*
b^2*Sinh[2*c + 3*d*x] - 80*b^3*Sinh[2*c + 3*d*x] - 90*a^3*Sinh[4*c + 3*d*x]
 - 90*a^2*b*Sinh[4*c + 3*d*x] + 46*a^3*Sinh[4*c + 5*d*x] + 18*a^2*b*Sinh[4*
c + 5*d*x] - 12*a*b^2*Sinh[4*c + 5*d*x] + 16*b^3*Sinh[4*c + 5*d*x]))/(480*d
)
```

fricas [B] time = 0.41, size = 521, normalized size = 7.55

$$-\frac{\left(23\,a^3+9\,a^2b-6\,ab^2+8\,b^3\right)\cosh\left(dx+c\right)^5+5\left(23\,a^3+9\,a^2b-6\,ab^2+8\,b^3\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^4-\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^6*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/15*((23*a^3 + 9*a^2*b - 6*a*b^2 + 8*b^3)*cosh(d*x + c)^5 + 5*(23*a^3 + 9
*a^2*b - 6*a*b^2 + 8*b^3)*cosh(d*x + c)*sinh(d*x + c)^4 - (15*a^3*d*x + 23*
```

```
a^3 + 9*a^2*b - 6*a*b^2 + 8*b^3)*sinh(d*x + c)^5 - 5*(5*a^3 - 9*a^2*b - 6*a
*b^2 + 8*b^3)*cosh(d*x + c)^3 + 5*(15*a^3*d*x + 23*a^3 + 9*a^2*b - 6*a*b^2
+ 8*b^3 - 2*(15*a^3*d*x + 23*a^3 + 9*a^2*b - 6*a*b^2 + 8*b^3)*cosh(d*x + c)
^2)*sinh(d*x + c)^3 + 5*(2*(23*a^3 + 9*a^2*b - 6*a*b^2 + 8*b^3)*cosh(d*x +
c)^3 - 3*(5*a^3 - 9*a^2*b - 6*a*b^2 + 8*b^3)*cosh(d*x + c))*sinh(d*x + c)^2
 + 10*(5*a^3 + 9*a^2*b + 12*a*b^2 + 8*b^3)*cosh(d*x + c) - 5*(30*a^3*d*x +
(15*a^3*d*x + 23*a^3 + 9*a^2*b - 6*a*b^2 + 8*b^3)*cosh(d*x + c)^4 + 46*a^3
+ 18*a^2*b - 12*a*b^2 + 16*b^3 - 3*(15*a^3*d*x + 23*a^3 + 9*a^2*b - 6*a*b^2
 + 8*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*sinh(d*x + c)^5 + 5*(2*d*cosh(
d*x + c)^2 - d)*sinh(d*x + c)^3 + 5*(d*cosh(d*x + c)^4 - 3*d*cosh(d*x + c)^
2 + 2*d)*sinh(d*x + c))
```

giac [B] time = 0.43, size = 210, normalized size = 3.04

$$\frac{15\,a^3dx - \frac{2\left(45\,a^3e^{(8\,dx+8\,c)}+45\,a^2be^{(8\,dx+8\,c)}-90\,a^3e^{(6\,dx+6\,c)}+90\,ab^2e^{(6\,dx+6\,c)}+140\,a^3e^{(4\,dx+4\,c)}+90\,a^2be^{(4\,dx+4\,c)}+30\,ab^2e^{(4\,dx+4\,c)}+80\,b^3e^{(4\,dx+4\,c)}-7\cdots\right.}{\left(e^{(2\,dx+2\,c)}-1\right)^5}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^6*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/15*(15*a^3*d*x - 2*(45*a^3*e^(8*d*x + 8*c) + 45*a^2*b*e^(8*d*x + 8*c) - 9
0*a^3*e^(6*d*x + 6*c) + 90*a*b^2*e^(6*d*x + 6*c) + 140*a^3*e^(4*d*x + 4*c)
+ 90*a^2*b*e^(4*d*x + 4*c) + 30*a*b^2*e^(4*d*x + 4*c) + 80*b^3*e^(4*d*x + 4
*c) - 70*a^3*e^(2*d*x + 2*c) + 30*a*b^2*e^(2*d*x + 2*c) - 40*b^3*e^(2*d*x +
 2*c) + 23*a^3 + 9*a^2*b - 6*a*b^2 + 8*b^3)/(e^(2*d*x + 2*c) - 1)^5)/d
```

maple [B] time = 0.42, size = 199, normalized size = 2.88

$$a^3\left(dx+c-\coth\left(dx+c\right)-\frac{\left(\coth^3(dx+c)\right)}{3}-\frac{\left(\coth^5(dx+c)\right)}{5}\right)+3a^2b\left(-\frac{\cosh^3(dx+c)}{2\sinh(dx+c)^5}+\frac{3\cosh(dx+c)}{8\sinh(dx+c)^5}+\frac{3\left(-\frac{8}{15}-\frac{\operatorname{csch}(dx+c)^4}{5}+\frac{4\operatorname{csch}(\cdots}{1\cdots}\right.}{8}\right.\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^6*(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(a^3*(d*x+c-coth(d*x+c)-1/3*coth(d*x+c)^3-1/5*coth(d*x+c)^5)+3*a^2*b*(-
1/2/sinh(d*x+c)^5*cosh(d*x+c)^3+3/8/sinh(d*x+c)^5*cosh(d*x+c)+3/8*(-8/15-1/
5*csch(d*x+c)^4+4/15*csch(d*x+c)^2)*coth(d*x+c))+3*a*b^2*(-1/4/sinh(d*x+c)^
5*cosh(d*x+c)-1/4*(-8/15-1/5*csch(d*x+c)^4+4/15*csch(d*x+c)^2)*coth(d*x+c))
+b^3*(-8/15-1/5*csch(d*x+c)^4+4/15*csch(d*x+c)^2)*coth(d*x+c))
```

maxima [B] time = 0.36, size = 826, normalized size = 11.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^6*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/15*a^3*(15*x + 15*c/d - 2*(70*e^(-2*d*x - 2*c) - 140*e^(-4*d*x - 4*c) + 9
0*e^(-6*d*x - 6*c) - 45*e^(-8*d*x - 8*c) - 23)/(d*(5*e^(-2*d*x - 2*c) - 10*
e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x -
10*c) - 1))) + 4/5*a*b^2*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) - 10*e^
(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10
*c) - 1)) + 5*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c)
 + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)) + 15
*e^(-6*d*x - 6*c)/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d
*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)) - 1/(d*(5*e^(-2*d
*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c)
+ e^(-10*d*x - 10*c) - 1))) - 16/15*b^3*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x
 - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) +
e^(-10*d*x - 10*c) - 1)) - 10*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) - 10*
e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x -
10*c) - 1)) - 1/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x
 - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1))) + 6/5*a^2*b*(10*e^
(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x
- 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)) + 5*e^(-8*d*x - 8*c)
/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(
-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)) + 1/(d*(5*e^(-2*d*x - 2*c) - 10*e^
(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10
*c) - 1)))
```

mupad [B] time = 1.60, size = 547, normalized size = 7.93

$$a^3\,x-\frac{\frac{6\left(a^3+b\,a^2\right)}{5\,d}+\frac{24\,{\mathrm{e}}^{2\,c+2\,d\,x}\left(a^2\,b+a\,b^2\right)}{5\,d}+\frac{24\,{\mathrm{e}}^{6\,c+6\,d\,x}\left(a^2\,b+a\,b^2\right)}{5\,d}+\frac{6\,{\mathrm{e}}^{8\,c+8\,d\,x}\left(a^3+b\,a^2\right)}{5\,d}+\frac{4\,{\mathrm{e}}^{4\,c+4\,d\,x}\left(5\,a^3+9\,a^2\,b+12\,a\,b^2+8\,b^3\right)}{5\,d}}{5\,{\mathrm{e}}^{2\,c+2\,d\,x}-10\,{\mathrm{e}}^{4\,c+4\,d\,x}+10\,{\mathrm{e}}^{6\,c+6\,d\,x}-5\,{\mathrm{e}}^{8\,c+8\,d\,x}+{\mathrm{e}}^{10\,c+10\,d\,x}-1}-\frac{\frac{6\left(a^2\,b \right.}{5}}{\mathrm{e}^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^6*(a + b/cosh(c + d*x)^2)^3,x)
```

```
[Out] a^3*x - ((6*(a^2*b + a^3))/(5*d) + (24*exp(2*c + 2*d*x)*(a*b^2 + a^2*b))/(5
*d) + (24*exp(6*c + 6*d*x)*(a*b^2 + a^2*b))/(5*d) + (6*exp(8*c + 8*d*x)*(a^
2*b + a^3))/(5*d) + (4*exp(4*c + 4*d*x)*(12*a*b^2 + 9*a^2*b + 5*a^3 + 8*b^3
))/(5*d))/(5*exp(2*c + 2*d*x) - 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) -
 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) - 1) - ((6*(a*b^2 + a^2*b))/(5*d)
+ (6*exp(2*c + 2*d*x)*(a^2*b + a^3))/(5*d))/(exp(4*c + 4*d*x) - 2*exp(2*c +
 2*d*x) + 1) - ((6*(a*b^2 + a^2*b))/(5*d) + (18*exp(4*c + 4*d*x)*(a*b^2 + a
^2*b))/(5*d) + (6*exp(6*c + 6*d*x)*(a^2*b + a^3))/(5*d) + (2*exp(2*c + 2*d*
x)*(12*a*b^2 + 9*a^2*b + 5*a^3 + 8*b^3))/(5*d))/(6*exp(4*c + 4*d*x) - 4*exp
```

```
(2*c + 2*d*x) - 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) - ((2*(12*a*b^2
+ 9*a^2*b + 5*a^3 + 8*b^3))/(15*d) + (12*exp(2*c + 2*d*x)*(a*b^2 + a^2*b))/
(5*d) + (6*exp(4*c + 4*d*x)*(a^2*b + a^3))/(5*d))/(3*exp(2*c + 2*d*x) - 3*e
xp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1) - (6*(a^2*b + a^3))/(5*d*(exp(2*c +
 2*d*x) - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**6*(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

3.135 $\int \coth^7(c+dx)\left(a+b\,\mathbf{sech}^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=77

$$\frac{a^3\log(\sinh(c+dx))}{d}-\frac{3a^2(a+b)\operatorname{csch}^2(c+dx)}{2d}-\frac{(a+b)^3\operatorname{csch}^6(c+dx)}{6d}-\frac{3a(a+b)^2\operatorname{csch}^4(c+dx)}{4d}$$

```
[Out] -3/2*a^2*(a+b)*csch(d*x+c)^2/d-3/4*a*(a+b)^2*csch(d*x+c)^4/d-1/6*(a+b)^3*cs
ch(d*x+c)^6/d+a^3*ln(sinh(d*x+c))/d
```

Rubi [A] time = 0.12, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4138, 444, 43}

$$-\frac{3a^2(a+b)\operatorname{csch}^2(c+dx)}{2d}+\frac{a^3\log(\sinh(c+dx))}{d}-\frac{(a+b)^3\operatorname{csch}^6(c+dx)}{6d}-\frac{3a(a+b)^2\operatorname{csch}^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^7*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (-3*a^2*(a + b)*Csch[c + d*x]^2)/(2*d) - (3*a*(a + b)^2*Csch[c + d*x]^4)/(4
*d) - ((a + b)^3*Csch[c + d*x]^6)/(6*d) + (a^3*Log[Sinh[c + d*x]])/d
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \coth^7(c+dx)\left(a+b\,\text{sech}^2(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{x\left(b+ax^2\right)^3}{\left(1-x^2\right)^4}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{(b+ax)^3}{(1-x)^4}\,dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{(a+b)^3}{(-1+x)^4}+\frac{3a(a+b)^2}{(-1+x)^3}+\frac{3a^2(a+b)}{(-1+x)^2}+\frac{a^3}{-1+x}\right)dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= -\frac{3a^2(a+b)\text{csch}^2(c+dx)}{2d} - \frac{3a(a+b)^2\text{csch}^4(c+dx)}{4d} - \frac{(a+b)^3\text{csch}}{6d}$$

Mathematica [A] time = 0.80, size = 98, normalized size = 1.27

$$-\frac{2\left(a\cosh^2(c+dx)+b\right)^3\left(-12a^3\log(\sinh(c+dx))+18a^2(a+b)\text{csch}^2(c+dx)+2(a+b)^3\text{csch}^6(c+dx)+9a(a\right.}{3d(a\cosh(2(c+dx))+a+2b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^7*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (-2*(b + a*Cosh[c + d*x]^2)^3*(18*a^2*(a + b)*Csch[c + d*x]^2 + 9*a*(a + b)
^2*Csch[c + d*x]^4 + 2*(a + b)^3*Csch[c + d*x]^6 - 12*a^3*Log[Sinh[c + d*x]
]))/(3*d*(a + 2*b + a*Cosh[2*(c + d*x)])^3)
```

fricas [B] time = 0.46, size = 2632, normalized size = 34.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^7*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/3*(3*a^3*d*x*cosh(d*x + c)^12 + 36*a^3*d*x*cosh(d*x + c)*sinh(d*x + c)^1
1 + 3*a^3*d*x*sinh(d*x + c)^12 - 18*(a^3*d*x - a^3 - a^2*b)*cosh(d*x + c)^1
0 + 18*(11*a^3*d*x*cosh(d*x + c)^2 - a^3*d*x + a^3 + a^2*b)*sinh(d*x + c)^1
0 + 60*(11*a^3*d*x*cosh(d*x + c)^3 - 3*(a^3*d*x - a^3 - a^2*b)*cosh(d*x + c
))*sinh(d*x + c)^9 + 9*(5*a^3*d*x - 4*a^3 + 4*a*b^2)*cosh(d*x + c)^8 + 9*(1
65*a^3*d*x*cosh(d*x + c)^4 + 5*a^3*d*x - 4*a^3 + 4*a*b^2 - 90*(a^3*d*x - a^
3 - a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 72*(33*a^3*d*x*cosh(d*x + c)^
5 - 30*(a^3*d*x - a^3 - a^2*b)*cosh(d*x + c)^3 + (5*a^3*d*x - 4*a^3 + 4*a*b
```

^2)*cosh(d*x + c))*sinh(d*x + c)^7 - 4*(15*a^3*d*x - 17*a^3 - 15*a^2*b - 6*
a*b^2 - 8*b^3)*cosh(d*x + c)^6 + 4*(693*a^3*d*x*cosh(d*x + c)^6 - 15*a^3*d*
x - 945*(a^3*d*x - a^3 - a^2*b)*cosh(d*x + c)^4 + 17*a^3 + 15*a^2*b + 6*a*b
^2 + 8*b^3 + 63*(5*a^3*d*x - 4*a^3 + 4*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c
)^6 + 24*(99*a^3*d*x*cosh(d*x + c)^7 - 189*(a^3*d*x - a^3 - a^2*b)*cosh(d*x
 + c)^5 + 21*(5*a^3*d*x - 4*a^3 + 4*a*b^2)*cosh(d*x + c)^3 - (15*a^3*d*x -
17*a^3 - 15*a^2*b - 6*a*b^2 - 8*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 3*a^3
*d*x + 9*(5*a^3*d*x - 4*a^3 + 4*a*b^2)*cosh(d*x + c)^4 + 3*(495*a^3*d*x*cos
h(d*x + c)^8 - 1260*(a^3*d*x - a^3 - a^2*b)*cosh(d*x + c)^6 + 15*a^3*d*x +
210*(5*a^3*d*x - 4*a^3 + 4*a*b^2)*cosh(d*x + c)^4 - 12*a^3 + 12*a*b^2 - 20*
(15*a^3*d*x - 17*a^3 - 15*a^2*b - 6*a*b^2 - 8*b^3)*cosh(d*x + c)^2)*sinh(d*
x + c)^4 + 4*(165*a^3*d*x*cosh(d*x + c)^9 - 540*(a^3*d*x - a^3 - a^2*b)*cos
h(d*x + c)^7 + 126*(5*a^3*d*x - 4*a^3 + 4*a*b^2)*cosh(d*x + c)^5 - 20*(15*a
^3*d*x - 17*a^3 - 15*a^2*b - 6*a*b^2 - 8*b^3)*cosh(d*x + c)^3 + 9*(5*a^3*d*
x - 4*a^3 + 4*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - 18*(a^3*d*x - a^3 - a
^2*b)*cosh(d*x + c)^2 + 6*(33*a^3*d*x*cosh(d*x + c)^10 - 135*(a^3*d*x - a^3
 - a^2*b)*cosh(d*x + c)^8 + 42*(5*a^3*d*x - 4*a^3 + 4*a*b^2)*cosh(d*x + c)^
6 - 3*a^3*d*x - 10*(15*a^3*d*x - 17*a^3 - 15*a^2*b - 6*a*b^2 - 8*b^3)*cosh(
d*x + c)^4 + 3*a^3 + 3*a^2*b + 9*(5*a^3*d*x - 4*a^3 + 4*a*b^2)*cosh(d*x + c
)^2)*sinh(d*x + c)^2 - 3*(a^3*cosh(d*x + c)^12 + 12*a^3*cosh(d*x + c)*sinh(
d*x + c)^11 + a^3*sinh(d*x + c)^12 - 6*a^3*cosh(d*x + c)^10 + 15*a^3*cosh(d
x + c)^8 + 6(11*a^3*cosh(d*x + c)^2 - a^3)*sinh(d*x + c)^10 + 20*(11*a^3*
cosh(d*x + c)^3 - 3*a^3*cosh(d*x + c))*sinh(d*x + c)^9 - 20*a^3*cosh(d*x +
c)^6 + 15*(33*a^3*cosh(d*x + c)^4 - 18*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x
+ c)^8 + 24*(33*a^3*cosh(d*x + c)^5 - 30*a^3*cosh(d*x + c)^3 + 5*a^3*cosh(d
*x + c))*sinh(d*x + c)^7 + 15*a^3*cosh(d*x + c)^4 + 4*(231*a^3*cosh(d*x + c
)^6 - 315*a^3*cosh(d*x + c)^4 + 105*a^3*cosh(d*x + c)^2 - 5*a^3)*sinh(d*x +
 c)^6 + 24*(33*a^3*cosh(d*x + c)^7 - 63*a^3*cosh(d*x + c)^5 + 35*a^3*cosh(d
*x + c)^3 - 5*a^3*cosh(d*x + c))*sinh(d*x + c)^5 - 6*a^3*cosh(d*x + c)^2 +
15*(33*a^3*cosh(d*x + c)^8 - 84*a^3*cosh(d*x + c)^6 + 70*a^3*cosh(d*x + c)^
4 - 20*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^4 + 20*(11*a^3*cosh(d*x + c
)^9 - 36*a^3*cosh(d*x + c)^7 + 42*a^3*cosh(d*x + c)^5 - 20*a^3*cosh(d*x + c
)^3 + 3*a^3*cosh(d*x + c))*sinh(d*x + c)^3 + a^3 + 6*(11*a^3*cosh(d*x + c)^
10 - 45*a^3*cosh(d*x + c)^8 + 70*a^3*cosh(d*x + c)^6 - 50*a^3*cosh(d*x + c)
^4 + 15*a^3*cosh(d*x + c)^2 - a^3)*sinh(d*x + c)^2 + 12*(a^3*cosh(d*x + c)^
11 - 5*a^3*cosh(d*x + c)^9 + 10*a^3*cosh(d*x + c)^7 - 10*a^3*cosh(d*x + c)^
5 + 5*a^3*cosh(d*x + c)^3 - a^3*cosh(d*x + c))*sinh(d*x + c))*log(2*sinh(d*
x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 12*(3*a^3*d*x*cosh(d*x + c)^11 -
15*(a^3*d*x - a^3 - a^2*b)*cosh(d*x + c)^9 + 6*(5*a^3*d*x - 4*a^3 + 4*a*b^2
)*cosh(d*x + c)^7 - 2*(15*a^3*d*x - 17*a^3 - 15*a^2*b - 6*a*b^2 - 8*b^3)*co
sh(d*x + c)^5 + 3*(5*a^3*d*x - 4*a^3 + 4*a*b^2)*cosh(d*x + c)^3 - 3*(a^3*d*
x - a^3 - a^2*b)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^12 + 12*d*c
osh(d*x + c)*sinh(d*x + c)^11 + d*sinh(d*x + c)^12 - 6*d*cosh(d*x + c)^10 +
 6*(11*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^10 + 20*(11*d*cosh(d*x + c)^3 -
 3*d*cosh(d*x + c))*sinh(d*x + c)^9 + 15*d*cosh(d*x + c)^8 + 15*(33*d*cosh(

```
d*x + c)^4 - 18*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^8 + 24*(33*d*cosh(d*x
+ c)^5 - 30*d*cosh(d*x + c)^3 + 5*d*cosh(d*x + c))*sinh(d*x + c)^7 - 20*d*c
osh(d*x + c)^6 + 4*(231*d*cosh(d*x + c)^6 - 315*d*cosh(d*x + c)^4 + 105*d*c
osh(d*x + c)^2 - 5*d)*sinh(d*x + c)^6 + 24*(33*d*cosh(d*x + c)^7 - 63*d*cos
h(d*x + c)^5 + 35*d*cosh(d*x + c)^3 - 5*d*cosh(d*x + c))*sinh(d*x + c)^5 +
15*d*cosh(d*x + c)^4 + 15*(33*d*cosh(d*x + c)^8 - 84*d*cosh(d*x + c)^6 + 70
*d*cosh(d*x + c)^4 - 20*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^4 + 20*(11*d*c
osh(d*x + c)^9 - 36*d*cosh(d*x + c)^7 + 42*d*cosh(d*x + c)^5 - 20*d*cosh(d*
x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^3 - 6*d*cosh(d*x + c)^2 + 6*(11
*d*cosh(d*x + c)^10 - 45*d*cosh(d*x + c)^8 + 70*d*cosh(d*x + c)^6 - 50*d*co
sh(d*x + c)^4 + 15*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^2 + 12*(d*cosh(d*x
+ c)^11 - 5*d*cosh(d*x + c)^9 + 10*d*cosh(d*x + c)^7 - 10*d*cosh(d*x + c)^5
 + 5*d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c) + d)
```

giac [B] time = 0.59, size = 239, normalized size = 3.10

$$-\frac{60\,a^3dx-60\,a^3\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)+\frac{147\,a^3e^{(12\,dx+12\,c)}-522\,a^3e^{(10\,dx+10\,c)}+360\,a^2be^{(10\,dx+10\,c)}+1485\,a^3e^{(8\,dx+8\,c)}+720\,ab^2e^{(8\,dx+8\,c)}-158}{}}{60}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^7*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/60*(60*a^3*d*x - 60*a^3*log(abs(e^(2*d*x + 2*c) - 1)) + (147*a^3*e^(12*d
*x + 12*c) - 522*a^3*e^(10*d*x + 10*c) + 360*a^2*b*e^(10*d*x + 10*c) + 1485
*a^3*e^(8*d*x + 8*c) + 720*a*b^2*e^(8*d*x + 8*c) - 1580*a^3*e^(6*d*x + 6*c)
 + 1200*a^2*b*e^(6*d*x + 6*c) + 480*a*b^2*e^(6*d*x + 6*c) + 640*b^3*e^(6*d*
x + 6*c) + 1485*a^3*e^(4*d*x + 4*c) + 720*a*b^2*e^(4*d*x + 4*c) - 522*a^3*e
^(2*d*x + 2*c) + 360*a^2*b*e^(2*d*x + 2*c) + 147*a^3)/(e^(2*d*x + 2*c) - 1)
^6)/d
```

maple [B] time = 0.36, size = 189, normalized size = 2.45

$$\frac{a^3\ln\left(\sinh\left(dx+c\right)\right)}{d}-\frac{a^3\left(\coth^2\left(dx+c\right)\right)}{2d}-\frac{a^3\left(\coth^4\left(dx+c\right)\right)}{4d}-\frac{a^3\left(\coth^6\left(dx+c\right)\right)}{6d}-\frac{3a^2b\left(\cosh^4\left(dx+c\right)\right)}{2d\sinh\left(dx+c\right)^6}+\frac{3a^2b}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^7*(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] a^3*ln(sinh(d*x+c))/d-1/2*a^3*coth(d*x+c)^2/d-1/4*a^3*coth(d*x+c)^4/d-1/6*a
^3*coth(d*x+c)^6/d-3/2/d*a^2*b/sinh(d*x+c)^6*cosh(d*x+c)^4+3/2/d*a^2*b/sinh
(d*x+c)^6*cosh(d*x+c)^2-1/2/d*a^2*b/sinh(d*x+c)^6-3/4/d*a*b^2/sinh(d*x+c)^6
*cosh(d*x+c)^2+1/4/d*a*b^2/sinh(d*x+c)^6-1/6/d/sinh(d*x+c)^6*b^3
```

maxima [B] time = 0.42, size = 727, normalized size = 9.44

$$\frac{1}{3}a^3\left(3x+\frac{3c}{d}+\frac{3\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{3\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(9e^{(-2dx-2c)}-18e^{(-4dx-4c)}+34e^{(-6dx-}\right.}{d\left(6e^{(-2dx-2c)}-15e^{(-4dx-4c)}+20e^{(-6dx-6c)}-15e\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^7*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/3*a^3*(3*x + 3*c/d + 3*log(e^(-d*x - c) + 1)/d + 3*log(e^(-d*x - c) - 1)/
d + 2*(9*e^(-2*d*x - 2*c) - 18*e^(-4*d*x - 4*c) + 34*e^(-6*d*x - 6*c) - 18*
e^(-8*d*x - 8*c) + 9*e^(-10*d*x - 10*c))/(d*(6*e^(-2*d*x - 2*c) - 15*e^(-4*
d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*
c) - e^(-12*d*x - 12*c) - 1))) + 2*a^2*b*(3*e^(-2*d*x - 2*c)/(d*(6*e^(-2*d*
x - 2*c) - 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c)
+ 6*e^(-10*d*x - 10*c) - e^(-12*d*x - 12*c) - 1)) + 10*e^(-6*d*x - 6*c)/(d*
(6*e^(-2*d*x - 2*c) - 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*
d*x - 8*c) + 6*e^(-10*d*x - 10*c) - e^(-12*d*x - 12*c) - 1)) + 3*e^(-10*d*x
 - 10*c)/(d*(6*e^(-2*d*x - 2*c) - 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c)
 - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) - e^(-12*d*x - 12*c) - 1))) +
 4*a*b^2*(3*e^(-4*d*x - 4*c)/(d*(6*e^(-2*d*x - 2*c) - 15*e^(-4*d*x - 4*c) +
 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) - e^(-12*
d*x - 12*c) - 1)) + 2*e^(-6*d*x - 6*c)/(d*(6*e^(-2*d*x - 2*c) - 15*e^(-4*d*
x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c)
 - e^(-12*d*x - 12*c) - 1)) + 3*e^(-8*d*x - 8*c)/(d*(6*e^(-2*d*x - 2*c) - 1
5*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d
*x - 10*c) - e^(-12*d*x - 12*c) - 1))) - 32/3*b^3/(d*(e^(d*x + c) - e^(-d*x
 - c))^6)
```

mupad [B] time = 1.61, size = 411, normalized size = 5.34

$$\frac{a^3\ln\left(e^{2c}e^{2dx}-1\right)}{d}-\frac{32\left(a^3+3a^2b+3ab^2+b^3\right)}{d\left(5e^{2c+2dx}-10e^{4c+4dx}+10e^{6c+6dx}-5e^{8c+8dx}+e^{10c+10dx}-1\right)}-\frac{}{3d\left(15e^{4c+4dx}-6\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^7*(a + b/cosh(c + d*x)^2)^3,x)
```

```
[Out] (a^3*log(exp(2*c)*exp(2*d*x) - 1))/d - (32*(3*a*b^2 + 3*a^2*b + a^3 + b^3))
/(d*(5*exp(2*c + 2*d*x) - 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) - 5*exp
(8*c + 8*d*x) + exp(10*c + 10*d*x) - 1)) - (32*(3*a*b^2 + 3*a^2*b + a^3 + b
^3))/(3*d*(15*exp(4*c + 4*d*x) - 6*exp(2*c + 2*d*x) - 20*exp(6*c + 6*d*x) +
 15*exp(8*c + 8*d*x) - 6*exp(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1)) - (6
*(a^2*b + a^3))/(d*(exp(2*c + 2*d*x) - 1)) - (6*(2*a*b^2 + 5*a^2*b + 3*a^3)
```

```
)/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) - (8*(21*a*b^2 + 30*a^2*b
 + 13*a^3 + 4*b^3))/(3*d*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c
 + 6*d*x) - 1)) - (4*(27*a*b^2 + 30*a^2*b + 11*a^3 + 8*b^3))/(d*(6*exp(4*c
+ 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1))
 - a^3*x
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**7*(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

3.136 $\int \left(a+b\,\mathbf{sech}^2(c+dx)\right)^4\,dx$

Optimal. Leaf size=111

$$a^4x-\frac{b^2\left(6a^2+8ab+3b^2\right)\tanh^3(c+dx)}{3d}+\frac{b(2a+b)\left(2a^2+2ab+b^2\right)\tanh(c+dx)}{d}+\frac{b^3(4a+3b)\tanh^5(c+dx)}{5d}-$$

```
[Out] a^4*x+b*(2*a+b)*(2*a^2+2*a*b+b^2)*tanh(d*x+c)/d-1/3*b^2*(6*a^2+8*a*b+3*b^2)
*tanh(d*x+c)^3/d+1/5*b^3*(4*a+3*b)*tanh(d*x+c)^5/d-1/7*b^4*tanh(d*x+c)^7/d
```

Rubi [A] time = 0.07, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4128, 390, 206}

$$-\frac{b^2\left(6a^2+8ab+3b^2\right)\tanh^3(c+dx)}{3d}+\frac{b(2a+b)\left(2a^2+2ab+b^2\right)\tanh(c+dx)}{d}+a^4x+\frac{b^3(4a+3b)\tanh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^4,x]
```

```
[Out] a^4*x + (b*(2*a + b)*(2*a^2 + 2*a*b + b^2)*Tanh[c + d*x])/d - (b^2*(6*a^2 +
 8*a*b + 3*b^2)*Tanh[c + d*x]^3)/(3*d) + (b^3*(4*a + 3*b)*Tanh[c + d*x]^5)/
(5*d) - (b^4*Tanh[c + d*x]^7)/(7*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \left(a+b\text{sech}^2(c+dx)\right)^4 dx = \frac{\text{Subst}\left(\int \frac{\left(a+b-bx^2\right)^4}{1-x^2} dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(b(2a+b)\left(2a^2+2ab+b^2\right)-b^2\left(6a^2+8ab+3b^2\right)x^2+b^3(4a+3b)x^4-\right.\right.}{d}$$

$$= \frac{b(2a+b)\left(2a^2+2ab+b^2\right)\tanh(c+dx)}{d} - \frac{b^2\left(6a^2+8ab+3b^2\right)\tanh^3(c+dx)}{3d} + \cdots$$

$$= a^4x + \frac{b(2a+b)\left(2a^2+2ab+b^2\right)\tanh(c+dx)}{d} - \frac{b^2\left(6a^2+8ab+3b^2\right)\tanh^3(c+\cdots}{3d}$$

Mathematica [B] time = 1.67, size = 455, normalized size = 4.10

$$\frac{\text{sech}(c)\text{sech}^7(c+dx)\left(3675a^4dx\cosh(2c+dx)+2205a^4dx\cosh(2c+3dx)+2205a^4dx\cosh(4c+3dx)+735a^4\cdots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^4,x]
```

```
[Out] (Sech[c]*Sech[c + d*x]^7*(3675*a^4*d*x*Cosh[d*x] + 3675*a^4*d*x*Cosh[2*c +
d*x] + 2205*a^4*d*x*Cosh[2*c + 3*d*x] + 2205*a^4*d*x*Cosh[4*c + 3*d*x] + 73
5*a^4*d*x*Cosh[4*c + 5*d*x] + 735*a^4*d*x*Cosh[6*c + 5*d*x] + 105*a^4*d*x*C
osh[6*c + 7*d*x] + 105*a^4*d*x*Cosh[8*c + 7*d*x] + 16800*a^3*b*Sinh[d*x] +
18480*a^2*b^2*Sinh[d*x] + 11200*a*b^3*Sinh[d*x] + 3360*b^4*Sinh[d*x] - 1260
0*a^3*b*Sinh[2*c + d*x] - 10920*a^2*b^2*Sinh[2*c + d*x] - 4480*a*b^3*Sinh[2
*c + d*x] + 12600*a^3*b*Sinh[2*c + 3*d*x] + 15120*a^2*b^2*Sinh[2*c + 3*d*x]
 + 9408*a*b^3*Sinh[2*c + 3*d*x] + 2016*b^4*Sinh[2*c + 3*d*x] - 5040*a^3*b*S
inh[4*c + 3*d*x] - 2520*a^2*b^2*Sinh[4*c + 3*d*x] + 5040*a^3*b*Sinh[4*c + 5
*d*x] + 5880*a^2*b^2*Sinh[4*c + 5*d*x] + 3136*a*b^3*Sinh[4*c + 5*d*x] + 672
*b^4*Sinh[4*c + 5*d*x] - 840*a^3*b*Sinh[6*c + 5*d*x] + 840*a^3*b*Sinh[6*c +
 7*d*x] + 840*a^2*b^2*Sinh[6*c + 7*d*x] + 448*a*b^3*Sinh[6*c + 7*d*x] + 96*
b^4*Sinh[6*c + 7*d*x]))/(13440*d)
```

fricas [B] time = 0.42, size = 941, normalized size = 8.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^4,x, algorithm="fricas")
```

```
[Out] 1/105*((105*a^4*d*x - 420*a^3*b - 420*a^2*b^2 - 224*a*b^3 - 48*b^4)*cosh(d*
x + c)^7 + 7*(105*a^4*d*x - 420*a^3*b - 420*a^2*b^2 - 224*a*b^3 - 48*b^4)*c
osh(d*x + c)*sinh(d*x + c)^6 + 4*(105*a^3*b + 105*a^2*b^2 + 56*a*b^3 + 12*b
^4)*sinh(d*x + c)^7 + 7*(105*a^4*d*x - 420*a^3*b - 420*a^2*b^2 - 224*a*b^3
- 48*b^4)*cosh(d*x + c)^5 + 28*(75*a^3*b + 105*a^2*b^2 + 56*a*b^3 + 12*b^4
+ 3*(105*a^3*b + 105*a^2*b^2 + 56*a*b^3 + 12*b^4)*cosh(d*x + c)^2)*sinh(d*x
 + c)^5 + 35*((105*a^4*d*x - 420*a^3*b - 420*a^2*b^2 - 224*a*b^3 - 48*b^4)*
cosh(d*x + c)^3 + (105*a^4*d*x - 420*a^3*b - 420*a^2*b^2 - 224*a*b^3 - 48*b
^4)*cosh(d*x + c))*sinh(d*x + c)^4 + 21*(105*a^4*d*x - 420*a^3*b - 420*a^2*
b^2 - 224*a*b^3 - 48*b^4)*cosh(d*x + c)^3 + 28*(5*(105*a^3*b + 105*a^2*b^2
+ 56*a*b^3 + 12*b^4)*cosh(d*x + c)^4 + 135*a^3*b + 225*a^2*b^2 + 168*a*b^3
+ 36*b^4 + 10*(75*a^3*b + 105*a^2*b^2 + 56*a*b^3 + 12*b^4)*cosh(d*x + c)^2)
*sinh(d*x + c)^3 + 7*(3*(105*a^4*d*x - 420*a^3*b - 420*a^2*b^2 - 224*a*b^3
- 48*b^4)*cosh(d*x + c)^5 + 10*(105*a^4*d*x - 420*a^3*b - 420*a^2*b^2 - 224
*a*b^3 - 48*b^4)*cosh(d*x + c)^3 + 9*(105*a^4*d*x - 420*a^3*b - 420*a^2*b^2
 - 224*a*b^3 - 48*b^4)*cosh(d*x + c))*sinh(d*x + c)^2 + 35*(105*a^4*d*x - 4
20*a^3*b - 420*a^2*b^2 - 224*a*b^3 - 48*b^4)*cosh(d*x + c) + 28*((105*a^3*b
 + 105*a^2*b^2 + 56*a*b^3 + 12*b^4)*cosh(d*x + c)^6 + 5*(75*a^3*b + 105*a^2
*b^2 + 56*a*b^3 + 12*b^4)*cosh(d*x + c)^4 + 75*a^3*b + 135*a^2*b^2 + 120*a*
b^3 + 60*b^4 + 9*(45*a^3*b + 75*a^2*b^2 + 56*a*b^3 + 12*b^4)*cosh(d*x + c)^
2)*sinh(d*x + c))/(d*cosh(d*x + c)^7 + 7*d*cosh(d*x + c)*sinh(d*x + c)^6 +
7*d*cosh(d*x + c)^5 + 35*(d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c
)^4 + 21*d*cosh(d*x + c)^3 + 7*(3*d*cosh(d*x + c)^5 + 10*d*cosh(d*x + c)^3
+ 9*d*cosh(d*x + c))*sinh(d*x + c)^2 + 35*d*cosh(d*x + c))
```

giac [B] time = 0.15, size = 334, normalized size = 3.01

$$105\,(dx+c)a^4 - \frac{8\left(105\,a^3be^{(12\,dx+12\,c)}+630\,a^3be^{(10\,dx+10\,c)}+315\,a^2b^2e^{(10\,dx+10\,c)}+1575\,a^3be^{(8\,dx+8\,c)}+1365\,a^2b^2e^{(8\,dx+8\,c)}+560\,ab^3e^{(8\,dx+8\,c)}+21\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^4,x, algorithm="giac")
```

```
[Out] 1/105*(105*(d*x + c)*a^4 - 8*(105*a^3*b*e^(12*d*x + 12*c) + 630*a^3*b*e^(10
*d*x + 10*c) + 315*a^2*b^2*e^(10*d*x + 10*c) + 1575*a^3*b*e^(8*d*x + 8*c) +
 1365*a^2*b^2*e^(8*d*x + 8*c) + 560*a*b^3*e^(8*d*x + 8*c) + 2100*a^3*b*e^(6
*d*x + 6*c) + 2310*a^2*b^2*e^(6*d*x + 6*c) + 1400*a*b^3*e^(6*d*x + 6*c) + 4
20*b^4*e^(6*d*x + 6*c) + 1575*a^3*b*e^(4*d*x + 4*c) + 1890*a^2*b^2*e^(4*d*x
 + 4*c) + 1176*a*b^3*e^(4*d*x + 4*c) + 252*b^4*e^(4*d*x + 4*c) + 630*a^3*b*
e^(2*d*x + 2*c) + 735*a^2*b^2*e^(2*d*x + 2*c) + 392*a*b^3*e^(2*d*x + 2*c) +
 84*b^4*e^(2*d*x + 2*c) + 105*a^3*b + 105*a^2*b^2 + 56*a*b^3 + 12*b^4)/(e^(
2*d*x + 2*c) + 1)^7)/d
```

maple [A] time = 0.52, size = 129, normalized size = 1.16

$$\frac{a^4(dx+c)+4a^3b\tanh(dx+c)+6a^2b^2\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)+4a\,b^3\left(\frac{8}{15}+\frac{\operatorname{sech}(dx+c)^4}{5}+\frac{4\operatorname{sech}(dx+c)^2}{15}\right)\tan}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)^4,x)
```

```
[Out] 1/d*(a^4*(d*x+c)+4*a^3*b*tanh(d*x+c)+6*a^2*b^2*(2/3+1/3*sech(d*x+c)^2)*tanh
(d*x+c)+4*a*b^3*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)*tanh(d*x+c)+b^4
*(16/35+1/7*sech(d*x+c)^6+6/35*sech(d*x+c)^4+8/35*sech(d*x+c)^2)*tanh(d*x+c
))
```

maxima [B] time = 0.57, size = 703, normalized size = 6.33

$$a^4x+\frac{32}{35}b^4\left(\frac{7\,e^{(-2\,dx-2\,c)}}{d\left(7\,e^{(-2\,dx-2\,c)}+21\,e^{(-4\,dx-4\,c)}+35\,e^{(-6\,dx-6\,c)}+35\,e^{(-8\,dx-8\,c)}+21\,e^{(-10\,dx-10\,c)}+7\,e^{(-12\,dx-12\,c)}+e^{(-14\,dx}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^4,x, algorithm="maxima")
```

```
[Out] a^4*x + 32/35*b^4*(7*e^(-2*d*x - 2*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x
 - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c)
 + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 21*e^(-4*d*x - 4*c)/(d
*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8
*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 1
4*c) + 1)) + 35*e^(-6*d*x - 6*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*
c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*
e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 1/(d*(7*e^(-2*d*x - 2*c) +
21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10
*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1))) + 64/15*a*b
^3*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^
(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 10*e^(-4*d
*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c
) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*d*x - 2*c
) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10
*d*x - 10*c) + 1))) + 8*a^2*b^2*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c)
+ 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) +
3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + 8*a^3*b/(d*(e^(-2*d*x - 2*c)
 + 1))
```

mupad [B] time = 0.20, size = 1083, normalized size = 9.76

$$a^4\,x-\frac{\frac{8\left(a^3\,b+a^2\,b^2\right)}{7\,d}+\frac{8\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(15\,a^3\,b+24\,a^2\,b^2+16\,a\,b^3\right)}{21\,d}+\frac{16\,{\mathrm{e}}^{6\,c+6\,d\,x}\,\left(15\,a^3\,b+24\,a^2\,b^2+16\,a\,b^3\right)}{21\,d}+\frac{16\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(5\,a^3\,b+9\,a^2\,b^2+8\,a\,b^3+4\,b^4\right)}{7\,d}}{6\,{\mathrm{e}}^{2\,c+2\,d\,x}+15\,{\mathrm{e}}^{4\,c+4\,d\,x}+20\,{\mathrm{e}}^{6\,c+6\,d\,x}+15\,{\mathrm{e}}^{8\,c+8\,d\,x}+6\,{\mathrm{e}}^{10\,c+10\,d\,x}+{\mathrm{e}}^{12\,c+12\,d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cosh(c + d*x)^2)^4,x)
```

```
[Out] a^4*x - ((8*(a^3*b + a^2*b^2))/(7*d) + (8*exp(2*c + 2*d*x)*(16*a*b^3 + 15*a
^3*b + 24*a^2*b^2))/(21*d) + (16*exp(6*c + 6*d*x)*(16*a*b^3 + 15*a^3*b + 24
*a^2*b^2))/(21*d) + (16*exp(4*c + 4*d*x)*(8*a*b^3 + 5*a^3*b + 4*b^4 + 9*a^2
*b^2))/(7*d) + (40*exp(8*c + 8*d*x)*(a^3*b + a^2*b^2))/(7*d) + (8*a^3*b*exp
(10*c + 10*d*x))/(7*d))/(6*exp(2*c + 2*d*x) + 15*exp(4*c + 4*d*x) + 20*exp(
6*c + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*exp(10*c + 10*d*x) + exp(12*c + 12*d
*x) + 1) - ((8*exp(4*c + 4*d*x)*(16*a*b^3 + 15*a^3*b + 24*a^2*b^2))/(7*d) +
 (8*exp(8*c + 8*d*x)*(16*a*b^3 + 15*a^3*b + 24*a^2*b^2))/(7*d) + (8*a^3*b)/
(7*d) + (32*exp(6*c + 6*d*x)*(8*a*b^3 + 5*a^3*b + 4*b^4 + 9*a^2*b^2))/(7*d)
 + (48*exp(2*c + 2*d*x)*(a^3*b + a^2*b^2))/(7*d) + (48*exp(10*c + 10*d*x)*(
a^3*b + a^2*b^2))/(7*d) + (8*a^3*b*exp(12*c + 12*d*x))/(7*d))/(7*exp(2*c +
2*d*x) + 21*exp(4*c + 4*d*x) + 35*exp(6*c + 6*d*x) + 35*exp(8*c + 8*d*x) +
21*exp(10*c + 10*d*x) + 7*exp(12*c + 12*d*x) + exp(14*c + 14*d*x) + 1) - ((
8*(a^3*b + a^2*b^2))/(7*d) + (8*a^3*b*exp(2*c + 2*d*x))/(7*d))/(2*exp(2*c +
 2*d*x) + exp(4*c + 4*d*x) + 1) - ((8*(16*a*b^3 + 15*a^3*b + 24*a^2*b^2))/(
105*d) + (16*exp(4*c + 4*d*x)*(16*a*b^3 + 15*a^3*b + 24*a^2*b^2))/(35*d) +
(32*exp(2*c + 2*d*x)*(8*a*b^3 + 5*a^3*b + 4*b^4 + 9*a^2*b^2))/(35*d) + (32*
exp(6*c + 6*d*x)*(a^3*b + a^2*b^2))/(7*d) + (8*a^3*b*exp(8*c + 8*d*x))/(7*d
))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(
8*c + 8*d*x) + exp(10*c + 10*d*x) + 1) - ((8*(8*a*b^3 + 5*a^3*b + 4*b^4 + 9
*a^2*b^2))/(35*d) + (8*exp(2*c + 2*d*x)*(16*a*b^3 + 15*a^3*b + 24*a^2*b^2))
/(35*d) + (24*exp(4*c + 4*d*x)*(a^3*b + a^2*b^2))/(7*d) + (8*a^3*b*exp(6*c
+ 6*d*x))/(7*d))/(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d
*x) + exp(8*c + 8*d*x) + 1) - ((8*(16*a*b^3 + 15*a^3*b + 24*a^2*b^2))/(105*
d) + (16*exp(2*c + 2*d*x)*(a^3*b + a^2*b^2))/(7*d) + (8*a^3*b*exp(4*c + 4*d
*x))/(7*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1
) - (8*a^3*b)/(7*d*(exp(2*c + 2*d*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\operatorname{sech}^{2}\left(c + dx\right)\right)^{4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)**4,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**4, x)
```

3.137 $\int \left(a+b\,\mathbf{sech}^2(c+dx)\right)^5\,dx$

Optimal. Leaf size=163

$$a^5x+\frac{b^3\left(10a^2+15ab+6b^2\right)\tanh^5(c+dx)}{5d}-\frac{b^2\left(10a^3+20a^2b+15ab^2+4b^3\right)\tanh^3(c+dx)}{3d}+\frac{b\left(5a^4+10a^3b+1\right.}{}$$

```
[Out] a^5*x+b*(5*a^4+10*a^3*b+10*a^2*b^2+5*a*b^3+b^4)*tanh(d*x+c)/d-1/3*b^2*(10*a
^3+20*a^2*b+15*a*b^2+4*b^3)*tanh(d*x+c)^3/d+1/5*b^3*(10*a^2+15*a*b+6*b^2)*t
anh(d*x+c)^5/d-1/7*b^4*(5*a+4*b)*tanh(d*x+c)^7/d+1/9*b^5*tanh(d*x+c)^9/d
```

Rubi [A] time = 0.10, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4128, 390, 206}

$$\frac{b^3\left(10a^2+15ab+6b^2\right)\tanh^5(c+dx)}{5d}-\frac{b^2\left(20a^2b+10a^3+15ab^2+4b^3\right)\tanh^3(c+dx)}{3d}+\frac{b\left(10a^2b^2+10a^3b+5a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^5,x]
```

```
[Out] a^5*x + (b*(5*a^4 + 10*a^3*b + 10*a^2*b^2 + 5*a*b^3 + b^4)*Tanh[c + d*x])/d
 - (b^2*(10*a^3 + 20*a^2*b + 15*a*b^2 + 4*b^3)*Tanh[c + d*x]^3)/(3*d) + (b^
3*(10*a^2 + 15*a*b + 6*b^2)*Tanh[c + d*x]^5)/(5*d) - (b^4*(5*a + 4*b)*Tanh[
c + d*x]^7)/(7*d) + (b^5*Tanh[c + d*x]^9)/(9*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
```

```
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \left(a+b\text{sech}^2(c+dx)\right)^5\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b-bx^2\right)^5}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(b\left(5a^4+10a^3b+10a^2b^2+5ab^3+b^4\right)-b^2\left(10a^3+20a^2b+15ab^2+4b^3\right.\right.\right.}{}$$

$$= \frac{b\left(5a^4+10a^3b+10a^2b^2+5ab^3+b^4\right)\tanh(c+dx)}{d} - \frac{b^2\left(10a^3+20a^2b+15ab^2+\right.}{3d}$$

$$= a^5x + \frac{b\left(5a^4+10a^3b+10a^2b^2+5ab^3+b^4\right)\tanh(c+dx)}{d} - \frac{b^2\left(10a^3+20a^2b+15\right.}{}$$

Mathematica [B] time = 6.57, size = 724, normalized size = 4.44

$$\frac{32a^5x\cosh^{10}(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^5}{(a\cosh(2c+2dx)+a+2b)^5} + \frac{64\text{sech}(c)\left(105a^2b^3\sinh(c)+45ab^4\sinh(c)+8b^5\sinh(c)\right)\cosh^6(c}{105d(a\cosh(2c+2dx)+a+2b)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^5,x]
```

```
[Out] (32*a^5*x*Cosh[c + d*x]^10*(a + b*Sech[c + d*x]^2)^5)/(a + 2*b + a*Cosh[2*c
 + 2*d*x])^5 + (32*Cosh[c + d*x]^4*Sech[c]*(a + b*Sech[c + d*x]^2)^5*(45*a*
b^4*Sinh[c] + 8*b^5*Sinh[c]))/(63*d*(a + 2*b + a*Cosh[2*c + 2*d*x])^5) + (6
4*Cosh[c + d*x]^6*Sech[c]*(a + b*Sech[c + d*x]^2)^5*(105*a^2*b^3*Sinh[c] +
45*a*b^4*Sinh[c] + 8*b^5*Sinh[c]))/(105*d*(a + 2*b + a*Cosh[2*c + 2*d*x])^5
) + (64*Cosh[c + d*x]^8*Sech[c]*(a + b*Sech[c + d*x]^2)^5*(525*a^3*b^2*Sinh
[c] + 420*a^2*b^3*Sinh[c] + 180*a*b^4*Sinh[c] + 32*b^5*Sinh[c]))/(315*d*(a
+ 2*b + a*Cosh[2*c + 2*d*x])^5) + (32*b^5*Cosh[c + d*x]*Sech[c]*(a + b*Sech
[c + d*x]^2)^5*Sinh[d*x])/(9*d*(a + 2*b + a*Cosh[2*c + 2*d*x])^5) + (32*Cos
h[c + d*x]^3*Sech[c]*(a + b*Sech[c + d*x]^2)^5*(45*a*b^4*Sinh[d*x] + 8*b^5*
Sinh[d*x]))/(63*d*(a + 2*b + a*Cosh[2*c + 2*d*x])^5) + (64*Cosh[c + d*x]^5*
Sech[c]*(a + b*Sech[c + d*x]^2)^5*(105*a^2*b^3*Sinh[d*x] + 45*a*b^4*Sinh[d*
x] + 8*b^5*Sinh[d*x]))/(105*d*(a + 2*b + a*Cosh[2*c + 2*d*x])^5) + (64*Cosh
[c + d*x]^7*Sech[c]*(a + b*Sech[c + d*x]^2)^5*(525*a^3*b^2*Sinh[d*x] + 420*
a^2*b^3*Sinh[d*x] + 180*a*b^4*Sinh[d*x] + 32*b^5*Sinh[d*x]))/(315*d*(a + 2*
b + a*Cosh[2*c + 2*d*x])^5) + (32*Cosh[c + d*x]^9*Sech[c]*(a + b*Sech[c + d
```

```
*x]^2)^5*(1575*a^4*b*Sinh[d*x] + 2100*a^3*b^2*Sinh[d*x] + 1680*a^2*b^3*Sinh
[d*x] + 720*a*b^4*Sinh[d*x] + 128*b^5*Sinh[d*x]))/(315*d*(a + 2*b + a*Cosh[
2*c + 2*d*x])^5) + (32*b^5*Cosh[c + d*x]^2*(a + b*Sech[c + d*x]^2)^5*Tanh[c
])/(9*d*(a + 2*b + a*Cosh[2*c + 2*d*x])^5)
```

fricas [B] time = 0.43, size = 1652, normalized size = 10.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^5,x, algorithm="fricas")
```

```
[Out] 1/315*((315*a^5*d*x - 1575*a^4*b - 2100*a^3*b^2 - 1680*a^2*b^3 - 720*a*b^4
- 128*b^5)*cosh(d*x + c)^9 + 9*(315*a^5*d*x - 1575*a^4*b - 2100*a^3*b^2 - 1
680*a^2*b^3 - 720*a*b^4 - 128*b^5)*cosh(d*x + c)*sinh(d*x + c)^8 + (1575*a^
4*b + 2100*a^3*b^2 + 1680*a^2*b^3 + 720*a*b^4 + 128*b^5)*sinh(d*x + c)^9 +
9*(315*a^5*d*x - 1575*a^4*b - 2100*a^3*b^2 - 1680*a^2*b^3 - 720*a*b^4 - 128
*b^5)*cosh(d*x + c)^7 + 9*(1225*a^4*b + 2100*a^3*b^2 + 1680*a^2*b^3 + 720*a
*b^4 + 128*b^5 + 4*(1575*a^4*b + 2100*a^3*b^2 + 1680*a^2*b^3 + 720*a*b^4 +
128*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^7 + 21*(4*(315*a^5*d*x - 1575*a^4*b
 - 2100*a^3*b^2 - 1680*a^2*b^3 - 720*a*b^4 - 128*b^5)*cosh(d*x + c)^3 + 3*(
315*a^5*d*x - 1575*a^4*b - 2100*a^3*b^2 - 1680*a^2*b^3 - 720*a*b^4 - 128*b^
5)*cosh(d*x + c))*sinh(d*x + c)^6 + 36*(315*a^5*d*x - 1575*a^4*b - 2100*a^3
*b^2 - 1680*a^2*b^3 - 720*a*b^4 - 128*b^5)*cosh(d*x + c)^5 + 9*(3500*a^4*b
+ 7000*a^3*b^2 + 6720*a^2*b^3 + 2880*a*b^4 + 512*b^5 + 14*(1575*a^4*b + 210
0*a^3*b^2 + 1680*a^2*b^3 + 720*a*b^4 + 128*b^5)*cosh(d*x + c)^4 + 21*(1225*
a^4*b + 2100*a^3*b^2 + 1680*a^2*b^3 + 720*a*b^4 + 128*b^5)*cosh(d*x + c)^2)
*sinh(d*x + c)^5 + 9*(14*(315*a^5*d*x - 1575*a^4*b - 2100*a^3*b^2 - 1680*a^
2*b^3 - 720*a*b^4 - 128*b^5)*cosh(d*x + c)^5 + 35*(315*a^5*d*x - 1575*a^4*b
 - 2100*a^3*b^2 - 1680*a^2*b^3 - 720*a*b^4 - 128*b^5)*cosh(d*x + c)^3 + 20*
(315*a^5*d*x - 1575*a^4*b - 2100*a^3*b^2 - 1680*a^2*b^3 - 720*a*b^4 - 128*b
^5)*cosh(d*x + c))*sinh(d*x + c)^4 + 84*(315*a^5*d*x - 1575*a^4*b - 2100*a^
3*b^2 - 1680*a^2*b^3 - 720*a*b^4 - 128*b^5)*cosh(d*x + c)^3 + 3*(28*(1575*a
^4*b + 2100*a^3*b^2 + 1680*a^2*b^3 + 720*a*b^4 + 128*b^5)*cosh(d*x + c)^6 +
 14700*a^4*b + 32200*a^3*b^2 + 35840*a^2*b^3 + 20160*a*b^4 + 3584*b^5 + 105
*(1225*a^4*b + 2100*a^3*b^2 + 1680*a^2*b^3 + 720*a*b^4 + 128*b^5)*cosh(d*x
+ c)^4 + 120*(875*a^4*b + 1750*a^3*b^2 + 1680*a^2*b^3 + 720*a*b^4 + 128*b^5
)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 9*(4*(315*a^5*d*x - 1575*a^4*b - 2100*
a^3*b^2 - 1680*a^2*b^3 - 720*a*b^4 - 128*b^5)*cosh(d*x + c)^7 + 21*(315*a^5
*d*x - 1575*a^4*b - 2100*a^3*b^2 - 1680*a^2*b^3 - 720*a*b^4 - 128*b^5)*cosh
(d*x + c)^5 + 40*(315*a^5*d*x - 1575*a^4*b - 2100*a^3*b^2 - 1680*a^2*b^3 -
720*a*b^4 - 128*b^5)*cosh(d*x + c)^3 + 28*(315*a^5*d*x - 1575*a^4*b - 2100*
a^3*b^2 - 1680*a^2*b^3 - 720*a*b^4 - 128*b^5)*cosh(d*x + c))*sinh(d*x + c)^
2 + 126*(315*a^5*d*x - 1575*a^4*b - 2100*a^3*b^2 - 1680*a^2*b^3 - 720*a*b^4
 - 128*b^5)*cosh(d*x + c) + 9*((1575*a^4*b + 2100*a^3*b^2 + 1680*a^2*b^3 +
```

```
720*a*b^4 + 128*b^5)*cosh(d*x + c)^8 + 7*(1225*a^4*b + 2100*a^3*b^2 + 1680*
a^2*b^3 + 720*a*b^4 + 128*b^5)*cosh(d*x + c)^6 + 2450*a^4*b + 5600*a^3*b^2
+ 6720*a^2*b^3 + 4480*a*b^4 + 1792*b^5 + 20*(875*a^4*b + 1750*a^3*b^2 + 168
0*a^2*b^3 + 720*a*b^4 + 128*b^5)*cosh(d*x + c)^4 + 28*(525*a^4*b + 1150*a^3
*b^2 + 1280*a^2*b^3 + 720*a*b^4 + 128*b^5)*cosh(d*x + c)^2)*sinh(d*x + c))/
(d*cosh(d*x + c)^9 + 9*d*cosh(d*x + c)*sinh(d*x + c)^8 + 9*d*cosh(d*x + c)^
7 + 21*(4*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^6 + 36*d*cos
h(d*x + c)^5 + 9*(14*d*cosh(d*x + c)^5 + 35*d*cosh(d*x + c)^3 + 20*d*cosh(d
*x + c))*sinh(d*x + c)^4 + 84*d*cosh(d*x + c)^3 + 9*(4*d*cosh(d*x + c)^7 +
21*d*cosh(d*x + c)^5 + 40*d*cosh(d*x + c)^3 + 28*d*cosh(d*x + c))*sinh(d*x
+ c)^2 + 126*d*cosh(d*x + c))
```

giac [B] time = 0.17, size = 537, normalized size = 3.29

$$315\,(dx+c)a^5 - \frac{2\left(1575\,a^4be^{(16\,dx+16\,c)}+12600\,a^4be^{(14\,dx+14\,c)}+6300\,a^3b^2e^{(14\,dx+14\,c)}+44100\,a^4be^{(12\,dx+12\,c)}+39900\,a^3b^2e^{(12\,dx+12\,c)}+16800\,a^2b^3e\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^5,x, algorithm="giac")
```

```
[Out] 1/315*(315*(d*x + c)*a^5 - 2*(1575*a^4*b*e^(16*d*x + 16*c) + 12600*a^4*b*e^
(14*d*x + 14*c) + 6300*a^3*b^2*e^(14*d*x + 14*c) + 44100*a^4*b*e^(12*d*x +
12*c) + 39900*a^3*b^2*e^(12*d*x + 12*c) + 16800*a^2*b^3*e^(12*d*x + 12*c) +
 88200*a^4*b*e^(10*d*x + 10*c) + 107100*a^3*b^2*e^(10*d*x + 10*c) + 75600*a
^2*b^3*e^(10*d*x + 10*c) + 25200*a*b^4*e^(10*d*x + 10*c) + 110250*a^4*b*e^(
8*d*x + 8*c) + 157500*a^3*b^2*e^(8*d*x + 8*c) + 136080*a^2*b^3*e^(8*d*x + 8
*c) + 65520*a*b^4*e^(8*d*x + 8*c) + 16128*b^5*e^(8*d*x + 8*c) + 88200*a^4*b
*e^(6*d*x + 6*c) + 136500*a^3*b^2*e^(6*d*x + 6*c) + 124320*a^2*b^3*e^(6*d*x
 + 6*c) + 60480*a*b^4*e^(6*d*x + 6*c) + 10752*b^5*e^(6*d*x + 6*c) + 44100*a
^4*b*e^(4*d*x + 4*c) + 69300*a^3*b^2*e^(4*d*x + 4*c) + 60480*a^2*b^3*e^(4*d
*x + 4*c) + 25920*a*b^4*e^(4*d*x + 4*c) + 4608*b^5*e^(4*d*x + 4*c) + 12600*
a^4*b*e^(2*d*x + 2*c) + 18900*a^3*b^2*e^(2*d*x + 2*c) + 15120*a^2*b^3*e^(2*
d*x + 2*c) + 6480*a*b^4*e^(2*d*x + 2*c) + 1152*b^5*e^(2*d*x + 2*c) + 1575*a
^4*b + 2100*a^3*b^2 + 1680*a^2*b^3 + 720*a*b^4 + 128*b^5)/(e^(2*d*x + 2*c)
+ 1)^9)/d
```

maple [A] time = 0.61, size = 185, normalized size = 1.13

$$a^5\,(dx+c)+5a^4b\tanh\,(dx+c)+10a^3b^2\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh\,(dx+c)+10a^2b^3\left(\frac{8}{15}+\frac{\operatorname{sech}(dx+c)^4}{5}+\frac{4\operatorname{sech}(dx+c)^2}{15}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)^5,x)
```

```
[Out] 1/d*(a^5*(d*x+c)+5*a^4*b*tanh(d*x+c)+10*a^3*b^2*(2/3+1/3*sech(d*x+c)^2)*tan
h(d*x+c)+10*a^2*b^3*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)*tanh(d*x+c)
+5*a*b^4*(16/35+1/7*sech(d*x+c)^6+6/35*sech(d*x+c)^4+8/35*sech(d*x+c)^2)*ta
nh(d*x+c)+b^5*(128/315+1/9*sech(d*x+c)^8+8/63*sech(d*x+c)^6+16/105*sech(d*x
+c)^4+64/315*sech(d*x+c)^2)*tanh(d*x+c))
```

maxima [B] time = 0.35, size = 1277, normalized size = 7.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^5,x, algorithm="maxima")
```

```
[Out] a^5*x + 256/315*b^5*(9*e^(-2*d*x - 2*c)/(d*(9*e^(-2*d*x - 2*c) + 36*e^(-4*d
*x - 4*c) + 84*e^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 1
0*c) + 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) + 9*e^(-16*d*x - 16*c)
 + e^(-18*d*x - 18*c) + 1)) + 36*e^(-4*d*x - 4*c)/(d*(9*e^(-2*d*x - 2*c) +
36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*c) + 126*e^(-
10*d*x - 10*c) + 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) + 9*e^(-16*d
*x - 16*c) + e^(-18*d*x - 18*c) + 1)) + 84*e^(-6*d*x - 6*c)/(d*(9*e^(-2*d*x
 - 2*c) + 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*c)
+ 126*e^(-10*d*x - 10*c) + 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) +
9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) + 1)) + 126*e^(-8*d*x - 8*c)/(d*(
9*e^(-2*d*x - 2*c) + 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) + 126*e^(-8*
d*x - 8*c) + 126*e^(-10*d*x - 10*c) + 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x
 - 14*c) + 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) + 1)) + 1/(d*(9*e^(-2*
d*x - 2*c) + 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*
c) + 126*e^(-10*d*x - 10*c) + 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c)
 + 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) + 1))) + 32/7*a*b^4*(7*e^(-2*d
*x - 2*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c
) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^
(-14*d*x - 14*c) + 1)) + 21*e^(-4*d*x - 4*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^
(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x
- 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 35*e^(-6*d*x -
6*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 3
5*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*
d*x - 14*c) + 1)) + 1/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(
-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x
- 12*c) + e^(-14*d*x - 14*c) + 1))) + 32/3*a^2*b^3*(5*e^(-2*d*x - 2*c)/(d*(
5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*
x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 10*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x -
 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^
(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 1
```

```
0*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) + 40/3*
a^3*b^2*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e
^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(
-6*d*x - 6*c) + 1))) + 10*a^4*b/(d*(e^(-2*d*x - 2*c) + 1))
```

mupad [B]   time = 0.32, size = 1952, normalized size = 11.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cosh(c + d*x)^2)^5,x)
```

```
[Out] a^5*x - ((10*(8*a*b^4 + 7*a^4*b + 16*a^2*b^3 + 15*a^3*b^2))/(63*d) + (10*ex
p(2*c + 2*d*x)*(7*a^4*b + 8*a^2*b^3 + 12*a^3*b^2))/(21*d) + (10*exp(4*c + 4
*d*x)*(a^4*b + a^3*b^2))/(3*d) + (10*a^4*b*exp(6*c + 6*d*x))/(9*d))/(4*exp(
2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) +
 1) - ((80*exp(6*c + 6*d*x)*(8*a*b^4 + 7*a^4*b + 16*a^2*b^3 + 15*a^3*b^2))/
(9*d) + (80*exp(10*c + 10*d*x)*(8*a*b^4 + 7*a^4*b + 16*a^2*b^3 + 15*a^3*b^2
))/(9*d) + (4*exp(8*c + 8*d*x)*(320*a*b^4 + 175*a^4*b + 128*b^5 + 480*a^2*b
^3 + 400*a^3*b^2))/(9*d) + (10*a^4*b)/(9*d) + (40*exp(4*c + 4*d*x)*(7*a^4*b
 + 8*a^2*b^3 + 12*a^3*b^2))/(9*d) + (40*exp(12*c + 12*d*x)*(7*a^4*b + 8*a^2
*b^3 + 12*a^3*b^2))/(9*d) + (80*exp(2*c + 2*d*x)*(a^4*b + a^3*b^2))/(9*d) +
 (80*exp(14*c + 14*d*x)*(a^4*b + a^3*b^2))/(9*d) + (10*a^4*b*exp(16*c + 16*
d*x))/(9*d))/(9*exp(2*c + 2*d*x) + 36*exp(4*c + 4*d*x) + 84*exp(6*c + 6*d*x
) + 126*exp(8*c + 8*d*x) + 126*exp(10*c + 10*d*x) + 84*exp(12*c + 12*d*x) +
 36*exp(14*c + 14*d*x) + 9*exp(16*c + 16*d*x) + exp(18*c + 18*d*x) + 1) - (
(10*(a^4*b + a^3*b^2))/(9*d) + (10*a^4*b*exp(2*c + 2*d*x))/(9*d))/(2*exp(2*
c + 2*d*x) + exp(4*c + 4*d*x) + 1) - ((10*(7*a^4*b + 8*a^2*b^3 + 12*a^3*b^2
))/(63*d) + (20*exp(2*c + 2*d*x)*(8*a*b^4 + 7*a^4*b + 16*a^2*b^3 + 15*a^3*b
^2))/(21*d) + (200*exp(6*c + 6*d*x)*(8*a*b^4 + 7*a^4*b + 16*a^2*b^3 + 15*a^
3*b^2))/(63*d) + (2*exp(4*c + 4*d*x)*(320*a*b^4 + 175*a^4*b + 128*b^5 + 480
*a^2*b^3 + 400*a^3*b^2))/(21*d) + (50*exp(8*c + 8*d*x)*(7*a^4*b + 8*a^2*b^3
 + 12*a^3*b^2))/(21*d) + (20*exp(10*c + 10*d*x)*(a^4*b + a^3*b^2))/(3*d) +
(10*a^4*b*exp(12*c + 12*d*x))/(9*d))/(7*exp(2*c + 2*d*x) + 21*exp(4*c + 4*d
*x) + 35*exp(6*c + 6*d*x) + 35*exp(8*c + 8*d*x) + 21*exp(10*c + 10*d*x) + 7
*exp(12*c + 12*d*x) + exp(14*c + 14*d*x) + 1) - ((2*(320*a*b^4 + 175*a^4*b
+ 128*b^5 + 480*a^2*b^3 + 400*a^3*b^2))/(315*d) + (40*exp(2*c + 2*d*x)*(8*a
*b^4 + 7*a^4*b + 16*a^2*b^3 + 15*a^3*b^2))/(63*d) + (20*exp(4*c + 4*d*x)*(7
*a^4*b + 8*a^2*b^3 + 12*a^3*b^2))/(21*d) + (40*exp(6*c + 6*d*x)*(a^4*b + a^
3*b^2))/(9*d) + (10*a^4*b*exp(8*c + 8*d*x))/(9*d))/(5*exp(2*c + 2*d*x) + 10
*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 1
0*d*x) + 1) - ((10*(a^4*b + a^3*b^2))/(9*d) + (10*exp(4*c + 4*d*x)*(8*a*b^4
 + 7*a^4*b + 16*a^2*b^3 + 15*a^3*b^2))/(3*d) + (50*exp(8*c + 8*d*x)*(8*a*b^
4 + 7*a^4*b + 16*a^2*b^3 + 15*a^3*b^2))/(9*d) + (2*exp(6*c + 6*d*x)*(320*a*
b^4 + 175*a^4*b + 128*b^5 + 480*a^2*b^3 + 400*a^3*b^2))/(9*d) + (10*exp(2*c
```

```
 + 2*d*x)*(7*a^4*b + 8*a^2*b^3 + 12*a^3*b^2))/(9*d) + (10*exp(10*c + 10*d*x
)*(7*a^4*b + 8*a^2*b^3 + 12*a^3*b^2))/(3*d) + (70*exp(12*c + 12*d*x)*(a^4*b
 + a^3*b^2))/(9*d) + (10*a^4*b*exp(14*c + 14*d*x))/(9*d))/(8*exp(2*c + 2*d*
x) + 28*exp(4*c + 4*d*x) + 56*exp(6*c + 6*d*x) + 70*exp(8*c + 8*d*x) + 56*e
xp(10*c + 10*d*x) + 28*exp(12*c + 12*d*x) + 8*exp(14*c + 14*d*x) + exp(16*c
 + 16*d*x) + 1) - ((10*(8*a*b^4 + 7*a^4*b + 16*a^2*b^3 + 15*a^3*b^2))/(63*d
) + (100*exp(4*c + 4*d*x)*(8*a*b^4 + 7*a^4*b + 16*a^2*b^3 + 15*a^3*b^2))/(6
3*d) + (2*exp(2*c + 2*d*x)*(320*a*b^4 + 175*a^4*b + 128*b^5 + 480*a^2*b^3 +
 400*a^3*b^2))/(63*d) + (100*exp(6*c + 6*d*x)*(7*a^4*b + 8*a^2*b^3 + 12*a^3
*b^2))/(63*d) + (50*exp(8*c + 8*d*x)*(a^4*b + a^3*b^2))/(9*d) + (10*a^4*b*e
xp(10*c + 10*d*x))/(9*d))/(6*exp(2*c + 2*d*x) + 15*exp(4*c + 4*d*x) + 20*ex
p(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*exp(10*c + 10*d*x) + exp(12*c + 12
*d*x) + 1) - ((10*(7*a^4*b + 8*a^2*b^3 + 12*a^3*b^2))/(63*d) + (20*exp(2*c
+ 2*d*x)*(a^4*b + a^3*b^2))/(9*d) + (10*a^4*b*exp(4*c + 4*d*x))/(9*d))/(3*e
xp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) - (10*a^4*b)/(
9*d*(exp(2*c + 2*d*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\operatorname{sech}^2(c + dx)\right)^5 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)**5,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**5, x)
```

3.138 $\int \frac{\tanh^5(c+dx)}{a+b\,\mathbf{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=70

$$\frac{(a+b)^2\log\left(a\cosh^2(c+dx)+b\right)}{2ab^2d}-\frac{(a+2b)\log(\cosh(c+dx))}{b^2d}-\frac{\mathrm{sech}^2(c+dx)}{2bd}$$

```
[Out] -(a+2*b)*ln(cosh(d*x+c))/b^2/d+1/2*(a+b)^2*ln(b+a*cosh(d*x+c)^2)/a/b^2/d-1/
2*sech(d*x+c)^2/b/d
```

Rubi [A] time = 0.11, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4138, 446, 88}

$$\frac{(a+b)^2\log\left(a\cosh^2(c+dx)+b\right)}{2ab^2d}-\frac{(a+2b)\log(\cosh(c+dx))}{b^2d}-\frac{\mathrm{sech}^2(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^5/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -(((a + 2*b)*Log[Cosh[c + d*x]])/(b^2*d)) + ((a + b)^2*Log[b + a*Cosh[c + d
*x]^2])/(2*a*b^2*d) - Sech[c + d*x]^2/(2*b*d)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
```

```
x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^5(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{x^3\left(b+ax^2\right)}\,dx, x, \cosh(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{(1-x)^2}{x^2(b+ax)}\,dx, x, \cosh^2(c+dx)\right)}{2d}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{bx^2}+\frac{-a-2b}{b^2x}+\frac{(a+b)^2}{b^2(b+ax)}\right)dx, x, \cosh^2(c+dx)\right)}{2d}\\ &= -\frac{(a+2b)\log(\cosh(c+dx))}{b^2d}+\frac{(a+b)^2\log\left(b+a\cosh^2(c+dx)\right)}{2ab^2d}-\frac{\text{sech}^2(c+dx)}{2bd}\end{aligned}$$

Mathematica [A] time = 0.31, size = 98, normalized size = 1.40

$$-\frac{\text{sech}^2(c+dx)(a\cosh(2(c+dx))+a+2b)\left(ab\text{sech}^2(c+dx)+(a+b)^2\left(-\log\left(a\sinh^2(c+dx)+a+b\right)\right)+2a(\ldots}{4ab^2d\left(a+b\text{sech}^2(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^5/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -1/4*((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*(2*a*(a + 2*b)*Log[Co
sh[c + d*x]] - (a + b)^2*Log[a + b + a*Sinh[c + d*x]^2] + a*b*Sech[c + d*x]
^2))/(a*b^2*d*(a + b*Sech[c + d*x]^2))
```

fricas [B] time = 0.48, size = 736, normalized size = 10.51

$$-\frac{2\,b^2dx\cosh\left(dx+c\right)^4+8\,b^2dx\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3+2\,b^2dx\sinh\left(dx+c\right)^4+2\,b^2dx+4\left(b^2dx+ab\right)\mathrm{co}\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^2*d*x*cosh(d*x + c)^4 + 8*b^2*d*x*cosh(d*x + c)*sinh(d*x + c)^3 +
 2*b^2*d*x*sinh(d*x + c)^4 + 2*b^2*d*x + 4*(b^2*d*x + a*b)*cosh(d*x + c)^2
```

```
+ 4*(3*b^2*d*x*cosh(d*x + c)^2 + b^2*d*x + a*b)*sinh(d*x + c)^2 - ((a^2 + 2
*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x
+ c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b + b^2)*cosh(d
*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b + b^2)*s
inh(d*x + c)^2 + a^2 + 2*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3
 + (a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c))*log(2*(a*cosh(d*x + c)
^2 + a*sinh(d*x + c)^2 + a + 2*b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d
*x + c) + sinh(d*x + c)^2)) + 2*((a^2 + 2*a*b)*cosh(d*x + c)^4 + 4*(a^2 + 2
*a*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b)*sinh(d*x + c)^4 + 2*(a^
2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b)*cosh(d*x + c)^2 + a^2 + 2*a
*b)*sinh(d*x + c)^2 + a^2 + 2*a*b + 4*((a^2 + 2*a*b)*cosh(d*x + c)^3 + (a^2
 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c)
- sinh(d*x + c))) + 8*(b^2*d*x*cosh(d*x + c)^3 + (b^2*d*x + a*b)*cosh(d*x +
 c))*sinh(d*x + c))/(a*b^2*d*cosh(d*x + c)^4 + 4*a*b^2*d*cosh(d*x + c)*sinh
(d*x + c)^3 + a*b^2*d*sinh(d*x + c)^4 + 2*a*b^2*d*cosh(d*x + c)^2 + a*b^2*d
 + 2*(3*a*b^2*d*cosh(d*x + c)^2 + a*b^2*d)*sinh(d*x + c)^2 + 4*(a*b^2*d*cos
h(d*x + c)^3 + a*b^2*d*cosh(d*x + c))*sinh(d*x + c))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error index.cc index_gcd Error: Bad Argument
 ValueError index.cc index_gcd Error: Bad Argument ValueError index.cc inde
x_gcd Error: Bad Argument ValueError index.cc index_gcd Error: Bad Argument
 ValueError index.cc index_gcd Error: Bad Argument ValueError index.cc inde
x_gcd Error: Bad Argument ValueError index.cc index_gcd Error: Bad Argument
 ValueError index.cc index_gcd Error: Bad Argument ValueError index.cc inde
x_gcd Error: Bad Argument ValueError index.cc index_gcd Error: Bad Argument
 ValueEvaluation time: 0.46Done
```

maple [B] time = 0.32, size = 331, normalized size = 4.73

$$-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{da}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{da}+\frac{a\ln\left(\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\left(\tanh^2\left(\frac{dx}{2}\right.\right.\right.}{2d\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^5/(a+b*sech(d*x+c)^2),x)
```

```
[Out] -1/d/a*ln(tanh(1/2*d*x+1/2*c)-1)-1/d/a*ln(tanh(1/2*d*x+1/2*c)+1)+1/2/d*a/b^
2*ln(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^
2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)+1/d/b*ln(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(
1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)+1
/2/d/a*ln(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/
2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)+2/d/b/(tanh(1/2*d*x+1/2*c)^2+1)-1/d
/b^2*ln(tanh(1/2*d*x+1/2*c)^2+1)*a-2/d/b*ln(tanh(1/2*d*x+1/2*c)^2+1)-2/d/b/
(tanh(1/2*d*x+1/2*c)^2+1)^2
```

maxima [A] time = 0.46, size = 131, normalized size = 1.87

$$\frac{dx + c}{ad} - \frac{2\,e^{(-2\,dx-2\,c)}}{\left(2\,be^{(-2\,dx-2\,c)} + be^{(-4\,dx-4\,c)} + b\right)d} - \frac{(a + 2\,b)\log\left(e^{(-2\,dx-2\,c)} + 1\right)}{b^2 d} + \frac{\left(a^2 + 2\,ab + b^2\right)\log\left(2\,(a + 2\,b)e^{(-2\,dx-}\right.}{2\,ab^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] (d*x + c)/(a*d) - 2*e^(-2*d*x - 2*c)/((2*b*e^(-2*d*x - 2*c) + b*e^(-4*d*x -
 4*c) + b)*d) - (a + 2*b)*log(e^(-2*d*x - 2*c) + 1)/(b^2*d) + 1/2*(a^2 + 2*
a*b + b^2)*log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/(a*b^
2*d)
```

mupad [B] time = 1.81, size = 421, normalized size = 6.01

$$\frac{2}{b\,d\,\left(2\,e^{2\,c+2\,d\,x} + e^{4\,c+4\,d\,x} + 1\right)} - \frac{2}{b\,d\,\left(e^{2\,c+2\,d\,x} + 1\right)} - \frac{x}{a} - \frac{\ln\left(39\,a\,b^7 + 243\,a^7\,b + 27\,a^8 + 2\,b^8 + 289\,a^2\,b^6 + 1017\,a^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^5/(a + b/cosh(c + d*x)^2),x)
```

```
[Out] 2/(b*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - 2/(b*d*(exp(2*c + 2*d
*x) + 1)) - x/a - (log(39*a*b^7 + 243*a^7*b + 27*a^8 + 2*b^8 + 289*a^2*b^6
+ 1017*a^3*b^5 + 1791*a^4*b^4 + 1701*a^5*b^3 + 891*a^6*b^2 + 27*a^8*exp(2*c
)*exp(2*d*x) + 2*b^8*exp(2*c)*exp(2*d*x) + 39*a*b^7*exp(2*c)*exp(2*d*x) + 2
43*a^7*b*exp(2*c)*exp(2*d*x) + 289*a^2*b^6*exp(2*c)*exp(2*d*x) + 1017*a^3*b
^5*exp(2*c)*exp(2*d*x) + 1791*a^4*b^4*exp(2*c)*exp(2*d*x) + 1701*a^5*b^3*ex
p(2*c)*exp(2*d*x) + 891*a^6*b^2*exp(2*c)*exp(2*d*x))*(a + 2*b))/(b^2*d) + (
log(a*b^2 + 6*a^2*b + 3*a^3 + 6*a^3*exp(2*c)*exp(2*d*x) + 3*a^3*exp(4*c)*ex
p(4*d*x) + 4*b^3*exp(2*c)*exp(2*d*x) + 26*a*b^2*exp(2*c)*exp(2*d*x) + 24*a^
2*b*exp(2*c)*exp(2*d*x) + a*b^2*exp(4*c)*exp(4*d*x) + 6*a^2*b*exp(4*c)*exp(
4*d*x))*(2*a*b + a^2 + b^2))/(2*a*b^2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^5(c+dx)}{a+b\,\mathrm{sech}^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**5/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(tanh(c + d*x)**5/(a + b*sech(c + d*x)**2), x)
```

3.139 $\int \frac{\tanh^4(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=59

$$-\frac{(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{ab^{3/2}d}+\frac{x}{a}+\frac{\tanh(c+dx)}{bd}$$

```
[Out] x/a-(a+b)^(3/2)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))/a/b^(3/2)/d+tanh(d
*x+c)/b/d
```

Rubi [A] time = 0.18, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4141, 1975, 479, 522, 206, 208}

$$-\frac{(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{ab^{3/2}d}+\frac{x}{a}+\frac{\tanh(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^4/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] x/a - ((a + b)^(3/2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a*b^(3/
2)*d) + Tanh[c + d*x]/(b*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
```

```
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\tanh^4(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)\left(a+b(1-x^2)\right)}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)(a+b-bx^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\tanh(c+dx)}{bd} - \frac{\text{Subst}\left(\int \frac{a+b+(-a-2b)x^2}{(1-x^2)(a+b-bx^2)}\,dx, x, \tanh(c+dx)\right)}{bd}$$

$$= \frac{\tanh(c+dx)}{bd} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{ad} - \frac{(a+b)^2\,\text{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x,\right.}{abd}$$

$$= \frac{x}{a} - \frac{(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a+b}}\right)}{ab^{3/2}d} + \frac{\tanh(c+dx)}{bd}$$

Mathematica [B] time = 1.09, size = 196, normalized size = 3.32

$$\frac{\operatorname{sech}^2(c+dx)(a\cosh(2(c+dx))+a+2b)\left(\sqrt{a+b}\sqrt{b(\cosh(c)-\sinh(c))^4}\,(a\operatorname{sech}(c)\sinh(dx)\operatorname{sech}(c+dx)+bd\right.}{2abd\sqrt{a+b}\sqrt{b(\cosh(c)-\sinh(c))^4}\,\left(a\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^4/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*((a + b)^2*ArcTanh[(Sech[d
*x]*(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*S
qrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(-Cosh[2*c] + Sinh[2*c]) + Sqrt[
a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4]*(b*d*x + a*Sech[c]*Sech[c + d*x]*Sinh[
d*x])))/(2*a*b*Sqrt[a + b]*d*(a + b*Sech[c + d*x]^2)*Sqrt[b*(Cosh[c] - Sinh
[c])^4])
```

fricas [B] time = 0.45, size = 683, normalized size = 11.58

$$\left[\frac{2\,bdx\cosh\left(dx+c\right)^2+4\,bdx\cosh\left(dx+c\right)\sinh\left(dx+c\right)+2\,bdx\sinh\left(dx+c\right)^2+2\,bdx+\left(\left(a+b\right)\cosh\left(dx+c\right)\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*b*d*x*cosh(d*x + c)^2 + 4*b*d*x*cosh(d*x + c)*sinh(d*x + c) + 2*b*d
*x*sinh(d*x + c)^2 + 2*b*d*x + ((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*
x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a + b)*sqrt((a + b)/b)*log
((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x
+ c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 +
 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a
^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) + 4*(a*b*cosh(d*x + c)^2 + 2*a*b*c
osh(d*x + c)*sinh(d*x + c) + a*b*sinh(d*x + c)^2 + a*b + 2*b^2)*sqrt((a + b
)/b))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x +
 c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sin
h(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c
) + a)) - 4*a)/(a*b*d*cosh(d*x + c)^2 + 2*a*b*d*cosh(d*x + c)*sinh(d*x + c)
 + a*b*d*sinh(d*x + c)^2 + a*b*d), (b*d*x*cosh(d*x + c)^2 + 2*b*d*x*cosh(d*
x + c)*sinh(d*x + c) + b*d*x*sinh(d*x + c)^2 + b*d*x - ((a + b)*cosh(d*x +
c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a
+ b)*sqrt(-(a + b)/b)*arctan(1/2*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sin
h(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-(a + b)/b)/(a + b)) - 2*a)/
```

```
(a*b*d*cosh(d*x + c)^2 + 2*a*b*d*cosh(d*x + c)*sinh(d*x + c) + a*b*d*sinh(d
*x + c)^2 + a*b*d)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error index.cc index_gcd Error: Bad Argument
 ValueError index.cc index_gcd Error: Bad Argument ValueError index.cc inde
x_gcd Error: Bad Argument ValueError index.cc index_gcd Error: Bad Argument
 ValueError index.cc index_gcd Error: Bad Argument ValueError index.cc inde
x_gcd Error: Bad Argument ValueDone
```

maple [B] time = 0.35, size = 386, normalized size = 6.54

$$-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{da}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{da}+\frac{a\ln\left(\sqrt{a+b}\,\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\sqrt{b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{a+\cdots}\right.}{2d\,b^{\frac{3}{2}}\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^4/(a+b*sech(d*x+c)^2),x)
```

```
[Out] -1/d/a*ln(tanh(1/2*d*x+1/2*c)-1)+1/d/a*ln(tanh(1/2*d*x+1/2*c)+1)+1/2/d*a/b^
(3/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*b^(1/2)*tanh(1/2*d
*x+1/2*c)+(a+b)^(1/2))+1/d/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+
1/2*c)^2-2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))+1/2/d/a*b^(1/2)/(a+b)^(
1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+
b)^(1/2))-1/2/d*a/b^(3/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+
2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))-1/d/b^(1/2)/(a+b)^(1/2)*ln((a+b)
^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))-1/2
/d/a*b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tan
h(1/2*d*x+1/2*c)+(a+b)^(1/2))+2/d/b*tanh(1/2*d*x+1/2*c)/(tanh(1/2*d*x+1/2*c
)^2+1)
```

maxima [B] time = 0.64, size = 637, normalized size = 10.80

$$-\frac{(a+2b)\log\left(\frac{ae^{(2dx+2c)}+a+2b-2\sqrt{(a+b)b}}{ae^{(2dx+2c)}+a+2b+2\sqrt{(a+b)b}}\right)}{8\sqrt{(a+b)b}\,bd}+\frac{(a+2b)\log\left(\frac{ae^{(-2dx-2c)}+a+2b-2\sqrt{(a+b)b}}{ae^{(-2dx-2c)}+a+2b+2\sqrt{(a+b)b}}\right)}{8\sqrt{(a+b)b}\,bd}+\frac{3a\log\left(\frac{ae^{(-2dx-2c)}+a+2b-2\sqrt{(a+b)\cdots}}{ae^{(-2dx-2c)}+a+2b+2\sqrt{(a+b)\cdots}}\right.}{16\sqrt{(a+b)b}\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/8*(a + 2*b)*log((a*e^(2*d*x + 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(
2*d*x + 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*b*d) + 1/8*(a
 + 2*b)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x
 - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*b*d) + 3/16*a*log(
(a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a
+ 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*b*d) + 1/8*(a + 2*b)*log(a*e^(
4*d*x + 4*c) + 2*(a + 2*b)*e^(2*d*x + 2*c) + a)/(a*b*d) + 1/4*log(a*e^(4*d*
x + 4*c) + 2*(a + 2*b)*e^(2*d*x + 2*c) + a)/(b*d) - 1/8*(a + 2*b)*log(2*(a
+ 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/(a*b*d) - 1/4*log(2*(a +
2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/(b*d) - 3/4*log(e^(2*d*x +
2*c) + 1)/(b*d) + 3/4*log(e^(-2*d*x - 2*c) + 1)/(b*d) - 1/32*(a^2 + 8*a*b +
 8*b^2)*log((a*e^(2*d*x + 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(2*d*x +
 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*a*b*d) + 1/32*(a^2 +
 8*a*b + 8*b^2)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e
^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*a*b*d) - 5
/8/((b*e^(2*d*x + 2*c) + b)*d) + 11/8/((b*e^(-2*d*x - 2*c) + b)*d)
```

mupad [B] time = 1.87, size = 183, normalized size = 3.10

$$\frac{x}{a}-\frac{2}{b\,d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)}+\frac{\ln\left(\frac{4\,\mathrm{e}^{2\,c+2\,d\,x}\,{\left(a+b\right)}^2}{a^2\,b}-\frac{2\,{\left(a+b\right)}^{3/2}\,\left(a+a\,\mathrm{e}^{2\,c+2\,d\,x}+2\,b\,\mathrm{e}^{2\,c+2\,d\,x}\right)}{a^2\,b^{3/2}}\right)\,{\left(a+b\right)}^{3/2}}{2\,a\,b^{3/2}\,d}-\frac{\ln\left(\frac{4\,\mathrm{e}^{2\,c+2\,d\,x}\,{\left(a+b\right)}^2}{a^2\,b}+\frac{2\,{\left(a+b\right)}^{3/2}}{}\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^4/(a + b/cosh(c + d*x)^2),x)
```

```
[Out] x/a - 2/(b*d*(exp(2*c + 2*d*x) + 1)) + (log((4*exp(2*c + 2*d*x)*(a + b)^2)/
(a^2*b) - (2*(a + b)^(3/2)*(a + a*exp(2*c + 2*d*x) + 2*b*exp(2*c + 2*d*x)))
/(a^2*b^(3/2)))*(a + b)^(3/2))/(2*a*b^(3/2)*d) - (log((4*exp(2*c + 2*d*x)*(
a + b)^2)/(a^2*b) + (2*(a + b)^(3/2)*(a + a*exp(2*c + 2*d*x) + 2*b*exp(2*c
+ 2*d*x)))/(a^2*b^(3/2)))*(a + b)^(3/2))/(2*a*b^(3/2)*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^4(c+dx)}{a+b\,\mathrm{sech}^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**4/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(tanh(c + d*x)**4/(a + b*sech(c + d*x)**2), x)
```

3.140 $\int \frac{\tanh^3(c+dx)}{a+b\,\mathbf{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=45

$$\frac{(a+b)\log\left(a\cosh^2(c+dx)+b\right)}{2abd}-\frac{\log(\cosh(c+dx))}{bd}$$

```
[Out] -ln(cosh(d*x+c))/b/d+1/2*(a+b)*ln(b+a*cosh(d*x+c)^2)/a/b/d
```

Rubi [A] time = 0.09, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4138, 446, 72}

$$\frac{(a+b)\log\left(a\cosh^2(c+dx)+b\right)}{2abd}-\frac{\log(\cosh(c+dx))}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^3/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -(Log[Cosh[c + d*x]]/(b*d)) + ((a + b)*Log[b + a*Cosh[c + d*x]^2])/(2*a*b*d
)
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^3(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx &= -\frac{\operatorname{Subst}\left(\int \frac{1-x^2}{x\left(b+ax^2\right)}\,dx, x, \cosh(c+dx)\right)}{d}\\
&= -\frac{\operatorname{Subst}\left(\int \frac{1-x}{x(b+ax)}\,dx, x, \cosh^2(c+dx)\right)}{2d}\\
&= -\frac{\operatorname{Subst}\left(\int \left(\frac{1}{bx}+\frac{-a-b}{b(b+ax)}\right)dx, x, \cosh^2(c+dx)\right)}{2d}\\
&= -\frac{\log(\cosh(c+dx))}{bd}+\frac{(a+b)\log\left(b+a\cosh^2(c+dx)\right)}{2abd}
\end{aligned}$$

Mathematica [A] time = 0.12, size = 41, normalized size = 0.91

$$\frac{(a+b)\log\left(a\cosh^2(c+dx)+b\right)-2a\log(\cosh(c+dx))}{2abd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^3/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] (-2*a*Log[Cosh[c + d*x]] + (a + b)*Log[b + a*Cosh[c + d*x]^2])/(2*a*b*d)
```

fricas [B] time = 0.46, size = 112, normalized size = 2.49

$$-\frac{2\,bdx-(a+b)\log\left(\frac{2\left(a\cosh(dx+c)^2+a\sinh(dx+c)^2+a+2\,b\right)}{\cosh(dx+c)^2-2\,\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^2}\right)+2\,a\log\left(\frac{2\,\cosh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)}{2\,abd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*d*x - (a + b)*log(2*(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + a +
2*b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) +
 2*a*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))))/(a*b*d)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error index.cc index_gcd Error: Bad Argument
 ValueError index.cc index_gcd Error: Bad Argument ValueError index.cc inde
x_gcd Error: Bad Argument ValueError index.cc index_gcd Error: Bad Argument
 ValueDone
```

maple [B] time = 0.33, size = 196, normalized size = 4.36

$$-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{da}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{da}+\frac{\ln\left(\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\left(\tanh^2\left(\frac{dx}{2}+\right.\right.\right.}{2db}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^3/(a+b*sech(d*x+c)^2),x)
```

```
[Out] -1/d/a*ln(tanh(1/2*d*x+1/2*c)-1)-1/d/a*ln(tanh(1/2*d*x+1/2*c)+1)+1/2/d/b*ln
(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-
2*tanh(1/2*d*x+1/2*c)^2*b+a+b)+1/2/d/a*ln(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/
2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)-1/d
/b*ln(tanh(1/2*d*x+1/2*c)^2+1)
```

maxima [A] time = 0.47, size = 77, normalized size = 1.71

$$\frac{dx+c}{ad}+\frac{(a+b)\log\left(2\,(a+2\,b)e^{(-2\,dx-2\,c)}+ae^{(-4\,dx-4\,c)}+a\right)}{2\,abd}-\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] (d*x + c)/(a*d) + 1/2*(a + b)*log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*
x - 4*c) + a)/(a*b*d) - log(e^(-2*d*x - 2*c) + 1)/(b*d)
```

mupad [B] time = 1.65, size = 238, normalized size = 5.29

$$\frac{\ln\left(a\,b+3\,a^2+6\,a^2\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}+3\,a^2\,\mathrm{e}^{4\,c}\,\mathrm{e}^{4\,d\,x}+4\,b^2\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}+14\,a\,b\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}+a\,b\,\mathrm{e}^{4\,c}\,\mathrm{e}^{4\,d\,x}\right)\,(a+b)}{2\,a\,b\,d}-\frac{\ln\left(21\,a\,b^4+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^3/(a + b/cosh(c + d*x)^2),x)
```

```
[Out] (log(a*b + 3*a^2 + 6*a^2*exp(2*c)*exp(2*d*x) + 3*a^2*exp(4*c)*exp(4*d*x) +
4*b^2*exp(2*c)*exp(2*d*x) + 14*a*b*exp(2*c)*exp(2*d*x) + a*b*exp(4*c)*exp(4
*d*x))*(a + b))/(2*a*b*d) - log(21*a*b^4 + 108*a^4*b + 27*a^5 + 2*b^5 + 82*
```

```
a^2*b^3 + 144*a^3*b^2 + 27*a^5*exp(2*c)*exp(2*d*x) + 2*b^5*exp(2*c)*exp(2*d
*x) + 21*a*b^4*exp(2*c)*exp(2*d*x) + 108*a^4*b*exp(2*c)*exp(2*d*x) + 82*a^2
*b^3*exp(2*c)*exp(2*d*x) + 144*a^3*b^2*exp(2*c)*exp(2*d*x))/(b*d) - x/a
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^3(c+dx)}{a+b\,\mathrm{sech}^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**3/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(tanh(c + d*x)**3/(a + b*sech(c + d*x)**2), x)
```

3.141 $\int \frac{\tanh^2(c+dx)}{a+b\,\mathbf{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=46

$$\frac{x}{a}-\frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a\sqrt{b}\,d}$$

```
[Out] x/a-arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))*(a+b)^(1/2)/a/d/b^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4141, 1975, 481, 206, 208}

$$\frac{x}{a}-\frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a\sqrt{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^2/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] x/a - (Sqrt[a + b]*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a*Sqrt[b]
*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh^2(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{x^2}{(1-x^2)\left(a+b(1-x^2)\right)}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{\operatorname{Subst}\left(\int \frac{x^2}{(1-x^2)(a+b-bx^2)}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{ad} - \frac{(a+b)\operatorname{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \tanh(c+dx)\right)}{ad}\\
&= \frac{x}{a} - \frac{\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a\sqrt{b}\,d}
\end{aligned}
$$

Mathematica [B] time = 0.30, size = 174, normalized size = 3.78

$$
\frac{\operatorname{sech}^2(c+dx)(a\cosh(2(c+dx))+a+2b)\left(dx\sqrt{a+b}\sqrt{b(\cosh(c)-\sinh(c))^4}+(a+b)(\sinh(2c)-\cosh(2c))\,\text{ta}\right.}{2ad\sqrt{a+b}\sqrt{b(\cosh(c)-\sinh(c))^4}\left(a+b\operatorname{sech}^2(c+dx)\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^2/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*(Sqrt[a + b]*d*x*Sqrt[b*(C
osh[c] - Sinh[c])^4] + (a + b)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*(
(a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] -
```

```
Sinh[c])^4])]*(-Cosh[2*c] + Sinh[2*c])))/(2*a*Sqrt[a + b]*d*(a + b*Sech[c +
 d*x]^2)*Sqrt[b*(Cosh[c] - Sinh[c])^4])
```

fricas [B] time = 0.45, size = 419, normalized size = 9.11

$$\left[\frac{2\,dx + \sqrt{\frac{a+b}{b}}\,\log\left(\frac{a^2\cosh(dx+c)^4+4\,a^2\cosh(dx+c)\sinh(dx+c)^3+a^2\sinh(dx+c)^4+2\left(a^2+2\,ab\right)\cosh(dx+c)^2+2\left(3\,a^2\cosh(dx+c)^2+a^2+2\,ab\right)\sinh}{a\cosh(dx+c)^4+4\,a\cosh(dx+c)\sinh(dx+c)^3+a\sinh(dx+c)^4+2\,(a+2\,b)\,c}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*d*x + sqrt((a + b)/b)*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c
)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 +
 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*
b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) +
 4*(a*b*cosh(d*x + c)^2 + 2*a*b*cosh(d*x + c)*sinh(d*x + c) + a*b*sinh(d*x
+ c)^2 + a*b + 2*b^2)*sqrt((a + b)/b))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x +
c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3
*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a +
 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)))/(a*d), (d*x - sqrt(-(a + b)/b)*ar
ctan(1/2*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x
+ c)^2 + a + 2*b)*sqrt(-(a + b)/b)/(a + b)))/(a*d)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error index.cc index_gcd Error: Bad Argument
 ValueError index.cc index_gcd Error: Bad Argument ValueError index.cc inde
x_gcd Error: Bad Argument ValueError index.cc index_gcd Error: Bad Argument
 ValueError index.cc index_gcd Error: Bad Argument ValueError index.cc inde
x_gcd Error: Bad Argument ValueDone
```

maple [B] time = 0.36, size = 251, normalized size = 5.46

$$-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{da}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{da}+\frac{\ln\left(\sqrt{a+b}\,\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\sqrt{b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{a+b}\right.}{2d\sqrt{b}\,\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^2/(a+b*sech(d*x+c)^2),x)
```

```
[Out] -1/d/a*ln(tanh(1/2*d*x+1/2*c)-1)+1/d/a*ln(tanh(1/2*d*x+1/2*c)+1)+1/2/d/b^(1
/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*b^(1/2)*tanh(1/2*d*x
+1/2*c)+(a+b)^(1/2))-1/2/d/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+
1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))+1/2/d/a*b^(1/2)/(a+b)^(
1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+
b)^(1/2))-1/2/d/a*b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+
2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))
```

maxima [B] time = 0.50, size = 291, normalized size = 6.33

$$-\frac{(a+2b)\log\left(\frac{ae^{(2dx+2c)}+a+2b-2\sqrt{(a+b)b}}{ae^{(2dx+2c)}+a+2b+2\sqrt{(a+b)b}}\right)}{8\sqrt{(a+b)b}\,ad}+\frac{\log\left(\frac{ae^{(-2dx-2c)}+a+2b-2\sqrt{(a+b)b}}{ae^{(-2dx-2c)}+a+2b+2\sqrt{(a+b)b}}\right)}{4\sqrt{(a+b)b}\,d}+\frac{(a+2b)\log\left(\frac{ae^{(-2dx-2c)}+a+2b-2\sqrt{(a+b)b}}{ae^{(-2dx-2c)}+a+2b+2\sqrt{(a+b)b}}\right)}{8\sqrt{(a+b)b}\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/8*(a + 2*b)*log((a*e^(2*d*x + 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(
2*d*x + 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*a*d) + 1/4*lo
g((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) +
a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*d) + 1/8*(a + 2*b)*log((a*e^
(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b
 + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*a*d) + 1/4*log(a*e^(4*d*x + 4*c) +
2*(a + 2*b)*e^(2*d*x + 2*c) + a)/(a*d) - 1/4*log(2*(a + 2*b)*e^(-2*d*x - 2*
c) + a*e^(-4*d*x - 4*c) + a)/(a*d)
```

mupad [B] time = 0.43, size = 105, normalized size = 2.28

$$\frac{x}{a}+\frac{\operatorname{atan}\left(\frac{\sqrt{-a^2\,b\,d^2}}{a\,d\,\sqrt{a+b}}+\frac{\sqrt{-a^2\,b\,d^2}}{2\,b\,d\,\sqrt{a+b}}+\frac{\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}\,\sqrt{-a^2\,b\,d^2}}{2\,b\,d\,\sqrt{a+b}}\right)\sqrt{a+b}}{\sqrt{-a^2\,b\,d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^2/(a + b/cosh(c + d*x)^2),x)
```

```
[Out] x/a + (atan((-a^2*b*d^2)^(1/2)/(a*d*(a + b)^(1/2)) + (-a^2*b*d^2)^(1/2)/(2*
b*d*(a + b)^(1/2)) + (exp(2*c)*exp(2*d*x)*(-a^2*b*d^2)^(1/2))/(2*b*d*(a + b
)^(1/2)))*(a + b)^(1/2))/(-a^2*b*d^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^2(c+dx)}{a+b\,\mathrm{sech}^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**2/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(tanh(c + d*x)**2/(a + b*sech(c + d*x)**2), x)
```

3.142 $\int \frac{\tanh(c+dx)}{a+b\,\mathbf{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=23

$$\frac{\log\left(a\cosh^2(c+dx)+b\right)}{2ad}$$

```
[Out] 1/2*ln(b+a*cosh(d*x+c)^2)/a/d
```

Rubi [A] time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4138, 260}

$$\frac{\log\left(a\cosh^2(c+dx)+b\right)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] Log[b + a*Cosh[c + d*x]^2]/(2*a*d)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{x}{b+ax^2}\,dx,x,\cosh(c+dx)\right)}{d}\\ &= \frac{\log\left(b+a\cosh^2(c+dx)\right)}{2ad}\end{aligned}$$

Mathematica [**A**] time = 0.18, size = 26, normalized size = 1.13

$$\frac{\log(a\cosh(2(c+dx))+a+2b)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] Log[a + 2*b + a*Cosh[2*(c + d*x)]]/(2*a*d)
```

fricas [**B**] time = 0.42, size = 76, normalized size = 3.30

$$-\frac{2\,dx-\log\left(\frac{2\left(a\cosh(dx+c)^2+a\sinh(dx+c)^2+a+2\,b\right)}{\cosh(dx+c)^2-2\,\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^2}\right)}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*d*x - log(2*(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + a + 2*b)/(cosh
(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)))/(a*d)
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error index.cc index_gcd Error: Bad Argument
 ValueError index.cc index_gcd Error: Bad Argument ValueDone
```

maple [**A**] time = 0.15, size = 38, normalized size = 1.65

$$\frac{\ln\left(a+b\operatorname{sech}(dx+c)^2\right)}{2da}-\frac{\ln(\operatorname{sech}(dx+c))}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)/(a+b*sech(d*x+c)^2),x)
```

```
[Out] 1/2/d/a*ln(a+b*sech(d*x+c)^2)-1/d/a*ln(sech(d*x+c))
```

maxima [B] time = 0.38, size = 51, normalized size = 2.22

$$\frac{dx+c}{ad}+\frac{\log\left(2\,(a+2\,b)e^{(-2\,dx-2\,c)}+ae^{(-4\,dx-4\,c)}+a\right)}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] (d*x + c)/(a*d) + 1/2*log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c)
 + a)/(a*d)
```

mupad [B] time = 0.35, size = 51, normalized size = 2.22

$$\frac{\ln\left(a+2\,a\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}+a\,\mathrm{e}^{4\,c}\,\mathrm{e}^{4\,d\,x}+4\,b\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}\right)-2\,d\,x}{2\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)/(a + b/cosh(c + d*x)^2),x)
```

```
[Out] (log(a + 2*a*exp(2*c)*exp(2*d*x) + a*exp(4*c)*exp(4*d*x) + 4*b*exp(2*c)*exp
(2*d*x)) - 2*d*x)/(2*a*d)
```

sympy [A] time = 7.04, size = 124, normalized size = 5.39

$$\begin{cases} \frac{\tilde{\infty} x\tanh(c)}{\operatorname{sech}^2(c)} & \text{for } a=0 \wedge b=0 \wedge d=0 \\ \frac{1}{2bd\operatorname{sech}^2(c+dx)} & \text{for } a=0 \\ \frac{x\tanh(c)}{a+b\operatorname{sech}^2(c)} & \text{for } d=0 \\ \frac{x-\frac{\log(\tanh(c+dx)+1)}{d}}{a} & \text{for } b=0 \\ \frac{x}{a}+\frac{\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\operatorname{sech}(c+dx)\right)}{2ad}+\frac{\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\operatorname{sech}(c+dx)\right)}{2ad}-\frac{\log(\tanh(c+dx)+1)}{ad} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Piecewise((zoo*x*tanh(c)/sech(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (1/(2
*b*d*sech(c + d*x)**2), Eq(a, 0)), (x*tanh(c)/(a + b*sech(c)**2), Eq(d, 0))
, ((x - log(tanh(c + d*x) + 1)/d)/a, Eq(b, 0)), (x/a + log(-I*sqrt(a)*sqrt(
1/b) + sech(c + d*x))/(2*a*d) + log(I*sqrt(a)*sqrt(1/b) + sech(c + d*x))/(2
*a*d) - log(tanh(c + d*x) + 1)/(a*d), True))
```

3.143 $\int \frac{1}{a+b\,\mathbf{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=46

$$\frac{x}{a}-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{ad\sqrt{a+b}}$$

[Out] x/a-arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))*b^(1/2)/a/d/(a+b)^(1/2)

Rubi [A] time = 0.05, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4127, 3181, 208}

$$\frac{x}{a}-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{ad\sqrt{a+b}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sech[c + d*x]^2)^(-1),x]

[Out] x/a - (Sqrt[b]*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a*Sqrt[a + b]*d)

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 4127

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> Simp[x/a, x
] - Dist[b/a, Int[1/(b + a*Cos[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f}, x
] && NeQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{a+b\text{sech}^2(c+dx)}\,dx=\frac{x}{a}-\frac{b\int \frac{1}{b+a\cosh^2(c+dx)}\,dx}{a}$$
$$=\frac{x}{a}-\frac{b\,\text{Subst}\left(\int \frac{1}{b-(a+b)x^2}\,dx,x,\coth(c+dx)\right)}{ad}$$
$$=\frac{x}{a}-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}\,d}$$

Mathematica [B] time = 0.25, size = 172, normalized size = 3.74

$$\frac{\text{sech}^2(c+dx)(a\cosh(2(c+dx))+a+2b)\left(dx\sqrt{a+b}\sqrt{b(\cosh(c)-\sinh(c))^4}+b(\sinh(2c)-\cosh(2c))\tanh^{-1}\right.}{2ad\sqrt{a+b}\sqrt{b(\cosh(c)-\sinh(c))^4}\left(a+b\text{sech}^2(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^(-1),x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*(Sqrt[a + b]*d*x*Sqrt[b*(C
osh[c] - Sinh[c])^4] + b*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*((a + 2
*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c
])^4])]*(-Cosh[2*c] + Sinh[2*c])))/(2*a*Sqrt[a + b]*d*(a + b*Sech[c + d*x]^
2)*Sqrt[b*(Cosh[c] - Sinh[c])^4])
```

fricas [B] time = 0.44, size = 436, normalized size = 9.48

$$\left[\frac{2\,dx+\sqrt{\frac{b}{a+b}}\log\left(\frac{a^2\cosh(dx+c)^4+4\,a^2\cosh(dx+c)\sinh(dx+c)^3+a^2\sinh(dx+c)^4+2\left(a^2+2\,ab\right)\cosh(dx+c)^2+2\left(3\,a^2\cosh(dx+c)^2+a^2+2\,ab\right)\sin}{a\cosh(dx+c)^4+4\,a\cosh(dx+c)\sinh(dx+c)^3+a\sinh(dx+c}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*d*x + sqrt(b/(a + b))*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c
)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 +
 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*
b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) +
 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c)
```

```
 + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 + 3*a*b + 2*b^2)*sqrt(b/(a + b)))/(a*c
osh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*
(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)
^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)))/(
a*d), (d*x - sqrt(-b/(a + b))*arctan(1/2*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x
+ c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-b/(a + b))/b))/(a*d
)]
```

giac [A] time = 0.39, size = 64, normalized size = 1.39

$$-\frac{\frac{b\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}\,a}-\frac{dx+c}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -(b*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/(sqrt(-a*b -
 b^2)*a) - (d*x + c)/a)/d
```

maple [B] time = 0.28, size = 149, normalized size = 3.24

$$-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{da}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{da}+\frac{\sqrt{b}\,\ln\left(\sqrt{a+b}\,\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\sqrt{b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{a}\cdots\right)}{2da\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sech(d*x+c)^2),x)
```

```
[Out] -1/d/a*ln(tanh(1/2*d*x+1/2*c)-1)+1/d/a*ln(tanh(1/2*d*x+1/2*c)+1)+1/2/d/a*b^
(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*b^(1/2)*tanh(1/2*d
*x+1/2*c)+(a+b)^(1/2))-1/2/d/a*b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*
d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))
```

maxima [B] time = 0.47, size = 83, normalized size = 1.80

$$\frac{b\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{2\,\sqrt{(a+b)b}\,ad}+\frac{dx+c}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*b*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x -
 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*a*d) + (d*x + c)/(a*
d)
```

mupad [B] time = 2.16, size = 470, normalized size = 10.22

$$\frac{x}{a}+\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\left(a^5\sqrt{-a^3d^2-ba^2d^2}+a^4b\sqrt{-a^3d^2-ba^2d^2}\right)\left(e^{2c}e^{2dx}\left(\frac{2\left(a^2+8ab+8b^2\right)\left(8b^{5/2}\sqrt{-a^3d^2-ba^2d^2}+8ab^{3/2}\sqrt{-a^3d^2-ba^2d^2}+a^2\sqrt{b}\sqrt{-a^3d^2-\cdots}\right.}{a^8d(a+b)^2\sqrt{-a^3d^2-ba^2d^2}}\cdots\right.\right.}{\cdots}\right)}{\sqrt{-a^3\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cosh(c + d*x)^2),x)
```

```
[Out] x/a + (b^(1/2)*atan(((a^5*(- a^3*d^2 - a^2*b*d^2)^(1/2) + a^4*b*(- a^3*d^2
- a^2*b*d^2)^(1/2))*(exp(2*c)*exp(2*d*x)*((2*(8*a*b + a^2 + 8*b^2)*(8*b^(5/
2)*(- a^3*d^2 - a^2*b*d^2)^(1/2) + 8*a*b^(3/2)*(- a^3*d^2 - a^2*b*d^2)^(1/2
) + a^2*b^(1/2)*(- a^3*d^2 - a^2*b*d^2)^(1/2)))/(a^8*d*(a + b)^2*(- a^3*d^2
 - a^2*b*d^2)^(1/2)) + (4*b^(1/2)*(2*a + 4*b)*(12*a^2*b^2*d + 8*a*b^3*d + 4
*a^3*b*d))/(a^7*(a + b)*(- a^3*d^2 - a^2*b*d^2)^(1/2)*(-a^2*d^2*(a + b))^(1
/2))) + (2*(2*a*b^(3/2)*(- a^3*d^2 - a^2*b*d^2)^(1/2) + a^2*b^(1/2)*(- a^3*
d^2 - a^2*b*d^2)^(1/2))*(8*a*b + a^2 + 8*b^2))/(a^8*d*(a + b)^2*(- a^3*d^2
- a^2*b*d^2)^(1/2)) + (4*b^(1/2)*(2*a^2*b^2*d + 2*a^3*b*d)*(2*a + 4*b))/(a^
7*(a + b)*(- a^3*d^2 - a^2*b*d^2)^(1/2)*(-a^2*d^2*(a + b))^(1/2))))/(4*b)))
/(- a^3*d^2 - a^2*b*d^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a+b\,\mathrm{sech}^2\,(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(1/(a + b*sech(c + d*x)**2), x)
```

3.144 $\int \frac{\coth(c+dx)}{a+b\mathbf{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=46

$$\frac{\log(\sinh(c+dx))}{d(a+b)}+\frac{b\log\left(a\cosh^2(c+dx)+b\right)}{2ad(a+b)}$$

```
[Out] 1/2*b*ln(b+a*cosh(d*x+c)^2)/a/d/(a+b)+ln(sinh(d*x+c))/(a+b)/d
```

Rubi [A] time = 0.09, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4138, 446, 72}

$$\frac{\log(\sinh(c+dx))}{d(a+b)}+\frac{b\log\left(a\cosh^2(c+dx)+b\right)}{2ad(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] (b*Log[b + a*Cosh[c + d*x]^2])/(2*a*(a + b)*d) + Log[Sinh[c + d*x]]/((a + b
)*d)
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\coth(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{x^3}{(1-x^2)(b+ax^2)}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \frac{x}{(1-x)(b+ax)}\,dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{(-a-b)(-1+x)} - \frac{b}{(a+b)(b+ax)}\right)dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= \frac{b\log\left(b+a\cosh^2(c+dx)\right)}{2a(a+b)d} + \frac{\log(\sinh(c+dx))}{(a+b)d}$$

Mathematica [A] time = 0.09, size = 42, normalized size = 0.91

$$\frac{b\log\left(a\sinh^2(c+dx)+a+b\right)+2a\log(\sinh(c+dx))}{2a^2d+2abd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] (2*a*Log[Sinh[c + d*x]] + b*Log[a + b + a*Sinh[c + d*x]^2])/(2*a^2*d + 2*a*
b*d)
```

fricas [B] time = 0.46, size = 115, normalized size = 2.50

$$-\frac{2(a+b)dx - b\log\left(\frac{2\left(a\cosh(dx+c)^2+a\sinh(dx+c)^2+a+2b\right)}{\cosh(dx+c)^2-2\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^2}\right) - 2a\log\left(\frac{2\sinh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)}{2\left(a^2+ab\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a + b)*d*x - b*log(2*(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + a +
2*b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) -
 2*a*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))))/((a^2 + a*b)*d)
```

giac [B] time = 0.33, size = 97, normalized size = 2.11

$$-\frac{\frac{2\,dx}{a} - \frac{b\log\left(ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+a\right)}{a^2+ab} - \frac{2\,e^{(2\,c)}\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)}{ae^{(2\,c)}+be^{(2\,c)}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/2*(2*d*x/a - b*log(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*
x + 2*c) + a)/(a^2 + a*b) - 2*e^(2*c)*log(abs(e^(2*d*x + 2*c) - 1))/(a*e^(2
*c) + b*e^(2*c)))/d
```

maple [B] time = 0.38, size = 133, normalized size = 2.89

$$-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{da}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{da}+\frac{b\ln\left(\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\left(\tanh^2\left(\frac{dx}{2}+\right.\right.\right.}{2da\,(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)/(a+b*sech(d*x+c)^2),x)
```

```
[Out] -1/d/a*ln(tanh(1/2*d*x+1/2*c)-1)-1/d/a*ln(tanh(1/2*d*x+1/2*c)+1)+1/2/d*b/a/
(a+b)*ln(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2
*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)+1/d/(a+b)*ln(tanh(1/2*d*x+1/2*c))
```

maxima [B] time = 0.42, size = 100, normalized size = 2.17

$$\frac{b\log\left(2\,(a+2\,b)e^{(-2\,dx-2\,c)}+ae^{(-4\,dx-4\,c)}+a\right)}{2\left(a^2+ab\right)d}+\frac{dx+c}{ad}+\frac{\log\left(e^{(-dx-c)}+1\right)}{(a+b)d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{(a+b)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*b*log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/((a^2 + a*
b)*d) + (d*x + c)/(a*d) + log(e^(-d*x - c) + 1)/((a + b)*d) + log(e^(-d*x -
 c) - 1)/((a + b)*d)
```

mupad [B] time = 1.82, size = 228, normalized size = 4.96

$$\frac{\ln\left(8\,a\,b^5-b^6-24\,a^2\,b^4+32\,a^3\,b^3-16\,a^4\,b^2+b^6\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}-8\,a\,b^5\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}+24\,a^2\,b^4\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}-32\,a^3\,b^3\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\right.}}{a\,d+b\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)/(a + b/cosh(c + d*x)^2),x)
```

```
[Out] log(8*a*b^5 - b^6 - 24*a^2*b^4 + 32*a^3*b^3 - 16*a^4*b^2 + b^6*exp(2*c)*exp
(2*d*x) - 8*a*b^5*exp(2*c)*exp(2*d*x) + 24*a^2*b^4*exp(2*c)*exp(2*d*x) - 32
*a^3*b^3*exp(2*c)*exp(2*d*x) + 16*a^4*b^2*exp(2*c)*exp(2*d*x))/(a*d + b*d)
```

```
- x/a + (b*log(2*a^2 - a*b + 4*a^2*exp(2*c)*exp(2*d*x) + 2*a^2*exp(4*c)*exp
(4*d*x) - 4*b^2*exp(2*c)*exp(2*d*x) + 6*a*b*exp(2*c)*exp(2*d*x) - a*b*exp(4
*c)*exp(4*d*x)))/(2*a^2*d + 2*a*b*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(c+dx)}{a+b\,\mathrm{sech}^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(coth(c + d*x)/(a + b*sech(c + d*x)**2), x)
```

3.145 $\int \frac{\coth^2(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=62

$$-\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{ad(a+b)^{3/2}}-\frac{\coth(c+dx)}{d(a+b)}+\frac{x}{a}$$

[Out] x/a-b^(3/2)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))/a/(a+b)^(3/2)/d-coth(d*x+c)/(a+b)/d

Rubi [A] time = 0.18, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4141, 1975, 480, 522, 206, 208}

$$-\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{ad(a+b)^{3/2}}-\frac{\coth(c+dx)}{d(a+b)}+\frac{x}{a}$$

Antiderivative was successfully verified.

[In] Int[Coth[c + d*x]^2/(a + b*Sech[c + d*x]^2),x]

[Out] x/a - (b^(3/2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a*(a + b)^(3/2)*d) - Coth[c + d*x]/((a + b)*d)

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
```

```
, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^2(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{x^2\left(1-x^2\right)\left(a+b\left(1-x^2\right)\right)}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{\operatorname{Subst}\left(\int \frac{1}{x^2\left(1-x^2\right)\left(a+b-bx^2\right)}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= -\frac{\coth(c+dx)}{(a+b)d} + \frac{\operatorname{Subst}\left(\int \frac{a+2b-bx^2}{\left(1-x^2\right)\left(a+b-bx^2\right)}\,dx, x, \tanh(c+dx)\right)}{(a+b)d}\\
&= -\frac{\coth(c+dx)}{(a+b)d} + \frac{\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{ad} - \frac{b^2\operatorname{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \tan\right.}{a(a+b)d}\\
&= \frac{x}{a} - \frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a(a+b)^{3/2}d} - \frac{\coth(c+dx)}{(a+b)d}
\end{aligned}$$

Mathematica [B] time = 1.16, size = 193, normalized size = 3.11

$$\frac{\operatorname{sech}^2(c+dx)(a\cosh(2(c+dx))+a+2b)\left(b^2(\sinh(2c)-\cosh(2c))\tanh^{-1}\left(\frac{(\cosh(2c)-\sinh(2c))\operatorname{sech}(dx)((a+2b)\sinh(dx)-a}{2\sqrt{a+b}\sqrt{b(\cosh(c)-\sinh(c))^4}}\right.\right.}{2ad(a+b)^{3/2}\sqrt{b(\cosh(c)-\sinh(c))^4}\,(a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^2/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*(b^2*ArcTanh[(Sech[d*x]*(C
osh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a
+ b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(-Cosh[2*c] + Sinh[2*c]) + Sqrt[a + b]
*Sqrt[b*(Cosh[c] - Sinh[c])^4]*((a + b)*d*x + a*Csch[c]*Csch[c + d*x]*Sinh[
d*x])))/(2*a*(a + b)^(3/2)*d*(a + b*Sech[c + d*x]^2)*Sqrt[b*(Cosh[c] - Sinh
[c])^4])
```

fricas [B] time = 0.46, size = 749, normalized size = 12.08

$$\left[\frac{2\,(a+b)dx\cosh\left(dx+c\right)^2+4\,(a+b)dx\cosh\left(dx+c\right)\sinh\left(dx+c\right)+2\,(a+b)dx\sinh\left(dx+c\right)^2-2\,(a+b)dx+(}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*(a + b)*d*x*cosh(d*x + c)^2 + 4*(a + b)*d*x*cosh(d*x + c)*sinh(d*x
+ c) + 2*(a + b)*d*x*sinh(d*x + c)^2 - 2*(a + b)*d*x + (b*cosh(d*x + c)^2 +
 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)*sqrt(b/(a + b))*l
og((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*
x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2
 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 +
(a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) + 4*((a^2 + a*b)*cosh(d*x + c)^2
 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2
+ a^2 + 3*a*b + 2*b^2)*sqrt(b/(a + b)))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x +
 c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(
3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a
+ 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) - 4*a)/((a^2 + a*b)*d*cosh(d*x +
c)^2 + 2*(a^2 + a*b)*d*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*d*sinh(d*x
 + c)^2 - (a^2 + a*b)*d), ((a + b)*d*x*cosh(d*x + c)^2 + 2*(a + b)*d*x*cosh
(d*x + c)*sinh(d*x + c) + (a + b)*d*x*sinh(d*x + c)^2 - (a + b)*d*x - (b*co
sh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)*sq
rt(-b/(a + b))*arctan(1/2*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x +
```

```
 c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-b/(a + b))/b) - 2*a)/((a^2 + a*b)*
d*cosh(d*x + c)^2 + 2*(a^2 + a*b)*d*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*
b)*d*sinh(d*x + c)^2 - (a^2 + a*b)*d)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error index.cc index_gcd Error: Bad Argument
 ValueError index.cc index_gcd Error: Bad Argument ValueError index.cc inde
x_gcd Error: Bad Argument ValueError index.cc index_gcd Error: Bad Argument
 ValueDone
```

maple [B] time = 0.46, size = 189, normalized size = 3.05

$$-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,(a+b)}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{da}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{da}-\frac{1}{2d\,(a+b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{b^{\frac{3}{2}}\ln\left(\sqrt{a+b}\ \left(\mathrm{ta}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^2/(a+b*sech(d*x+c)^2),x)
```

```
[Out] -1/2/d/(a+b)*tanh(1/2*d*x+1/2*c)-1/d/a*ln(tanh(1/2*d*x+1/2*c)-1)+1/d/a*ln(t
anh(1/2*d*x+1/2*c)+1)-1/2/d/(a+b)/tanh(1/2*d*x+1/2*c)+1/2/d*b^(3/2)/a/(a+b)
^(3/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(
a+b)^(1/2))-1/2/d*b^(3/2)/a/(a+b)^(3/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^
2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))
```

maxima [B] time = 0.48, size = 429, normalized size = 6.92

$$\frac{b\log\left(ae^{(4\,dx+4\,c)}+2\,(a+2\,b)e^{(2\,dx+2\,c)}+a\right)}{4\left(a^2+ab\right)d}-\frac{b\log\left(2\,(a+2\,b)e^{(-2\,dx-2\,c)}+ae^{(-4\,dx-4\,c)}+a\right)}{4\left(a^2+ab\right)d}-\frac{\left(ab+2\,b^2\right)\log\left(\frac{ae^{(2\,c}}{ae^{(2\,c}}\right.}{8\left(a^2+ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/4*b*log(a*e^(4*d*x + 4*c) + 2*(a + 2*b)*e^(2*d*x + 2*c) + a)/((a^2 + a*b)
*d) - 1/4*b*log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/((a^
```

```
2 + a*b)*d) - 1/8*(a*b + 2*b^2)*log((a*e^(2*d*x + 2*c) + a + 2*b - 2*sqrt((
a + b)*b))/(a*e^(2*d*x + 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^2 + a*b)*
sqrt((a + b)*b)*d) + 1/8*(a*b + 2*b^2)*log((a*e^(-2*d*x - 2*c) + a + 2*b -
2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^
2 + a*b)*sqrt((a + b)*b)*d) - 1/4*b*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*s
qrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((
a + b)*b)*(a + b)*d) + 1/2*log(e^(2*d*x + 2*c) - 1)/((a + b)*d) - 1/2*log(e
^(-2*d*x - 2*c) - 1)/((a + b)*d) - 1/2/(((a + b)*e^(2*d*x + 2*c) - a - b)*d
) + 3/2/(((a + b)*e^(-2*d*x - 2*c) - a - b)*d)
```

mupad [B] time = 3.39, size = 977, normalized size = 15.76

$$\frac{x}{a}-\frac{2}{\left(e^{2c+2dx}-1\right)(a\,d+b\,d)}+\frac{\mathrm{atan}\left(\frac{\left(e^{2c}\,e^{2dx}\left(\frac{8\,(a+2\,b)\left(4\,d\,a^4\,b^2+16\,d\,a^3\,b^3+20\,d\,a^2\,b^4+8\,d\,a\,b^5\right)}{a^6\,(a+b)\left(a^3+2\,a^2\,b+a\,b^2\right)\sqrt{-a^2\,d^2\,(a+b)^3}\,\sqrt{-a^5\,d^2-3\,a^4\,b\,d^2-3\,a^3\,b^2\,d^2-a^2\,b^3\,d^2}}+\frac{2\,\sqrt{b^3}\left(a^2+8\,a\,b+8\,b^2\right)\cdots}{\cdots}\right.\right.}{\cdots}\right)}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^2/(a + b/cosh(c + d*x)^2),x)
```

```
[Out] x/a - 2/((exp(2*c + 2*d*x) - 1)*(a*d + b*d)) + (atan(((exp(2*c)*exp(2*d*x)*
((8*(a + 2*b)*(20*a^2*b^4*d + 16*a^3*b^3*d + 4*a^4*b^2*d + 8*a*b^5*d))/(a^6
*(a + b)*(a*b^2 + 2*a^2*b + a^3)*(-a^2*d^2*(a + b)^3)^(1/2)*(- a^5*d^2 - 3*
a^4*b*d^2 - a^2*b^3*d^2 - 3*a^3*b^2*d^2)^(1/2)) + (2*(b^3)^(1/2)*(8*a*b + a
^2 + 8*b^2)*(a^2*(b^3)^(1/2)*(- a^5*d^2 - 3*a^4*b*d^2 - a^2*b^3*d^2 - 3*a^3
*b^2*d^2)^(1/2) + 8*b^2*(b^3)^(1/2)*(- a^5*d^2 - 3*a^4*b*d^2 - a^2*b^3*d^2
- 3*a^3*b^2*d^2)^(1/2) + 8*a*b*(b^3)^(1/2)*(- a^5*d^2 - 3*a^4*b*d^2 - a^2*b
^3*d^2 - 3*a^3*b^2*d^2)^(1/2)))/(a^7*b^2*d*(a + b)^3*(a*b^2 + 2*a^2*b + a^3
)*(- a^5*d^2 - 3*a^4*b*d^2 - a^2*b^3*d^2 - 3*a^3*b^2*d^2)^(1/2))) + (8*(a +
 2*b)*(2*a^2*b^4*d + 4*a^3*b^3*d + 2*a^4*b^2*d))/(a^6*(a + b)*(a*b^2 + 2*a^
2*b + a^3)*(-a^2*d^2*(a + b)^3)^(1/2)*(- a^5*d^2 - 3*a^4*b*d^2 - a^2*b^3*d^
2 - 3*a^3*b^2*d^2)^(1/2)) + (2*(a^2*(b^3)^(1/2)*(- a^5*d^2 - 3*a^4*b*d^2 -
a^2*b^3*d^2 - 3*a^3*b^2*d^2)^(1/2) + 2*a*b*(b^3)^(1/2)*(- a^5*d^2 - 3*a^4*b
*d^2 - a^2*b^3*d^2 - 3*a^3*b^2*d^2)^(1/2))*(b^3)^(1/2)*(8*a*b + a^2 + 8*b^2
))/(a^7*b^2*d*(a + b)^3*(a*b^2 + 2*a^2*b + a^3)*(- a^5*d^2 - 3*a^4*b*d^2 -
a^2*b^3*d^2 - 3*a^3*b^2*d^2)^(1/2)))*(a^7*(- a^5*d^2 - 3*a^4*b*d^2 - a^2*b^
3*d^2 - 3*a^3*b^2*d^2)^(1/2) + a^4*b^3*(- a^5*d^2 - 3*a^4*b*d^2 - a^2*b^3*d
^2 - 3*a^3*b^2*d^2)^(1/2) + 3*a^5*b^2*(- a^5*d^2 - 3*a^4*b*d^2 - a^2*b^3*d^
2 - 3*a^3*b^2*d^2)^(1/2) + 3*a^6*b*(- a^5*d^2 - 3*a^4*b*d^2 - a^2*b^3*d^2 -
 3*a^3*b^2*d^2)^(1/2)))/(4*(b^3)^(1/2)))*(b^3)^(1/2))/(- a^5*d^2 - 3*a^4*b*
d^2 - a^2*b^3*d^2 - 3*a^3*b^2*d^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**2/(a+b*sech(d*x+c)**2),x)
[Out] Integral(coth(c + d*x)**2/(a + b*sech(c + d*x)**2), x)
```

3.146 $\int \frac{\coth^3(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=73

$$\frac{b^2\log\left(a\cosh^2(c+dx)+b\right)}{2ad(a+b)^2}-\frac{\operatorname{csch}^2(c+dx)}{2d(a+b)}+\frac{(a+2b)\log(\sinh(c+dx))}{d(a+b)^2}$$

```
[Out] -1/2*csch(d*x+c)^2/(a+b)/d+1/2*b^2*ln(b+a*cosh(d*x+c)^2)/a/(a+b)^2/d+(a+2*b
)*ln(sinh(d*x+c))/(a+b)^2/d
```

Rubi [A] time = 0.12, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4138, 446, 88}

$$\frac{b^2\log\left(a\cosh^2(c+dx)+b\right)}{2ad(a+b)^2}-\frac{\operatorname{csch}^2(c+dx)}{2d(a+b)}+\frac{(a+2b)\log(\sinh(c+dx))}{d(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^3/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -Csch[c + d*x]^2/(2*(a + b)*d) + (b^2*Log[b + a*Cosh[c + d*x]^2])/(2*a*(a +
 b)^2*d) + ((a + 2*b)*Log[Sinh[c + d*x]])/((a + b)^2*d)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
```

```
x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\coth^3(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^5}{\left(1-x^2\right)^2\left(b+ax^2\right)}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{x^2}{(1-x)^2(b+ax)}\,dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{(a+b)(-1+x)^2}+\frac{a+2b}{(a+b)^2(-1+x)}+\frac{b^2}{(a+b)^2(b+ax)}\right)dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= -\frac{\text{csch}^2(c+dx)}{2(a+b)d}+\frac{b^2\log\left(b+a\cosh^2(c+dx)\right)}{2a(a+b)^2d}+\frac{(a+2b)\log(\sinh(c+dx))}{(a+b)^2d}$$

Mathematica [A] time = 0.24, size = 100, normalized size = 1.37

$$-\frac{\text{sech}^2(c+dx)(a\cosh(2(c+dx))+a+2b)\left(b^2\left(-\log\left(a\sinh^2(c+dx)+a+b\right)\right)+a(a+b)\text{csch}^2(c+dx)-2a(a$$
$$4ad(a+b)^2\left(a+b\text{sech}^2(c+dx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^3/(a + b*Sech[c + d*x]^2),x]

[Out] -1/4*((a + 2*b + a*Cosh[2*(c + d*x)])*(a*(a + b)*Csch[c + d*x]^2 - 2*a*(a +
 2*b)*Log[Sinh[c + d*x]] - b^2*Log[a + b + a*Sinh[c + d*x]^2])*Sech[c + d*x
]^2)/(a*(a + b)^2*d*(a + b*Sech[c + d*x]^2))
```

fricas [B] time = 0.52, size = 862, normalized size = 11.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="fricas")

[Out] -1/2*(2*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^4 + 8*(a^2 + 2*a*b + b^2)*d*x
*cosh(d*x + c)*sinh(d*x + c)^3 + 2*(a^2 + 2*a*b + b^2)*d*x*sinh(d*x + c)^4
+ 2*(a^2 + 2*a*b + b^2)*d*x - 4*((a^2 + 2*a*b + b^2)*d*x - a^2 - a*b)*cosh(
d*x + c)^2 + 4*(3*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^2 - (a^2 + 2*a*b +
```

```
b^2)*d*x + a^2 + a*b)*sinh(d*x + c)^2 - (b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d
*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4 - 2*b^2*cosh(d*x + c)^2 + 2*(
3*b^2*cosh(d*x + c)^2 - b^2)*sinh(d*x + c)^2 + b^2 + 4*(b^2*cosh(d*x + c)^3
 - b^2*cosh(d*x + c))*sinh(d*x + c))*log(2*(a*cosh(d*x + c)^2 + a*sinh(d*x
+ c)^2 + a + 2*b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d
*x + c)^2)) - 2*((a^2 + 2*a*b)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b)*cosh(d*x +
 c)*sinh(d*x + c)^3 + (a^2 + 2*a*b)*sinh(d*x + c)^4 - 2*(a^2 + 2*a*b)*cosh(
d*x + c)^2 + 2*(3*(a^2 + 2*a*b)*cosh(d*x + c)^2 - a^2 - 2*a*b)*sinh(d*x + c
)^2 + a^2 + 2*a*b + 4*((a^2 + 2*a*b)*cosh(d*x + c)^3 - (a^2 + 2*a*b)*cosh(d
*x + c))*sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))
) + 8*((a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^3 - ((a^2 + 2*a*b + b^2)*d*x -
 a^2 - a*b)*cosh(d*x + c))*sinh(d*x + c))/((a^3 + 2*a^2*b + a*b^2)*d*cosh(d
*x + c)^4 + 4*(a^3 + 2*a^2*b + a*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^
3 + 2*a^2*b + a*b^2)*d*sinh(d*x + c)^4 - 2*(a^3 + 2*a^2*b + a*b^2)*d*cosh(d
*x + c)^2 + 2*(3*(a^3 + 2*a^2*b + a*b^2)*d*cosh(d*x + c)^2 - (a^3 + 2*a^2*b
 + a*b^2)*d)*sinh(d*x + c)^2 + (a^3 + 2*a^2*b + a*b^2)*d + 4*((a^3 + 2*a^2*
b + a*b^2)*d*cosh(d*x + c)^3 - (a^3 + 2*a^2*b + a*b^2)*d*cosh(d*x + c))*sin
h(d*x + c))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error index.cc index_gcd Error: Bad Argument
 ValueError index.cc index_gcd Error: Bad Argument ValueError index.cc inde
x_gcd Error: Bad Argument ValueError index.cc index_gcd Error: Bad Argument
 ValueEvaluation time: 0.58Done
```

maple [B] time = 0.48, size = 199, normalized size = 2.73

$$-\frac{\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,(a+b)}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{da}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{da}+\frac{b^2\ln\left(\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^3/(a+b*sech(d*x+c)^2),x)
```

```
[Out] -1/8/d*tanh(1/2*d*x+1/2*c)^2/(a+b)-1/d/a*ln(tanh(1/2*d*x+1/2*c)-1)-1/d/a*ln
(tanh(1/2*d*x+1/2*c)+1)+1/2/d*b^2/a/(a+b)^2*ln(tanh(1/2*d*x+1/2*c)^4*a+b*ta
nh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b
```

```
)-1/8/d/(a+b)/tanh(1/2*d*x+1/2*c)^2+1/d/(a+b)^2*ln(tanh(1/2*d*x+1/2*c))*a+2
/d/(a+b)^2*ln(tanh(1/2*d*x+1/2*c))*b
```

maxima [B] time = 0.44, size = 187, normalized size = 2.56

$$\frac{b^2\log\left(2\,(a+2\,b)e^{(-2\,dx-2\,c)}+ae^{(-4\,dx-4\,c)}+a\right)}{2\left(a^3+2\,a^2b+ab^2\right)d}+\frac{(a+2\,b)\log\left(e^{(-dx-c)}+1\right)}{\left(a^2+2\,ab+b^2\right)d}+\frac{(a+2\,b)\log\left(e^{(-dx-c)}-1\right)}{\left(a^2+2\,ab+b^2\right)d}+\frac{dx+c}{ad}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*b^2*log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/((a^3 +
2*a^2*b + a*b^2)*d) + (a + 2*b)*log(e^(-d*x - c) + 1)/((a^2 + 2*a*b + b^2)*
d) + (a + 2*b)*log(e^(-d*x - c) - 1)/((a^2 + 2*a*b + b^2)*d) + (d*x + c)/(a
*d) + 2*e^(-2*d*x - 2*c)/((2*(a + b)*e^(-2*d*x - 2*c) - (a + b)*e^(-4*d*x -
 4*c) - a - b)*d)
```

mupad [B] time = 2.07, size = 523, normalized size = 7.16

$$\frac{\ln\left(23\,a\,b^7+8\,a^7\,b-2\,b^8-72\,a^2\,b^6-10\,a^3\,b^5+184\,a^4\,b^4+180\,a^5\,b^3+64\,a^6\,b^2+2\,b^8\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}-23\,a\,b^7\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d}\right.}{d\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^3/(a + b/cosh(c + d*x)^2),x)
```

```
[Out] (log(23*a*b^7 + 8*a^7*b - 2*b^8 - 72*a^2*b^6 - 10*a^3*b^5 + 184*a^4*b^4 + 1
80*a^5*b^3 + 64*a^6*b^2 + 2*b^8*exp(2*c)*exp(2*d*x) - 23*a*b^7*exp(2*c)*exp
(2*d*x) - 8*a^7*b*exp(2*c)*exp(2*d*x) + 72*a^2*b^6*exp(2*c)*exp(2*d*x) + 10
*a^3*b^5*exp(2*c)*exp(2*d*x) - 184*a^4*b^4*exp(2*c)*exp(2*d*x) - 180*a^5*b^
3*exp(2*c)*exp(2*d*x) - 64*a^6*b^2*exp(2*c)*exp(2*d*x))*(a + 2*b))/(a^2*d +
 b^2*d + 2*a*b*d) - x/a - 2/((a*d + b*d)*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*
d*x) + 1)) + (b^2*log(a*b^4 + 16*a^4*b + 4*a^5 - 8*a^2*b^3 + 12*a^3*b^2 + 8
*a^5*exp(2*c)*exp(2*d*x) + 4*a^5*exp(4*c)*exp(4*d*x) + 4*b^5*exp(2*c)*exp(2
*d*x) - 30*a*b^4*exp(2*c)*exp(2*d*x) + 48*a^4*b*exp(2*c)*exp(2*d*x) + a*b^4
*exp(4*c)*exp(4*d*x) + 16*a^4*b*exp(4*c)*exp(4*d*x) + 32*a^2*b^3*exp(2*c)*e
xp(2*d*x) + 88*a^3*b^2*exp(2*c)*exp(2*d*x) - 8*a^2*b^3*exp(4*c)*exp(4*d*x)
+ 12*a^3*b^2*exp(4*c)*exp(4*d*x)))/(2*a^3*d + 2*a*b^2*d + 4*a^2*b*d) - (2*(
a*b + a^2))/(a*(exp(2*c + 2*d*x) - 1)*(a + b)*(a*d + b*d))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\coth^3(c+dx)}{a+b\,\mathrm{sech}^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**3/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(coth(c + d*x)**3/(a + b*sech(c + d*x)**2), x)
```

3.147 $\int \frac{\coth^4(c+dx)}{a+b\mathbf{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=87

$$-\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{ad(a+b)^{5/2}}-\frac{\coth^3(c+dx)}{3d(a+b)}-\frac{(a+2b)\coth(c+dx)}{d(a+b)^2}+\frac{x}{a}$$

```
[Out] x/a-b^(5/2)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))/a/(a+b)^(5/2)/d-(a+2*b
)*coth(d*x+c)/(a+b)^2/d-1/3*coth(d*x+c)^3/(a+b)/d
```

Rubi [A] time = 0.29, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4141, 1975, 480, 583, 522, 206, 208}

$$-\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{ad(a+b)^{5/2}}-\frac{\coth^3(c+dx)}{3d(a+b)}-\frac{(a+2b)\coth(c+dx)}{d(a+b)^2}+\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^4/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] x/a - (b^(5/2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a*(a + b)^(5/
2)*d) - ((a + 2*b)*Coth[c + d*x])/((a + b)^2*d) - Coth[c + d*x]^3/(3*(a + b
)*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
```

```
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\coth^4(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^4(1-x^2)\left(a+b(1-x^2)\right)}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^4(1-x^2)(a+b-bx^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{\coth^3(c+dx)}{3(a+b)d} + \frac{\text{Subst}\left(\int \frac{3(a+2b)-3bx^2}{x^2(1-x^2)(a+b-bx^2)}\,dx, x, \tanh(c+dx)\right)}{3(a+b)d}$$

$$= -\frac{(a+2b)\coth(c+dx)}{(a+b)^2d} - \frac{\coth^3(c+dx)}{3(a+b)d} - \frac{\text{Subst}\left(\int \frac{-3(a^2+3ab+3b^2)+3b(a+2b)x^2}{(1-x^2)(a+b-bx^2)}\,dx, x, \text{ta}\right.}{3(a+b)^2d}$$

$$= -\frac{(a+2b)\coth(c+dx)}{(a+b)^2d} - \frac{\coth^3(c+dx)}{3(a+b)d} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{ad} - \frac{b^3}{}$$

$$= \frac{x}{a} - \frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a(a+b)^{5/2}d} - \frac{(a+2b)\coth(c+dx)}{(a+b)^2d} - \frac{\coth^3(c+dx)}{3(a+b)d}$$

Mathematica [B] time = 3.32, size = 380, normalized size = 4.37

$$\frac{\text{sech}^2(c+dx)(a\cosh(2(c+dx))+a+2b)\left(\frac{1}{8}\sqrt{a+b}\,\text{csch}(c)\sqrt{b(\cosh(c)-\sinh(c))^4}\,\text{csch}^3(c+dx)\left(-12a^2\sinh(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^4/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*(3*b^3*ArcTanh[(Sech[d*x]*
(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[
a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(-Cosh[2*c] + Sinh[2*c]) + (Sqrt[a +
 b]*Csch[c]*Csch[c + d*x]^3*Sqrt[b*(Cosh[c] - Sinh[c])^4]*(9*(a + b)^2*d*x*
Cosh[d*x] - 9*(a + b)^2*d*x*Cosh[2*c + d*x] - 3*a^2*d*x*Cosh[2*c + 3*d*x] -
 6*a*b*d*x*Cosh[2*c + 3*d*x] - 3*b^2*d*x*Cosh[2*c + 3*d*x] + 3*a^2*d*x*Cosh
[4*c + 3*d*x] + 6*a*b*d*x*Cosh[4*c + 3*d*x] + 3*b^2*d*x*Cosh[4*c + 3*d*x] -
 12*a^2*Sinh[d*x] - 24*a*b*Sinh[d*x] - 12*a^2*Sinh[2*c + d*x] - 18*a*b*Sinh
[2*c + d*x] + 8*a^2*Sinh[2*c + 3*d*x] + 14*a*b*Sinh[2*c + 3*d*x]))/8))/(6*a
*(a + b)^(5/2)*d*(a + b*Sech[c + d*x]^2)*Sqrt[b*(Cosh[c] - Sinh[c])^4])
```

fricas [B] time = 0.46, size = 2705, normalized size = 31.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/6*(6*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^6 + 36*(a^2 + 2*a*b + b^2)*d*
x*cosh(d*x + c)*sinh(d*x + c)^5 + 6*(a^2 + 2*a*b + b^2)*d*x*sinh(d*x + c)^6
 - 6*(3*(a^2 + 2*a*b + b^2)*d*x + 4*a^2 + 6*a*b)*cosh(d*x + c)^4 + 6*(15*(a
^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^2 - 3*(a^2 + 2*a*b + b^2)*d*x - 4*a^2 -
 6*a*b)*sinh(d*x + c)^4 + 24*(5*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^3 - (
3*(a^2 + 2*a*b + b^2)*d*x + 4*a^2 + 6*a*b)*cosh(d*x + c))*sinh(d*x + c)^3 -
 6*(a^2 + 2*a*b + b^2)*d*x + 6*(3*(a^2 + 2*a*b + b^2)*d*x + 4*a^2 + 8*a*b)*
cosh(d*x + c)^2 + 6*(15*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^4 + 3*(a^2 +
2*a*b + b^2)*d*x - 6*(3*(a^2 + 2*a*b + b^2)*d*x + 4*a^2 + 6*a*b)*cosh(d*x +
 c)^2 + 4*a^2 + 8*a*b)*sinh(d*x + c)^2 + 3*(b^2*cosh(d*x + c)^6 + 6*b^2*cos
h(d*x + c)*sinh(d*x + c)^5 + b^2*sinh(d*x + c)^6 - 3*b^2*cosh(d*x + c)^4 +
3*(5*b^2*cosh(d*x + c)^2 - b^2)*sinh(d*x + c)^4 + 3*b^2*cosh(d*x + c)^2 + 4
*(5*b^2*cosh(d*x + c)^3 - 3*b^2*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*b^2*c
osh(d*x + c)^4 - 6*b^2*cosh(d*x + c)^2 + b^2)*sinh(d*x + c)^2 - b^2 + 6*(b^
2*cosh(d*x + c)^5 - 2*b^2*cosh(d*x + c)^3 + b^2*cosh(d*x + c))*sinh(d*x + c
))*sqrt(b/(a + b))*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x
+ c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*c
osh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^
2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) + 4*((a^2 +
a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a
*b)*sinh(d*x + c)^2 + a^2 + 3*a*b + 2*b^2)*sqrt(b/(a + b)))/(a*cosh(d*x + c
)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*c
osh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*c
osh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) - 16*a^2 - 28
*a*b + 12*(3*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^5 - 2*(3*(a^2 + 2*a*b +
b^2)*d*x + 4*a^2 + 6*a*b)*cosh(d*x + c)^3 + (3*(a^2 + 2*a*b + b^2)*d*x + 4*
a^2 + 8*a*b)*cosh(d*x + c))*sinh(d*x + c))/((a^3 + 2*a^2*b + a*b^2)*d*cosh(
d*x + c)^6 + 6*(a^3 + 2*a^2*b + a*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a
^3 + 2*a^2*b + a*b^2)*d*sinh(d*x + c)^6 - 3*(a^3 + 2*a^2*b + a*b^2)*d*cosh(
d*x + c)^4 + 3*(5*(a^3 + 2*a^2*b + a*b^2)*d*cosh(d*x + c)^2 - (a^3 + 2*a^2*
b + a*b^2)*d)*sinh(d*x + c)^4 + 3*(a^3 + 2*a^2*b + a*b^2)*d*cosh(d*x + c)^2
 + 4*(5*(a^3 + 2*a^2*b + a*b^2)*d*cosh(d*x + c)^3 - 3*(a^3 + 2*a^2*b + a*b^
2)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*(a^3 + 2*a^2*b + a*b^2)*d*cosh(d
*x + c)^4 - 6*(a^3 + 2*a^2*b + a*b^2)*d*cosh(d*x + c)^2 + (a^3 + 2*a^2*b +
a*b^2)*d)*sinh(d*x + c)^2 - (a^3 + 2*a^2*b + a*b^2)*d + 6*((a^3 + 2*a^2*b +
 a*b^2)*d*cosh(d*x + c)^5 - 2*(a^3 + 2*a^2*b + a*b^2)*d*cosh(d*x + c)^3 + (
a^3 + 2*a^2*b + a*b^2)*d*cosh(d*x + c))*sinh(d*x + c)), 1/3*(3*(a^2 + 2*a*b
```

```
 + b^2)*d*x*cosh(d*x + c)^6 + 18*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)*sinh
(d*x + c)^5 + 3*(a^2 + 2*a*b + b^2)*d*x*sinh(d*x + c)^6 - 3*(3*(a^2 + 2*a*b
 + b^2)*d*x + 4*a^2 + 6*a*b)*cosh(d*x + c)^4 + 3*(15*(a^2 + 2*a*b + b^2)*d*
x*cosh(d*x + c)^2 - 3*(a^2 + 2*a*b + b^2)*d*x - 4*a^2 - 6*a*b)*sinh(d*x + c
)^4 + 12*(5*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^3 - (3*(a^2 + 2*a*b + b^2
)*d*x + 4*a^2 + 6*a*b)*cosh(d*x + c))*sinh(d*x + c)^3 - 3*(a^2 + 2*a*b + b^
2)*d*x + 3*(3*(a^2 + 2*a*b + b^2)*d*x + 4*a^2 + 8*a*b)*cosh(d*x + c)^2 + 3*
(15*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^4 + 3*(a^2 + 2*a*b + b^2)*d*x - 6
*(3*(a^2 + 2*a*b + b^2)*d*x + 4*a^2 + 6*a*b)*cosh(d*x + c)^2 + 4*a^2 + 8*a*
b)*sinh(d*x + c)^2 - 3*(b^2*cosh(d*x + c)^6 + 6*b^2*cosh(d*x + c)*sinh(d*x
+ c)^5 + b^2*sinh(d*x + c)^6 - 3*b^2*cosh(d*x + c)^4 + 3*(5*b^2*cosh(d*x +
c)^2 - b^2)*sinh(d*x + c)^4 + 3*b^2*cosh(d*x + c)^2 + 4*(5*b^2*cosh(d*x + c
)^3 - 3*b^2*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*b^2*cosh(d*x + c)^4 - 6*b
^2*cosh(d*x + c)^2 + b^2)*sinh(d*x + c)^2 - b^2 + 6*(b^2*cosh(d*x + c)^5 -
2*b^2*cosh(d*x + c)^3 + b^2*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b/(a + b))*
arctan(1/2*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*
x + c)^2 + a + 2*b)*sqrt(-b/(a + b))/b) - 8*a^2 - 14*a*b + 6*(3*(a^2 + 2*a*
b + b^2)*d*x*cosh(d*x + c)^5 - 2*(3*(a^2 + 2*a*b + b^2)*d*x + 4*a^2 + 6*a*b
)*cosh(d*x + c)^3 + (3*(a^2 + 2*a*b + b^2)*d*x + 4*a^2 + 8*a*b)*cosh(d*x +
c))*sinh(d*x + c))/((a^3 + 2*a^2*b + a*b^2)*d*cosh(d*x + c)^6 + 6*(a^3 + 2*
a^2*b + a*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a^3 + 2*a^2*b + a*b^2)*d*
sinh(d*x + c)^6 - 3*(a^3 + 2*a^2*b + a*b^2)*d*cosh(d*x + c)^4 + 3*(5*(a^3 +
 2*a^2*b + a*b^2)*d*cosh(d*x + c)^2 - (a^3 + 2*a^2*b + a*b^2)*d)*sinh(d*x +
 c)^4 + 3*(a^3 + 2*a^2*b + a*b^2)*d*cosh(d*x + c)^2 + 4*(5*(a^3 + 2*a^2*b +
 a*b^2)*d*cosh(d*x + c)^3 - 3*(a^3 + 2*a^2*b + a*b^2)*d*cosh(d*x + c))*sinh
(d*x + c)^3 + 3*(5*(a^3 + 2*a^2*b + a*b^2)*d*cosh(d*x + c)^4 - 6*(a^3 + 2*a
^2*b + a*b^2)*d*cosh(d*x + c)^2 + (a^3 + 2*a^2*b + a*b^2)*d)*sinh(d*x + c)^
2 - (a^3 + 2*a^2*b + a*b^2)*d + 6*((a^3 + 2*a^2*b + a*b^2)*d*cosh(d*x + c)^
5 - 2*(a^3 + 2*a^2*b + a*b^2)*d*cosh(d*x + c)^3 + (a^3 + 2*a^2*b + a*b^2)*d
*cosh(d*x + c))*sinh(d*x + c))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error index.cc index_gcd Error: Bad Argument
 ValueError index.cc index_gcd Error: Bad Argument ValueError index.cc inde
x_gcd Error: Bad Argument ValueError index.cc index_gcd Error: Bad Argument
 ValueError index.cc index_gcd Error: Bad Argument ValueError index.cc inde
x_gcd Error: Bad Argument ValueEvaluation time: 0.68Done
```

maple [B] time = 0.49, size = 298, normalized size = 3.43

$$-\frac{a\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d\,(a+b)^2}-\frac{\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b}{24d\,(a+b)^2}-\frac{5a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,(a+b)^2}-\frac{9\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)b}{8d\,(a+b)^2}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{da}+\frac{\ln\left(\tan\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^4/(a+b*sech(d*x+c)^2),x)
```

```
[Out] -1/24/d/(a+b)^2*a*tanh(1/2*d*x+1/2*c)^3-1/24/d/(a+b)^2*tanh(1/2*d*x+1/2*c)^
3*b-5/8/d/(a+b)^2*a*tanh(1/2*d*x+1/2*c)-9/8/d/(a+b)^2*tanh(1/2*d*x+1/2*c)*b
-1/d/a*ln(tanh(1/2*d*x+1/2*c)-1)+1/d/a*ln(tanh(1/2*d*x+1/2*c)+1)+1/2/d*b^(5
/2)/a/(a+b)^(5/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*b^(1/2)*tanh(1/2*d
*x+1/2*c)+(a+b)^(1/2))-1/2/d*b^(5/2)/a/(a+b)^(5/2)*ln((a+b)^(1/2)*tanh(1/2*
d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))-1/24/d/(a+b)/tanh(1
/2*d*x+1/2*c)^3-5/8/d/(a+b)^2/tanh(1/2*d*x+1/2*c)*a-9/8/d/(a+b)^2/tanh(1/2*
d*x+1/2*c)*b
```

maxima [B] time = 0.63, size = 1435, normalized size = 16.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 3/16*a*b*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*
x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^2 + 2*a*b + b^2)*sqrt((a + b)*
b)*d) + 1/8*(a*b + 2*b^2)*log(a*e^(4*d*x + 4*c) + 2*(a + 2*b)*e^(2*d*x + 2*
c) + a)/((a^3 + 2*a^2*b + a*b^2)*d) - 1/4*b*log(a*e^(4*d*x + 4*c) + 2*(a +
2*b)*e^(2*d*x + 2*c) + a)/((a^2 + 2*a*b + b^2)*d) - 1/8*(a*b + 2*b^2)*log(2
*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/((a^3 + 2*a^2*b + a*b
^2)*d) + 1/4*b*log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/(
(a^2 + 2*a*b + b^2)*d) + 1/4*(2*a + 3*b)*log(e^(2*d*x + 2*c) - 1)/((a^2 + 2
*a*b + b^2)*d) + 1/2*b*log(e^(2*d*x + 2*c) - 1)/((a^2 + 2*a*b + b^2)*d) - 1
/4*(2*a + 3*b)*log(e^(-2*d*x - 2*c) - 1)/((a^2 + 2*a*b + b^2)*d) - 1/2*b*lo
g(e^(-2*d*x - 2*c) - 1)/((a^2 + 2*a*b + b^2)*d) - 1/32*(a^2*b + 8*a*b^2 + 8
*b^3)*log((a*e^(2*d*x + 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(2*d*x + 2
*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^3 + 2*a^2*b + a*b^2)*sqrt((a + b)*b
)*d) + 1/8*(a*b + 2*b^2)*log((a*e^(2*d*x + 2*c) + a + 2*b - 2*sqrt((a + b)*
b))/(a*e^(2*d*x + 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^2 + 2*a*b + b^2)
*sqrt((a + b)*b)*d) + 1/32*(a^2*b + 8*a*b^2 + 8*b^3)*log((a*e^(-2*d*x - 2*c
) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a
+ b)*b)))/((a^3 + 2*a^2*b + a*b^2)*sqrt((a + b)*b)*d) - 1/8*(a*b + 2*b^2)*l
```

```
og((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) +
 a + 2*b + 2*sqrt((a + b)*b)))/((a^2 + 2*a*b + b^2)*sqrt((a + b)*b)*d) + 1/
24*(3*(12*a + 13*b)*e^(4*d*x + 4*c) - 6*(9*a + 10*b)*e^(2*d*x + 2*c) + 22*a
 + 25*b)/((a^2 + 2*a*b + b^2 - (a^2 + 2*a*b + b^2)*e^(6*d*x + 6*c) + 3*(a^2
 + 2*a*b + b^2)*e^(4*d*x + 4*c) - 3*(a^2 + 2*a*b + b^2)*e^(2*d*x + 2*c))*d)
 + 1/6*(3*(4*a + 5*b)*e^(4*d*x + 4*c) - 6*(2*a + 3*b)*e^(2*d*x + 2*c) + 4*a
 + 7*b)/((a^2 + 2*a*b + b^2 - (a^2 + 2*a*b + b^2)*e^(6*d*x + 6*c) + 3*(a^2
+ 2*a*b + b^2)*e^(4*d*x + 4*c) - 3*(a^2 + 2*a*b + b^2)*e^(2*d*x + 2*c))*d)
+ 1/24*(6*(9*a + 10*b)*e^(-2*d*x - 2*c) - 3*(12*a + 13*b)*e^(-4*d*x - 4*c)
- 22*a - 25*b)/((a^2 + 2*a*b + b^2 - 3*(a^2 + 2*a*b + b^2)*e^(-2*d*x - 2*c)
 + 3*(a^2 + 2*a*b + b^2)*e^(-4*d*x - 4*c) - (a^2 + 2*a*b + b^2)*e^(-6*d*x -
 6*c))*d) + 1/6*(6*(2*a + 3*b)*e^(-2*d*x - 2*c) - 3*(4*a + 5*b)*e^(-4*d*x -
 4*c) - 4*a - 7*b)/((a^2 + 2*a*b + b^2 - 3*(a^2 + 2*a*b + b^2)*e^(-2*d*x -
2*c) + 3*(a^2 + 2*a*b + b^2)*e^(-4*d*x - 4*c) - (a^2 + 2*a*b + b^2)*e^(-6*d
*x - 6*c))*d) - 1/4*(6*a*e^(-2*d*x - 2*c) + 3*b*e^(-4*d*x - 4*c) - 2*a + b)
/((a^2 + 2*a*b + b^2 - 3*(a^2 + 2*a*b + b^2)*e^(-2*d*x - 2*c) + 3*(a^2 + 2*
a*b + b^2)*e^(-4*d*x - 4*c) - (a^2 + 2*a*b + b^2)*e^(-6*d*x - 6*c))*d)
```

mupad [B] time = 4.15, size = 779, normalized size = 8.95

$$\frac{x}{a}-\frac{8}{3\,\left(a\,d+b\,d\right)\,\left(3\,{\mathrm{e}}^{2\,c+2\,d\,x}-3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}-1\right)}-\frac{\mathrm{atan}\left(\left(\left({\mathrm{e}}^{2\,c}\,{\mathrm{e}}^{2\,d\,x}\,\left(\frac{4\,b^3}{a^3\,d\,{\left(a+b\right)}^2\,\sqrt{b^5}\,\left(a^3+2\,a^2\,b+a\,b^2\right)}+\frac{}{a^2\,b^3\,\left(a^3+2\,a\right.}\right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^4/(a + b/cosh(c + d*x)^2),x)
```

```
[Out] x/a - 8/(3*(a*d + b*d)*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c +
 6*d*x) - 1)) - (atan((exp(2*c)*exp(2*d*x)*((4*b^3)/(a^3*d*(a + b)^2*(b^5)^
(1/2)*(a*b^2 + 2*a^2*b + a^3)) + ((a + 2*b)*(a^4*d*(b^5)^(1/2) + 2*a*b^3*d*
(b^5)^(1/2) + 4*a^3*b*d*(b^5)^(1/2) + 5*a^2*b^2*d*(b^5)^(1/2)))/(a^2*b^3*(a
*b^2 + 2*a^2*b + a^3)*(-a^2*d^2*(a + b)^5)^(1/2)*(- a^7*d^2 - 5*a^6*b*d^2 -
 a^2*b^5*d^2 - 5*a^3*b^4*d^2 - 10*a^4*b^3*d^2 - 10*a^5*b^2*d^2)^(1/2))) + (
(a + 2*b)*(a^4*d*(b^5)^(1/2) + 2*a^3*b*d*(b^5)^(1/2) + a^2*b^2*d*(b^5)^(1/2
)))/(a^2*b^3*(a*b^2 + 2*a^2*b + a^3)*(-a^2*d^2*(a + b)^5)^(1/2)*(- a^7*d^2
- 5*a^6*b*d^2 - a^2*b^5*d^2 - 5*a^3*b^4*d^2 - 10*a^4*b^3*d^2 - 10*a^5*b^2*d
^2)^(1/2)))*((a^4*(- a^7*d^2 - 5*a^6*b*d^2 - a^2*b^5*d^2 - 5*a^3*b^4*d^2 -
10*a^4*b^3*d^2 - 10*a^5*b^2*d^2)^(1/2))/2 + (a^2*b^2*(- a^7*d^2 - 5*a^6*b*d
^2 - a^2*b^5*d^2 - 5*a^3*b^4*d^2 - 10*a^4*b^3*d^2 - 10*a^5*b^2*d^2)^(1/2))/
2 + a^3*b*(- a^7*d^2 - 5*a^6*b*d^2 - a^2*b^5*d^2 - 5*a^3*b^4*d^2 - 10*a^4*b
^3*d^2 - 10*a^5*b^2*d^2)^(1/2)))*(b^5)^(1/2))/(- a^7*d^2 - 5*a^6*b*d^2 - a^
2*b^5*d^2 - 5*a^3*b^4*d^2 - 10*a^4*b^3*d^2 - 10*a^5*b^2*d^2)^(1/2) - (4*(a*
b + a^2))/(a*(a + b)*(a*d + b*d)*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1
)) - (2*(3*a*b + 2*a^2))/(a*(exp(2*c + 2*d*x) - 1)*(a + b)*(a*d + b*d))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^4(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**4/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(coth(c + d*x)**4/(a + b*sech(c + d*x)**2), x)
```

3.148 $\int \frac{\tanh^5(c+dx)}{\left(a+b\,\mathbf{sech}^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=76

$$\frac{\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\log\left(a\cosh^2(c+dx)+b\right)}{2d}+\frac{(a+b)^2}{2a^2bd\left(a\cosh^2(c+dx)+b\right)}+\frac{\log(\cosh(c+dx))}{b^2d}$$

```
[Out] 1/2*(a+b)^2/a^2/b/d/(b+a*cosh(d*x+c)^2)+ln(cosh(d*x+c))/b^2/d+1/2*(1/a^2-1/
b^2)*ln(b+a*cosh(d*x+c)^2)/d
```

Rubi [A] time = 0.12, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4138, 446, 88}

$$\frac{\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\log\left(a\cosh^2(c+dx)+b\right)}{2d}+\frac{(a+b)^2}{2a^2bd\left(a\cosh^2(c+dx)+b\right)}+\frac{\log(\cosh(c+dx))}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^5/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (a + b)^2/(2*a^2*b*d*(b + a*Cosh[c + d*x]^2)) + Log[Cosh[c + d*x]]/(b^2*d)
+ ((a^(-2) - b^(-2))*Log[b + a*Cosh[c + d*x]^2])/(2*d)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
```

```
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\tanh^5(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{x\left(b+ax^2\right)^2}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{(1-x)^2}{x(b+ax)^2}\,dx, x, \cosh^2(c+dx)\right)}{2d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{b^2x} - \frac{(a+b)^2}{ab(b+ax)^2} + \frac{-a^2+b^2}{ab^2(b+ax)}\right)dx, x, \cosh^2(c+dx)\right)}{2d}$$

$$= \frac{(a+b)^2}{2a^2bd\left(b+a\cosh^2(c+dx)\right)} + \frac{\log(\cosh(c+dx))}{b^2d} + \frac{\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\log\left(b+a\cosh^2(c\right.}{2d}$$

Mathematica [A] time = 0.45, size = 109, normalized size = 1.43

$$\frac{\text{sech}^4(c+dx)(a\cosh(2c+2dx)+a+2b)^2\left(\left(\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\log\left(a\cosh^2(c+dx)+b\right)+\frac{(a+b)^2}{a^2b\left(a\cosh^2(c+dx)+b\right)}+\frac{2\log(\cosh(c+}{b^2}\right.\right.}{8d\left(a+b\text{sech}^2(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^5/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*c + 2*d*x])^2*((a + b)^2/(a^2*b*(b + a*Cosh[c + d*x]^2
)) + (2*Log[Cosh[c + d*x]])/b^2 + (a^(-2) - b^(-2))*Log[b + a*Cosh[c + d*x]
^2])*Sech[c + d*x]^4)/(8*d*(a + b*Sech[c + d*x]^2)^2)
```

fricas [B] time = 0.50, size = 853, normalized size = 11.22

$$-\frac{2ab^2dx\cosh(dx+c)^4 + 8ab^2dx\cosh(dx+c)\sinh(dx+c)^3 + 2ab^2dx\sinh(dx+c)^4 + 2ab^2dx - 4\left(a^2b+2ab\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*b^2*d*x*cosh(d*x + c)^4 + 8*a*b^2*d*x*cosh(d*x + c)*sinh(d*x + c)
^3 + 2*a*b^2*d*x*sinh(d*x + c)^4 + 2*a*b^2*d*x - 4*(a^2*b + 2*a*b^2 + b^3 -
 (a*b^2 + 2*b^3)*d*x)*cosh(d*x + c)^2 + 4*(3*a*b^2*d*x*cosh(d*x + c)^2 - a^
2*b - 2*a*b^2 - b^3 + (a*b^2 + 2*b^3)*d*x)*sinh(d*x + c)^2 + ((a^3 - a*b^2)
*cosh(d*x + c)^4 + 4*(a^3 - a*b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3 - a
*b^2)*sinh(d*x + c)^4 + a^3 - a*b^2 + 2*(a^3 + 2*a^2*b - a*b^2 - 2*b^3)*cos
h(d*x + c)^2 + 2*(a^3 + 2*a^2*b - a*b^2 - 2*b^3 + 3*(a^3 - a*b^2)*cosh(d*x
+ c)^2)*sinh(d*x + c)^2 + 4*((a^3 - a*b^2)*cosh(d*x + c)^3 + (a^3 + 2*a^2*b
 - a*b^2 - 2*b^3)*cosh(d*x + c))*sinh(d*x + c))*log(2*(a*cosh(d*x + c)^2 +
a*sinh(d*x + c)^2 + a + 2*b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x +
c) + sinh(d*x + c)^2)) - 2*(a^3*cosh(d*x + c)^4 + 4*a^3*cosh(d*x + c)*sinh(
d*x + c)^3 + a^3*sinh(d*x + c)^4 + a^3 + 2*(a^3 + 2*a^2*b)*cosh(d*x + c)^2
+ 2*(3*a^3*cosh(d*x + c)^2 + a^3 + 2*a^2*b)*sinh(d*x + c)^2 + 4*(a^3*cosh(d
*x + c)^3 + (a^3 + 2*a^2*b)*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x +
c)/(cosh(d*x + c) - sinh(d*x + c))) + 8*(a*b^2*d*x*cosh(d*x + c)^3 - (a^2*b
 + 2*a*b^2 + b^3 - (a*b^2 + 2*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c))/(a^3*
b^2*d*cosh(d*x + c)^4 + 4*a^3*b^2*d*cosh(d*x + c)*sinh(d*x + c)^3 + a^3*b^2
*d*sinh(d*x + c)^4 + a^3*b^2*d + 2*(a^3*b^2 + 2*a^2*b^3)*d*cosh(d*x + c)^2
+ 2*(3*a^3*b^2*d*cosh(d*x + c)^2 + (a^3*b^2 + 2*a^2*b^3)*d)*sinh(d*x + c)^2
 + 4*(a^3*b^2*d*cosh(d*x + c)^3 + (a^3*b^2 + 2*a^2*b^3)*d*cosh(d*x + c))*si
nh(d*x + c))
```

giac [B] time = 1.38, size = 208, normalized size = 2.74

$$-\frac{\frac{2\,dx}{a^2} - \frac{2\,\log\left(e^{(2\,dx+2\,c)}+1\right)}{b^2} + \frac{\left(a^2-b^2\right)\log\left(ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+a\right)}{a^2b^2} - \frac{a^2e^{(4\,dx+4\,c)}-b^2e^{(4\,dx+4\,c)}+2\,a^2e^{(2\,dx+2\,c)}+8\,abe^{(2\,dx+2\,c)}+\cdots}{\left(ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+\cdots\right.}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*d*x/a^2 - 2*log(e^(2*d*x + 2*c) + 1)/b^2 + (a^2 - b^2)*log(a*e^(4*d
*x + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) + a)/(a^2*b^2) - (a^2
*e^(4*d*x + 4*c) - b^2*e^(4*d*x + 4*c) + 2*a^2*e^(2*d*x + 2*c) + 8*a*b*e^(2
*d*x + 2*c) + 6*b^2*e^(2*d*x + 2*c) + a^2 - b^2)/((a*e^(4*d*x + 4*c) + 2*a*
e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) + a)*a*b^2))/d
```

maple [B] time = 0.37, size = 351, normalized size = 4.62

$$-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d\,a^2}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d\,a^2}-\frac{2\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cdots}{db\left(\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\left(\tanh^2\left(\frac{dx}{2}\cdots\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^5/(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] -1/d/a^2*ln(tanh(1/2*d*x+1/2*c)-1)-1/d/a^2*ln(tanh(1/2*d*x+1/2*c)+1)-2/d/b*
tanh(1/2*d*x+1/2*c)^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*ta
nh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)-1/2/d/b^2*ln(tanh(1/2*
d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2
*d*x+1/2*c)^2*b+a+b)-2/d/a*tanh(1/2*d*x+1/2*c)^2/(tanh(1/2*d*x+1/2*c)^4*a+b
*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+
a+b)+1/2/d/a^2*ln(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/
2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)+1/d/b^2*ln(tanh(1/2*d*x+1/2
*c)^2+1)
```

maxima [B] time = 0.45, size = 154, normalized size = 2.03

$$\frac{2\left(a^2+2\,ab+b^2\right)e^{(-2\,dx-2\,c)}}{\left(a^3be^{(-4\,dx-4\,c)}+a^3b+2\left(a^3b+2\,a^2b^2\right)e^{(-2\,dx-2\,c)}\right)d}+\frac{dx+c}{a^2d}+\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{b^2d}-\frac{\left(a^2-b^2\right)\log\left(2\,(a+2\,b)e^{(-2\,dx\cdots}\right.}{2\,a^2b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 2*(a^2 + 2*a*b + b^2)*e^(-2*d*x - 2*c)/((a^3*b*e^(-4*d*x - 4*c) + a^3*b + 2
*(a^3*b + 2*a^2*b^2)*e^(-2*d*x - 2*c))*d) + (d*x + c)/(a^2*d) + log(e^(-2*d
*x - 2*c) + 1)/(b^2*d) - 1/2*(a^2 - b^2)*log(2*(a + 2*b)*e^(-2*d*x - 2*c) +
 a*e^(-4*d*x - 4*c) + a)/(a^2*b^2*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh\left(c+d\,x\right)^4\,\tanh\left(c+d\,x\right)^5}{{\left(a\,\cosh\left(c+d\,x\right)^2+b\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^5/(a + b/cosh(c + d*x)^2)^2,x)
```

```
[Out] int((cosh(c + d*x)^4*tanh(c + d*x)^5)/(b + a*cosh(c + d*x)^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^{5}{\left(c+dx\right)}}{\left(a+b\operatorname{sech}^{2}{\left(c+dx\right)}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**5/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral(tanh(c + d*x)**5/(a + b*sech(c + d*x)**2)**2, x)
```

3.149 $\int \frac{\tanh^4(c+dx)}{\left(a+b\,\mathbf{sech}^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=91

$$\frac{(a-2b)\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^2b^{3/2}d}+\frac{x}{a^2}-\frac{(a+b)\tanh(c+dx)}{2abd\left(a-b\tanh^2(c+dx)+b\right)}$$

```
[Out] x/a^2+1/2*(a-2*b)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))*(a+b)^(1/2)/a^2/
b^(3/2)/d-1/2*(a+b)*tanh(d*x+c)/a/b/d/(a+b-b*tanh(d*x+c)^2)
```

Rubi [A] time = 0.19, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4141, 1975, 470, 522, 206, 208}

$$\frac{(a-2b)\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^2b^{3/2}d}+\frac{x}{a^2}-\frac{(a+b)\tanh(c+dx)}{2abd\left(a-b\tanh^2(c+dx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^4/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] x/a^2 + ((a - 2*b)*Sqrt[a + b]*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]]
)/(2*a^2*b^(3/2)*d) - ((a + b)*Tanh[c + d*x])/(2*a*b*d*(a + b - b*Tanh[c +
d*x]^2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
```

```
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\tanh^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)\left(a+b(1-x^2)\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)\left(a+b-bx^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{(a+b)\tanh(c+dx)}{2abd\left(a+b-b\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{a+b+(-a+b)x^2}{(1-x^2)\left(a+b-bx^2\right)}\,dx, x, \tanh(c+dx)\right)}{2abd}$$

$$= -\frac{(a+b)\tanh(c+dx)}{2abd\left(a+b-b\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{a^2d} + \frac{((a-2b}{}$$

$$= \frac{x}{a^2} + \frac{(a-2b)\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^2b^{3/2}d} - \frac{(a+b)\tanh(c+dx)}{2abd\left(a+b-b\tanh^2(c+dx)\right)}$$

Mathematica [B] time = 2.13, size = 228, normalized size = 2.51

$$\frac{\operatorname{sech}^4(c+dx)(a\cosh(2(c+dx))+a+2b)\left(\frac{\left(a^2-ab-2b^2\right)(\cosh(2c)-\sinh(2c))(a\cosh(2(c+dx))+a+2b)\tanh^{-1}\left(\frac{(\cosh(2c)-\sinh(2c))\operatorname{sech}(dx)}{2\sqrt{a+b}\sqrt{b(c}}\right.}{bd\sqrt{a+b}\sqrt{b(\cosh(c)-\sinh(c))^4}}\right.}{8a^2\left(a+b\operatorname{sech}^2(c+dx\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^4/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^4*(2*x*(a + 2*b + a*Cosh[2*(
c + d*x)]) + ((a^2 - a*b - 2*b^2)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c]
)*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c]
 - Sinh[c])^4])]*(a + 2*b + a*Cosh[2*(c + d*x)])*(Cosh[2*c] - Sinh[2*c]))/(
b*Sqrt[a + b]*d*Sqrt[b*(Cosh[c] - Sinh[c])^4]) + ((a + b)*Sech[2*c]*((a + 2
*b)*Sinh[2*c] - a*Sinh[2*d*x]))/(b*d)))/(8*a^2*(a + b*Sech[c + d*x]^2)^2)
```

fricas [B] time = 0.45, size = 1479, normalized size = 16.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*a*b*d*x*cosh(d*x + c)^4 + 16*a*b*d*x*cosh(d*x + c)*sinh(d*x + c)^3
+ 4*a*b*d*x*sinh(d*x + c)^4 + 4*a*b*d*x + 4*(2*(a*b + 2*b^2)*d*x + a^2 + 3*
a*b + 2*b^2)*cosh(d*x + c)^2 + 4*(6*a*b*d*x*cosh(d*x + c)^2 + 2*(a*b + 2*b^
2)*d*x + a^2 + 3*a*b + 2*b^2)*sinh(d*x + c)^2 - ((a^2 - 2*a*b)*cosh(d*x + c
)^4 + 4*(a^2 - 2*a*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 - 2*a*b)*sinh(d*
x + c)^4 + 2*(a^2 - 4*b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 - 2*a*b)*cosh(d*x +
c)^2 + a^2 - 4*b^2)*sinh(d*x + c)^2 + a^2 - 2*a*b + 4*((a^2 - 2*a*b)*cosh(d
*x + c)^3 + (a^2 - 4*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a + b)/b)*log
((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x
+ c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 +
 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a
^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) + 4*(a*b*cosh(d*x + c)^2 + 2*a*b*c
osh(d*x + c)*sinh(d*x + c) + a*b*sinh(d*x + c)^2 + a*b + 2*b^2)*sqrt((a + b
)/b))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x +
 c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sin
h(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c
) + a)) + 4*a^2 + 4*a*b + 8*(2*a*b*d*x*cosh(d*x + c)^3 + (2*(a*b + 2*b^2)*d
*x + a^2 + 3*a*b + 2*b^2)*cosh(d*x + c))*sinh(d*x + c))/(a^3*b*d*cosh(d*x +
 c)^4 + 4*a^3*b*d*cosh(d*x + c)*sinh(d*x + c)^3 + a^3*b*d*sinh(d*x + c)^4 +
 a^3*b*d + 2*(a^3*b + 2*a^2*b^2)*d*cosh(d*x + c)^2 + 2*(3*a^3*b*d*cosh(d*x
+ c)^2 + (a^3*b + 2*a^2*b^2)*d)*sinh(d*x + c)^2 + 4*(a^3*b*d*cosh(d*x + c)^
3 + (a^3*b + 2*a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c)), 1/2*(2*a*b*d*x*cos
h(d*x + c)^4 + 8*a*b*d*x*cosh(d*x + c)*sinh(d*x + c)^3 + 2*a*b*d*x*sinh(d*x
 + c)^4 + 2*a*b*d*x + 2*(2*(a*b + 2*b^2)*d*x + a^2 + 3*a*b + 2*b^2)*cosh(d*
x + c)^2 + 2*(6*a*b*d*x*cosh(d*x + c)^2 + 2*(a*b + 2*b^2)*d*x + a^2 + 3*a*b
 + 2*b^2)*sinh(d*x + c)^2 + ((a^2 - 2*a*b)*cosh(d*x + c)^4 + 4*(a^2 - 2*a*b
)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 - 2*a*b)*sinh(d*x + c)^4 + 2*(a^2 -
4*b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 - 2*a*b)*cosh(d*x + c)^2 + a^2 - 4*b^2)*
sinh(d*x + c)^2 + a^2 - 2*a*b + 4*((a^2 - 2*a*b)*cosh(d*x + c)^3 + (a^2 - 4
*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-(a + b)/b)*arctan(1/2*(a*cosh(d*x
 + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sq
rt(-(a + b)/b)/(a + b)) + 2*a^2 + 2*a*b + 4*(2*a*b*d*x*cosh(d*x + c)^3 + (2
*(a*b + 2*b^2)*d*x + a^2 + 3*a*b + 2*b^2)*cosh(d*x + c))*sinh(d*x + c))/(a^
3*b*d*cosh(d*x + c)^4 + 4*a^3*b*d*cosh(d*x + c)*sinh(d*x + c)^3 + a^3*b*d*s
inh(d*x + c)^4 + a^3*b*d + 2*(a^3*b + 2*a^2*b^2)*d*cosh(d*x + c)^2 + 2*(3*a
^3*b*d*cosh(d*x + c)^2 + (a^3*b + 2*a^2*b^2)*d)*sinh(d*x + c)^2 + 4*(a^3*b*
d*cosh(d*x + c)^3 + (a^3*b + 2*a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c))]
```

giac [B] time = 1.16, size = 187, normalized size = 2.05

$$\frac{\frac{2\,dx}{a^2}+\frac{\left(a^2e^{(2\,c)}-abe^{(2\,c)}-2\,b^2e^{(2\,c)}\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)e^{(-2\,c)}}{\sqrt{-ab-b^2}\,a^2b}+\frac{2\left(a^2e^{(2\,dx+2\,c)}+3\,abe^{(2\,dx+2\,c)}+2\,b^2e^{(2\,dx+2\,c)}+a^2+ab\right)}{\left(ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+a\right)a^2b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*d*x/a^2 + (a^2*e^(2*c) - a*b*e^(2*c) - 2*b^2*e^(2*c))*arctan(1/2*(a*
e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))*e^(-2*c)/(sqrt(-a*b - b^2)*a^2
*b) + 2*(a^2*e^(2*d*x + 2*c) + 3*a*b*e^(2*d*x + 2*c) + 2*b^2*e^(2*d*x + 2*c
) + a^2 + a*b)/((a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2
*c) + a)*a^2*b))/d
```

maple [B] time = 0.40, size = 676, normalized size = 7.43

$$-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d\,a^2}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d\,a^2}-\frac{\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\left(\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\left(\tanh^2\left(\frac{dx}{2}+\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] -1/d/a^2*ln(tanh(1/2*d*x+1/2*c)-1)+1/d/a^2*ln(tanh(1/2*d*x+1/2*c)+1)-1/d/(t
anh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*
tanh(1/2*d*x+1/2*c)^2*b+a+b)/b*tanh(1/2*d*x+1/2*c)^3-1/d/a/(tanh(1/2*d*x+1/
2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1
/2*c)^2*b+a+b)*tanh(1/2*d*x+1/2*c)^3-1/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/
2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)/b*t
anh(1/2*d*x+1/2*c)-1/d/a/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2
*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)*tanh(1/2*d*x+1/2*c)
-1/4/d/b^(3/2)/(a+b)^(1/2)*ln(-(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*
tanh(1/2*d*x+1/2*c)-(a+b)^(1/2))+1/4/d/a/b^(1/2)/(a+b)^(1/2)*ln(-(a+b)^(1/2
)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)-(a+b)^(1/2))+1/2/d/a^
2*b^(1/2)/(a+b)^(1/2)*ln(-(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(
1/2*d*x+1/2*c)-(a+b)^(1/2))+1/4/d/b^(3/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1
/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))-1/4/d/a/b^(1/2)/
(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2
*c)+(a+b)^(1/2))-1/2/d/a^2*b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+
1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))
```

maxima [B] time = 0.84, size = 1053, normalized size = 11.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/64*(a^3 - 6*a^2*b - 24*a*b^2 - 16*b^3)*log((a*e^(2*d*x + 2*c) + a + 2*b -
 2*sqrt((a + b)*b))/(a*e^(2*d*x + 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^
3*b + a^2*b^2)*sqrt((a + b)*b)*d) + 1/16*a*log((a*e^(2*d*x + 2*c) + a + 2*b
 - 2*sqrt((a + b)*b))/(a*e^(2*d*x + 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(s
qrt((a + b)*b)*(a*b + b^2)*d) - 1/64*(a^3 - 6*a^2*b - 24*a*b^2 - 16*b^3)*lo
g((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) +
a + 2*b + 2*sqrt((a + b)*b)))/((a^3*b + a^2*b^2)*sqrt((a + b)*b)*d) - 3/32*
(a + 2*b)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d
*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*(a*b + b^2)*d) -
 1/16*a*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x
 - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*(a*b + b^2)*d) + 1
/16*(a^3 + 8*a^2*b + 8*a*b^2 + (a^3 + 18*a^2*b + 48*a*b^2 + 32*b^3)*e^(2*d*
x + 2*c))/((a^4*b + a^3*b^2 + (a^4*b + a^3*b^2)*e^(4*d*x + 4*c) + 2*(a^4*b
+ 3*a^3*b^2 + 2*a^2*b^3)*e^(2*d*x + 2*c))*d) - 1/16*(a^3 + 8*a^2*b + 8*a*b^
2 + (a^3 + 18*a^2*b + 48*a*b^2 + 32*b^3)*e^(-2*d*x - 2*c))/((a^4*b + a^3*b^
2 + 2*(a^4*b + 3*a^3*b^2 + 2*a^2*b^3)*e^(-2*d*x - 2*c) + (a^4*b + a^3*b^2)*
e^(-4*d*x - 4*c))*d) + 1/4*(a^2 + 2*a*b + (a^2 + 8*a*b + 8*b^2)*e^(2*d*x +
2*c))/((a^3*b + a^2*b^2 + (a^3*b + a^2*b^2)*e^(4*d*x + 4*c) + 2*(a^3*b + 3*
a^2*b^2 + 2*a*b^3)*e^(2*d*x + 2*c))*d) - 1/4*(a^2 + 2*a*b + (a^2 + 8*a*b +
8*b^2)*e^(-2*d*x - 2*c))/((a^3*b + a^2*b^2 + 2*(a^3*b + 3*a^2*b^2 + 2*a*b^3
)*e^(-2*d*x - 2*c) + (a^3*b + a^2*b^2)*e^(-4*d*x - 4*c))*d) - 3/8*((a + 2*b
)*e^(-2*d*x - 2*c) + a)/((a^2*b + a*b^2 + 2*(a^2*b + 3*a*b^2 + 2*b^3)*e^(-2
*d*x - 2*c) + (a^2*b + a*b^2)*e^(-4*d*x - 4*c))*d) + 1/4*log(a*e^(4*d*x + 4
*c) + 2*(a + 2*b)*e^(2*d*x + 2*c) + a)/(a^2*d) - 1/4*log(2*(a + 2*b)*e^(-2*
d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/(a^2*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\cosh\left(c+d\,x\right)^2-1\right)^2}{\left(a\cosh\left(c+d\,x\right)^2+b\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^4/(a + b/cosh(c + d*x)^2)^2,x)
```

```
[Out] int((cosh(c + d*x)^2 - 1)^2/(b + a*cosh(c + d*x)^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^4\left(c+dx\right)}{\left(a+b\operatorname{sech}^2\left(c+dx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**4/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral(tanh(c + d*x)**4/(a + b*sech(c + d*x)**2)**2, x)
```

3.150 $\int \frac{\tanh^3(c+dx)}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=51

$$\frac{a+b}{2a^2d\left(a\cosh^2(c+dx)+b\right)}+\frac{\log\left(a\cosh^2(c+dx)+b\right)}{2a^2d}$$

```
[Out] 1/2*(a+b)/a^2/d/(b+a*cosh(d*x+c)^2)+1/2*ln(b+a*cosh(d*x+c)^2)/a^2/d
```

Rubi [A] time = 0.09, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4138, 444, 43}

$$\frac{a+b}{2a^2d\left(a\cosh^2(c+dx)+b\right)}+\frac{\log\left(a\cosh^2(c+dx)+b\right)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^3/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (a + b)/(2*a^2*d*(b + a*Cosh[c + d*x]^2)) + Log[b + a*Cosh[c + d*x]^2]/(2*a
^2*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
```

```
x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\tanh^3(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{x(1-x^2)}{(b+ax^2)^2}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \frac{1-x}{(b+ax)^2}\,dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{a+b}{a(b+ax)^2}-\frac{1}{a(b+ax)}\right)dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= \frac{a+b}{2a^2d\left(b+a\cosh^2(c+dx)\right)}+\frac{\log\left(b+a\cosh^2(c+dx)\right)}{2a^2d}$$

Mathematica [A] time = 0.76, size = 81, normalized size = 1.59

$$\frac{(a+2b)\log(a\cosh(2(c+dx))+a+2b)+a\cosh(2(c+dx))\log(a\cosh(2(c+dx))+a+2b)+2(a+b)}{2a^2d(a\cosh(2(c+dx))+a+2b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^3/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (2*(a + b) + (a + 2*b)*Log[a + 2*b + a*Cosh[2*(c + d*x)]] + a*Cosh[2*(c + d
*x)]*Log[a + 2*b + a*Cosh[2*(c + d*x)]])/(2*a^2*d*(a + 2*b + a*Cosh[2*(c +
d*x)]))
```

fricas [B] time = 0.42, size = 485, normalized size = 9.51

$$-\frac{2\,adx\cosh(dx+c)^4+8\,adx\cosh(dx+c)\sinh(dx+c)^3+2\,adx\sinh(dx+c)^4+2\,adx+4\,((a+2\,b)dx-a-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*d*x*cosh(d*x + c)^4 + 8*a*d*x*cosh(d*x + c)*sinh(d*x + c)^3 + 2*a
*d*x*sinh(d*x + c)^4 + 2*a*d*x + 4*((a + 2*b)*d*x - a - b)*cosh(d*x + c)^2
```

```
+ 4*(3*a*d*x*cosh(d*x + c)^2 + (a + 2*b)*d*x - a - b)*sinh(d*x + c)^2 - (a*
cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2
*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c
)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)*lo
g(2*(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + a + 2*b)/(cosh(d*x + c)^2 - 2*
cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) + 8*(a*d*x*cosh(d*x + c)^3
+ ((a + 2*b)*d*x - a - b)*cosh(d*x + c))*sinh(d*x + c))/(a^3*d*cosh(d*x + c
)^4 + 4*a^3*d*cosh(d*x + c)*sinh(d*x + c)^3 + a^3*d*sinh(d*x + c)^4 + a^3*d
 + 2*(a^3 + 2*a^2*b)*d*cosh(d*x + c)^2 + 2*(3*a^3*d*cosh(d*x + c)^2 + (a^3
+ 2*a^2*b)*d)*sinh(d*x + c)^2 + 4*(a^3*d*cosh(d*x + c)^3 + (a^3 + 2*a^2*b)*
d*cosh(d*x + c))*sinh(d*x + c))
```

giac [B] time = 0.89, size = 121, normalized size = 2.37

$$-\frac{\frac{2\,dx}{a^2}+\frac{e^{(4\,dx+4\,c)}-2\,e^{(2\,dx+2\,c)}+1}{\left(ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+a\right)a}-\frac{\log\left(ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+a\right)}{a^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*d*x/a^2 + (e^(4*d*x + 4*c) - 2*e^(2*d*x + 2*c) + 1)/((a*e^(4*d*x +
4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) + a)*a) - log(a*e^(4*d*x +
 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) + a)/a^2)/d
```

maple [B] time = 0.35, size = 186, normalized size = 3.65

$$-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d\,a^2}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d\,a^2}-\frac{2\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da\left(\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\left(\tanh^2\left(\frac{dx}{2}+\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] -1/d/a^2*ln(tanh(1/2*d*x+1/2*c)-1)-1/d/a^2*ln(tanh(1/2*d*x+1/2*c)+1)-2/d/a*
tanh(1/2*d*x+1/2*c)^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*ta
nh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)+1/2/d/a^2*ln(tanh(1/2*
d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2
*d*x+1/2*c)^2*b+a+b)
```

maxima [B] time = 0.38, size = 108, normalized size = 2.12

$$\frac{2\,(a+b)e^{(-2\,dx-2\,c)}}{\left(a^3e^{(-4\,dx-4\,c)}+a^3+2\left(a^3+2\,a^2b\right)e^{(-2\,dx-2\,c)}\right)d}+\frac{dx+c}{a^2d}+\frac{\log\left(2\,(a+2\,b)e^{(-2\,dx-2\,c)}+ae^{(-4\,dx-4\,c)}+a\right)}{2\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 2*(a + b)*e^(-2*d*x - 2*c)/((a^3*e^(-4*d*x - 4*c) + a^3 + 2*(a^3 + 2*a^2*b)
*e^(-2*d*x - 2*c))*d) + (d*x + c)/(a^2*d) + 1/2*log(2*(a + 2*b)*e^(-2*d*x -
 2*c) + a*e^(-4*d*x - 4*c) + a)/(a^2*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cosh(c+dx)^4 \tanh(c+dx)^3}{\left(a\cosh(c+dx)^2+b\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^3/(a + b/cosh(c + d*x)^2)^2,x)
```

```
[Out] int((cosh(c + d*x)^4*tanh(c + d*x)^3)/(b + a*cosh(c + d*x)^2)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**3/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

3.151 $\int \frac{\tanh^2(c+dx)}{\left(a+b\,\mathbf{sech}^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=85

$$-\frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^2\sqrt{b}\,d\sqrt{a+b}}+\frac{x}{a^2}-\frac{\tanh(c+dx)}{2ad\left(a-b\tanh^2(c+dx)+b\right)}$$

[Out] x/a^2-1/2*(a+2*b)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))/a^2/d/b^(1/2)/(a+b)^(1/2)-1/2*tanh(d*x+c)/a/d/(a+b-b*tanh(d*x+c)^2)

Rubi [A] time = 0.17, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4141, 1975, 471, 522, 206, 208}

$$-\frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^2\sqrt{b}\,d\sqrt{a+b}}+\frac{x}{a^2}-\frac{\tanh(c+dx)}{2ad\left(a-b\tanh^2(c+dx)+b\right)}$$

Antiderivative was successfully verified.

[In] Int[Tanh[c + d*x]^2/(a + b*Sech[c + d*x]^2)^2,x]

[Out] x/a^2 - ((a + 2*b)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(2*a^2*Sqrt[b]*Sqrt[a + b]*d) - Tanh[c + d*x]/(2*a*d*(a + b - b*Tanh[c + d*x]^2))

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
```

```
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\tanh^2(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1-x^2)\left(a+b(1-x^2)\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{x^2}{(1-x^2)(a+b-bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{\tanh(c+dx)}{2ad\left(a+b-b\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1+x^2}{(1-x^2)(a+b-bx^2)}\,dx, x, \tanh(c+dx)\right)}{2ad}$$

$$= -\frac{\tanh(c+dx)}{2ad\left(a+b-b\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{a^2d} - \frac{(a+2b)\,\text{S}}{}$$

$$= \frac{x}{a^2} - \frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^2\sqrt{b}\sqrt{a+b}\,d} - \frac{\tanh(c+dx)}{2ad\left(a+b-b\tanh^2(c+dx)\right)}$$

Mathematica [B] time = 4.63, size = 326, normalized size = 3.84

$$\frac{\text{sech}^4(c+dx)(a\cosh(2(c+dx))+a+2b)^2\left(\frac{(a^2+8ab+8b^2)\text{sech}(2c)((a+2b)\sinh(2c)-a\sinh(2dx))}{a^2bd(a+b)(a\cosh(2(c+dx))+a+2b)} + \frac{16x}{a^2} + \frac{(a^3-6a^2b-24ab^2-16b^3)(\text{c}}{}\right.}{64\left(a+b\text{se}\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tanh[c + d*x]^2/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])^2*Sech[c + d*x]^4*((16*x)/a^2 - ((a + 2*b)
*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(b^(3/2)*(a + b)^(3/2)*d) +
((a^3 - 6*a^2*b - 24*a*b^2 - 16*b^3)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2
*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh
[c] - Sinh[c])^4])]*(Cosh[2*c] - Sinh[2*c]))/(a^2*b*(a + b)^(3/2)*d*Sqrt[b*
(Cosh[c] - Sinh[c])^4]) + ((a^2 + 8*a*b + 8*b^2)*Sech[2*c]*((a + 2*b)*Sinh[
2*c] - a*Sinh[2*d*x]))/(a^2*b*(a + b)*d*(a + 2*b + a*Cosh[2*(c + d*x)])) +
(a*Sinh[2*(c + d*x)])/(b*(a + b)*d*(a + 2*b + a*Cosh[2*(c + d*x)]))))/(64*(
a + b*Sech[c + d*x]^2)^2)
```

fricas [B] time = 0.47, size = 1846, normalized size = 21.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a^2*b + a*b^2)*d*x*cosh(d*x + c)^4 + 16*(a^2*b + a*b^2)*d*x*cosh(d
*x + c)*sinh(d*x + c)^3 + 4*(a^2*b + a*b^2)*d*x*sinh(d*x + c)^4 + 4*a^2*b +
 4*a*b^2 + 4*(a^2*b + a*b^2)*d*x + 4*(a^2*b + 3*a*b^2 + 2*b^3 + 2*(a^2*b +
3*a*b^2 + 2*b^3)*d*x)*cosh(d*x + c)^2 + 4*(6*(a^2*b + a*b^2)*d*x*cosh(d*x +
 c)^2 + a^2*b + 3*a*b^2 + 2*b^3 + 2*(a^2*b + 3*a*b^2 + 2*b^3)*d*x)*sinh(d*x
 + c)^2 + ((a^2 + 2*a*b)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b)*cosh(d*x + c)*si
nh(d*x + c)^3 + (a^2 + 2*a*b)*sinh(d*x + c)^4 + 2*(a^2 + 4*a*b + 4*b^2)*cos
h(d*x + c)^2 + 2*(3*(a^2 + 2*a*b)*cosh(d*x + c)^2 + a^2 + 4*a*b + 4*b^2)*si
nh(d*x + c)^2 + a^2 + 2*a*b + 4*((a^2 + 2*a*b)*cosh(d*x + c)^3 + (a^2 + 4*a
*b + 4*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a*b + b^2)*log((a^2*cosh(d*x
 + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^
2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d
*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*co
sh(d*x + c))*sinh(d*x + c) + 4*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(
d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(a*b + b^2))/(a*cosh(d*x + c)^4
 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh
(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh
(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) + 8*(2*(a^2*b +
a*b^2)*d*x*cosh(d*x + c)^3 + (a^2*b + 3*a*b^2 + 2*b^3 + 2*(a^2*b + 3*a*b^2
+ 2*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a^4*b + a^3*b^2)*d*cosh(d*x +
 c)^4 + 4*(a^4*b + a^3*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4*b + a^3*
b^2)*d*sinh(d*x + c)^4 + 2*(a^4*b + 3*a^3*b^2 + 2*a^2*b^3)*d*cosh(d*x + c)^
2 + 2*(3*(a^4*b + a^3*b^2)*d*cosh(d*x + c)^2 + (a^4*b + 3*a^3*b^2 + 2*a^2*b
^3)*d)*sinh(d*x + c)^2 + (a^4*b + a^3*b^2)*d + 4*((a^4*b + a^3*b^2)*d*cosh(
d*x + c)^3 + (a^4*b + 3*a^3*b^2 + 2*a^2*b^3)*d*cosh(d*x + c))*sinh(d*x + c)
), 1/2*(2*(a^2*b + a*b^2)*d*x*cosh(d*x + c)^4 + 8*(a^2*b + a*b^2)*d*x*cosh(
d*x + c)*sinh(d*x + c)^3 + 2*(a^2*b + a*b^2)*d*x*sinh(d*x + c)^4 + 2*a^2*b
+ 2*a*b^2 + 2*(a^2*b + a*b^2)*d*x + 2*(a^2*b + 3*a*b^2 + 2*b^3 + 2*(a^2*b +
 3*a*b^2 + 2*b^3)*d*x)*cosh(d*x + c)^2 + 2*(6*(a^2*b + a*b^2)*d*x*cosh(d*x
+ c)^2 + a^2*b + 3*a*b^2 + 2*b^3 + 2*(a^2*b + 3*a*b^2 + 2*b^3)*d*x)*sinh(d*
x + c)^2 - ((a^2 + 2*a*b)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b)*cosh(d*x + c)*s
inh(d*x + c)^3 + (a^2 + 2*a*b)*sinh(d*x + c)^4 + 2*(a^2 + 4*a*b + 4*b^2)*co
sh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b)*cosh(d*x + c)^2 + a^2 + 4*a*b + 4*b^2)*s
inh(d*x + c)^2 + a^2 + 2*a*b + 4*((a^2 + 2*a*b)*cosh(d*x + c)^3 + (a^2 + 4*
a*b + 4*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a*b - b^2)*arctan(1/2*(a*c
osh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a +
2*b)*sqrt(-a*b - b^2)/(a*b + b^2)) + 4*(2*(a^2*b + a*b^2)*d*x*cosh(d*x + c)
```

^3 + (a^2*b + 3*a*b^2 + 2*b^3 + 2*(a^2*b + 3*a*b^2 + 2*b^3)*d*x)*cosh(d*x +
 c))*sinh(d*x + c))/((a^4*b + a^3*b^2)*d*cosh(d*x + c)^4 + 4*(a^4*b + a^3*b
^2)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4*b + a^3*b^2)*d*sinh(d*x + c)^4 +
 2*(a^4*b + 3*a^3*b^2 + 2*a^2*b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^4*b + a^3*b^
2)*d*cosh(d*x + c)^2 + (a^4*b + 3*a^3*b^2 + 2*a^2*b^3)*d)*sinh(d*x + c)^2 +
 (a^4*b + a^3*b^2)*d + 4*((a^4*b + a^3*b^2)*d*cosh(d*x + c)^3 + (a^4*b + 3*
a^3*b^2 + 2*a^2*b^3)*d*cosh(d*x + c))*sinh(d*x + c))]

giac [A] time = 0.72, size = 147, normalized size = 1.73

$$-\frac{\frac{\left(ae^{(2c)}+2\,be^{(2c)}\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)e^{(-2\,c)}}{\sqrt{-ab-b^2}\,a^2}-\frac{2\,dx}{a^2}-\frac{2\left(ae^{(2\,dx+2\,c)}+2\,be^{(2\,dx+2\,c)}+a\right)}{\left(ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+a\right)a^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")

[Out] -1/2*((a*e^(2*c) + 2*b*e^(2*c))*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sq
rt(-a*b - b^2))*e^(-2*c)/(sqrt(-a*b - b^2)*a^2) - 2*d*x/a^2 - 2*(a*e^(2*d*x
 + 2*c) + 2*b*e^(2*d*x + 2*c) + a)/((a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c
) + 4*b*e^(2*d*x + 2*c) + a)*a^2))/d

maple [B] time = 0.35, size = 411, normalized size = 4.84

$$-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d\,a^2}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d\,a^2}-\frac{\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{da\left(\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\left(\tanh^2\left(\frac{dx}{2}+\cdots\right)\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x)

[Out] -1/d/a^2*ln(tanh(1/2*d*x+1/2*c)-1)+1/d/a^2*ln(tanh(1/2*d*x+1/2*c)+1)-1/d/a/
(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-
2*tanh(1/2*d*x+1/2*c)^2*b+a+b)*tanh(1/2*d*x+1/2*c)^3-1/d/a/(tanh(1/2*d*x+1/
2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1
/2*c)^2*b+a+b)*tanh(1/2*d*x+1/2*c)+1/4/d/a/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/
2)*tanh(1/2*d*x+1/2*c)^2-2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))-1/4/d/a
/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/
2*d*x+1/2*c)+(a+b)^(1/2))+1/2/d/a^2*b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh
(1/2*d*x+1/2*c)^2-2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))-1/2/d/a^2*b^(1
/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x
+1/2*c)+(a+b)^(1/2))

maxima [B] time = 0.55, size = 597, normalized size = 7.02

$$-\frac{\left(a^2+6\,ab+4\,b^2\right)\log\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(2\,dx+2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{16\left(a^3+a^2b\right)\sqrt{(a+b)b}\,d}+\frac{\left(a^2+6\,ab+4\,b^2\right)\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{16\left(a^3+a^2b\right)\sqrt{(a+b)b}\,d}+\frac{}{4\left(a^4+a^3b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/16*(a^2 + 6*a*b + 4*b^2)*log((a*e^(2*d*x + 2*c) + a + 2*b - 2*sqrt((a +
b)*b))/(a*e^(2*d*x + 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^3 + a^2*b)*sq
rt((a + b)*b)*d) + 1/16*(a^2 + 6*a*b + 4*b^2)*log((a*e^(-2*d*x - 2*c) + a +
 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)
))/((a^3 + a^2*b)*sqrt((a + b)*b)*d) + 1/4*(a^2 + 2*a*b + (a^2 + 8*a*b + 8*
b^2)*e^(2*d*x + 2*c))/((a^4 + a^3*b + (a^4 + a^3*b)*e^(4*d*x + 4*c) + 2*(a^
4 + 3*a^3*b + 2*a^2*b^2)*e^(2*d*x + 2*c))*d) - 1/4*(a^2 + 2*a*b + (a^2 + 8*
a*b + 8*b^2)*e^(-2*d*x - 2*c))/((a^4 + a^3*b + 2*(a^4 + 3*a^3*b + 2*a^2*b^2
)*e^(-2*d*x - 2*c) + (a^4 + a^3*b)*e^(-4*d*x - 4*c))*d) - 1/2*((a + 2*b)*e^
(-2*d*x - 2*c) + a)/((a^3 + a^2*b + 2*(a^3 + 3*a^2*b + 2*a*b^2)*e^(-2*d*x -
 2*c) + (a^3 + a^2*b)*e^(-4*d*x - 4*c))*d) + 1/8*log((a*e^(-2*d*x - 2*c) +
a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)
*b)))/(sqrt((a + b)*b)*(a + b)*d) + 1/4*log(a*e^(4*d*x + 4*c) + 2*(a + 2*b)
*e^(2*d*x + 2*c) + a)/(a^2*d) - 1/4*log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^
(-4*d*x - 4*c) + a)/(a^2*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh\left(c+d\,x\right)^2\,\left(\cosh\left(c+d\,x\right)^2-1\right)}{{\left(a\cosh\left(c+d\,x\right)^2+b\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^2/(a + b/cosh(c + d*x)^2)^2,x)
```

```
[Out] int((cosh(c + d*x)^2*(cosh(c + d*x)^2 - 1))/(b + a*cosh(c + d*x)^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^2\left(c+dx\right)}{\left(a+b\operatorname{sech}^2\left(c+dx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**2/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral(tanh(c + d*x)**2/(a + b*sech(c + d*x)**2)**2, x)
```

3.152 $\int \frac{\tanh(c+dx)}{\left(a+b\,\mathbf{sech}^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=49

$$\frac{b}{2a^2d\left(a\cosh^2(c+dx)+b\right)}+\frac{\log\left(a\cosh^2(c+dx)+b\right)}{2a^2d}$$

```
[Out] 1/2*b/a^2/d/(b+a*cosh(d*x+c)^2)+1/2*ln(b+a*cosh(d*x+c)^2)/a^2/d
```

Rubi [A] time = 0.06, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4138, 266, 43}

$$\frac{b}{2a^2d\left(a\cosh^2(c+dx)+b\right)}+\frac{\log\left(a\cosh^2(c+dx)+b\right)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] b/(2*a^2*d*(b + a*Cosh[c + d*x]^2)) + Log[b + a*Cosh[c + d*x]^2]/(2*a^2*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\tanh(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx = \frac{\operatorname{Subst}\left(\int \frac{x^3}{\left(b+ax^2\right)^2}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= \frac{\operatorname{Subst}\left(\int \frac{x}{(b+ax)^2}\,dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= \frac{\operatorname{Subst}\left(\int \left(-\frac{b}{a(b+ax)^2}+\frac{1}{a(b+ax)}\right)dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= \frac{b}{2a^2d\left(b+a\cosh^2(c+dx)\right)}+\frac{\log\left(b+a\cosh^2(c+dx)\right)}{2a^2d}$$

Mathematica [A] time = 0.59, size = 79, normalized size = 1.61

$$\frac{(a+2b)\log(a\cosh(2(c+dx))+a+2b)+a\cosh(2(c+dx))\log(a\cosh(2(c+dx))+a+2b)+2b}{2a^2d(a\cosh(2(c+dx))+a+2b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (2*b + (a + 2*b)*Log[a + 2*b + a*Cosh[2*(c + d*x)]] + a*Cosh[2*(c + d*x)]*L
og[a + 2*b + a*Cosh[2*(c + d*x)]])/(2*a^2*d*(a + 2*b + a*Cosh[2*(c + d*x)])
)
```

fricas [B] time = 0.42, size = 476, normalized size = 9.71

$$-\frac{2\,adx\cosh(dx+c)^4+8\,adx\cosh(dx+c)\sinh(dx+c)^3+2\,adx\sinh(dx+c)^4+2\,adx+4\,((a+2\,b)dx-b)\,\mathrm{c}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*d*x*cosh(d*x + c)^4 + 8*a*d*x*cosh(d*x + c)*sinh(d*x + c)^3 + 2*a
*d*x*sinh(d*x + c)^4 + 2*a*d*x + 4*((a + 2*b)*d*x - b)*cosh(d*x + c)^2 + 4*
(3*a*d*x*cosh(d*x + c)^2 + (a + 2*b)*d*x - b)*sinh(d*x + c)^2 - (a*cosh(d*x
 + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*
```

```
b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*
(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)*log(2*(a*c
osh(d*x + c)^2 + a*sinh(d*x + c)^2 + a + 2*b)/(cosh(d*x + c)^2 - 2*cosh(d*x
 + c)*sinh(d*x + c) + sinh(d*x + c)^2)) + 8*(a*d*x*cosh(d*x + c)^3 + ((a +
2*b)*d*x - b)*cosh(d*x + c))*sinh(d*x + c))/(a^3*d*cosh(d*x + c)^4 + 4*a^3*
d*cosh(d*x + c)*sinh(d*x + c)^3 + a^3*d*sinh(d*x + c)^4 + a^3*d + 2*(a^3 +
2*a^2*b)*d*cosh(d*x + c)^2 + 2*(3*a^3*d*cosh(d*x + c)^2 + (a^3 + 2*a^2*b)*d
)*sinh(d*x + c)^2 + 4*(a^3*d*cosh(d*x + c)^3 + (a^3 + 2*a^2*b)*d*cosh(d*x +
 c))*sinh(d*x + c))
```

giac [B] time = 0.42, size = 121, normalized size = 2.47

$$-\frac{\frac{2\,dx}{a^2} + \frac{e^{(4\,dx+4\,c)}+2\,e^{(2\,dx+2\,c)}+1}{\left(ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+a\right)a} - \frac{\log\left(ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+a\right)}{a^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*d*x/a^2 + (e^(4*d*x + 4*c) + 2*e^(2*d*x + 2*c) + 1)/((a*e^(4*d*x +
4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) + a)*a) - log(a*e^(4*d*x +
 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) + a)/a^2)/d
```

maple [A] time = 0.18, size = 60, normalized size = 1.22

$$\frac{\ln\left(a + b\mathrm{sech}\left(dx + c\right)^2\right)}{2d\,a^2} - \frac{1}{2da\left(a + b\mathrm{sech}\left(dx + c\right)^2\right)} - \frac{\ln\left(\mathrm{sech}\left(dx + c\right)\right)}{d\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)/(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] 1/2/d/a^2*ln(a+b*sech(d*x+c)^2)-1/2/d/a/(a+b*sech(d*x+c)^2)-1/d/a^2*ln(sech
(d*x+c))
```

maxima [B] time = 0.42, size = 106, normalized size = 2.16

$$\frac{2\,be^{(-2\,dx-2\,c)}}{\left(a^3e^{(-4\,dx-4\,c)} + a^3 + 2\left(a^3 + 2\,a^2b\right)e^{(-2\,dx-2\,c)}\right)d} + \frac{dx + c}{a^2d} + \frac{\log\left(2\,(a + 2\,b)e^{(-2\,dx-2\,c)} + ae^{(-4\,dx-4\,c)} + a\right)}{2\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 2*b*e^(-2*d*x - 2*c)/((a^3*e^(-4*d*x - 4*c) + a^3 + 2*(a^3 + 2*a^2*b)*e^(-2
*d*x - 2*c))*d) + (d*x + c)/(a^2*d) + 1/2*log(2*(a + 2*b)*e^(-2*d*x - 2*c)
+ a*e^(-4*d*x - 4*c) + a)/(a^2*d)
```

mupad [B] time = 1.73, size = 53, normalized size = 1.08

$$\frac{\ln\left(\cosh\left(c+d\,x\right)^2\,\left(a+\frac{b}{\cosh\left(c+d\,x\right)^2}\right)\right)}{2\,a^2\,d}-\frac{1}{2\,a\,d\,\left(a+\frac{b}{\cosh\left(c+d\,x\right)^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)/(a + b/cosh(c + d*x)^2)^2,x)
```

```
[Out] log(cosh(c + d*x)^2*(a + b/cosh(c + d*x)^2))/(2*a^2*d) - 1/(2*a*d*(a + b/co
sh(c + d*x)^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

3.153 $\int \frac{1}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=93

$$-\frac{\sqrt{b}\,(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^2d(a+b)^{3/2}}+\frac{x}{a^2}-\frac{b\tanh(c+dx)}{2ad(a+b)\left(a-b\tanh^2(c+dx)+b\right)}$$

```
[Out] x/a^2-1/2*(3*a+2*b)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))*b^(1/2)/a^2/(a
+b)^(3/2)/d-1/2*b*tanh(d*x+c)/a/(a+b)/d/(a+b-b*tanh(d*x+c)^2)
```

Rubi [A] time = 0.10, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {4128, 414, 522, 206, 208}

$$-\frac{\sqrt{b}\,(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^2d(a+b)^{3/2}}+\frac{x}{a^2}-\frac{b\tanh(c+dx)}{2ad(a+b)\left(a-b\tanh^2(c+dx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^(-2),x]
```

```
[Out] x/a^2 - (Sqrt[b]*(3*a + 2*b)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/
(2*a^2*(a + b)^(3/2)*d) - (b*Tanh[c + d*x])/(2*a*(a + b)*d*(a + b - b*Tanh[
c + d*x]^2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
```

```
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{\left(1-x^2\right)\left(a+b-bx^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{b\tanh(c+dx)}{2a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)} - \frac{\operatorname{Subst}\left(\int \frac{-2a-b-bx^2}{\left(1-x^2\right)\left(a+b-bx^2\right)}\,dx, x, \tanh(c+\right.}{2a(a+b)d}$$

$$= -\frac{b\tanh(c+dx)}{2a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)} + \frac{\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{a^2d} - \frac{(b}{}$$

$$= \frac{x}{a^2} - \frac{\sqrt{b}\,(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{3/2}d} - \frac{b\tanh(c+dx)}{2a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)}$$

Mathematica [B] time = 2.04, size = 221, normalized size = 2.38

$$\frac{\operatorname{sech}^4(c+dx)(a\cosh(2(c+dx))+a+2b)\left(2x(a\cosh(2(c+dx))+a+2b)+\frac{b\operatorname{sech}(2c)((a+2b)\sinh(2c)-a\sinh(2dx))}{d(a+b)}-\frac{b(3a\ldots}{\ldots}\right.}{8a^2\left(a+b\operatorname{sech}^2(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^(-2),x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^4*(2*x*(a + 2*b + a*Cosh[2*(
c + d*x)]) - (b*(3*a + 2*b)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*((a
+ 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sin
h[c])^4])]*(a + 2*b + a*Cosh[2*(c + d*x)])*(Cosh[2*c] - Sinh[2*c]))/((a + b
)^(3/2)*d*Sqrt[b*(Cosh[c] - Sinh[c])^4]) + (b*Sech[2*c]*((a + 2*b)*Sinh[2*c
] - a*Sinh[2*d*x]))/((a + b)*d)))/(8*a^2*(a + b*Sech[c + d*x]^2)^2)
```

fricas [B] time = 0.46, size = 1690, normalized size = 18.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a^2 + a*b)*d*x*cosh(d*x + c)^4 + 16*(a^2 + a*b)*d*x*cosh(d*x + c)*
sinh(d*x + c)^3 + 4*(a^2 + a*b)*d*x*sinh(d*x + c)^4 + 4*(a^2 + a*b)*d*x + 4
*(2*(a^2 + 3*a*b + 2*b^2)*d*x + a*b + 2*b^2)*cosh(d*x + c)^2 + 4*(6*(a^2 +
a*b)*d*x*cosh(d*x + c)^2 + 2*(a^2 + 3*a*b + 2*b^2)*d*x + a*b + 2*b^2)*sinh(
d*x + c)^2 + ((3*a^2 + 2*a*b)*cosh(d*x + c)^4 + 4*(3*a^2 + 2*a*b)*cosh(d*x
+ c)*sinh(d*x + c)^3 + (3*a^2 + 2*a*b)*sinh(d*x + c)^4 + 2*(3*a^2 + 8*a*b +
 4*b^2)*cosh(d*x + c)^2 + 2*(3*(3*a^2 + 2*a*b)*cosh(d*x + c)^2 + 3*a^2 + 8*
a*b + 4*b^2)*sinh(d*x + c)^2 + 3*a^2 + 2*a*b + 4*((3*a^2 + 2*a*b)*cosh(d*x
+ c)^3 + (3*a^2 + 8*a*b + 4*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(b/(a +
b))*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*si
nh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2
+ a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)
^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) + 4*((a^2 + a*b)*cosh(d*x +
 c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x +
c)^2 + a^2 + 3*a*b + 2*b^2)*sqrt(b/(a + b)))/(a*cosh(d*x + c)^4 + 4*a*cosh(
d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2
+ 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3
+ (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) + 4*a*b + 8*(2*(a^2 + a*b)*d
```

```
*x*cosh(d*x + c)^3 + (2*(a^2 + 3*a*b + 2*b^2)*d*x + a*b + 2*b^2)*cosh(d*x +
 c))*sinh(d*x + c))/((a^4 + a^3*b)*d*cosh(d*x + c)^4 + 4*(a^4 + a^3*b)*d*co
sh(d*x + c)*sinh(d*x + c)^3 + (a^4 + a^3*b)*d*sinh(d*x + c)^4 + 2*(a^4 + 3*
a^3*b + 2*a^2*b^2)*d*cosh(d*x + c)^2 + 2*(3*(a^4 + a^3*b)*d*cosh(d*x + c)^2
 + (a^4 + 3*a^3*b + 2*a^2*b^2)*d)*sinh(d*x + c)^2 + (a^4 + a^3*b)*d + 4*((a
^4 + a^3*b)*d*cosh(d*x + c)^3 + (a^4 + 3*a^3*b + 2*a^2*b^2)*d*cosh(d*x + c)
)*sinh(d*x + c)), 1/2*(2*(a^2 + a*b)*d*x*cosh(d*x + c)^4 + 8*(a^2 + a*b)*d*
x*cosh(d*x + c)*sinh(d*x + c)^3 + 2*(a^2 + a*b)*d*x*sinh(d*x + c)^4 + 2*(a^
2 + a*b)*d*x + 2*(2*(a^2 + 3*a*b + 2*b^2)*d*x + a*b + 2*b^2)*cosh(d*x + c)^
2 + 2*(6*(a^2 + a*b)*d*x*cosh(d*x + c)^2 + 2*(a^2 + 3*a*b + 2*b^2)*d*x + a*
b + 2*b^2)*sinh(d*x + c)^2 - ((3*a^2 + 2*a*b)*cosh(d*x + c)^4 + 4*(3*a^2 +
2*a*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a^2 + 2*a*b)*sinh(d*x + c)^4 + 2*
(3*a^2 + 8*a*b + 4*b^2)*cosh(d*x + c)^2 + 2*(3*(3*a^2 + 2*a*b)*cosh(d*x + c
)^2 + 3*a^2 + 8*a*b + 4*b^2)*sinh(d*x + c)^2 + 3*a^2 + 2*a*b + 4*((3*a^2 +
2*a*b)*cosh(d*x + c)^3 + (3*a^2 + 8*a*b + 4*b^2)*cosh(d*x + c))*sinh(d*x +
c))*sqrt(-b/(a + b))*arctan(1/2*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh
(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-b/(a + b))/b) + 2*a*b + 4*(2
*(a^2 + a*b)*d*x*cosh(d*x + c)^3 + (2*(a^2 + 3*a*b + 2*b^2)*d*x + a*b + 2*b
^2)*cosh(d*x + c))*sinh(d*x + c))/((a^4 + a^3*b)*d*cosh(d*x + c)^4 + 4*(a^4
 + a^3*b)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 + a^3*b)*d*sinh(d*x + c)^4
 + 2*(a^4 + 3*a^3*b + 2*a^2*b^2)*d*cosh(d*x + c)^2 + 2*(3*(a^4 + a^3*b)*d*c
osh(d*x + c)^2 + (a^4 + 3*a^3*b + 2*a^2*b^2)*d)*sinh(d*x + c)^2 + (a^4 + a^
3*b)*d + 4*((a^4 + a^3*b)*d*cosh(d*x + c)^3 + (a^4 + 3*a^3*b + 2*a^2*b^2)*d
*cosh(d*x + c))*sinh(d*x + c))]
```

giac [A] time = 0.44, size = 163, normalized size = 1.75

$$-\frac{\frac{\left(3\,ab+2\,b^{2}\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^{2}}}\right)}{\left(a^{3}+a^{2}b\right)\sqrt{-ab-b^{2}}}-\frac{2\left(abe^{(2\,dx+2\,c)}+2\,b^{2}e^{(2\,dx+2\,c)}+ab\right)}{\left(a^{3}+a^{2}b\right)\left(ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+a\right)}-\frac{2\,(dx+c)}{a^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*((3*a*b + 2*b^2)*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b -
b^2))/((a^3 + a^2*b)*sqrt(-a*b - b^2)) - 2*(a*b*e^(2*d*x + 2*c) + 2*b^2*e^(
2*d*x + 2*c) + a*b)/((a^3 + a^2*b)*(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c)
 + 4*b*e^(2*d*x + 2*c) + a)) - 2*(d*x + c)/a^2)/d
```

maple [B] time = 0.38, size = 423, normalized size = 4.55

$$-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d\,a^{2}}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d\,a^{2}}-\frac{b\left(\tanh^{3}\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{d\left(\left(\tanh^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+b\left(\tanh^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\left(\tanh^{2}\left(\frac{dx}{2}+\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] -1/d/a^2*ln(tanh(1/2*d*x+1/2*c)-1)+1/d/a^2*ln(tanh(1/2*d*x+1/2*c)+1)-1/d/a*
b/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*
a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)/(a+b)*tanh(1/2*d*x+1/2*c)^3-1/d/a*b/(tanh(
1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh
(1/2*d*x+1/2*c)^2*b+a+b)/(a+b)*tanh(1/2*d*x+1/2*c)+3/4/d/a*b^(1/2)/(a+b)^(3
/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b
)^(1/2))-3/4/d/a*b^(1/2)/(a+b)^(3/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2
*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))+1/2/d/a^2*b^(3/2)/(a+b)^(3/2)*ln(
(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2)
)-1/2/d/a^2*b^(3/2)/(a+b)^(3/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1
/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))
```

maxima [B] time = 0.69, size = 187, normalized size = 2.01

$$\frac{\left(3\,ab+2\,b^{2}\right)\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{4\left(a^{3}+a^{2}b\right)\sqrt{(a+b)b}\,d}-\frac{ab+\left(ab+2\,b^{2}\right)e^{(-2\,dx-2\,c)}}{\left(a^{4}+a^{3}b+2\left(a^{4}+3\,a^{3}b+2\,a^{2}b^{2}\right)e^{(-2\,dx-2\,c)}+\left(a^{4}+a^{3}b\right)e^{(-4\,dx-4\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(3*a*b + 2*b^2)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/
(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^3 + a^2*b)*sqrt((a
+ b)*b)*d) - (a*b + (a*b + 2*b^2)*e^(-2*d*x - 2*c))/((a^4 + a^3*b + 2*(a^4
+ 3*a^3*b + 2*a^2*b^2)*e^(-2*d*x - 2*c) + (a^4 + a^3*b)*e^(-4*d*x - 4*c))*d
) + (d*x + c)/(a^2*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+\frac{b}{\cosh(c+d\,x)^{2}}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cosh(c + d*x)^2)^2,x)
```

```
[Out] int(1/(a + b/cosh(c + d*x)^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\,\mathrm{sech}^{2}\left(c+dx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**(-2), x)
```

3.154 $\int \frac{\coth(c+dx)}{\left(a+b\,\mathbf{sech}^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=83

$$\frac{b^2}{2a^2d(a+b)\left(a\cosh^2(c+dx)+b\right)}+\frac{b(2a+b)\log\left(a\cosh^2(c+dx)+b\right)}{2a^2d(a+b)^2}+\frac{\log(\sinh(c+dx))}{d(a+b)^2}$$

```
[Out] 1/2*b^2/a^2/(a+b)/d/(b+a*cosh(d*x+c)^2)+1/2*b*(2*a+b)*ln(b+a*cosh(d*x+c)^2)
/a^2/(a+b)^2/d+ln(sinh(d*x+c))/(a+b)^2/d
```

Rubi [A] time = 0.13, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4138, 446, 88}

$$\frac{b^2}{2a^2d(a+b)\left(a\cosh^2(c+dx)+b\right)}+\frac{b(2a+b)\log\left(a\cosh^2(c+dx)+b\right)}{2a^2d(a+b)^2}+\frac{\log(\sinh(c+dx))}{d(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] b^2/(2*a^2*(a + b)*d*(b + a*Cosh[c + d*x]^2)) + (b*(2*a + b)*Log[b + a*Cosh
[c + d*x]^2])/(2*a^2*(a + b)^2*d) + Log[Sinh[c + d*x]]/((a + b)^2*d)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
```

```
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\coth(c+dx)}{\left(a+b\mathrm{sech}^2(c+dx)\right)^2}\,dx = -\frac{\mathrm{Subst}\left(\int \frac{x^5}{(1-x^2)(b+ax^2)^2}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= -\frac{\mathrm{Subst}\left(\int \frac{x^2}{(1-x)(b+ax)^2}\,dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= -\frac{\mathrm{Subst}\left(\int \left(-\frac{1}{(a+b)^2(-1+x)}+\frac{b^2}{a(a+b)(b+ax)^2}-\frac{b(2a+b)}{a(a+b)^2(b+ax)}\right)dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= \frac{b^2}{2a^2(a+b)d\left(b+a\cosh^2(c+dx)\right)}+\frac{b(2a+b)\log\left(b+a\cosh^2(c+dx)\right)}{2a^2(a+b)^2d}+\frac{\log(s}{(}$$

Mathematica [A] time = 0.30, size = 115, normalized size = 1.39

$$\frac{a\sinh^2(c+dx)\left(2a^2\log(\sinh(c+dx))+b(2a+b)\log\left(a\sinh^2(c+dx)+a+b\right)\right)+(a+b)\left(2a^2\log(\sinh(c+dx\right.}{a^2d(a+b)^2(a\cosh(2(c+dx))+a+2b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + b)*(2*a^2*Log[Sinh[c + d*x]] + b*(b + (2*a + b)*Log[a + b + a*Sinh[c
+ d*x]^2])) + a*(2*a^2*Log[Sinh[c + d*x]] + b*(2*a + b)*Log[a + b + a*Sinh[
c + d*x]^2])*Sinh[c + d*x]^2)/(a^2*(a + b)^2*d*(a + 2*b + a*Cosh[2*(c + d*x
)]))
```

fricas [B] time = 0.55, size = 1031, normalized size = 12.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a^3 + 2*a^2*b + a*b^2)*d*x*cosh(d*x + c)^4 + 8*(a^3 + 2*a^2*b + a*
b^2)*d*x*cosh(d*x + c)*sinh(d*x + c)^3 + 2*(a^3 + 2*a^2*b + a*b^2)*d*x*sinh
```

```
(d*x + c)^4 + 2*(a^3 + 2*a^2*b + a*b^2)*d*x - 4*(a*b^2 + b^3 - (a^3 + 4*a^2
*b + 5*a*b^2 + 2*b^3)*d*x)*cosh(d*x + c)^2 + 4*(3*(a^3 + 2*a^2*b + a*b^2)*d
*x*cosh(d*x + c)^2 - a*b^2 - b^3 + (a^3 + 4*a^2*b + 5*a*b^2 + 2*b^3)*d*x)*s
inh(d*x + c)^2 - ((2*a^2*b + a*b^2)*cosh(d*x + c)^4 + 4*(2*a^2*b + a*b^2)*c
osh(d*x + c)*sinh(d*x + c)^3 + (2*a^2*b + a*b^2)*sinh(d*x + c)^4 + 2*a^2*b
+ a*b^2 + 2*(2*a^2*b + 5*a*b^2 + 2*b^3)*cosh(d*x + c)^2 + 2*(2*a^2*b + 5*a*
b^2 + 2*b^3 + 3*(2*a^2*b + a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((2*
a^2*b + a*b^2)*cosh(d*x + c)^3 + (2*a^2*b + 5*a*b^2 + 2*b^3)*cosh(d*x + c))
*sinh(d*x + c))*log(2*(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + a + 2*b)/(co
sh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) - 2*(a^3*
cosh(d*x + c)^4 + 4*a^3*cosh(d*x + c)*sinh(d*x + c)^3 + a^3*sinh(d*x + c)^4
 + a^3 + 2*(a^3 + 2*a^2*b)*cosh(d*x + c)^2 + 2*(3*a^3*cosh(d*x + c)^2 + a^3
 + 2*a^2*b)*sinh(d*x + c)^2 + 4*(a^3*cosh(d*x + c)^3 + (a^3 + 2*a^2*b)*cosh
(d*x + c))*sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c
))) + 8*((a^3 + 2*a^2*b + a*b^2)*d*x*cosh(d*x + c)^3 - (a*b^2 + b^3 - (a^3
+ 4*a^2*b + 5*a*b^2 + 2*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a^5 + 2*a
^4*b + a^3*b^2)*d*cosh(d*x + c)^4 + 4*(a^5 + 2*a^4*b + a^3*b^2)*d*cosh(d*x
+ c)*sinh(d*x + c)^3 + (a^5 + 2*a^4*b + a^3*b^2)*d*sinh(d*x + c)^4 + 2*(a^5
 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^5 + 2*a^4*b
 + a^3*b^2)*d*cosh(d*x + c)^2 + (a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3)*d)*
sinh(d*x + c)^2 + (a^5 + 2*a^4*b + a^3*b^2)*d + 4*((a^5 + 2*a^4*b + a^3*b^2
)*d*cosh(d*x + c)^3 + (a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3)*d*cosh(d*x +
c))*sinh(d*x + c))
```

giac [B] time = 0.56, size = 246, normalized size = 2.96

$$\frac{\frac{\left(2\,ab+b^2\right)\log\left(ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+a\right)}{a^4+2\,a^3b+a^2b^2}+\frac{2\,e^{(2\,c)}\log\left(\left|-e^{(2\,dx+2\,c)}+1\right|\right)}{a^2e^{(2\,c)}+2\,abe^{(2\,c)}+b^2e^{(2\,c)}}-\frac{2\,dx}{a^2}-\frac{2\,abe^{(4\,dx+4\,c)}+b^2e^{(4\,dx+4\,c)}+4\,abe^{(2\,dx+2\,c)}+6\,b^2e^{(2\,dx}}{\left(a^3+2\,a^2b+ab^2\right)\left(ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+4\,be^{(2}\right.}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((2*a*b + b^2)*log(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d
*x + 2*c) + a)/(a^4 + 2*a^3*b + a^2*b^2) + 2*e^(2*c)*log(abs(-e^(2*d*x + 2*
c) + 1))/(a^2*e^(2*c) + 2*a*b*e^(2*c) + b^2*e^(2*c)) - 2*d*x/a^2 - (2*a*b*e
^(4*d*x + 4*c) + b^2*e^(4*d*x + 4*c) + 4*a*b*e^(2*d*x + 2*c) + 6*b^2*e^(2*d
*x + 2*c) + 2*a*b + b^2)/((a^3 + 2*a^2*b + a*b^2)*(a*e^(4*d*x + 4*c) + 2*a*
e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) + a)))/d
```

maple [B] time = 0.42, size = 292, normalized size = 3.52

$$-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d\,a^2}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d\,a^2}-\frac{2b^2\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{da\left(a+b\right)^2\left(\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\left(\tanh\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)/(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] -1/d/a^2*ln(tanh(1/2*d*x+1/2*c)-1)-1/d/a^2*ln(tanh(1/2*d*x+1/2*c)+1)-2/d*b^
2/a/(a+b)^2*tanh(1/2*d*x+1/2*c)^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1
/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)+1/d*b/a/(a
+b)^2*ln(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2
*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)+1/2/d*b^2/a^2/(a+b)^2*ln(tanh(1/2*d*
x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d
*x+1/2*c)^2*b+a+b)+1/d/(a+b)^2*ln(tanh(1/2*d*x+1/2*c))
```

maxima [B] time = 0.35, size = 209, normalized size = 2.52

$$\frac{2\,b^2e^{(-2\,dx-2\,c)}}{\left(a^4+a^3b+2\left(a^4+3\,a^3b+2\,a^2b^2\right)e^{(-2\,dx-2\,c)}+\left(a^4+a^3b\right)e^{(-4\,dx-4\,c)}\right)d}+\frac{\left(2\,ab+b^2\right)\log\left(2\,(a+2\,b)e^{(-2\,dx-2\,c)}+a\right.}{2\left(a^4+2\,a^3b+a^2b^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 2*b^2*e^(-2*d*x - 2*c)/((a^4 + a^3*b + 2*(a^4 + 3*a^3*b + 2*a^2*b^2)*e^(-2*
d*x - 2*c) + (a^4 + a^3*b)*e^(-4*d*x - 4*c))*d) + 1/2*(2*a*b + b^2)*log(2*(
a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/((a^4 + 2*a^3*b + a^2*b
^2)*d) + log(e^(-d*x - c) + 1)/((a^2 + 2*a*b + b^2)*d) + log(e^(-d*x - c) -
 1)/((a^2 + 2*a*b + b^2)*d) + (d*x + c)/(a^2*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh\left(c+d\,x\right)^4\,\coth\left(c+d\,x\right)}{{\left(a\,\cosh\left(c+d\,x\right)^2+b\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)/(a + b/cosh(c + d*x)^2)^2,x)
```

```
[Out] int((cosh(c + d*x)^4*coth(c + d*x))/(b + a*cosh(c + d*x)^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth\left(c+dx\right)}{\left(a+b\operatorname{sech}^2\left(c+dx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral(coth(c + d*x)/(a + b*sech(c + d*x)**2)**2, x)
```

3.155 $\int \frac{\coth^2(c+dx)}{\left(a+b\,\mathbf{sech}^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=121

$$-\frac{b^{3/2}(5a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^2d(a+b)^{5/2}}+\frac{x}{a^2}-\frac{(2a-b)\coth(c+dx)}{2ad(a+b)^2}-\frac{b\coth(c+dx)}{2ad(a+b)\left(a-b\tanh^2(c+dx)+b\right)}$$

```
[Out] x/a^2-1/2*b^(3/2)*(5*a+2*b)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))/a^2/(a
+b)^(5/2)/d-1/2*(2*a-b)*coth(d*x+c)/a/(a+b)^2/d-1/2*b*coth(d*x+c)/a/(a+b)/d
/(a+b-b*tanh(d*x+c)^2)
```

Rubi [A] time = 0.28, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4141, 1975, 472, 583, 522, 206, 208}

$$-\frac{b^{3/2}(5a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^2d(a+b)^{5/2}}+\frac{x}{a^2}-\frac{(2a-b)\coth(c+dx)}{2ad(a+b)^2}-\frac{b\coth(c+dx)}{2ad(a+b)\left(a-b\tanh^2(c+dx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^2/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] x/a^2 - (b^(3/2)*(5*a + 2*b)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/
(2*a^2*(a + b)^(5/2)*d) - ((2*a - b)*Coth[c + d*x])/(2*a*(a + b)^2*d) - (b*
Coth[c + d*x])/(2*a*(a + b)*d*(a + b - b*Tanh[c + d*x]^2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
```

```
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\coth^2(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2(1-x^2)\left(a+b(1-x^2)\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^2(1-x^2)\left(a+b-bx^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{b\coth(c+dx)}{2a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{-2a+b-3bx^2}{x^2(1-x^2)(a+b-bx^2)}\,dx, x, \tanh(c+\right.}{2a(a+b)d}$$

$$= -\frac{(2a-b)\coth(c+dx)}{2a(a+b)^2d} - \frac{b\coth(c+dx)}{2a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{2a^2+6ab}{(1-x^2}\right.}{}$$

$$= -\frac{(2a-b)\coth(c+dx)}{2a(a+b)^2d} - \frac{b\coth(c+dx)}{2a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx\right.}{a}$$

$$= \frac{x}{a^2} - \frac{b^{3/2}(5a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{5/2}d} - \frac{(2a-b)\coth(c+dx)}{2a(a+b)^2d} - \frac{b\,\text{c}}{2a(a+b)d\left(a\right.}$$

Mathematica [**B**] time = 2.82, size = 268, normalized size = 2.21

$$\frac{\text{sech}^4(c+dx)(a\cosh(2(c+dx))+a+2b)\left(\frac{b^2\text{sech}(2c)((a+2b)\sinh(2c)-a\sinh(2dx))}{a^2d(a+b)^2} - \frac{b^2(5a+2b)(\cosh(2c)-\sinh(2c))(a\cosh(2(c+dx))}{a^2d(a+}\right.}{8\left(a+b\text{s}\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Coth[c + d*x]^2/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^4*((2*x*(a + 2*b + a*Cosh[2*
(c + d*x)]))/a^2 - (b^2*(5*a + 2*b)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*
c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh[
c] - Sinh[c])^4])]*(a + 2*b + a*Cosh[2*(c + d*x)])*(Cosh[2*c] - Sinh[2*c]))
/(a^2*(a + b)^(5/2)*d*Sqrt[b*(Cosh[c] - Sinh[c])^4]) + (2*(a + 2*b + a*Cosh
[2*(c + d*x)])*Csch[c]*Csch[c + d*x]*Sinh[d*x])/((a + b)^2*d) + (b^2*Sech[2
```

```
*c]*((a + 2*b)*Sinh[2*c] - a*Sinh[2*d*x]))/(a^2*(a + b)^2*d)))/(8*(a + b*Se
ch[c + d*x]^2)^2)
```

fricas [B] time = 0.51, size = 3624, normalized size = 29.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a^3 + 2*a^2*b + a*b^2)*d*x*cosh(d*x + c)^6 + 24*(a^3 + 2*a^2*b + a
*b^2)*d*x*cosh(d*x + c)*sinh(d*x + c)^5 + 4*(a^3 + 2*a^2*b + a*b^2)*d*x*sin
h(d*x + c)^6 - 4*(2*a^3 - a*b^2 - 2*b^3 - (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)
*d*x)*cosh(d*x + c)^4 + 4*(15*(a^3 + 2*a^2*b + a*b^2)*d*x*cosh(d*x + c)^2 -
 2*a^3 + a*b^2 + 2*b^3 + (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*d*x)*sinh(d*x +
c)^4 + 16*(5*(a^3 + 2*a^2*b + a*b^2)*d*x*cosh(d*x + c)^3 - (2*a^3 - a*b^2 -
 2*b^3 - (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c
)^3 - 8*a^3 - 4*a*b^2 - 4*(a^3 + 2*a^2*b + a*b^2)*d*x - 4*(4*a^3 + 8*a^2*b
+ 2*b^3 + (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*d*x)*cosh(d*x + c)^2 + 4*(15*(a
^3 + 2*a^2*b + a*b^2)*d*x*cosh(d*x + c)^4 - 4*a^3 - 8*a^2*b - 2*b^3 - (a^3
+ 6*a^2*b + 9*a*b^2 + 4*b^3)*d*x - 6*(2*a^3 - a*b^2 - 2*b^3 - (a^3 + 6*a^2*
b + 9*a*b^2 + 4*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((5*a^2*b + 2*
a*b^2)*cosh(d*x + c)^6 + 6*(5*a^2*b + 2*a*b^2)*cosh(d*x + c)*sinh(d*x + c)^
5 + (5*a^2*b + 2*a*b^2)*sinh(d*x + c)^6 + (5*a^2*b + 22*a*b^2 + 8*b^3)*cosh
(d*x + c)^4 + (5*a^2*b + 22*a*b^2 + 8*b^3 + 15*(5*a^2*b + 2*a*b^2)*cosh(d*x
 + c)^2)*sinh(d*x + c)^4 + 4*(5*(5*a^2*b + 2*a*b^2)*cosh(d*x + c)^3 + (5*a^
2*b + 22*a*b^2 + 8*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 - 5*a^2*b - 2*a*b^2
- (5*a^2*b + 22*a*b^2 + 8*b^3)*cosh(d*x + c)^2 + (15*(5*a^2*b + 2*a*b^2)*co
sh(d*x + c)^4 - 5*a^2*b - 22*a*b^2 - 8*b^3 + 6*(5*a^2*b + 22*a*b^2 + 8*b^3)
*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(3*(5*a^2*b + 2*a*b^2)*cosh(d*x + c)^
5 + 2*(5*a^2*b + 22*a*b^2 + 8*b^3)*cosh(d*x + c)^3 - (5*a^2*b + 22*a*b^2 +
8*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(b/(a + b))*log((a^2*cosh(d*x + c)
^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2
*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x +
c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*
x + c))*sinh(d*x + c) + 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh
(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 + 3*a*b + 2*b^2
)*sqrt(b/(a + b)))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 +
 a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 +
 a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))
*sinh(d*x + c) + a)) + 8*(3*(a^3 + 2*a^2*b + a*b^2)*d*x*cosh(d*x + c)^5 - 2
*(2*a^3 - a*b^2 - 2*b^3 - (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*d*x)*cosh(d*x +
 c)^3 - (4*a^3 + 8*a^2*b + 2*b^3 + (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*d*x)*c
osh(d*x + c))*sinh(d*x + c))/((a^5 + 2*a^4*b + a^3*b^2)*d*cosh(d*x + c)^6 +
 6*(a^5 + 2*a^4*b + a^3*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a^5 + 2*a^4
```

```
*b + a^3*b^2)*d*sinh(d*x + c)^6 + (a^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*d
*cosh(d*x + c)^4 + (15*(a^5 + 2*a^4*b + a^3*b^2)*d*cosh(d*x + c)^2 + (a^5 +
 6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*d)*sinh(d*x + c)^4 - (a^5 + 6*a^4*b + 9*a
^3*b^2 + 4*a^2*b^3)*d*cosh(d*x + c)^2 + 4*(5*(a^5 + 2*a^4*b + a^3*b^2)*d*co
sh(d*x + c)^3 + (a^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*d*cosh(d*x + c))*si
nh(d*x + c)^3 + (15*(a^5 + 2*a^4*b + a^3*b^2)*d*cosh(d*x + c)^4 + 6*(a^5 +
6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*d*cosh(d*x + c)^2 - (a^5 + 6*a^4*b + 9*a^3
*b^2 + 4*a^2*b^3)*d)*sinh(d*x + c)^2 - (a^5 + 2*a^4*b + a^3*b^2)*d + 2*(3*(
a^5 + 2*a^4*b + a^3*b^2)*d*cosh(d*x + c)^5 + 2*(a^5 + 6*a^4*b + 9*a^3*b^2 +
 4*a^2*b^3)*d*cosh(d*x + c)^3 - (a^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*d*c
osh(d*x + c))*sinh(d*x + c)), 1/2*(2*(a^3 + 2*a^2*b + a*b^2)*d*x*cosh(d*x +
 c)^6 + 12*(a^3 + 2*a^2*b + a*b^2)*d*x*cosh(d*x + c)*sinh(d*x + c)^5 + 2*(a
^3 + 2*a^2*b + a*b^2)*d*x*sinh(d*x + c)^6 - 2*(2*a^3 - a*b^2 - 2*b^3 - (a^3
 + 6*a^2*b + 9*a*b^2 + 4*b^3)*d*x)*cosh(d*x + c)^4 + 2*(15*(a^3 + 2*a^2*b +
 a*b^2)*d*x*cosh(d*x + c)^2 - 2*a^3 + a*b^2 + 2*b^3 + (a^3 + 6*a^2*b + 9*a*
b^2 + 4*b^3)*d*x)*sinh(d*x + c)^4 + 8*(5*(a^3 + 2*a^2*b + a*b^2)*d*x*cosh(d
*x + c)^3 - (2*a^3 - a*b^2 - 2*b^3 - (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*d*x)
*cosh(d*x + c))*sinh(d*x + c)^3 - 4*a^3 - 2*a*b^2 - 2*(a^3 + 2*a^2*b + a*b^
2)*d*x - 2*(4*a^3 + 8*a^2*b + 2*b^3 + (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*d*x
)*cosh(d*x + c)^2 + 2*(15*(a^3 + 2*a^2*b + a*b^2)*d*x*cosh(d*x + c)^4 - 4*a
^3 - 8*a^2*b - 2*b^3 - (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*d*x - 6*(2*a^3 - a
*b^2 - 2*b^3 - (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*d*x)*cosh(d*x + c)^2)*sinh
(d*x + c)^2 - ((5*a^2*b + 2*a*b^2)*cosh(d*x + c)^6 + 6*(5*a^2*b + 2*a*b^2)*
cosh(d*x + c)*sinh(d*x + c)^5 + (5*a^2*b + 2*a*b^2)*sinh(d*x + c)^6 + (5*a^
2*b + 22*a*b^2 + 8*b^3)*cosh(d*x + c)^4 + (5*a^2*b + 22*a*b^2 + 8*b^3 + 15*
(5*a^2*b + 2*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(5*a^2*b + 2*a*
b^2)*cosh(d*x + c)^3 + (5*a^2*b + 22*a*b^2 + 8*b^3)*cosh(d*x + c))*sinh(d*x
 + c)^3 - 5*a^2*b - 2*a*b^2 - (5*a^2*b + 22*a*b^2 + 8*b^3)*cosh(d*x + c)^2
+ (15*(5*a^2*b + 2*a*b^2)*cosh(d*x + c)^4 - 5*a^2*b - 22*a*b^2 - 8*b^3 + 6*
(5*a^2*b + 22*a*b^2 + 8*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(3*(5*a^2
*b + 2*a*b^2)*cosh(d*x + c)^5 + 2*(5*a^2*b + 22*a*b^2 + 8*b^3)*cosh(d*x + c
)^3 - (5*a^2*b + 22*a*b^2 + 8*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b/(a
 + b))*arctan(1/2*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*
sinh(d*x + c)^2 + a + 2*b)*sqrt(-b/(a + b))/b) + 4*(3*(a^3 + 2*a^2*b + a*b^
2)*d*x*cosh(d*x + c)^5 - 2*(2*a^3 - a*b^2 - 2*b^3 - (a^3 + 6*a^2*b + 9*a*b^
2 + 4*b^3)*d*x)*cosh(d*x + c)^3 - (4*a^3 + 8*a^2*b + 2*b^3 + (a^3 + 6*a^2*b
 + 9*a*b^2 + 4*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a^5 + 2*a^4*b + a^
3*b^2)*d*cosh(d*x + c)^6 + 6*(a^5 + 2*a^4*b + a^3*b^2)*d*cosh(d*x + c)*sinh
(d*x + c)^5 + (a^5 + 2*a^4*b + a^3*b^2)*d*sinh(d*x + c)^6 + (a^5 + 6*a^4*b
+ 9*a^3*b^2 + 4*a^2*b^3)*d*cosh(d*x + c)^4 + (15*(a^5 + 2*a^4*b + a^3*b^2)*
d*cosh(d*x + c)^2 + (a^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*d)*sinh(d*x + c
)^4 - (a^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*d*cosh(d*x + c)^2 + 4*(5*(a^5
 + 2*a^4*b + a^3*b^2)*d*cosh(d*x + c)^3 + (a^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^
2*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^3 + (15*(a^5 + 2*a^4*b + a^3*b^2)*d*c
osh(d*x + c)^4 + 6*(a^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*d*cosh(d*x + c)^
```

```
2 - (a^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*d)*sinh(d*x + c)^2 - (a^5 + 2*a
^4*b + a^3*b^2)*d + 2*(3*(a^5 + 2*a^4*b + a^3*b^2)*d*cosh(d*x + c)^5 + 2*(a
^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*d*cosh(d*x + c)^3 - (a^5 + 6*a^4*b +
9*a^3*b^2 + 4*a^2*b^3)*d*cosh(d*x + c))*sinh(d*x + c))]
```

giac [B] time = 0.96, size = 282, normalized size = 2.33

$$\frac{\frac{\left(5\,ab^2e^{(2\,c)}+2\,b^3e^{(2\,c)}\right)\arctan\left(-\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)e^{(-2\,c)}}{\left(a^4+2\,a^3b+a^2b^2\right)\sqrt{-ab-b^2}}+\frac{2\,dx}{a^2}-\frac{2\left(2\,a^3e^{(4\,dx+4\,c)}-ab^2e^{(4\,dx+4\,c)}-2\,b^3e^{(4\,dx+4\,c)}+4\,a^3e^{(2\,dx+2\,c)}+8\,a^2be^{(2\,dx+2\,c)}+2\,b^3e\cdots\right.}{\left(a^4+2\,a^3b+a^2b^2\right)\left(ae^{(6\,dx+6\,c)}+ae^{(4\,dx+4\,c)}+4\,be^{(4\,dx+4\,c)}-ae^{(2\,dx+2\,c)}-4\,be^{(2\cdots}\right.}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((5*a*b^2*e^(2*c) + 2*b^3*e^(2*c))*arctan(-1/2*(a*e^(2*d*x + 2*c) + a +
 2*b)/sqrt(-a*b - b^2))*e^(-2*c)/((a^4 + 2*a^3*b + a^2*b^2)*sqrt(-a*b - b^2
)) + 2*d*x/a^2 - 2*(2*a^3*e^(4*d*x + 4*c) - a*b^2*e^(4*d*x + 4*c) - 2*b^3*e
^(4*d*x + 4*c) + 4*a^3*e^(2*d*x + 2*c) + 8*a^2*b*e^(2*d*x + 2*c) + 2*b^3*e^
(2*d*x + 2*c) + 2*a^3 + a*b^2)/((a^4 + 2*a^3*b + a^2*b^2)*(a*e^(6*d*x + 6*c
) + a*e^(4*d*x + 4*c) + 4*b*e^(4*d*x + 4*c) - a*e^(2*d*x + 2*c) - 4*b*e^(2*
d*x + 2*c) - a)))/d
```

maple [B] time = 0.46, size = 481, normalized size = 3.98

$$-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\left(a^2+2ab+b^2\right)}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d\,a^2}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d\,a^2}-\frac{\cdots}{da\left(a+b\right)^2\left(\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+b\left(\tan\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] -1/2/d/(a^2+2*a*b+b^2)*tanh(1/2*d*x+1/2*c)-1/d/a^2*ln(tanh(1/2*d*x+1/2*c)-1
)+1/d/a^2*ln(tanh(1/2*d*x+1/2*c)+1)-1/d*b^2/a/(a+b)^2/(tanh(1/2*d*x+1/2*c)^
4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)
^2*b+a+b)*tanh(1/2*d*x+1/2*c)^3-1/d*b^2/a/(a+b)^2/(tanh(1/2*d*x+1/2*c)^4*a+
b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b
+a+b)*tanh(1/2*d*x+1/2*c)+5/4/d*b^(3/2)/a/(a+b)^(5/2)*ln(-(a+b)^(1/2)*tanh(
1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)-(a+b)^(1/2))-5/4/d*b^(3/2)/a
/(a+b)^(5/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/
2*c)+(a+b)^(1/2))+1/2/d*b^(5/2)/a^2/(a+b)^(5/2)*ln(-(a+b)^(1/2)*tanh(1/2*d*
x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)-(a+b)^(1/2))-1/2/d*b^(5/2)/a^2/(a+
b)^(5/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)
+(a+b)^(1/2))-1/2/d/(a+b)^2/tanh(1/2*d*x+1/2*c)
```

maxima [B] time = 0.92, size = 1070, normalized size = 8.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*a*b + b^2)*log(a*e^(4*d*x + 4*c) + 2*(a + 2*b)*e^(2*d*x + 2*c) + a)/
((a^4 + 2*a^3*b + a^2*b^2)*d) - 1/4*(2*a*b + b^2)*log(2*(a + 2*b)*e^(-2*d*x
 - 2*c) + a*e^(-4*d*x - 4*c) + a)/((a^4 + 2*a^3*b + a^2*b^2)*d) - 1/16*(3*a
^2*b + 10*a*b^2 + 4*b^3)*log((a*e^(2*d*x + 2*c) + a + 2*b - 2*sqrt((a + b)*
b))/(a*e^(2*d*x + 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^4 + 2*a^3*b + a^
2*b^2)*sqrt((a + b)*b)*d) + 1/16*(3*a^2*b + 10*a*b^2 + 4*b^3)*log((a*e^(-2*
d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2
*sqrt((a + b)*b)))/((a^4 + 2*a^3*b + a^2*b^2)*sqrt((a + b)*b)*d) - 3/8*b*lo
g((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) +
a + 2*b + 2*sqrt((a + b)*b)))/((a^2 + 2*a*b + b^2)*sqrt((a + b)*b)*d) + 1/4
*(2*a^3 + a^2*b + 2*a*b^2 + (2*a^3 - a^2*b - 8*a*b^2 - 8*b^3)*e^(4*d*x + 4*
c) + 2*(2*a^3 + 4*a^2*b + 3*a*b^2 + 4*b^3)*e^(2*d*x + 2*c))/((a^5 + 2*a^4*b
 + a^3*b^2 - (a^5 + 2*a^4*b + a^3*b^2)*e^(6*d*x + 6*c) - (a^5 + 6*a^4*b + 9
*a^3*b^2 + 4*a^2*b^3)*e^(4*d*x + 4*c) + (a^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^2*
b^3)*e^(2*d*x + 2*c))*d) - 1/4*(2*a^3 + a^2*b + 2*a*b^2 + 2*(2*a^3 + 4*a^2*
b + 3*a*b^2 + 4*b^3)*e^(-2*d*x - 2*c) + (2*a^3 - a^2*b - 8*a*b^2 - 8*b^3)*e
^(-4*d*x - 4*c))/((a^5 + 2*a^4*b + a^3*b^2 + (a^5 + 6*a^4*b + 9*a^3*b^2 + 4
*a^2*b^3)*e^(-2*d*x - 2*c) - (a^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*e^(-4*
d*x - 4*c) - (a^5 + 2*a^4*b + a^3*b^2)*e^(-6*d*x - 6*c))*d) - 1/2*(2*a^2 -
a*b + 2*(2*a^2 + 4*a*b - b^2)*e^(-2*d*x - 2*c) + (2*a^2 + a*b + 2*b^2)*e^(-
4*d*x - 4*c))/((a^4 + 2*a^3*b + a^2*b^2 + (a^4 + 6*a^3*b + 9*a^2*b^2 + 4*a*
b^3)*e^(-2*d*x - 2*c) - (a^4 + 6*a^3*b + 9*a^2*b^2 + 4*a*b^3)*e^(-4*d*x - 4
*c) - (a^4 + 2*a^3*b + a^2*b^2)*e^(-6*d*x - 6*c))*d) + 1/2*log(e^(2*d*x + 2
*c) - 1)/((a^2 + 2*a*b + b^2)*d) - 1/2*log(e^(-2*d*x - 2*c) - 1)/((a^2 + 2*
a*b + b^2)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh\left(c+d\,x\right)^4\,\mathrm{coth}\left(c+d\,x\right)^2}{{\left(a\,{\cosh\left(c+d\,x\right)}^2+b\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^2/(a + b/cosh(c + d*x)^2)^2,x)
```

```
[Out] int((cosh(c + d*x)^4*coth(c + d*x)^2)/(b + a*cosh(c + d*x)^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(c+dx)}{\left(a+b\,\text{sech}^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**2/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral(coth(c + d*x)**2/(a + b*sech(c + d*x)**2)**2, x)
```

3.156 $\int \frac{\coth^3(c+dx)}{\left(a+b\,\mathbf{sech}^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=110

$$\frac{b^3}{2a^2d(a+b)^2\left(a\cosh^2(c+dx)+b\right)}+\frac{b^2(3a+b)\log\left(a\cosh^2(c+dx)+b\right)}{2a^2d(a+b)^3}-\frac{\operatorname{csch}^2(c+dx)}{2d(a+b)^2}+\frac{(a+3b)\log(\sinh(c+$$

```
[Out] 1/2*b^3/a^2/(a+b)^2/d/(b+a*cosh(d*x+c)^2)-1/2*csch(d*x+c)^2/(a+b)^2/d+1/2*b
^2*(3*a+b)*ln(b+a*cosh(d*x+c)^2)/a^2/(a+b)^3/d+(a+3*b)*ln(sinh(d*x+c))/d/(a
+b)^3
```

Rubi [A] time = 0.17, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4138, 446, 88}

$$\frac{b^3}{2a^2d(a+b)^2\left(a\cosh^2(c+dx)+b\right)}+\frac{b^2(3a+b)\log\left(a\cosh^2(c+dx)+b\right)}{2a^2d(a+b)^3}-\frac{\operatorname{csch}^2(c+dx)}{2d(a+b)^2}+\frac{(a+3b)\log(\sinh(c+$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^3/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] b^3/(2*a^2*(a + b)^2*d*(b + a*Cosh[c + d*x]^2)) - Csch[c + d*x]^2/(2*(a + b
)^2*d) + (b^2*(3*a + b)*Log[b + a*Cosh[c + d*x]^2])/(2*a^2*(a + b)^3*d) + (
(a + 3*b)*Log[Sinh[c + d*x]])/((a + b)^3*d)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
```

```
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\coth^3(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx = \frac{\operatorname{Subst}\left(\int \frac{x^7}{\left(1-x^2\right)^2\left(b+ax^2\right)^2}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \frac{x^3}{(1-x)^2(b+ax)^2}\,dx, x, \cosh^2(c+dx)\right)}{2d}$$

$$= \frac{\operatorname{Subst}\left(\int \left(\frac{1}{(a+b)^2(-1+x)^2}+\frac{a+3b}{(a+b)^3(-1+x)}-\frac{b^3}{a(a+b)^2(b+ax)^2}+\frac{b^2(3a+b)}{a(a+b)^3(b+ax)}\right)dx, x, \cosh^2(c\right.}{2d}$$

$$= \frac{b^3}{2a^2(a+b)^2d\left(b+a\cosh^2(c+dx)\right)}-\frac{\operatorname{csch}^2(c+dx)}{2(a+b)^2d}+\frac{b^2(3a+b)\log\left(b+a\cosh\right.}{2a^2(a+b)^3d}$$

Mathematica [A] time = 1.28, size = 130, normalized size = 1.18

$$\frac{\operatorname{sech}^4(c+dx)(a\cosh(2(c+dx))+a+2b)^2\left(\frac{b^3(a+b)}{a^2\left(a\sinh^2(c+dx)+a+b\right)}+\frac{b^2(3a+b)\log\left(a\sinh^2(c+dx)+a+b\right)}{a^2}-(a+b)\operatorname{csch}^2(c+\right.}{8d(a+b)^3\left(a+b\operatorname{sech}^2(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^3/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])^2*Sech[c + d*x]^4*(-((a + b)*Csch[c + d*x]
^2) + 2*(a + 3*b)*Log[Sinh[c + d*x]] + (b^2*(3*a + b)*Log[a + b + a*Sinh[c
+ d*x]^2])/a^2 + (b^3*(a + b))/(a^2*(a + b + a*Sinh[c + d*x]^2))))/(8*(a +
b)^3*d*(a + b*Sech[c + d*x]^2)^2)
```

fricas [B] time = 0.72, size = 3624, normalized size = 32.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*cosh(d*x + c)^8 + 16*(a^4 +
 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + 2*(a^4 +
3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*sinh(d*x + c)^8 + 4*(a^4 + a^3*b - a*b^3 -
 b^4 + 2*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d*x)*cosh(d*x + c)^6 + 4*(14*(
a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*cosh(d*x + c)^2 + a^4 + a^3*b - a*b^
3 - b^4 + 2*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d*x)*sinh(d*x + c)^6 + 8*(1
4*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*cosh(d*x + c)^3 + 3*(a^4 + a^3*b
- a*b^3 - b^4 + 2*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d*x)*cosh(d*x + c))*s
inh(d*x + c)^5 + 4*(2*a^4 + 6*a^3*b + 4*a^2*b^2 + 2*a*b^3 + 2*b^4 - (a^4 +
7*a^3*b + 15*a^2*b^2 + 13*a*b^3 + 4*b^4)*d*x)*cosh(d*x + c)^4 + 4*(35*(a^4
+ 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*cosh(d*x + c)^4 + 2*a^4 + 6*a^3*b + 4*a^
2*b^2 + 2*a*b^3 + 2*b^4 - (a^4 + 7*a^3*b + 15*a^2*b^2 + 13*a*b^3 + 4*b^4)*d
*x + 15*(a^4 + a^3*b - a*b^3 - b^4 + 2*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*
d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 16*(7*(a^4 + 3*a^3*b + 3*a^2*b^2 +
a*b^3)*d*x*cosh(d*x + c)^5 + 5*(a^4 + a^3*b - a*b^3 - b^4 + 2*(a^3*b + 3*a^
2*b^2 + 3*a*b^3 + b^4)*d*x)*cosh(d*x + c)^3 + (2*a^4 + 6*a^3*b + 4*a^2*b^2
+ 2*a*b^3 + 2*b^4 - (a^4 + 7*a^3*b + 15*a^2*b^2 + 13*a*b^3 + 4*b^4)*d*x)*co
sh(d*x + c))*sinh(d*x + c)^3 + 2*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x +
4*(a^4 + a^3*b - a*b^3 - b^4 + 2*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d*x)*c
osh(d*x + c)^2 + 4*(14*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*cosh(d*x + c
)^6 + 15*(a^4 + a^3*b - a*b^3 - b^4 + 2*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)
*d*x)*cosh(d*x + c)^4 + a^4 + a^3*b - a*b^3 - b^4 + 2*(a^3*b + 3*a^2*b^2 +
3*a*b^3 + b^4)*d*x + 6*(2*a^4 + 6*a^3*b + 4*a^2*b^2 + 2*a*b^3 + 2*b^4 - (a^
4 + 7*a^3*b + 15*a^2*b^2 + 13*a*b^3 + 4*b^4)*d*x)*cosh(d*x + c)^2)*sinh(d*x
 + c)^2 - ((3*a^2*b^2 + a*b^3)*cosh(d*x + c)^8 + 8*(3*a^2*b^2 + a*b^3)*cosh
(d*x + c)*sinh(d*x + c)^7 + (3*a^2*b^2 + a*b^3)*sinh(d*x + c)^8 + 4*(3*a*b^
3 + b^4)*cosh(d*x + c)^6 + 4*(3*a*b^3 + b^4 + 7*(3*a^2*b^2 + a*b^3)*cosh(d*
x + c)^2)*sinh(d*x + c)^6 + 8*(7*(3*a^2*b^2 + a*b^3)*cosh(d*x + c)^3 + 3*(3
*a*b^3 + b^4)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(3*a^2*b^2 + 13*a*b^3 + 4*
b^4)*cosh(d*x + c)^4 + 2*(35*(3*a^2*b^2 + a*b^3)*cosh(d*x + c)^4 - 3*a^2*b^
2 - 13*a*b^3 - 4*b^4 + 30*(3*a*b^3 + b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4
+ 3*a^2*b^2 + a*b^3 + 8*(7*(3*a^2*b^2 + a*b^3)*cosh(d*x + c)^5 + 10*(3*a*b^
3 + b^4)*cosh(d*x + c)^3 - (3*a^2*b^2 + 13*a*b^3 + 4*b^4)*cosh(d*x + c))*si
nh(d*x + c)^3 + 4*(3*a*b^3 + b^4)*cosh(d*x + c)^2 + 4*(7*(3*a^2*b^2 + a*b^3
)*cosh(d*x + c)^6 + 15*(3*a*b^3 + b^4)*cosh(d*x + c)^4 + 3*a*b^3 + b^4 - 3*
(3*a^2*b^2 + 13*a*b^3 + 4*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((3*a^2
*b^2 + a*b^3)*cosh(d*x + c)^7 + 3*(3*a*b^3 + b^4)*cosh(d*x + c)^5 - (3*a^2*
b^2 + 13*a*b^3 + 4*b^4)*cosh(d*x + c)^3 + (3*a*b^3 + b^4)*cosh(d*x + c))*si
nh(d*x + c))*log(2*(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + a + 2*b)/(cosh(
d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) - 2*((a^4 +
3*a^3*b)*cosh(d*x + c)^8 + 8*(a^4 + 3*a^3*b)*cosh(d*x + c)*sinh(d*x + c)^7
+ (a^4 + 3*a^3*b)*sinh(d*x + c)^8 + 4*(a^3*b + 3*a^2*b^2)*cosh(d*x + c)^6 +
 4*(a^3*b + 3*a^2*b^2 + 7*(a^4 + 3*a^3*b)*cosh(d*x + c)^2)*sinh(d*x + c)^6
+ 8*(7*(a^4 + 3*a^3*b)*cosh(d*x + c)^3 + 3*(a^3*b + 3*a^2*b^2)*cosh(d*x + c
))*sinh(d*x + c)^5 - 2*(a^4 + 7*a^3*b + 12*a^2*b^2)*cosh(d*x + c)^4 + 2*(35
```

```
*(a^4 + 3*a^3*b)*cosh(d*x + c)^4 - a^4 - 7*a^3*b - 12*a^2*b^2 + 30*(a^3*b +
 3*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + a^4 + 3*a^3*b + 8*(7*(a^4 +
3*a^3*b)*cosh(d*x + c)^5 + 10*(a^3*b + 3*a^2*b^2)*cosh(d*x + c)^3 - (a^4 +
7*a^3*b + 12*a^2*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a^3*b + 3*a^2*b^2
)*cosh(d*x + c)^2 + 4*(7*(a^4 + 3*a^3*b)*cosh(d*x + c)^6 + 15*(a^3*b + 3*a^
2*b^2)*cosh(d*x + c)^4 + a^3*b + 3*a^2*b^2 - 3*(a^4 + 7*a^3*b + 12*a^2*b^2)
*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((a^4 + 3*a^3*b)*cosh(d*x + c)^7 + 3*
(a^3*b + 3*a^2*b^2)*cosh(d*x + c)^5 - (a^4 + 7*a^3*b + 12*a^2*b^2)*cosh(d*x
 + c)^3 + (a^3*b + 3*a^2*b^2)*cosh(d*x + c))*sinh(d*x + c))*log(2*sinh(d*x
+ c)/(cosh(d*x + c) - sinh(d*x + c))) + 8*(2*(a^4 + 3*a^3*b + 3*a^2*b^2 + a
*b^3)*d*x*cosh(d*x + c)^7 + 3*(a^4 + a^3*b - a*b^3 - b^4 + 2*(a^3*b + 3*a^2
*b^2 + 3*a*b^3 + b^4)*d*x)*cosh(d*x + c)^5 + 2*(2*a^4 + 6*a^3*b + 4*a^2*b^2
 + 2*a*b^3 + 2*b^4 - (a^4 + 7*a^3*b + 15*a^2*b^2 + 13*a*b^3 + 4*b^4)*d*x)*c
osh(d*x + c)^3 + (a^4 + a^3*b - a*b^3 - b^4 + 2*(a^3*b + 3*a^2*b^2 + 3*a*b^
3 + b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a
^3*b^3)*d*cosh(d*x + c)^8 + 8*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(
d*x + c)*sinh(d*x + c)^7 + (a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*sinh(d*x
 + c)^8 + 4*(a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)^6 + 4
*(7*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^2 + (a^5*b + 3*a^
4*b^2 + 3*a^3*b^3 + a^2*b^4)*d)*sinh(d*x + c)^6 - 2*(a^6 + 7*a^5*b + 15*a^4
*b^2 + 13*a^3*b^3 + 4*a^2*b^4)*d*cosh(d*x + c)^4 + 8*(7*(a^6 + 3*a^5*b + 3*
a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^3 + 3*(a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a
^2*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^6 + 3*a^5*b + 3*a^4*b^2
 + a^3*b^3)*d*cosh(d*x + c)^4 + 30*(a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4
)*d*cosh(d*x + c)^2 - (a^6 + 7*a^5*b + 15*a^4*b^2 + 13*a^3*b^3 + 4*a^2*b^4)
*d)*sinh(d*x + c)^4 + 4*(a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*d*cosh(d*
x + c)^2 + 8*(7*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^5 + 1
0*(a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)^3 - (a^6 + 7*a^
5*b + 15*a^4*b^2 + 13*a^3*b^3 + 4*a^2*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^3
 + 4*(7*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^6 + 15*(a^5*b
 + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)^4 - 3*(a^6 + 7*a^5*b +
15*a^4*b^2 + 13*a^3*b^3 + 4*a^2*b^4)*d*cosh(d*x + c)^2 + (a^5*b + 3*a^4*b^2
 + 3*a^3*b^3 + a^2*b^4)*d)*sinh(d*x + c)^2 + (a^6 + 3*a^5*b + 3*a^4*b^2 + a
^3*b^3)*d + 8*((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^7 + 3*
(a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)^5 - (a^6 + 7*a^5*
b + 15*a^4*b^2 + 13*a^3*b^3 + 4*a^2*b^4)*d*cosh(d*x + c)^3 + (a^5*b + 3*a^4
*b^2 + 3*a^3*b^3 + a^2*b^4)*d*cosh(d*x + c))*sinh(d*x + c))
```

giac [B] time = 1.32, size = 383, normalized size = 3.48

$$\frac{\frac{(3\,ab^2+b^3)\log\left(ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+a\right)}{a^5+3\,a^4b+3\,a^3b^2+a^2b^3}+\frac{2\left(ae^{(2\,c)}+3\,be^{(2\,c)}\right)\log\left(\left|-e^{(2\,dx+2\,c)}+1\right|\right)}{a^3e^{(2\,c)}+3\,a^2be^{(2\,c)}+3\,ab^2e^{(2\,c)}+b^3e^{(2\,c)}}-\frac{2\,dx}{a^2}-\frac{3\,ab^2e^{(4\,dx+4\,c)}+b^3e^{(4\,dx+4\,c)}+6\,ab^2e^{(2}}{\left(a^4+3\,a^3b+3\,a^2b^2+ab^3\right)\left(ae^{(4\,dx+4\,c}\right.}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((3*a*b^2 + b^3)*log(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2
*d*x + 2*c) + a)/(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3) + 2*(a*e^(2*c) + 3*b
*e^(2*c))*log(abs(-e^(2*d*x + 2*c) + 1))/(a^3*e^(2*c) + 3*a^2*b*e^(2*c) + 3
*a*b^2*e^(2*c) + b^3*e^(2*c)) - 2*d*x/a^2 - (3*a*b^2*e^(4*d*x + 4*c) + b^3*
e^(4*d*x + 4*c) + 6*a*b^2*e^(2*d*x + 2*c) + 10*b^3*e^(2*d*x + 2*c) + 3*a*b^
2 + b^3)/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*(a*e^(4*d*x + 4*c) + 2*a*e^(2
*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) + a)) - (3*a*e^(4*d*x + 4*c) + 9*b*e^(4*d
*x + 4*c) - 2*a*e^(2*d*x + 2*c) - 14*b*e^(2*d*x + 2*c) + 3*a + 9*b)/((a^3 +
 3*a^2*b + 3*a*b^2 + b^3)*(e^(2*d*x + 2*c) - 1)^2))/d
```

maple [B] time = 0.53, size = 367, normalized size = 3.34

$$-\frac{\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\left(a^2+2ab+b^2\right)}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d\,a^2}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d\,a^2}-\frac{}{da\,(a+b)^3\left(\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+b\left(\tanh\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] -1/8/d*tanh(1/2*d*x+1/2*c)^2/(a^2+2*a*b+b^2)-1/d/a^2*ln(tanh(1/2*d*x+1/2*c)
-1)-1/d/a^2*ln(tanh(1/2*d*x+1/2*c)+1)-2/d*b^3/a/(a+b)^3*tanh(1/2*d*x+1/2*c)
^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2
*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)+3/2/d*b^2/a/(a+b)^3*ln(tanh(1/2*d*x+1/2*c
)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*
c)^2*b+a+b)+1/2/d*b^3/a^2/(a+b)^3*ln(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x
+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)-1/8/d/(a
+b)^2/tanh(1/2*d*x+1/2*c)^2+1/d/(a+b)^3*ln(tanh(1/2*d*x+1/2*c))*a+3/d/(a+b)
^3*ln(tanh(1/2*d*x+1/2*c))*b
```

maxima [B] time = 0.47, size = 384, normalized size = 3.49

$$\frac{\left(3\,ab^2+b^3\right)\log\left(2\,(a+2\,b)e^{(-2\,dx-2\,c)}+ae^{(-4\,dx-4\,c)}+a\right)}{2\left(a^5+3\,a^4b+3\,a^3b^2+a^2b^3\right)d}+\frac{(a+3\,b)\log\left(e^{(-dx-c)}+1\right)}{\left(a^3+3\,a^2b+3\,ab^2+b^3\right)d}+\frac{(a+3\,b)\log\left(e^{(-dx-c)}-1\right)}{\left(a^3+3\,a^2b+3\,ab^2+b^3\right)d}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(3*a*b^2 + b^3)*log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) +
 a)/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d) + (a + 3*b)*log(e^(-d*x - c)
+ 1)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) + (a + 3*b)*log(e^(-d*x - c) - 1)/
((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) - 2*((a^3 - b^3)*e^(-2*d*x - 2*c) + 2*(
```

```
a^3 + 2*a^2*b + b^3)*e^(-4*d*x - 4*c) + (a^3 - b^3)*e^(-6*d*x - 6*c))/((a^5
 + 2*a^4*b + a^3*b^2 + 4*(a^4*b + 2*a^3*b^2 + a^2*b^3)*e^(-2*d*x - 2*c) - 2
*(a^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*e^(-4*d*x - 4*c) + 4*(a^4*b + 2*a^
3*b^2 + a^2*b^3)*e^(-6*d*x - 6*c) + (a^5 + 2*a^4*b + a^3*b^2)*e^(-8*d*x - 8
*c))*d) + (d*x + c)/(a^2*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh\left(c+d\,x\right)^4\,\coth\left(c+d\,x\right)^3}{{\left(a\,\cosh\left(c+d\,x\right)^2+b\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^3/(a + b/cosh(c + d*x)^2)^2,x)
```

```
[Out] int((cosh(c + d*x)^4*coth(c + d*x)^3)/(b + a*cosh(c + d*x)^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^{3}{\left(c+dx \right)}}{\left(a+b\operatorname{sech}^{2}{\left(c+dx \right)}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**3/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral(coth(c + d*x)**3/(a + b*sech(c + d*x)**2)**2, x)
```

3.157 $\int \frac{\coth^4(c+dx)}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=161

$$-\frac{b^{5/2}(7a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^2d(a+b)^{7/2}}-\frac{\left(2a^2+6ab-b^2\right)\coth(c+dx)}{2ad(a+b)^3}+\frac{x}{a^2}-\frac{(2a-3b)\coth^3(c+dx)}{6ad(a+b)^2}-\frac{b\,c}{2ad(a+b)\left(a\right.}$$

```
[Out] x/a^2-1/2*b^(5/2)*(7*a+2*b)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))/a^2/(a
+b)^(7/2)/d-1/2*(2*a^2+6*a*b-b^2)*coth(d*x+c)/a/(a+b)^3/d-1/6*(2*a-3*b)*cot
h(d*x+c)^3/a/(a+b)^2/d-1/2*b*coth(d*x+c)^3/a/(a+b)/d/(a+b-b*tanh(d*x+c)^2)
```

Rubi [A] time = 0.41, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4141, 1975, 472, 583, 522, 206, 208}

$$-\frac{b^{5/2}(7a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^2d(a+b)^{7/2}}-\frac{\left(2a^2+6ab-b^2\right)\coth(c+dx)}{2ad(a+b)^3}+\frac{x}{a^2}-\frac{(2a-3b)\coth^3(c+dx)}{6ad(a+b)^2}-\frac{b\,c}{2ad(a+b)\left(a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^4/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] x/a^2 - (b^(5/2)*(7*a + 2*b)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/
(2*a^2*(a + b)^(7/2)*d) - ((2*a^2 + 6*a*b - b^2)*Coth[c + d*x])/(2*a*(a + b
)^3*d) - ((2*a - 3*b)*Coth[c + d*x]^3)/(6*a*(a + b)^2*d) - (b*Coth[c + d*x]
^3)/(2*a*(a + b)*d*(a + b - b*Tanh[c + d*x]^2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
```

```
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\coth^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{x^4(1-x^2)\left(a+b(1-x^2)\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \frac{1}{x^4(1-x^2)\left(a+b-bx^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{b\coth^3(c+dx)}{2a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)} - \frac{\operatorname{Subst}\left(\int \frac{-2a+3b-5bx^2}{x^4(1-x^2)(a+b-bx^2)}\,dx, x, \tanh(c+\right.}{2a(a+b)d}$$

$$= -\frac{(2a-3b)\coth^3(c+dx)}{6a(a+b)^2d} - \frac{b\coth^3(c+dx)}{2a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)} + \frac{\operatorname{Subst}\left(\int \frac{3(2a^2}{x}\right.}{}$$

$$= -\frac{\left(2a^2+6ab-b^2\right)\coth(c+dx)}{2a(a+b)^3d} - \frac{(2a-3b)\coth^3(c+dx)}{6a(a+b)^2d} - \frac{b\coth^3(c}{2a(a+b)d\left(a+b-b\right.}$$

$$= -\frac{\left(2a^2+6ab-b^2\right)\coth(c+dx)}{2a(a+b)^3d} - \frac{(2a-3b)\coth^3(c+dx)}{6a(a+b)^2d} - \frac{b\coth^3(c}{2a(a+b)d\left(a+b-b\right.}$$

$$= \frac{x}{a^2} - \frac{b^{5/2}(7a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{7/2}d} - \frac{\left(2a^2+6ab-b^2\right)\coth(c+dx)}{2a(a+b)^3d} - \frac{(2a-}{}$$

Mathematica [B] time = 5.09, size = 350, normalized size = 2.17

$$\frac{\operatorname{sech}^4(c+dx)(a\cosh(2(c+dx))+a+2b)\left(\frac{3b^3\operatorname{sech}(2c)((a+2b)\sinh(2c)-a\sinh(2dx))}{a^2d(a+b)^3} - \frac{3b^3(7a+2b)(\cosh(2c)-\sinh(2c))(a\cosh(2(c+d}{a^2d(}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Coth[c + d*x]^4/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^4*((6*x*(a + 2*b + a*Cosh[2*
(c + d*x)]))/a^2 - (2*(a + 2*b + a*Cosh[2*(c + d*x)])*Coth[c]*Csch[c + d*x]
```

```
^2)/((a + b)^2*d) - (3*b^3*(7*a + 2*b)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh
[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Co
sh[c] - Sinh[c])^4])]*(a + 2*b + a*Cosh[2*(c + d*x)])*(Cosh[2*c] - Sinh[2*c
]))/(a^2*(a + b)^(7/2)*d*Sqrt[b*(Cosh[c] - Sinh[c])^4]) + (4*(2*a + 5*b)*(a
 + 2*b + a*Cosh[2*(c + d*x)])*Csch[c]*Csch[c + d*x]*Sinh[d*x])/((a + b)^3*d
) + (2*(a + 2*b + a*Cosh[2*(c + d*x)])*Csch[c]*Csch[c + d*x]^3*Sinh[d*x])/(
(a + b)^2*d) + (3*b^3*Sech[2*c]*((a + 2*b)*Sinh[2*c] - a*Sinh[2*d*x]))/(a^2
*(a + b)^3*d)))/(24*(a + b*Sech[c + d*x]^2)^2)
```

fricas [B] time = 0.61, size = 9849, normalized size = 61.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(12*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*cosh(d*x + c)^10 + 120*(a
^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*cosh(d*x + c)*sinh(d*x + c)^9 + 12*(a
^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*sinh(d*x + c)^10 - 12*(4*a^4 + 8*a^3*
b - a*b^3 - 2*b^4 + (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*x)*cosh(
d*x + c)^8 + 12*(45*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*cosh(d*x + c)^2
 - 4*a^4 - 8*a^3*b + a*b^3 + 2*b^4 - (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 -
4*b^4)*d*x)*sinh(d*x + c)^8 + 96*(15*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*
x*cosh(d*x + c)^3 - (4*a^4 + 8*a^3*b - a*b^3 - 2*b^4 + (a^4 - a^3*b - 9*a^2
*b^2 - 11*a*b^3 - 4*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c)^7 - 24*(2*a^4 +
10*a^3*b + 16*a^2*b^2 + a*b^3 + 3*b^4 + (a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*
b^3 + 6*b^4)*d*x)*cosh(d*x + c)^6 + 24*(105*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*
b^3)*d*x*cosh(d*x + c)^4 - 2*a^4 - 10*a^3*b - 16*a^2*b^2 - a*b^3 - 3*b^4 -
(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*x - 14*(4*a^4 + 8*a^3*b -
 a*b^3 - 2*b^4 + (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*x)*cosh(d*x
 + c)^2)*sinh(d*x + c)^6 + 48*(63*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*c
osh(d*x + c)^5 - 14*(4*a^4 + 8*a^3*b - a*b^3 - 2*b^4 + (a^4 - a^3*b - 9*a^2
*b^2 - 11*a*b^3 - 4*b^4)*d*x)*cosh(d*x + c)^3 - 3*(2*a^4 + 10*a^3*b + 16*a^
2*b^2 + a*b^3 + 3*b^4 + (a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*x
)*cosh(d*x + c))*sinh(d*x + c)^5 + 8*(2*a^4 + 38*a^3*b + 72*a^2*b^2 + 9*b^4
 + 3*(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*x)*cosh(d*x + c)^4 +
 8*(315*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*cosh(d*x + c)^6 - 105*(4*a^
4 + 8*a^3*b - a*b^3 - 2*b^4 + (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*
d*x)*cosh(d*x + c)^4 + 2*a^4 + 38*a^3*b + 72*a^2*b^2 + 9*b^4 + 3*(a^4 + 9*a
^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*x - 45*(2*a^4 + 10*a^3*b + 16*a^2*b
^2 + a*b^3 + 3*b^4 + (a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*x)*c
osh(d*x + c)^2)*sinh(d*x + c)^4 - 32*a^4 - 80*a^3*b - 12*a*b^3 + 32*(45*(a^
4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*cosh(d*x + c)^7 - 21*(4*a^4 + 8*a^3*b
- a*b^3 - 2*b^4 + (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*x)*cosh(d*
x + c)^5 - 15*(2*a^4 + 10*a^3*b + 16*a^2*b^2 + a*b^3 + 3*b^4 + (a^4 + 9*a^3
```

*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*x)*cosh(d*x + c)^3 + (2*a^4 + 38*a^3*
b + 72*a^2*b^2 + 9*b^4 + 3*(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*
d*x)*cosh(d*x + c))*sinh(d*x + c)^3 - 12*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3
)*d*x - 4*(4*a^4 + 36*a^3*b + 80*a^2*b^2 - 6*a*b^3 + 6*b^4 - 3*(a^4 - a^3*b
 - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*x)*cosh(d*x + c)^2 + 4*(135*(a^4 + 3*a^3
*b + 3*a^2*b^2 + a*b^3)*d*x*cosh(d*x + c)^8 - 84*(4*a^4 + 8*a^3*b - a*b^3 -
 2*b^4 + (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*x)*cosh(d*x + c)^6
- 90*(2*a^4 + 10*a^3*b + 16*a^2*b^2 + a*b^3 + 3*b^4 + (a^4 + 9*a^3*b + 21*a
^2*b^2 + 19*a*b^3 + 6*b^4)*d*x)*cosh(d*x + c)^4 - 4*a^4 - 36*a^3*b - 80*a^2
*b^2 + 6*a*b^3 - 6*b^4 + 3*(a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*x
 + 12*(2*a^4 + 38*a^3*b + 72*a^2*b^2 + 9*b^4 + 3*(a^4 + 9*a^3*b + 21*a^2*b^
2 + 19*a*b^3 + 6*b^4)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 3*((7*a^2*b^2
 + 2*a*b^3)*cosh(d*x + c)^10 + 10*(7*a^2*b^2 + 2*a*b^3)*cosh(d*x + c)*sinh(
d*x + c)^9 + (7*a^2*b^2 + 2*a*b^3)*sinh(d*x + c)^10 - (7*a^2*b^2 - 26*a*b^3
 - 8*b^4)*cosh(d*x + c)^8 - (7*a^2*b^2 - 26*a*b^3 - 8*b^4 - 45*(7*a^2*b^2 +
 2*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 8*(15*(7*a^2*b^2 + 2*a*b^3)*co
sh(d*x + c)^3 - (7*a^2*b^2 - 26*a*b^3 - 8*b^4)*cosh(d*x + c))*sinh(d*x + c)
^7 - 2*(7*a^2*b^2 + 44*a*b^3 + 12*b^4)*cosh(d*x + c)^6 + 2*(105*(7*a^2*b^2
+ 2*a*b^3)*cosh(d*x + c)^4 - 7*a^2*b^2 - 44*a*b^3 - 12*b^4 - 14*(7*a^2*b^2
- 26*a*b^3 - 8*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(63*(7*a^2*b^2 + 2
*a*b^3)*cosh(d*x + c)^5 - 14*(7*a^2*b^2 - 26*a*b^3 - 8*b^4)*cosh(d*x + c)^3
 - 3*(7*a^2*b^2 + 44*a*b^3 + 12*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(7*
a^2*b^2 + 44*a*b^3 + 12*b^4)*cosh(d*x + c)^4 + 2*(105*(7*a^2*b^2 + 2*a*b^3)
*cosh(d*x + c)^6 - 35*(7*a^2*b^2 - 26*a*b^3 - 8*b^4)*cosh(d*x + c)^4 + 7*a^
2*b^2 + 44*a*b^3 + 12*b^4 - 15*(7*a^2*b^2 + 44*a*b^3 + 12*b^4)*cosh(d*x + c
)^2)*sinh(d*x + c)^4 - 7*a^2*b^2 - 2*a*b^3 + 8*(15*(7*a^2*b^2 + 2*a*b^3)*co
sh(d*x + c)^7 - 7*(7*a^2*b^2 - 26*a*b^3 - 8*b^4)*cosh(d*x + c)^5 - 5*(7*a^2
*b^2 + 44*a*b^3 + 12*b^4)*cosh(d*x + c)^3 + (7*a^2*b^2 + 44*a*b^3 + 12*b^4)
*cosh(d*x + c))*sinh(d*x + c)^3 + (7*a^2*b^2 - 26*a*b^3 - 8*b^4)*cosh(d*x +
 c)^2 + (45*(7*a^2*b^2 + 2*a*b^3)*cosh(d*x + c)^8 - 28*(7*a^2*b^2 - 26*a*b^
3 - 8*b^4)*cosh(d*x + c)^6 - 30*(7*a^2*b^2 + 44*a*b^3 + 12*b^4)*cosh(d*x +
c)^4 + 7*a^2*b^2 - 26*a*b^3 - 8*b^4 + 12*(7*a^2*b^2 + 44*a*b^3 + 12*b^4)*co
sh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(5*(7*a^2*b^2 + 2*a*b^3)*cosh(d*x + c)^9
 - 4*(7*a^2*b^2 - 26*a*b^3 - 8*b^4)*cosh(d*x + c)^7 - 6*(7*a^2*b^2 + 44*a*b
^3 + 12*b^4)*cosh(d*x + c)^5 + 4*(7*a^2*b^2 + 44*a*b^3 + 12*b^4)*cosh(d*x +
 c)^3 + (7*a^2*b^2 - 26*a*b^3 - 8*b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(b
/(a + b))*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 +
a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x +
 c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*
x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) + 4*((a^2 + a*b)*cosh
(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(
d*x + c)^2 + a^2 + 3*a*b + 2*b^2)*sqrt(b/(a + b)))/(a*cosh(d*x + c)^4 + 4*a
*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x +
 c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x +
 c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) + 8*(15*(a^4 + 3*a^3*b

+ 3*a^2*b^2 + a*b^3)*d*x*cosh(d*x + c)^9 - 12*(4*a^4 + 8*a^3*b - a*b^3 - 2
*b^4 + (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*x)*cosh(d*x + c)^7 -
18*(2*a^4 + 10*a^3*b + 16*a^2*b^2 + a*b^3 + 3*b^4 + (a^4 + 9*a^3*b + 21*a^2
*b^2 + 19*a*b^3 + 6*b^4)*d*x)*cosh(d*x + c)^5 + 4*(2*a^4 + 38*a^3*b + 72*a^
2*b^2 + 9*b^4 + 3*(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*x)*cosh
(d*x + c)^3 - (4*a^4 + 36*a^3*b + 80*a^2*b^2 - 6*a*b^3 + 6*b^4 - 3*(a^4 - a
^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a
^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^10 + 10*(a^6 + 3*a^5*b
+ 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^9 + (a^6 + 3*a^5*b + 3
*a^4*b^2 + a^3*b^3)*d*sinh(d*x + c)^10 - (a^6 - a^5*b - 9*a^4*b^2 - 11*a^3*
b^3 - 4*a^2*b^4)*d*cosh(d*x + c)^8 + (45*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b
^3)*d*cosh(d*x + c)^2 - (a^6 - a^5*b - 9*a^4*b^2 - 11*a^3*b^3 - 4*a^2*b^4)*
d)*sinh(d*x + c)^8 - 2*(a^6 + 9*a^5*b + 21*a^4*b^2 + 19*a^3*b^3 + 6*a^2*b^4
)*d*cosh(d*x + c)^6 + 8*(15*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*
x + c)^3 - (a^6 - a^5*b - 9*a^4*b^2 - 11*a^3*b^3 - 4*a^2*b^4)*d*cosh(d*x +
c))*sinh(d*x + c)^7 + 2*(105*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d
x + c)^4 - 14(a^6 - a^5*b - 9*a^4*b^2 - 11*a^3*b^3 - 4*a^2*b^4)*d*cosh(d*
x + c)^2 - (a^6 + 9*a^5*b + 21*a^4*b^2 + 19*a^3*b^3 + 6*a^2*b^4)*d)*sinh(d*
x + c)^6 + 2*(a^6 + 9*a^5*b + 21*a^4*b^2 + 19*a^3*b^3 + 6*a^2*b^4)*d*cosh(d
x + c)^4 + 4(63*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^5 -
14*(a^6 - a^5*b - 9*a^4*b^2 - 11*a^3*b^3 - 4*a^2*b^4)*d*cosh(d*x + c)^3 -
3*(a^6 + 9*a^5*b + 21*a^4*b^2 + 19*a^3*b^3 + 6*a^2*b^4)*d*cosh(d*x + c))*si
nh(d*x + c)^5 + 2*(105*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c
)^6 - 35*(a^6 - a^5*b - 9*a^4*b^2 - 11*a^3*b^3 - 4*a^2*b^4)*d*cosh(d*x + c)
^4 - 15*(a^6 + 9*a^5*b + 21*a^4*b^2 + 19*a^3*b^3 + 6*a^2*b^4)*d*cosh(d*x +
c)^2 + (a^6 + 9*a^5*b + 21*a^4*b^2 + 19*a^3*b^3 + 6*a^2*b^4)*d)*sinh(d*x +
c)^4 + (a^6 - a^5*b - 9*a^4*b^2 - 11*a^3*b^3 - 4*a^2*b^4)*d*cosh(d*x + c)^2
+ 8*(15*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^7 - 7*(a^6 -
a^5*b - 9*a^4*b^2 - 11*a^3*b^3 - 4*a^2*b^4)*d*cosh(d*x + c)^5 - 5*(a^6 + 9
*a^5*b + 21*a^4*b^2 + 19*a^3*b^3 + 6*a^2*b^4)*d*cosh(d*x + c)^3 + (a^6 + 9*
a^5*b + 21*a^4*b^2 + 19*a^3*b^3 + 6*a^2*b^4)*d*cosh(d*x + c))*sinh(d*x + c)
^3 + (45*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^8 - 28*(a^6
- a^5*b - 9*a^4*b^2 - 11*a^3*b^3 - 4*a^2*b^4)*d*cosh(d*x + c)^6 - 30*(a^6 +
9*a^5*b + 21*a^4*b^2 + 19*a^3*b^3 + 6*a^2*b^4)*d*cosh(d*x + c)^4 + 12*(a^6
+ 9*a^5*b + 21*a^4*b^2 + 19*a^3*b^3 + 6*a^2*b^4)*d*cosh(d*x + c)^2 + (a^6
- a^5*b - 9*a^4*b^2 - 11*a^3*b^3 - 4*a^2*b^4)*d)*sinh(d*x + c)^2 - (a^6 + 3
*a^5*b + 3*a^4*b^2 + a^3*b^3)*d + 2*(5*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3
)*d*cosh(d*x + c)^9 - 4*(a^6 - a^5*b - 9*a^4*b^2 - 11*a^3*b^3 - 4*a^2*b^4)*
d*cosh(d*x + c)^7 - 6*(a^6 + 9*a^5*b + 21*a^4*b^2 + 19*a^3*b^3 + 6*a^2*b^4)
*d*cosh(d*x + c)^5 + 4*(a^6 + 9*a^5*b + 21*a^4*b^2 + 19*a^3*b^3 + 6*a^2*b^4
)*d*cosh(d*x + c)^3 + (a^6 - a^5*b - 9*a^4*b^2 - 11*a^3*b^3 - 4*a^2*b^4)*d*
cosh(d*x + c))*sinh(d*x + c)), 1/6*(6*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d
*x*cosh(d*x + c)^10 + 60*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*cosh(d*x +
c)*sinh(d*x + c)^9 + 6*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*sinh(d*x +
c)^10 - 6*(4*a^4 + 8*a^3*b - a*b^3 - 2*b^4 + (a^4 - a^3*b - 9*a^2*b^2 - 11*

a*b^3 - 4*b^4)*d*x)*cosh(d*x + c)^8 + 6*(45*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*
b^3)*d*x*cosh(d*x + c)^2 - 4*a^4 - 8*a^3*b + a*b^3 + 2*b^4 - (a^4 - a^3*b -
9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*x)*sinh(d*x + c)^8 + 48*(15*(a^4 + 3*a^3*b
+ 3*a^2*b^2 + a*b^3)*d*x*cosh(d*x + c)^3 - (4*a^4 + 8*a^3*b - a*b^3 - 2*b^
4 + (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*x)*cosh(d*x + c))*sinh(d
x + c)^7 - 12(2*a^4 + 10*a^3*b + 16*a^2*b^2 + a*b^3 + 3*b^4 + (a^4 + 9*a^
3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*x)*cosh(d*x + c)^6 + 12*(105*(a^4 +
3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*cosh(d*x + c)^4 - 2*a^4 - 10*a^3*b - 16*a^
2*b^2 - a*b^3 - 3*b^4 - (a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*x
- 14*(4*a^4 + 8*a^3*b - a*b^3 - 2*b^4 + (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^
3 - 4*b^4)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 24*(63*(a^4 + 3*a^3*b +
3*a^2*b^2 + a*b^3)*d*x*cosh(d*x + c)^5 - 14*(4*a^4 + 8*a^3*b - a*b^3 - 2*b^
4 + (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*x)*cosh(d*x + c)^3 - 3*(
2*a^4 + 10*a^3*b + 16*a^2*b^2 + a*b^3 + 3*b^4 + (a^4 + 9*a^3*b + 21*a^2*b^2
+ 19*a*b^3 + 6*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 + 4*(2*a^4 + 38*a^
3*b + 72*a^2*b^2 + 9*b^4 + 3*(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4
)*d*x)*cosh(d*x + c)^4 + 4*(315*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*cos
h(d*x + c)^6 - 105*(4*a^4 + 8*a^3*b - a*b^3 - 2*b^4 + (a^4 - a^3*b - 9*a^2*
b^2 - 11*a*b^3 - 4*b^4)*d*x)*cosh(d*x + c)^4 + 2*a^4 + 38*a^3*b + 72*a^2*b^
2 + 9*b^4 + 3*(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*x - 45*(2*a
^4 + 10*a^3*b + 16*a^2*b^2 + a*b^3 + 3*b^4 + (a^4 + 9*a^3*b + 21*a^2*b^2 +
19*a*b^3 + 6*b^4)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 - 16*a^4 - 40*a^3*b
- 6*a*b^3 + 16*(45*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*cosh(d*x + c)^7
- 21*(4*a^4 + 8*a^3*b - a*b^3 - 2*b^4 + (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^
3 - 4*b^4)*d*x)*cosh(d*x + c)^5 - 15*(2*a^4 + 10*a^3*b + 16*a^2*b^2 + a*b^3
+ 3*b^4 + (a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*x)*cosh(d*x +
c)^3 + (2*a^4 + 38*a^3*b + 72*a^2*b^2 + 9*b^4 + 3*(a^4 + 9*a^3*b + 21*a^2*b
^2 + 19*a*b^3 + 6*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 - 6*(a^4 + 3*a^3
*b + 3*a^2*b^2 + a*b^3)*d*x - 2*(4*a^4 + 36*a^3*b + 80*a^2*b^2 - 6*a*b^3 +
6*b^4 - 3*(a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*x)*cosh(d*x + c)^2
+ 2*(135*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*cosh(d*x + c)^8 - 84*(4*a
^4 + 8*a^3*b - a*b^3 - 2*b^4 + (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)
*d*x)*cosh(d*x + c)^6 - 90*(2*a^4 + 10*a^3*b + 16*a^2*b^2 + a*b^3 + 3*b^4 +
(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*x)*cosh(d*x + c)^4 - 4*a
^4 - 36*a^3*b - 80*a^2*b^2 + 6*a*b^3 - 6*b^4 + 3*(a^4 - a^3*b - 9*a^2*b^2 -
11*a*b^3 - 4*b^4)*d*x + 12*(2*a^4 + 38*a^3*b + 72*a^2*b^2 + 9*b^4 + 3*(a^4
+ 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*x)*cosh(d*x + c)^2)*sinh(d*x
+ c)^2 - 3*((7*a^2*b^2 + 2*a*b^3)*cosh(d*x + c)^10 + 10*(7*a^2*b^2 + 2*a*b^
3)*cosh(d*x + c)*sinh(d*x + c)^9 + (7*a^2*b^2 + 2*a*b^3)*sinh(d*x + c)^10 -
(7*a^2*b^2 - 26*a*b^3 - 8*b^4)*cosh(d*x + c)^8 - (7*a^2*b^2 - 26*a*b^3 - 8
b^4 - 45(7*a^2*b^2 + 2*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 8*(15*(7
*a^2*b^2 + 2*a*b^3)*cosh(d*x + c)^3 - (7*a^2*b^2 - 26*a*b^3 - 8*b^4)*cosh(d
*x + c))*sinh(d*x + c)^7 - 2*(7*a^2*b^2 + 44*a*b^3 + 12*b^4)*cosh(d*x + c)^
6 + 2*(105*(7*a^2*b^2 + 2*a*b^3)*cosh(d*x + c)^4 - 7*a^2*b^2 - 44*a*b^3 - 1
2*b^4 - 14*(7*a^2*b^2 - 26*a*b^3 - 8*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6

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+ 4*(63*(7*a^2*b^2 + 2*a*b^3)*cosh(d*x + c)^5 - 14*(7*a^2*b^2 - 26*a*b^3 -
8*b^4)*cosh(d*x + c)^3 - 3*(7*a^2*b^2 + 44*a*b^3 + 12*b^4)*cosh(d*x + c))*s
inh(d*x + c)^5 + 2*(7*a^2*b^2 + 44*a*b^3 + 12*b^4)*cosh(d*x + c)^4 + 2*(105
*(7*a^2*b^2 + 2*a*b^3)*cosh(d*x + c)^6 - 35*(7*a^2*b^2 - 26*a*b^3 - 8*b^4)*
cosh(d*x + c)^4 + 7*a^2*b^2 + 44*a*b^3 + 12*b^4 - 15*(7*a^2*b^2 + 44*a*b^3
+ 12*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 - 7*a^2*b^2 - 2*a*b^3 + 8*(15*(7
*a^2*b^2 + 2*a*b^3)*cosh(d*x + c)^7 - 7*(7*a^2*b^2 - 26*a*b^3 - 8*b^4)*cosh
(d*x + c)^5 - 5*(7*a^2*b^2 + 44*a*b^3 + 12*b^4)*cosh(d*x + c)^3 + (7*a^2*b^
2 + 44*a*b^3 + 12*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + (7*a^2*b^2 - 26*a*b
^3 - 8*b^4)*cosh(d*x + c)^2 + (45*(7*a^2*b^2 + 2*a*b^3)*cosh(d*x + c)^8 - 2
8*(7*a^2*b^2 - 26*a*b^3 - 8*b^4)*cosh(d*x + c)^6 - 30*(7*a^2*b^2 + 44*a*b^3
 + 12*b^4)*cosh(d*x + c)^4 + 7*a^2*b^2 - 26*a*b^3 - 8*b^4 + 12*(7*a^2*b^2 +
 44*a*b^3 + 12*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(5*(7*a^2*b^2 + 2*
a*b^3)*cosh(d*x + c)^9 - 4*(7*a^2*b^2 - 26*a*b^3 - 8*b^4)*cosh(d*x + c)^7 -
 6*(7*a^2*b^2 + 44*a*b^3 + 12*b^4)*cosh(d*x + c)^5 + 4*(7*a^2*b^2 + 44*a*b^
3 + 12*b^4)*cosh(d*x + c)^3 + (7*a^2*b^2 - 26*a*b^3 - 8*b^4)*cosh(d*x + c))
*sinh(d*x + c))*sqrt(-b/(a + b))*arctan(1/2*(a*cosh(d*x + c)^2 + 2*a*cosh(d
*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-b/(a + b))/b) +
4*(15*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*cosh(d*x + c)^9 - 12*(4*a^4 +
 8*a^3*b - a*b^3 - 2*b^4 + (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*x
)*cosh(d*x + c)^7 - 18*(2*a^4 + 10*a^3*b + 16*a^2*b^2 + a*b^3 + 3*b^4 + (a^
4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*x)*cosh(d*x + c)^5 + 4*(2*a^
4 + 38*a^3*b + 72*a^2*b^2 + 9*b^4 + 3*(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^
3 + 6*b^4)*d*x)*cosh(d*x + c)^3 - (4*a^4 + 36*a^3*b + 80*a^2*b^2 - 6*a*b^3
+ 6*b^4 - 3*(a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*x)*cosh(d*x + c)
)*sinh(d*x + c))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^10
+ 10*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^9
+ (a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*sinh(d*x + c)^10 - (a^6 - a^5*b -
 9*a^4*b^2 - 11*a^3*b^3 - 4*a^2*b^4)*d*cosh(d*x + c)^8 + (45*(a^6 + 3*a^5*b
 + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^2 - (a^6 - a^5*b - 9*a^4*b^2 - 11*a
^3*b^3 - 4*a^2*b^4)*d)*sinh(d*x + c)^8 - 2*(a^6 + 9*a^5*b + 21*a^4*b^2 + 19
*a^3*b^3 + 6*a^2*b^4)*d*cosh(d*x + c)^6 + 8*(15*(a^6 + 3*a^5*b + 3*a^4*b^2
+ a^3*b^3)*d*cosh(d*x + c)^3 - (a^6 - a^5*b - 9*a^4*b^2 - 11*a^3*b^3 - 4*a^
2*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(105*(a^6 + 3*a^5*b + 3*a^4*b^2
 + a^3*b^3)*d*cosh(d*x + c)^4 - 14*(a^6 - a^5*b - 9*a^4*b^2 - 11*a^3*b^3 -
4*a^2*b^4)*d*cosh(d*x + c)^2 - (a^6 + 9*a^5*b + 21*a^4*b^2 + 19*a^3*b^3 + 6
*a^2*b^4)*d)*sinh(d*x + c)^6 + 2*(a^6 + 9*a^5*b + 21*a^4*b^2 + 19*a^3*b^3 +
 6*a^2*b^4)*d*cosh(d*x + c)^4 + 4*(63*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)
*d*cosh(d*x + c)^5 - 14*(a^6 - a^5*b - 9*a^4*b^2 - 11*a^3*b^3 - 4*a^2*b^4)*
d*cosh(d*x + c)^3 - 3*(a^6 + 9*a^5*b + 21*a^4*b^2 + 19*a^3*b^3 + 6*a^2*b^4)
*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(105*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3
*b^3)*d*cosh(d*x + c)^6 - 35*(a^6 - a^5*b - 9*a^4*b^2 - 11*a^3*b^3 - 4*a^2*
b^4)*d*cosh(d*x + c)^4 - 15*(a^6 + 9*a^5*b + 21*a^4*b^2 + 19*a^3*b^3 + 6*a^
2*b^4)*d*cosh(d*x + c)^2 + (a^6 + 9*a^5*b + 21*a^4*b^2 + 19*a^3*b^3 + 6*a^2
*b^4)*d)*sinh(d*x + c)^4 + (a^6 - a^5*b - 9*a^4*b^2 - 11*a^3*b^3 - 4*a^2*b^
```

```
4)*d*cosh(d*x + c)^2 + 8*(15*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d
*x + c)^7 - 7*(a^6 - a^5*b - 9*a^4*b^2 - 11*a^3*b^3 - 4*a^2*b^4)*d*cosh(d*x
 + c)^5 - 5*(a^6 + 9*a^5*b + 21*a^4*b^2 + 19*a^3*b^3 + 6*a^2*b^4)*d*cosh(d*
x + c)^3 + (a^6 + 9*a^5*b + 21*a^4*b^2 + 19*a^3*b^3 + 6*a^2*b^4)*d*cosh(d*x
 + c))*sinh(d*x + c)^3 + (45*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d
*x + c)^8 - 28*(a^6 - a^5*b - 9*a^4*b^2 - 11*a^3*b^3 - 4*a^2*b^4)*d*cosh(d*
x + c)^6 - 30*(a^6 + 9*a^5*b + 21*a^4*b^2 + 19*a^3*b^3 + 6*a^2*b^4)*d*cosh(
d*x + c)^4 + 12*(a^6 + 9*a^5*b + 21*a^4*b^2 + 19*a^3*b^3 + 6*a^2*b^4)*d*cos
h(d*x + c)^2 + (a^6 - a^5*b - 9*a^4*b^2 - 11*a^3*b^3 - 4*a^2*b^4)*d)*sinh(d
*x + c)^2 - (a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d + 2*(5*(a^6 + 3*a^5*b +
 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^9 - 4*(a^6 - a^5*b - 9*a^4*b^2 - 11*a
^3*b^3 - 4*a^2*b^4)*d*cosh(d*x + c)^7 - 6*(a^6 + 9*a^5*b + 21*a^4*b^2 + 19*
a^3*b^3 + 6*a^2*b^4)*d*cosh(d*x + c)^5 + 4*(a^6 + 9*a^5*b + 21*a^4*b^2 + 19
*a^3*b^3 + 6*a^2*b^4)*d*cosh(d*x + c)^3 + (a^6 - a^5*b - 9*a^4*b^2 - 11*a^3
*b^3 - 4*a^2*b^4)*d*cosh(d*x + c))*sinh(d*x + c))]
```

giac [B] time = 1.60, size = 302, normalized size = 1.88

$$-\frac{\frac{3\left(7\,ab^3e^{(2\,c)}+2\,b^4e^{(2\,c)}\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)e^{(-2\,c)}}{\left(a^5+3\,a^4b+3\,a^3b^2+a^2b^3\right)\sqrt{-ab-b^2}}-\frac{6\,dx}{a^2}-\frac{6\left(ab^3e^{(2\,dx+2\,c)}+2\,b^4e^{(2\,dx+2\,c)}+ab^3\right)}{\left(a^5+3\,a^4b+3\,a^3b^2+a^2b^3\right)\left(ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+a\right)}+\frac{8\left(3\,ae^{(4\,dx}\cdots\right.}{\cdots}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/6*(3*(7*a*b^3*e^(2*c) + 2*b^4*e^(2*c))*arctan(1/2*(a*e^(2*d*x + 2*c) + a
 + 2*b)/sqrt(-a*b - b^2))*e^(-2*c)/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*s
qrt(-a*b - b^2)) - 6*d*x/a^2 - 6*(a*b^3*e^(2*d*x + 2*c) + 2*b^4*e^(2*d*x +
2*c) + a*b^3)/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*(a*e^(4*d*x + 4*c) + 2
*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) + a)) + 8*(3*a*e^(4*d*x + 4*c) + 6
*b*e^(4*d*x + 4*c) - 3*a*e^(2*d*x + 2*c) - 9*b*e^(2*d*x + 2*c) + 2*a + 5*b)
/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(e^(2*d*x + 2*c) - 1)^3))/d
```

maple [B] time = 0.52, size = 634, normalized size = 3.94

$$-\frac{a\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d\left(a^2+2ab+b^2\right)(a+b)}-\frac{\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b}{24d\left(a^2+2ab+b^2\right)(a+b)}-\frac{5a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\left(a^2+2ab+b^2\right)(a+b)}-\frac{13\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)b}{8d\left(a^2+2ab+b^2\right)(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x)
```

```
[Out] -1/24/d/(a^2+2*a*b+b^2)/(a+b)*a*tanh(1/2*d*x+1/2*c)^3-1/24/d/(a^2+2*a*b+b^2
)/(a+b)*tanh(1/2*d*x+1/2*c)^3*b-5/8/d/(a^2+2*a*b+b^2)/(a+b)*a*tanh(1/2*d*x+
1/2*c)-13/8/d/(a^2+2*a*b+b^2)/(a+b)*tanh(1/2*d*x+1/2*c)*b-1/d/a^2*ln(tanh(1
/2*d*x+1/2*c)-1)+1/d/a^2*ln(tanh(1/2*d*x+1/2*c)+1)-1/d*b^3/a/(a+b)^3/(tanh(
1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh
(1/2*d*x+1/2*c)^2*b+a+b)*tanh(1/2*d*x+1/2*c)^3-1/d*b^3/a/(a+b)^3/(tanh(1/2*
d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2
*d*x+1/2*c)^2*b+a+b)*tanh(1/2*d*x+1/2*c)+7/4/d*b^(5/2)/a/(a+b)^(7/2)*ln(-(a
+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)-(a+b)^(1/2))-
7/4/d*b^(5/2)/a/(a+b)^(7/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*
tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))+1/2/d*b^(7/2)/a^2/(a+b)^(7/2)*ln(-(a+b)^(1
/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)-(a+b)^(1/2))-1/2/d*
b^(7/2)/a^2/(a+b)^(7/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh
(1/2*d*x+1/2*c)+(a+b)^(1/2))-1/24/d/(a+b)^2/tanh(1/2*d*x+1/2*c)^3-5/8/d/(a+
b)^3/tanh(1/2*d*x+1/2*c)*a-13/8/d/(a+b)^3/tanh(1/2*d*x+1/2*c)*b
```

maxima [B] time = 1.41, size = 2961, normalized size = 18.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(a^2*b + 3*a*b^2 + b^3)*log(a*e^(4*d*x + 4*c) + 2*(a + 2*b)*e^(2*d*x +
2*c) + a)/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d) - 1/2*b*log(a*e^(4*d*x
+ 4*c) + 2*(a + 2*b)*e^(2*d*x + 2*c) + a)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*
d) - 1/4*(a^2*b + 3*a*b^2 + b^3)*log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4
*d*x - 4*c) + a)/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d) + 1/2*b*log(2*(a
 + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/((a^3 + 3*a^2*b + 3*a*b^
2 + b^3)*d) + 1/2*(a + 2*b)*log(e^(2*d*x + 2*c) - 1)/((a^3 + 3*a^2*b + 3*a*
b^2 + b^3)*d) + b*log(e^(2*d*x + 2*c) - 1)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)
*d) - 1/2*(a + 2*b)*log(e^(-2*d*x - 2*c) - 1)/((a^3 + 3*a^2*b + 3*a*b^2 + b
^3)*d) - b*log(e^(-2*d*x - 2*c) - 1)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) -
1/64*(3*a^3*b + 38*a^2*b^2 + 56*a*b^3 + 16*b^4)*log((a*e^(2*d*x + 2*c) + a
+ 2*b - 2*sqrt((a + b)*b))/(a*e^(2*d*x + 2*c) + a + 2*b + 2*sqrt((a + b)*b)
))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*sqrt((a + b)*b)*d) + 1/16*(3*a*b
+ 8*b^2)*log((a*e^(2*d*x + 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(2*d*x
+ 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqr
t((a + b)*b)*d) + 1/64*(3*a^3*b + 38*a^2*b^2 + 56*a*b^3 + 16*b^4)*log((a*e^
(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b
 + 2*sqrt((a + b)*b)))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*sqrt((a + b)*
b)*d) - 1/16*(3*a*b + 8*b^2)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a
+ b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^3 + 3*a^2*
b + 3*a*b^2 + b^3)*sqrt((a + b)*b)*d) + 3/32*(3*a*b - 2*b^2)*log((a*e^(-2*d
*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*
```

sqrt((a + b)*b)))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt((a + b)*b)*d) + 1/4
8*(44*a^4 + 59*a^3*b + 24*a^2*b^2 + 24*a*b^3 + 3*(24*a^4 + 27*a^3*b - 18*a^
2*b^2 - 48*a*b^3 - 32*b^4)*e^(8*d*x + 8*c) + 6*(6*a^4 + 55*a^3*b + 79*a^2*b
^2 + 68*a*b^3 + 48*b^4)*e^(6*d*x + 6*c) - 2*(50*a^4 + 278*a^3*b + 309*a^2*b
^2 + 180*a*b^3 + 144*b^4)*e^(4*d*x + 4*c) - 2*(10*a^4 - 75*a^3*b - 103*a^2*
b^2 - 36*a*b^3 - 48*b^4)*e^(2*d*x + 2*c))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3
*b^3 - (a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*e^(10*d*x + 10*c) + (a^6 - a^5
*b - 9*a^4*b^2 - 11*a^3*b^3 - 4*a^2*b^4)*e^(8*d*x + 8*c) + 2*(a^6 + 9*a^5*b
 + 21*a^4*b^2 + 19*a^3*b^3 + 6*a^2*b^4)*e^(6*d*x + 6*c) - 2*(a^6 + 9*a^5*b
+ 21*a^4*b^2 + 19*a^3*b^3 + 6*a^2*b^4)*e^(4*d*x + 4*c) - (a^6 - a^5*b - 9*a
^4*b^2 - 11*a^3*b^3 - 4*a^2*b^4)*e^(2*d*x + 2*c))*d) - 1/48*(44*a^4 + 59*a^
3*b + 24*a^2*b^2 + 24*a*b^3 - 2*(10*a^4 - 75*a^3*b - 103*a^2*b^2 - 36*a*b^3
 - 48*b^4)*e^(-2*d*x - 2*c) - 2*(50*a^4 + 278*a^3*b + 309*a^2*b^2 + 180*a*b
^3 + 144*b^4)*e^(-4*d*x - 4*c) + 6*(6*a^4 + 55*a^3*b + 79*a^2*b^2 + 68*a*b^
3 + 48*b^4)*e^(-6*d*x - 6*c) + 3*(24*a^4 + 27*a^3*b - 18*a^2*b^2 - 48*a*b^3
 - 32*b^4)*e^(-8*d*x - 8*c))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3 - (a^6 -
 a^5*b - 9*a^4*b^2 - 11*a^3*b^3 - 4*a^2*b^4)*e^(-2*d*x - 2*c) - 2*(a^6 + 9*
a^5*b + 21*a^4*b^2 + 19*a^3*b^3 + 6*a^2*b^4)*e^(-4*d*x - 4*c) + 2*(a^6 + 9*
a^5*b + 21*a^4*b^2 + 19*a^3*b^3 + 6*a^2*b^4)*e^(-6*d*x - 6*c) + (a^6 - a^5*
b - 9*a^4*b^2 - 11*a^3*b^3 - 4*a^2*b^4)*e^(-8*d*x - 8*c) - (a^6 + 3*a^5*b +
 3*a^4*b^2 + a^3*b^3)*e^(-10*d*x - 10*c))*d) + 1/12*(8*a^3 + 17*a^2*b - 6*a
b^2 + 3(8*a^3 + 13*a^2*b + 8*a*b^2 + 8*b^3)*e^(8*d*x + 8*c) + 6*(4*a^3 +
19*a^2*b + 13*a*b^2 - 12*b^3)*e^(6*d*x + 6*c) - 2*(8*a^3 + 68*a^2*b + 69*a*
b^2 - 36*b^3)*e^(4*d*x + 4*c) - 2*(4*a^3 - 15*a^2*b - 37*a*b^2 + 12*b^3)*e^
(2*d*x + 2*c))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3 - (a^5 + 3*a^4*b + 3*a
^3*b^2 + a^2*b^3)*e^(10*d*x + 10*c) + (a^5 - a^4*b - 9*a^3*b^2 - 11*a^2*b^3
 - 4*a*b^4)*e^(8*d*x + 8*c) + 2*(a^5 + 9*a^4*b + 21*a^3*b^2 + 19*a^2*b^3 +
6*a*b^4)*e^(6*d*x + 6*c) - 2*(a^5 + 9*a^4*b + 21*a^3*b^2 + 19*a^2*b^3 + 6*a
*b^4)*e^(4*d*x + 4*c) - (a^5 - a^4*b - 9*a^3*b^2 - 11*a^2*b^3 - 4*a*b^4)*e^
(2*d*x + 2*c))*d) - 1/12*(8*a^3 + 17*a^2*b - 6*a*b^2 - 2*(4*a^3 - 15*a^2*b
- 37*a*b^2 + 12*b^3)*e^(-2*d*x - 2*c) - 2*(8*a^3 + 68*a^2*b + 69*a*b^2 - 36
*b^3)*e^(-4*d*x - 4*c) + 6*(4*a^3 + 19*a^2*b + 13*a*b^2 - 12*b^3)*e^(-6*d*x
 - 6*c) + 3*(8*a^3 + 13*a^2*b + 8*a*b^2 + 8*b^3)*e^(-8*d*x - 8*c))/((a^5 +
3*a^4*b + 3*a^3*b^2 + a^2*b^3 - (a^5 - a^4*b - 9*a^3*b^2 - 11*a^2*b^3 - 4*a
*b^4)*e^(-2*d*x - 2*c) - 2*(a^5 + 9*a^4*b + 21*a^3*b^2 + 19*a^2*b^3 + 6*a*b
^4)*e^(-4*d*x - 4*c) + 2*(a^5 + 9*a^4*b + 21*a^3*b^2 + 19*a^2*b^3 + 6*a*b^4
)*e^(-6*d*x - 6*c) + (a^5 - a^4*b - 9*a^3*b^2 - 11*a^2*b^3 - 4*a*b^4)*e^(-8
*d*x - 8*c) - (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*e^(-10*d*x - 10*c))*d)
+ 1/8*(4*a^2 - 11*a*b - 2*(2*a^2 - 9*a*b + 19*b^2)*e^(-2*d*x - 2*c) - 2*(10
*a^2 + 22*a*b - 33*b^2)*e^(-4*d*x - 4*c) - 6*(2*a^2 + 3*a*b + 11*b^2)*e^(-6
*d*x - 6*c) - 3*(3*a*b - 2*b^2)*e^(-8*d*x - 8*c))/((a^4 + 3*a^3*b + 3*a^2*b
^2 + a*b^3 - (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*e^(-2*d*x - 2*c)
- 2*(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*e^(-4*d*x - 4*c) + 2*(a
^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*e^(-6*d*x - 6*c) + (a^4 - a^3
*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*e^(-8*d*x - 8*c) - (a^4 + 3*a^3*b + 3*a^

```
2*b^2 + a*b^3)*e^(-10*d*x - 10*c))*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh (c+d\,x)^4\,\coth (c+d\,x)^4}{\left(a\cosh (c+d\,x)^2+b\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^4/(a + b/cosh(c + d*x)^2)^2,x)
[Out] int((cosh(c + d*x)^4*coth(c + d*x)^4)/(b + a*cosh(c + d*x)^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^4(c+dx)}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**4/(a+b*sech(d*x+c)**2)**2,x)
[Out] Integral(coth(c + d*x)**4/(a + b*sech(c + d*x)**2)**2, x)
```

3.158 $\int \frac{\tanh^6(c+dx)}{\left(a+b\,\mathbf{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=148

$$\frac{x}{a^3}+\frac{(3a-4b)(a+b)\tanh(c+dx)}{8a^2b^2d\left(a-b\tanh^2(c+dx)+b\right)}-\frac{\sqrt{a+b}\left(3a^2-4ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^3b^{5/2}d}-\frac{(a+b)\tanh^3(c+dx)}{4abd\left(a-b\tanh^2(c+dx)+b\right)^2}$$

```
[Out] x/a^3-1/8*(3*a^2-4*a*b+8*b^2)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))*(a+b
)^(1/2)/a^3/b^(5/2)/d-1/4*(a+b)*tanh(d*x+c)^3/a/b/d/(a+b-b*tanh(d*x+c)^2)^2
+1/8*(3*a-4*b)*(a+b)*tanh(d*x+c)/a^2/b^2/d/(a+b-b*tanh(d*x+c)^2)
```

Rubi [A] time = 0.32, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4141, 1975, 470, 578, 522, 206, 208}

$$\frac{(3a-4b)(a+b)\tanh(c+dx)}{8a^2b^2d\left(a-b\tanh^2(c+dx)+b\right)}-\frac{\sqrt{a+b}\left(3a^2-4ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^3b^{5/2}d}+\frac{x}{a^3}-\frac{(a+b)\tanh^3(c+dx)}{4abd\left(a-b\tanh^2(c+dx)+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^6/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] x/a^3 - (Sqrt[a + b]*(3*a^2 - 4*a*b + 8*b^2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x]
)/Sqrt[a + b]])/(8*a^3*b^(5/2)*d) - ((a + b)*Tanh[c + d*x]^3)/(4*a*b*d*(a +
 b - b*Tanh[c + d*x]^2)^2) + ((3*a - 4*b)*(a + b)*Tanh[c + d*x])/(8*a^2*b^2
*d*(a + b - b*Tanh[c + d*x]^2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
```

```
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\tanh^6(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{(1-x^2)\left(a+b(1-x^2)\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{x^6}{(1-x^2)(a+b-bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{(a+b)\tanh^3(c+dx)}{4abd\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{\text{Subst}\left(\int \frac{x^2(3(a+b)+(-3a+b)x^2)}{(1-x^2)(a+b-bx^2)^2}\,dx, x, \tanh(c+dx\right.}{4abd}$$

$$= -\frac{(a+b)\tanh^3(c+dx)}{4abd\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{(3a-4b)(a+b)\tanh(c+dx)}{8a^2b^2d\left(a+b-b\tanh^2(c+dx)\right)} - \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{(a+b)\tanh^3(c+dx)}{4abd\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{(3a-4b)(a+b)\tanh(c+dx)}{8a^2b^2d\left(a+b-b\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= \frac{x}{a^3} - \frac{\sqrt{a+b}\left(3a^2-4ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^3b^{5/2}d} - \frac{(a+b)\tanh^3(c+dx}{4abd\left(a+b-b\tanh^2(c+\right.}$$

Mathematica [B] time = 5.78, size = 515, normalized size = 3.48

$$\frac{\text{sech}^6(c+dx)(a\cosh(2(c+dx))+a+2b)\left(\text{sech}(2c)\left(3a^4\sinh(2(c+2dx))-3a^4\sinh(4c+2dx)-9a^4\sinh(2c)+\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tanh[c + d*x]^6/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^6*((-2*(3*a^3 - a^2*b + 4*a*
b^2 + 8*b^3)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x
] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(a +
 2*b + a*Cosh[2*(c + d*x)])^2*(Cosh[2*c] - Sinh[2*c]))/(Sqrt[a + b]*Sqrt[b*
(Cosh[c] - Sinh[c])^4]) + Sech[2*c]*(8*b^2*(3*a^2 + 8*a*b + 8*b^2)*d*x*Cosh
```

```
[2*c] + 16*a*b^2*(a + 2*b)*d*x*Cosh[2*d*x] + 4*a^2*b^2*d*x*Cosh[2*(c + 2*d*
x)] + 16*a^2*b^2*d*x*Cosh[4*c + 2*d*x] + 32*a*b^3*d*x*Cosh[4*c + 2*d*x] + 4
*a^2*b^2*d*x*Cosh[6*c + 4*d*x] - 9*a^4*Sinh[2*c] - 15*a^3*b*Sinh[2*c] + 18*
a^2*b^2*Sinh[2*c] + 72*a*b^3*Sinh[2*c] + 48*b^4*Sinh[2*c] + 9*a^4*Sinh[2*d*
x] + 13*a^3*b*Sinh[2*d*x] - 28*a^2*b^2*Sinh[2*d*x] - 32*a*b^3*Sinh[2*d*x] +
 3*a^4*Sinh[2*(c + 2*d*x)] - 3*a^3*b*Sinh[2*(c + 2*d*x)] - 6*a^2*b^2*Sinh[2
*(c + 2*d*x)] - 3*a^4*Sinh[4*c + 2*d*x] + a^3*b*Sinh[4*c + 2*d*x] + 20*a^2*
b^2*Sinh[4*c + 2*d*x] + 16*a*b^3*Sinh[4*c + 2*d*x])))/(128*a^3*b^2*d*(a + b
*Sech[c + d*x]^2)^3)
```

fricas [B] time = 0.52, size = 5463, normalized size = 36.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^6/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(16*a^2*b^2*d*x*cosh(d*x + c)^8 + 128*a^2*b^2*d*x*cosh(d*x + c)*sinh(
d*x + c)^7 + 16*a^2*b^2*d*x*sinh(d*x + c)^8 - 4*(3*a^4 - a^3*b - 20*a^2*b^2
 - 16*a*b^3 - 16*(a^2*b^2 + 2*a*b^3)*d*x)*cosh(d*x + c)^6 + 4*(112*a^2*b^2*
d*x*cosh(d*x + c)^2 - 3*a^4 + a^3*b + 20*a^2*b^2 + 16*a*b^3 + 16*(a^2*b^2 +
 2*a*b^3)*d*x)*sinh(d*x + c)^6 + 16*a^2*b^2*d*x + 8*(112*a^2*b^2*d*x*cosh(d
*x + c)^3 - 3*(3*a^4 - a^3*b - 20*a^2*b^2 - 16*a*b^3 - 16*(a^2*b^2 + 2*a*b^
3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 - 4*(9*a^4 + 15*a^3*b - 18*a^2*b^2 -
 72*a*b^3 - 48*b^4 - 8*(3*a^2*b^2 + 8*a*b^3 + 8*b^4)*d*x)*cosh(d*x + c)^4 +
 4*(280*a^2*b^2*d*x*cosh(d*x + c)^4 - 9*a^4 - 15*a^3*b + 18*a^2*b^2 + 72*a*
b^3 + 48*b^4 + 8*(3*a^2*b^2 + 8*a*b^3 + 8*b^4)*d*x - 15*(3*a^4 - a^3*b - 20
*a^2*b^2 - 16*a*b^3 - 16*(a^2*b^2 + 2*a*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x
 + c)^4 - 12*a^4 + 12*a^3*b + 24*a^2*b^2 + 16*(56*a^2*b^2*d*x*cosh(d*x + c)
^5 - 5*(3*a^4 - a^3*b - 20*a^2*b^2 - 16*a*b^3 - 16*(a^2*b^2 + 2*a*b^3)*d*x)
*cosh(d*x + c)^3 - (9*a^4 + 15*a^3*b - 18*a^2*b^2 - 72*a*b^3 - 48*b^4 - 8*(
3*a^2*b^2 + 8*a*b^3 + 8*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 - 4*(9*a^4
 + 13*a^3*b - 28*a^2*b^2 - 32*a*b^3 - 16*(a^2*b^2 + 2*a*b^3)*d*x)*cosh(d*x
+ c)^2 + 4*(112*a^2*b^2*d*x*cosh(d*x + c)^6 - 15*(3*a^4 - a^3*b - 20*a^2*b^
2 - 16*a*b^3 - 16*(a^2*b^2 + 2*a*b^3)*d*x)*cosh(d*x + c)^4 - 9*a^4 - 13*a^3
*b + 28*a^2*b^2 + 32*a*b^3 + 16*(a^2*b^2 + 2*a*b^3)*d*x - 6*(9*a^4 + 15*a^3
*b - 18*a^2*b^2 - 72*a*b^3 - 48*b^4 - 8*(3*a^2*b^2 + 8*a*b^3 + 8*b^4)*d*x)*
cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((3*a^4 - 4*a^3*b + 8*a^2*b^2)*cosh(d*x
+ c)^8 + 8*(3*a^4 - 4*a^3*b + 8*a^2*b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (3
*a^4 - 4*a^3*b + 8*a^2*b^2)*sinh(d*x + c)^8 + 4*(3*a^4 + 2*a^3*b + 16*a*b^3
)*cosh(d*x + c)^6 + 4*(3*a^4 + 2*a^3*b + 16*a*b^3 + 7*(3*a^4 - 4*a^3*b + 8*
a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(3*a^4 - 4*a^3*b + 8*a^2*b
^2)*cosh(d*x + c)^3 + 3*(3*a^4 + 2*a^3*b + 16*a*b^3)*cosh(d*x + c))*sinh(d*
x + c)^5 + 2*(9*a^4 + 12*a^3*b + 16*a^2*b^2 + 32*a*b^3 + 64*b^4)*cosh(d*x +
 c)^4 + 2*(35*(3*a^4 - 4*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^4 + 9*a^4 + 12*a^
```

3*b + 16*a^2*b^2 + 32*a*b^3 + 64*b^4 + 30*(3*a^4 + 2*a^3*b + 16*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 3*a^4 - 4*a^3*b + 8*a^2*b^2 + 8*(7*(3*a^4 - 4*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^5 + 10*(3*a^4 + 2*a^3*b + 16*a*b^3)*cosh(d*x + c)^3 + (9*a^4 + 12*a^3*b + 16*a^2*b^2 + 32*a*b^3 + 64*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(3*a^4 + 2*a^3*b + 16*a*b^3)*cosh(d*x + c)^2 + 4*(7*(3*a^4 - 4*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^6 + 15*(3*a^4 + 2*a^3*b + 16*a*b^3)*cosh(d*x + c)^4 + 3*a^4 + 2*a^3*b + 16*a*b^3 + 3*(9*a^4 + 12*a^3*b + 16*a^2*b^2 + 32*a*b^3 + 64*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((3*a^4 - 4*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^7 + 3*(3*a^4 + 2*a^3*b + 16*a*b^3)*cosh(d*x + c)^5 + (9*a^4 + 12*a^3*b + 16*a^2*b^2 + 32*a*b^3 + 64*b^4)*cosh(d*x + c)^3 + (3*a^4 + 2*a^3*b + 16*a*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a + b)/b)*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) + 4*(a*b*cosh(d*x + c)^2 + 2*a*b*cosh(d*x + c)*sinh(d*x + c) + a*b*sinh(d*x + c)^2 + a*b + 2*b^2)*sqrt((a + b)/b))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) + 8*(16*a^2*b^2*d*x*cosh(d*x + c)^7 - 3*(3*a^4 - a^3*b - 20*a^2*b^2 - 16*a*b^3 - 16*(a^2*b^2 + 2*a*b^3)*d*x)*cosh(d*x + c)^5 - 2*(9*a^4 + 15*a^3*b - 18*a^2*b^2 - 72*a*b^3 - 48*b^4 - 8*(3*a^2*b^2 + 8*a*b^3 + 8*b^4)*d*x)*cosh(d*x + c)^3 - (9*a^4 + 13*a^3*b - 28*a^2*b^2 - 32*a*b^3 - 16*(a^2*b^2 + 2*a*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c))/(a^5*b^2*d*cosh(d*x + c)^8 + 8*a^5*b^2*d*cosh(d*x + c)*sinh(d*x + c)^7 + a^5*b^2*d*sinh(d*x + c)^8 + a^5*b^2*d + 4*(a^5*b^2 + 2*a^4*b^3)*d*cosh(d*x + c)^6 + 4*(7*a^5*b^2*d*cosh(d*x + c)^2 + (a^5*b^2 + 2*a^4*b^3)*d)*sinh(d*x + c)^6 + 2*(3*a^5*b^2 + 8*a^4*b^3 + 8*a^3*b^4)*d*cosh(d*x + c)^4 + 8*(7*a^5*b^2*d*cosh(d*x + c)^3 + 3*(a^5*b^2 + 2*a^4*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*a^5*b^2*d*cosh(d*x + c)^4 + 30*(a^5*b^2 + 2*a^4*b^3)*d*cosh(d*x + c)^2 + (3*a^5*b^2 + 8*a^4*b^3 + 8*a^3*b^4)*d)*sinh(d*x + c)^4 + 4*(a^5*b^2 + 2*a^4*b^3)*d*cosh(d*x + c)^2 + 8*(7*a^5*b^2*d*cosh(d*x + c)^5 + 10*(a^5*b^2 + 2*a^4*b^3)*d*cosh(d*x + c)^3 + (3*a^5*b^2 + 8*a^4*b^3 + 8*a^3*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*a^5*b^2*d*cosh(d*x + c)^6 + 15*(a^5*b^2 + 2*a^4*b^3)*d*cosh(d*x + c)^4 + 3*(3*a^5*b^2 + 8*a^4*b^3 + 8*a^3*b^4)*d*cosh(d*x + c)^2 + (a^5*b^2 + 2*a^4*b^3)*d)*sinh(d*x + c)^2 + 8*(a^5*b^2*d*cosh(d*x + c)^7 + 3*(a^5*b^2 + 2*a^4*b^3)*d*cosh(d*x + c)^5 + (3*a^5*b^2 + 8*a^4*b^3 + 8*a^3*b^4)*d*cosh(d*x + c)^3 + (a^5*b^2 + 2*a^4*b^3)*d*cosh(d*x + c))*sinh(d*x + c)), 1/8*(8*a^2*b^2*d*x*cosh(d*x + c)^8 + 64*a^2*b^2*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + 8*a^2*b^2*d*x*sinh(d*x + c)^8 - 2*(3*a^4 - a^3*b - 20*a^2*b^2 - 16*a*b^3 - 16*(a^2*b^2 + 2*a*b^3)*d*x)*cosh(d*x + c)^6 + 2*(112*a^2*b^2*d*x*cosh(d*x + c)^2 - 3*a^4 + a^3*b + 20*a^2*b^2 + 16*a*b^3 + 16*(a^2*b^2 + 2*a*b^3)*d*x)*sinh(d*x + c)^6 + 8*a^2*b^2*d*x + 4*(112*a^2*b^2*d*x*cosh(d*x + c)^3 - 3*(3*a^4 - a^3*b - 20*a^2*b^2 - 16*a*b^3 - 16*(a^2*b^2 + 2*a*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(9*a^4 + 15*a^3*b - 18

*a^2*b^2 - 72*a*b^3 - 48*b^4 - 8*(3*a^2*b^2 + 8*a*b^3 + 8*b^4)*d*x)*cosh(d*
x + c)^4 + 2*(280*a^2*b^2*d*x*cosh(d*x + c)^4 - 9*a^4 - 15*a^3*b + 18*a^2*b
^2 + 72*a*b^3 + 48*b^4 + 8*(3*a^2*b^2 + 8*a*b^3 + 8*b^4)*d*x - 15*(3*a^4 -
a^3*b - 20*a^2*b^2 - 16*a*b^3 - 16*(a^2*b^2 + 2*a*b^3)*d*x)*cosh(d*x + c)^2
)*sinh(d*x + c)^4 - 6*a^4 + 6*a^3*b + 12*a^2*b^2 + 8*(56*a^2*b^2*d*x*cosh(d
x + c)^5 - 5(3*a^4 - a^3*b - 20*a^2*b^2 - 16*a*b^3 - 16*(a^2*b^2 + 2*a*b^
3)*d*x)*cosh(d*x + c)^3 - (9*a^4 + 15*a^3*b - 18*a^2*b^2 - 72*a*b^3 - 48*b^
4 - 8*(3*a^2*b^2 + 8*a*b^3 + 8*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 - 2
*(9*a^4 + 13*a^3*b - 28*a^2*b^2 - 32*a*b^3 - 16*(a^2*b^2 + 2*a*b^3)*d*x)*co
sh(d*x + c)^2 + 2*(112*a^2*b^2*d*x*cosh(d*x + c)^6 - 15*(3*a^4 - a^3*b - 20
*a^2*b^2 - 16*a*b^3 - 16*(a^2*b^2 + 2*a*b^3)*d*x)*cosh(d*x + c)^4 - 9*a^4 -
13*a^3*b + 28*a^2*b^2 + 32*a*b^3 + 16*(a^2*b^2 + 2*a*b^3)*d*x - 6*(9*a^4 +
15*a^3*b - 18*a^2*b^2 - 72*a*b^3 - 48*b^4 - 8*(3*a^2*b^2 + 8*a*b^3 + 8*b^4
)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - ((3*a^4 - 4*a^3*b + 8*a^2*b^2)*co
sh(d*x + c)^8 + 8*(3*a^4 - 4*a^3*b + 8*a^2*b^2)*cosh(d*x + c)*sinh(d*x + c)
^7 + (3*a^4 - 4*a^3*b + 8*a^2*b^2)*sinh(d*x + c)^8 + 4*(3*a^4 + 2*a^3*b + 1
6*a*b^3)*cosh(d*x + c)^6 + 4*(3*a^4 + 2*a^3*b + 16*a*b^3 + 7*(3*a^4 - 4*a^3
*b + 8*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(3*a^4 - 4*a^3*b +
8*a^2*b^2)*cosh(d*x + c)^3 + 3*(3*a^4 + 2*a^3*b + 16*a*b^3)*cosh(d*x + c))*
sinh(d*x + c)^5 + 2*(9*a^4 + 12*a^3*b + 16*a^2*b^2 + 32*a*b^3 + 64*b^4)*cos
h(d*x + c)^4 + 2*(35*(3*a^4 - 4*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^4 + 9*a^4
+ 12*a^3*b + 16*a^2*b^2 + 32*a*b^3 + 64*b^4 + 30*(3*a^4 + 2*a^3*b + 16*a*b^
3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 3*a^4 - 4*a^3*b + 8*a^2*b^2 + 8*(7*(3
*a^4 - 4*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^5 + 10*(3*a^4 + 2*a^3*b + 16*a*b^
3)*cosh(d*x + c)^3 + (9*a^4 + 12*a^3*b + 16*a^2*b^2 + 32*a*b^3 + 64*b^4)*co
sh(d*x + c))*sinh(d*x + c)^3 + 4*(3*a^4 + 2*a^3*b + 16*a*b^3)*cosh(d*x + c)
^2 + 4*(7*(3*a^4 - 4*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^6 + 15*(3*a^4 + 2*a^3
*b + 16*a*b^3)*cosh(d*x + c)^4 + 3*a^4 + 2*a^3*b + 16*a*b^3 + 3*(9*a^4 + 12
*a^3*b + 16*a^2*b^2 + 32*a*b^3 + 64*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 +
8*((3*a^4 - 4*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^7 + 3*(3*a^4 + 2*a^3*b + 16
*a*b^3)*cosh(d*x + c)^5 + (9*a^4 + 12*a^3*b + 16*a^2*b^2 + 32*a*b^3 + 64*b^
4)*cosh(d*x + c)^3 + (3*a^4 + 2*a^3*b + 16*a*b^3)*cosh(d*x + c))*sinh(d*x +
c))*sqrt(-(a + b)/b)*arctan(1/2*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sin
h(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-(a + b)/b)/(a + b)) + 4*(16
*a^2*b^2*d*x*cosh(d*x + c)^7 - 3*(3*a^4 - a^3*b - 20*a^2*b^2 - 16*a*b^3 - 1
6*(a^2*b^2 + 2*a*b^3)*d*x)*cosh(d*x + c)^5 - 2*(9*a^4 + 15*a^3*b - 18*a^2*b
^2 - 72*a*b^3 - 48*b^4 - 8*(3*a^2*b^2 + 8*a*b^3 + 8*b^4)*d*x)*cosh(d*x + c)
^3 - (9*a^4 + 13*a^3*b - 28*a^2*b^2 - 32*a*b^3 - 16*(a^2*b^2 + 2*a*b^3)*d*x
)*cosh(d*x + c))*sinh(d*x + c))/(a^5*b^2*d*cosh(d*x + c)^8 + 8*a^5*b^2*d*co
sh(d*x + c)*sinh(d*x + c)^7 + a^5*b^2*d*sinh(d*x + c)^8 + a^5*b^2*d + 4*(a^
5*b^2 + 2*a^4*b^3)*d*cosh(d*x + c)^6 + 4*(7*a^5*b^2*d*cosh(d*x + c)^2 + (a^
5*b^2 + 2*a^4*b^3)*d)*sinh(d*x + c)^6 + 2*(3*a^5*b^2 + 8*a^4*b^3 + 8*a^3*b^
4)*d*cosh(d*x + c)^4 + 8*(7*a^5*b^2*d*cosh(d*x + c)^3 + 3*(a^5*b^2 + 2*a^4*
b^3)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*a^5*b^2*d*cosh(d*x + c)^4 + 3
0*(a^5*b^2 + 2*a^4*b^3)*d*cosh(d*x + c)^2 + (3*a^5*b^2 + 8*a^4*b^3 + 8*a^3*

```
b^4)*d)*sinh(d*x + c)^4 + 4*(a^5*b^2 + 2*a^4*b^3)*d*cosh(d*x + c)^2 + 8*(7*
a^5*b^2*d*cosh(d*x + c)^5 + 10*(a^5*b^2 + 2*a^4*b^3)*d*cosh(d*x + c)^3 + (3
*a^5*b^2 + 8*a^4*b^3 + 8*a^3*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*a
^5*b^2*d*cosh(d*x + c)^6 + 15*(a^5*b^2 + 2*a^4*b^3)*d*cosh(d*x + c)^4 + 3*(
3*a^5*b^2 + 8*a^4*b^3 + 8*a^3*b^4)*d*cosh(d*x + c)^2 + (a^5*b^2 + 2*a^4*b^3
)*d)*sinh(d*x + c)^2 + 8*(a^5*b^2*d*cosh(d*x + c)^7 + 3*(a^5*b^2 + 2*a^4*b^
3)*d*cosh(d*x + c)^5 + (3*a^5*b^2 + 8*a^4*b^3 + 8*a^3*b^4)*d*cosh(d*x + c)^
3 + (a^5*b^2 + 2*a^4*b^3)*d*cosh(d*x + c))*sinh(d*x + c))]
```

giac [B] time = 2.25, size = 371, normalized size = 2.51

$$\frac{\frac{8\,dx}{a^3} - \frac{\left(3\,a^3e^{(2\,c)} - a^2be^{(2\,c)} + 4\,ab^2e^{(2\,c)} + 8\,b^3e^{(2\,c)}\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)} + a + 2\,b}{2\,\sqrt{-ab-b^2}}\right)e^{(-2\,c)}}{\sqrt{-ab-b^2}\,a^3b^2} - \frac{2\left(3\,a^4e^{(6\,dx+6\,c)} - a^3be^{(6\,dx+6\,c)} - 20\,a^2b^2e^{(6\,dx+6\,c)} - 16\,ab^3e^{(6\,dx+6\,c)} - \cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^6/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*(8*d*x/a^3 - (3*a^3*e^(2*c) - a^2*b*e^(2*c) + 4*a*b^2*e^(2*c) + 8*b^3*e
^(2*c))*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))*e^(-2*c)
/(sqrt(-a*b - b^2)*a^3*b^2) - 2*(3*a^4*e^(6*d*x + 6*c) - a^3*b*e^(6*d*x + 6
*c) - 20*a^2*b^2*e^(6*d*x + 6*c) - 16*a*b^3*e^(6*d*x + 6*c) + 9*a^4*e^(4*d*
x + 4*c) + 15*a^3*b*e^(4*d*x + 4*c) - 18*a^2*b^2*e^(4*d*x + 4*c) - 72*a*b^3
*e^(4*d*x + 4*c) - 48*b^4*e^(4*d*x + 4*c) + 9*a^4*e^(2*d*x + 2*c) + 13*a^3*
b*e^(2*d*x + 2*c) - 28*a^2*b^2*e^(2*d*x + 2*c) - 32*a*b^3*e^(2*d*x + 2*c) +
 3*a^4 - 3*a^3*b - 6*a^2*b^2)/((a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + 4
*b*e^(2*d*x + 2*c) + a)^2*a^3*b^2))/d
```

maple [B] time = 0.45, size = 1713, normalized size = 11.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^6/(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] 1/d/a^3*ln(tanh(1/2*d*x+1/2*c)+1)-1/d/a^3*ln(tanh(1/2*d*x+1/2*c)-1)-5/4/d/a
/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a
-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^7-19/4/d/a/(tanh(1/2*
d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2
*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^5-19/4/d/a/(tanh(1/2*d*x+1/2*c)^
4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)
^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^3-5/4/d/a/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1
/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*
tanh(1/2*d*x+1/2*c)+1/2/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+
```

```
2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/b*tanh(1/2*d*x+1
/2*c)^7-7/2/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d
*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/b*tanh(1/2*d*x+1/2*c)^5-7/2/
d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*
a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/b*tanh(1/2*d*x+1/2*c)^3+1/2/d/(tanh(1/2*
d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2
*d*x+1/2*c)^2*b+a+b)^2/b*tanh(1/2*d*x+1/2*c)+3/16/d/b^(5/2)/(a+b)^(1/2)*ln(
(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2)
)-3/16/d/b^(5/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)
*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))+1/4/d/a^2/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1
/2)*tanh(1/2*d*x+1/2*c)^2-2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))+1/16/d
/a/b^(3/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(
1/2*d*x+1/2*c)+(a+b)^(1/2))-1/4/d/a^2/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*ta
nh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))+3/4/d*a/(tan
h(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*ta
nh(1/2*d*x+1/2*c)^2*b+a+b)^2/b^2*tanh(1/2*d*x+1/2*c)^7+9/4/d*a/(tanh(1/2*d*
x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d
*x+1/2*c)^2*b+a+b)^2/b^2*tanh(1/2*d*x+1/2*c)^5+9/4/d*a/(tanh(1/2*d*x+1/2*c)
^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c
)^2*b+a+b)^2/b^2*tanh(1/2*d*x+1/2*c)^3+3/4/d*a/(tanh(1/2*d*x+1/2*c)^4*a+b*t
anh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+
b)^2/b^2*tanh(1/2*d*x+1/2*c)-1/16/d/a/b^(3/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*ta
nh(1/2*d*x+1/2*c)^2-2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))-1/2/d/a^3*b^
(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d
*x+1/2*c)+(a+b)^(1/2))+1/d/a^2*b/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/
2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*
d*x+1/2*c)^5+1/2/d/a^3*b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*
c)^2-2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))-1/d/a^2*b/(tanh(1/2*d*x+1/2
*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/
2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^7+1/d/a^2*b/(tanh(1/2*d*x+1/2*c)^4*a+b*
tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a
+b)^2*tanh(1/2*d*x+1/2*c)^3-1/d/a^2*b/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d
*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh
(1/2*d*x+1/2*c)
```

maxima [B] time = 1.99, size = 3239, normalized size = 21.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^6/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -45/1024*(a + 2*b)*a*log((a*e^(2*d*x + 2*c) + a + 2*b - 2*sqrt((a + b)*b))/
(a*e^(2*d*x + 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^2*b^2 + 2*a*b^3 + b^
4)*sqrt((a + b)*b)*d) - 9/512*a^2*log((a*e^(2*d*x + 2*c) + a + 2*b - 2*sqrt
```

((a + b)*b))/(a*e^(2*d*x + 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^2*b^2 + 2*a*b^3 + b^4)*sqrt((a + b)*b)*d) + 45/1024*(a + 2*b)*a*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt ((a + b)*b)))/((a^2*b^2 + 2*a*b^3 + b^4)*sqrt((a + b)*b)*d) + 9/512*a^2*log ((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^2*b^2 + 2*a*b^3 + b^4)*sqrt((a + b)*b)*d) - 1/1024*(3*a^5 - 10*a^4*b + 80*a^3*b^2 + 480*a^2*b^3 + 640*a*b^4 + 256*b^5)*log((a*e^(2*d*x + 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(2*d*x + 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^5*b^2 + 2*a^4*b^3 + a^3*b^4)*sqrt((a + b)*b)*d) + 1/1024*(3*a^5 - 10*a^4*b + 80*a^3*b^2 + 480*a^2*b^3 + 640*a*b^4 + 256*b^5)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2* d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^5*b^2 + 2*a^4*b^3 + a^3*b^4) *sqrt((a + b)*b)*d) + 5/256*(3*a^2 + 8*a*b + 8*b^2)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^2*b^2 + 2*a*b^3 + b^4)*sqrt((a + b)*b)*d) - 1/256*(3*a^6 - 12* a^5*b - 204*a^4*b^2 - 384*a^3*b^3 - 192*a^2*b^4 + (3*a^6 - 10*a^5*b - 560*a ^4*b^2 - 2080*a^3*b^3 - 2560*a^2*b^4 - 1024*a*b^5)*e^(6*d*x + 6*c) + (9*a^6 - 12*a^5*b - 1100*a^4*b^2 - 5248*a^3*b^3 - 10304*a^2*b^4 - 9216*a*b^5 - 30 72*b^6)*e^(4*d*x + 4*c) + (9*a^6 - 14*a^5*b - 864*a^4*b^2 - 3136*a^3*b^3 - 3840*a^2*b^4 - 1536*a*b^5)*e^(2*d*x + 2*c))/((a^7*b^2 + 2*a^6*b^3 + a^5*b^4 + (a^7*b^2 + 2*a^6*b^3 + a^5*b^4)*e^(8*d*x + 8*c) + 4*(a^7*b^2 + 4*a^6*b^3 + 5*a^5*b^4 + 2*a^4*b^5)*e^(6*d*x + 6*c) + 2*(3*a^7*b^2 + 14*a^6*b^3 + 27* a^5*b^4 + 24*a^4*b^5 + 8*a^3*b^6)*e^(4*d*x + 4*c) + 4*(a^7*b^2 + 4*a^6*b^3 + 5*a^5*b^4 + 2*a^4*b^5)*e^(2*d*x + 2*c))*d) + 1/256*(3*a^6 - 12*a^5*b - 20 4*a^4*b^2 - 384*a^3*b^3 - 192*a^2*b^4 + (9*a^6 - 14*a^5*b - 864*a^4*b^2 - 3 136*a^3*b^3 - 3840*a^2*b^4 - 1536*a*b^5)*e^(-2*d*x - 2*c) + (9*a^6 - 12*a^5 *b - 1100*a^4*b^2 - 5248*a^3*b^3 - 10304*a^2*b^4 - 9216*a*b^5 - 3072*b^6)*e ^(-4*d*x - 4*c) + (3*a^6 - 10*a^5*b - 560*a^4*b^2 - 2080*a^3*b^3 - 2560*a^2 *b^4 - 1024*a*b^5)*e^(-6*d*x - 6*c))/((a^7*b^2 + 2*a^6*b^3 + a^5*b^4 + 4*(a ^7*b^2 + 4*a^6*b^3 + 5*a^5*b^4 + 2*a^4*b^5)*e^(-2*d*x - 2*c) + 2*(3*a^7*b^2 + 14*a^6*b^3 + 27*a^5*b^4 + 24*a^4*b^5 + 8*a^3*b^6)*e^(-4*d*x - 4*c) + 4*(a^7*b^2 + 4*a^6*b^3 + 5*a^5*b^4 + 2*a^4*b^5)*e^(-6*d*x - 6*c) + (a^7*b^2 + 2*a^6*b^3 + a^5*b^4)*e^(-8*d*x - 8*c))*d) - 3/128*(3*a^5 - 2*a^4*b - 24*a^3 *b^2 - 16*a^2*b^3 + (3*a^5 - 128*a^3*b^2 - 256*a^2*b^3 - 128*a*b^4)*e^(6*d* x + 6*c) + (9*a^5 + 18*a^4*b - 128*a^3*b^2 - 512*a^2*b^3 - 640*a*b^4 - 256* b^5)*e^(4*d*x + 4*c) + (9*a^5 + 16*a^4*b - 112*a^3*b^2 - 256*a^2*b^3 - 128* a*b^4)*e^(2*d*x + 2*c))/((a^6*b^2 + 2*a^5*b^3 + a^4*b^4 + (a^6*b^2 + 2*a^5* b^3 + a^4*b^4)*e^(8*d*x + 8*c) + 4*(a^6*b^2 + 4*a^5*b^3 + 5*a^4*b^4 + 2*a^3 *b^5)*e^(6*d*x + 6*c) + 2*(3*a^6*b^2 + 14*a^5*b^3 + 27*a^4*b^4 + 24*a^3*b^5 + 8*a^2*b^6)*e^(4*d*x + 4*c) + 4*(a^6*b^2 + 4*a^5*b^3 + 5*a^4*b^4 + 2*a^3* b^5)*e^(2*d*x + 2*c))*d) + 3/128*(3*a^5 - 2*a^4*b - 24*a^3*b^2 - 16*a^2*b^3 + (9*a^5 + 16*a^4*b - 112*a^3*b^2 - 256*a^2*b^3 - 128*a*b^4)*e^(-2*d*x - 2 *c) + (9*a^5 + 18*a^4*b - 128*a^3*b^2 - 512*a^2*b^3 - 640*a*b^4 - 256*b^5)* e^(-4*d*x - 4*c) + (3*a^5 - 128*a^3*b^2 - 256*a^2*b^3 - 128*a*b^4)*e^(-6*d* x - 6*c))/((a^6*b^2 + 2*a^5*b^3 + a^4*b^4 + 4*(a^6*b^2 + 4*a^5*b^3 + 5*a^4*

```
b^4 + 2*a^3*b^5)*e^(-2*d*x - 2*c) + 2*(3*a^6*b^2 + 14*a^5*b^3 + 27*a^4*b^4
+ 24*a^3*b^5 + 8*a^2*b^6)*e^(-4*d*x - 4*c) + 4*(a^6*b^2 + 4*a^5*b^3 + 5*a^4
*b^4 + 2*a^3*b^5)*e^(-6*d*x - 6*c) + (a^6*b^2 + 2*a^5*b^3 + a^4*b^4)*e^(-8*
d*x - 8*c))*d) - 15/256*(3*a^4 + 4*a^3*b + 4*a^2*b^2 + 3*(a^4 + 2*a^3*b)*e^
(6*d*x + 6*c) + (9*a^4 + 36*a^3*b + 100*a^2*b^2 + 128*a*b^3 + 64*b^4)*e^(4*
d*x + 4*c) + (9*a^4 + 34*a^3*b + 48*a^2*b^2 + 32*a*b^3)*e^(2*d*x + 2*c))/((
a^5*b^2 + 2*a^4*b^3 + a^3*b^4 + (a^5*b^2 + 2*a^4*b^3 + a^3*b^4)*e^(8*d*x +
8*c) + 4*(a^5*b^2 + 4*a^4*b^3 + 5*a^3*b^4 + 2*a^2*b^5)*e^(6*d*x + 6*c) + 2*
(3*a^5*b^2 + 14*a^4*b^3 + 27*a^3*b^4 + 24*a^2*b^5 + 8*a*b^6)*e^(4*d*x + 4*c
) + 4*(a^5*b^2 + 4*a^4*b^3 + 5*a^3*b^4 + 2*a^2*b^5)*e^(2*d*x + 2*c))*d) + 1
5/256*(3*a^4 + 4*a^3*b + 4*a^2*b^2 + (9*a^4 + 34*a^3*b + 48*a^2*b^2 + 32*a*
b^3)*e^(-2*d*x - 2*c) + (9*a^4 + 36*a^3*b + 100*a^2*b^2 + 128*a*b^3 + 64*b^
4)*e^(-4*d*x - 4*c) + 3*(a^4 + 2*a^3*b)*e^(-6*d*x - 6*c))/((a^5*b^2 + 2*a^4
*b^3 + a^3*b^4 + 4*(a^5*b^2 + 4*a^4*b^3 + 5*a^3*b^4 + 2*a^2*b^5)*e^(-2*d*x
- 2*c) + 2*(3*a^5*b^2 + 14*a^4*b^3 + 27*a^3*b^4 + 24*a^2*b^5 + 8*a*b^6)*e^(
-4*d*x - 4*c) + 4*(a^5*b^2 + 4*a^4*b^3 + 5*a^3*b^4 + 2*a^2*b^5)*e^(-6*d*x -
 6*c) + (a^5*b^2 + 2*a^4*b^3 + a^3*b^4)*e^(-8*d*x - 8*c))*d) + 5/64*(3*a^3
+ 6*a^2*b + (9*a^3 + 40*a^2*b + 40*a*b^2)*e^(-2*d*x - 2*c) + 3*(3*a^3 + 14*
a^2*b + 24*a*b^2 + 16*b^3)*e^(-4*d*x - 4*c) + (3*a^3 + 8*a^2*b + 8*a*b^2)*e
^(-6*d*x - 6*c))/((a^4*b^2 + 2*a^3*b^3 + a^2*b^4 + 4*(a^4*b^2 + 4*a^3*b^3 +
 5*a^2*b^4 + 2*a*b^5)*e^(-2*d*x - 2*c) + 2*(3*a^4*b^2 + 14*a^3*b^3 + 27*a^2
*b^4 + 24*a*b^5 + 8*b^6)*e^(-4*d*x - 4*c) + 4*(a^4*b^2 + 4*a^3*b^3 + 5*a^2*
b^4 + 2*a*b^5)*e^(-6*d*x - 6*c) + (a^4*b^2 + 2*a^3*b^3 + a^2*b^4)*e^(-8*d*x
 - 8*c))*d) + 1/4*log(a*e^(4*d*x + 4*c) + 2*(a + 2*b)*e^(2*d*x + 2*c) + a)/
(a^3*d) - 1/4*log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/(a
^3*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\cosh\left(c+d\,x\right)^2-1\right)^3}{\left(a\cosh\left(c+d\,x\right)^2+b\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^6/(a + b/cosh(c + d*x)^2)^3,x)
```

```
[Out] int((cosh(c + d*x)^2 - 1)^3/(b + a*cosh(c + d*x)^2)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**6/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

3.159 $\int \frac{\tanh^5(c+dx)}{\left(a+b\mathbf{sech}^2(c+dx)\right)^3} dx$

Optimal. Leaf size=77

$$-\frac{(a+b)^2}{4a^3d\left(a\cosh^2(c+dx)+b\right)^2}+\frac{a+b}{a^3d\left(a\cosh^2(c+dx)+b\right)}+\frac{\log\left(a\cosh^2(c+dx)+b\right)}{2a^3d}$$

```
[Out] -1/4*(a+b)^2/a^3/d/(b+a*cosh(d*x+c)^2)^2+(a+b)/a^3/d/(b+a*cosh(d*x+c)^2)+1/
2*ln(b+a*cosh(d*x+c)^2)/a^3/d
```

Rubi [A] time = 0.12, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4138, 444, 43}

$$-\frac{(a+b)^2}{4a^3d\left(a\cosh^2(c+dx)+b\right)^2}+\frac{a+b}{a^3d\left(a\cosh^2(c+dx)+b\right)}+\frac{\log\left(a\cosh^2(c+dx)+b\right)}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^5/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] -(a + b)^2/(4*a^3*d*(b + a*Cosh[c + d*x]^2)^2) + (a + b)/(a^3*d*(b + a*Cosh
[c + d*x]^2)) + Log[b + a*Cosh[c + d*x]^2]/(2*a^3*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
```

```
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\tanh^5(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x\left(1-x^2\right)^2}{\left(b+ax^2\right)^3}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{(1-x)^2}{(b+ax)^3}\,dx, x, \cosh^2(c+dx)\right)}{2d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{(a+b)^2}{a^2(b+ax)^3} - \frac{2(a+b)}{a^2(b+ax)^2} + \frac{1}{a^2(b+ax)}\right) dx, x, \cosh^2(c+dx)\right)}{2d}$$

$$= -\frac{(a+b)^2}{4a^3d\left(b+a\cosh^2(c+dx)\right)^2} + \frac{a+b}{a^3d\left(b+a\cosh^2(c+dx)\right)} + \frac{\log\left(b+a\cosh^2(c\right.}{2a^3d}$$

Mathematica [A] time = 2.17, size = 136, normalized size = 1.77

$$\frac{2\left(a^2+4ab+3b^2\right)+a^2\cosh^2(2(c+dx))\log(a\cosh(2(c+dx))+a+2b)+(a+2b)^2\log(a\cosh(2(c+dx))+a}{2a^3d(a\cosh(2(c+dx))+a+2b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^5/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (2*(a^2 + 4*a*b + 3*b^2) + (a + 2*b)^2*Log[a + 2*b + a*Cosh[2*(c + d*x)]] +
 a^2*Cosh[2*(c + d*x)]^2*Log[a + 2*b + a*Cosh[2*(c + d*x)]] + 2*a*Cosh[2*(c
 + d*x)]*(2*(a + b) + (a + 2*b)*Log[a + 2*b + a*Cosh[2*(c + d*x)]]))/(2*a^3
*d*(a + 2*b + a*Cosh[2*(c + d*x)])^2)
```

fricas [B] time = 0.46, size = 1741, normalized size = 22.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^2*d*x*cosh(d*x + c)^8 + 16*a^2*d*x*cosh(d*x + c)*sinh(d*x + c)^7
+ 2*a^2*d*x*sinh(d*x + c)^8 + 8*((a^2 + 2*a*b)*d*x - a^2 - a*b)*cosh(d*x +
c)^6 + 8*(7*a^2*d*x*cosh(d*x + c)^2 + (a^2 + 2*a*b)*d*x - a^2 - a*b)*sinh(d
*x + c)^6 + 16*(7*a^2*d*x*cosh(d*x + c)^3 + 3*((a^2 + 2*a*b)*d*x - a^2 - a*
b)*cosh(d*x + c))*sinh(d*x + c)^5 + 4*((3*a^2 + 8*a*b + 8*b^2)*d*x - 2*a^2
- 8*a*b - 6*b^2)*cosh(d*x + c)^4 + 4*(35*a^2*d*x*cosh(d*x + c)^4 + (3*a^2 +
 8*a*b + 8*b^2)*d*x + 30*((a^2 + 2*a*b)*d*x - a^2 - a*b)*cosh(d*x + c)^2 -
2*a^2 - 8*a*b - 6*b^2)*sinh(d*x + c)^4 + 2*a^2*d*x + 16*(7*a^2*d*x*cosh(d*x
 + c)^5 + 10*((a^2 + 2*a*b)*d*x - a^2 - a*b)*cosh(d*x + c)^3 + ((3*a^2 + 8*
a*b + 8*b^2)*d*x - 2*a^2 - 8*a*b - 6*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 +
8*((a^2 + 2*a*b)*d*x - a^2 - a*b)*cosh(d*x + c)^2 + 8*(7*a^2*d*x*cosh(d*x +
 c)^6 + 15*((a^2 + 2*a*b)*d*x - a^2 - a*b)*cosh(d*x + c)^4 + (a^2 + 2*a*b)*
d*x + 3*((3*a^2 + 8*a*b + 8*b^2)*d*x - 2*a^2 - 8*a*b - 6*b^2)*cosh(d*x + c)
^2 - a^2 - a*b)*sinh(d*x + c)^2 - (a^2*cosh(d*x + c)^8 + 8*a^2*cosh(d*x + c
)*sinh(d*x + c)^7 + a^2*sinh(d*x + c)^8 + 4*(a^2 + 2*a*b)*cosh(d*x + c)^6 +
 4*(7*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^6 + 8*(7*a^2*cosh(d*
x + c)^3 + 3*(a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*a^2 + 8*a*
b + 8*b^2)*cosh(d*x + c)^4 + 2*(35*a^2*cosh(d*x + c)^4 + 30*(a^2 + 2*a*b)*c
osh(d*x + c)^2 + 3*a^2 + 8*a*b + 8*b^2)*sinh(d*x + c)^4 + 8*(7*a^2*cosh(d*x
 + c)^5 + 10*(a^2 + 2*a*b)*cosh(d*x + c)^3 + (3*a^2 + 8*a*b + 8*b^2)*cosh(d
*x + c))*sinh(d*x + c)^3 + 4*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 4*(7*a^2*cosh(
d*x + c)^6 + 15*(a^2 + 2*a*b)*cosh(d*x + c)^4 + 3*(3*a^2 + 8*a*b + 8*b^2)*c
osh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*(a^2*cosh(d*x + c)^
7 + 3*(a^2 + 2*a*b)*cosh(d*x + c)^5 + (3*a^2 + 8*a*b + 8*b^2)*cosh(d*x + c)
^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c))*log(2*(a*cosh(d*x + c)^2 +
 a*sinh(d*x + c)^2 + a + 2*b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x +
 c) + sinh(d*x + c)^2)) + 16*(a^2*d*x*cosh(d*x + c)^7 + 3*((a^2 + 2*a*b)*d*
x - a^2 - a*b)*cosh(d*x + c)^5 + ((3*a^2 + 8*a*b + 8*b^2)*d*x - 2*a^2 - 8*a
*b - 6*b^2)*cosh(d*x + c)^3 + ((a^2 + 2*a*b)*d*x - a^2 - a*b)*cosh(d*x + c)
)*sinh(d*x + c))/(a^5*d*cosh(d*x + c)^8 + 8*a^5*d*cosh(d*x + c)*sinh(d*x +
c)^7 + a^5*d*sinh(d*x + c)^8 + 4*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^6 + 4*(7*a
^5*d*cosh(d*x + c)^2 + (a^5 + 2*a^4*b)*d)*sinh(d*x + c)^6 + a^5*d + 2*(3*a^
5 + 8*a^4*b + 8*a^3*b^2)*d*cosh(d*x + c)^4 + 8*(7*a^5*d*cosh(d*x + c)^3 + 3
*(a^5 + 2*a^4*b)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*a^5*d*cosh(d*x +
c)^4 + 30*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^2 + (3*a^5 + 8*a^4*b + 8*a^3*b^2)
*d)*sinh(d*x + c)^4 + 4*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^2 + 8*(7*a^5*d*cosh
(d*x + c)^5 + 10*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^3 + (3*a^5 + 8*a^4*b + 8*a
^3*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*a^5*d*cosh(d*x + c)^6 + 15*
(a^5 + 2*a^4*b)*d*cosh(d*x + c)^4 + 3*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*d*cosh(
d*x + c)^2 + (a^5 + 2*a^4*b)*d)*sinh(d*x + c)^2 + 8*(a^5*d*cosh(d*x + c)^7
+ 3*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^5 + (3*a^5 + 8*a^4*b + 8*a^3*b^2)*d*cos
h(d*x + c)^3 + (a^5 + 2*a^4*b)*d*cosh(d*x + c))*sinh(d*x + c))
```

giac [B] time = 1.88, size = 187, normalized size = 2.43

$$-\frac{\frac{4\,dx}{a^3} - \frac{2\,\log\left(ae^{(4\,dx+4\,c)} + 2\,ae^{(2\,dx+2\,c)} + 4\,be^{(2\,dx+2\,c)} + a\right)}{a^3} + \frac{3\,ae^{(8\,dx+8\,c)} - 4\,ae^{(6\,dx+6\,c)} + 8\,be^{(6\,dx+6\,c)} + 2\,ae^{(4\,dx+4\,c)} - 16\,be^{(4\,dx+4\,c)} - 4\,ae^{(2\,dx+2\,c)} + 8\,be}{\left(ae^{(4\,dx+4\,c)} + 2\,ae^{(2\,dx+2\,c)} + 4\,be^{(2\,dx+2\,c)} + a\right)^2 a^2}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/4*(4*d*x/a^3 - 2*log(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*
d*x + 2*c) + a)/a^3 + (3*a*e^(8*d*x + 8*c) - 4*a*e^(6*d*x + 6*c) + 8*b*e^(6
*d*x + 6*c) + 2*a*e^(4*d*x + 4*c) - 16*b*e^(4*d*x + 4*c) - 4*a*e^(2*d*x + 2
*c) + 8*b*e^(2*d*x + 2*c) + 3*a)/((a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c)
+ 4*b*e^(2*d*x + 2*c) + a)^2*a^2))/d
```

maple [B] time = 0.42, size = 579, normalized size = 7.52

$$-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d\,a^3} - \frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d\,a^3} - \frac{2\left(\tanh^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da\left(\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a + b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right) + 2\left(\tanh^2\left(\frac{dx}{2}\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^5/(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] -1/d/a^3*ln(tanh(1/2*d*x+1/2*c)-1)-1/d/a^3*ln(tanh(1/2*d*x+1/2*c)+1)-2/d/a/
(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-
2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^6-2/d/a^2*b/(tanh(1/2*
d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2
*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^6-8/d/a/(tanh(1/2*d*x+1/2*c)^4*a
+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*
b+a+b)^2*tanh(1/2*d*x+1/2*c)^4+4/d/a^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*
d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tan
h(1/2*d*x+1/2*c)^4*b-2/d/a/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4
+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/
2*c)^2-2/d/a^2*b/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/
2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^2+1/2
/d/a^3*ln(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/
2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)
```

maxima [B] time = 0.42, size = 206, normalized size = 2.68

$$\frac{4\left(\left(a^2+ab\right)e^{(-2\,dx-2\,c)} + \left(a^2+4\,ab+3\,b^2\right)e^{(-4\,dx-4\,c)} + \left(a^2+ab\right)e^{(-6\,dx-6\,c)}\right)}{\left(a^5e^{(-8\,dx-8\,c)} + a^5 + 4\left(a^5+2\,a^4b\right)e^{(-2\,dx-2\,c)} + 2\left(3\,a^5+8\,a^4b+8\,a^3b^2\right)e^{(-4\,dx-4\,c)} + 4\left(a^5+2\,a^4b\right)e^{(-6\,dx-6\,c)}\right)d} + \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 4*((a^2 + a*b)*e^(-2*d*x - 2*c) + (a^2 + 4*a*b + 3*b^2)*e^(-4*d*x - 4*c) +
(a^2 + a*b)*e^(-6*d*x - 6*c))/((a^5*e^(-8*d*x - 8*c) + a^5 + 4*(a^5 + 2*a^4
*b)*e^(-2*d*x - 2*c) + 2*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*e^(-4*d*x - 4*c) + 4
*(a^5 + 2*a^4*b)*e^(-6*d*x - 6*c))*d) + (d*x + c)/(a^3*d) + 1/2*log(2*(a +
2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/(a^3*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+d\,x)^6\,\tanh(c+d\,x)^5}{\left(a\cosh(c+d\,x)^2+b\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^5/(a + b/cosh(c + d*x)^2)^3,x)
```

```
[Out] int((cosh(c + d*x)^6*tanh(c + d*x)^5)/(b + a*cosh(c + d*x)^2)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**5/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

3.160 $\int \frac{\tanh^4(c+dx)}{\left(a+b\,\mathbf{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=139

$$\frac{x}{a^3}+\frac{(a-4b)\tanh(c+dx)}{8a^2bd\left(a-b\tanh^2(c+dx)+b\right)}+\frac{\left(a^2-4ab-8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^3b^{3/2}d\sqrt{a+b}}-\frac{(a+b)\tanh(c+dx)}{4abd\left(a-b\tanh^2(c+dx)+b\right)^2}$$

```
[Out] x/a^3+1/8*(a^2-4*a*b-8*b^2)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))/a^3/b^
(3/2)/d/(a+b)^(1/2)-1/4*(a+b)*tanh(d*x+c)/a/b/d/(a+b-b*tanh(d*x+c)^2)^2+1/8
*(a-4*b)*tanh(d*x+c)/a^2/b/d/(a+b-b*tanh(d*x+c)^2)
```

Rubi [A] time = 0.29, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4141, 1975, 470, 527, 522, 206, 208}

$$\frac{\left(a^2-4ab-8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^3b^{3/2}d\sqrt{a+b}}+\frac{(a-4b)\tanh(c+dx)}{8a^2bd\left(a-b\tanh^2(c+dx)+b\right)}+\frac{x}{a^3}-\frac{(a+b)\tanh(c+dx)}{4abd\left(a-b\tanh^2(c+dx)+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^4/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] x/a^3 + ((a^2 - 4*a*b - 8*b^2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]]
)/(8*a^3*b^(3/2)*Sqrt[a + b]*d) - ((a + b)*Tanh[c + d*x])/(4*a*b*d*(a + b -
 b*Tanh[c + d*x]^2)^2) + ((a - 4*b)*Tanh[c + d*x])/(8*a^2*b*d*(a + b - b*Ta
nh[c + d*x]^2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
```

```
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\tanh^4(c+dx)}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^3}\,dx = \frac{\mathrm{Subst}\left(\int \frac{x^4}{(1-x^2)\left(a+b(1-x^2)\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\mathrm{Subst}\left(\int \frac{x^4}{(1-x^2)(a+b-bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{(a+b)\tanh(c+dx)}{4abd\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{\mathrm{Subst}\left(\int \frac{a+b+(-a+3b)x^2}{(1-x^2)(a+b-bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{4abd}$$

$$= -\frac{(a+b)\tanh(c+dx)}{4abd\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{(a-4b)\tanh(c+dx)}{8a^2bd\left(a+b-b\tanh^2(c+dx)\right)} - \frac{\mathrm{Subst}\left(\int\right.}{}$$

$$= -\frac{(a+b)\tanh(c+dx)}{4abd\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{(a-4b)\tanh(c+dx)}{8a^2bd\left(a+b-b\tanh^2(c+dx)\right)} + \frac{\mathrm{Subst}\left(\int\right.}{}$$

$$= \frac{x}{a^3} + \frac{(a^2-4ab-8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^3b^{3/2}\sqrt{a+b}\,d} - \frac{(a+b)\tanh(c+dx)}{4abd\left(a+b-b\tanh^2(c+dx)\right)^2}$$

Mathematica [B] time = 13.09, size = 1457, normalized size = 10.48

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tanh[c + d*x]^4/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*c + 2*d*x])^3*Sech[c + d*x]^6*(((3*a^2 + 8*a*b + 8*b^2
)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a + b)^(5/2) - (a*Sqrt[b]*
(3*a^2 + 16*a*b + 16*b^2 + 3*a*(a + 2*b)*Cosh[2*(c + d*x)])*Sinh[2*(c + d*x
)])/((a + b)^2*(a + 2*b + a*Cosh[2*(c + d*x)])^2)))/(1024*b^(5/2)*d*(a + b*
Sech[c + d*x]^2)^3) - ((a + 2*b + a*Cosh[2*c + 2*d*x])^3*Sech[c + d*x]^6*((
-3*a*(a + 2*b)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a + b)^(5/2)
+ (Sqrt[b]*(3*a^3 + 14*a^2*b + 24*a*b^2 + 16*b^3 + a*(3*a^2 + 4*a*b + 4*b^2
)*Cosh[2*(c + d*x)])*Sinh[2*(c + d*x)])/((a + b)^2*(a + 2*b + a*Cosh[2*(c +
 d*x)])^2)))/(2048*b^(5/2)*d*(a + b*Sech[c + d*x]^2)^3) + ((a + 2*b + a*Cos
h[2*c + 2*d*x])^3*Sech[c + d*x]^6*((-2*(3*a^5 - 10*a^4*b + 80*a^3*b^2 + 480
```

```
*a^2*b^3 + 640*a*b^4 + 256*b^5)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*
((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] -
 Sinh[c])^4])]*(Cosh[2*c] - Sinh[2*c]))/(Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh
[c])^4]) + (Sech[2*c]*(256*b^2*(a + b)^2*(3*a^2 + 8*a*b + 8*b^2)*d*x*Cosh[2
*c] + 512*a*b^2*(a + b)^2*(a + 2*b)*d*x*Cosh[2*d*x] + 128*a^4*b^2*d*x*Cosh[
2*(c + 2*d*x)] + 256*a^3*b^3*d*x*Cosh[2*(c + 2*d*x)] + 128*a^2*b^4*d*x*Cosh
[2*(c + 2*d*x)] + 512*a^4*b^2*d*x*Cosh[4*c + 2*d*x] + 2048*a^3*b^3*d*x*Cosh
[4*c + 2*d*x] + 2560*a^2*b^4*d*x*Cosh[4*c + 2*d*x] + 1024*a*b^5*d*x*Cosh[4*
c + 2*d*x] + 128*a^4*b^2*d*x*Cosh[6*c + 4*d*x] + 256*a^3*b^3*d*x*Cosh[6*c +
 4*d*x] + 128*a^2*b^4*d*x*Cosh[6*c + 4*d*x] - 9*a^6*Sinh[2*c] + 12*a^5*b*Si
nh[2*c] + 684*a^4*b^2*Sinh[2*c] + 2880*a^3*b^3*Sinh[2*c] + 5280*a^2*b^4*Sin
h[2*c] + 4608*a*b^5*Sinh[2*c] + 1536*b^6*Sinh[2*c] + 9*a^6*Sinh[2*d*x] - 14
*a^5*b*Sinh[2*d*x] - 608*a^4*b^2*Sinh[2*d*x] - 2112*a^3*b^3*Sinh[2*d*x] - 2
560*a^2*b^4*Sinh[2*d*x] - 1024*a*b^5*Sinh[2*d*x] + 3*a^6*Sinh[2*(c + 2*d*x)
] - 12*a^5*b*Sinh[2*(c + 2*d*x)] - 204*a^4*b^2*Sinh[2*(c + 2*d*x)] - 384*a^
3*b^3*Sinh[2*(c + 2*d*x)] - 192*a^2*b^4*Sinh[2*(c + 2*d*x)] - 3*a^6*Sinh[4*
c + 2*d*x] + 10*a^5*b*Sinh[4*c + 2*d*x] + 304*a^4*b^2*Sinh[4*c + 2*d*x] + 1
056*a^3*b^3*Sinh[4*c + 2*d*x] + 1280*a^2*b^4*Sinh[4*c + 2*d*x] + 512*a*b^5*
Sinh[4*c + 2*d*x]))/(a + 2*b + a*Cosh[2*(c + d*x)])^2))/(4096*a^3*b^2*(a +
b)^2*d*(a + b*Sech[c + d*x]^2)^3) - ((a + 2*b + a*Cosh[2*c + 2*d*x])^3*Sech
[c + d*x]^6*((6*a^2*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*S
inh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4]
)]*(Cosh[2*c] - Sinh[2*c]))/(Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4]) + (
a*Sech[2*c]*((-9*a^4 - 16*a^3*b + 48*a^2*b^2 + 128*a*b^3 + 64*b^4)*Sinh[2*d
*x] + a*(-3*a^3 + 2*a^2*b + 24*a*b^2 + 16*b^3)*Sinh[2*(c + 2*d*x)] + (3*a^4
 - 64*a^2*b^2 - 128*a*b^3 - 64*b^4)*Sinh[4*c + 2*d*x]) + (9*a^5 + 18*a^4*b
- 64*a^3*b^2 - 256*a^2*b^3 - 320*a*b^4 - 128*b^5)*Tanh[2*c])/(a^2*(a + 2*b
+ a*Cosh[2*(c + d*x)])^2)))/(2048*b^2*(a + b)^2*d*(a + b*Sech[c + d*x]^2)^3
)
```

<u>fricas</u> [B] time = 0.57, size = 6464, normalized size = 46.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(16*(a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^8 + 128*(a^3*b^2 + a^2*b^3)
*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + 16*(a^3*b^2 + a^2*b^3)*d*x*sinh(d*x +
c)^8 + 4*(a^4*b + 13*a^3*b^2 + 28*a^2*b^3 + 16*a*b^4 + 16*(a^3*b^2 + 3*a^2*
b^3 + 2*a*b^4)*d*x)*cosh(d*x + c)^6 + 4*(a^4*b + 13*a^3*b^2 + 28*a^2*b^3 +
16*a*b^4 + 112*(a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^2 + 16*(a^3*b^2 + 3*a^
2*b^3 + 2*a*b^4)*d*x)*sinh(d*x + c)^6 + 8*(112*(a^3*b^2 + a^2*b^3)*d*x*cosh
(d*x + c)^3 + 3*(a^4*b + 13*a^3*b^2 + 28*a^2*b^3 + 16*a*b^4 + 16*(a^3*b^2 +
 3*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 + 4*a^4*b + 28*a^
```

```
3*b^2 + 24*a^2*b^3 + 4*(3*a^4*b + 29*a^3*b^2 + 82*a^2*b^3 + 104*a*b^4 + 48*
b^5 + 8*(3*a^3*b^2 + 11*a^2*b^3 + 16*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c)^4 +
4*(280*(a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^4 + 3*a^4*b + 29*a^3*b^2 + 82*
a^2*b^3 + 104*a*b^4 + 48*b^5 + 8*(3*a^3*b^2 + 11*a^2*b^3 + 16*a*b^4 + 8*b^5
)*d*x + 15*(a^4*b + 13*a^3*b^2 + 28*a^2*b^3 + 16*a*b^4 + 16*(a^3*b^2 + 3*a^
2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 16*(56*(a^3*b^2 +
a^2*b^3)*d*x*cosh(d*x + c)^5 + 5*(a^4*b + 13*a^3*b^2 + 28*a^2*b^3 + 16*a*b^
4 + 16*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c)^3 + (3*a^4*b + 29
*a^3*b^2 + 82*a^2*b^3 + 104*a*b^4 + 48*b^5 + 8*(3*a^3*b^2 + 11*a^2*b^3 + 16
*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 + 16*(a^3*b^2 + a^2*b^3
)*d*x + 4*(3*a^4*b + 23*a^3*b^2 + 52*a^2*b^3 + 32*a*b^4 + 16*(a^3*b^2 + 3*a
^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c)^2 + 4*(112*(a^3*b^2 + a^2*b^3)*d*x*cos
h(d*x + c)^6 + 3*a^4*b + 23*a^3*b^2 + 52*a^2*b^3 + 32*a*b^4 + 15*(a^4*b + 1
3*a^3*b^2 + 28*a^2*b^3 + 16*a*b^4 + 16*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*d*x)
*cosh(d*x + c)^4 + 16*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*d*x + 6*(3*a^4*b + 29
*a^3*b^2 + 82*a^2*b^3 + 104*a*b^4 + 48*b^5 + 8*(3*a^3*b^2 + 11*a^2*b^3 + 16
*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - ((a^4 - 4*a^3*b - 8
*a^2*b^2)*cosh(d*x + c)^8 + 8*(a^4 - 4*a^3*b - 8*a^2*b^2)*cosh(d*x + c)*sin
h(d*x + c)^7 + (a^4 - 4*a^3*b - 8*a^2*b^2)*sinh(d*x + c)^8 + 4*(a^4 - 2*a^3
*b - 16*a^2*b^2 - 16*a*b^3)*cosh(d*x + c)^6 + 4*(a^4 - 2*a^3*b - 16*a^2*b^2
 - 16*a*b^3 + 7*(a^4 - 4*a^3*b - 8*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^
6 + 8*(7*(a^4 - 4*a^3*b - 8*a^2*b^2)*cosh(d*x + c)^3 + 3*(a^4 - 2*a^3*b - 1
6*a^2*b^2 - 16*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*a^4 - 4*a^3*b -
 48*a^2*b^2 - 96*a*b^3 - 64*b^4)*cosh(d*x + c)^4 + 2*(35*(a^4 - 4*a^3*b - 8
*a^2*b^2)*cosh(d*x + c)^4 + 3*a^4 - 4*a^3*b - 48*a^2*b^2 - 96*a*b^3 - 64*b^
4 + 30*(a^4 - 2*a^3*b - 16*a^2*b^2 - 16*a*b^3)*cosh(d*x + c)^2)*sinh(d*x +
c)^4 + a^4 - 4*a^3*b - 8*a^2*b^2 + 8*(7*(a^4 - 4*a^3*b - 8*a^2*b^2)*cosh(d*
x + c)^5 + 10*(a^4 - 2*a^3*b - 16*a^2*b^2 - 16*a*b^3)*cosh(d*x + c)^3 + (3*
a^4 - 4*a^3*b - 48*a^2*b^2 - 96*a*b^3 - 64*b^4)*cosh(d*x + c))*sinh(d*x + c
)^3 + 4*(a^4 - 2*a^3*b - 16*a^2*b^2 - 16*a*b^3)*cosh(d*x + c)^2 + 4*(7*(a^4
 - 4*a^3*b - 8*a^2*b^2)*cosh(d*x + c)^6 + 15*(a^4 - 2*a^3*b - 16*a^2*b^2 -
16*a*b^3)*cosh(d*x + c)^4 + a^4 - 2*a^3*b - 16*a^2*b^2 - 16*a*b^3 + 3*(3*a^
4 - 4*a^3*b - 48*a^2*b^2 - 96*a*b^3 - 64*b^4)*cosh(d*x + c)^2)*sinh(d*x + c
)^2 + 8*((a^4 - 4*a^3*b - 8*a^2*b^2)*cosh(d*x + c)^7 + 3*(a^4 - 2*a^3*b - 1
6*a^2*b^2 - 16*a*b^3)*cosh(d*x + c)^5 + (3*a^4 - 4*a^3*b - 48*a^2*b^2 - 96*
a*b^3 - 64*b^4)*cosh(d*x + c)^3 + (a^4 - 2*a^3*b - 16*a^2*b^2 - 16*a*b^3)*c
osh(d*x + c))*sinh(d*x + c))*sqrt(a*b + b^2)*log((a^2*cosh(d*x + c)^4 + 4*a
^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*co
sh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a
^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*
sinh(d*x + c) + 4*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*
sinh(d*x + c)^2 + a + 2*b)*sqrt(a*b + b^2))/(a*cosh(d*x + c)^4 + 4*a*cosh(d
*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 +
 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 +
 (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) + 8*(16*(a^3*b^2 + a^2*b^3)*d
```

```
*x*cosh(d*x + c)^7 + 3*(a^4*b + 13*a^3*b^2 + 28*a^2*b^3 + 16*a*b^4 + 16*(a^
3*b^2 + 3*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c)^5 + 2*(3*a^4*b + 29*a^3*b^2
 + 82*a^2*b^3 + 104*a*b^4 + 48*b^5 + 8*(3*a^3*b^2 + 11*a^2*b^3 + 16*a*b^4 +
 8*b^5)*d*x)*cosh(d*x + c)^3 + (3*a^4*b + 23*a^3*b^2 + 52*a^2*b^3 + 32*a*b^
4 + 16*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c))/(
(a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^8 + 8*(a^6*b^2 + a^5*b^3)*d*cosh(d*x +
c)*sinh(d*x + c)^7 + (a^6*b^2 + a^5*b^3)*d*sinh(d*x + c)^8 + 4*(a^6*b^2 + 3
*a^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c)^6 + 4*(7*(a^6*b^2 + a^5*b^3)*d*cosh(d
*x + c)^2 + (a^6*b^2 + 3*a^5*b^3 + 2*a^4*b^4)*d)*sinh(d*x + c)^6 + 2*(3*a^6
*b^2 + 11*a^5*b^3 + 16*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c)^4 + 8*(7*(a^6*b
^2 + a^5*b^3)*d*cosh(d*x + c)^3 + 3*(a^6*b^2 + 3*a^5*b^3 + 2*a^4*b^4)*d*cos
h(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^4 +
 30*(a^6*b^2 + 3*a^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c)^2 + (3*a^6*b^2 + 11*a
^5*b^3 + 16*a^4*b^4 + 8*a^3*b^5)*d)*sinh(d*x + c)^4 + 4*(a^6*b^2 + 3*a^5*b^
3 + 2*a^4*b^4)*d*cosh(d*x + c)^2 + 8*(7*(a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)
^5 + 10*(a^6*b^2 + 3*a^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c)^3 + (3*a^6*b^2 +
11*a^5*b^3 + 16*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(
7*(a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^6 + 15*(a^6*b^2 + 3*a^5*b^3 + 2*a^4*b
^4)*d*cosh(d*x + c)^4 + 3*(3*a^6*b^2 + 11*a^5*b^3 + 16*a^4*b^4 + 8*a^3*b^5)
*d*cosh(d*x + c)^2 + (a^6*b^2 + 3*a^5*b^3 + 2*a^4*b^4)*d)*sinh(d*x + c)^2 +
 (a^6*b^2 + a^5*b^3)*d + 8*((a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^7 + 3*(a^6*
b^2 + 3*a^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c)^5 + (3*a^6*b^2 + 11*a^5*b^3 +
16*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c)^3 + (a^6*b^2 + 3*a^5*b^3 + 2*a^4*b^
4)*d*cosh(d*x + c))*sinh(d*x + c)), 1/8*(8*(a^3*b^2 + a^2*b^3)*d*x*cosh(d*x
 + c)^8 + 64*(a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + 8*(a^3
*b^2 + a^2*b^3)*d*x*sinh(d*x + c)^8 + 2*(a^4*b + 13*a^3*b^2 + 28*a^2*b^3 +
16*a*b^4 + 16*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c)^6 + 2*(a^4
*b + 13*a^3*b^2 + 28*a^2*b^3 + 16*a*b^4 + 112*(a^3*b^2 + a^2*b^3)*d*x*cosh(
d*x + c)^2 + 16*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*d*x)*sinh(d*x + c)^6 + 4*(1
12*(a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^3 + 3*(a^4*b + 13*a^3*b^2 + 28*a^2
*b^3 + 16*a*b^4 + 16*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c))*si
nh(d*x + c)^5 + 2*a^4*b + 14*a^3*b^2 + 12*a^2*b^3 + 2*(3*a^4*b + 29*a^3*b^2
 + 82*a^2*b^3 + 104*a*b^4 + 48*b^5 + 8*(3*a^3*b^2 + 11*a^2*b^3 + 16*a*b^4 +
 8*b^5)*d*x)*cosh(d*x + c)^4 + 2*(280*(a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)
^4 + 3*a^4*b + 29*a^3*b^2 + 82*a^2*b^3 + 104*a*b^4 + 48*b^5 + 8*(3*a^3*b^2
+ 11*a^2*b^3 + 16*a*b^4 + 8*b^5)*d*x + 15*(a^4*b + 13*a^3*b^2 + 28*a^2*b^3
+ 16*a*b^4 + 16*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c)^2)*sinh(
d*x + c)^4 + 8*(56*(a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^5 + 5*(a^4*b + 13*
a^3*b^2 + 28*a^2*b^3 + 16*a*b^4 + 16*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*d*x)*c
osh(d*x + c)^3 + (3*a^4*b + 29*a^3*b^2 + 82*a^2*b^3 + 104*a*b^4 + 48*b^5 +
8*(3*a^3*b^2 + 11*a^2*b^3 + 16*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c))*sinh(d*x
+ c)^3 + 8*(a^3*b^2 + a^2*b^3)*d*x + 2*(3*a^4*b + 23*a^3*b^2 + 52*a^2*b^3 +
 32*a*b^4 + 16*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c)^2 + 2*(11
2*(a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^6 + 3*a^4*b + 23*a^3*b^2 + 52*a^2*b
^3 + 32*a*b^4 + 15*(a^4*b + 13*a^3*b^2 + 28*a^2*b^3 + 16*a*b^4 + 16*(a^3*b^
```

2 + 3*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c)^4 + 16*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*d*x + 6*(3*a^4*b + 29*a^3*b^2 + 82*a^2*b^3 + 104*a*b^4 + 48*b^5 + 8*(3*a^3*b^2 + 11*a^2*b^3 + 16*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((a^4 - 4*a^3*b - 8*a^2*b^2)*cosh(d*x + c)^8 + 8*(a^4 - 4*a^3*b - 8*a^2*b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^4 - 4*a^3*b - 8*a^2*b^2)*sinh(d*x + c)^8 + 4*(a^4 - 2*a^3*b - 16*a^2*b^2 - 16*a*b^3)*cosh(d*x + c)^6 + 4*(a^4 - 2*a^3*b - 16*a^2*b^2 - 16*a*b^3 + 7*(a^4 - 4*a^3*b - 8*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(a^4 - 4*a^3*b - 8*a^2*b^2)*cosh(d*x + c)^3 + 3*(a^4 - 2*a^3*b - 16*a^2*b^2 - 16*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*a^4 - 4*a^3*b - 48*a^2*b^2 - 96*a*b^3 - 64*b^4)*cosh(d*x + c)^4 + 2*(35*(a^4 - 4*a^3*b - 8*a^2*b^2)*cosh(d*x + c)^4 + 3*a^4 - 4*a^3*b - 48*a^2*b^2 - 96*a*b^3 - 64*b^4 + 30*(a^4 - 2*a^3*b - 16*a^2*b^2 - 16*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + a^4 - 4*a^3*b - 8*a^2*b^2 + 8*(7*(a^4 - 4*a^3*b - 8*a^2*b^2)*cosh(d*x + c)^5 + 10*(a^4 - 2*a^3*b - 16*a^2*b^2 - 16*a*b^3)*cosh(d*x + c)^3 + (3*a^4 - 4*a^3*b - 48*a^2*b^2 - 96*a*b^3 - 64*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a^4 - 2*a^3*b - 16*a^2*b^2 - 16*a*b^3)*cosh(d*x + c)^2 + 4*(7*(a^4 - 4*a^3*b - 8*a^2*b^2)*cosh(d*x + c)^6 + 15*(a^4 - 2*a^3*b - 16*a^2*b^2 - 16*a*b^3)*cosh(d*x + c)^4 + a^4 - 2*a^3*b - 16*a^2*b^2 - 16*a*b^3 + 3*(3*a^4 - 4*a^3*b - 48*a^2*b^2 - 96*a*b^3 - 64*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((a^4 - 4*a^3*b - 8*a^2*b^2)*cosh(d*x + c)^7 + 3*(a^4 - 2*a^3*b - 16*a^2*b^2 - 16*a*b^3)*cosh(d*x + c)^5 + (3*a^4 - 4*a^3*b - 48*a^2*b^2 - 96*a*b^3 - 64*b^4)*cosh(d*x + c)^3 + (a^4 - 2*a^3*b - 16*a^2*b^2 - 16*a*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a*b - b^2)*arctan(1/2*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-a*b - b^2)/(a*b + b^2)) + 4*(16*(a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^7 + 3*(a^4*b + 13*a^3*b^2 + 28*a^2*b^3 + 16*a*b^4 + 16*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c)^5 + 2*(3*a^4*b + 29*a^3*b^2 + 82*a^2*b^3 + 104*a*b^4 + 48*b^5 + 8*(3*a^3*b^2 + 11*a^2*b^3 + 16*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c)^3 + (3*a^4*b + 23*a^3*b^2 + 52*a^2*b^3 + 32*a*b^4 + 16*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^8 + 8*(a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^6*b^2 + a^5*b^3)*d*sinh(d*x + c)^8 + 4*(a^6*b^2 + 3*a^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c)^6 + 4*(7*(a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^2 + (a^6*b^2 + 3*a^5*b^3 + 2*a^4*b^4)*d)*sinh(d*x + c)^6 + 2*(3*a^6*b^2 + 11*a^5*b^3 + 16*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c)^4 + 8*(7*(a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^3 + 3*(a^6*b^2 + 3*a^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^4 + 30*(a^6*b^2 + 3*a^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c)^2 + (3*a^6*b^2 + 11*a^5*b^3 + 16*a^4*b^4 + 8*a^3*b^5)*d)*sinh(d*x + c)^4 + 4*(a^6*b^2 + 3*a^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c)^2 + 8*(7*(a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^5 + 10*(a^6*b^2 + 3*a^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c)^3 + (3*a^6*b^2 + 11*a^5*b^3 + 16*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^6 + 15*(a^6*b^2 + 3*a^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c)^4 + 3*(3*a^6*b^2 + 11*a^5*b^3 + 16*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c)^2 + (a^6*b^2 + 3*a^5*b^3 + 2*a^4*b^4)*d)*sinh(d*x + c)

```
^2 + (a^6*b^2 + a^5*b^3)*d + 8*((a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^7 + 3*(
a^6*b^2 + 3*a^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c)^5 + (3*a^6*b^2 + 11*a^5*b^
3 + 16*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c)^3 + (a^6*b^2 + 3*a^5*b^3 + 2*a^
4*b^4)*d*cosh(d*x + c))*sinh(d*x + c))]
```

giac [B] time = 1.61, size = 295, normalized size = 2.12

$$\frac{\frac{8\,dx}{a^3} + \frac{\left(a^2e^{(2\,c)} - 4\,abe^{(2\,c)} - 8\,b^2e^{(2\,c)}\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)e^{(-2\,c)}}{\sqrt{-ab-b^2}\,a^3b} + \frac{2\left(a^3e^{(6\,dx+6\,c)} + 12\,a^2be^{(6\,dx+6\,c)} + 16\,ab^2e^{(6\,dx+6\,c)} + 3\,a^3e^{(4\,dx+4\,c)} + 26\,a^2be^{(4\,dx+\cdots}\right.}{\left(ae^{(4\,dx+4\,c)} + 2\cdots\right.}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*(8*d*x/a^3 + (a^2*e^(2*c) - 4*a*b*e^(2*c) - 8*b^2*e^(2*c))*arctan(1/2*(
a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))*e^(-2*c)/(sqrt(-a*b - b^2)*a
^3*b) + 2*(a^3*e^(6*d*x + 6*c) + 12*a^2*b*e^(6*d*x + 6*c) + 16*a*b^2*e^(6*d
*x + 6*c) + 3*a^3*e^(4*d*x + 4*c) + 26*a^2*b*e^(4*d*x + 4*c) + 56*a*b^2*e^(
4*d*x + 4*c) + 48*b^3*e^(4*d*x + 4*c) + 3*a^3*e^(2*d*x + 2*c) + 20*a^2*b*e^
(2*d*x + 2*c) + 32*a*b^2*e^(2*d*x + 2*c) + a^3 + 6*a^2*b)/((a*e^(4*d*x + 4*
c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) + a)^2*a^3*b))/d
```

maple [B] time = 0.48, size = 1306, normalized size = 9.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] -1/d/a^3*ln(tanh(1/2*d*x+1/2*c)-1)+1/d/a^3*ln(tanh(1/2*d*x+1/2*c)+1)-1/4/d/
(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-
2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/b*tanh(1/2*d*x+1/2*c)^7-5/4/d/a/(tanh(1/2*
d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2
*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^7-1/d/a^2*b/(tanh(1/2*d*x+1/2*c)
^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c
)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^7-3/4/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/
2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/b
*tanh(1/2*d*x+1/2*c)^5-19/4/d/a/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2
*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d
*x+1/2*c)^5+1/d/a^2*b/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*ta
nh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^
5-3/4/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2
*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/b*tanh(1/2*d*x+1/2*c)^3-19/4/d/a/(
tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2
```

```
*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^3+1/d/a^2*b/(tanh(1/2*d
*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*
d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^3-1/4/d/(tanh(1/2*d*x+1/2*c)^4*a+
b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b
+a+b)^2/b*tanh(1/2*d*x+1/2*c)-5/4/d/a/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d
*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh
(1/2*d*x+1/2*c)-1/d/a^2*b/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+
2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2
*c)-1/16/d/a/b^(3/2)/(a+b)^(1/2)*ln(-(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^
(1/2)*tanh(1/2*d*x+1/2*c)-(a+b)^(1/2))+1/4/d/a^2/b^(1/2)/(a+b)^(1/2)*ln(-(a
+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)-(a+b)^(1/2))+
1/2/d/a^3*b^(1/2)/(a+b)^(1/2)*ln(-(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/
2)*tanh(1/2*d*x+1/2*c)-(a+b)^(1/2))+1/16/d/a/b^(3/2)/(a+b)^(1/2)*ln((a+b)^(
1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))-1/4/d
/a^2/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tan
h(1/2*d*x+1/2*c)+(a+b)^(1/2))-1/2/d/a^3*b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*
tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))
```

maxima [B] time = 1.02, size = 2201, normalized size = 15.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/256*(a^4 - 20*a^3*b - 120*a^2*b^2 - 160*a*b^3 - 64*b^4)*log((a*e^(2*d*x +
 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(2*d*x + 2*c) + a + 2*b + 2*sqrt(
(a + b)*b)))/((a^5*b + 2*a^4*b^2 + a^3*b^3)*sqrt((a + b)*b)*d) + 1/64*(a -
2*b)*log((a*e^(2*d*x + 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(2*d*x + 2*
c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^2*b + 2*a*b^2 + b^3)*sqrt((a + b)*b)
*d) - 1/256*(a^4 - 20*a^3*b - 120*a^2*b^2 - 160*a*b^3 - 64*b^4)*log((a*e^(-
2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b +
 2*sqrt((a + b)*b)))/((a^5*b + 2*a^4*b^2 + a^3*b^3)*sqrt((a + b)*b)*d) - 3/
128*(a + 4*b)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(
-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^2*b + 2*a*b^2 + b^3)*sqrt
((a + b)*b)*d) - 1/64*(a - 2*b)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt(
(a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^2*b + 2
*a*b^2 + b^3)*sqrt((a + b)*b)*d) + 1/64*(a^5 + 38*a^4*b + 88*a^3*b^2 + 48*a
^2*b^3 + (a^5 + 76*a^4*b + 392*a^3*b^2 + 576*a^2*b^3 + 256*a*b^4)*e^(6*d*x
+ 6*c) + (3*a^5 + 186*a^4*b + 1024*a^3*b^2 + 2240*a^2*b^3 + 2176*a*b^4 + 76
8*b^5)*e^(4*d*x + 4*c) + (3*a^5 + 148*a^4*b + 648*a^3*b^2 + 896*a^2*b^3 + 3
84*a*b^4)*e^(2*d*x + 2*c))/((a^7*b + 2*a^6*b^2 + a^5*b^3 + (a^7*b + 2*a^6*b
^2 + a^5*b^3)*e^(8*d*x + 8*c) + 4*(a^7*b + 4*a^6*b^2 + 5*a^5*b^3 + 2*a^4*b^
4)*e^(6*d*x + 6*c) + 2*(3*a^7*b + 14*a^6*b^2 + 27*a^5*b^3 + 24*a^4*b^4 + 8*
a^3*b^5)*e^(4*d*x + 4*c) + 4*(a^7*b + 4*a^6*b^2 + 5*a^5*b^3 + 2*a^4*b^4)*e^
```

```
(2*d*x + 2*c))*d) - 1/64*(a^5 + 38*a^4*b + 88*a^3*b^2 + 48*a^2*b^3 + (3*a^5
 + 148*a^4*b + 648*a^3*b^2 + 896*a^2*b^3 + 384*a*b^4)*e^(-2*d*x - 2*c) + (3
*a^5 + 186*a^4*b + 1024*a^3*b^2 + 2240*a^2*b^3 + 2176*a*b^4 + 768*b^5)*e^(-
4*d*x - 4*c) + (a^5 + 76*a^4*b + 392*a^3*b^2 + 576*a^2*b^3 + 256*a*b^4)*e^(
-6*d*x - 6*c))/((a^7*b + 2*a^6*b^2 + a^5*b^3 + 4*(a^7*b + 4*a^6*b^2 + 5*a^5
*b^3 + 2*a^4*b^4)*e^(-2*d*x - 2*c) + 2*(3*a^7*b + 14*a^6*b^2 + 27*a^5*b^3 +
 24*a^4*b^4 + 8*a^3*b^5)*e^(-4*d*x - 4*c) + 4*(a^7*b + 4*a^6*b^2 + 5*a^5*b^
3 + 2*a^4*b^4)*e^(-6*d*x - 6*c) + (a^7*b + 2*a^6*b^2 + a^5*b^3)*e^(-8*d*x -
 8*c))*d) + 1/16*(a^4 + 8*a^3*b + 4*a^2*b^2 + (a^4 + 30*a^3*b + 64*a^2*b^2
+ 32*a*b^3)*e^(6*d*x + 6*c) + (3*a^4 + 64*a^3*b + 180*a^2*b^2 + 192*a*b^3 +
 64*b^4)*e^(4*d*x + 4*c) + (3*a^4 + 42*a^3*b + 80*a^2*b^2 + 32*a*b^3)*e^(2*
d*x + 2*c))/((a^6*b + 2*a^5*b^2 + a^4*b^3 + (a^6*b + 2*a^5*b^2 + a^4*b^3)*e
^(8*d*x + 8*c) + 4*(a^6*b + 4*a^5*b^2 + 5*a^4*b^3 + 2*a^3*b^4)*e^(6*d*x + 6
*c) + 2*(3*a^6*b + 14*a^5*b^2 + 27*a^4*b^3 + 24*a^3*b^4 + 8*a^2*b^5)*e^(4*d
*x + 4*c) + 4*(a^6*b + 4*a^5*b^2 + 5*a^4*b^3 + 2*a^3*b^4)*e^(2*d*x + 2*c))*
d) - 1/16*(a^4 + 8*a^3*b + 4*a^2*b^2 + (3*a^4 + 42*a^3*b + 80*a^2*b^2 + 32*
a*b^3)*e^(-2*d*x - 2*c) + (3*a^4 + 64*a^3*b + 180*a^2*b^2 + 192*a*b^3 + 64*
b^4)*e^(-4*d*x - 4*c) + (a^4 + 30*a^3*b + 64*a^2*b^2 + 32*a*b^3)*e^(-6*d*x
- 6*c))/((a^6*b + 2*a^5*b^2 + a^4*b^3 + 4*(a^6*b + 4*a^5*b^2 + 5*a^4*b^3 +
2*a^3*b^4)*e^(-2*d*x - 2*c) + 2*(3*a^6*b + 14*a^5*b^2 + 27*a^4*b^3 + 24*a^3
*b^4 + 8*a^2*b^5)*e^(-4*d*x - 4*c) + 4*(a^6*b + 4*a^5*b^2 + 5*a^4*b^3 + 2*a
^3*b^4)*e^(-6*d*x - 6*c) + (a^6*b + 2*a^5*b^2 + a^4*b^3)*e^(-8*d*x - 8*c))*
d) - 3/32*(a^3 - 2*a^2*b + (3*a^3 - 4*a^2*b - 16*a*b^2)*e^(-2*d*x - 2*c) +
(3*a^3 + 2*a^2*b - 8*a*b^2 - 16*b^3)*e^(-4*d*x - 4*c) + (a^3 + 4*a^2*b)*e^(
-6*d*x - 6*c))/((a^5*b + 2*a^4*b^2 + a^3*b^3 + 4*(a^5*b + 4*a^4*b^2 + 5*a^3
*b^3 + 2*a^2*b^4)*e^(-2*d*x - 2*c) + 2*(3*a^5*b + 14*a^4*b^2 + 27*a^3*b^3 +
 24*a^2*b^4 + 8*a*b^5)*e^(-4*d*x - 4*c) + 4*(a^5*b + 4*a^4*b^2 + 5*a^3*b^3
+ 2*a^2*b^4)*e^(-6*d*x - 6*c) + (a^5*b + 2*a^4*b^2 + a^3*b^3)*e^(-8*d*x - 8
*c))*d) + 1/4*log(a*e^(4*d*x + 4*c) + 2*(a + 2*b)*e^(2*d*x + 2*c) + a)/(a^3
*d) - 1/4*log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/(a^3*d
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+dx)^6 \tanh(c+dx)^4}{\left(a\cosh(c+dx)^2+b\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^4/(a + b/cosh(c + d*x)^2)^3,x)
```

```
[Out] int((cosh(c + d*x)^6*tanh(c + d*x)^4)/(b + a*cosh(c + d*x)^2)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**4/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

3.161 $\int \frac{\tanh^3(c+dx)}{\left(a+b\mathbf{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=81

$$-\frac{b(a+b)}{4a^3d\left(a\cosh^2(c+dx)+b\right)^2}+\frac{a+2b}{2a^3d\left(a\cosh^2(c+dx)+b\right)}+\frac{\log\left(a\cosh^2(c+dx)+b\right)}{2a^3d}$$

```
[Out] -1/4*b*(a+b)/a^3/d/(b+a*cosh(d*x+c)^2)^2+1/2*(a+2*b)/a^3/d/(b+a*cosh(d*x+c)
^2)+1/2*ln(b+a*cosh(d*x+c)^2)/a^3/d
```

Rubi [A] time = 0.13, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4138, 446, 77}

$$-\frac{b(a+b)}{4a^3d\left(a\cosh^2(c+dx)+b\right)^2}+\frac{a+2b}{2a^3d\left(a\cosh^2(c+dx)+b\right)}+\frac{\log\left(a\cosh^2(c+dx)+b\right)}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^3/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] -(b*(a + b))/(4*a^3*d*(b + a*Cosh[c + d*x]^2)^2) + (a + 2*b)/(2*a^3*d*(b +
a*Cosh[c + d*x]^2)) + Log[b + a*Cosh[c + d*x]^2]/(2*a^3*d)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\tanh^3(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{x^3(1-x^2)}{(b+ax^2)^3}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \frac{(1-x)x}{(b+ax)^3}\,dx, x, \cosh^2(c+dx)\right)}{2d}$$

$$= -\frac{\text{Subst}\left(\int \left(-\frac{b(a+b)}{a^2(b+ax)^3}+\frac{a+2b}{a^2(b+ax)^2}-\frac{1}{a^2(b+ax)}\right)dx, x, \cosh^2(c+dx)\right)}{2d}$$

$$= -\frac{b(a+b)}{4a^3d\left(b+a\cosh^2(c+dx)\right)^2}+\frac{a+2b}{2a^3d\left(b+a\cosh^2(c+dx)\right)}+\frac{\log\left(b+a\cosh^2(\right.}{2a^3d}$$

Mathematica [A] time = 1.57, size = 131, normalized size = 1.62

$$\frac{2\left(a^2+3ab+3b^2\right)+a^2\cosh^2(2(c+dx))\log(a\cosh(2(c+dx))+a+2b)+(a+2b)^2\log(a\cosh(2(c+dx))+a}{2a^3d(a\cosh(2(c+dx))+a+2b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^3/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (2*(a^2 + 3*a*b + 3*b^2) + (a + 2*b)^2*Log[a + 2*b + a*Cosh[2*(c + d*x)]] +
 a^2*Cosh[2*(c + d*x)]^2*Log[a + 2*b + a*Cosh[2*(c + d*x)]] + 2*a*(a + 2*b)
*Cosh[2*(c + d*x)]*(1 + Log[a + 2*b + a*Cosh[2*(c + d*x)]]))/(2*a^3*d*(a +
2*b + a*Cosh[2*(c + d*x)])^2)
```

fricas [B] time = 0.45, size = 1753, normalized size = 21.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^2*d*x*cosh(d*x + c)^8 + 16*a^2*d*x*cosh(d*x + c)*sinh(d*x + c)^7
+ 2*a^2*d*x*sinh(d*x + c)^8 + 4*(2*(a^2 + 2*a*b)*d*x - a^2 - 2*a*b)*cosh(d*
x + c)^6 + 4*(14*a^2*d*x*cosh(d*x + c)^2 + 2*(a^2 + 2*a*b)*d*x - a^2 - 2*a*
b)*sinh(d*x + c)^6 + 8*(14*a^2*d*x*cosh(d*x + c)^3 + 3*(2*(a^2 + 2*a*b)*d*x
 - a^2 - 2*a*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 4*((3*a^2 + 8*a*b + 8*b^2)
*d*x - 2*a^2 - 6*a*b - 6*b^2)*cosh(d*x + c)^4 + 4*(35*a^2*d*x*cosh(d*x + c)
^4 + (3*a^2 + 8*a*b + 8*b^2)*d*x + 15*(2*(a^2 + 2*a*b)*d*x - a^2 - 2*a*b)*c
osh(d*x + c)^2 - 2*a^2 - 6*a*b - 6*b^2)*sinh(d*x + c)^4 + 2*a^2*d*x + 16*(7
*a^2*d*x*cosh(d*x + c)^5 + 5*(2*(a^2 + 2*a*b)*d*x - a^2 - 2*a*b)*cosh(d*x +
 c)^3 + ((3*a^2 + 8*a*b + 8*b^2)*d*x - 2*a^2 - 6*a*b - 6*b^2)*cosh(d*x + c)
)*sinh(d*x + c)^3 + 4*(2*(a^2 + 2*a*b)*d*x - a^2 - 2*a*b)*cosh(d*x + c)^2 +
 4*(14*a^2*d*x*cosh(d*x + c)^6 + 15*(2*(a^2 + 2*a*b)*d*x - a^2 - 2*a*b)*cos
h(d*x + c)^4 + 2*(a^2 + 2*a*b)*d*x + 6*((3*a^2 + 8*a*b + 8*b^2)*d*x - 2*a^2
 - 6*a*b - 6*b^2)*cosh(d*x + c)^2 - a^2 - 2*a*b)*sinh(d*x + c)^2 - (a^2*cos
h(d*x + c)^8 + 8*a^2*cosh(d*x + c)*sinh(d*x + c)^7 + a^2*sinh(d*x + c)^8 +
4*(a^2 + 2*a*b)*cosh(d*x + c)^6 + 4*(7*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*s
inh(d*x + c)^6 + 8*(7*a^2*cosh(d*x + c)^3 + 3*(a^2 + 2*a*b)*cosh(d*x + c))*
sinh(d*x + c)^5 + 2*(3*a^2 + 8*a*b + 8*b^2)*cosh(d*x + c)^4 + 2*(35*a^2*cos
h(d*x + c)^4 + 30*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 3*a^2 + 8*a*b + 8*b^2)*si
nh(d*x + c)^4 + 8*(7*a^2*cosh(d*x + c)^5 + 10*(a^2 + 2*a*b)*cosh(d*x + c)^3
 + (3*a^2 + 8*a*b + 8*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a^2 + 2*a*b)
*cosh(d*x + c)^2 + 4*(7*a^2*cosh(d*x + c)^6 + 15*(a^2 + 2*a*b)*cosh(d*x + c
)^4 + 3*(3*a^2 + 8*a*b + 8*b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c
)^2 + a^2 + 8*(a^2*cosh(d*x + c)^7 + 3*(a^2 + 2*a*b)*cosh(d*x + c)^5 + (3*a
^2 + 8*a*b + 8*b^2)*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x
 + c))*log(2*(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + a + 2*b)/(cosh(d*x +
c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) + 8*(2*a^2*d*x*cos
h(d*x + c)^7 + 3*(2*(a^2 + 2*a*b)*d*x - a^2 - 2*a*b)*cosh(d*x + c)^5 + 2*((
3*a^2 + 8*a*b + 8*b^2)*d*x - 2*a^2 - 6*a*b - 6*b^2)*cosh(d*x + c)^3 + (2*(a
^2 + 2*a*b)*d*x - a^2 - 2*a*b)*cosh(d*x + c))*sinh(d*x + c))/(a^5*d*cosh(d*
x + c)^8 + 8*a^5*d*cosh(d*x + c)*sinh(d*x + c)^7 + a^5*d*sinh(d*x + c)^8 +
4*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^6 + 4*(7*a^5*d*cosh(d*x + c)^2 + (a^5 + 2
*a^4*b)*d)*sinh(d*x + c)^6 + a^5*d + 2*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*d*cosh
(d*x + c)^4 + 8*(7*a^5*d*cosh(d*x + c)^3 + 3*(a^5 + 2*a^4*b)*d*cosh(d*x + c
))*sinh(d*x + c)^5 + 2*(35*a^5*d*cosh(d*x + c)^4 + 30*(a^5 + 2*a^4*b)*d*cos
h(d*x + c)^2 + (3*a^5 + 8*a^4*b + 8*a^3*b^2)*d)*sinh(d*x + c)^4 + 4*(a^5 +
2*a^4*b)*d*cosh(d*x + c)^2 + 8*(7*a^5*d*cosh(d*x + c)^5 + 10*(a^5 + 2*a^4*b
)*d*cosh(d*x + c)^3 + (3*a^5 + 8*a^4*b + 8*a^3*b^2)*d*cosh(d*x + c))*sinh(d
*x + c)^3 + 4*(7*a^5*d*cosh(d*x + c)^6 + 15*(a^5 + 2*a^4*b)*d*cosh(d*x + c)
^4 + 3*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*d*cosh(d*x + c)^2 + (a^5 + 2*a^4*b)*d)
*sinh(d*x + c)^2 + 8*(a^5*d*cosh(d*x + c)^7 + 3*(a^5 + 2*a^4*b)*d*cosh(d*x
+ c)^5 + (3*a^5 + 8*a^4*b + 8*a^3*b^2)*d*cosh(d*x + c)^3 + (a^5 + 2*a^4*b)*
d*cosh(d*x + c))*sinh(d*x + c))
```

giac [B] time = 1.22, size = 175, normalized size = 2.16

$$-\frac{\frac{4\,dx}{a^3} - \frac{2\,\log\left(ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+a\right)}{a^3} + \frac{3\,ae^{(8\,dx+8\,c)}+4\,ae^{(6\,dx+6\,c)}+8\,be^{(6\,dx+6\,c)}+2\,ae^{(4\,dx+4\,c)}+4\,ae^{(2\,dx+2\,c)}+8\,be^{(2\,dx+2\,c)}+3\,a}{\left(ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+a\right)^2 a^2}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/4*(4*d*x/a^3 - 2*log(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*
d*x + 2*c) + a)/a^3 + (3*a*e^(8*d*x + 8*c) + 4*a*e^(6*d*x + 6*c) + 8*b*e^(6
*d*x + 6*c) + 2*a*e^(4*d*x + 4*c) + 4*a*e^(2*d*x + 2*c) + 8*b*e^(2*d*x + 2*
c) + 3*a)/((a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) +
 a)^2*a^2))/d
```

maple [B] time = 0.40, size = 672, normalized size = 8.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] -1/d/a^3*ln(tanh(1/2*d*x+1/2*c)-1)-1/d/a^3*ln(tanh(1/2*d*x+1/2*c)+1)-2/d/a/
(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-
2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^6-2/d/a^2*b/(tanh(1/2*
d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2
*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^6-4/d/(tanh(1/2*d*x+1/2*c)^4*a+b
*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+
a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)^4-4/d/a*b/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(
1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2
/(a+b)*tanh(1/2*d*x+1/2*c)^4+4/d/a^2*b^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/
2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(
a+b)*tanh(1/2*d*x+1/2*c)^4-2/d/a/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/
2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*
d*x+1/2*c)^2-2/d/a^2*b/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*t
anh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)
^2+1/2/d/a^3*ln(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*
d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)
```

maxima [B] time = 0.34, size = 209, normalized size = 2.58

$$\frac{2\left(\left(a^2+2\,ab\right)e^{(-2\,dx-2\,c)}+2\left(a^2+3\,ab+3\,b^2\right)e^{(-4\,dx-4\,c)}+\left(a^2+2\,ab\right)e^{(-6\,dx-6\,c)}\right)}{\left(a^5e^{(-8\,dx-8\,c)}+a^5+4\left(a^5+2\,a^4b\right)e^{(-2\,dx-2\,c)}+2\left(3\,a^5+8\,a^4b+8\,a^3b^2\right)e^{(-4\,dx-4\,c)}+4\left(a^5+2\,a^4b\right)e^{(-6\,dx-6\,c)}\right)d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 2*((a^2 + 2*a*b)*e^(-2*d*x - 2*c) + 2*(a^2 + 3*a*b + 3*b^2)*e^(-4*d*x - 4*c
) + (a^2 + 2*a*b)*e^(-6*d*x - 6*c))/((a^5*e^(-8*d*x - 8*c) + a^5 + 4*(a^5 +
 2*a^4*b)*e^(-2*d*x - 2*c) + 2*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*e^(-4*d*x - 4*
c) + 4*(a^5 + 2*a^4*b)*e^(-6*d*x - 6*c))*d) + (d*x + c)/(a^3*d) + 1/2*log(2
*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/(a^3*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh\left(c+d\,x\right)^6\,\tanh\left(c+d\,x\right)^3}{{\left(a\,\cosh\left(c+d\,x\right)^2+b\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^3/(a + b/cosh(c + d*x)^2)^3,x)
```

```
[Out] int((cosh(c + d*x)^6*tanh(c + d*x)^3)/(b + a*cosh(c + d*x)^2)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**3/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

3.162 $\int \frac{\tanh^2(c+dx)}{\left(a+b\,\mathbf{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=139

$$\frac{x}{a^3}-\frac{(3a+4b)\tanh(c+dx)}{8a^2d(a+b)\left(a-b\tanh^2(c+dx)+b\right)}-\frac{\left(3a^2+12ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^3\sqrt{b}\,d(a+b)^{3/2}}-\frac{\tanh(c+dx)}{4ad\left(a-b\tanh^2(c+dx)\right.}$$

```
[Out] x/a^3-1/8*(3*a^2+12*a*b+8*b^2)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))/a^3
/(a+b)^(3/2)/d/b^(1/2)-1/4*tanh(d*x+c)/a/d/(a+b-b*tanh(d*x+c)^2)^2-1/8*(3*a
+4*b)*tanh(d*x+c)/a^2/(a+b)/d/(a+b-b*tanh(d*x+c)^2)
```

Rubi [A] time = 0.25, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4141, 1975, 471, 527, 522, 206, 208}

$$-\frac{\left(3a^2+12ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^3\sqrt{b}\,d(a+b)^{3/2}}-\frac{(3a+4b)\tanh(c+dx)}{8a^2d(a+b)\left(a-b\tanh^2(c+dx)+b\right)}+\frac{x}{a^3}-\frac{\tanh(c+dx)}{4ad\left(a-b\tanh^2(c+d\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^2/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] x/a^3 - ((3*a^2 + 12*a*b + 8*b^2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a +
b]])/(8*a^3*Sqrt[b]*(a + b)^(3/2)*d) - Tanh[c + d*x]/(4*a*d*(a + b - b*Tanh
[c + d*x]^2)^2) - ((3*a + 4*b)*Tanh[c + d*x])/(8*a^2*(a + b)*d*(a + b - b*T
anh[c + d*x]^2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
```

```
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\tanh^2(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = \frac{\operatorname{Subst}\left(\int \frac{x^2}{(1-x^2)\left(a+b(1-x^2)\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \frac{x^2}{(1-x^2)(a+b-bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{\tanh(c+dx)}{4ad\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{\operatorname{Subst}\left(\int \frac{1+3x^2}{(1-x^2)(a+b-bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{4ad}$$

$$= -\frac{\tanh(c+dx)}{4ad\left(a+b-b\tanh^2(c+dx)\right)^2} - \frac{(3a+4b)\tanh(c+dx)}{8a^2(a+b)d\left(a+b-b\tanh^2(c+dx)\right)} - \frac{\operatorname{Subs}}{}$$

$$= -\frac{\tanh(c+dx)}{4ad\left(a+b-b\tanh^2(c+dx)\right)^2} - \frac{(3a+4b)\tanh(c+dx)}{8a^2(a+b)d\left(a+b-b\tanh^2(c+dx)\right)} + \frac{\operatorname{Subs}}{}$$

$$= \frac{x}{a^3} - \frac{\left(3a^2+12ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^3\sqrt{b}\,(a+b)^{3/2}d} - \frac{\tanh(c+dx)}{4ad\left(a+b-b\tanh^2(c+dx)\right)^2}$$

Mathematica [B] time = 12.06, size = 1457, normalized size = 10.48

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tanh[c + d*x]^2/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] -1/1024*((a + 2*b + a*Cosh[2*c + 2*d*x])^3*Sech[c + d*x]^6*(((3*a^2 + 8*a*b
 + 8*b^2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a + b)^(5/2) - (a*
Sqrt[b]*(3*a^2 + 16*a*b + 16*b^2 + 3*a*(a + 2*b)*Cosh[2*(c + d*x)])*Sinh[2*
(c + d*x)])/((a + b)^2*(a + 2*b + a*Cosh[2*(c + d*x)])^2)))/(b^(5/2)*d*(a +
 b*Sech[c + d*x]^2)^3) - ((a + 2*b + a*Cosh[2*c + 2*d*x])^3*Sech[c + d*x]^6
*((-3*a*(a + 2*b)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a + b)^(5/
2) + (Sqrt[b]*(3*a^3 + 14*a^2*b + 24*a*b^2 + 16*b^3 + a*(3*a^2 + 4*a*b + 4*
b^2)*Cosh[2*(c + d*x)])*Sinh[2*(c + d*x)])/((a + b)^2*(a + 2*b + a*Cosh[2*(
c + d*x)])^2)))/(2048*b^(5/2)*d*(a + b*Sech[c + d*x]^2)^3) + ((a + 2*b + a*
Cosh[2*c + 2*d*x])^3*Sech[c + d*x]^6*((-2*(3*a^5 - 10*a^4*b + 80*a^3*b^2 +
```

```
480*a^2*b^3 + 640*a*b^4 + 256*b^5)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c
])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c
] - Sinh[c])^4])]*(Cosh[2*c] - Sinh[2*c]))/(Sqrt[a + b]*Sqrt[b*(Cosh[c] - S
inh[c])^4]) + (Sech[2*c]*(256*b^2*(a + b)^2*(3*a^2 + 8*a*b + 8*b^2)*d*x*Cos
h[2*c] + 512*a*b^2*(a + b)^2*(a + 2*b)*d*x*Cosh[2*d*x] + 128*a^4*b^2*d*x*Co
sh[2*(c + 2*d*x)] + 256*a^3*b^3*d*x*Cosh[2*(c + 2*d*x)] + 128*a^2*b^4*d*x*C
osh[2*(c + 2*d*x)] + 512*a^4*b^2*d*x*Cosh[4*c + 2*d*x] + 2048*a^3*b^3*d*x*C
osh[4*c + 2*d*x] + 2560*a^2*b^4*d*x*Cosh[4*c + 2*d*x] + 1024*a*b^5*d*x*Cosh
[4*c + 2*d*x] + 128*a^4*b^2*d*x*Cosh[6*c + 4*d*x] + 256*a^3*b^3*d*x*Cosh[6*
c + 4*d*x] + 128*a^2*b^4*d*x*Cosh[6*c + 4*d*x] - 9*a^6*Sinh[2*c] + 12*a^5*b
*Sinh[2*c] + 684*a^4*b^2*Sinh[2*c] + 2880*a^3*b^3*Sinh[2*c] + 5280*a^2*b^4*
Sinh[2*c] + 4608*a*b^5*Sinh[2*c] + 1536*b^6*Sinh[2*c] + 9*a^6*Sinh[2*d*x] -
 14*a^5*b*Sinh[2*d*x] - 608*a^4*b^2*Sinh[2*d*x] - 2112*a^3*b^3*Sinh[2*d*x]
- 2560*a^2*b^4*Sinh[2*d*x] - 1024*a*b^5*Sinh[2*d*x] + 3*a^6*Sinh[2*(c + 2*d
*x)] - 12*a^5*b*Sinh[2*(c + 2*d*x)] - 204*a^4*b^2*Sinh[2*(c + 2*d*x)] - 384
*a^3*b^3*Sinh[2*(c + 2*d*x)] - 192*a^2*b^4*Sinh[2*(c + 2*d*x)] - 3*a^6*Sinh
[4*c + 2*d*x] + 10*a^5*b*Sinh[4*c + 2*d*x] + 304*a^4*b^2*Sinh[4*c + 2*d*x]
+ 1056*a^3*b^3*Sinh[4*c + 2*d*x] + 1280*a^2*b^4*Sinh[4*c + 2*d*x] + 512*a*b
^5*Sinh[4*c + 2*d*x]))/(a + 2*b + a*Cosh[2*(c + d*x)])^2))/(4096*a^3*b^2*(a
 + b)^2*d*(a + b*Sech[c + d*x]^2)^3) + ((a + 2*b + a*Cosh[2*c + 2*d*x])^3*S
ech[c + d*x]^6*((6*a^2*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*((a + 2*b
)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])
^4])]*(Cosh[2*c] - Sinh[2*c]))/(Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])
+ (a*Sech[2*c]*((-9*a^4 - 16*a^3*b + 48*a^2*b^2 + 128*a*b^3 + 64*b^4)*Sinh[
2*d*x] + a*(-3*a^3 + 2*a^2*b + 24*a*b^2 + 16*b^3)*Sinh[2*(c + 2*d*x)] + (3*
a^4 - 64*a^2*b^2 - 128*a*b^3 - 64*b^4)*Sinh[4*c + 2*d*x]) + (9*a^5 + 18*a^4
*b - 64*a^3*b^2 - 256*a^2*b^3 - 320*a*b^4 - 128*b^5)*Tanh[2*c])/(a^2*(a + 2
*b + a*Cosh[2*(c + d*x)])^2)))/(2048*b^2*(a + b)^2*d*(a + b*Sech[c + d*x]^2
)^3)
```

fricas [B] time = 0.57, size = 7158, normalized size = 51.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(16*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^8 + 128*(a^4*b +
2*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + 16*(a^4*b + 2*a^3*
b^2 + a^2*b^3)*d*x*sinh(d*x + c)^8 + 4*(5*a^4*b + 25*a^3*b^2 + 36*a^2*b^3 +
 16*a*b^4 + 16*(a^4*b + 4*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c)
^6 + 4*(5*a^4*b + 25*a^3*b^2 + 36*a^2*b^3 + 16*a*b^4 + 112*(a^4*b + 2*a^3*b
^2 + a^2*b^3)*d*x*cosh(d*x + c)^2 + 16*(a^4*b + 4*a^3*b^2 + 5*a^2*b^3 + 2*a
*b^4)*d*x)*sinh(d*x + c)^6 + 8*(112*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*x*cosh(
d*x + c)^3 + 3*(5*a^4*b + 25*a^3*b^2 + 36*a^2*b^3 + 16*a*b^4 + 16*(a^4*b +
```

4*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 + 20*a
^4*b + 44*a^3*b^2 + 24*a^2*b^3 + 4*(15*a^4*b + 73*a^3*b^2 + 146*a^2*b^3 + 1
36*a*b^4 + 48*b^5 + 8*(3*a^4*b + 14*a^3*b^2 + 27*a^2*b^3 + 24*a*b^4 + 8*b^5
)*d*x)*cosh(d*x + c)^4 + 4*(280*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x
+ c)^4 + 15*a^4*b + 73*a^3*b^2 + 146*a^2*b^3 + 136*a*b^4 + 48*b^5 + 8*(3*a^
4*b + 14*a^3*b^2 + 27*a^2*b^3 + 24*a*b^4 + 8*b^5)*d*x + 15*(5*a^4*b + 25*a^
3*b^2 + 36*a^2*b^3 + 16*a*b^4 + 16*(a^4*b + 4*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4
)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 16*(56*(a^4*b + 2*a^3*b^2 + a^2*b
^3)*d*x*cosh(d*x + c)^5 + 5*(5*a^4*b + 25*a^3*b^2 + 36*a^2*b^3 + 16*a*b^4 +
16*(a^4*b + 4*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c)^3 + (15*a^
4*b + 73*a^3*b^2 + 146*a^2*b^3 + 136*a*b^4 + 48*b^5 + 8*(3*a^4*b + 14*a^3*b
^2 + 27*a^2*b^3 + 24*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 + 1
6*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*x + 4*(15*a^4*b + 59*a^3*b^2 + 76*a^2*b^3
+ 32*a*b^4 + 16*(a^4*b + 4*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x +
c)^2 + 4*(112*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^6 + 15*a^4*b
+ 59*a^3*b^2 + 76*a^2*b^3 + 32*a*b^4 + 15*(5*a^4*b + 25*a^3*b^2 + 36*a^2*b^
3 + 16*a*b^4 + 16*(a^4*b + 4*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x +
c)^4 + 16*(a^4*b + 4*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4)*d*x + 6*(15*a^4*b + 73
*a^3*b^2 + 146*a^2*b^3 + 136*a*b^4 + 48*b^5 + 8*(3*a^4*b + 14*a^3*b^2 + 27*
a^2*b^3 + 24*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((3*a^4
+ 12*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^8 + 8*(3*a^4 + 12*a^3*b + 8*a^2*b^2)
*cosh(d*x + c)*sinh(d*x + c)^7 + (3*a^4 + 12*a^3*b + 8*a^2*b^2)*sinh(d*x +
c)^8 + 4*(3*a^4 + 18*a^3*b + 32*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^6 + 4*(3*
a^4 + 18*a^3*b + 32*a^2*b^2 + 16*a*b^3 + 7*(3*a^4 + 12*a^3*b + 8*a^2*b^2)*c
osh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(3*a^4 + 12*a^3*b + 8*a^2*b^2)*cosh(
d*x + c)^3 + 3*(3*a^4 + 18*a^3*b + 32*a^2*b^2 + 16*a*b^3)*cosh(d*x + c))*si
nh(d*x + c)^5 + 2*(9*a^4 + 60*a^3*b + 144*a^2*b^2 + 160*a*b^3 + 64*b^4)*cos
h(d*x + c)^4 + 2*(35*(3*a^4 + 12*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^4 + 9*a^4
+ 60*a^3*b + 144*a^2*b^2 + 160*a*b^3 + 64*b^4 + 30*(3*a^4 + 18*a^3*b + 32*
a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 3*a^4 + 12*a^3*b + 8
*a^2*b^2 + 8*(7*(3*a^4 + 12*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^5 + 10*(3*a^4
+ 18*a^3*b + 32*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^3 + (9*a^4 + 60*a^3*b + 1
44*a^2*b^2 + 160*a*b^3 + 64*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(3*a^4
+ 18*a^3*b + 32*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^2 + 4*(7*(3*a^4 + 12*a^3*
b + 8*a^2*b^2)*cosh(d*x + c)^6 + 15*(3*a^4 + 18*a^3*b + 32*a^2*b^2 + 16*a*b
^3)*cosh(d*x + c)^4 + 3*a^4 + 18*a^3*b + 32*a^2*b^2 + 16*a*b^3 + 3*(9*a^4 +
60*a^3*b + 144*a^2*b^2 + 160*a*b^3 + 64*b^4)*cosh(d*x + c)^2)*sinh(d*x + c
)^2 + 8*((3*a^4 + 12*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^7 + 3*(3*a^4 + 18*a^3
*b + 32*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^5 + (9*a^4 + 60*a^3*b + 144*a^2*b
^2 + 160*a*b^3 + 64*b^4)*cosh(d*x + c)^3 + (3*a^4 + 18*a^3*b + 32*a^2*b^2 +
16*a*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a*b + b^2)*log((a^2*cosh(d*x
+ c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2
+ 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*
x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cos
h(d*x + c))*sinh(d*x + c) + 4*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d

```
*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(a*b + b^2))/(a*cosh(d*x + c)^4
+ 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(
d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(
d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) + 8*(16*(a^4*b +
2*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^7 + 3*(5*a^4*b + 25*a^3*b^2 + 36*a^2
*b^3 + 16*a*b^4 + 16*(a^4*b + 4*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*
x + c)^5 + 2*(15*a^4*b + 73*a^3*b^2 + 146*a^2*b^3 + 136*a*b^4 + 48*b^5 + 8*
(3*a^4*b + 14*a^3*b^2 + 27*a^2*b^3 + 24*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c)^3
 + (15*a^4*b + 59*a^3*b^2 + 76*a^2*b^3 + 32*a*b^4 + 16*(a^4*b + 4*a^3*b^2 +
 5*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a^7*b + 2*a^6*b^
2 + a^5*b^3)*d*cosh(d*x + c)^8 + 8*(a^7*b + 2*a^6*b^2 + a^5*b^3)*d*cosh(d*x
 + c)*sinh(d*x + c)^7 + (a^7*b + 2*a^6*b^2 + a^5*b^3)*d*sinh(d*x + c)^8 + 4
*(a^7*b + 4*a^6*b^2 + 5*a^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c)^6 + 4*(7*(a^7*
b + 2*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^2 + (a^7*b + 4*a^6*b^2 + 5*a^5*b^3
 + 2*a^4*b^4)*d)*sinh(d*x + c)^6 + 2*(3*a^7*b + 14*a^6*b^2 + 27*a^5*b^3 + 2
4*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c)^4 + 8*(7*(a^7*b + 2*a^6*b^2 + a^5*b^
3)*d*cosh(d*x + c)^3 + 3*(a^7*b + 4*a^6*b^2 + 5*a^5*b^3 + 2*a^4*b^4)*d*cosh
(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^7*b + 2*a^6*b^2 + a^5*b^3)*d*cosh(d*x
 + c)^4 + 30*(a^7*b + 4*a^6*b^2 + 5*a^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c)^2
+ (3*a^7*b + 14*a^6*b^2 + 27*a^5*b^3 + 24*a^4*b^4 + 8*a^3*b^5)*d)*sinh(d*x
+ c)^4 + 4*(a^7*b + 4*a^6*b^2 + 5*a^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c)^2 +
8*(7*(a^7*b + 2*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^5 + 10*(a^7*b + 4*a^6*b^
2 + 5*a^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c)^3 + (3*a^7*b + 14*a^6*b^2 + 27*a
^5*b^3 + 24*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a
^7*b + 2*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^6 + 15*(a^7*b + 4*a^6*b^2 + 5*a
^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c)^4 + 3*(3*a^7*b + 14*a^6*b^2 + 27*a^5*b^
3 + 24*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c)^2 + (a^7*b + 4*a^6*b^2 + 5*a^5*
b^3 + 2*a^4*b^4)*d)*sinh(d*x + c)^2 + (a^7*b + 2*a^6*b^2 + a^5*b^3)*d + 8*(
(a^7*b + 2*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^7 + 3*(a^7*b + 4*a^6*b^2 + 5*
a^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c)^5 + (3*a^7*b + 14*a^6*b^2 + 27*a^5*b^3
 + 24*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c)^3 + (a^7*b + 4*a^6*b^2 + 5*a^5*b
^3 + 2*a^4*b^4)*d*cosh(d*x + c))*sinh(d*x + c)), 1/8*(8*(a^4*b + 2*a^3*b^2
+ a^2*b^3)*d*x*cosh(d*x + c)^8 + 64*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*x*cosh(
d*x + c)*sinh(d*x + c)^7 + 8*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*x*sinh(d*x + c
)^8 + 2*(5*a^4*b + 25*a^3*b^2 + 36*a^2*b^3 + 16*a*b^4 + 16*(a^4*b + 4*a^3*b
^2 + 5*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c)^6 + 2*(5*a^4*b + 25*a^3*b^2 +
36*a^2*b^3 + 16*a*b^4 + 112*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)
^2 + 16*(a^4*b + 4*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4)*d*x)*sinh(d*x + c)^6 + 4*
(112*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^3 + 3*(5*a^4*b + 25*a^
3*b^2 + 36*a^2*b^3 + 16*a*b^4 + 16*(a^4*b + 4*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4
)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 + 10*a^4*b + 22*a^3*b^2 + 12*a^2*b^3
+ 2*(15*a^4*b + 73*a^3*b^2 + 146*a^2*b^3 + 136*a*b^4 + 48*b^5 + 8*(3*a^4*b
+ 14*a^3*b^2 + 27*a^2*b^3 + 24*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c)^4 + 2*(280
*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^4 + 15*a^4*b + 73*a^3*b^2
+ 146*a^2*b^3 + 136*a*b^4 + 48*b^5 + 8*(3*a^4*b + 14*a^3*b^2 + 27*a^2*b^3 +
```

24*a*b^4 + 8*b^5)*d*x + 15*(5*a^4*b + 25*a^3*b^2 + 36*a^2*b^3 + 16*a*b^4 + 16*(a^4*b + 4*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(56*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^5 + 5*(5*a^4*b + 25*a^3*b^2 + 36*a^2*b^3 + 16*a*b^4 + 16*(a^4*b + 4*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c)^3 + (15*a^4*b + 73*a^3*b^2 + 146*a^2*b^3 + 136*a*b^4 + 48*b^5 + 8*(3*a^4*b + 14*a^3*b^2 + 27*a^2*b^3 + 24*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 + 8*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*x + 2*(15*a^4*b + 59*a^3*b^2 + 76*a^2*b^3 + 32*a*b^4 + 16*(a^4*b + 4*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c)^2 + 2*(112*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^6 + 15*a^4*b + 59*a^3*b^2 + 76*a^2*b^3 + 32*a*b^4 + 15*(5*a^4*b + 25*a^3*b^2 + 36*a^2*b^3 + 16*a*b^4 + 16*(a^4*b + 4*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c)^4 + 16*(a^4*b + 4*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4)*d*x + 6*(15*a^4*b + 73*a^3*b^2 + 146*a^2*b^3 + 136*a*b^4 + 48*b^5 + 8*(3*a^4*b + 14*a^3*b^2 + 27*a^2*b^3 + 24*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - ((3*a^4 + 12*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^8 + 8*(3*a^4 + 12*a^3*b + 8*a^2*b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (3*a^4 + 12*a^3*b + 8*a^2*b^2)*sinh(d*x + c)^8 + 4*(3*a^4 + 18*a^3*b + 32*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^6 + 4*(3*a^4 + 18*a^3*b + 32*a^2*b^2 + 16*a*b^3 + 7*(3*a^4 + 12*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(3*a^4 + 12*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^3 + 3*(3*a^4 + 18*a^3*b + 32*a^2*b^2 + 16*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(9*a^4 + 60*a^3*b + 144*a^2*b^2 + 160*a*b^3 + 64*b^4)*cosh(d*x + c)^4 + 2*(35*(3*a^4 + 12*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^4 + 9*a^4 + 60*a^3*b + 144*a^2*b^2 + 160*a*b^3 + 64*b^4 + 30*(3*a^4 + 18*a^3*b + 32*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 3*a^4 + 12*a^3*b + 8*a^2*b^2 + 8*(7*(3*a^4 + 12*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^5 + 10*(3*a^4 + 18*a^3*b + 32*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^3 + (9*a^4 + 60*a^3*b + 144*a^2*b^2 + 160*a*b^3 + 64*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(3*a^4 + 18*a^3*b + 32*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^2 + 4*(7*(3*a^4 + 12*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^6 + 15*(3*a^4 + 18*a^3*b + 32*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^4 + 3*a^4 + 18*a^3*b + 32*a^2*b^2 + 16*a*b^3 + 3*(9*a^4 + 60*a^3*b + 144*a^2*b^2 + 160*a*b^3 + 64*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((3*a^4 + 12*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^7 + 3*(3*a^4 + 18*a^3*b + 32*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^5 + (9*a^4 + 60*a^3*b + 144*a^2*b^2 + 160*a*b^3 + 64*b^4)*cosh(d*x + c)^3 + (3*a^4 + 18*a^3*b + 32*a^2*b^2 + 16*a*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a*b - b^2)*arctan(1/2*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-a*b - b^2)/(a*b + b^2)) + 4*(16*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^7 + 3*(5*a^4*b + 25*a^3*b^2 + 36*a^2*b^3 + 16*a*b^4 + 16*(a^4*b + 4*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c)^5 + 2*(15*a^4*b + 73*a^3*b^2 + 146*a^2*b^3 + 136*a*b^4 + 48*b^5 + 8*(3*a^4*b + 14*a^3*b^2 + 27*a^2*b^3 + 24*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c)^3 + (15*a^4*b + 59*a^3*b^2 + 76*a^2*b^3 + 32*a*b^4 + 16*(a^4*b + 4*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a^7*b + 2*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^8 + 8*(a^7*b + 2*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^7*b + 2*a^6*b^2 + a^5*b^3)*d*si

```
nh(d*x + c)^8 + 4*(a^7*b + 4*a^6*b^2 + 5*a^5*b^3 + 2*a^4*b^4)*d*cosh(d*x +
c)^6 + 4*(7*(a^7*b + 2*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^2 + (a^7*b + 4*a^
6*b^2 + 5*a^5*b^3 + 2*a^4*b^4)*d)*sinh(d*x + c)^6 + 2*(3*a^7*b + 14*a^6*b^2
 + 27*a^5*b^3 + 24*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c)^4 + 8*(7*(a^7*b + 2
*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^3 + 3*(a^7*b + 4*a^6*b^2 + 5*a^5*b^3 +
2*a^4*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^7*b + 2*a^6*b^2 + a^
5*b^3)*d*cosh(d*x + c)^4 + 30*(a^7*b + 4*a^6*b^2 + 5*a^5*b^3 + 2*a^4*b^4)*d
*cosh(d*x + c)^2 + (3*a^7*b + 14*a^6*b^2 + 27*a^5*b^3 + 24*a^4*b^4 + 8*a^3*
b^5)*d)*sinh(d*x + c)^4 + 4*(a^7*b + 4*a^6*b^2 + 5*a^5*b^3 + 2*a^4*b^4)*d*c
osh(d*x + c)^2 + 8*(7*(a^7*b + 2*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^5 + 10*
(a^7*b + 4*a^6*b^2 + 5*a^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c)^3 + (3*a^7*b +
14*a^6*b^2 + 27*a^5*b^3 + 24*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c))*sinh(d*x
 + c)^3 + 4*(7*(a^7*b + 2*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^6 + 15*(a^7*b
+ 4*a^6*b^2 + 5*a^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c)^4 + 3*(3*a^7*b + 14*a^
6*b^2 + 27*a^5*b^3 + 24*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c)^2 + (a^7*b + 4
*a^6*b^2 + 5*a^5*b^3 + 2*a^4*b^4)*d)*sinh(d*x + c)^2 + (a^7*b + 2*a^6*b^2 +
 a^5*b^3)*d + 8*((a^7*b + 2*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^7 + 3*(a^7*b
 + 4*a^6*b^2 + 5*a^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c)^5 + (3*a^7*b + 14*a^6
*b^2 + 27*a^5*b^3 + 24*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c)^3 + (a^7*b + 4*
a^6*b^2 + 5*a^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c))*sinh(d*x + c))]
```

giac [B] time = 0.97, size = 309, normalized size = 2.22

$$-\frac{\frac{\left(3\,a^2e^{(2\,c)}+12\,abe^{(2\,c)}+8\,b^2e^{(2\,c)}\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)e^{(-2\,c)}}{\left(a^4+a^3b\right)\sqrt{-ab-b^2}}-\frac{8\,dx}{a^3}-\frac{2\left(5\,a^3e^{(6\,dx+6\,c)}+20\,a^2be^{(6\,dx+6\,c)}+16\,ab^2e^{(6\,dx+6\,c)}+15\,a^3e^{(4\,dx+4\,c)}+58\,a^2\cdots\right.}{\left(a^4+a^3b\right)\cdots}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*((3*a^2*e^(2*c) + 12*a*b*e^(2*c) + 8*b^2*e^(2*c))*arctan(1/2*(a*e^(2*d
*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))*e^(-2*c)/((a^4 + a^3*b)*sqrt(-a*b -
b^2)) - 8*d*x/a^3 - 2*(5*a^3*e^(6*d*x + 6*c) + 20*a^2*b*e^(6*d*x + 6*c) + 1
6*a*b^2*e^(6*d*x + 6*c) + 15*a^3*e^(4*d*x + 4*c) + 58*a^2*b*e^(4*d*x + 4*c)
 + 88*a*b^2*e^(4*d*x + 4*c) + 48*b^3*e^(4*d*x + 4*c) + 15*a^3*e^(2*d*x + 2*
c) + 44*a^2*b*e^(2*d*x + 2*c) + 32*a*b^2*e^(2*d*x + 2*c) + 5*a^3 + 6*a^2*b)
/((a^4 + a^3*b)*(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2
*c) + a)^2))/d
```

maple [B] time = 0.43, size = 1173, normalized size = 8.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] -1/d/a^3*ln(tanh(1/2*d*x+1/2*c)-1)+1/d/a^3*ln(tanh(1/2*d*x+1/2*c)+1)-5/4/d/
a/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*
a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^7-1/d/a^2*b/(tanh(1/
2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1
/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^7-15/4/d/(tanh(1/2*d*x+1/2*c)^
4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)
^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)^5-15/4/d*b/a/(tanh(1/2*d*x+1/2*c)^4*a
+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*
b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)^5+1/d*b^2/a^2/(tanh(1/2*d*x+1/2*c)^4*a+b
*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+
a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)^5-15/4/d/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1
/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/
(a+b)*tanh(1/2*d*x+1/2*c)^3-15/4/d*b/a/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*
d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+
b)*tanh(1/2*d*x+1/2*c)^3+1/d*b^2/a^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*
x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)
*tanh(1/2*d*x+1/2*c)^3-5/4/d/a/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*
c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*
x+1/2*c)-1/d/a^2*b/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(
1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)+3/1
6/d/a/(a+b)^(3/2)/b^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*b^(1/2)*ta
nh(1/2*d*x+1/2*c)+(a+b)^(1/2))+3/4/d/a^2/(a+b)^(3/2)*b^(1/2)*ln((a+b)^(1/2)
*tanh(1/2*d*x+1/2*c)^2-2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))+1/2/d/a^3
/(a+b)^(3/2)*b^(3/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*b^(1/2)*tanh(1/
2*d*x+1/2*c)+(a+b)^(1/2))-3/16/d/a/(a+b)^(3/2)/b^(1/2)*ln((a+b)^(1/2)*tanh(
1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))-3/4/d*b^(1/2)/a
^2/(a+b)^(3/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+
1/2*c)+(a+b)^(1/2))-1/2/d*b^(3/2)/a^3/(a+b)^(3/2)*ln((a+b)^(1/2)*tanh(1/2*d
*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))
```

maxima [B] time = 0.93, size = 1255, normalized size = 9.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/64*(3*a^3 + 30*a^2*b + 40*a*b^2 + 16*b^3)*log((a*e^(2*d*x + 2*c) + a + 2
*b - 2*sqrt((a + b)*b))/(a*e^(2*d*x + 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/
((a^5 + 2*a^4*b + a^3*b^2)*sqrt((a + b)*b)*d) + 1/64*(3*a^3 + 30*a^2*b + 40
*a*b^2 + 16*b^3)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*
e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^5 + 2*a^4*b + a^3*b^2)
*sqrt((a + b)*b)*d) + 1/16*(5*a^4 + 20*a^3*b + 12*a^2*b^2 + (5*a^4 + 66*a^3
```

```
*b + 128*a^2*b^2 + 64*a*b^3)*e^(6*d*x + 6*c) + (15*a^4 + 164*a^3*b + 460*a^
2*b^2 + 512*a*b^3 + 192*b^4)*e^(4*d*x + 4*c) + (15*a^4 + 118*a^3*b + 208*a^
2*b^2 + 96*a*b^3)*e^(2*d*x + 2*c))/((a^7 + 2*a^6*b + a^5*b^2 + (a^7 + 2*a^6
*b + a^5*b^2)*e^(8*d*x + 8*c) + 4*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*e
^(6*d*x + 6*c) + 2*(3*a^7 + 14*a^6*b + 27*a^5*b^2 + 24*a^4*b^3 + 8*a^3*b^4)
*e^(4*d*x + 4*c) + 4*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*e^(2*d*x + 2*c
))*d) - 1/16*(5*a^4 + 20*a^3*b + 12*a^2*b^2 + (15*a^4 + 118*a^3*b + 208*a^2
*b^2 + 96*a*b^3)*e^(-2*d*x - 2*c) + (15*a^4 + 164*a^3*b + 460*a^2*b^2 + 512
*a*b^3 + 192*b^4)*e^(-4*d*x - 4*c) + (5*a^4 + 66*a^3*b + 128*a^2*b^2 + 64*a
*b^3)*e^(-6*d*x - 6*c))/((a^7 + 2*a^6*b + a^5*b^2 + 4*(a^7 + 4*a^6*b + 5*a^
5*b^2 + 2*a^4*b^3)*e^(-2*d*x - 2*c) + 2*(3*a^7 + 14*a^6*b + 27*a^5*b^2 + 24
*a^4*b^3 + 8*a^3*b^4)*e^(-4*d*x - 4*c) + 4*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a
^4*b^3)*e^(-6*d*x - 6*c) + (a^7 + 2*a^6*b + a^5*b^2)*e^(-8*d*x - 8*c))*d) -
 1/8*(5*a^3 + 2*a^2*b + (15*a^3 + 32*a^2*b + 8*a*b^2)*e^(-2*d*x - 2*c) + (1
5*a^3 + 46*a^2*b + 56*a*b^2 + 16*b^3)*e^(-4*d*x - 4*c) + (5*a^3 + 16*a^2*b
+ 8*a*b^2)*e^(-6*d*x - 6*c))/((a^6 + 2*a^5*b + a^4*b^2 + 4*(a^6 + 4*a^5*b +
 5*a^4*b^2 + 2*a^3*b^3)*e^(-2*d*x - 2*c) + 2*(3*a^6 + 14*a^5*b + 27*a^4*b^2
 + 24*a^3*b^3 + 8*a^2*b^4)*e^(-4*d*x - 4*c) + 4*(a^6 + 4*a^5*b + 5*a^4*b^2
+ 2*a^3*b^3)*e^(-6*d*x - 6*c) + (a^6 + 2*a^5*b + a^4*b^2)*e^(-8*d*x - 8*c))
*d) + 3/32*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*
d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^2 + 2*a*b + b^2)*sqrt((a + b
)*b)*d) + 1/4*log(a*e^(4*d*x + 4*c) + 2*(a + 2*b)*e^(2*d*x + 2*c) + a)/(a^3
*d) - 1/4*log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/(a^3*d
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+d\,x)^4\,\left(\cosh(c+d\,x)^2-1\right)}{\left(a\cosh(c+d\,x)^2+b\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^2/(a + b/cosh(c + d*x)^2)^3,x)
[Out] int((cosh(c + d*x)^4*(cosh(c + d*x)^2 - 1))/(b + a*cosh(c + d*x)^2)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**2/(a+b*sech(d*x+c)**2)**3,x)
[Out] Timed out
```

3.163 $\int \frac{\tanh(c+dx)}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=73

$$-\frac{b^2}{4a^3d\left(a\cosh^2(c+dx)+b\right)^2}+\frac{b}{a^3d\left(a\cosh^2(c+dx)+b\right)}+\frac{\log\left(a\cosh^2(c+dx)+b\right)}{2a^3d}$$

```
[Out] -1/4*b^2/a^3/d/(b+a*cosh(d*x+c)^2)^2+b/a^3/d/(b+a*cosh(d*x+c)^2)+1/2*ln(b+a
*cosh(d*x+c)^2)/a^3/d
```

Rubi [A] time = 0.08, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4138, 266, 43}

$$-\frac{b^2}{4a^3d\left(a\cosh^2(c+dx)+b\right)^2}+\frac{b}{a^3d\left(a\cosh^2(c+dx)+b\right)}+\frac{\log\left(a\cosh^2(c+dx)+b\right)}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] -b^2/(4*a^3*d*(b + a*Cosh[c + d*x]^2)^2) + b/(a^3*d*(b + a*Cosh[c + d*x]^2)
) + Log[b + a*Cosh[c + d*x]^2]/(2*a^3*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
```

```
x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\tanh(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^5}{\left(b+ax^2\right)^3}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{x^2}{(b+ax)^3}\,dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{b^2}{a^2(b+ax)^3} - \frac{2b}{a^2(b+ax)^2} + \frac{1}{a^2(b+ax)}\right)dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= -\frac{b^2}{4a^3d\left(b+a\cosh^2(c+dx)\right)^2} + \frac{b}{a^3d\left(b+a\cosh^2(c+dx)\right)} + \frac{\log\left(b+a\cosh^2(c+\right.}{2a^3d}$$

Mathematica [A] time = 2.01, size = 129, normalized size = 1.77

$$\frac{a^2\cosh^2(2(c+dx))\log(a\cosh(2(c+dx))+a+2b)+(a+2b)^2\log(a\cosh(2(c+dx))+a+2b)+2a\cosh(2(c+}{2a^3d(a\cosh(2(c+dx))+a+2b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (2*b*(2*a + 3*b) + (a + 2*b)^2*Log[a + 2*b + a*Cosh[2*(c + d*x)]] + a^2*Cos
h[2*(c + d*x)]^2*Log[a + 2*b + a*Cosh[2*(c + d*x)]] + 2*a*Cosh[2*(c + d*x)]
*(2*b + (a + 2*b)*Log[a + 2*b + a*Cosh[2*(c + d*x)]]))/(2*a^3*d*(a + 2*b +
a*Cosh[2*(c + d*x)])^2)
```

fricas [B] time = 0.45, size = 1666, normalized size = 22.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^2*d*x*cosh(d*x + c)^8 + 16*a^2*d*x*cosh(d*x + c)*sinh(d*x + c)^7
+ 2*a^2*d*x*sinh(d*x + c)^8 + 8*((a^2 + 2*a*b)*d*x - a*b)*cosh(d*x + c)^6 +
 8*(7*a^2*d*x*cosh(d*x + c)^2 + (a^2 + 2*a*b)*d*x - a*b)*sinh(d*x + c)^6 +
```

```
16*(7*a^2*d*x*cosh(d*x + c)^3 + 3*((a^2 + 2*a*b)*d*x - a*b)*cosh(d*x + c))*
sinh(d*x + c)^5 + 4*((3*a^2 + 8*a*b + 8*b^2)*d*x - 4*a*b - 6*b^2)*cosh(d*x
+ c)^4 + 4*(35*a^2*d*x*cosh(d*x + c)^4 + (3*a^2 + 8*a*b + 8*b^2)*d*x + 30*(
(a^2 + 2*a*b)*d*x - a*b)*cosh(d*x + c)^2 - 4*a*b - 6*b^2)*sinh(d*x + c)^4 +
 2*a^2*d*x + 16*(7*a^2*d*x*cosh(d*x + c)^5 + 10*((a^2 + 2*a*b)*d*x - a*b)*c
osh(d*x + c)^3 + ((3*a^2 + 8*a*b + 8*b^2)*d*x - 4*a*b - 6*b^2)*cosh(d*x + c
))*sinh(d*x + c)^3 + 8*((a^2 + 2*a*b)*d*x - a*b)*cosh(d*x + c)^2 + 8*(7*a^2
*d*x*cosh(d*x + c)^6 + 15*((a^2 + 2*a*b)*d*x - a*b)*cosh(d*x + c)^4 + (a^2
+ 2*a*b)*d*x + 3*((3*a^2 + 8*a*b + 8*b^2)*d*x - 4*a*b - 6*b^2)*cosh(d*x + c
)^2 - a*b)*sinh(d*x + c)^2 - (a^2*cosh(d*x + c)^8 + 8*a^2*cosh(d*x + c)*sin
h(d*x + c)^7 + a^2*sinh(d*x + c)^8 + 4*(a^2 + 2*a*b)*cosh(d*x + c)^6 + 4*(7
*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^6 + 8*(7*a^2*cosh(d*x + c
)^3 + 3*(a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*a^2 + 8*a*b + 8
*b^2)*cosh(d*x + c)^4 + 2*(35*a^2*cosh(d*x + c)^4 + 30*(a^2 + 2*a*b)*cosh(d
*x + c)^2 + 3*a^2 + 8*a*b + 8*b^2)*sinh(d*x + c)^4 + 8*(7*a^2*cosh(d*x + c)
^5 + 10*(a^2 + 2*a*b)*cosh(d*x + c)^3 + (3*a^2 + 8*a*b + 8*b^2)*cosh(d*x +
c))*sinh(d*x + c)^3 + 4*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 4*(7*a^2*cosh(d*x +
 c)^6 + 15*(a^2 + 2*a*b)*cosh(d*x + c)^4 + 3*(3*a^2 + 8*a*b + 8*b^2)*cosh(d
*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*(a^2*cosh(d*x + c)^7 + 3
*(a^2 + 2*a*b)*cosh(d*x + c)^5 + (3*a^2 + 8*a*b + 8*b^2)*cosh(d*x + c)^3 +
(a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c))*log(2*(a*cosh(d*x + c)^2 + a*si
nh(d*x + c)^2 + a + 2*b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) +
 sinh(d*x + c)^2)) + 16*(a^2*d*x*cosh(d*x + c)^7 + 3*((a^2 + 2*a*b)*d*x - a
*b)*cosh(d*x + c)^5 + ((3*a^2 + 8*a*b + 8*b^2)*d*x - 4*a*b - 6*b^2)*cosh(d*
x + c)^3 + ((a^2 + 2*a*b)*d*x - a*b)*cosh(d*x + c))*sinh(d*x + c))/(a^5*d*c
osh(d*x + c)^8 + 8*a^5*d*cosh(d*x + c)*sinh(d*x + c)^7 + a^5*d*sinh(d*x + c
)^8 + 4*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^6 + 4*(7*a^5*d*cosh(d*x + c)^2 + (a
^5 + 2*a^4*b)*d)*sinh(d*x + c)^6 + a^5*d + 2*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*
d*cosh(d*x + c)^4 + 8*(7*a^5*d*cosh(d*x + c)^3 + 3*(a^5 + 2*a^4*b)*d*cosh(d
*x + c))*sinh(d*x + c)^5 + 2*(35*a^5*d*cosh(d*x + c)^4 + 30*(a^5 + 2*a^4*b)
*d*cosh(d*x + c)^2 + (3*a^5 + 8*a^4*b + 8*a^3*b^2)*d)*sinh(d*x + c)^4 + 4*(
a^5 + 2*a^4*b)*d*cosh(d*x + c)^2 + 8*(7*a^5*d*cosh(d*x + c)^5 + 10*(a^5 + 2
*a^4*b)*d*cosh(d*x + c)^3 + (3*a^5 + 8*a^4*b + 8*a^3*b^2)*d*cosh(d*x + c))*
sinh(d*x + c)^3 + 4*(7*a^5*d*cosh(d*x + c)^6 + 15*(a^5 + 2*a^4*b)*d*cosh(d*
x + c)^4 + 3*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*d*cosh(d*x + c)^2 + (a^5 + 2*a^4
*b)*d)*sinh(d*x + c)^2 + 8*(a^5*d*cosh(d*x + c)^7 + 3*(a^5 + 2*a^4*b)*d*cos
h(d*x + c)^5 + (3*a^5 + 8*a^4*b + 8*a^3*b^2)*d*cosh(d*x + c)^3 + (a^5 + 2*a
^4*b)*d*cosh(d*x + c))*sinh(d*x + c))
```

giac [B] time = 0.53, size = 187, normalized size = 2.56

$$-\frac{\frac{4\,dx}{a^3} - \frac{2\,\log\left(ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+a\right)}{a^3} + \frac{3\,ae^{(8\,dx+8\,c)}+12\,ae^{(6\,dx+6\,c)}+8\,be^{(6\,dx+6\,c)}+18\,ae^{(4\,dx+4\,c)}+16\,be^{(4\,dx+4\,c)}+12\,ae^{(2\,dx+2\,c)}+}{\left(ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+a\right)^2a^2}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/4*(4*d*x/a^3 - 2*log(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*
d*x + 2*c) + a)/a^3 + (3*a*e^(8*d*x + 8*c) + 12*a*e^(6*d*x + 6*c) + 8*b*e^(
6*d*x + 6*c) + 18*a*e^(4*d*x + 4*c) + 16*b*e^(4*d*x + 4*c) + 12*a*e^(2*d*x
+ 2*c) + 8*b*e^(2*d*x + 2*c) + 3*a)/((a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*
c) + 4*b*e^(2*d*x + 2*c) + a)^2*a^2))/d
```

maple [A] time = 0.21, size = 82, normalized size = 1.12

$$\frac{\ln\left(a+b\operatorname{sech}(dx+c)^2\right)}{2d\,a^3}-\frac{1}{4da\left(a+b\operatorname{sech}(dx+c)^2\right)^2}-\frac{1}{2d\,a^2\left(a+b\operatorname{sech}(dx+c)^2\right)}-\frac{\ln(\operatorname{sech}(dx+c))}{d\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)/(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] 1/2/d/a^3*ln(a+b*sech(d*x+c)^2)-1/4/d/a/(a+b*sech(d*x+c)^2)^2-1/2/d/a^2/(a+
b*sech(d*x+c)^2)-1/d/a^3*ln(sech(d*x+c))
```

maxima [B] time = 0.42, size = 193, normalized size = 2.64

$$\frac{4\left(abe^{(-2\,dx-2\,c)}+abe^{(-6\,dx-6\,c)}+\left(2\,ab+3\,b^2\right)e^{(-4\,dx-4\,c)}\right)}{\left(a^5e^{(-8\,dx-8\,c)}+a^5+4\left(a^5+2\,a^4b\right)e^{(-2\,dx-2\,c)}+2\left(3\,a^5+8\,a^4b+8\,a^3b^2\right)e^{(-4\,dx-4\,c)}+4\left(a^5+2\,a^4b\right)e^{(-6\,dx-6\,c)}\right)d}+\frac{dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 4*(a*b*e^(-2*d*x - 2*c) + a*b*e^(-6*d*x - 6*c) + (2*a*b + 3*b^2)*e^(-4*d*x
- 4*c))/((a^5*e^(-8*d*x - 8*c) + a^5 + 4*(a^5 + 2*a^4*b)*e^(-2*d*x - 2*c) +
 2*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*e^(-4*d*x - 4*c) + 4*(a^5 + 2*a^4*b)*e^(-6
*d*x - 6*c))*d) + (d*x + c)/(a^3*d) + 1/2*log(2*(a + 2*b)*e^(-2*d*x - 2*c)
+ a*e^(-4*d*x - 4*c) + a)/(a^3*d)
```

mupad [B] time = 1.60, size = 94, normalized size = 1.29

$$\frac{\ln\left(\cosh(c+d\,x)^2\,\left(a+\frac{b}{\cosh(c+d\,x)^2}\right)\right)}{2\,a^3\,d}-\frac{b^2}{4\,a^3\,d\cosh(c+d\,x)^4\left(a+\frac{b}{\cosh(c+d\,x)^2}\right)^2}+\frac{b}{a^3\,d\cosh(c+d\,x)^2\,\left(a+\frac{b}{\cosh(c+d\,x}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)/(a + b/cosh(c + d*x)^2)^3,x)
```

```
[Out] log(cosh(c + d*x)^2*(a + b/cosh(c + d*x)^2))/(2*a^3*d) - b^2/(4*a^3*d*cosh(
c + d*x)^4*(a + b/cosh(c + d*x)^2)^2) + b/(a^3*d*cosh(c + d*x)^2*(a + b/cos
h(c + d*x)^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

3.164 $\int \frac{1}{\left(a+b\,\mathbf{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=146

$$\frac{x}{a^3}-\frac{b(7a+4b)\tanh(c+dx)}{8a^2d(a+b)^2\left(a-b\tanh^2(c+dx)+b\right)}-\frac{\sqrt{b}\left(15a^2+20ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^3d(a+b)^{5/2}}-\frac{b\tanh(c}{4ad(a+b)\left(a-b\tan\right.}$$

```
[Out] x/a^3-1/8*(15*a^2+20*a*b+8*b^2)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))*b^
(1/2)/a^3/(a+b)^(5/2)/d-1/4*b*tanh(d*x+c)/a/(a+b)/d/(a+b-b*tanh(d*x+c)^2)^2
-1/8*b*(7*a+4*b)*tanh(d*x+c)/a^2/(a+b)^2/d/(a+b-b*tanh(d*x+c)^2)
```

Rubi [A] time = 0.18, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4128, 414, 527, 522, 206, 208}

$$-\frac{\sqrt{b}\left(15a^2+20ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^3d(a+b)^{5/2}}-\frac{b(7a+4b)\tanh(c+dx)}{8a^2d(a+b)^2\left(a-b\tanh^2(c+dx)+b\right)}+\frac{x}{a^3}-\frac{b\tanh(}{4ad(a+b)\left(a-b\,t\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^(-3),x]
```

```
[Out] x/a^3 - (Sqrt[b]*(15*a^2 + 20*a*b + 8*b^2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/
Sqrt[a + b]])/(8*a^3*(a + b)^(5/2)*d) - (b*Tanh[c + d*x])/(4*a*(a + b)*d*(a
 + b - b*Tanh[c + d*x]^2)^2) - (b*(7*a + 4*b)*Tanh[c + d*x])/(8*a^2*(a + b)
^2*d*(a + b - b*Tanh[c + d*x]^2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
```

```
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{(1-x^2)(a+b-bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{b\tanh(c+dx)}{4a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2} - \frac{\operatorname{Subst}\left(\int \frac{-4a-b-3bx^2}{(1-x^2)(a+b-bx^2)^2}\,dx, x, \tanh(c+\right.}{4a(a+b)d}$$

$$= -\frac{b\tanh(c+dx)}{4a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2} - \frac{b(7a+4b)\tanh(c+dx)}{8a^2(a+b)^2d\left(a+b-b\tanh^2(c+dx)\right)}$$

$$= -\frac{b\tanh(c+dx)}{4a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2} - \frac{b(7a+4b)\tanh(c+dx)}{8a^2(a+b)^2d\left(a+b-b\tanh^2(c+dx)\right)}$$

$$= \frac{x}{a^3} - \frac{\sqrt{b}\left(15a^2+20ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^3(a+b)^{5/2}d} - \frac{b\tanh(c+dx)}{4a(a+b)d\left(a+b-b\tanh^2\right.}$$

Mathematica [B] time = 6.21, size = 301, normalized size = 2.06

$$\operatorname{sech}^6(c+dx)(a\cosh(2(c+dx))+a+2b)\left(\frac{b\operatorname{sech}(2c)\left(\left(9a^2+28ab+16b^2\right)\sinh(2c)-3a(3a+2b)\sinh(2dx)\right)(a\cosh(2(c+dx))+a+2b)}{d(a+b)^2} - \frac{b(1}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^(-3),x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^6*(8*x*(a + 2*b + a*Cosh[2*(
c + d*x)])^2 - (b*(15*a^2 + 20*a*b + 8*b^2)*ArcTanh[(Sech[d*x]*(Cosh[2*c] -
 Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[
b*(Cosh[c] - Sinh[c])^4])]*(a + 2*b + a*Cosh[2*(c + d*x)])^2*(Cosh[2*c] - S
inh[2*c]))/((a + b)^(5/2)*d*Sqrt[b*(Cosh[c] - Sinh[c])^4]) - (4*b^2*Sech[2*
c]*((a + 2*b)*Sinh[2*c] - a*Sinh[2*d*x]))/((a + b)*d) + (b*(a + 2*b + a*Cos
h[2*(c + d*x)])*Sech[2*c]*((9*a^2 + 28*a*b + 16*b^2)*Sinh[2*c] - 3*a*(3*a +
 2*b)*Sinh[2*d*x]))/((a + b)^2*d)))/(64*a^3*(a + b*Sech[c + d*x]^2)^3)
```

fricas [B] time = 0.57, size = 6538, normalized size = 44.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(16*(a^4 + 2*a^3*b + a^2*b^2)*d*x*cosh(d*x + c)^8 + 128*(a^4 + 2*a^3*
b + a^2*b^2)*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + 16*(a^4 + 2*a^3*b + a^2*b^
2)*d*x*sinh(d*x + c)^8 + 4*(9*a^3*b + 28*a^2*b^2 + 16*a*b^3 + 16*(a^4 + 4*a
^3*b + 5*a^2*b^2 + 2*a*b^3)*d*x)*cosh(d*x + c)^6 + 4*(112*(a^4 + 2*a^3*b +
a^2*b^2)*d*x*cosh(d*x + c)^2 + 9*a^3*b + 28*a^2*b^2 + 16*a*b^3 + 16*(a^4 +
4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d*x)*sinh(d*x + c)^6 + 8*(112*(a^4 + 2*a^3*b
 + a^2*b^2)*d*x*cosh(d*x + c)^3 + 3*(9*a^3*b + 28*a^2*b^2 + 16*a*b^3 + 16*(
a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 +
4*(27*a^3*b + 90*a^2*b^2 + 120*a*b^3 + 48*b^4 + 8*(3*a^4 + 14*a^3*b + 27*a^
2*b^2 + 24*a*b^3 + 8*b^4)*d*x)*cosh(d*x + c)^4 + 4*(280*(a^4 + 2*a^3*b + a^
2*b^2)*d*x*cosh(d*x + c)^4 + 27*a^3*b + 90*a^2*b^2 + 120*a*b^3 + 48*b^4 + 8
*(3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4)*d*x + 15*(9*a^3*b + 28*
a^2*b^2 + 16*a*b^3 + 16*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d*x)*cosh(d*x
 + c)^2)*sinh(d*x + c)^4 + 36*a^3*b + 24*a^2*b^2 + 16*(56*(a^4 + 2*a^3*b +
a^2*b^2)*d*x*cosh(d*x + c)^5 + 5*(9*a^3*b + 28*a^2*b^2 + 16*a*b^3 + 16*(a^4
 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d*x)*cosh(d*x + c)^3 + (27*a^3*b + 90*a^2
*b^2 + 120*a*b^3 + 48*b^4 + 8*(3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8
*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 + 16*(a^4 + 2*a^3*b + a^2*b^2)*d*
x + 4*(27*a^3*b + 68*a^2*b^2 + 32*a*b^3 + 16*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2
*a*b^3)*d*x)*cosh(d*x + c)^2 + 4*(112*(a^4 + 2*a^3*b + a^2*b^2)*d*x*cosh(d*
x + c)^6 + 15*(9*a^3*b + 28*a^2*b^2 + 16*a*b^3 + 16*(a^4 + 4*a^3*b + 5*a^2*
b^2 + 2*a*b^3)*d*x)*cosh(d*x + c)^4 + 27*a^3*b + 68*a^2*b^2 + 32*a*b^3 + 16
*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d*x + 6*(27*a^3*b + 90*a^2*b^2 + 120
*a*b^3 + 48*b^4 + 8*(3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4)*d*x)
*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((15*a^4 + 20*a^3*b + 8*a^2*b^2)*cosh(d
*x + c)^8 + 8*(15*a^4 + 20*a^3*b + 8*a^2*b^2)*cosh(d*x + c)*sinh(d*x + c)^7
 + (15*a^4 + 20*a^3*b + 8*a^2*b^2)*sinh(d*x + c)^8 + 4*(15*a^4 + 50*a^3*b +
 48*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^6 + 4*(15*a^4 + 50*a^3*b + 48*a^2*b^2
 + 16*a*b^3 + 7*(15*a^4 + 20*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x +
 c)^6 + 8*(7*(15*a^4 + 20*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^3 + 3*(15*a^4 +
50*a^3*b + 48*a^2*b^2 + 16*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(45*a^
4 + 180*a^3*b + 304*a^2*b^2 + 224*a*b^3 + 64*b^4)*cosh(d*x + c)^4 + 2*(35*(
15*a^4 + 20*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^4 + 45*a^4 + 180*a^3*b + 304*a
^2*b^2 + 224*a*b^3 + 64*b^4 + 30*(15*a^4 + 50*a^3*b + 48*a^2*b^2 + 16*a*b^3
)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 15*a^4 + 20*a^3*b + 8*a^2*b^2 + 8*(7*(
15*a^4 + 20*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^5 + 10*(15*a^4 + 50*a^3*b + 48
*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^3 + (45*a^4 + 180*a^3*b + 304*a^2*b^2 +
```

224*a*b^3 + 64*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(15*a^4 + 50*a^3*b + 48*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^2 + 4*(7*(15*a^4 + 20*a^3*b + 8*a^2*b ^2)*cosh(d*x + c)^6 + 15*(15*a^4 + 50*a^3*b + 48*a^2*b^2 + 16*a*b^3)*cosh(d *x + c)^4 + 15*a^4 + 50*a^3*b + 48*a^2*b^2 + 16*a*b^3 + 3*(45*a^4 + 180*a^3 *b + 304*a^2*b^2 + 224*a*b^3 + 64*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8 *((15*a^4 + 20*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^7 + 3*(15*a^4 + 50*a^3*b + 48*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^5 + (45*a^4 + 180*a^3*b + 304*a^2*b^2 + 224*a*b^3 + 64*b^4)*cosh(d*x + c)^3 + (15*a^4 + 50*a^3*b + 48*a^2*b^2 + 1 6*a*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(b/(a + b))*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) + 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*co sh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 + 3*a*b + 2*b ^2)*sqrt(b/(a + b)))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) + 8*(16*(a^4 + 2*a^3*b + a^2*b^2)*d*x*cosh(d*x + c)^ 7 + 3*(9*a^3*b + 28*a^2*b^2 + 16*a*b^3 + 16*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2* a*b^3)*d*x)*cosh(d*x + c)^5 + 2*(27*a^3*b + 90*a^2*b^2 + 120*a*b^3 + 48*b^4 + 8*(3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4)*d*x)*cosh(d*x + c)^ 3 + (27*a^3*b + 68*a^2*b^2 + 32*a*b^3 + 16*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a *b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a^7 + 2*a^6*b + a^5*b^2)*d*cosh(d*x + c)^8 + 8*(a^7 + 2*a^6*b + a^5*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^7 + 2*a^6*b + a^5*b^2)*d*sinh(d*x + c)^8 + 4*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*d*cosh(d*x + c)^6 + 4*(7*(a^7 + 2*a^6*b + a^5*b^2)*d*cosh(d*x + c)^2 + (a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*d)*sinh(d*x + c)^6 + 2*(3* a^7 + 14*a^6*b + 27*a^5*b^2 + 24*a^4*b^3 + 8*a^3*b^4)*d*cosh(d*x + c)^4 + 8 *(7*(a^7 + 2*a^6*b + a^5*b^2)*d*cosh(d*x + c)^3 + 3*(a^7 + 4*a^6*b + 5*a^5* b^2 + 2*a^4*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^7 + 2*a^6*b + a^5*b^2)*d*cosh(d*x + c)^4 + 30*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*d*c osh(d*x + c)^2 + (3*a^7 + 14*a^6*b + 27*a^5*b^2 + 24*a^4*b^3 + 8*a^3*b^4)*d)*sinh(d*x + c)^4 + 4*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*d*cosh(d*x + c)^2 + 8*(7*(a^7 + 2*a^6*b + a^5*b^2)*d*cosh(d*x + c)^5 + 10*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*d*cosh(d*x + c)^3 + (3*a^7 + 14*a^6*b + 27*a^5*b^ 2 + 24*a^4*b^3 + 8*a^3*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^7 + 2*a^6*b + a^5*b^2)*d*cosh(d*x + c)^6 + 15*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^ 4*b^3)*d*cosh(d*x + c)^4 + 3*(3*a^7 + 14*a^6*b + 27*a^5*b^2 + 24*a^4*b^3 + 8*a^3*b^4)*d*cosh(d*x + c)^2 + (a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*d)*s inh(d*x + c)^2 + (a^7 + 2*a^6*b + a^5*b^2)*d + 8*((a^7 + 2*a^6*b + a^5*b^2) *d*cosh(d*x + c)^7 + 3*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*d*cosh(d*x + c)^5 + (3*a^7 + 14*a^6*b + 27*a^5*b^2 + 24*a^4*b^3 + 8*a^3*b^4)*d*cosh(d*x + c)^3 + (a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*d*cosh(d*x + c))*sinh(d*x + c)), 1/8*(8*(a^4 + 2*a^3*b + a^2*b^2)*d*x*cosh(d*x + c)^8 + 64*(a^4 + 2* a^3*b + a^2*b^2)*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + 8*(a^4 + 2*a^3*b + a^2

```
*b^2)*d*x*sinh(d*x + c)^8 + 2*(9*a^3*b + 28*a^2*b^2 + 16*a*b^3 + 16*(a^4 +
4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d*x)*cosh(d*x + c)^6 + 2*(112*(a^4 + 2*a^3*b
 + a^2*b^2)*d*x*cosh(d*x + c)^2 + 9*a^3*b + 28*a^2*b^2 + 16*a*b^3 + 16*(a^4
 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d*x)*sinh(d*x + c)^6 + 4*(112*(a^4 + 2*a^
3*b + a^2*b^2)*d*x*cosh(d*x + c)^3 + 3*(9*a^3*b + 28*a^2*b^2 + 16*a*b^3 + 1
6*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5
 + 2*(27*a^3*b + 90*a^2*b^2 + 120*a*b^3 + 48*b^4 + 8*(3*a^4 + 14*a^3*b + 27
*a^2*b^2 + 24*a*b^3 + 8*b^4)*d*x)*cosh(d*x + c)^4 + 2*(280*(a^4 + 2*a^3*b +
 a^2*b^2)*d*x*cosh(d*x + c)^4 + 27*a^3*b + 90*a^2*b^2 + 120*a*b^3 + 48*b^4
+ 8*(3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4)*d*x + 15*(9*a^3*b +
28*a^2*b^2 + 16*a*b^3 + 16*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d*x)*cosh(
d*x + c)^2)*sinh(d*x + c)^4 + 18*a^3*b + 12*a^2*b^2 + 8*(56*(a^4 + 2*a^3*b
+ a^2*b^2)*d*x*cosh(d*x + c)^5 + 5*(9*a^3*b + 28*a^2*b^2 + 16*a*b^3 + 16*(a
^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d*x)*cosh(d*x + c)^3 + (27*a^3*b + 90*a
^2*b^2 + 120*a*b^3 + 48*b^4 + 8*(3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 +
 8*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 + 8*(a^4 + 2*a^3*b + a^2*b^2)*d
*x + 2*(27*a^3*b + 68*a^2*b^2 + 32*a*b^3 + 16*(a^4 + 4*a^3*b + 5*a^2*b^2 +
2*a*b^3)*d*x)*cosh(d*x + c)^2 + 2*(112*(a^4 + 2*a^3*b + a^2*b^2)*d*x*cosh(d
*x + c)^6 + 15*(9*a^3*b + 28*a^2*b^2 + 16*a*b^3 + 16*(a^4 + 4*a^3*b + 5*a^2
*b^2 + 2*a*b^3)*d*x)*cosh(d*x + c)^4 + 27*a^3*b + 68*a^2*b^2 + 32*a*b^3 + 1
6*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d*x + 6*(27*a^3*b + 90*a^2*b^2 + 12
0*a*b^3 + 48*b^4 + 8*(3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4)*d*x
)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - ((15*a^4 + 20*a^3*b + 8*a^2*b^2)*cosh(
d*x + c)^8 + 8*(15*a^4 + 20*a^3*b + 8*a^2*b^2)*cosh(d*x + c)*sinh(d*x + c)^
7 + (15*a^4 + 20*a^3*b + 8*a^2*b^2)*sinh(d*x + c)^8 + 4*(15*a^4 + 50*a^3*b
+ 48*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^6 + 4*(15*a^4 + 50*a^3*b + 48*a^2*b^
2 + 16*a*b^3 + 7*(15*a^4 + 20*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x
+ c)^6 + 8*(7*(15*a^4 + 20*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^3 + 3*(15*a^4 +
 50*a^3*b + 48*a^2*b^2 + 16*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(45*a
^4 + 180*a^3*b + 304*a^2*b^2 + 224*a*b^3 + 64*b^4)*cosh(d*x + c)^4 + 2*(35*
(15*a^4 + 20*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^4 + 45*a^4 + 180*a^3*b + 304*
a^2*b^2 + 224*a*b^3 + 64*b^4 + 30*(15*a^4 + 50*a^3*b + 48*a^2*b^2 + 16*a*b^
3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 15*a^4 + 20*a^3*b + 8*a^2*b^2 + 8*(7*
(15*a^4 + 20*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^5 + 10*(15*a^4 + 50*a^3*b + 4
8*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^3 + (45*a^4 + 180*a^3*b + 304*a^2*b^2 +
 224*a*b^3 + 64*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(15*a^4 + 50*a^3*b
+ 48*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^2 + 4*(7*(15*a^4 + 20*a^3*b + 8*a^2*
b^2)*cosh(d*x + c)^6 + 15*(15*a^4 + 50*a^3*b + 48*a^2*b^2 + 16*a*b^3)*cosh(
d*x + c)^4 + 15*a^4 + 50*a^3*b + 48*a^2*b^2 + 16*a*b^3 + 3*(45*a^4 + 180*a^
3*b + 304*a^2*b^2 + 224*a*b^3 + 64*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 +
8*((15*a^4 + 20*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^7 + 3*(15*a^4 + 50*a^3*b +
 48*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^5 + (45*a^4 + 180*a^3*b + 304*a^2*b^2
 + 224*a*b^3 + 64*b^4)*cosh(d*x + c)^3 + (15*a^4 + 50*a^3*b + 48*a^2*b^2 +
16*a*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b/(a + b))*arctan(1/2*(a*cosh
(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b
```

```
)*sqrt(-b/(a + b))/b) + 4*(16*(a^4 + 2*a^3*b + a^2*b^2)*d*x*cosh(d*x + c)^7
 + 3*(9*a^3*b + 28*a^2*b^2 + 16*a*b^3 + 16*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a
*b^3)*d*x)*cosh(d*x + c)^5 + 2*(27*a^3*b + 90*a^2*b^2 + 120*a*b^3 + 48*b^4
+ 8*(3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4)*d*x)*cosh(d*x + c)^3
 + (27*a^3*b + 68*a^2*b^2 + 32*a*b^3 + 16*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*
b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a^7 + 2*a^6*b + a^5*b^2)*d*cosh(d
*x + c)^8 + 8*(a^7 + 2*a^6*b + a^5*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (
a^7 + 2*a^6*b + a^5*b^2)*d*sinh(d*x + c)^8 + 4*(a^7 + 4*a^6*b + 5*a^5*b^2 +
 2*a^4*b^3)*d*cosh(d*x + c)^6 + 4*(7*(a^7 + 2*a^6*b + a^5*b^2)*d*cosh(d*x +
 c)^2 + (a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*d)*sinh(d*x + c)^6 + 2*(3*a
^7 + 14*a^6*b + 27*a^5*b^2 + 24*a^4*b^3 + 8*a^3*b^4)*d*cosh(d*x + c)^4 + 8*
(7*(a^7 + 2*a^6*b + a^5*b^2)*d*cosh(d*x + c)^3 + 3*(a^7 + 4*a^6*b + 5*a^5*b
^2 + 2*a^4*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^7 + 2*a^6*b + a
^5*b^2)*d*cosh(d*x + c)^4 + 30*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*d*co
sh(d*x + c)^2 + (3*a^7 + 14*a^6*b + 27*a^5*b^2 + 24*a^4*b^3 + 8*a^3*b^4)*d)
*sinh(d*x + c)^4 + 4*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*d*cosh(d*x + c
)^2 + 8*(7*(a^7 + 2*a^6*b + a^5*b^2)*d*cosh(d*x + c)^5 + 10*(a^7 + 4*a^6*b
+ 5*a^5*b^2 + 2*a^4*b^3)*d*cosh(d*x + c)^3 + (3*a^7 + 14*a^6*b + 27*a^5*b^2
 + 24*a^4*b^3 + 8*a^3*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^7 + 2
*a^6*b + a^5*b^2)*d*cosh(d*x + c)^6 + 15*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4
*b^3)*d*cosh(d*x + c)^4 + 3*(3*a^7 + 14*a^6*b + 27*a^5*b^2 + 24*a^4*b^3 + 8
*a^3*b^4)*d*cosh(d*x + c)^2 + (a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*d)*si
nh(d*x + c)^2 + (a^7 + 2*a^6*b + a^5*b^2)*d + 8*((a^7 + 2*a^6*b + a^5*b^2)*
d*cosh(d*x + c)^7 + 3*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*d*cosh(d*x +
c)^5 + (3*a^7 + 14*a^6*b + 27*a^5*b^2 + 24*a^4*b^3 + 8*a^3*b^4)*d*cosh(d*x
+ c)^3 + (a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*d*cosh(d*x + c))*sinh(d*x
+ c))]
```

giac [B] time = 0.86, size = 327, normalized size = 2.24

$$-\frac{\frac{\left(15\,a^{2}b+20\,ab^{2}+8\,b^{3}\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^{2}}}\right)}{\left(a^{5}+2\,a^{4}b+a^{3}b^{2}\right)\sqrt{-ab-b^{2}}}-\frac{2\left(9\,a^{3}be^{(6\,dx+6\,c)}+28\,a^{2}b^{2}e^{(6\,dx+6\,c)}+16\,ab^{3}e^{(6\,dx+6\,c)}+27\,a^{3}be^{(4\,dx+4\,c)}+90\,a^{2}b^{2}e^{(4\,dx+4\,c)}+120\,ab^{3}\cdots\right.}{\left(a^{5}+2\,a^{4}b+a^{3}b^{2}\right)\left(ae^{(4\,dx+4\,c)}+2\,a\cdots\right.}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*((15*a^2*b + 20*a*b^2 + 8*b^3)*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b
)/sqrt(-a*b - b^2))/((a^5 + 2*a^4*b + a^3*b^2)*sqrt(-a*b - b^2)) - 2*(9*a^3
*b*e^(6*d*x + 6*c) + 28*a^2*b^2*e^(6*d*x + 6*c) + 16*a*b^3*e^(6*d*x + 6*c)
+ 27*a^3*b*e^(4*d*x + 4*c) + 90*a^2*b^2*e^(4*d*x + 4*c) + 120*a*b^3*e^(4*d*
x + 4*c) + 48*b^4*e^(4*d*x + 4*c) + 27*a^3*b*e^(2*d*x + 2*c) + 68*a^2*b^2*e
^(2*d*x + 2*c) + 32*a*b^3*e^(2*d*x + 2*c) + 9*a^3*b + 6*a^2*b^2)/((a^5 + 2*
```

```
a^4*b + a^3*b^2)*(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x +
2*c) + a)^2) - 8*(d*x + c)/a^3)/d
```

maple [B] time = 0.41, size = 1283, normalized size = 8.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] -1/d/a^3*ln(tanh(1/2*d*x+1/2*c)-1)+1/d/a^3*ln(tanh(1/2*d*x+1/2*c)+1)-9/4/d/
a*b/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^
2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)^7-1/d/a^2*b^
2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*
a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)^7-27/4/d*b/(ta
nh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*t
anh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^5-11/4/d/a*b^2/(t
anh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*
tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^5+1/d/a^2*b^3/(t
anh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*
tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^5-27/4/d*b/(tanh
(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tan
h(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^3-11/4/d/a*b^2/(tan
h(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*ta
nh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^3+1/d/a^2*b^3/(tan
h(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*ta
nh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)^2*tanh(1/2*d*x+1/2*c)^3-9/4/d/a*b/(tanh(
1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh
(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)-1/d/a^2*b^2/(tanh(1/2*
d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2
*d*x+1/2*c)^2*b+a+b)^2/(a+b)*tanh(1/2*d*x+1/2*c)+15/16/d/a*b^(1/2)/(a^2+2*a
*b+b^2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*b^(1/2)*tanh(1/2
*d*x+1/2*c)+(a+b)^(1/2))+5/4/d/a^2*b^(3/2)/(a^2+2*a*b+b^2)/(a+b)^(1/2)*ln((
a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))
+1/2/d/a^3*b^(5/2)/(a^2+2*a*b+b^2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+
1/2*c)^2-2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))-15/16/d/a*b^(1/2)/(a^2+
2*a*b+b^2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(
1/2*d*x+1/2*c)+(a+b)^(1/2))-5/4/d/a^2*b^(3/2)/(a^2+2*a*b+b^2)/(a+b)^(1/2)*l
n((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/
2))-1/2/d/a^3*b^(5/2)/(a^2+2*a*b+b^2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d
*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))
```

maxima [B] time = 0.53, size = 402, normalized size = 2.75

$$\frac{\left(15\,a^{2}b+20\,ab^{2}+8\,b^{3}\right)\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{16\left(a^{5}+2\,a^{4}b+a^{3}b^{2}\right)\sqrt{(a+b)b}\,d}-\frac{9\,a^{3}b+6\,a^{2}b^{2}+\left(27\,a^{3}b+6\right.}{4\left(a^{7}+2\,a^{6}b+a^{5}b^{2}+4\left(a^{7}+4\,a^{6}b+5\,a^{5}b^{2}+2\,a^{4}b^{3}\right)e^{(-}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(15*a^2*b + 20*a*b^2 + 8*b^3)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sq
rt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^5 +
2*a^4*b + a^3*b^2)*sqrt((a + b)*b)*d) - 1/4*(9*a^3*b + 6*a^2*b^2 + (27*a^3*
b + 68*a^2*b^2 + 32*a*b^3)*e^(-2*d*x - 2*c) + 3*(9*a^3*b + 30*a^2*b^2 + 40*
a*b^3 + 16*b^4)*e^(-4*d*x - 4*c) + (9*a^3*b + 28*a^2*b^2 + 16*a*b^3)*e^(-6*
d*x - 6*c))/((a^7 + 2*a^6*b + a^5*b^2 + 4*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^
4*b^3)*e^(-2*d*x - 2*c) + 2*(3*a^7 + 14*a^6*b + 27*a^5*b^2 + 24*a^4*b^3 + 8
*a^3*b^4)*e^(-4*d*x - 4*c) + 4*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*e^(-
6*d*x - 6*c) + (a^7 + 2*a^6*b + a^5*b^2)*e^(-8*d*x - 8*c))*d) + (d*x + c)/(
a^3*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+\frac{b}{\cosh(c+d\,x)^{2}}\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cosh(c + d*x)^2)^3,x)
```

```
[Out] int(1/(a + b/cosh(c + d*x)^2)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\,\mathrm{sech}^{2}\,(c+dx)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**(-3), x)
```

3.165 $\int \frac{\coth(c+dx)}{\left(a+b\,\mathbf{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=130

$$-\frac{b^3}{4a^3d(a+b)\left(a\cosh^2(c+dx)+b\right)^2}+\frac{b^2(3a+2b)}{2a^3d(a+b)^2\left(a\cosh^2(c+dx)+b\right)}+\frac{b\left(3a^2+3ab+b^2\right)\log\left(a\cosh^2(c+\ldots\right.}{2a^3d(a+b)^3}$$

```
[Out] -1/4*b^3/a^3/(a+b)/d/(b+a*cosh(d*x+c)^2)^2+1/2*b^2*(3*a+2*b)/a^3/(a+b)^2/d/
(b+a*cosh(d*x+c)^2)+1/2*b*(3*a^2+3*a*b+b^2)*ln(b+a*cosh(d*x+c)^2)/a^3/(a+b)
^3/d+ln(sinh(d*x+c))/d/(a+b)^3
```

Rubi [A] time = 0.19, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4138, 446, 88}

$$-\frac{b^3}{4a^3d(a+b)\left(a\cosh^2(c+dx)+b\right)^2}+\frac{b^2(3a+2b)}{2a^3d(a+b)^2\left(a\cosh^2(c+dx)+b\right)}+\frac{b\left(3a^2+3ab+b^2\right)\log\left(a\cosh^2(c+\ldots\right.}{2a^3d(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] -b^3/(4*a^3*(a + b)*d*(b + a*Cosh[c + d*x]^2)^2) + (b^2*(3*a + 2*b))/(2*a^3
*(a + b)^2*d*(b + a*Cosh[c + d*x]^2)) + (b*(3*a^2 + 3*a*b + b^2)*Log[b + a*
Cosh[c + d*x]^2])/(2*a^3*(a + b)^3*d) + Log[Sinh[c + d*x]]/((a + b)^3*d)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\coth(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{x^7}{(1-x^2)\left(b+ax^2\right)^3}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \frac{x^3}{(1-x)(b+ax)^3}\,dx, x, \cosh^2(c+dx)\right)}{2d}$$

$$= -\frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)^3(-1+x)} - \frac{b^3}{a^2(a+b)(b+ax)^3} + \frac{b^2(3a+2b)}{a^2(a+b)^2(b+ax)^2} - \frac{b\left(3a^2+3ab+b^2\right)}{a^2(a+b)^3(b+ax)}\right) dx, x, \cos\right.}{2d}$$

$$= -\frac{b^3}{4a^3(a+b)d\left(b+a\cosh^2(c+dx)\right)^2} + \frac{b^2(3a+2b)}{2a^3(a+b)^2d\left(b+a\cosh^2(c+dx)\right)} + \frac{b\,(3a}{}$$

Mathematica [A] time = 1.06, size = 155, normalized size = 1.19

$$\frac{\text{sech}^6(c+dx)(a\cosh(2(c+dx))+a+2b)^3\left(-\frac{b^3(a+b)^2}{a^3\left(a\sinh^2(c+dx)+a+b\right)^2} + \frac{2b^2(a+b)(3a+2b)}{a^3\left(a\sinh^2(c+dx)+a+b\right)} + \frac{2b\left(3a^2+3ab+b^2\right)\log\left(a\sinh^2(c+\right.}{a^3}\right.}{32d(a+b)^3\left(a+b\text{sech}^2(c+dx)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])^3*Sech[c + d*x]^6*(4*Log[Sinh[c + d*x]] +
(2*b*(3*a^2 + 3*a*b + b^2)*Log[a + b + a*Sinh[c + d*x]^2])/a^3 - (b^3*(a +
b)^2)/(a^3*(a + b + a*Sinh[c + d*x]^2)^2) + (2*b^2*(a + b)*(3*a + 2*b))/(a^
3*(a + b + a*Sinh[c + d*x]^2))))/(32*(a + b)^3*d*(a + b*Sech[c + d*x]^2)^3)
```

fricas [B] time = 0.85, size = 4132, normalized size = 31.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")

[Out] -1/2*(2*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^8 + 16*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + 2*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*sinh(d*x + c)^8 - 4*(3*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4 - 2*(a^5 + 5*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c)^6 - 4*(3*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4 - 14*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^2 - 2*(a^5 + 5*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4)*d*x)*sinh(d*x + c)^6 + 8*(14*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^3 - 3*(3*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4 - 2*(a^5 + 5*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 - 4*(6*a^3*b^2 + 20*a^2*b^3 + 20*a*b^4 + 6*b^5 - (3*a^5 + 17*a^4*b + 41*a^3*b^2 + 51*a^2*b^3 + 32*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c)^4 + 4*(35*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^4 - 6*a^3*b^2 - 20*a^2*b^3 - 20*a*b^4 - 6*b^5 + (3*a^5 + 17*a^4*b + 41*a^3*b^2 + 51*a^2*b^3 + 32*a*b^4 + 8*b^5)*d*x - 15*(3*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4 - 2*(a^5 + 5*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 16*(7*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^5 - 5*(3*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4 - 2*(a^5 + 5*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c)^3 - (6*a^3*b^2 + 20*a^2*b^3 + 20*a*b^4 + 6*b^5 - (3*a^5 + 17*a^4*b + 41*a^3*b^2 + 51*a^2*b^3 + 32*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x - 4*(3*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4 - 2*(a^5 + 5*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c)^2 + 4*(14*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^6 - 3*a^3*b^2 - 5*a^2*b^3 - 2*a*b^4 - 15*(3*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4 - 2*(a^5 + 5*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c)^4 + 2*(a^5 + 5*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4)*d*x - 6*(6*a^3*b^2 + 20*a^2*b^3 + 20*a*b^4 + 6*b^5 - (3*a^5 + 17*a^4*b + 41*a^3*b^2 + 51*a^2*b^3 + 32*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - ((3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)^8 + 8*(3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)*sinh(d*x + c)^7 + (3*a^4*b + 3*a^3*b^2 + a^2*b^3)*sinh(d*x + c)^8 + 4*(3*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4)*cosh(d*x + c)^6 + 4*(3*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4 + 7*(3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)^3 + 3*(3*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3 + 2*(9*a^4*b + 33*a^3*b^2 + 51*a^2*b^3 + 32*a*b^4 + 8*b^5)*cosh(d*x + c)^4 + 2*(9*a^4*b + 33*a^3*b^2 + 51*a^2*b^3 + 32*a*b^4 + 8*b^5 + 35*(3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)^4 + 30*(3*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)^5 + 10*(3*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4)*cosh(d*x + c)^3 + (9*a^4*b + 33*a^3*b^2 + 51*a^2*b^3 + 32*a*b^4 + 8*b^5)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(3*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4)*cosh(d*x + c)^2 + 4*(7*(3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)^6 + 3*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4 + 15*(3*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 +

2*a*b^4)*cosh(d*x + c)^4 + 3*(9*a^4*b + 33*a^3*b^2 + 51*a^2*b^3 + 32*a*b^4 + 8*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)^7 + 3*(3*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4)*cosh(d*x + c)^5 + (9*a^4*b + 33*a^3*b^2 + 51*a^2*b^3 + 32*a*b^4 + 8*b^5)*cosh(d*x + c)^3 + (3*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4)*cosh(d*x + c))*sinh(d*x + c))*log(2*(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + a + 2*b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) - 2*(a^5*cosh(d*x + c)^8 + 8*a^5*cosh(d*x + c)*sinh(d*x + c)^7 + a^5*sinh(d*x + c)^8 + 4*(a^5 + 2*a^4*b)*cosh(d*x + c)^6 + 4*(7*a^5*cosh(d*x + c)^2 + a^5 + 2*a^4*b)*sinh(d*x + c)^6 + 8*(7*a^5*cosh(d*x + c)^3 + 3*(a^5 + 2*a^4*b)*cosh(d*x + c))*sinh(d*x + c)^5 + a^5 + 2*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*cosh(d*x + c)^4 + 2*(35*a^5*cosh(d*x + c)^4 + 3*a^5 + 8*a^4*b + 8*a^3*b^2 + 30*(a^5 + 2*a^4*b)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*a^5*cosh(d*x + c)^5 + 10*(a^5 + 2*a^4*b)*cosh(d*x + c)^3 + (3*a^5 + 8*a^4*b + 8*a^3*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a^5 + 2*a^4*b)*cosh(d*x + c)^2 + 4*(7*a^5*cosh(d*x + c)^6 + a^5 + 2*a^4*b + 15*(a^5 + 2*a^4*b)*cosh(d*x + c)^4 + 3*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*(a^5*cosh(d*x + c)^7 + 3*(a^5 + 2*a^4*b)*cosh(d*x + c)^5 + (3*a^5 + 8*a^4*b + 8*a^3*b^2)*cosh(d*x + c)^3 + (a^5 + 2*a^4*b)*cosh(d*x + c))*sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 8*(2*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^7 - 3*(3*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4 - 2*(a^5 + 5*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c)^5 - 2*(6*a^3*b^2 + 20*a^2*b^3 + 20*a*b^4 + 6*b^5 - (3*a^5 + 17*a^4*b + 41*a^3*b^2 + 51*a^2*b^3 + 32*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c)^3 - (3*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4 - 2*(a^5 + 5*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^8 + 8*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*sinh(d*x + c)^8 + 4*(a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c)^6 + 4*(7*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^2 + (a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a^5*b^3 + 2*a^4*b^4)*d)*sinh(d*x + c)^6 + 2*(3*a^8 + 17*a^7*b + 41*a^6*b^2 + 51*a^5*b^3 + 32*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c)^4 + 8*(7*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^3 + 3*(a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^4 + 30*(a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c)^2 + (3*a^8 + 17*a^7*b + 41*a^6*b^2 + 51*a^5*b^3 + 32*a^4*b^4 + 8*a^3*b^5)*d)*sinh(d*x + c)^4 + 4*(a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c)^2 + 8*(7*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^5 + 10*(a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c)^3 + (3*a^8 + 17*a^7*b + 41*a^6*b^2 + 51*a^5*b^3 + 32*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^6 + 15*(a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c)^4 + 3*(3*a^8 + 17*a^7*b + 41*a^6*b^2 + 51*a^5*b^3 + 32*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c)^2 + (a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a^5*b^3 + 2*a^4*b^4)*d)*sinh(d*

```
x + c)^2 + (a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d + 8*((a^8 + 3*a^7*b + 3*
a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^7 + 3*(a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a^5
*b^3 + 2*a^4*b^4)*d*cosh(d*x + c)^5 + (3*a^8 + 17*a^7*b + 41*a^6*b^2 + 51*a
^5*b^3 + 32*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c)^3 + (a^8 + 5*a^7*b + 9*a^6
*b^2 + 7*a^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c))*sinh(d*x + c))
```

giac [B] time = 0.72, size = 475, normalized size = 3.65

$$\frac{2\left(3a^2b+3ab^2+b^3\right)\log\left(ae^{(4dx+4c)}+2ae^{(2dx+2c)}+4be^{(2dx+2c)}+a\right)}{a^6+3a^5b+3a^4b^2+a^3b^3}+\frac{4e^{(2c)}\log\left(\left|-e^{(2dx+2c)}+1\right|\right)}{a^3e^{(2c)}+3a^2be^{(2c)}+3ab^2e^{(2c)}+b^3e^{(2c)}}-\frac{4dx}{a^3}-\frac{9a^3be^{(8dx+8c)}+9a^2b^2e^{(8dx+8}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/4*(2*(3*a^2*b + 3*a*b^2 + b^3)*log(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c
) + 4*b*e^(2*d*x + 2*c) + a)/(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3) + 4*e^(2
*c)*log(abs(-e^(2*d*x + 2*c) + 1))/(a^3*e^(2*c) + 3*a^2*b*e^(2*c) + 3*a*b^2
*e^(2*c) + b^3*e^(2*c)) - 4*d*x/a^3 - (9*a^3*b*e^(8*d*x + 8*c) + 9*a^2*b^2*
e^(8*d*x + 8*c) + 3*a*b^3*e^(8*d*x + 8*c) + 36*a^3*b*e^(6*d*x + 6*c) + 84*a
^2*b^2*e^(6*d*x + 6*c) + 44*a*b^3*e^(6*d*x + 6*c) + 8*b^4*e^(6*d*x + 6*c) +
 54*a^3*b*e^(4*d*x + 4*c) + 150*a^2*b^2*e^(4*d*x + 4*c) + 146*a*b^3*e^(4*d*
x + 4*c) + 32*b^4*e^(4*d*x + 4*c) + 36*a^3*b*e^(2*d*x + 2*c) + 84*a^2*b^2*e
^(2*d*x + 2*c) + 44*a*b^3*e^(2*d*x + 2*c) + 8*b^4*e^(2*d*x + 2*c) + 9*a^3*b
 + 9*a^2*b^2 + 3*a*b^3)/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*(a*e^(4*d*x
+ 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) + a)^2))/d
```

maple [B] time = 0.47, size = 1046, normalized size = 8.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)/(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] -1/d/a^3*ln(tanh(1/2*d*x+1/2*c)-1)-1/d/a^3*ln(tanh(1/2*d*x+1/2*c)+1)-6/d*b^
2/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1
/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^6-8/d*b^3/(a
+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c
)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/a*tanh(1/2*d*x+1/2*c)^6-2/d*b^4/(a+b
)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^
2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/a^2*tanh(1/2*d*x+1/2*c)^6-12/d*b^2/(a+
b)^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)
^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^4+4/d*b^3/(a+b)^3
/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a
```

```
-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/a*tanh(1/2*d*x+1/2*c)^4+4/d*b^4/(a+b)^3/(
tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2
*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/a^2*tanh(1/2*d*x+1/2*c)^4-6/d*b^2/(a+b)^3/(
tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2
*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^2-8/d*b^3/(a+b)^3/(tanh
(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tan
h(1/2*d*x+1/2*c)^2*b+a+b)^2/a*tanh(1/2*d*x+1/2*c)^2-2/d*b^4/(a+b)^3/(tanh(1
/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(
1/2*d*x+1/2*c)^2*b+a+b)^2/a^2*tanh(1/2*d*x+1/2*c)^2+3/2/d*b/a/(a+b)^3*ln(ta
nh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*t
anh(1/2*d*x+1/2*c)^2*b+a+b)+3/2/d*b^2/a^2/(a+b)^3*ln(tanh(1/2*d*x+1/2*c)^4*
a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2
*b+a+b)+1/2/d*b^3/a^3/(a+b)^3*ln(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2
*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)+1/d/(a+b)^3*
ln(tanh(1/2*d*x+1/2*c))
```

maxima [B] time = 0.72, size = 419, normalized size = 3.22

$$\frac{\left(3\,a^{2}b+3\,ab^{2}+b^{3}\right)\log\left(2\,(a+2\,b)e^{(-2\,dx-2\,c)}+ae^{(-4\,dx-4\,c)}+a\right)}{2\left(a^{6}+3\,a^{5}b+3\,a^{4}b^{2}+a^{3}b^{3}\right)d}+\frac{}{\left(a^{7}+2\,a^{6}b+a^{5}b^{2}+4\left(a^{7}+4\,a^{6}b+5\,a^{5}b^{2}+2\,a\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(3*a^2*b + 3*a*b^2 + b^3)*log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*
x - 4*c) + a)/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d) + 2*((3*a^2*b^2 + 2
*a*b^3)*e^(-2*d*x - 2*c) + 2*(3*a^2*b^2 + 7*a*b^3 + 3*b^4)*e^(-4*d*x - 4*c)
 + (3*a^2*b^2 + 2*a*b^3)*e^(-6*d*x - 6*c))/((a^7 + 2*a^6*b + a^5*b^2 + 4*(a
^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*e^(-2*d*x - 2*c) + 2*(3*a^7 + 14*a^6*
b + 27*a^5*b^2 + 24*a^4*b^3 + 8*a^3*b^4)*e^(-4*d*x - 4*c) + 4*(a^7 + 4*a^6*
b + 5*a^5*b^2 + 2*a^4*b^3)*e^(-6*d*x - 6*c) + (a^7 + 2*a^6*b + a^5*b^2)*e^(
-8*d*x - 8*c))*d) + log(e^(-d*x - c) + 1)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*
d) + log(e^(-d*x - c) - 1)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) + (d*x + c)/
(a^3*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh\left(c+d\,x\right)^{6}\,\mathrm{coth}\left(c+d\,x\right)}{{\left(a\,{\cosh\left(c+d\,x\right)}^2+b\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)/(a + b/cosh(c + d*x)^2)^3,x)
```

```
[Out] int((cosh(c + d*x)^6*coth(c + d*x))/(b + a*cosh(c + d*x)^2)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(c+dx)}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Integral(coth(c + d*x)/(a + b*sech(c + d*x)**2)**3, x)
```

3.166 $\int \frac{\coth^2(c+dx)}{\left(a+b\,\mathbf{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=182

$$\frac{x}{a^3}-\frac{\left(8a^2-11ab-4b^2\right)\coth(c+dx)}{8a^2d(a+b)^3}-\frac{b(9a+4b)\coth(c+dx)}{8a^2d(a+b)^2\left(a-b\tanh^2(c+dx)+b\right)}-\frac{b^{3/2}\left(35a^2+28ab+8b^2\right)\tanh^{-1}\left(\right.}{8a^3d(a+b)^{7/2}}$$

```
[Out] x/a^3-1/8*b^(3/2)*(35*a^2+28*a*b+8*b^2)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(
1/2))/a^3/(a+b)^(7/2)/d-1/8*(8*a^2-11*a*b-4*b^2)*coth(d*x+c)/a^2/(a+b)^3/d-
1/4*b*coth(d*x+c)/a/(a+b)/d/(a+b-b*tanh(d*x+c)^2)^2-1/8*b*(9*a+4*b)*coth(d*
x+c)/a^2/(a+b)^2/d/(a+b-b*tanh(d*x+c)^2)
```

Rubi [A] time = 0.40, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {4141, 1975, 472, 579, 583, 522, 206, 208}

$$-\frac{b^{3/2}\left(35a^2+28ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^3d(a+b)^{7/2}}-\frac{\left(8a^2-11ab-4b^2\right)\coth(c+dx)}{8a^2d(a+b)^3}-\frac{b(9a+4b)\coth(c+d}{8a^2d(a+b)^2\left(a-b\tanh^2(c+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^2/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] x/a^3 - (b^(3/2)*(35*a^2 + 28*a*b + 8*b^2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/
Sqrt[a + b]])/(8*a^3*(a + b)^(7/2)*d) - ((8*a^2 - 11*a*b - 4*b^2)*Coth[c +
d*x])/(8*a^2*(a + b)^3*d) - (b*Coth[c + d*x])/(4*a*(a + b)*d*(a + b - b*Tan
h[c + d*x]^2)^2) - (b*(9*a + 4*b)*Coth[c + d*x])/(8*a^2*(a + b)^2*d*(a + b
- b*Tanh[c + d*x]^2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
```

```
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\coth^2(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2(1-x^2)\left(a+b(1-x^2)\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^2(1-x^2)\left(a+b-bx^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{b\coth(c+dx)}{4a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{-4a+b-5bx^2}{x^2(1-x^2)\left(a+b-bx^2\right)^2}\,dx, x, \tanh(c\right.}{4a(a+b)d}$$

$$= -\frac{b\coth(c+dx)}{4a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2} - \frac{b(9a+4b)\coth(c+dx)}{8a^2(a+b)^2d\left(a+b-b\tanh^2(c+dx)\right)}$$

$$= -\frac{\left(8a^2-11ab-4b^2\right)\coth(c+dx)}{8a^2(a+b)^3d} - \frac{b\coth(c+dx)}{4a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2} - \frac{}{8a^2(}$$

$$= -\frac{\left(8a^2-11ab-4b^2\right)\coth(c+dx)}{8a^2(a+b)^3d} - \frac{b\coth(c+dx)}{4a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2} - \frac{}{8a^2(}$$

$$= \frac{x}{a^3} - \frac{b^{3/2}\left(35a^2+28ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^3(a+b)^{7/2}d} - \frac{\left(8a^2-11ab-4b^2\right)\coth(c}{8a^2(a+b)^3d}$$

Mathematica [C] time = 7.23, size = 2083, normalized size = 11.45

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Coth[c + d*x]^2/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((35*a^2 + 28*a*b + 8*b^2)*(a + 2*b + a*Cosh[2*c + 2*d*x])^3*Sech[c + d*x]^
6*(((I/64)*b^2*ArcTan[Sech[d*x]*(((-1/2*I)*Cosh[2*c])/(Sqrt[a + b]*Sqrt[b*C
osh[4*c] - b*Sinh[4*c]]) + ((I/2)*Sinh[2*c])/(Sqrt[a + b]*Sqrt[b*Cosh[4*c]
 - b*Sinh[4*c]]))*(-(a*Sinh[d*x]) - 2*b*Sinh[d*x] + a*Sinh[2*c + d*x])]*Cosh
[2*c])/(a^3*Sqrt[a + b]*d*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]]) - ((I/64)*b^2*Ar
cTan[Sech[d*x]*(((-1/2*I)*Cosh[2*c])/(Sqrt[a + b]*Sqrt[b*Cosh[4*c] - b*Sinh
[4*c]]) + ((I/2)*Sinh[2*c])/(Sqrt[a + b]*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]]))*
(-(a*Sinh[d*x]) - 2*b*Sinh[d*x] + a*Sinh[2*c + d*x])]*Sinh[2*c])/(a^3*Sqrt[
a + b]*d*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]])))/((a + b)^3*(a + b*Sech[c + d*x]
^2)^3) + ((a + 2*b + a*Cosh[2*c + 2*d*x])*Csch[c]*Csch[c + d*x]*Sech[2*c]*S
ech[c + d*x]^6*(8*a^5*d*x*Cosh[d*x] + 56*a^4*b*d*x*Cosh[d*x] + 184*a^3*b^2*
d*x*Cosh[d*x] + 296*a^2*b^3*d*x*Cosh[d*x] + 224*a*b^4*d*x*Cosh[d*x] + 64*b^
5*d*x*Cosh[d*x] - 12*a^5*d*x*Cosh[3*d*x] - 68*a^4*b*d*x*Cosh[3*d*x] - 132*a
^3*b^2*d*x*Cosh[3*d*x] - 108*a^2*b^3*d*x*Cosh[3*d*x] - 32*a*b^4*d*x*Cosh[3*
d*x] - 8*a^5*d*x*Cosh[2*c - d*x] - 56*a^4*b*d*x*Cosh[2*c - d*x] - 184*a^3*b
^2*d*x*Cosh[2*c - d*x] - 296*a^2*b^3*d*x*Cosh[2*c - d*x] - 224*a*b^4*d*x*Co
sh[2*c - d*x] - 64*b^5*d*x*Cosh[2*c - d*x] - 8*a^5*d*x*Cosh[2*c + d*x] - 56
*a^4*b*d*x*Cosh[2*c + d*x] - 184*a^3*b^2*d*x*Cosh[2*c + d*x] - 296*a^2*b^3*
d*x*Cosh[2*c + d*x] - 224*a*b^4*d*x*Cosh[2*c + d*x] - 64*b^5*d*x*Cosh[2*c +
 d*x] + 8*a^5*d*x*Cosh[4*c + d*x] + 56*a^4*b*d*x*Cosh[4*c + d*x] + 184*a^3*
b^2*d*x*Cosh[4*c + d*x] + 296*a^2*b^3*d*x*Cosh[4*c + d*x] + 224*a*b^4*d*x*C
osh[4*c + d*x] + 64*b^5*d*x*Cosh[4*c + d*x] + 12*a^5*d*x*Cosh[2*c + 3*d*x]
 + 68*a^4*b*d*x*Cosh[2*c + 3*d*x] + 132*a^3*b^2*d*x*Cosh[2*c + 3*d*x] + 108*
a^2*b^3*d*x*Cosh[2*c + 3*d*x] + 32*a*b^4*d*x*Cosh[2*c + 3*d*x] - 12*a^5*d*x
*Cosh[4*c + 3*d*x] - 68*a^4*b*d*x*Cosh[4*c + 3*d*x] - 132*a^3*b^2*d*x*Cosh[
4*c + 3*d*x] - 108*a^2*b^3*d*x*Cosh[4*c + 3*d*x] - 32*a*b^4*d*x*Cosh[4*c +
3*d*x] + 12*a^5*d*x*Cosh[6*c + 3*d*x] + 68*a^4*b*d*x*Cosh[6*c + 3*d*x] + 13
2*a^3*b^2*d*x*Cosh[6*c + 3*d*x] + 108*a^2*b^3*d*x*Cosh[6*c + 3*d*x] + 32*a*
b^4*d*x*Cosh[6*c + 3*d*x] - 4*a^5*d*x*Cosh[2*c + 5*d*x] - 12*a^4*b*d*x*Cosh
[2*c + 5*d*x] - 12*a^3*b^2*d*x*Cosh[2*c + 5*d*x] - 4*a^2*b^3*d*x*Cosh[2*c +
 5*d*x] + 4*a^5*d*x*Cosh[4*c + 5*d*x] + 12*a^4*b*d*x*Cosh[4*c + 5*d*x] + 12
*a^3*b^2*d*x*Cosh[4*c + 5*d*x] + 4*a^2*b^3*d*x*Cosh[4*c + 5*d*x] - 4*a^5*d*
x*Cosh[6*c + 5*d*x] - 12*a^4*b*d*x*Cosh[6*c + 5*d*x] - 12*a^3*b^2*d*x*Cosh[
6*c + 5*d*x] - 4*a^2*b^3*d*x*Cosh[6*c + 5*d*x] + 4*a^5*d*x*Cosh[8*c + 5*d*x
] + 12*a^4*b*d*x*Cosh[8*c + 5*d*x] + 12*a^3*b^2*d*x*Cosh[8*c + 5*d*x] + 4*a
^2*b^3*d*x*Cosh[8*c + 5*d*x] - 32*a^5*Sinh[d*x] - 64*a^4*b*Sinh[d*x] - 30*a
^2*b^3*Sinh[d*x] - 120*a*b^4*Sinh[d*x] - 48*b^5*Sinh[d*x] + 32*a^5*Sinh[3*d
*x] + 64*a^4*b*Sinh[3*d*x] + 26*a^3*b^2*Sinh[3*d*x] + 86*a^2*b^3*Sinh[3*d*x
] + 32*a*b^4*Sinh[3*d*x] - 48*a^5*Sinh[2*c - d*x] - 128*a^4*b*Sinh[2*c - d*
x] - 128*a^3*b^2*Sinh[2*c - d*x] - 30*a^2*b^3*Sinh[2*c - d*x] - 120*a*b^4*S
inh[2*c - d*x] - 48*b^5*Sinh[2*c - d*x] + 48*a^5*Sinh[2*c + d*x] + 128*a^4*
b*Sinh[2*c + d*x] + 102*a^3*b^2*Sinh[2*c + d*x] - 86*a^2*b^3*Sinh[2*c + d*x
] - 136*a*b^4*Sinh[2*c + d*x] - 48*b^5*Sinh[2*c + d*x] - 32*a^5*Sinh[4*c +
d*x] - 64*a^4*b*Sinh[4*c + d*x] + 26*a^3*b^2*Sinh[4*c + d*x] + 86*a^2*b^3*S
inh[4*c + d*x] + 136*a*b^4*Sinh[4*c + d*x] + 48*b^5*Sinh[4*c + d*x] - 8*a^5
```

```
*Sinh[2*c + 3*d*x] - 26*a^3*b^2*Sinh[2*c + 3*d*x] - 86*a^2*b^3*Sinh[2*c + 3
*d*x] - 32*a*b^4*Sinh[2*c + 3*d*x] + 32*a^5*Sinh[4*c + 3*d*x] + 64*a^4*b*Si
nh[4*c + 3*d*x] - 13*a^3*b^2*Sinh[4*c + 3*d*x] - 36*a^2*b^3*Sinh[4*c + 3*d*
x] - 16*a*b^4*Sinh[4*c + 3*d*x] - 8*a^5*Sinh[6*c + 3*d*x] + 13*a^3*b^2*Sinh
[6*c + 3*d*x] + 36*a^2*b^3*Sinh[6*c + 3*d*x] + 16*a*b^4*Sinh[6*c + 3*d*x] +
 8*a^5*Sinh[2*c + 5*d*x] + 13*a^3*b^2*Sinh[2*c + 5*d*x] + 6*a^2*b^3*Sinh[2*
c + 5*d*x] - 13*a^3*b^2*Sinh[4*c + 5*d*x] - 6*a^2*b^3*Sinh[4*c + 5*d*x] + 8
*a^5*Sinh[6*c + 5*d*x]))/(512*a^3*(a + b)^3*d*(a + b*Sech[c + d*x]^2)^3)
```

fricas [B] time = 0.63, size = 11606, normalized size = 63.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")

[Out] [1/16*(16*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^10 + 160*
(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)*sinh(d*x + c)^9 + 1
6*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*sinh(d*x + c)^10 - 4*(8*a^5 - 1
3*a^3*b^2 - 36*a^2*b^3 - 16*a*b^4 - 4*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a
^2*b^3 + 8*a*b^4)*d*x)*cosh(d*x + c)^8 - 4*(8*a^5 - 13*a^3*b^2 - 36*a^2*b^3
 - 16*a*b^4 - 180*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^2
 - 4*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*d*x)*sinh(d*x +
 c)^8 + 32*(60*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^3 -
(8*a^5 - 13*a^3*b^2 - 36*a^2*b^3 - 16*a*b^4 - 4*(3*a^5 + 17*a^4*b + 33*a^3*
b^2 + 27*a^2*b^3 + 8*a*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c)^7 - 8*(16*a^5
 + 32*a^4*b - 13*a^3*b^2 - 43*a^2*b^3 - 68*a*b^4 - 24*b^5 - 4*(a^5 + 7*a^4*
b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c)^6 + 8*(4
20*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^4 - 16*a^5 - 32*
a^4*b + 13*a^3*b^2 + 43*a^2*b^3 + 68*a*b^4 + 24*b^5 + 4*(a^5 + 7*a^4*b + 23
*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*d*x - 14*(8*a^5 - 13*a^3*b^2 - 36
*a^2*b^3 - 16*a*b^4 - 4*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b
^4)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 16*(252*(a^5 + 3*a^4*b + 3*a^3*
b^2 + a^2*b^3)*d*x*cosh(d*x + c)^5 - 14*(8*a^5 - 13*a^3*b^2 - 36*a^2*b^3 -
16*a*b^4 - 4*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*d*x)*co
sh(d*x + c)^3 - 3*(16*a^5 + 32*a^4*b - 13*a^3*b^2 - 43*a^2*b^3 - 68*a*b^4 -
 24*b^5 - 4*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*d*
x)*cosh(d*x + c))*sinh(d*x + c)^5 - 32*a^5 - 52*a^3*b^2 - 24*a^2*b^3 - 8*(2
4*a^5 + 64*a^4*b + 64*a^3*b^2 + 15*a^2*b^3 + 60*a*b^4 + 24*b^5 + 4*(a^5 + 7
*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c)^4 +
 8*(420*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^6 - 24*a^5
- 64*a^4*b - 64*a^3*b^2 - 15*a^2*b^3 - 60*a*b^4 - 24*b^5 - 35*(8*a^5 - 13*a
^3*b^2 - 36*a^2*b^3 - 16*a*b^4 - 4*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*
b^3 + 8*a*b^4)*d*x)*cosh(d*x + c)^4 - 4*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^
2*b^3 + 28*a*b^4 + 8*b^5)*d*x - 15*(16*a^5 + 32*a^4*b - 13*a^3*b^2 - 43*a^2
```

*b^3 - 68*a*b^4 - 24*b^5 - 4*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*
a*b^4 + 8*b^5)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 32*(60*(a^5 + 3*a^4*
b + 3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^7 - 7*(8*a^5 - 13*a^3*b^2 - 36*a
^2*b^3 - 16*a*b^4 - 4*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4
)*d*x)*cosh(d*x + c)^5 - 5*(16*a^5 + 32*a^4*b - 13*a^3*b^2 - 43*a^2*b^3 - 6
8*a*b^4 - 24*b^5 - 4*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 +
8*b^5)*d*x)*cosh(d*x + c)^3 - (24*a^5 + 64*a^4*b + 64*a^3*b^2 + 15*a^2*b^3
+ 60*a*b^4 + 24*b^5 + 4*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4
 + 8*b^5)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 - 16*(a^5 + 3*a^4*b + 3*a^3*b
^2 + a^2*b^3)*d*x - 8*(16*a^5 + 32*a^4*b + 13*a^3*b^2 + 43*a^2*b^3 + 16*a*b
^4 + 2*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*d*x)*cosh(d*x
 + c)^2 + 8*(90*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^8 -
 14*(8*a^5 - 13*a^3*b^2 - 36*a^2*b^3 - 16*a*b^4 - 4*(3*a^5 + 17*a^4*b + 33*
a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*d*x)*cosh(d*x + c)^6 - 16*a^5 - 32*a^4*b -
13*a^3*b^2 - 43*a^2*b^3 - 16*a*b^4 - 15*(16*a^5 + 32*a^4*b - 13*a^3*b^2 - 4
3*a^2*b^3 - 68*a*b^4 - 24*b^5 - 4*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3
+ 28*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c)^4 - 2*(3*a^5 + 17*a^4*b + 33*a^3*b^2
 + 27*a^2*b^3 + 8*a*b^4)*d*x - 6*(24*a^5 + 64*a^4*b + 64*a^3*b^2 + 15*a^2*b
^3 + 60*a*b^4 + 24*b^5 + 4*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*
b^4 + 8*b^5)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((35*a^4*b + 28*a^3*b^
2 + 8*a^2*b^3)*cosh(d*x + c)^10 + 10*(35*a^4*b + 28*a^3*b^2 + 8*a^2*b^3)*co
sh(d*x + c)*sinh(d*x + c)^9 + (35*a^4*b + 28*a^3*b^2 + 8*a^2*b^3)*sinh(d*x
+ c)^10 + (105*a^4*b + 364*a^3*b^2 + 248*a^2*b^3 + 64*a*b^4)*cosh(d*x + c)^
8 + (105*a^4*b + 364*a^3*b^2 + 248*a^2*b^3 + 64*a*b^4 + 45*(35*a^4*b + 28*a
^3*b^2 + 8*a^2*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 8*(15*(35*a^4*b + 28
*a^3*b^2 + 8*a^2*b^3)*cosh(d*x + c)^3 + (105*a^4*b + 364*a^3*b^2 + 248*a^2*
b^3 + 64*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(35*a^4*b + 168*a^3*b^2
+ 400*a^2*b^3 + 256*a*b^4 + 64*b^5)*cosh(d*x + c)^6 + 2*(35*a^4*b + 168*a^3
*b^2 + 400*a^2*b^3 + 256*a*b^4 + 64*b^5 + 105*(35*a^4*b + 28*a^3*b^2 + 8*a^
2*b^3)*cosh(d*x + c)^4 + 14*(105*a^4*b + 364*a^3*b^2 + 248*a^2*b^3 + 64*a*b
^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(63*(35*a^4*b + 28*a^3*b^2 + 8*a^2
*b^3)*cosh(d*x + c)^5 + 14*(105*a^4*b + 364*a^3*b^2 + 248*a^2*b^3 + 64*a*b^
4)*cosh(d*x + c)^3 + 3*(35*a^4*b + 168*a^3*b^2 + 400*a^2*b^3 + 256*a*b^4 +
64*b^5)*cosh(d*x + c))*sinh(d*x + c)^5 - 35*a^4*b - 28*a^3*b^2 - 8*a^2*b^3
- 2*(35*a^4*b + 168*a^3*b^2 + 400*a^2*b^3 + 256*a*b^4 + 64*b^5)*cosh(d*x +
c)^4 + 2*(105*(35*a^4*b + 28*a^3*b^2 + 8*a^2*b^3)*cosh(d*x + c)^6 - 35*a^4*
b - 168*a^3*b^2 - 400*a^2*b^3 - 256*a*b^4 - 64*b^5 + 35*(105*a^4*b + 364*a^
3*b^2 + 248*a^2*b^3 + 64*a*b^4)*cosh(d*x + c)^4 + 15*(35*a^4*b + 168*a^3*b^
2 + 400*a^2*b^3 + 256*a*b^4 + 64*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*
(15*(35*a^4*b + 28*a^3*b^2 + 8*a^2*b^3)*cosh(d*x + c)^7 + 7*(105*a^4*b + 36
4*a^3*b^2 + 248*a^2*b^3 + 64*a*b^4)*cosh(d*x + c)^5 + 5*(35*a^4*b + 168*a^3
*b^2 + 400*a^2*b^3 + 256*a*b^4 + 64*b^5)*cosh(d*x + c)^3 - (35*a^4*b + 168*
a^3*b^2 + 400*a^2*b^3 + 256*a*b^4 + 64*b^5)*cosh(d*x + c))*sinh(d*x + c)^3
- (105*a^4*b + 364*a^3*b^2 + 248*a^2*b^3 + 64*a*b^4)*cosh(d*x + c)^2 + (45*
(35*a^4*b + 28*a^3*b^2 + 8*a^2*b^3)*cosh(d*x + c)^8 + 28*(105*a^4*b + 364*a

^3*b^2 + 248*a^2*b^3 + 64*a*b^4)*cosh(d*x + c)^6 - 105*a^4*b - 364*a^3*b^2 - 248*a^2*b^3 - 64*a*b^4 + 30*(35*a^4*b + 168*a^3*b^2 + 400*a^2*b^3 + 256*a*b^4 + 64*b^5)*cosh(d*x + c)^4 - 12*(35*a^4*b + 168*a^3*b^2 + 400*a^2*b^3 + 256*a*b^4 + 64*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(5*(35*a^4*b + 28*a^3*b^2 + 8*a^2*b^3)*cosh(d*x + c)^9 + 4*(105*a^4*b + 364*a^3*b^2 + 248*a^2*b^3 + 64*a*b^4)*cosh(d*x + c)^7 + 6*(35*a^4*b + 168*a^3*b^2 + 400*a^2*b^3 + 256*a*b^4 + 64*b^5)*cosh(d*x + c)^5 - 4*(35*a^4*b + 168*a^3*b^2 + 400*a^2*b^3 + 256*a*b^4 + 64*b^5)*cosh(d*x + c)^3 - (105*a^4*b + 364*a^3*b^2 + 248*a^2*b^3 + 64*a*b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(b/(a + b))*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) + 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 + 3*a*b + 2*b^2)*sqrt(b/(a + b)))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) + 16*(10*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^9 - 2*(8*a^5 - 13*a^3*b^2 - 36*a^2*b^3 - 16*a*b^4 - 4*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*d*x)*cosh(d*x + c)^7 - 3*(16*a^5 + 32*a^4*b - 13*a^3*b^2 - 43*a^2*b^3 - 68*a*b^4 - 24*b^5 - 4*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c)^5 - 2*(24*a^5 + 64*a^4*b + 64*a^3*b^2 + 15*a^2*b^3 + 60*a*b^4 + 24*b^5 + 4*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c)^3 - (16*a^5 + 32*a^4*b + 13*a^3*b^2 + 43*a^2*b^3 + 16*a*b^4 + 2*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^10 + 10*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^9 + (a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*sinh(d*x + c)^10 + (3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*d*cosh(d*x + c)^8 + (45*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^2 + (3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*d)*sinh(d*x + c)^8 + 2*(a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c)^6 + 8*(15*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^3 + (3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(105*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^4 + 14*(3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*d*cosh(d*x + c)^2 + (a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*d)*sinh(d*x + c)^6 - 2*(a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c)^4 + 4*(63*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^5 + 14*(3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*d*cosh(d*x + c)^3 + 3*(a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(105*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^6 + 35*(3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*d*cosh(d*x + c)^4 + 15*(a^8 + 7*a^7*b + 23*a^6*b^2 + 37* a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c)^2 - (a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*d)*sinh(d*x + c)^4 - (3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*d*cosh(d*x + c)^2 + 8*(15*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^7 + 7*(3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*d*cosh(d*x + c)^5 + 5*(a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c)^3 - (a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^3 + (45*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^8 + 28*(3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*d*cosh(d*x + c)^6 + 30*(a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c)^4 - 12*(a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c)^2 - (3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*d)*sinh(d*x + c)^2 - (a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d + 2*(5*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^9 + 4*(3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*d*cosh(d*x + c)^7 + 6*(a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c)^5 - 4*(a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c)^3 - (3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*d*cosh(d*x + c))*sinh(d*x + c)), 1/8*(8*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^10 + 80*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)*sinh(d*x + c)^9 + 8*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*sinh(d*x + c)^10 - 2*(8*a^5 - 13*a^3*b^2 - 36*a^2*b^3 - 16*a*b^4 - 4*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*d*x)*cosh(d*x + c)^8 - 2*(8*a^5 - 13*a^3*b^2 - 36*a^2*b^3 - 16*a*b^4 - 180*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^2 - 4*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*d*x)*sinh(d*x + c)^8 + 16*(60*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^3 - (8*a^5 - 13*a^3*b^2 - 36*a^2*b^3 - 16*a*b^4 - 4*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c)^7 - 4*(16*a^5 + 32*a^4*b - 13*a^3*b^2 - 43*a^2*b^3 - 68*a*b^4 - 24*b^5 - 4*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c)^6 + 4*(420*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^4 - 16*a^5 - 32*a^4*b + 13*a^3*b^2 + 43*a^2*b^3 + 68*a*b^4 + 24*b^5 + 4*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*d*x - 14*(8*a^5 - 13*a^3*b^2 - 36*a^2*b^3 - 16*a*b^4 - 4*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(252*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^5 - 14*(8*a^5 - 13*a^3*b^2 - 36*a^2*b^3 - 16*a*b^4 - 4*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*d*x)*cosh(d*x + c)^3 - 3*(16*a^5 + 32*a^4*b - 13*a^3*b^2 - 43*a^2*b^3 - 68*a*b^4 - 24*b^5 - 4*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 - 16*a^5 - 26*a^3*b^2 - 12*a^2*b^3 - 4*(24*a^5 + 64*a^4*b + 64*a^3*b^2 + 15*a^2*b^3 + 60*a*b^4 + 24*b^5 + 4*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c)^4 + 4*(420*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^6 - 24*a^5 - 64*a^4*b - 64*a^3*b^2 - 15*a^2*b^3 - 60*a*b^4 - 24*b^5 - 35*(8*a^5 - 13*a^3*b^2 - 36*a^2*b^3 - 16*a

b^4 - 4(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*d*x)*cosh(d
x + c)^4 - 4(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*
d*x - 15*(16*a^5 + 32*a^4*b - 13*a^3*b^2 - 43*a^2*b^3 - 68*a*b^4 - 24*b^5 -
4*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*d*x)*cosh(d
*x + c)^2)*sinh(d*x + c)^4 + 16*(60*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d
*x*cosh(d*x + c)^7 - 7*(8*a^5 - 13*a^3*b^2 - 36*a^2*b^3 - 16*a*b^4 - 4*(3*a
^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*d*x)*cosh(d*x + c)^5 - 5
*(16*a^5 + 32*a^4*b - 13*a^3*b^2 - 43*a^2*b^3 - 68*a*b^4 - 24*b^5 - 4*(a^5
+ 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c)^
3 - (24*a^5 + 64*a^4*b + 64*a^3*b^2 + 15*a^2*b^3 + 60*a*b^4 + 24*b^5 + 4*(a
^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*d*x)*cosh(d*x +
c))*sinh(d*x + c)^3 - 8*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x - 4*(16*a
^5 + 32*a^4*b + 13*a^3*b^2 + 43*a^2*b^3 + 16*a*b^4 + 2*(3*a^5 + 17*a^4*b +
33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*d*x)*cosh(d*x + c)^2 + 4*(90*(a^5 + 3*a^
4*b + 3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^8 - 14*(8*a^5 - 13*a^3*b^2 - 3
6*a^2*b^3 - 16*a*b^4 - 4*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*
b^4)*d*x)*cosh(d*x + c)^6 - 16*a^5 - 32*a^4*b - 13*a^3*b^2 - 43*a^2*b^3 - 1
6*a*b^4 - 15*(16*a^5 + 32*a^4*b - 13*a^3*b^2 - 43*a^2*b^3 - 68*a*b^4 - 24*b
^5 - 4*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*d*x)*co
sh(d*x + c)^4 - 2*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*d*
x - 6*(24*a^5 + 64*a^4*b + 64*a^3*b^2 + 15*a^2*b^3 + 60*a*b^4 + 24*b^5 + 4*
(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*d*x)*cosh(d*x
+ c)^2)*sinh(d*x + c)^2 - ((35*a^4*b + 28*a^3*b^2 + 8*a^2*b^3)*cosh(d*x + c
)^10 + 10*(35*a^4*b + 28*a^3*b^2 + 8*a^2*b^3)*cosh(d*x + c)*sinh(d*x + c)^9
+ (35*a^4*b + 28*a^3*b^2 + 8*a^2*b^3)*sinh(d*x + c)^10 + (105*a^4*b + 364*
a^3*b^2 + 248*a^2*b^3 + 64*a*b^4)*cosh(d*x + c)^8 + (105*a^4*b + 364*a^3*b^
2 + 248*a^2*b^3 + 64*a*b^4 + 45*(35*a^4*b + 28*a^3*b^2 + 8*a^2*b^3)*cosh(d*
x + c)^2)*sinh(d*x + c)^8 + 8*(15*(35*a^4*b + 28*a^3*b^2 + 8*a^2*b^3)*cosh(
d*x + c)^3 + (105*a^4*b + 364*a^3*b^2 + 248*a^2*b^3 + 64*a*b^4)*cosh(d*x +
c))*sinh(d*x + c)^7 + 2*(35*a^4*b + 168*a^3*b^2 + 400*a^2*b^3 + 256*a*b^4 +
64*b^5)*cosh(d*x + c)^6 + 2*(35*a^4*b + 168*a^3*b^2 + 400*a^2*b^3 + 256*a*
b^4 + 64*b^5 + 105*(35*a^4*b + 28*a^3*b^2 + 8*a^2*b^3)*cosh(d*x + c)^4 + 14
*(105*a^4*b + 364*a^3*b^2 + 248*a^2*b^3 + 64*a*b^4)*cosh(d*x + c)^2)*sinh(d
x + c)^6 + 4(63*(35*a^4*b + 28*a^3*b^2 + 8*a^2*b^3)*cosh(d*x + c)^5 + 14*
(105*a^4*b + 364*a^3*b^2 + 248*a^2*b^3 + 64*a*b^4)*cosh(d*x + c)^3 + 3*(35*
a^4*b + 168*a^3*b^2 + 400*a^2*b^3 + 256*a*b^4 + 64*b^5)*cosh(d*x + c))*sinh
(d*x + c)^5 - 35*a^4*b - 28*a^3*b^2 - 8*a^2*b^3 - 2*(35*a^4*b + 168*a^3*b^2
+ 400*a^2*b^3 + 256*a*b^4 + 64*b^5)*cosh(d*x + c)^4 + 2*(105*(35*a^4*b + 2
8*a^3*b^2 + 8*a^2*b^3)*cosh(d*x + c)^6 - 35*a^4*b - 168*a^3*b^2 - 400*a^2*b
^3 - 256*a*b^4 - 64*b^5 + 35*(105*a^4*b + 364*a^3*b^2 + 248*a^2*b^3 + 64*a*
b^4)*cosh(d*x + c)^4 + 15*(35*a^4*b + 168*a^3*b^2 + 400*a^2*b^3 + 256*a*b^4
+ 64*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(15*(35*a^4*b + 28*a^3*b^2
+ 8*a^2*b^3)*cosh(d*x + c)^7 + 7*(105*a^4*b + 364*a^3*b^2 + 248*a^2*b^3 + 6
4*a*b^4)*cosh(d*x + c)^5 + 5*(35*a^4*b + 168*a^3*b^2 + 400*a^2*b^3 + 256*a*
b^4 + 64*b^5)*cosh(d*x + c)^3 - (35*a^4*b + 168*a^3*b^2 + 400*a^2*b^3 + 256

*a*b^4 + 64*b^5)*cosh(d*x + c))*sinh(d*x + c)^3 - (105*a^4*b + 364*a^3*b^2 + 248*a^2*b^3 + 64*a*b^4)*cosh(d*x + c)^2 + (45*(35*a^4*b + 28*a^3*b^2 + 8*a^2*b^3)*cosh(d*x + c)^8 + 28*(105*a^4*b + 364*a^3*b^2 + 248*a^2*b^3 + 64*a*b^4)*cosh(d*x + c)^6 - 105*a^4*b - 364*a^3*b^2 - 248*a^2*b^3 - 64*a*b^4 + 30*(35*a^4*b + 168*a^3*b^2 + 400*a^2*b^3 + 256*a*b^4 + 64*b^5)*cosh(d*x + c)^4 - 12*(35*a^4*b + 168*a^3*b^2 + 400*a^2*b^3 + 256*a*b^4 + 64*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(5*(35*a^4*b + 28*a^3*b^2 + 8*a^2*b^3)*cosh(d*x + c)^9 + 4*(105*a^4*b + 364*a^3*b^2 + 248*a^2*b^3 + 64*a*b^4)*cosh(d*x + c)^7 + 6*(35*a^4*b + 168*a^3*b^2 + 400*a^2*b^3 + 256*a*b^4 + 64*b^5)*cosh(d*x + c)^5 - 4*(35*a^4*b + 168*a^3*b^2 + 400*a^2*b^3 + 256*a*b^4 + 64*b^5)*cosh(d*x + c)^3 - (105*a^4*b + 364*a^3*b^2 + 248*a^2*b^3 + 64*a*b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b/(a + b))*arctan(1/2*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-b/(a + b))/b) + 8*(10*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^9 - 2*(8*a^5 - 13*a^3*b^2 - 36*a^2*b^3 - 16*a*b^4 - 4*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*d*x)*cosh(d*x + c)^7 - 3*(16*a^5 + 32*a^4*b - 13*a^3*b^2 - 43*a^2*b^3 - 68*a*b^4 - 24*b^5 - 4*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c)^5 - 2*(24*a^5 + 64*a^4*b + 64*a^3*b^2 + 15*a^2*b^3 + 60*a*b^4 + 24*b^5 + 4*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c)^3 - (16*a^5 + 32*a^4*b + 13*a^3*b^2 + 43*a^2*b^3 + 16*a*b^4 + 2*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^10 + 10*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^9 + (a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*sinh(d*x + c)^10 + (3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*d*cosh(d*x + c)^8 + (45*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^2 + (3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*d)*sinh(d*x + c)^8 + 2*(a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c)^6 + 8*(15*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^3 + (3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(105*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^4 + 14*(3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*d*cosh(d*x + c)^2 + (a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*d)*sinh(d*x + c)^6 - 2*(a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c)^4 + 4*(63*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^5 + 14*(3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*d*cosh(d*x + c)^3 + 3*(a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(105*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^6 + 35*(3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*d*cosh(d*x + c)^4 + 15*(a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c)^2 - (a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*d)*sinh(d*x + c)^4 - (3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*d*cosh(d*x + c)^2 + 8*(15*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^7 + 7*(3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 +

```
 8*a^4*b^4)*d*cosh(d*x + c)^5 + 5*(a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3
+ 28*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c)^3 - (a^8 + 7*a^7*b + 23*a^6*b^2 +
 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^3 + (4
5*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^8 + 28*(3*a^8 + 17*
a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*d*cosh(d*x + c)^6 + 30*(a^8 +
7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c)
^4 - 12*(a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*
d*cosh(d*x + c)^2 - (3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4
)*d)*sinh(d*x + c)^2 - (a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d + 2*(5*(a^8
+ 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^9 + 4*(3*a^8 + 17*a^7*b +
33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*d*cosh(d*x + c)^7 + 6*(a^8 + 7*a^7*b +
 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c)^5 - 4*(a
^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x
 + c)^3 - (3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*d*cosh(d
*x + c))*sinh(d*x + c))]
```

giac [B] time = 1.27, size = 402, normalized size = 2.21

$$-\frac{\frac{\left(35\,a^{2}b^{2}e^{(2\,c)}+28\,ab^{3}e^{(2\,c)}+8\,b^{4}e^{(2\,c)}\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^{2}}}\right)e^{(-2\,c)}}{\left(a^{6}+3\,a^{5}b+3\,a^{4}b^{2}+a^{3}b^{3}\right)\sqrt{-ab-b^{2}}}-\frac{8\,dx}{a^{3}}-\frac{2\left(13\,a^{3}b^{2}e^{(6\,dx+6\,c)}+36\,a^{2}b^{3}e^{(6\,dx+6\,c)}+16\,ab^{4}e^{(6\,dx+6\,c)}+39\,a^{3}b^{2}e^{(4\,dx}\cdots}{\left(a^{6}\cdots\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*((35*a^2*b^2*e^(2*c) + 28*a*b^3*e^(2*c) + 8*b^4*e^(2*c))*arctan(1/2*(a
*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))*e^(-2*c)/((a^6 + 3*a^5*b + 3*
a^4*b^2 + a^3*b^3)*sqrt(-a*b - b^2)) - 8*d*x/a^3 - 2*(13*a^3*b^2*e^(6*d*x +
 6*c) + 36*a^2*b^3*e^(6*d*x + 6*c) + 16*a*b^4*e^(6*d*x + 6*c) + 39*a^3*b^2*
e^(4*d*x + 4*c) + 122*a^2*b^3*e^(4*d*x + 4*c) + 152*a*b^4*e^(4*d*x + 4*c) +
 48*b^5*e^(4*d*x + 4*c) + 39*a^3*b^2*e^(2*d*x + 2*c) + 92*a^2*b^3*e^(2*d*x
+ 2*c) + 32*a*b^4*e^(2*d*x + 2*c) + 13*a^3*b^2 + 6*a^2*b^3)/((a^6 + 3*a^5*b
 + 3*a^4*b^2 + a^3*b^3)*(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2
*d*x + 2*c) + a)^2) + 16/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(e^(2*d*x + 2*c)
- 1)))/d
```

maple [B] time = 0.51, size = 1433, normalized size = 7.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] -1/2/d/(a^3+3*a^2*b+3*a*b^2+b^3)*tanh(1/2*d*x+1/2*c)-1/d/a^3*ln(tanh(1/2*d*
x+1/2*c)-1)+1/d/a^3*ln(tanh(1/2*d*x+1/2*c)+1)-13/4/d*b^2/(a+b)^3/(tanh(1/2*
d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2
*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^7-17/4/d*b^3/a/(a+b)^3/(tanh(1/2
*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/
2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^7-1/d*b^4/a^2/(a+b)^3/(tanh(1/2
*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/
2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^7-39/4/d*b^2/(a+b)^3/(tanh(1/2*
d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2
*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^5-7/4/d*b^3/a/(a+b)^3/(tanh(1/2*
d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2
*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^5+1/d*b^4/a^2/(a+b)^3/(tanh(1/2*
d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2
*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^5-39/4/d*b^2/(a+b)^3/(tanh(1/2*d
*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*
d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^3-7/4/d*b^3/a/(a+b)^3/(tanh(1/2*d
*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*
d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^3+1/d*b^4/a^2/(a+b)^3/(tanh(1/2*d
*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*
d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^3-13/4/d*b^2/(a+b)^3/(tanh(1/2*d*
x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d
*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)-17/4/d*b^3/a/(a+b)^3/(tanh(1/2*d*x
+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*
x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)-1/d*b^4/a^2/(a+b)^3/(tanh(1/2*d*x+1
/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+
1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)+35/16/d*b^(3/2)/a/(a+b)^(7/2)*ln((a+b
)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))-35
/16/d*b^(3/2)/a/(a+b)^(7/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*
tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))+7/4/d*b^(5/2)/a^2/(a+b)^(7/2)*ln((a+b)^(1/
2)*tanh(1/2*d*x+1/2*c)^2-2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))-7/4/d*b
^(5/2)/a^2/(a+b)^(7/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(
1/2*d*x+1/2*c)+(a+b)^(1/2))+1/2/d*b^(7/2)/a^3/(a+b)^(7/2)*ln((a+b)^(1/2)*ta
nh(1/2*d*x+1/2*c)^2-2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))-1/2/d*b^(7/2
)/a^3/(a+b)^(7/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d
*x+1/2*c)+(a+b)^(1/2))-1/2/d/(a+b)^3/tanh(1/2*d*x+1/2*c)
```

maxima [B] time = 0.94, size = 1971, normalized size = 10.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(3*a^2*b + 3*a*b^2 + b^3)*log(a*e^(4*d*x + 4*c) + 2*(a + 2*b)*e^(2*d*x
+ 2*c) + a)/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d) - 1/4*(3*a^2*b + 3*a*
```

```
b^2 + b^3)*log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/((a^6
 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d) - 1/64*(15*a^3*b + 70*a^2*b^2 + 56*a*b
^3 + 16*b^4)*log((a*e^(2*d*x + 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(2*
d*x + 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^
3*b^3)*sqrt((a + b)*b)*d) + 1/64*(15*a^3*b + 70*a^2*b^2 + 56*a*b^3 + 16*b^4
)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c
) + a + 2*b + 2*sqrt((a + b)*b)))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*sq
rt((a + b)*b)*d) - 15/32*b*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a +
b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^3 + 3*a^2*b
+ 3*a*b^2 + b^3)*sqrt((a + b)*b)*d) + 1/16*(8*a^5 + 9*a^4*b + 28*a^3*b^2 +
12*a^2*b^3 + (8*a^5 - 9*a^4*b - 98*a^3*b^2 - 160*a^2*b^3 - 64*a*b^4)*e^(8*d
*x + 8*c) + 2*(16*a^5 + 23*a^4*b - 77*a^3*b^2 - 246*a^2*b^3 - 288*a*b^4 - 9
6*b^5)*e^(6*d*x + 6*c) + 2*(24*a^5 + 64*a^4*b + 99*a^3*b^2 + 190*a^2*b^3 +
272*a*b^4 + 96*b^5)*e^(4*d*x + 4*c) + 2*(16*a^5 + 41*a^4*b + 77*a^3*b^2 + 1
30*a^2*b^3 + 48*a*b^4)*e^(2*d*x + 2*c))/((a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b
^3 - (a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*e^(10*d*x + 10*c) - (3*a^8 + 17*
a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*e^(8*d*x + 8*c) - 2*(a^8 + 7*a
^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*e^(6*d*x + 6*c) +
2*(a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*e^(4*d
*x + 4*c) + (3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*e^(2*d
*x + 2*c))*d) - 1/16*(8*a^5 + 9*a^4*b + 28*a^3*b^2 + 12*a^2*b^3 + 2*(16*a^5
 + 41*a^4*b + 77*a^3*b^2 + 130*a^2*b^3 + 48*a*b^4)*e^(-2*d*x - 2*c) + 2*(24
*a^5 + 64*a^4*b + 99*a^3*b^2 + 190*a^2*b^3 + 272*a*b^4 + 96*b^5)*e^(-4*d*x
- 4*c) + 2*(16*a^5 + 23*a^4*b - 77*a^3*b^2 - 246*a^2*b^3 - 288*a*b^4 - 96*b
^5)*e^(-6*d*x - 6*c) + (8*a^5 - 9*a^4*b - 98*a^3*b^2 - 160*a^2*b^3 - 64*a*b
^4)*e^(-8*d*x - 8*c))/((a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3 + (3*a^8 + 17*a
^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*e^(-2*d*x - 2*c) + 2*(a^8 + 7*a
^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*e^(-4*d*x - 4*c) -
 2*(a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*e^(-6
*d*x - 6*c) - (3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*e^(-
8*d*x - 8*c) - (a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*e^(-10*d*x - 10*c))*d)
 - 1/8*(8*a^4 - 9*a^3*b - 2*a^2*b^2 + 2*(16*a^4 + 23*a^3*b - 27*a^2*b^2 - 4
*a*b^3)*e^(-2*d*x - 2*c) + 2*(24*a^4 + 64*a^3*b + 53*a^2*b^2 - 40*a*b^3 - 8
*b^4)*e^(-4*d*x - 4*c) + 2*(16*a^4 + 41*a^3*b + 27*a^2*b^2 + 40*a*b^3 + 8*b
^4)*e^(-6*d*x - 6*c) + (8*a^4 + 9*a^3*b + 24*a^2*b^2 + 8*a*b^3)*e^(-8*d*x -
 8*c))/((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3 + (3*a^7 + 17*a^6*b + 33*a^5*b
^2 + 27*a^4*b^3 + 8*a^3*b^4)*e^(-2*d*x - 2*c) + 2*(a^7 + 7*a^6*b + 23*a^5*b
^2 + 37*a^4*b^3 + 28*a^3*b^4 + 8*a^2*b^5)*e^(-4*d*x - 4*c) - 2*(a^7 + 7*a^6
*b + 23*a^5*b^2 + 37*a^4*b^3 + 28*a^3*b^4 + 8*a^2*b^5)*e^(-6*d*x - 6*c) - (
3*a^7 + 17*a^6*b + 33*a^5*b^2 + 27*a^4*b^3 + 8*a^3*b^4)*e^(-8*d*x - 8*c) -
(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*e^(-10*d*x - 10*c))*d) + 1/2*log(e^(2
*d*x + 2*c) - 1)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) - 1/2*log(e^(-2*d*x -
2*c) - 1)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh (c+d\,x)^6\,\coth (c+d\,x)^2}{\left(a\cosh (c+d\,x)^2+b\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^2/(a + b/cosh(c + d*x)^2)^3,x)
```

```
[Out] int((cosh(c + d*x)^6*coth(c + d*x)^2)/(b + a*cosh(c + d*x)^2)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^2 (c+dx)}{\left(a+b\,\mathrm{sech}^2 (c+dx)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**2/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Integral(coth(c + d*x)**2/(a + b*sech(c + d*x)**2)**3, x)
```

3.167 $\int \frac{\coth^3(c+dx)}{\left(a+b\,\mathbf{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=152

$$-\frac{b^4}{4a^3d(a+b)^2\left(a\cosh^2(c+dx)+b\right)^2}+\frac{b^3(2a+b)}{a^3d(a+b)^3\left(a\cosh^2(c+dx)+b\right)}+\frac{b^2\left(6a^2+4ab+b^2\right)\log\left(a\cosh^2(c+d\right.}{2a^3d(a+b)^4}$$

```
[Out] -1/4*b^4/a^3/(a+b)^2/d/(b+a*cosh(d*x+c)^2)^2+b^3*(2*a+b)/a^3/(a+b)^3/d/(b+a
*cosh(d*x+c)^2)-1/2*csch(d*x+c)^2/d/(a+b)^3+1/2*b^2*(6*a^2+4*a*b+b^2)*ln(b+
a*cosh(d*x+c)^2)/a^3/(a+b)^4/d+(a+4*b)*ln(sinh(d*x+c))/d/(a+b)^4
```

Rubi [A] time = 0.24, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4138, 446, 88}

$$-\frac{b^4}{4a^3d(a+b)^2\left(a\cosh^2(c+dx)+b\right)^2}+\frac{b^3(2a+b)}{a^3d(a+b)^3\left(a\cosh^2(c+dx)+b\right)}+\frac{b^2\left(6a^2+4ab+b^2\right)\log\left(a\cosh^2(c+d\right.}{2a^3d(a+b)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^3/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] -b^4/(4*a^3*(a + b)^2*d*(b + a*Cosh[c + d*x]^2)^2) + (b^3*(2*a + b))/(a^3*(
a + b)^3*d*(b + a*Cosh[c + d*x]^2)) - Csch[c + d*x]^2/(2*(a + b)^3*d) + (b^
2*(6*a^2 + 4*a*b + b^2)*Log[b + a*Cosh[c + d*x]^2])/(2*a^3*(a + b)^4*d) + (
(a + 4*b)*Log[Sinh[c + d*x]])/((a + b)^4*d)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\coth^3(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^9}{\left(1-x^2\right)^2\left(b+ax^2\right)^3}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{x^4}{(1-x)^2(b+ax)^3}\,dx, x, \cosh^2(c+dx)\right)}{2d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{(a+b)^3(-1+x)^2} + \frac{a+4b}{(a+b)^4(-1+x)} + \frac{b^4}{a^2(a+b)^2(b+ax)^3} - \frac{2b^3(2a+b)}{a^2(a+b)^3(b+ax)^2} + \frac{b^2\left(6a^2+4ab-\right.}{a^2(a+b)^4(b+}\right.\right.}{2d}$$

$$= -\frac{b^4}{4a^3(a+b)^2d\left(b+a\cosh^2(c+dx)\right)^2} + \frac{b^3(2a+b)}{a^3(a+b)^3d\left(b+a\cosh^2(c+dx)\right)} - \frac{\text{csc}}{2(}$$

Mathematica [A] time = 1.92, size = 172, normalized size = 1.13

$$-\frac{\text{sech}^6(c+dx)(a\cosh(2(c+dx))+a+2b)^3\left(\frac{b^4(a+b)^2}{a^3\left(a\sinh^2(c+dx)+a+b\right)^2} - \frac{4b^3(a+b)(2a+b)}{a^3\left(a\sinh^2(c+dx)+a+b\right)} - \frac{2b^2\left(6a^2+4ab+b^2\right)\log\left(a\sinh^2\right.}{a^3}\right.}{32d(a+b)^4\left(a+b\text{sech}^2(c+dx)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^3/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] -1/32*((a + 2*b + a*Cosh[2*(c + d*x)])^3*Sech[c + d*x]^6*(2*(a + b)*Csch[c
+ d*x]^2 - 4*(a + 4*b)*Log[Sinh[c + d*x]] - (2*b^2*(6*a^2 + 4*a*b + b^2)*Lo
g[a + b + a*Sinh[c + d*x]^2])/a^3 + (b^4*(a + b)^2)/(a^3*(a + b + a*Sinh[c
+ d*x]^2)^2) - (4*b^3*(a + b)*(2*a + b))/(a^3*(a + b + a*Sinh[c + d*x]^2)))
)/((a + b)^4*d*(a + b*Sech[c + d*x]^2)^3)
```

fricas [B] time = 1.34, size = 10255, normalized size = 67.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")

[Out] -1/2*(2*(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*x*cosh(d*x + c)^12 + 24*(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*x*cosh(d*x + c)*sinh(d*x + c)^11 + 2*(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*x*sinh(d*x + c)^12 + 4*(a^6 + a^5*b - 4*a^3*b^3 - 6*a^2*b^4 - 2*a*b^5 + (a^6 + 8*a^5*b + 22*a^4*b^2 + 28*a^3*b^3 + 17*a^2*b^4 + 4*a*b^5)*d*x)*cosh(d*x + c)^10 + 4*(a^6 + a^5*b - 4*a^3*b^3 - 6*a^2*b^4 - 2*a*b^5 + 33*(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*x*cosh(d*x + c)^2 + (a^6 + 8*a^5*b + 22*a^4*b^2 + 28*a^3*b^3 + 17*a^2*b^4 + 4*a*b^5)*d*x)*sinh(d*x + c)^10 + 40*(11*(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*x*cosh(d*x + c)^3 + (a^6 + a^5*b - 4*a^3*b^3 - 6*a^2*b^4 - 2*a*b^5 + (a^6 + 8*a^5*b + 22*a^4*b^2 + 28*a^3*b^3 + 17*a^2*b^4 + 4*a*b^5)*d*x)*cosh(d*x + c))*sinh(d*x + c)^9 + 2*(8*a^6 + 24*a^5*b + 16*a^4*b^2 - 28*a^2*b^4 - 40*a*b^5 - 12*b^6 - (a^6 + 4*a^5*b - 10*a^4*b^2 - 60*a^3*b^3 - 95*a^2*b^4 - 64*a*b^5 - 16*b^6)*d*x)*cosh(d*x + c)^8 + 2*(495*(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*x*cosh(d*x + c)^4 + 8*a^6 + 24*a^5*b + 16*a^4*b^2 - 28*a^2*b^4 - 40*a*b^5 - 12*b^6 - (a^6 + 4*a^5*b - 10*a^4*b^2 - 60*a^3*b^3 - 95*a^2*b^4 - 64*a*b^5 - 16*b^6)*d*x + 90*(a^6 + a^5*b - 4*a^3*b^3 - 6*a^2*b^4 - 2*a*b^5 + (a^6 + 8*a^5*b + 22*a^4*b^2 + 28*a^3*b^3 + 17*a^2*b^4 + 4*a*b^5)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 16*(99*(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*x*cosh(d*x + c)^5 + 30*(a^6 + a^5*b - 4*a^3*b^3 - 6*a^2*b^4 - 2*a*b^5 + (a^6 + 8*a^5*b + 22*a^4*b^2 + 28*a^3*b^3 + 17*a^2*b^4 + 4*a*b^5)*d*x)*cosh(d*x + c)^3 + (8*a^6 + 24*a^5*b + 16*a^4*b^2 - 28*a^2*b^4 - 40*a*b^5 - 12*b^6 - (a^6 + 4*a^5*b - 10*a^4*b^2 - 60*a^3*b^3 - 95*a^2*b^4 - 64*a*b^5 - 16*b^6)*d*x)*cosh(d*x + c))*sinh(d*x + c)^7 + 8*(3*a^6 + 11*a^5*b + 16*a^4*b^2 + 12*a^3*b^3 + 20*a^2*b^4 + 22*a*b^5 + 6*b^6 - (a^6 + 8*a^5*b + 30*a^4*b^2 + 60*a^3*b^3 + 65*a^2*b^4 + 36*a*b^5 + 8*b^6)*d*x)*cosh(d*x + c)^6 + 8*(231*(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*x*cosh(d*x + c)^6 + 3*a^6 + 11*a^5*b + 16*a^4*b^2 + 12*a^3*b^3 + 20*a^2*b^4 + 22*a*b^5 + 6*b^6 + 105*(a^6 + a^5*b - 4*a^3*b^3 - 6*a^2*b^4 - 2*a*b^5 + (a^6 + 8*a^5*b + 22*a^4*b^2 + 28*a^3*b^3 + 17*a^2*b^4 + 4*a*b^5)*d*x)*cosh(d*x + c)^4 - (a^6 + 8*a^5*b + 30*a^4*b^2 + 60*a^3*b^3 + 65*a^2*b^4 + 36*a*b^5 + 8*b^6)*d*x + 7*(8*a^6 + 24*a^5*b + 16*a^4*b^2 - 28*a^2*b^4 - 40*a*b^5 - 12*b^6 - (a^6 + 4*a^5*b - 10*a^4*b^2 - 60*a^3*b^3 - 95*a^2*b^4 - 64*a*b^5 - 16*b^6)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 16*(99*(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*x*cosh(d*x + c)^7 + 63*(a^6 + a^5*b - 4*a^3*b^3 - 6*a^2*b^4 - 2*a*b^5 + (a^6 + 8*a^5*b + 22*a^4*b^2 + 28*a^3*b^3 + 17*a^2*b^4 + 4*a*b^5)*d*x)*cosh(d*x + c)^5 + 7*(8*a^6 + 24*a^5*b + 16*a^4*b^2 - 28*a^2*b^4 - 40*a*b^5 - 12*b^6 - (a^6 + 4*a^5*b - 10*a^4*b^2 - 60*a^3*b^3 - 95*a^2*b^4 - 64*a*b^5 - 16*b^6)*d*x)*cosh(d*x + c)^3 + 3*(3*a^6 + 11*a^5*b + 16*a^4*b^2 + 12*a^3*b^3 + 20*a^2*b^4 + 22*a*b^5 + 6*b^6 - (a^6 + 8*a^5*b + 30*a^4*b^2 + 60*a^3*b^3 + 65*a^2*b^4 + 36*a*b^5 + 8*b^6)*d*x)*cosh(d*x + c))*s inh(d*x + c)^5 + 2*(8*a^6 + 24*a^5*b + 16*a^4*b^2 - 28*a^2*b^4 - 40*a*b^5 - 12*b^6 - (a^6 + 4*a^5*b - 10*a^4*b^2 - 60*a^3*b^3 - 95*a^2*b^4 - 64*a*b^5 - 16*b^6)*d*x)*cosh(d*x + c)^4 + 2*(495*(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*x*cosh(d*x + c)^8 + 420*(a^6 + a^5*b - 4*a^3*b^3 - 6*a^2*b^4 - 2*a*b^5 + (a^6 + 8*a^5*b + 22*a^4*b^2 + 28*a^3*b^3 + 17*a^2*b^4 + 4*a*b^5)*d*x)*cosh(d*x + c)^6 + 8*a^6 + 24*a^5*b + 16*a^4*b^2 - 28*a^2*b^4 - 40*a*b^5 - 12*b^6 + 70*(8*a^6 + 24*a^5*b + 16*a^4*b^2 - 28*a^2*b^4 - 40*a*b^5 - 12*b^6 - (a^6 + 4*a^5*b - 10*a^4*b^2 - 60*a^3*b^3 - 95*a^2*b^4 - 64*a*b^5 - 16*b^6)*d*x)*cosh(d*x + c)^4 - (a^6 + 4*a^5*b - 10*a^4*b^2 - 60*a^3*b^3 - 95*a^2*b^4 - 64*a*b^5 - 16*b^6)*d*x + 60*(3*a^6 + 11*a^5*b + 16*a^4*b^2 + 12*a^3*b^3 + 20*a^2*b^4 + 22*a*b^5 + 6*b^6 - (a^6 + 8*a^5*b + 30*a^4*b^2 + 60*a^3*b^3 + 65*a^2*b^4 + 36*a*b^5 + 8*b^6)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(55*(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*x*cosh(d*x + c)^9 + 60*(a^6 + a^5*b - 4*a^3*b^3 - 6*a^2*b^4 - 2*a*b^5 + (a^6 + 8*a^5*b + 22*a^4*b^2 + 28*a^3*b^3 + 17*a^2*b^4 + 4*a*b^5)*d*x)*cosh(d*x + c)^7 + 14*(8*a^6 + 24*a^5*b + 16*a^4*b^2 - 28*a^2*b^4 - 40*a*b^5 - 12*b^6 - (a^6 + 4*a^5*b - 10*a^4*b^2 - 60*a^3*b^3 - 95*a^2*b^4 - 64*a*b^5 - 16*b^6)*d*x)*cosh(d*x + c)^5 + 20*(3*a^6 + 11*a^5*b + 16*a^4*b^2 + 12*a^3*b^3 + 20*a^2*b^4 + 22*a*b^5 + 6*b^6 - (a^6 + 8*a^5*b + 30*a^4*b^2 + 60*a^3*b^3 + 65*a^2*b^4 + 36*a*b^5 + 8*b^6)*d*x)*cosh(d*x + c)^3 + (8*a^6 + 24*a^5*b + 16*a^4*b^2 - 28*a^2*b^4 - 40*a*b^5 - 12*b^6 - (a^6 + 4*a^5*b - 10*a^4*b^2 - 60*a^3*b^3 - 95*a^2*b^4 - 64*a*b^5 - 16*b^6)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*x + 4*(a^6 + a^5*b - 4*a^3*b^3 - 6*a^2*b^4 - 2*a*b^5 + (a^6 + 8*a^5*b + 22*a^4*b^2 + 28*a^3*b^3 + 17*a^2*b^4 + 4*a*b^5)*d*x)*cosh(d*x + c)^2 + 4*(33*(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*x*cosh(d*x + c)^10 + 45*(a^6 + a^5*b - 4*a^3*b^3 - 6*a^2*b^4 - 2*a*b^5 + (a^6 + 8*a^5*b + 22*a^4*b^2 + 28*a^3*b^3 + 17*a^2*b^4 + 4*a*b^5)*d*x)*cosh(d*x + c)^8 + 14*(8*a^6 + 24*a^5*b + 16*a^4*b^2 - 28*a^2*b^4 - 40*a*b^5 - 12*b^6 - (a^6 + 4*a^5*b - 10*a^4*b^2 - 60*a^3*b^3 - 95*a^2*b^4 - 64*a*b^5 - 16*b^6)*d*x)*cosh(d*x + c)^6 + a^6 + a^5*b - 4*a^3*b^3 - 6*a^2*b^4 - 2*a*b^5 + 30*(3*a^6 + 11*a^5*b + 16*a^4*b^2 + 12*a^3*b^3 + 20*a^2*b^4 + 22*a*b^5 + 6*b^6 - (a^6 + 8*a^5*b + 30*a^4*b^2 + 60*a^3*b^3 + 65*a^2*b^4 + 36*a*b^5 + 8*b^6)*d*x)*cosh(d*x + c)^4 + (a^6 + 8*a^5*b + 22*a^4*b^2 + 28*a^3*b^3 + 17*a^2*b^4 + 4*a*b^5)*d*x + 3*(8*a^6 + 24*a^5*b + 16*a^4*b^2 - 28*a^2*b^4 - 40*a*b^5 - 12*b^6 - (a^6 + 4*a^5*b - 10*a^4*b^2 - 60*a^3*b^3 - 95*a^2*b^4 - 64*a*b^5 - 16*b^6)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - ((6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*cosh(d*x + c)^12 + 12*(6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*cosh(d*x + c)*sinh(d*x + c)^11 + (6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*sinh(d*x + c)^12 + 2*(6*a^4*b^2 + 28*a^3*b^3 + 17*a^2*b^4 + 4*a*b^5)*cosh(d*x + c)^10 + 2*(6*a^4*b^2 + 28*a^3*b^3 + 17*a^2*b^4 + 4*a*b^5 + 33*(6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 20*(11*(6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*cosh(d*x + c)^3 + (6*a^4*b^2 + 28*a^3*b^3 + 17*a^2*b^4 + 4*a*b^5)*cosh(d*x + c))*sinh(d*x + c)^9 - (6*a^4*b^2 + 4*a^3*b^3 - 95*a^2*b^4 - 64*a*b^5 - 16*b^6)*cosh(d*x + c)^8 - (6*a^4*b^2 + 4*a^3*b^3 - 95*a^2*b^4 - 64*a*b^5 - 16*b^6 - 495*(6*a^4*b^2 +

4*a^3*b^3 + a^2*b^4)*cosh(d*x + c)^4 - 90*(6*a^4*b^2 + 28*a^3*b^3 + 17*a^2
*b^4 + 4*a*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 8*(99*(6*a^4*b^2 + 4*a^3
*b^3 + a^2*b^4)*cosh(d*x + c)^5 + 30*(6*a^4*b^2 + 28*a^3*b^3 + 17*a^2*b^4 +
4*a*b^5)*cosh(d*x + c)^3 - (6*a^4*b^2 + 4*a^3*b^3 - 95*a^2*b^4 - 64*a*b^5
- 16*b^6)*cosh(d*x + c))*sinh(d*x + c)^7 - 4*(6*a^4*b^2 + 28*a^3*b^3 + 65*a
^2*b^4 + 36*a*b^5 + 8*b^6)*cosh(d*x + c)^6 + 4*(231*(6*a^4*b^2 + 4*a^3*b^3
+ a^2*b^4)*cosh(d*x + c)^6 - 6*a^4*b^2 - 28*a^3*b^3 - 65*a^2*b^4 - 36*a*b^5
- 8*b^6 + 105*(6*a^4*b^2 + 28*a^3*b^3 + 17*a^2*b^4 + 4*a*b^5)*cosh(d*x + c
)^4 - 7*(6*a^4*b^2 + 4*a^3*b^3 - 95*a^2*b^4 - 64*a*b^5 - 16*b^6)*cosh(d*x +
c)^2)*sinh(d*x + c)^6 + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4 + 8*(99*(6*a^4*b^2
+ 4*a^3*b^3 + a^2*b^4)*cosh(d*x + c)^7 + 63*(6*a^4*b^2 + 28*a^3*b^3 + 17*a
^2*b^4 + 4*a*b^5)*cosh(d*x + c)^5 - 7*(6*a^4*b^2 + 4*a^3*b^3 - 95*a^2*b^4 -
64*a*b^5 - 16*b^6)*cosh(d*x + c)^3 - 3*(6*a^4*b^2 + 28*a^3*b^3 + 65*a^2*b^
4 + 36*a*b^5 + 8*b^6)*cosh(d*x + c))*sinh(d*x + c)^5 - (6*a^4*b^2 + 4*a^3*b
^3 - 95*a^2*b^4 - 64*a*b^5 - 16*b^6)*cosh(d*x + c)^4 + (495*(6*a^4*b^2 + 4*
a^3*b^3 + a^2*b^4)*cosh(d*x + c)^8 + 420*(6*a^4*b^2 + 28*a^3*b^3 + 17*a^2*b
^4 + 4*a*b^5)*cosh(d*x + c)^6 - 6*a^4*b^2 - 4*a^3*b^3 + 95*a^2*b^4 + 64*a*b
^5 + 16*b^6 - 70*(6*a^4*b^2 + 4*a^3*b^3 - 95*a^2*b^4 - 64*a*b^5 - 16*b^6)*c
osh(d*x + c)^4 - 60*(6*a^4*b^2 + 28*a^3*b^3 + 65*a^2*b^4 + 36*a*b^5 + 8*b^6
)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(55*(6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4
)*cosh(d*x + c)^9 + 60*(6*a^4*b^2 + 28*a^3*b^3 + 17*a^2*b^4 + 4*a*b^5)*cosh
(d*x + c)^7 - 14*(6*a^4*b^2 + 4*a^3*b^3 - 95*a^2*b^4 - 64*a*b^5 - 16*b^6)*c
osh(d*x + c)^5 - 20*(6*a^4*b^2 + 28*a^3*b^3 + 65*a^2*b^4 + 36*a*b^5 + 8*b^6
)*cosh(d*x + c)^3 - (6*a^4*b^2 + 4*a^3*b^3 - 95*a^2*b^4 - 64*a*b^5 - 16*b^6
)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(6*a^4*b^2 + 28*a^3*b^3 + 17*a^2*b^4 +
4*a*b^5)*cosh(d*x + c)^2 + 2*(33*(6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*cosh(d*
x + c)^10 + 45*(6*a^4*b^2 + 28*a^3*b^3 + 17*a^2*b^4 + 4*a*b^5)*cosh(d*x + c
)^8 - 14*(6*a^4*b^2 + 4*a^3*b^3 - 95*a^2*b^4 - 64*a*b^5 - 16*b^6)*cosh(d*x
+ c)^6 + 6*a^4*b^2 + 28*a^3*b^3 + 17*a^2*b^4 + 4*a*b^5 - 30*(6*a^4*b^2 + 28
*a^3*b^3 + 65*a^2*b^4 + 36*a*b^5 + 8*b^6)*cosh(d*x + c)^4 - 3*(6*a^4*b^2 +
4*a^3*b^3 - 95*a^2*b^4 - 64*a*b^5 - 16*b^6)*cosh(d*x + c)^2)*sinh(d*x + c)^
2 + 4*(3*(6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*cosh(d*x + c)^11 + 5*(6*a^4*b^2
+ 28*a^3*b^3 + 17*a^2*b^4 + 4*a*b^5)*cosh(d*x + c)^9 - 2*(6*a^4*b^2 + 4*a^3
*b^3 - 95*a^2*b^4 - 64*a*b^5 - 16*b^6)*cosh(d*x + c)^7 - 6*(6*a^4*b^2 + 28*
a^3*b^3 + 65*a^2*b^4 + 36*a*b^5 + 8*b^6)*cosh(d*x + c)^5 - (6*a^4*b^2 + 4*a
^3*b^3 - 95*a^2*b^4 - 64*a*b^5 - 16*b^6)*cosh(d*x + c)^3 + (6*a^4*b^2 + 28*
a^3*b^3 + 17*a^2*b^4 + 4*a*b^5)*cosh(d*x + c))*sinh(d*x + c))*log(2*(a*cosh
(d*x + c)^2 + a*sinh(d*x + c)^2 + a + 2*b)/(cosh(d*x + c)^2 - 2*cosh(d*x +
c)*sinh(d*x + c) + sinh(d*x + c)^2)) - 2*((a^6 + 4*a^5*b)*cosh(d*x + c)^12
+ 12*(a^6 + 4*a^5*b)*cosh(d*x + c)*sinh(d*x + c)^11 + (a^6 + 4*a^5*b)*sinh(
d*x + c)^12 + 2*(a^6 + 8*a^5*b + 16*a^4*b^2)*cosh(d*x + c)^10 + 2*(a^6 + 8*
a^5*b + 16*a^4*b^2 + 33*(a^6 + 4*a^5*b)*cosh(d*x + c)^2)*sinh(d*x + c)^10 +
20*(11*(a^6 + 4*a^5*b)*cosh(d*x + c)^3 + (a^6 + 8*a^5*b + 16*a^4*b^2)*cosh
(d*x + c))*sinh(d*x + c)^9 - (a^6 + 4*a^5*b - 16*a^4*b^2 - 64*a^3*b^3)*cosh
(d*x + c)^8 - (a^6 + 4*a^5*b - 16*a^4*b^2 - 64*a^3*b^3 - 495*(a^6 + 4*a^5*b

)*cosh(d*x + c)^4 - 90*(a^6 + 8*a^5*b + 16*a^4*b^2)*cosh(d*x + c)^2)*sinh(d
x + c)^8 + 8(99*(a^6 + 4*a^5*b)*cosh(d*x + c)^5 + 30*(a^6 + 8*a^5*b + 16*
a^4*b^2)*cosh(d*x + c)^3 - (a^6 + 4*a^5*b - 16*a^4*b^2 - 64*a^3*b^3)*cosh(d
*x + c))*sinh(d*x + c)^7 - 4*(a^6 + 8*a^5*b + 24*a^4*b^2 + 32*a^3*b^3)*cosh
(d*x + c)^6 + 4*(231*(a^6 + 4*a^5*b)*cosh(d*x + c)^6 - a^6 - 8*a^5*b - 24*a
^4*b^2 - 32*a^3*b^3 + 105*(a^6 + 8*a^5*b + 16*a^4*b^2)*cosh(d*x + c)^4 - 7*
(a^6 + 4*a^5*b - 16*a^4*b^2 - 64*a^3*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6
+ a^6 + 4*a^5*b + 8*(99*(a^6 + 4*a^5*b)*cosh(d*x + c)^7 + 63*(a^6 + 8*a^5*b
 + 16*a^4*b^2)*cosh(d*x + c)^5 - 7*(a^6 + 4*a^5*b - 16*a^4*b^2 - 64*a^3*b^3
)*cosh(d*x + c)^3 - 3*(a^6 + 8*a^5*b + 24*a^4*b^2 + 32*a^3*b^3)*cosh(d*x +
c))*sinh(d*x + c)^5 - (a^6 + 4*a^5*b - 16*a^4*b^2 - 64*a^3*b^3)*cosh(d*x +
c)^4 + (495*(a^6 + 4*a^5*b)*cosh(d*x + c)^8 + 420*(a^6 + 8*a^5*b + 16*a^4*b
^2)*cosh(d*x + c)^6 - a^6 - 4*a^5*b + 16*a^4*b^2 + 64*a^3*b^3 - 70*(a^6 + 4
*a^5*b - 16*a^4*b^2 - 64*a^3*b^3)*cosh(d*x + c)^4 - 60*(a^6 + 8*a^5*b + 24*
a^4*b^2 + 32*a^3*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(55*(a^6 + 4*a^5
*b)*cosh(d*x + c)^9 + 60*(a^6 + 8*a^5*b + 16*a^4*b^2)*cosh(d*x + c)^7 - 14*
(a^6 + 4*a^5*b - 16*a^4*b^2 - 64*a^3*b^3)*cosh(d*x + c)^5 - 20*(a^6 + 8*a^5
*b + 24*a^4*b^2 + 32*a^3*b^3)*cosh(d*x + c)^3 - (a^6 + 4*a^5*b - 16*a^4*b^2
 - 64*a^3*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(a^6 + 8*a^5*b + 16*a^4*b
^2)*cosh(d*x + c)^2 + 2*(33*(a^6 + 4*a^5*b)*cosh(d*x + c)^10 + 45*(a^6 + 8*
a^5*b + 16*a^4*b^2)*cosh(d*x + c)^8 - 14*(a^6 + 4*a^5*b - 16*a^4*b^2 - 64*a
^3*b^3)*cosh(d*x + c)^6 + a^6 + 8*a^5*b + 16*a^4*b^2 - 30*(a^6 + 8*a^5*b +
24*a^4*b^2 + 32*a^3*b^3)*cosh(d*x + c)^4 - 3*(a^6 + 4*a^5*b - 16*a^4*b^2 -
64*a^3*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*(3*(a^6 + 4*a^5*b)*cosh(d*
x + c)^11 + 5*(a^6 + 8*a^5*b + 16*a^4*b^2)*cosh(d*x + c)^9 - 2*(a^6 + 4*a^5
*b - 16*a^4*b^2 - 64*a^3*b^3)*cosh(d*x + c)^7 - 6*(a^6 + 8*a^5*b + 24*a^4*b
^2 + 32*a^3*b^3)*cosh(d*x + c)^5 - (a^6 + 4*a^5*b - 16*a^4*b^2 - 64*a^3*b^3
)*cosh(d*x + c)^3 + (a^6 + 8*a^5*b + 16*a^4*b^2)*cosh(d*x + c))*sinh(d*x +
c))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 8*(3*(a^6 + 4*a^
5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*x*cosh(d*x + c)^11 + 5*(a^6 + a^5*
b - 4*a^3*b^3 - 6*a^2*b^4 - 2*a*b^5 + (a^6 + 8*a^5*b + 22*a^4*b^2 + 28*a^3*
b^3 + 17*a^2*b^4 + 4*a*b^5)*d*x)*cosh(d*x + c)^9 + 2*(8*a^6 + 24*a^5*b + 16
*a^4*b^2 - 28*a^2*b^4 - 40*a*b^5 - 12*b^6 - (a^6 + 4*a^5*b - 10*a^4*b^2 - 6
0*a^3*b^3 - 95*a^2*b^4 - 64*a*b^5 - 16*b^6)*d*x)*cosh(d*x + c)^7 + 6*(3*a^6
 + 11*a^5*b + 16*a^4*b^2 + 12*a^3*b^3 + 20*a^2*b^4 + 22*a*b^5 + 6*b^6 - (a^
6 + 8*a^5*b + 30*a^4*b^2 + 60*a^3*b^3 + 65*a^2*b^4 + 36*a*b^5 + 8*b^6)*d*x)
*cosh(d*x + c)^5 + (8*a^6 + 24*a^5*b + 16*a^4*b^2 - 28*a^2*b^4 - 40*a*b^5 -
 12*b^6 - (a^6 + 4*a^5*b - 10*a^4*b^2 - 60*a^3*b^3 - 95*a^2*b^4 - 64*a*b^5
- 16*b^6)*d*x)*cosh(d*x + c)^3 + (a^6 + a^5*b - 4*a^3*b^3 - 6*a^2*b^4 - 2*a
*b^5 + (a^6 + 8*a^5*b + 22*a^4*b^2 + 28*a^3*b^3 + 17*a^2*b^4 + 4*a*b^5)*d*x
)*cosh(d*x + c))*sinh(d*x + c))/((a^9 + 4*a^8*b + 6*a^7*b^2 + 4*a^6*b^3 + a
^5*b^4)*d*cosh(d*x + c)^12 + 12*(a^9 + 4*a^8*b + 6*a^7*b^2 + 4*a^6*b^3 + a^
5*b^4)*d*cosh(d*x + c)*sinh(d*x + c)^11 + (a^9 + 4*a^8*b + 6*a^7*b^2 + 4*a^
6*b^3 + a^5*b^4)*d*sinh(d*x + c)^12 + 2*(a^9 + 8*a^8*b + 22*a^7*b^2 + 28*a^
6*b^3 + 17*a^5*b^4 + 4*a^4*b^5)*d*cosh(d*x + c)^10 + 2*(33*(a^9 + 4*a^8*b +

6*a^7*b^2 + 4*a^6*b^3 + a^5*b^4)*d*cosh(d*x + c)^2 + (a^9 + 8*a^8*b + 22*a
^7*b^2 + 28*a^6*b^3 + 17*a^5*b^4 + 4*a^4*b^5)*d)*sinh(d*x + c)^10 - (a^9 +
4*a^8*b - 10*a^7*b^2 - 60*a^6*b^3 - 95*a^5*b^4 - 64*a^4*b^5 - 16*a^3*b^6)*d
*cosh(d*x + c)^8 + 20*(11*(a^9 + 4*a^8*b + 6*a^7*b^2 + 4*a^6*b^3 + a^5*b^4)
*d*cosh(d*x + c)^3 + (a^9 + 8*a^8*b + 22*a^7*b^2 + 28*a^6*b^3 + 17*a^5*b^4
+ 4*a^4*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^9 + (495*(a^9 + 4*a^8*b + 6*a^7
*b^2 + 4*a^6*b^3 + a^5*b^4)*d*cosh(d*x + c)^4 + 90*(a^9 + 8*a^8*b + 22*a^7*
b^2 + 28*a^6*b^3 + 17*a^5*b^4 + 4*a^4*b^5)*d*cosh(d*x + c)^2 - (a^9 + 4*a^8
*b - 10*a^7*b^2 - 60*a^6*b^3 - 95*a^5*b^4 - 64*a^4*b^5 - 16*a^3*b^6)*d)*sin
h(d*x + c)^8 - 4*(a^9 + 8*a^8*b + 30*a^7*b^2 + 60*a^6*b^3 + 65*a^5*b^4 + 36
*a^4*b^5 + 8*a^3*b^6)*d*cosh(d*x + c)^6 + 8*(99*(a^9 + 4*a^8*b + 6*a^7*b^2
+ 4*a^6*b^3 + a^5*b^4)*d*cosh(d*x + c)^5 + 30*(a^9 + 8*a^8*b + 22*a^7*b^2 +
28*a^6*b^3 + 17*a^5*b^4 + 4*a^4*b^5)*d*cosh(d*x + c)^3 - (a^9 + 4*a^8*b -
10*a^7*b^2 - 60*a^6*b^3 - 95*a^5*b^4 - 64*a^4*b^5 - 16*a^3*b^6)*d*cosh(d*x
+ c))*sinh(d*x + c)^7 + 4*(231*(a^9 + 4*a^8*b + 6*a^7*b^2 + 4*a^6*b^3 + a^5
*b^4)*d*cosh(d*x + c)^6 + 105*(a^9 + 8*a^8*b + 22*a^7*b^2 + 28*a^6*b^3 + 17
*a^5*b^4 + 4*a^4*b^5)*d*cosh(d*x + c)^4 - 7*(a^9 + 4*a^8*b - 10*a^7*b^2 - 6
0*a^6*b^3 - 95*a^5*b^4 - 64*a^4*b^5 - 16*a^3*b^6)*d*cosh(d*x + c)^2 - (a^9
+ 8*a^8*b + 30*a^7*b^2 + 60*a^6*b^3 + 65*a^5*b^4 + 36*a^4*b^5 + 8*a^3*b^6)*
d)*sinh(d*x + c)^6 - (a^9 + 4*a^8*b - 10*a^7*b^2 - 60*a^6*b^3 - 95*a^5*b^4
- 64*a^4*b^5 - 16*a^3*b^6)*d*cosh(d*x + c)^4 + 8*(99*(a^9 + 4*a^8*b + 6*a^7
*b^2 + 4*a^6*b^3 + a^5*b^4)*d*cosh(d*x + c)^7 + 63*(a^9 + 8*a^8*b + 22*a^7*
b^2 + 28*a^6*b^3 + 17*a^5*b^4 + 4*a^4*b^5)*d*cosh(d*x + c)^5 - 7*(a^9 + 4*a
^8*b - 10*a^7*b^2 - 60*a^6*b^3 - 95*a^5*b^4 - 64*a^4*b^5 - 16*a^3*b^6)*d*co
sh(d*x + c)^3 - 3*(a^9 + 8*a^8*b + 30*a^7*b^2 + 60*a^6*b^3 + 65*a^5*b^4 + 3
6*a^4*b^5 + 8*a^3*b^6)*d*cosh(d*x + c))*sinh(d*x + c)^5 + (495*(a^9 + 4*a^8
*b + 6*a^7*b^2 + 4*a^6*b^3 + a^5*b^4)*d*cosh(d*x + c)^8 + 420*(a^9 + 8*a^8*
b + 22*a^7*b^2 + 28*a^6*b^3 + 17*a^5*b^4 + 4*a^4*b^5)*d*cosh(d*x + c)^6 - 7
0*(a^9 + 4*a^8*b - 10*a^7*b^2 - 60*a^6*b^3 - 95*a^5*b^4 - 64*a^4*b^5 - 16*a
^3*b^6)*d*cosh(d*x + c)^4 - 60*(a^9 + 8*a^8*b + 30*a^7*b^2 + 60*a^6*b^3 + 6
5*a^5*b^4 + 36*a^4*b^5 + 8*a^3*b^6)*d*cosh(d*x + c)^2 - (a^9 + 4*a^8*b - 10
*a^7*b^2 - 60*a^6*b^3 - 95*a^5*b^4 - 64*a^4*b^5 - 16*a^3*b^6)*d)*sinh(d*x +
c)^4 + 2*(a^9 + 8*a^8*b + 22*a^7*b^2 + 28*a^6*b^3 + 17*a^5*b^4 + 4*a^4*b^5
)*d*cosh(d*x + c)^2 + 4*(55*(a^9 + 4*a^8*b + 6*a^7*b^2 + 4*a^6*b^3 + a^5*b^
4)*d*cosh(d*x + c)^9 + 60*(a^9 + 8*a^8*b + 22*a^7*b^2 + 28*a^6*b^3 + 17*a^5
*b^4 + 4*a^4*b^5)*d*cosh(d*x + c)^7 - 14*(a^9 + 4*a^8*b - 10*a^7*b^2 - 60*a
^6*b^3 - 95*a^5*b^4 - 64*a^4*b^5 - 16*a^3*b^6)*d*cosh(d*x + c)^5 - 20*(a^9
+ 8*a^8*b + 30*a^7*b^2 + 60*a^6*b^3 + 65*a^5*b^4 + 36*a^4*b^5 + 8*a^3*b^6)*
d*cosh(d*x + c)^3 - (a^9 + 4*a^8*b - 10*a^7*b^2 - 60*a^6*b^3 - 95*a^5*b^4 -
64*a^4*b^5 - 16*a^3*b^6)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(33*(a^9 + 4
*a^8*b + 6*a^7*b^2 + 4*a^6*b^3 + a^5*b^4)*d*cosh(d*x + c)^10 + 45*(a^9 + 8*
a^8*b + 22*a^7*b^2 + 28*a^6*b^3 + 17*a^5*b^4 + 4*a^4*b^5)*d*cosh(d*x + c)^8
- 14*(a^9 + 4*a^8*b - 10*a^7*b^2 - 60*a^6*b^3 - 95*a^5*b^4 - 64*a^4*b^5 -
16*a^3*b^6)*d*cosh(d*x + c)^6 - 30*(a^9 + 8*a^8*b + 30*a^7*b^2 + 60*a^6*b^3
+ 65*a^5*b^4 + 36*a^4*b^5 + 8*a^3*b^6)*d*cosh(d*x + c)^4 - 3*(a^9 + 4*a^8*

```
b - 10*a^7*b^2 - 60*a^6*b^3 - 95*a^5*b^4 - 64*a^4*b^5 - 16*a^3*b^6)*d*cosh(
d*x + c)^2 + (a^9 + 8*a^8*b + 22*a^7*b^2 + 28*a^6*b^3 + 17*a^5*b^4 + 4*a^4*
b^5)*d)*sinh(d*x + c)^2 + (a^9 + 4*a^8*b + 6*a^7*b^2 + 4*a^6*b^3 + a^5*b^4)
*d + 4*(3*(a^9 + 4*a^8*b + 6*a^7*b^2 + 4*a^6*b^3 + a^5*b^4)*d*cosh(d*x + c)
^11 + 5*(a^9 + 8*a^8*b + 22*a^7*b^2 + 28*a^6*b^3 + 17*a^5*b^4 + 4*a^4*b^5)*
d*cosh(d*x + c)^9 - 2*(a^9 + 4*a^8*b - 10*a^7*b^2 - 60*a^6*b^3 - 95*a^5*b^4
 - 64*a^4*b^5 - 16*a^3*b^6)*d*cosh(d*x + c)^7 - 6*(a^9 + 8*a^8*b + 30*a^7*b
^2 + 60*a^6*b^3 + 65*a^5*b^4 + 36*a^4*b^5 + 8*a^3*b^6)*d*cosh(d*x + c)^5 -
(a^9 + 4*a^8*b - 10*a^7*b^2 - 60*a^6*b^3 - 95*a^5*b^4 - 64*a^4*b^5 - 16*a^3
*b^6)*d*cosh(d*x + c)^3 + (a^9 + 8*a^8*b + 22*a^7*b^2 + 28*a^6*b^3 + 17*a^5
*b^4 + 4*a^4*b^5)*d*cosh(d*x + c))*sinh(d*x + c))
```

giac [B] time = 2.47, size = 766, normalized size = 5.04

$$\frac{\left(6\,a^{2}b^{2}+4\,ab^{3}+b^{4}\right)\log\left(ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+a\right)}{a^{7}+4\,a^{6}b+6\,a^{5}b^{2}+4\,a^{4}b^{3}+a^{3}b^{4}}+\frac{2\left(ae^{(2\,c)}+4\,be^{(2\,c)}\right)\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)}{a^{4}e^{(2\,c)}+4\,a^{3}be^{(2\,c)}+6\,a^{2}b^{2}e^{(2\,c)}+4\,ab^{3}e^{(2\,c)}+b^{4}e^{(2\,c)}}-\frac{2\,dx}{a^{3}}-\frac{a^{5}e^{(12\,dx+12\,c)}+3\,\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/2*((6*a^2*b^2 + 4*a*b^3 + b^4)*log(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c
) + 4*b*e^(2*d*x + 2*c) + a)/(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b
^4) + 2*(a*e^(2*c) + 4*b*e^(2*c))*log(abs(e^(2*d*x + 2*c) - 1))/(a^4*e^(2*c
) + 4*a^3*b*e^(2*c) + 6*a^2*b^2*e^(2*c) + 4*a*b^3*e^(2*c) + b^4*e^(2*c)) -
2*d*x/a^3 - (a^5*e^(12*d*x + 12*c) + 3*a^4*b*e^(12*d*x + 12*c) + 3*a^3*b^2*
e^(12*d*x + 12*c) + a^2*b^3*e^(12*d*x + 12*c) + 6*a^5*e^(10*d*x + 10*c) + 1
4*a^4*b*e^(10*d*x + 10*c) + 30*a^3*b^2*e^(10*d*x + 10*c) + 10*a^2*b^3*e^(10
*d*x + 10*c) + 15*a^5*e^(8*d*x + 8*c) + 29*a^4*b*e^(8*d*x + 8*c) + 13*a^3*b
^2*e^(8*d*x + 8*c) + 47*a^2*b^3*e^(8*d*x + 8*c) - 8*a*b^4*e^(8*d*x + 8*c) -
 8*b^5*e^(8*d*x + 8*c) + 20*a^5*e^(6*d*x + 6*c) + 36*a^4*b*e^(6*d*x + 6*c)
- 28*a^3*b^2*e^(6*d*x + 6*c) - 116*a^2*b^3*e^(6*d*x + 6*c) + 16*a*b^4*e^(6*
d*x + 6*c) + 16*b^5*e^(6*d*x + 6*c) + 15*a^5*e^(4*d*x + 4*c) + 29*a^4*b*e^(
4*d*x + 4*c) + 13*a^3*b^2*e^(4*d*x + 4*c) + 47*a^2*b^3*e^(4*d*x + 4*c) - 8*
a*b^4*e^(4*d*x + 4*c) - 8*b^5*e^(4*d*x + 4*c) + 6*a^5*e^(2*d*x + 2*c) + 14*
a^4*b*e^(2*d*x + 2*c) + 30*a^3*b^2*e^(2*d*x + 2*c) + 10*a^2*b^3*e^(2*d*x +
2*c) + a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)/((a^6 + 3*a^5*b + 3*a^4*b^2 + a
^3*b^3)*(a*e^(6*d*x + 6*c) + a*e^(4*d*x + 4*c) + 4*b*e^(4*d*x + 4*c) - a*e^
(2*d*x + 2*c) - 4*b*e^(2*d*x + 2*c) - a)^2))/d
```

maple [B] time = 0.57, size = 1128, normalized size = 7.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] -1/8/d*tanh(1/2*d*x+1/2*c)^2/(a^3+3*a^2*b+3*a*b^2+b^3)-1/d/a^3*ln(tanh(1/2*
d*x+1/2*c)-1)-1/d/a^3*ln(tanh(1/2*d*x+1/2*c)+1)-8/d*b^3/(a+b)^4/(tanh(1/2*d
*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*
d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^6-10/d*b^4/(a+b)^4/(tanh(1/2*d*x+
1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x
+1/2*c)^2*b+a+b)^2/a*tanh(1/2*d*x+1/2*c)^6-2/d*b^5/(a+b)^4/a^2/(tanh(1/2*d*
x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d
*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^6-16/d*b^3/(a+b)^4/(tanh(1/2*d*x+1
/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+
1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^4+8/d*b^4/(a+b)^4/(tanh(1/2*d*x+1/2*c
)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*
c)^2*b+a+b)^2/a*tanh(1/2*d*x+1/2*c)^4+4/d*b^5/(a+b)^4/a^2/(tanh(1/2*d*x+1/2
*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/
2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^4-8/d*b^3/(a+b)^4/(tanh(1/2*d*x+1/2*c)^
4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)
^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^2-10/d*b^4/(a+b)^4/(tanh(1/2*d*x+1/2*c)^4*a
+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*
b+a+b)^2/a*tanh(1/2*d*x+1/2*c)^2-2/d*b^5/(a+b)^4/a^2/(tanh(1/2*d*x+1/2*c)^4
*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^
2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^2+3/d*b^2/(a+b)^4/a*ln(tanh(1/2*d*x+1/2*c)^4
*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^
2*b+a+b)+2/d*b^3/(a+b)^4/a^2*ln(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*
c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)+1/2/d*b^4/(a+
b)^4/a^3*ln(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+
1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)-1/8/d/(a+b)^3/tanh(1/2*d*x+1/2*c)
^2+1/d/(a+b)^4*ln(tanh(1/2*d*x+1/2*c))*a+4/d/(a+b)^4*ln(tanh(1/2*d*x+1/2*c)
)*b
```

maxima [B] time = 0.54, size = 692, normalized size = 4.55

$$\frac{\left(6\,a^2b^2+4\,ab^3+b^4\right)\log\left(2\,(a+2\,b)e^{(-2\,dx-2\,c)}+ae^{(-4\,dx-4\,c)}+a\right)}{2\left(a^7+4\,a^6b+6\,a^5b^2+4\,a^4b^3+a^3b^4\right)d}+\frac{(a+4\,b)\log\left(e^{(-dx-c)}+1\right)}{\left(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4\right)d}+\frac{(a+}{\left(a^4+4\,a^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(6*a^2*b^2 + 4*a*b^3 + b^4)*log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*
d*x - 4*c) + a)/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d) + (a
+ 4*b)*log(e^(-d*x - c) + 1)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d
) + (a + 4*b)*log(e^(-d*x - c) - 1)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 +
 b^4)*d) - 2*((a^5 - 4*a^2*b^3 - 2*a*b^4)*e^(-2*d*x - 2*c) + 2*(2*a^5 + 4*a
^4*b - 7*a*b^4 - 3*b^5)*e^(-4*d*x - 4*c) + 2*(3*a^5 + 8*a^4*b + 8*a^3*b^2 +
 4*a^2*b^3 + 16*a*b^4 + 6*b^5)*e^(-6*d*x - 6*c) + 2*(2*a^5 + 4*a^4*b - 7*a*
```

```
b^4 - 3*b^5)*e^(-8*d*x - 8*c) + (a^5 - 4*a^2*b^3 - 2*a*b^4)*e^(-10*d*x - 10
*c))/((a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3 + 2*(a^8 + 7*a^7*b + 15*a^6*b^2
+ 13*a^5*b^3 + 4*a^4*b^4)*e^(-2*d*x - 2*c) - (a^8 + 3*a^7*b - 13*a^6*b^2 -
47*a^5*b^3 - 48*a^4*b^4 - 16*a^3*b^5)*e^(-4*d*x - 4*c) - 4*(a^8 + 7*a^7*b +
 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*e^(-6*d*x - 6*c) - (a^8
+ 3*a^7*b - 13*a^6*b^2 - 47*a^5*b^3 - 48*a^4*b^4 - 16*a^3*b^5)*e^(-8*d*x -
8*c) + 2*(a^8 + 7*a^7*b + 15*a^6*b^2 + 13*a^5*b^3 + 4*a^4*b^4)*e^(-10*d*x -
 10*c) + (a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*e^(-12*d*x - 12*c))*d) + (d*
x + c)/(a^3*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh\left(c+d\,x\right)^6\,\coth\left(c+d\,x\right)^3}{{\left(a\,\cosh\left(c+d\,x\right)^2+b\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^3/(a + b/cosh(c + d*x)^2)^3,x)
```

```
[Out] int((cosh(c + d*x)^6*coth(c + d*x)^3)/(b + a*cosh(c + d*x)^2)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^{3}\left(c+dx\right)}{\left(a+b\operatorname{sech}^{2}\left(c+dx\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**3/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Integral(coth(c + d*x)**3/(a + b*sech(c + d*x)**2)**3, x)
```

3.168 $\int \frac{\coth^4(c+dx)}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=232

$$\frac{x}{a^3}-\frac{\left(8a^2-39ab-12b^2\right)\coth^3(c+dx)}{24a^2d(a+b)^3}-\frac{b(11a+4b)\coth^3(c+dx)}{8a^2d(a+b)^2\left(a-b\tanh^2(c+dx)+b\right)}-\frac{b^{5/2}\left(63a^2+36ab+8b^2\right)\tanh^{-1}}{8a^3d(a+b)^{9/2}}$$

```
[Out] x/a^3-1/8*b^(5/2)*(63*a^2+36*a*b+8*b^2)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(
1/2))/a^3/(a+b)^(9/2)/d-1/8*(8*a^3+32*a^2*b-15*a*b^2-4*b^3)*coth(d*x+c)/a^2
/(a+b)^4/d-1/24*(8*a^2-39*a*b-12*b^2)*coth(d*x+c)^3/a^2/(a+b)^3/d-1/4*b*cot
h(d*x+c)^3/a/(a+b)/d/(a+b-b*tanh(d*x+c)^2)^2-1/8*b*(11*a+4*b)*coth(d*x+c)^3
/a^2/(a+b)^2/d/(a+b-b*tanh(d*x+c)^2)
```

Rubi [A] time = 0.51, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {4141, 1975, 472, 579, 583, 522, 206, 208}

$$-\frac{b^{5/2}\left(63a^2+36ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^3d(a+b)^{9/2}}-\frac{\left(8a^2-39ab-12b^2\right)\coth^3(c+dx)}{24a^2d(a+b)^3}-\frac{\left(32a^2b+8a^3-15ab^2-4b\right.}{8a^2d(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^4/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] x/a^3 - (b^(5/2)*(63*a^2 + 36*a*b + 8*b^2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/
Sqrt[a + b]])/(8*a^3*(a + b)^(9/2)*d) - ((8*a^3 + 32*a^2*b - 15*a*b^2 - 4*b
^3)*Coth[c + d*x])/(8*a^2*(a + b)^4*d) - ((8*a^2 - 39*a*b - 12*b^2)*Coth[c
+ d*x]^3)/(24*a^2*(a + b)^3*d) - (b*Coth[c + d*x]^3)/(4*a*(a + b)*d*(a + b
- b*Tanh[c + d*x]^2)^2) - (b*(11*a + 4*b)*Coth[c + d*x]^3)/(8*a^2*(a + b)^2
*d*(a + b - b*Tanh[c + d*x]^2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\coth^4(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^4(1-x^2)\left(a+b(1-x^2)\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^4(1-x^2)\left(a+b-bx^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{b\coth^3(c+dx)}{4a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{-4a+3b-7bx^2}{x^4(1-x^2)\left(a+b-bx^2\right)^2}\,dx, x, \tanh(c\right.}{4a(a+b)d}$$

$$= -\frac{b\coth^3(c+dx)}{4a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2} - \frac{b(11a+4b)\coth^3(c+dx)}{8a^2(a+b)^2d\left(a+b-b\tanh^2(c+dx)\right)}$$

$$= -\frac{\left(8a^2-39ab-12b^2\right)\coth^3(c+dx)}{24a^2(a+b)^3d} - \frac{b\coth^3(c+dx)}{4a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2} - \frac{}{8a}$$

$$= -\frac{\left(8a^3+32a^2b-15ab^2-4b^3\right)\coth(c+dx)}{8a^2(a+b)^4d} - \frac{\left(8a^2-39ab-12b^2\right)\coth^3(c+dx)}{24a^2(a+b)^3d}$$

$$= -\frac{\left(8a^3+32a^2b-15ab^2-4b^3\right)\coth(c+dx)}{8a^2(a+b)^4d} - \frac{\left(8a^2-39ab-12b^2\right)\coth^3(c+dx)}{24a^2(a+b)^3d}$$

$$= \frac{x}{a^3} - \frac{b^{5/2}\left(63a^2+36ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^3(a+b)^{9/2}d} - \frac{\left(8a^3+32a^2b-15ab^2-4b\right.}{8a^2(a+b)^4}$$

Mathematica [C] time = 7.74, size = 3334, normalized size = 14.37

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Coth[c + d*x]^4/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((63*a^2 + 36*a*b + 8*b^2)*(a + 2*b + a*Cosh[2*c + 2*d*x])^3*Sech[c + d*x]^
6*(((I/64)*b^3*ArcTan[Sech[d*x]*(((-1/2*I)*Cosh[2*c])/(Sqrt[a + b]*Sqrt[b*C
osh[4*c] - b*Sinh[4*c]]) + ((I/2)*Sinh[2*c])/(Sqrt[a + b]*Sqrt[b*Cosh[4*c]
- b*Sinh[4*c]]))*(-(a*Sinh[d*x]) - 2*b*Sinh[d*x] + a*Sinh[2*c + d*x])]*Cosh
[2*c])/(a^3*Sqrt[a + b]*d*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]]) - ((I/64)*b^3*Ar
cTan[Sech[d*x]*(((-1/2*I)*Cosh[2*c])/(Sqrt[a + b]*Sqrt[b*Cosh[4*c] - b*Sinh
[4*c]]) + ((I/2)*Sinh[2*c])/(Sqrt[a + b]*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]]))*
(-(a*Sinh[d*x]) - 2*b*Sinh[d*x] + a*Sinh[2*c + d*x])]*Sinh[2*c])/(a^3*Sqrt[
a + b]*d*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]])))/((a + b)^4*(a + b*Sech[c + d*x]
^2)^3) + ((a + 2*b + a*Cosh[2*c + 2*d*x])*Csch[c]*Csch[c + d*x]^3*Sech[2*c]
*Sech[c + d*x]^6*(-36*a^6*d*x*Cosh[d*x] - 336*a^5*b*d*x*Cosh[d*x] - 1560*a^
4*b^2*d*x*Cosh[d*x] - 3600*a^3*b^3*d*x*Cosh[d*x] - 4260*a^2*b^4*d*x*Cosh[d*
x] - 2496*a*b^5*d*x*Cosh[d*x] - 576*b^6*d*x*Cosh[d*x] + 36*a^6*d*x*Cosh[3*d
*x] + 240*a^5*b*d*x*Cosh[3*d*x] + 408*a^4*b^2*d*x*Cosh[3*d*x] - 48*a^3*b^3*
d*x*Cosh[3*d*x] - 732*a^2*b^4*d*x*Cosh[3*d*x] - 672*a*b^5*d*x*Cosh[3*d*x] -
 192*b^6*d*x*Cosh[3*d*x] + 36*a^6*d*x*Cosh[2*c - d*x] + 336*a^5*b*d*x*Cosh[
2*c - d*x] + 1560*a^4*b^2*d*x*Cosh[2*c - d*x] + 3600*a^3*b^3*d*x*Cosh[2*c -
 d*x] + 4260*a^2*b^4*d*x*Cosh[2*c - d*x] + 2496*a*b^5*d*x*Cosh[2*c - d*x] +
 576*b^6*d*x*Cosh[2*c - d*x] + 36*a^6*d*x*Cosh[2*c + d*x] + 336*a^5*b*d*x*C
osh[2*c + d*x] + 1560*a^4*b^2*d*x*Cosh[2*c + d*x] + 3600*a^3*b^3*d*x*Cosh[2
*c + d*x] + 4260*a^2*b^4*d*x*Cosh[2*c + d*x] + 2496*a*b^5*d*x*Cosh[2*c + d*
x] + 576*b^6*d*x*Cosh[2*c + d*x] - 36*a^6*d*x*Cosh[4*c + d*x] - 336*a^5*b*d
*x*Cosh[4*c + d*x] - 1560*a^4*b^2*d*x*Cosh[4*c + d*x] - 3600*a^3*b^3*d*x*Co
sh[4*c + d*x] - 4260*a^2*b^4*d*x*Cosh[4*c + d*x] - 2496*a*b^5*d*x*Cosh[4*c
+ d*x] - 576*b^6*d*x*Cosh[4*c + d*x] - 36*a^6*d*x*Cosh[2*c + 3*d*x] - 240*a
^5*b*d*x*Cosh[2*c + 3*d*x] - 408*a^4*b^2*d*x*Cosh[2*c + 3*d*x] + 48*a^3*b^3
*d*x*Cosh[2*c + 3*d*x] + 732*a^2*b^4*d*x*Cosh[2*c + 3*d*x] + 672*a*b^5*d*x*
Cosh[2*c + 3*d*x] + 192*b^6*d*x*Cosh[2*c + 3*d*x] + 36*a^6*d*x*Cosh[4*c + 3
*d*x] + 240*a^5*b*d*x*Cosh[4*c + 3*d*x] + 408*a^4*b^2*d*x*Cosh[4*c + 3*d*x]
 - 48*a^3*b^3*d*x*Cosh[4*c + 3*d*x] - 732*a^2*b^4*d*x*Cosh[4*c + 3*d*x] - 6
72*a*b^5*d*x*Cosh[4*c + 3*d*x] - 192*b^6*d*x*Cosh[4*c + 3*d*x] - 36*a^6*d*x
*Cosh[6*c + 3*d*x] - 240*a^5*b*d*x*Cosh[6*c + 3*d*x] - 408*a^4*b^2*d*x*Cosh
[6*c + 3*d*x] + 48*a^3*b^3*d*x*Cosh[6*c + 3*d*x] + 732*a^2*b^4*d*x*Cosh[6*c
 + 3*d*x] + 672*a*b^5*d*x*Cosh[6*c + 3*d*x] + 192*b^6*d*x*Cosh[6*c + 3*d*x]
 - 12*a^6*d*x*Cosh[2*c + 5*d*x] - 144*a^5*b*d*x*Cosh[2*c + 5*d*x] - 456*a^4
*b^2*d*x*Cosh[2*c + 5*d*x] - 624*a^3*b^3*d*x*Cosh[2*c + 5*d*x] - 396*a^2*b^
4*d*x*Cosh[2*c + 5*d*x] - 96*a*b^5*d*x*Cosh[2*c + 5*d*x] + 12*a^6*d*x*Cosh[
```

4*c + 5*d*x] + 144*a^5*b*d*x*Cosh[4*c + 5*d*x] + 456*a^4*b^2*d*x*Cosh[4*c + 5*d*x] + 624*a^3*b^3*d*x*Cosh[4*c + 5*d*x] + 396*a^2*b^4*d*x*Cosh[4*c + 5*d*x] + 96*a*b^5*d*x*Cosh[4*c + 5*d*x] - 12*a^6*d*x*Cosh[6*c + 5*d*x] - 144*a^5*b*d*x*Cosh[6*c + 5*d*x] - 456*a^4*b^2*d*x*Cosh[6*c + 5*d*x] - 624*a^3*b^3*d*x*Cosh[6*c + 5*d*x] - 396*a^2*b^4*d*x*Cosh[6*c + 5*d*x] - 96*a*b^5*d*x*Cosh[6*c + 5*d*x] + 12*a^6*d*x*Cosh[8*c + 5*d*x] + 144*a^5*b*d*x*Cosh[8*c + 5*d*x] + 456*a^4*b^2*d*x*Cosh[8*c + 5*d*x] + 624*a^3*b^3*d*x*Cosh[8*c + 5*d*x] + 396*a^2*b^4*d*x*Cosh[8*c + 5*d*x] + 96*a*b^5*d*x*Cosh[8*c + 5*d*x] - 12*a^6*d*x*Cosh[4*c + 7*d*x] - 48*a^5*b*d*x*Cosh[4*c + 7*d*x] - 72*a^4*b^2*d*x*Cosh[4*c + 7*d*x] - 48*a^3*b^3*d*x*Cosh[4*c + 7*d*x] - 12*a^2*b^4*d*x*Cosh[4*c + 7*d*x] + 12*a^6*d*x*Cosh[6*c + 7*d*x] + 48*a^5*b*d*x*Cosh[6*c + 7*d*x] + 72*a^4*b^2*d*x*Cosh[6*c + 7*d*x] + 48*a^3*b^3*d*x*Cosh[6*c + 7*d*x] + 12*a^2*b^4*d*x*Cosh[6*c + 7*d*x] - 12*a^6*d*x*Cosh[8*c + 7*d*x] - 48*a^5*b*d*x*Cosh[8*c + 7*d*x] - 72*a^4*b^2*d*x*Cosh[8*c + 7*d*x] - 48*a^3*b^3*d*x*Cosh[8*c + 7*d*x] - 12*a^2*b^4*d*x*Cosh[8*c + 7*d*x] + 12*a^6*d*x*Cosh[10*c + 7*d*x] + 48*a^5*b*d*x*Cosh[10*c + 7*d*x] + 72*a^4*b^2*d*x*Cosh[10*c + 7*d*x] + 48*a^3*b^3*d*x*Cosh[10*c + 7*d*x] + 12*a^2*b^4*d*x*Cosh[10*c + 7*d*x] - 128*a^6*Sinh[d*x] - 440*a^5*b*Sinh[d*x] - 1152*a^4*b^2*Sinh[d*x] - 1920*a^3*b^3*Sinh[d*x] + 228*a^2*b^4*Sinh[d*x] + 1320*a*b^5*Sinh[d*x] + 432*b^6*Sinh[d*x] + 48*a^6*Sinh[3*d*x] + 104*a^5*b*Sinh[3*d*x] + 640*a^4*b^2*Sinh[3*d*x] + 1511*a^3*b^3*Sinh[3*d*x] - 528*a^2*b^4*Sinh[3*d*x] + 264*a*b^5*Sinh[3*d*x] + 144*b^6*Sinh[3*d*x] - 32*a^6*Sinh[2*c - d*x] + 384*a^5*b*Sinh[2*c - d*x] + 2048*a^4*b^2*Sinh[2*c - d*x] + 3072*a^3*b^3*Sinh[2*c - d*x] + 228*a^2*b^4*Sinh[2*c - d*x] + 1320*a*b^5*Sinh[2*c - d*x] + 432*b^6*Sinh[2*c - d*x] + 32*a^6*Sinh[2*c + d*x] - 384*a^5*b*Sinh[2*c + d*x] - 2048*a^4*b^2*Sinh[2*c + d*x] - 2919*a^3*b^3*Sinh[2*c + d*x] + 642*a^2*b^4*Sinh[2*c + d*x] + 1416*a*b^5*Sinh[2*c + d*x] + 432*b^6*Sinh[2*c + d*x] - 128*a^6*Sinh[4*c + d*x] - 440*a^5*b*Sinh[4*c + d*x] - 1152*a^4*b^2*Sinh[4*c + d*x] - 2073*a^3*b^3*Sinh[4*c + d*x] - 642*a^2*b^4*Sinh[4*c + d*x] - 1416*a*b^5*Sinh[4*c + d*x] - 432*b^6*Sinh[4*c + d*x] - 144*a^6*Sinh[2*c + 3*d*x] - 672*a^5*b*Sinh[2*c + 3*d*x] - 960*a^4*b^2*Sinh[2*c + 3*d*x] + 153*a^3*b^3*Sinh[2*c + 3*d*x] + 528*a^2*b^4*Sinh[2*c + 3*d*x] - 264*a*b^5*Sinh[2*c + 3*d*x] - 144*b^6*Sinh[2*c + 3*d*x] + 48*a^6*Sinh[4*c + 3*d*x] + 104*a^5*b*Sinh[4*c + 3*d*x] + 640*a^4*b^2*Sinh[4*c + 3*d*x] + 1664*a^3*b^3*Sinh[4*c + 3*d*x] - 66*a^2*b^4*Sinh[4*c + 3*d*x] - 408*a*b^5*Sinh[4*c + 3*d*x] - 144*b^6*Sinh[4*c + 3*d*x] - 144*a^6*Sinh[6*c + 3*d*x] - 672*a^5*b*Sinh[6*c + 3*d*x] - 960*a^4*b^2*Sinh[6*c + 3*d*x] + 66*a^2*b^4*Sinh[6*c + 3*d*x] + 408*a*b^5*Sinh[6*c + 3*d*x] + 144*b^6*Sinh[6*c + 3*d*x] + 80*a^6*Sinh[2*c + 5*d*x] + 480*a^5*b*Sinh[2*c + 5*d*x] + 832*a^4*b^2*Sinh[2*c + 5*d*x] + 294*a^2*b^4*Sinh[2*c + 5*d*x] + 96*a*b^5*Sinh[2*c + 5*d*x] - 48*a^6*Sinh[4*c + 5*d*x] - 120*a^5*b*Sinh[4*c + 5*d*x] - 294*a^2*b^4*Sinh[4*c + 5*d*x] - 96*a*b^5*Sinh[4*c + 5*d*x] + 80*a^6*Sinh[6*c + 5*d*x] + 480*a^5*b*Sinh[6*c + 5*d*x] + 832*a^4*b^2*Sinh[6*c + 5*d*x] - 51*a^3*b^3*Sinh[6*c + 5*d*x] - 132*a^2*b^4*Sinh[6*c + 5*d*x] - 48*a*b^5*Sinh[6*c + 5*d*x] - 48*a^6*Sinh[8*c + 5*d*x] - 120*a^5*b*Sinh[8*c + 5*d*x] + 51*a^3*b^3*Sinh[8*c + 5*d*x] + 132*a^2*b^4*Sinh[8*c +

```
5*d*x] + 48*a*b^5*Sinh[8*c + 5*d*x] + 32*a^6*Sinh[4*c + 7*d*x] + 104*a^5*b*
Sinh[4*c + 7*d*x] + 51*a^3*b^3*Sinh[4*c + 7*d*x] + 18*a^2*b^4*Sinh[4*c + 7*
d*x] - 51*a^3*b^3*Sinh[6*c + 7*d*x] - 18*a^2*b^4*Sinh[6*c + 7*d*x] + 32*a^6
*Sinh[8*c + 7*d*x] + 104*a^5*b*Sinh[8*c + 7*d*x]))/(6144*a^3*(a + b)^4*d*(a
 + b*Sech[c + d*x]^2)^3)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 2.59, size = 482, normalized size = 2.08

$$-\frac{\frac{3\left(63\,a^2b^3e^{(2\,c)}+36\,ab^4e^{(2\,c)}+8\,b^5e^{(2\,c)}\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)e^{(-2\,c)}}{\left(a^7+4\,a^6b+6\,a^5b^2+4\,a^4b^3+a^3b^4\right)\sqrt{-ab-b^2}}-\frac{24\,dx}{a^3}-\frac{6\left(17\,a^3b^3e^{(6\,dx+6\,c)}+44\,a^2b^4e^{(6\,dx+6\,c)}+16\,ab^5e^{(6\,dx+6\,c)}+51\,a^3b^3\cdots\right.}{\left(a\cdots\right.}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/24*(3*(63*a^2*b^3*e^(2*c) + 36*a*b^4*e^(2*c) + 8*b^5*e^(2*c))*arctan(1/2
*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))*e^(-2*c)/((a^7 + 4*a^6*b +
 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*sqrt(-a*b - b^2)) - 24*d*x/a^3 - 6*(17*a^
3*b^3*e^(6*d*x + 6*c) + 44*a^2*b^4*e^(6*d*x + 6*c) + 16*a*b^5*e^(6*d*x + 6*
c) + 51*a^3*b^3*e^(4*d*x + 4*c) + 154*a^2*b^4*e^(4*d*x + 4*c) + 184*a*b^5*e
^(4*d*x + 4*c) + 48*b^6*e^(4*d*x + 4*c) + 51*a^3*b^3*e^(2*d*x + 2*c) + 116*
a^2*b^4*e^(2*d*x + 2*c) + 32*a*b^5*e^(2*d*x + 2*c) + 17*a^3*b^3 + 6*a^2*b^4
)/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*(a*e^(4*d*x + 4*c) + 2
*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) + a)^2) + 16*(6*a*e^(4*d*x + 4*c)
+ 15*b*e^(4*d*x + 4*c) - 6*a*e^(2*d*x + 2*c) - 24*b*e^(2*d*x + 2*c) + 4*a +
 13*b)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*(e^(2*d*x + 2*c) - 1)^3
))/d
```

maple [B] time = 0.57, size = 1610, normalized size = 6.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x)
```

```
[Out] 1/d/a^3*ln(tanh(1/2*d*x+1/2*c)+1)-1/d/a^3*ln(tanh(1/2*d*x+1/2*c)-1)-5/8/d/(
a^3+3*a^2*b+3*a*b^2+b^3)/(a+b)*a*tanh(1/2*d*x+1/2*c)-17/8/d/(a^3+3*a^2*b+3*
a*b^2+b^3)/(a+b)*tanh(1/2*d*x+1/2*c)*b-1/24/d/(a+b)^3/tanh(1/2*d*x+1/2*c)^3
-5/8/d/(a+b)^4/tanh(1/2*d*x+1/2*c)*a-17/8/d/(a+b)^4/tanh(1/2*d*x+1/2*c)*b-1
/2/d*b^(9/2)/(a+b)^(9/2)/a^3*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)
*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))+63/16/d*b^(5/2)/(a+b)^(9/2)/a*ln((a+b)^(1
/2)*tanh(1/2*d*x+1/2*c)^2-2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))-63/16/
d*b^(5/2)/(a+b)^(9/2)/a*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh
(1/2*d*x+1/2*c)+(a+b)^(1/2))+9/4/d*b^(7/2)/(a+b)^(9/2)/a^2*ln((a+b)^(1/2)*t
anh(1/2*d*x+1/2*c)^2-2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))-9/4/d*b^(7/
2)/(a+b)^(9/2)/a^2*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*
d*x+1/2*c)+(a+b)^(1/2))+1/2/d*b^(9/2)/(a+b)^(9/2)/a^3*ln((a+b)^(1/2)*tanh(1
/2*d*x+1/2*c)^2-2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))-17/4/d*b^3/(a+b)
^4/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2
*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)-17/4/d*b^3/(a+b)^4/
(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-
2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^7-51/4/d*b^3/(a+b)^4/(
tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2
*tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^5-51/4/d*b^3/(a+b)^4/(t
anh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*
tanh(1/2*d*x+1/2*c)^2*b+a+b)^2*tanh(1/2*d*x+1/2*c)^3-1/24/d/(a^3+3*a^2*b+3*
a*b^2+b^3)/(a+b)*a*tanh(1/2*d*x+1/2*c)^3-1/24/d/(a^3+3*a^2*b+3*a*b^2+b^3)/(
a+b)*tanh(1/2*d*x+1/2*c)^3*b-1/d*b^5/(a+b)^4/a^2/(tanh(1/2*d*x+1/2*c)^4*a+b
*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+
a+b)^2*tanh(1/2*d*x+1/2*c)-3/4/d*b^4/(a+b)^4/a/(tanh(1/2*d*x+1/2*c)^4*a+b*t
anh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+
b)^2*tanh(1/2*d*x+1/2*c)^3-3/4/d*b^4/(a+b)^4/a/(tanh(1/2*d*x+1/2*c)^4*a+b*t
anh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+
b)^2*tanh(1/2*d*x+1/2*c)^5-21/4/d*b^4/(a+b)^4/a/(tanh(1/2*d*x+1/2*c)^4*a+b*
tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a
+b)^2*tanh(1/2*d*x+1/2*c)^7-1/d*b^5/(a+b)^4/a^2/(tanh(1/2*d*x+1/2*c)^4*a+b*
tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a
+b)^2*tanh(1/2*d*x+1/2*c)^7+1/d*b^5/(a+b)^4/a^2/(tanh(1/2*d*x+1/2*c)^4*a+b*
tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a
+b)^2*tanh(1/2*d*x+1/2*c)^5+1/d*b^5/(a+b)^4/a^2/(tanh(1/2*d*x+1/2*c)^4*a+b*
tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a
+b)^2*tanh(1/2*d*x+1/2*c)^3-21/4/d*b^4/(a+b)^4/a/(tanh(1/2*d*x+1/2*c)^4*a+b
*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+
a+b)^2*tanh(1/2*d*x+1/2*c)
```

maxima [B] time = 1.84, size = 4920, normalized size = 21.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")

[Out] 1/8*(3*a^3*b + 12*a^2*b^2 + 8*a*b^3 + 2*b^4)*log(a*e^(4*d*x + 4*c) + 2*(a +
 2*b)*e^(2*d*x + 2*c) + a)/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^
4)*d) - 3/4*b*log(a*e^(4*d*x + 4*c) + 2*(a + 2*b)*e^(2*d*x + 2*c) + a)/((a^
4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d) - 1/8*(3*a^3*b + 12*a^2*b^2 + 8
*a*b^3 + 2*b^4)*log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/
((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d) + 3/4*b*log(2*(a + 2*
b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/((a^4 + 4*a^3*b + 6*a^2*b^2 +
 4*a*b^3 + b^4)*d) + 1/4*(2*a + 5*b)*log(e^(2*d*x + 2*c) - 1)/((a^4 + 4*a^3
*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d) + 3/2*b*log(e^(2*d*x + 2*c) - 1)/((a^4 +
 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d) - 1/4*(2*a + 5*b)*log(e^(-2*d*x -
2*c) - 1)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d) - 3/2*b*log(e^(-2
*d*x - 2*c) - 1)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d) - 1/256*(1
5*a^4*b + 260*a^3*b^2 + 504*a^2*b^3 + 288*a*b^4 + 64*b^5)*log((a*e^(2*d*x +
 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(2*d*x + 2*c) + a + 2*b + 2*sqrt(
(a + b)*b)))/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*sqrt((a + b
)*b)*d) + 5/64*(3*a*b + 10*b^2)*log((a*e^(2*d*x + 2*c) + a + 2*b - 2*sqrt((
a + b)*b))/(a*e^(2*d*x + 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^4 + 4*a^3
*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*sqrt((a + b)*b)*d) + 1/256*(15*a^4*b + 260*
a^3*b^2 + 504*a^2*b^3 + 288*a*b^4 + 64*b^5)*log((a*e^(-2*d*x - 2*c) + a + 2
*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))
/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*sqrt((a + b)*b)*d) - 5/
64*(3*a*b + 10*b^2)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/
(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^4 + 4*a^3*b + 6*a^2
*b^2 + 4*a*b^3 + b^4)*sqrt((a + b)*b)*d) + 15/128*(3*a*b - 4*b^2)*log((a*e^
(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b
 + 2*sqrt((a + b)*b)))/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*sqrt((a
 + b)*b)*d) + 1/192*(176*a^6 + 275*a^5*b + 306*a^4*b^2 + 456*a^3*b^3 + 144*
a^2*b^4 + 3*(96*a^6 + 111*a^5*b - 220*a^4*b^2 - 776*a^3*b^3 - 832*a^2*b^4 -
 256*a*b^5)*e^(12*d*x + 12*c) + 6*(120*a^6 + 528*a^5*b + 525*a^4*b^2 - 52*a
^3*b^3 - 896*a^2*b^4 - 1216*a*b^5 - 384*b^6)*e^(10*d*x + 10*c) + (176*a^6 +
 1337*a^5*b + 7554*a^4*b^2 + 16416*a^3*b^3 + 26880*a^2*b^4 + 25344*a*b^5 +
6912*b^6)*e^(8*d*x + 8*c) - 4*(184*a^6 + 1056*a^5*b + 2993*a^4*b^2 + 4122*a
^3*b^3 + 5892*a^2*b^4 + 6144*a*b^5 + 1728*b^6)*e^(6*d*x + 6*c) - (384*a^6 +
 1177*a^5*b - 736*a^4*b^2 + 112*a^3*b^3 - 624*a^2*b^4 - 6144*a*b^5 - 2304*b
^6)*e^(4*d*x + 4*c) + 2*(136*a^6 + 912*a^5*b + 1211*a^4*b^2 + 1440*a^3*b^3
+ 1896*a^2*b^4 + 576*a*b^5)*e^(2*d*x + 2*c))/((a^9 + 4*a^8*b + 6*a^7*b^2 +
4*a^6*b^3 + a^5*b^4 - (a^9 + 4*a^8*b + 6*a^7*b^2 + 4*a^6*b^3 + a^5*b^4)*e^(
14*d*x + 14*c) - (a^9 + 12*a^8*b + 38*a^7*b^2 + 52*a^6*b^3 + 33*a^5*b^4 + 8
*a^4*b^5)*e^(12*d*x + 12*c) + (3*a^9 + 20*a^8*b + 34*a^7*b^2 - 4*a^6*b^3 -
61*a^5*b^4 - 56*a^4*b^5 - 16*a^3*b^6)*e^(10*d*x + 10*c) + (3*a^9 + 28*a^8*b
 + 130*a^7*b^2 + 300*a^6*b^3 + 355*a^5*b^4 + 208*a^4*b^5 + 48*a^3*b^6)*e^(8
*d*x + 8*c) - (3*a^9 + 28*a^8*b + 130*a^7*b^2 + 300*a^6*b^3 + 355*a^5*b^4 +
 208*a^4*b^5 + 48*a^3*b^6)*e^(6*d*x + 6*c) - (3*a^9 + 20*a^8*b + 34*a^7*b^2
```

- 4*a^6*b^3 - 61*a^5*b^4 - 56*a^4*b^5 - 16*a^3*b^6)*e^(4*d*x + 4*c) + (a^9 + 12*a^8*b + 38*a^7*b^2 + 52*a^6*b^3 + 33*a^5*b^4 + 8*a^4*b^5)*e^(2*d*x + 2*c))*d) - 1/192*(176*a^6 + 275*a^5*b + 306*a^4*b^2 + 456*a^3*b^3 + 144*a^2*b^4 + 2*(136*a^6 + 912*a^5*b + 1211*a^4*b^2 + 1440*a^3*b^3 + 1896*a^2*b^4 + 576*a*b^5)*e^(-2*d*x - 2*c) - (384*a^6 + 1177*a^5*b - 736*a^4*b^2 + 112*a^3*b^3 - 624*a^2*b^4 - 6144*a*b^5 - 2304*b^6)*e^(-4*d*x - 4*c) - 4*(184*a^6 + 1056*a^5*b + 2993*a^4*b^2 + 4122*a^3*b^3 + 5892*a^2*b^4 + 6144*a*b^5 + 1728*b^6)*e^(-6*d*x - 6*c) + (176*a^6 + 1337*a^5*b + 7554*a^4*b^2 + 16416*a^3*b^3 + 26880*a^2*b^4 + 25344*a*b^5 + 6912*b^6)*e^(-8*d*x - 8*c) + 6*(120*a^6 + 528*a^5*b + 525*a^4*b^2 - 52*a^3*b^3 - 896*a^2*b^4 - 1216*a*b^5 - 384*b^6)*e^(-10*d*x - 10*c) + 3*(96*a^6 + 111*a^5*b - 220*a^4*b^2 - 776*a^3*b^3 - 832*a^2*b^4 - 256*a*b^5)*e^(-12*d*x - 12*c))/((a^9 + 4*a^8*b + 6*a^7*b^2 + 4*a^6*b^3 + a^5*b^4 + (a^9 + 12*a^8*b + 38*a^7*b^2 + 52*a^6*b^3 + 33*a^5*b^4 + 8*a^4*b^5)*e^(-2*d*x - 2*c) - (3*a^9 + 20*a^8*b + 34*a^7*b^2 - 4*a^6*b^3 - 61*a^5*b^4 - 56*a^4*b^5 - 16*a^3*b^6)*e^(-4*d*x - 4*c) - (3*a^9 + 28*a^8*b + 130*a^7*b^2 + 300*a^6*b^3 + 355*a^5*b^4 + 208*a^4*b^5 + 48*a^3*b^6)*e^(-6*d*x - 6*c) + (3*a^9 + 28*a^8*b + 130*a^7*b^2 + 300*a^6*b^3 + 355*a^5*b^4 + 208*a^4*b^5 + 48*a^3*b^6)*e^(-8*d*x - 8*c) + (3*a^9 + 20*a^8*b + 34*a^7*b^2 - 4*a^6*b^3 - 61*a^5*b^4 - 56*a^4*b^5 - 16*a^3*b^6)*e^(-10*d*x - 10*c) - (a^9 + 12*a^8*b + 38*a^7*b^2 + 52*a^6*b^3 + 33*a^5*b^4 + 8*a^4*b^5)*e^(-12*d*x - 12*c) - (a^9 + 4*a^8*b + 6*a^7*b^2 + 4*a^6*b^3 + a^5*b^4)*e^(-14*d*x - 14*c))*d) + 1/48*(32*a^5 + 77*a^4*b - 72*a^3*b^2 - 12*a^2*b^3 + 3*(32*a^5 + 65*a^4*b + 94*a^3*b^2 + 128*a^2*b^3 + 32*a*b^4)*e^(12*d*x + 12*c) + 6*(48*a^5 + 200*a^4*b + 203*a^3*b^2 + 90*a^2*b^3 + 176*a*b^4 + 32*b^5)*e^(10*d*x + 10*c) + (224*a^5 + 839*a^4*b + 1500*a^3*b^2 - 612*a^2*b^3 - 3648*a*b^4 - 576*b^5)*e^(8*d*x + 8*c) - 4*(16*a^5 + 216*a^4*b + 695*a^3*b^2 + 252*a^2*b^3 - 912*a*b^4 - 144*b^5)*e^(6*d*x + 6*c) - (96*a^5 + 343*a^4*b - 850*a^3*b^2 - 1808*a^2*b^3 + 1056*a*b^4 + 192*b^5)*e^(4*d*x + 4*c) + 2*(16*a^5 + 216*a^4*b + 269*a^3*b^2 - 294*a^2*b^3 - 48*a*b^4)*e^(2*d*x + 2*c))/((a^8 + 4*a^7*b + 6*a^6*b^2 + 4*a^5*b^3 + a^4*b^4 - (a^8 + 4*a^7*b + 6*a^6*b^2 + 4*a^5*b^3 + a^4*b^4)*e^(14*d*x + 14*c) - (a^8 + 12*a^7*b + 38*a^6*b^2 + 52*a^5*b^3 + 33*a^4*b^4 + 8*a^3*b^5)*e^(12*d*x + 12*c) + (3*a^8 + 20*a^7*b + 34*a^6*b^2 - 4*a^5*b^3 - 61*a^4*b^4 - 56*a^3*b^5 - 16*a^2*b^6)*e^(10*d*x + 10*c) + (3*a^8 + 28*a^7*b + 130*a^6*b^2 + 300*a^5*b^3 + 355*a^4*b^4 + 208*a^3*b^5 + 48*a^2*b^6)*e^(8*d*x + 8*c) - (3*a^8 + 28*a^7*b + 130*a^6*b^2 + 300*a^5*b^3 + 355*a^4*b^4 + 208*a^3*b^5 + 48*a^2*b^6)*e^(6*d*x + 6*c) - (3*a^8 + 20*a^7*b + 34*a^6*b^2 - 4*a^5*b^3 - 61*a^4*b^4 - 56*a^3*b^5 - 16*a^2*b^6)*e^(4*d*x + 4*c) + (a^8 + 12*a^7*b + 38*a^6*b^2 + 52*a^5*b^3 + 33*a^4*b^4 + 8*a^3*b^5)*e^(2*d*x + 2*c))*d) - 1/48*(32*a^5 + 77*a^4*b - 72*a^3*b^2 - 12*a^2*b^3 + 2*(16*a^5 + 216*a^4*b + 269*a^3*b^2 - 294*a^2*b^3 - 48*a*b^4)*e^(-2*d*x - 2*c) - (96*a^5 + 343*a^4*b - 850*a^3*b^2 - 1808*a^2*b^3 + 1056*a*b^4 + 192*b^5)*e^(-4*d*x - 4*c) - 4*(16*a^5 + 216*a^4*b + 695*a^3*b^2 + 252*a^2*b^3 - 912*a*b^4 - 144*b^5)*e^(-6*d*x - 6*c) + (224*a^5 + 839*a^4*b + 1500*a^3*b^2 - 612*a^2*b^3 - 3648*a*b^4 - 576*b^5)*e^(-8*d*x - 8*c) + 6*(48*a^5 + 200*a^4*b + 203*a^3*b^2 + 90*a^2*b^3 + 176*a*b^4 + 32*b^5)*e^(-10

```
*d*x - 10*c) + 3*(32*a^5 + 65*a^4*b + 94*a^3*b^2 + 128*a^2*b^3 + 32*a*b^4)*
e^(-12*d*x - 12*c))/((a^8 + 4*a^7*b + 6*a^6*b^2 + 4*a^5*b^3 + a^4*b^4 + (a^
8 + 12*a^7*b + 38*a^6*b^2 + 52*a^5*b^3 + 33*a^4*b^4 + 8*a^3*b^5)*e^(-2*d*x
- 2*c) - (3*a^8 + 20*a^7*b + 34*a^6*b^2 - 4*a^5*b^3 - 61*a^4*b^4 - 56*a^3*b
^5 - 16*a^2*b^6)*e^(-4*d*x - 4*c) - (3*a^8 + 28*a^7*b + 130*a^6*b^2 + 300*a
^5*b^3 + 355*a^4*b^4 + 208*a^3*b^5 + 48*a^2*b^6)*e^(-6*d*x - 6*c) + (3*a^8
+ 28*a^7*b + 130*a^6*b^2 + 300*a^5*b^3 + 355*a^4*b^4 + 208*a^3*b^5 + 48*a^2
*b^6)*e^(-8*d*x - 8*c) + (3*a^8 + 20*a^7*b + 34*a^6*b^2 - 4*a^5*b^3 - 61*a^
4*b^4 - 56*a^3*b^5 - 16*a^2*b^6)*e^(-10*d*x - 10*c) - (a^8 + 12*a^7*b + 38*
a^6*b^2 + 52*a^5*b^3 + 33*a^4*b^4 + 8*a^3*b^5)*e^(-12*d*x - 12*c) - (a^8 +
4*a^7*b + 6*a^6*b^2 + 4*a^5*b^3 + a^4*b^4)*e^(-14*d*x - 14*c))*d) + 1/32*(1
6*a^4 - 83*a^3*b + 6*a^2*b^2 + 2*(8*a^4 - 299*a^2*b^2 + 24*a*b^3)*e^(-2*d*x
 - 2*c) - (96*a^4 + 71*a^3*b - 344*a^2*b^2 + 1208*a*b^3 - 48*b^4)*e^(-4*d*x
 - 4*c) - 4*(56*a^4 + 144*a^3*b + 31*a^2*b^2 - 546*a*b^3 + 36*b^4)*e^(-6*d*
x - 6*c) - (176*a^4 + 569*a^3*b + 666*a^2*b^2 + 1704*a*b^3 - 144*b^4)*e^(-8
*d*x - 8*c) - 6*(8*a^4 + 32*a^3*b + 93*a^2*b^2 - 28*a*b^3 + 8*b^4)*e^(-10*d
*x - 10*c) - 15*(3*a^3*b - 4*a^2*b^2)*e^(-12*d*x - 12*c))/((a^7 + 4*a^6*b +
 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4 + (a^7 + 12*a^6*b + 38*a^5*b^2 + 52*a^4*b^
3 + 33*a^3*b^4 + 8*a^2*b^5)*e^(-2*d*x - 2*c) - (3*a^7 + 20*a^6*b + 34*a^5*b
^2 - 4*a^4*b^3 - 61*a^3*b^4 - 56*a^2*b^5 - 16*a*b^6)*e^(-4*d*x - 4*c) - (3*
a^7 + 28*a^6*b + 130*a^5*b^2 + 300*a^4*b^3 + 355*a^3*b^4 + 208*a^2*b^5 + 48
*a*b^6)*e^(-6*d*x - 6*c) + (3*a^7 + 28*a^6*b + 130*a^5*b^2 + 300*a^4*b^3 +
355*a^3*b^4 + 208*a^2*b^5 + 48*a*b^6)*e^(-8*d*x - 8*c) + (3*a^7 + 20*a^6*b
+ 34*a^5*b^2 - 4*a^4*b^3 - 61*a^3*b^4 - 56*a^2*b^5 - 16*a*b^6)*e^(-10*d*x -
 10*c) - (a^7 + 12*a^6*b + 38*a^5*b^2 + 52*a^4*b^3 + 33*a^3*b^4 + 8*a^2*b^5
)*e^(-12*d*x - 12*c) - (a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*e^
(-14*d*x - 14*c))*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c+d\,x)^6\coth(c+d\,x)^4}{\left(a\cosh(c+d\,x)^2+b\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^4/(a + b/cosh(c + d*x)^2)^3,x)
```

```
[Out] int((cosh(c + d*x)^6*coth(c + d*x)^4)/(b + a*cosh(c + d*x)^2)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**4/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Integral(coth(c + d*x)**4/(a + b*sech(c + d*x)**2)**3, x)
```

3.169 $\int \frac{1}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^4}\,dx$

Optimal. Leaf size=207

$$\frac{x}{a^4}-\frac{b(11a+6b)\tanh(c+dx)}{24a^2d(a+b)^2\left(a-b\tanh^2(c+dx)+b\right)^2}-\frac{b\left(19a^2+22ab+8b^2\right)\tanh(c+dx)}{16a^3d(a+b)^3\left(a-b\tanh^2(c+dx)+b\right)}-\frac{\sqrt{b}\left(35a^3+70a^2b+56\right.}{16}$$

[Out] x/a^4-1/16*(35*a^3+70*a^2*b+56*a*b^2+16*b^3)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))*b^(1/2)/a^4/(a+b)^(7/2)/d-1/6*b*tanh(d*x+c)/a/(a+b)/d/(a+b-b*tanh(d*x+c)^2)^3-1/24*b*(11*a+6*b)*tanh(d*x+c)/a^2/(a+b)^2/d/(a+b-b*tanh(d*x+c)^2)^2-1/16*b*(19*a^2+22*a*b+8*b^2)*tanh(d*x+c)/a^3/(a+b)^3/d/(a+b-b*tanh(d*x+c)^2)

Rubi [A] time = 0.35, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4128, 414, 527, 522, 206, 208}

$$-\frac{\sqrt{b}\left(70a^2b+35a^3+56ab^2+16b^3\right)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{16a^4d(a+b)^{7/2}}-\frac{b\left(19a^2+22ab+8b^2\right)\tanh(c+dx)}{16a^3d(a+b)^3\left(a-b\tanh^2(c+dx)+b\right)}-\frac{b}{24a^2d(a}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sech[c + d*x]^2)^(-4),x]

[Out] x/a^4 - (Sqrt[b]*(35*a^3 + 70*a^2*b + 56*a*b^2 + 16*b^3)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(16*a^4*(a + b)^(7/2)*d) - (b*Tanh[c + d*x])/(6*a*(a + b)*d*(a + b - b*Tanh[c + d*x]^2)^3) - (b*(11*a + 6*b)*Tanh[c + d*x])/(24*a^2*(a + b)^2*d*(a + b - b*Tanh[c + d*x]^2)^2) - (b*(19*a^2 + 22*a*b + 8*b^2)*Tanh[c + d*x])/(16*a^3*(a + b)^3*d*(a + b - b*Tanh[c + d*x]^2))

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+b\operatorname{sech}^2(c+dx)\right)^4}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{(1-x^2)\left(a+b-bx^2\right)^4}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{b\tanh(c+dx)}{6a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^3} - \frac{\operatorname{Subst}\left(\int \frac{-6a-b-5bx^2}{(1-x^2)\left(a+b-bx^2\right)^3}\,dx, x, \tanh(c\right.}{6a(a+b)d}$$

$$= -\frac{b\tanh(c+dx)}{6a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^3} - \frac{b(11a+6b)\tanh(c+dx)}{24a^2(a+b)^2d\left(a+b-b\tanh^2(c+dx\right.}$$

$$= -\frac{b\tanh(c+dx)}{6a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^3} - \frac{b(11a+6b)\tanh(c+dx)}{24a^2(a+b)^2d\left(a+b-b\tanh^2(c+dx\right.}$$

$$= -\frac{b\tanh(c+dx)}{6a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^3} - \frac{b(11a+6b)\tanh(c+dx)}{24a^2(a+b)^2d\left(a+b-b\tanh^2(c+dx\right.}$$

$$= \frac{x}{a^4} - \frac{\sqrt{b}\left(35a^3+70a^2b+56ab^2+16b^3\right)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{16a^4(a+b)^{7/2}d} - \frac{b\,\mathrm{ta}}{6a(a+b)d\left(a\right.}$$

Mathematica [C] time = 6.93, size = 1405, normalized size = 6.79

$$\frac{\left(35a^3+70ba^2+56b^2a+16b^3\right)(\cosh(2c+2dx)a+a+2b)^4\left(\frac{ib\tan^{-1}\left(\operatorname{sech}(dx)\left(\frac{i\sinh(2c)}{2\sqrt{a+b}\sqrt{b\cosh(4c)-b\sinh(4c)}}-\frac{i\cosh(2c)}{2\sqrt{a+b}\sqrt{b\cosh(4c)-l}}\right.\right.}{256a^4\sqrt{a+b}\,d\sqrt{b\cosh}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^(-4),x]
```

```
[Out] ((35*a^3 + 70*a^2*b + 56*a*b^2 + 16*b^3)*(a + 2*b + a*Cosh[2*c + 2*d*x])^4*
Sech[c + d*x]^8*(((I/256)*b*ArcTan[Sech[d*x]*(((-1/2*I)*Cosh[2*c])/(Sqrt[a
+ b]*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]]) + ((I/2)*Sinh[2*c])/(Sqrt[a + b]*Sqrt
[b*Cosh[4*c] - b*Sinh[4*c]]))*(-(a*Sinh[d*x]) - 2*b*Sinh[d*x] + a*Sinh[2*c
+ d*x])]*Cosh[2*c])/(a^4*Sqrt[a + b]*d*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]]) - (
```

```
(I/256)*b*ArcTan[Sech[d*x]*(((-1/2*I)*Cosh[2*c])/(Sqrt[a + b]*Sqrt[b*Cosh[4
*c] - b*Sinh[4*c]]) + ((I/2)*Sinh[2*c])/(Sqrt[a + b]*Sqrt[b*Cosh[4*c] - b*S
inh[4*c]]))*(-(a*Sinh[d*x]) - 2*b*Sinh[d*x] + a*Sinh[2*c + d*x])]*Sinh[2*c]
)/(a^4*Sqrt[a + b]*d*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]])))/((a + b)^3*(a + b*S
ech[c + d*x]^2)^4) + ((a + 2*b + a*Cosh[2*c + 2*d*x])*Sech[2*c]*Sech[c + d*
x]^8*(480*a^6*d*x*Cosh[2*c] + 3168*a^5*b*d*x*Cosh[2*c] + 8928*a^4*b^2*d*x*C
osh[2*c] + 14112*a^3*b^3*d*x*Cosh[2*c] + 13248*a^2*b^4*d*x*Cosh[2*c] + 6912
*a*b^5*d*x*Cosh[2*c] + 1536*b^6*d*x*Cosh[2*c] + 360*a^6*d*x*Cosh[2*d*x] + 2
232*a^5*b*d*x*Cosh[2*d*x] + 5688*a^4*b^2*d*x*Cosh[2*d*x] + 7272*a^3*b^3*d*x
*Cosh[2*d*x] + 4608*a^2*b^4*d*x*Cosh[2*d*x] + 1152*a*b^5*d*x*Cosh[2*d*x] +
360*a^6*d*x*Cosh[4*c + 2*d*x] + 2232*a^5*b*d*x*Cosh[4*c + 2*d*x] + 5688*a^4
*b^2*d*x*Cosh[4*c + 2*d*x] + 7272*a^3*b^3*d*x*Cosh[4*c + 2*d*x] + 4608*a^2*
b^4*d*x*Cosh[4*c + 2*d*x] + 1152*a*b^5*d*x*Cosh[4*c + 2*d*x] + 144*a^6*d*x*
Cosh[2*c + 4*d*x] + 720*a^5*b*d*x*Cosh[2*c + 4*d*x] + 1296*a^4*b^2*d*x*Cosh
[2*c + 4*d*x] + 1008*a^3*b^3*d*x*Cosh[2*c + 4*d*x] + 288*a^2*b^4*d*x*Cosh[2
*c + 4*d*x] + 144*a^6*d*x*Cosh[6*c + 4*d*x] + 720*a^5*b*d*x*Cosh[6*c + 4*d*
x] + 1296*a^4*b^2*d*x*Cosh[6*c + 4*d*x] + 1008*a^3*b^3*d*x*Cosh[6*c + 4*d*x
] + 288*a^2*b^4*d*x*Cosh[6*c + 4*d*x] + 24*a^6*d*x*Cosh[4*c + 6*d*x] + 72*a
^5*b*d*x*Cosh[4*c + 6*d*x] + 72*a^4*b^2*d*x*Cosh[4*c + 6*d*x] + 24*a^3*b^3*
d*x*Cosh[4*c + 6*d*x] + 24*a^6*d*x*Cosh[8*c + 6*d*x] + 72*a^5*b*d*x*Cosh[8*
c + 6*d*x] + 72*a^4*b^2*d*x*Cosh[8*c + 6*d*x] + 24*a^3*b^3*d*x*Cosh[8*c + 6
*d*x] + 870*a^5*b*Sinh[2*c] + 4292*a^4*b^2*Sinh[2*c] + 8792*a^3*b^3*Sinh[2*
c] + 9936*a^2*b^4*Sinh[2*c] + 5824*a*b^5*Sinh[2*c] + 1408*b^6*Sinh[2*c] - 8
70*a^5*b*Sinh[2*d*x] - 3792*a^4*b^2*Sinh[2*d*x] - 6432*a^3*b^3*Sinh[2*d*x]
- 4608*a^2*b^4*Sinh[2*d*x] - 1248*a*b^5*Sinh[2*d*x] + 435*a^5*b*Sinh[4*c +
2*d*x] + 2124*a^4*b^2*Sinh[4*c + 2*d*x] + 3972*a^3*b^3*Sinh[4*c + 2*d*x] +
3072*a^2*b^4*Sinh[4*c + 2*d*x] + 864*a*b^5*Sinh[4*c + 2*d*x] - 435*a^5*b*Si
nh[2*c + 4*d*x] - 1374*a^4*b^2*Sinh[2*c + 4*d*x] - 1248*a^3*b^3*Sinh[2*c +
4*d*x] - 384*a^2*b^4*Sinh[2*c + 4*d*x] + 87*a^5*b*Sinh[6*c + 4*d*x] + 366*a
^4*b^2*Sinh[6*c + 4*d*x] + 408*a^3*b^3*Sinh[6*c + 4*d*x] + 144*a^2*b^4*Sinh
[6*c + 4*d*x] - 87*a^5*b*Sinh[4*c + 6*d*x] - 116*a^4*b^2*Sinh[4*c + 6*d*x]
- 44*a^3*b^3*Sinh[4*c + 6*d*x]))/(3072*a^4*(a + b)^3*d*(a + b*Sech[c + d*x]
^2)^4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)^2)^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.85, size = 594, normalized size = 2.87

$$-\frac{\frac{3\left(35\,a^3b+70\,a^2b^2+56\,ab^3+16\,b^4\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{\left(a^7+3\,a^6b+3\,a^5b^2+a^4b^3\right)\sqrt{-ab-b^2}}-\frac{2\left(87\,a^5be^{(10\,dx+10\,c)}+366\,a^4b^2e^{(10\,dx+10\,c)}+408\,a^3b^3e^{(10\,dx+10\,c)}+144\,a^2b^4e^{(10\,dx+10\,c)}\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)^2)^4,x, algorithm="giac")
```

```
[Out] -1/48*(3*(35*a^3*b + 70*a^2*b^2 + 56*a*b^3 + 16*b^4)*arctan(1/2*(a*e^(2*d*x
 + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)
*sqrt(-a*b - b^2)) - 2*(87*a^5*b*e^(10*d*x + 10*c) + 366*a^4*b^2*e^(10*d*x
+ 10*c) + 408*a^3*b^3*e^(10*d*x + 10*c) + 144*a^2*b^4*e^(10*d*x + 10*c) + 4
35*a^5*b*e^(8*d*x + 8*c) + 2124*a^4*b^2*e^(8*d*x + 8*c) + 3972*a^3*b^3*e^(8
*d*x + 8*c) + 3072*a^2*b^4*e^(8*d*x + 8*c) + 864*a*b^5*e^(8*d*x + 8*c) + 87
0*a^5*b*e^(6*d*x + 6*c) + 4292*a^4*b^2*e^(6*d*x + 6*c) + 8792*a^3*b^3*e^(6*
d*x + 6*c) + 9936*a^2*b^4*e^(6*d*x + 6*c) + 5824*a*b^5*e^(6*d*x + 6*c) + 14
08*b^6*e^(6*d*x + 6*c) + 870*a^5*b*e^(4*d*x + 4*c) + 3792*a^4*b^2*e^(4*d*x
+ 4*c) + 6432*a^3*b^3*e^(4*d*x + 4*c) + 4608*a^2*b^4*e^(4*d*x + 4*c) + 1248
*a*b^5*e^(4*d*x + 4*c) + 435*a^5*b*e^(2*d*x + 2*c) + 1374*a^4*b^2*e^(2*d*x
+ 2*c) + 1248*a^3*b^3*e^(2*d*x + 2*c) + 384*a^2*b^4*e^(2*d*x + 2*c) + 87*a^
5*b + 116*a^4*b^2 + 44*a^3*b^3)/((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*(a*e
^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) + a)^3) - 48*(d*
x + c)/a^4)/d
```

maple [B] time = 0.45, size = 2880, normalized size = 13.91

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sech(d*x+c)^2)^4,x)
```

```
[Out] -2/d*b^5/a^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*
x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^2+2*a*b+b^2)*tanh(1/
2*d*x+1/2*c)^7-37/4/d*b^3/a/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^
4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^2+2*a
*b+b^2)*tanh(1/2*d*x+1/2*c)^7-1/2/d*b^4/a^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh
(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^
3/(a+b)/(a^2+2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7-145/4/d*b*a/(tanh(1/2*d*x+1/2
*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/
2*c)^2*b+a+b)^3/(a+b)/(a^2+2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7-145/8/d*b/(tanh
(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tan
h(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3-37/d*b^2/
```

(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^3+3*a^2*b+3*a*b^2+b^3)*tanh(1/2*d*x+1/2*c)^5-13/4/d*b^2/a^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)*tanh(1/2*d*x+1/2*c)^11-1/d*b^3/a^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)*tanh(1/2*d*x+1/2*c)^11-281/24/d*b^2/a/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^9+11/4/d*b^3/a^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^9+3/d*b^4/a^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^9-37/4/d*b^3/a/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^3+3*a^2*b+3*a*b^2+b^3)*tanh(1/2*d*x+1/2*c)^5-1/2/d*b^4/a^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^3+3*a^2*b+3*a*b^2+b^3)*tanh(1/2*d*x+1/2*c)^5-2/d*b^5/a^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^3+3*a^2*b+3*a*b^2+b^3)*tanh(1/2*d*x+1/2*c)^5-29/8/d*b/a/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)*tanh(1/2*d*x+1/2*c)^11-1/d/a^4*ln(tanh(1/2*d*x+1/2*c)-1)+1/d/a^4*ln(tanh(1/2*d*x+1/2*c)+1)-145/4/d*b*a/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^3+3*a^2*b+3*a*b^2+b^3)*tanh(1/2*d*x+1/2*c)^5-29/8/d*b/a/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)*tanh(1/2*d*x+1/2*c)-35/16/d*b^(3/2)/a^2/(a^3+3*a^2*b+3*a*b^2+b^3)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))-7/4/d*b^(5/2)/a^3/(a^3+3*a^2*b+3*a*b^2+b^3)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))-37/d*b^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^2+2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7+35/32/d*b^(1/2)/a/(a^3+3*a^2*b+3*a*b^2+b^3)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))-1/2/d*b^(7/2)/a^4/(a^3+3*a^2*b+3*a*b^2+b^3)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))+35/16/d*b^(3/2)/a^2/(a^3+3*a^2*b+3*a*b^2+b^3)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))+1/2/d*b^(7/2)/a^4/(a^3+3*a^2*b+3*a*b^2+b^3)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))-35/32/d*b^(1/2)/a/(a^3+3*a^2*b+3*a*b^2+b^3)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))+7/4/d*b^(5/2)/a^3/(a^3+3*a^2*b+3*a*b^2+b^3)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*b^(1/2)*tanh(1/2*d*x+1/2*c)+(a+b)^(1/2))-281/24/d*b^2/a/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)*tanh(1/2*d

```
*x+1/2*c)^3+11/4/d*b^3/a^2/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+1/2*c)^4
+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)
*tanh(1/2*d*x+1/2*c)^3+3/d*b^4/a^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x+
1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*
a*b+b^2)*tanh(1/2*d*x+1/2*c)^3-13/4/d*b^2/a^2/(tanh(1/2*d*x+1/2*c)^4*a+b*ta
nh(1/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b
)^3/(a+b)*tanh(1/2*d*x+1/2*c)-1/d*b^3/a^3/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1
/2*d*x+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^3/
(a+b)*tanh(1/2*d*x+1/2*c)-145/8/d*b/(tanh(1/2*d*x+1/2*c)^4*a+b*tanh(1/2*d*x
+1/2*c)^4+2*tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2
*a*b+b^2)*tanh(1/2*d*x+1/2*c)^9
```

maxima [B] time = 0.84, size = 718, normalized size = 3.47

$$\frac{\left(35\,a^3b+70\,a^2b^2+56\,ab^3+16\,b^4\right)\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{32\left(a^7+3\,a^6b+3\,a^5b^2+a^4b^3\right)\sqrt{(a+b)b}\,d}-\frac{87\,a^5b+116\,a^4b^2+\cdots}{24\left(a^{10}+3\,a^9b+3\,a^8b^2+a^7b^3+6\left(a^{10}+5\,\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)^2)^4,x, algorithm="maxima")
```

```
[Out] 1/32*(35*a^3*b + 70*a^2*b^2 + 56*a*b^3 + 16*b^4)*log((a*e^(-2*d*x - 2*c) +
a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)
*b)))/((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*sqrt((a + b)*b)*d) - 1/24*(87*
a^5*b + 116*a^4*b^2 + 44*a^3*b^3 + 3*(145*a^5*b + 458*a^4*b^2 + 416*a^3*b^3
 + 128*a^2*b^4)*e^(-2*d*x - 2*c) + 6*(145*a^5*b + 632*a^4*b^2 + 1072*a^3*b^
3 + 768*a^2*b^4 + 208*a*b^5)*e^(-4*d*x - 4*c) + 2*(435*a^5*b + 2146*a^4*b^2
 + 4396*a^3*b^3 + 4968*a^2*b^4 + 2912*a*b^5 + 704*b^6)*e^(-6*d*x - 6*c) + 3
*(145*a^5*b + 708*a^4*b^2 + 1324*a^3*b^3 + 1024*a^2*b^4 + 288*a*b^5)*e^(-8*
d*x - 8*c) + 3*(29*a^5*b + 122*a^4*b^2 + 136*a^3*b^3 + 48*a^2*b^4)*e^(-10*d
*x - 10*c))/((a^10 + 3*a^9*b + 3*a^8*b^2 + a^7*b^3 + 6*(a^10 + 5*a^9*b + 9*
a^8*b^2 + 7*a^7*b^3 + 2*a^6*b^4)*e^(-2*d*x - 2*c) + 3*(5*a^10 + 31*a^9*b +
79*a^8*b^2 + 101*a^7*b^3 + 64*a^6*b^4 + 16*a^5*b^5)*e^(-4*d*x - 4*c) + 4*(5
*a^10 + 33*a^9*b + 93*a^8*b^2 + 147*a^7*b^3 + 138*a^6*b^4 + 72*a^5*b^5 + 16
*a^4*b^6)*e^(-6*d*x - 6*c) + 3*(5*a^10 + 31*a^9*b + 79*a^8*b^2 + 101*a^7*b^
3 + 64*a^6*b^4 + 16*a^5*b^5)*e^(-8*d*x - 8*c) + 6*(a^10 + 5*a^9*b + 9*a^8*b
^2 + 7*a^7*b^3 + 2*a^6*b^4)*e^(-10*d*x - 10*c) + (a^10 + 3*a^9*b + 3*a^8*b^
2 + a^7*b^3)*e^(-12*d*x - 12*c))*d) + (d*x + c)/(a^4*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+\frac{b}{\cosh(c+d\,x)^2}\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cosh(c + d*x)^2)^4,x)
```

```
[Out] int(1/(a + b/cosh(c + d*x)^2)^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)**2)**4,x)
```

```
[Out] Timed out
```

3.170 $\int \left(1-\text{sech}^2(x)\right)^{3/2} \, dx$

Optimal. Leaf size=29

$$\sqrt{\tanh^2(x)} \coth(x) \log(\cosh(x)) - \frac{1}{2} \tanh^2(x)^{3/2} \coth(x)$$

```
[Out] coth(x)*ln(cosh(x))*(tanh(x)^2)^(1/2)-1/2*coth(x)*(tanh(x)^2)^(3/2)
```

Rubi [A] time = 0.03, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4121, 3658, 3473, 3475}

$$\sqrt{\tanh^2(x)} \coth(x) \log(\cosh(x)) - \frac{1}{2} \tanh^2(x)^{3/2} \coth(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sech[x]^2)^(3/2),x]
```

```
[Out] Coth[x]*Log[Cosh[x]]*Sqrt[Tanh[x]^2] - (Coth[x]*(Tanh[x]^2)^(3/2))/2
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 4121

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
```

```
[a + b, 0]
```

Rubi steps

$$\begin{aligned}\int \left(1-\operatorname{sech}^2(x)\right)^{3/2}\,dx &= \int \tanh^2(x)^{3/2}\,dx\\ &= \left(\coth(x)\sqrt{\tanh^2(x)}\right)\int \tanh^3(x)\,dx\\ &= -\frac{1}{2}\coth(x)\tanh^2(x)^{3/2}+\left(\coth(x)\sqrt{\tanh^2(x)}\right)\int \tanh(x)\,dx\\ &= \coth(x)\log(\cosh(x))\sqrt{\tanh^2(x)}-\frac{1}{2}\coth(x)\tanh^2(x)^{3/2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 25, normalized size = 0.86

$$\frac{1}{2}\sqrt{\tanh^2(x)}\,(\operatorname{csch}(x)\operatorname{sech}(x)+2\coth(x)\log(\cosh(x)))$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sech[x]^2)^(3/2),x]
[Out] ((2*Coth[x]*Log[Cosh[x]] + Csch[x]*Sech[x])*Sqrt[Tanh[x]^2])/2
```

fricas [B] time = 0.67, size = 183, normalized size = 6.31

$$-\frac{x\cosh(x)^4+4\,x\cosh(x)\sinh(x)^3+x\sinh(x)^4+2\,(x-1)\cosh(x)^2+2\left(3\,x\cosh(x)^2+x-1\right)\sinh(x)^2-\left(\cosh\right.}{\cosh(x)^4+4\,\cosh(x)\sin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sech(x)^2)^(3/2),x, algorithm="fricas")
[Out] -(x*cosh(x)^4 + 4*x*cosh(x)*sinh(x)^3 + x*sinh(x)^4 + 2*(x - 1)*cosh(x)^2 +
 2*(3*x*cosh(x)^2 + x - 1)*sinh(x)^2 - (cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + s
inh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cos
h(x))*sinh(x) + 1)*log(2*cosh(x)/(cosh(x) - sinh(x))) + 4*(x*cosh(x)^3 + (x
 - 1)*cosh(x))*sinh(x) + x)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 +
2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(
x) + 1)
```

giac [B] time = 0.13, size = 72, normalized size = 2.48

$$-x\operatorname{sgn}\left(e^{(4x)}-1\right)+\log\left(e^{(2x)}+1\right)\operatorname{sgn}\left(e^{(4x)}-1\right)-\frac{3\,e^{(4x)}\operatorname{sgn}\left(e^{(4x)}-1\right)+2\,e^{(2x)}\operatorname{sgn}\left(e^{(4x)}-1\right)+3\operatorname{sgn}\left(e^{(4x)}-1\right.}{2\left(e^{(2x)}+1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sech(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -x*sgn(e^(4*x) - 1) + log(e^(2*x) + 1)*sgn(e^(4*x) - 1) - 1/2*(3*e^(4*x)*sg
n(e^(4*x) - 1) + 2*e^(2*x)*sgn(e^(4*x) - 1) + 3*sgn(e^(4*x) - 1))/(e^(2*x)
+ 1)^2
```

maple [B] time = 0.28, size = 120, normalized size = 4.14

$$-\frac{\left(1+\mathrm{e}^{2x}\right)\sqrt{\frac{\left(\mathrm{e}^{2x}-1\right)^{2}}{\left(1+\mathrm{e}^{2x}\right)^{2}}}\,x}{\mathrm{e}^{2x}-1}+\frac{2\sqrt{\frac{\left(\mathrm{e}^{2x}-1\right)^{2}}{\left(1+\mathrm{e}^{2x}\right)^{2}}}\,\mathrm{e}^{2x}}{\left(\mathrm{e}^{2x}-1\right)\left(1+\mathrm{e}^{2x}\right)}+\frac{\left(1+\mathrm{e}^{2x}\right)\sqrt{\frac{\left(\mathrm{e}^{2x}-1\right)^{2}}{\left(1+\mathrm{e}^{2x}\right)^{2}}}\,\ln\left(1+\mathrm{e}^{2x}\right)}{\mathrm{e}^{2x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-sech(x)^2)^(3/2),x)
```

```
[Out] -1/(exp(2*x)-1)*(1+exp(2*x))*((exp(2*x)-1)^2/(1+exp(2*x))^2)^(1/2)*x+2/(exp
(2*x)-1)/(1+exp(2*x))*((exp(2*x)-1)^2/(1+exp(2*x))^2)^(1/2)*exp(2*x)+1/(exp
(2*x)-1)*(1+exp(2*x))*((exp(2*x)-1)^2/(1+exp(2*x))^2)^(1/2)*ln(1+exp(2*x))
```

maxima [A] time = 0.46, size = 33, normalized size = 1.14

$$-x-\frac{2\,e^{(-2\,x)}}{2\,e^{(-2\,x)}+e^{(-4\,x)}+1}-\log\left(e^{(-2\,x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sech(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -x - 2*e^(-2*x)/(2*e^(-2*x) + e^(-4*x) + 1) - log(e^(-2*x) + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \left(1-\frac{1}{\cosh(x)^{2}}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - 1/cosh(x)^2)^(3/2),x)
[Out] int((1 - 1/cosh(x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(1 - \operatorname{sech}^{2}(x)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sech(x)**2)**(3/2),x)
[Out] Integral((1 - sech(x)**2)**(3/2), x)
```

3.171 $\int \sqrt{1-\text{sech}^2(x)}\, dx$

Optimal. Leaf size=14

$$\sqrt{\tanh^2(x)}\coth(x)\log(\cosh(x))$$

```
[Out] coth(x)*ln(cosh(x))*(tanh(x)^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4121, 3658, 3475}

$$\sqrt{\tanh^2(x)}\coth(x)\log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - Sech[x]^2],x]
```

```
[Out] Coth[x]*Log[Cosh[x]]*Sqrt[Tanh[x]^2]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 4121

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rubi steps

$$\int \sqrt{1-\mathrm{sech}^2(x)}\,dx = \int \sqrt{\tanh^2(x)}\,dx$$
$$= \left(\coth(x)\sqrt{\tanh^2(x)}\right)\int \tanh(x)\,dx$$
$$= \coth(x)\log(\cosh(x))\sqrt{\tanh^2(x)}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.00

$$\sqrt{\tanh^2(x)}\coth(x)\log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - Sech[x]^2],x]
[Out] Coth[x]*Log[Cosh[x]]*Sqrt[Tanh[x]^2]
```

fricas [A] time = 0.77, size = 18, normalized size = 1.29

$$-x+\log\left(\frac{2\cosh(x)}{\cosh(x)-\sinh(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sech(x)^2)^(1/2),x, algorithm="fricas")
[Out] -x + log(2*cosh(x)/(cosh(x) - sinh(x)))
```

giac [B] time = 0.12, size = 26, normalized size = 1.86

$$-x\mathrm{sgn}\left(e^{(4x)}-1\right)+\log\left(e^{(2x)}+1\right)\mathrm{sgn}\left(e^{(4x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sech(x)^2)^(1/2),x, algorithm="giac")
[Out] -x*sgn(e^(4*x) - 1) + log(e^(2*x) + 1)*sgn(e^(4*x) - 1)
```

maple [B] time = 0.30, size = 79, normalized size = 5.64

$$-\frac{\left(1+e^{2x}\right)\sqrt{\frac{\left(e^{2x}-1\right)^2}{\left(1+e^{2x}\right)^2}}\,x}{e^{2x}-1}+\frac{\left(1+e^{2x}\right)\sqrt{\frac{\left(e^{2x}-1\right)^2}{\left(1+e^{2x}\right)^2}}\,\ln\left(1+e^{2x}\right)}{e^{2x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-sech(x)^2)^(1/2),x)
```

```
[Out] -1/(exp(2*x)-1)*(1+exp(2*x))*((exp(2*x)-1)^2/(1+exp(2*x))^2)^(1/2)*x+1/(exp
(2*x)-1)*(1+exp(2*x))*((exp(2*x)-1)^2/(1+exp(2*x))^2)^(1/2)*ln(1+exp(2*x))
```

maxima [A] time = 0.61, size = 13, normalized size = 0.93

$$-x - \log\left(e^{(-2x)} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sech(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -x - log(e^(-2*x) + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.07

$$\int \sqrt{1 - \frac{1}{\cosh(x)^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - 1/cosh(x)^2)^(1/2),x)
```

```
[Out] int((1 - 1/cosh(x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{1 - \operatorname{sech}^2(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sech(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(1 - sech(x)**2), x)
```

3.172 $\int \frac{1}{\sqrt{1-\mathbf{sech}^2(x)}}\,dx$

Optimal. Leaf size=14

$$\frac{\tanh(x)\log(\sinh(x))}{\sqrt{\tanh^2(x)}}$$

```
[Out] ln(sinh(x))*tanh(x)/(tanh(x)^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4121, 3658, 3475}

$$\frac{\tanh(x)\log(\sinh(x))}{\sqrt{\tanh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 - Sech[x]^2],x]
```

```
[Out] (Log[Sinh[x]]*Tanh[x])/Sqrt[Tanh[x]^2]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 4121

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{1-\operatorname{sech}^2(x)}}\,dx = \int \frac{1}{\sqrt{\tanh^2(x)}}\,dx$$
$$= \frac{\tanh(x)\int \coth(x)\,dx}{\sqrt{\tanh^2(x)}}$$
$$= \frac{\log(\sinh(x))\tanh(x)}{\sqrt{\tanh^2(x)}}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.00

$$\frac{\tanh(x)\log(\sinh(x))}{\sqrt{\tanh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 - Sech[x]^2],x]
[Out] (Log[Sinh[x]]*Tanh[x])/Sqrt[Tanh[x]^2]
```

fricas [A] time = 0.53, size = 18, normalized size = 1.29

$$-x+\log\left(\frac{2\sinh(x)}{\cosh(x)-\sinh(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sech(x)^2)^(1/2),x, algorithm="fricas")
[Out] -x + log(2*sinh(x)/(cosh(x) - sinh(x)))
```

giac [B] time = 0.13, size = 31, normalized size = 2.21

$$-\frac{x}{\operatorname{sgn}\left(e^{(4x)}-1\right)}+\frac{\log\left(\left|e^{(2x)}-1\right|\right)}{\operatorname{sgn}\left(e^{(4x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sech(x)^2)^(1/2),x, algorithm="giac")
[Out] -x/sgn(e^(4*x) - 1) + log(abs(e^(2*x) - 1))/sgn(e^(4*x) - 1)
```

maple [B] time = 0.26, size = 79, normalized size = 5.64

$$-\frac{\left(\mathrm{e}^{2x}-1\right)x}{\sqrt{\frac{\left(\mathrm{e}^{2x}-1\right)^{2}}{\left(1+\mathrm{e}^{2x}\right)^{2}}}\,\left(1+\mathrm{e}^{2x}\right)}+\frac{\left(\mathrm{e}^{2x}-1\right)\ln\left(\mathrm{e}^{2x}-1\right)}{\sqrt{\frac{\left(\mathrm{e}^{2x}-1\right)^{2}}{\left(1+\mathrm{e}^{2x}\right)^{2}}}\,\left(1+\mathrm{e}^{2x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-sech(x)^2)^(1/2),x)
```

```
[Out] -1/((exp(2*x)-1)^2/(1+exp(2*x))^2)^(1/2)/(1+exp(2*x))*(exp(2*x)-1)*x+1/((ex
p(2*x)-1)^2/(1+exp(2*x))^2)^(1/2)/(1+exp(2*x))*(exp(2*x)-1)*ln(exp(2*x)-1)
```

maxima [A] time = 0.67, size = 22, normalized size = 1.57

$$-x-\log\left(e^{(-x)}+1\right)-\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sech(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -x - log(e^(-x) + 1) - log(e^(-x) - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{1}{\sqrt{1-\frac{1}{\cosh(x)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1 - 1/cosh(x)^2)^(1/2),x)
```

```
[Out] int(1/(1 - 1/cosh(x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{1-\operatorname{sech}^{2}(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sech(x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(1 - sech(x)**2), x)
```

3.173 $\int \left(-1+\operatorname{sech}^2(x)\right)^{3/2} dx$

Optimal. Leaf size=34

$$\frac{1}{2}\tanh(x)\sqrt{-\tanh^2(x)}-\sqrt{-\tanh^2(x)}\coth(x)\log(\cosh(x))$$

```
[Out] -coth(x)*ln(cosh(x))*(-tanh(x)^2)^(1/2)+1/2*(-tanh(x)^2)^(1/2)*tanh(x)
```

Rubi [A] time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4121, 3658, 3473, 3475}

$$\frac{1}{2}\tanh(x)\sqrt{-\tanh^2(x)}-\sqrt{-\tanh^2(x)}\coth(x)\log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + Sech[x]^2)^(3/2),x]
```

```
[Out] -(Coth[x]*Log[Cosh[x]]*Sqrt[-Tanh[x]^2]) + (Tanh[x]*Sqrt[-Tanh[x]^2])/2
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 4121

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
```

```
[a + b, 0]
```

Rubi steps

$$
\begin{aligned}
\int \left(-1+\operatorname{sech}^2(x)\right)^{3/2}\,dx &= \int \left(-\tanh^2(x)\right)^{3/2}\,dx\\
&= -\left(\left(\coth(x)\sqrt{-\tanh^2(x)}\right)\int \tanh^3(x)\,dx\right)\\
&= \frac{1}{2}\tanh(x)\sqrt{-\tanh^2(x)}-\left(\coth(x)\sqrt{-\tanh^2(x)}\right)\int \tanh(x)\,dx\\
&= -\coth(x)\log(\cosh(x))\sqrt{-\tanh^2(x)}+\frac{1}{2}\tanh(x)\sqrt{-\tanh^2(x)}
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.79

$$-\frac{1}{2}\sqrt{-\tanh^2(x)}\,(\operatorname{csch}(x)\operatorname{sech}(x)+2\coth(x)\log(\cosh(x)))$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + Sech[x]^2)^(3/2),x]
[Out] -1/2*((2*Coth[x]*Log[Cosh[x]] + Csch[x]*Sech[x])*Sqrt[-Tanh[x]^2])
```

fricas [A] time = 0.68, size = 1, normalized size = 0.03

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sech(x)^2)^(3/2),x, algorithm="fricas")
[Out] 0
```

giac [C] time = 0.14, size = 83, normalized size = 2.44

$$-ix\operatorname{sgn}\left(-e^{(4x)}+1\right)+i\log\left(e^{(2x)}+1\right)\operatorname{sgn}\left(-e^{(4x)}+1\right)-\frac{i\left(3e^{(4x)}\operatorname{sgn}\left(-e^{(4x)}+1\right)+2e^{(2x)}\operatorname{sgn}\left(-e^{(4x)}+1\right)+3\operatorname{sgn}\right.}{2\left(e^{(2x)}+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sech(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -I*x*sgn(-e^(4*x) + 1) + I*log(e^(2*x) + 1)*sgn(-e^(4*x) + 1) - 1/2*I*(3*e^
(4*x)*sgn(-e^(4*x) + 1) + 2*e^(2*x)*sgn(-e^(4*x) + 1) + 3*sgn(-e^(4*x) + 1)
)/(e^(2*x) + 1)^2
```

maple [B] time = 0.32, size = 123, normalized size = 3.62

$$\frac{\left(1+e^{2x}\right)\sqrt{-\frac{\left(e^{2x}-1\right)^{2}}{\left(1+e^{2x}\right)^{2}}}\, x}{e^{2x}-1}-\frac{2\sqrt{-\frac{\left(e^{2x}-1\right)^{2}}{\left(1+e^{2x}\right)^{2}}}\, e^{2x}}{\left(e^{2x}-1\right)\left(1+e^{2x}\right)}-\frac{\left(1+e^{2x}\right)\sqrt{-\frac{\left(e^{2x}-1\right)^{2}}{\left(1+e^{2x}\right)^{2}}}\, \ln\left(1+e^{2x}\right)}{e^{2x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+sech(x)^2)^(3/2),x)
```

```
[Out] 1/(exp(2*x)-1)*(1+exp(2*x))*(-(exp(2*x)-1)^2/(1+exp(2*x))^2)^(1/2)*x-2/(exp
(2*x)-1)/(1+exp(2*x))*(-(exp(2*x)-1)^2/(1+exp(2*x))^2)^(1/2)*exp(2*x)-1/(ex
p(2*x)-1)*(1+exp(2*x))*(-(exp(2*x)-1)^2/(1+exp(2*x))^2)^(1/2)*ln(1+exp(2*x)
)
```

maxima [C] time = 0.49, size = 33, normalized size = 0.97

$$i\,x+\frac{2i\,e^{(-2\,x)}}{2\,e^{(-2\,x)}+e^{(-4\,x)}+1}+i\,\log\left(e^{(-2\,x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sech(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] I*x + 2*I*e^(-2*x)/(2*e^(-2*x) + e^(-4*x) + 1) + I*log(e^(-2*x) + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \left(\frac{1}{\cosh(x)^2}-1\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cosh(x)^2 - 1)^(3/2),x)
```

```
[Out] int((1/cosh(x)^2 - 1)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\operatorname{sech}^{2}(x)-1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sech(x)**2)**(3/2),x)
[Out] Integral((sech(x)**2 - 1)**(3/2), x)
```

3.174 $\int \sqrt{-1+\mathrm{sech}^2(x)}\,dx$

Optimal. Leaf size=16

$$\sqrt{-\tanh^2(x)}\,\coth(x)\log(\cosh(x))$$

```
[Out] coth(x)*ln(cosh(x))*(-tanh(x)^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4121, 3658, 3475}

$$\sqrt{-\tanh^2(x)}\,\coth(x)\log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + Sech[x]^2],x]
```

```
[Out] Coth[x]*Log[Cosh[x]]*Sqrt[-Tanh[x]^2]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 4121

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rubi steps

$$\int \sqrt{-1+\operatorname{sech}^2(x)}\,dx = \int \sqrt{-\tanh^2(x)}\,dx$$
$$= \left(\coth(x)\sqrt{-\tanh^2(x)}\right)\int \tanh(x)\,dx$$
$$= \coth(x)\log(\cosh(x))\sqrt{-\tanh^2(x)}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$\sqrt{-\tanh^2(x)}\,\coth(x)\log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + Sech[x]^2],x]
[Out] Coth[x]*Log[Cosh[x]]*Sqrt[-Tanh[x]^2]
```

fricas [A] time = 0.51, size = 1, normalized size = 0.06

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sech(x)^2)^(1/2),x, algorithm="fricas")
[Out] 0
```

giac [C] time = 0.13, size = 31, normalized size = 1.94

$$i\,x\operatorname{sgn}\left(-e^{(4\,x)}+1\right) - i\,\log\left(e^{(2\,x)}+1\right)\operatorname{sgn}\left(-e^{(4\,x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sech(x)^2)^(1/2),x, algorithm="giac")
[Out] I*x*sgn(-e^(4*x) + 1) - I*log(e^(2*x) + 1)*sgn(-e^(4*x) + 1)
```

maple [B] time = 0.32, size = 81, normalized size = 5.06

$$-\frac{\left(1+e^{2x}\right)\sqrt{-\frac{\left(e^{2x}-1\right)^2}{\left(1+e^{2x}\right)^2}}\,x}{e^{2x}-1}+\frac{\left(1+e^{2x}\right)\sqrt{-\frac{\left(e^{2x}-1\right)^2}{\left(1+e^{2x}\right)^2}}\,\ln\left(1+e^{2x}\right)}{e^{2x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+sech(x)^2)^(1/2),x)
```

```
[Out] -1/(exp(2*x)-1)*(1+exp(2*x))*(-(exp(2*x)-1)^2/(1+exp(2*x))^2)^(1/2)*x+1/(ex
p(2*x)-1)*(1+exp(2*x))*(-(exp(2*x)-1)^2/(1+exp(2*x))^2)^(1/2)*ln(1+exp(2*x)
)
```

maxima [C] time = 0.55, size = 13, normalized size = 0.81

$$-i\,x - i\,\log\left(e^{(-2\,x)} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sech(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -I*x - I*log(e^(-2*x) + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$\int \sqrt{\frac{1}{\cosh(x)^2} - 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cosh(x)^2 - 1)^(1/2),x)
```

```
[Out] int((1/cosh(x)^2 - 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\operatorname{sech}^2(x) - 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sech(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(sech(x)**2 - 1), x)
```

3.175 $\int \frac{1}{\sqrt{-1+\mathbf{sech}^2(x)}}\,dx$

Optimal. Leaf size=16

$$\frac{\tanh(x)\log(\sinh(x))}{\sqrt{-\tanh^2(x)}}$$

```
[Out] ln(sinh(x))*tanh(x)/(-tanh(x)^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4121, 3658, 3475}

$$\frac{\tanh(x)\log(\sinh(x))}{\sqrt{-\tanh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-1 + Sech[x]^2],x]
```

```
[Out] (Log[Sinh[x]]*Tanh[x])/Sqrt[-Tanh[x]^2]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 4121

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-1+\operatorname{sech}^2(x)}}\,dx = \int \frac{1}{\sqrt{-\tanh^2(x)}}\,dx$$
$$= \frac{\tanh(x)\int \coth(x)\,dx}{\sqrt{-\tanh^2(x)}}$$
$$= \frac{\log(\sinh(x))\tanh(x)}{\sqrt{-\tanh^2(x)}}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$\frac{\tanh(x)\log(\sinh(x))}{\sqrt{-\tanh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-1 + Sech[x]^2],x]
[Out] (Log[Sinh[x]]*Tanh[x])/Sqrt[-Tanh[x]^2]
```

fricas [A] time = 0.49, size = 1, normalized size = 0.06

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+sech(x)^2)^(1/2),x, algorithm="fricas")
[Out] 0
```

giac [C] time = 0.14, size = 37, normalized size = 2.31

$$-\frac{i\,x}{\operatorname{sgn}\left(-e^{(4\,x)}+1\right)} + \frac{i\,\log\left(-i\,e^{(2\,x)}+i\right)}{\operatorname{sgn}\left(-e^{(4\,x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+sech(x)^2)^(1/2),x, algorithm="giac")
[Out] -I*x/sgn(-e^(4*x) + 1) + I*log(-I*e^(2*x) + I)/sgn(-e^(4*x) + 1)
```

maple [B] time = 0.30, size = 81, normalized size = 5.06

$$-\frac{\left(\mathrm{e}^{2x}-1\right)x}{\sqrt{-\frac{\left(\mathrm{e}^{2x}-1\right)^{2}}{\left(1+\mathrm{e}^{2x}\right)^{2}}}\,\left(1+\mathrm{e}^{2x}\right)}+\frac{\left(\mathrm{e}^{2x}-1\right)\ln\left(\mathrm{e}^{2x}-1\right)}{\sqrt{-\frac{\left(\mathrm{e}^{2x}-1\right)^{2}}{\left(1+\mathrm{e}^{2x}\right)^{2}}}\,\left(1+\mathrm{e}^{2x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1+sech(x)^2)^(1/2),x)
[Out] -1/(-(exp(2*x)-1)^2/(1+exp(2*x))^2)^(1/2)/(1+exp(2*x))*(exp(2*x)-1)*x+1/(-(
exp(2*x)-1)^2/(1+exp(2*x))^2)^(1/2)/(1+exp(2*x))*(exp(2*x)-1)*ln(exp(2*x)-1
)
```

maxima [C] time = 0.44, size = 22, normalized size = 1.38

$$i\,x+i\,\log\left(e^{(-x)}+1\right)+i\,\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+sech(x)^2)^(1/2),x, algorithm="maxima")
[Out] I*x + I*log(e^(-x) + 1) + I*log(e^(-x) - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{\sqrt{\frac{1}{\cosh(x)^2}-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/cosh(x)^2 - 1)^(1/2),x)
[Out] int(1/(1/cosh(x)^2 - 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\operatorname{sech}^{2}(x)-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+sech(x)**2)**(1/2),x)
[Out] Integral(1/sqrt(sech(x)**2 - 1), x)
```

3.176 $\int \sqrt{a+b\,\mathbf{sech}^2(x)}\,\tanh^5(x)\,dx$

Optimal. Leaf size=83

$$-\frac{\left(a+b\,\mathrm{sech}^2(x)\right)^{5/2}}{5b^2}+\frac{(a+2b)\left(a+b\,\mathrm{sech}^2(x)\right)^{3/2}}{3b^2}-\sqrt{a+b\,\mathrm{sech}^2(x)}+\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}^2(x)}}{\sqrt{a}}\right)$$

```
[Out] 1/3*(a+2*b)*(a+b*sech(x)^2)^(3/2)/b^2-1/5*(a+b*sech(x)^2)^(5/2)/b^2+arctanh
((a+b*sech(x)^2)^(1/2)/a^(1/2))*a^(1/2)-(a+b*sech(x)^2)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {4139, 446, 88, 50, 63, 208}

$$-\frac{\left(a+b\,\mathrm{sech}^2(x)\right)^{5/2}}{5b^2}+\frac{(a+2b)\left(a+b\,\mathrm{sech}^2(x)\right)^{3/2}}{3b^2}-\sqrt{a+b\,\mathrm{sech}^2(x)}+\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}^2(x)}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sech[x]^2]*Tanh[x]^5,x]
```

```
[Out] Sqrt[a]*ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]] - Sqrt[a + b*Sech[x]^2] + ((
a + 2*b)*(a + b*Sech[x]^2)^(3/2))/(3*b^2) - (a + b*Sech[x]^2)^(5/2)/(5*b^2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \sqrt{a+b\text{sech}^2(x)}\,\tanh^5(x)\,dx = -\text{Subst}\left(\int \frac{\left(-1+x^2\right)^2\sqrt{a+bx^2}}{x}\,dx, x, \text{sech}(x)\right)$$
$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{(-1+x)^2\sqrt{a+bx}}{x}\,dx, x, \text{sech}^2(x)\right)\right)$$
$$= -\left(\frac{1}{2}\text{Subst}\left(\int \left(\frac{(-a-2b)\sqrt{a+bx}}{b} + \frac{\sqrt{a+bx}}{x} + \frac{(a+bx)^{3/2}}{b}\right) dx, x, \text{sech}^2(x\right.\right.$$
$$= \frac{(a+2b)\left(a+b\text{sech}^2(x)\right)^{3/2}}{3b^2} - \frac{\left(a+b\text{sech}^2(x)\right)^{5/2}}{5b^2} - \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx,\right.$$
$$= -\sqrt{a+b\text{sech}^2(x)} + \frac{(a+2b)\left(a+b\text{sech}^2(x)\right)^{3/2}}{3b^2} - \frac{\left(a+b\text{sech}^2(x)\right)^{5/2}}{5b^2} - \frac{1}{2}a\,\text{S}$$
$$= -\sqrt{a+b\text{sech}^2(x)} + \frac{(a+2b)\left(a+b\text{sech}^2(x)\right)^{3/2}}{3b^2} - \frac{\left(a+b\text{sech}^2(x)\right)^{5/2}}{5b^2} - \frac{a\,\text{Su}}{}$$
$$= \sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right) - \sqrt{a+b\text{sech}^2(x)} + \frac{(a+2b)\left(a+b\text{sech}^2(x)\right)}{3b^2}$$

Mathematica [A] time = 0.75, size = 114, normalized size = 1.37

$$\frac{1}{15}\cosh(x)\sqrt{a+b\text{sech}^2(x)}\left(\left(\frac{2a^2}{b^2}+\frac{10a}{b}-15\right)\text{sech}(x)+\left(10-\frac{a}{b}\right)\text{sech}^3(x)+\frac{15\sqrt{2}\sqrt{a}\log\left(\sqrt{a}\cosh(2x)+a\right.}{\sqrt{a}\cosh(2x)+}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sech[x]^2]*Tanh[x]^5,x]
```

```
[Out] (Cosh[x]*Sqrt[a + b*Sech[x]^2]*((15*Sqrt[2]*Sqrt[a]*Log[Sqrt[2]*Sqrt[a]*Cos
h[x] + Sqrt[a + 2*b + a*Cosh[2*x]]])/Sqrt[a + 2*b + a*Cosh[2*x]] + (-15 + (
2*a^2)/b^2 + (10*a)/b)*Sech[x] + (10 - a/b)*Sech[x]^3 - 3*Sech[x]^5))/15
```

fricas [B] time = 1.33, size = 4594, normalized size = 55.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(1/2)*tanh(x)^5,x, algorithm="fricas")
```

```
[Out] [1/60*(15*(b^2*cosh(x)^10 + 10*b^2*cosh(x)*sinh(x)^9 + b^2*sinh(x)^10 + 5*b
^2*cosh(x)^8 + 5*(9*b^2*cosh(x)^2 + b^2)*sinh(x)^8 + 10*b^2*cosh(x)^6 + 40*
(3*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x)^7 + 10*(21*b^2*cosh(x)^4 + 14*b^2*c
osh(x)^2 + b^2)*sinh(x)^6 + 10*b^2*cosh(x)^4 + 4*(63*b^2*cosh(x)^5 + 70*b^2
*cosh(x)^3 + 15*b^2*cosh(x))*sinh(x)^5 + 10*(21*b^2*cosh(x)^6 + 35*b^2*cosh
(x)^4 + 15*b^2*cosh(x)^2 + b^2)*sinh(x)^4 + 5*b^2*cosh(x)^2 + 40*(3*b^2*cos
h(x)^7 + 7*b^2*cosh(x)^5 + 5*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x)^3 + 5*(9*
b^2*cosh(x)^8 + 28*b^2*cosh(x)^6 + 30*b^2*cosh(x)^4 + 12*b^2*cosh(x)^2 + b^
2)*sinh(x)^2 + b^2 + 10*(b^2*cosh(x)^9 + 4*b^2*cosh(x)^7 + 6*b^2*cosh(x)^5
+ 4*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(a)*log(((a^3 + 2*a^2*b + a*b
^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^7 + (a^3 + 2*a^2*
b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^6 + 2*(2
*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh
(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + 3*(2*a^3 + 5*a^2*b + 4*a*
b^2 + b^3)*cosh(x))*sinh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^4 + (7
0*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 6*a^3 + 14*a^2*b + 9*a*b^2 + 30*(2*a^
3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + 2*a^2*b +
a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^3 + (6*a^3
+ 14*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(2*a^3 + 3*a^2*b)*cosh(x
)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 + 15*(2*a^3 + 5*a^2*b + 4*a*b
^2 + b^3)*cosh(x)^4 + 2*a^3 + 3*a^2*b + 3*(6*a^3 + 14*a^2*b + 9*a*b^2)*cosh
(x)^2)*sinh(x)^2 + sqrt(2)*((a^2 + 2*a*b + b^2)*cosh(x)^6 + 6*(a^2 + 2*a*b
+ b^2)*cosh(x)*sinh(x)^5 + (a^2 + 2*a*b + b^2)*sinh(x)^6 + 3*(a^2 + 2*a*b +
 b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 + 2*a*b + b^2)*s
inh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(
x))*sinh(x)^3 + (3*a^2 + 4*a*b)*cosh(x)^2 + (15*(a^2 + 2*a*b + b^2)*cosh(x)
^4 + 18*(a^2 + 2*a*b + b^2)*cosh(x)^2 + 3*a^2 + 4*a*b)*sinh(x)^2 + a^2 + 2*
(3*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 6*(a^2 + 2*a*b + b^2)*cosh(x)^3 + (3*a^2
 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2
*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + 2*a^2*b + a*
b^2)*cosh(x)^7 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^5 + (6*a^3 + 1
4*a^2*b + 9*a*b^2)*cosh(x)^3 + (2*a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)
^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3
+ 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + 15*(b^2*cosh
(x)^10 + 10*b^2*cosh(x)*sinh(x)^9 + b^2*sinh(x)^10 + 5*b^2*cosh(x)^8 + 5*(9
*b^2*cosh(x)^2 + b^2)*sinh(x)^8 + 10*b^2*cosh(x)^6 + 40*(3*b^2*cosh(x)^3 +
b^2*cosh(x))*sinh(x)^7 + 10*(21*b^2*cosh(x)^4 + 14*b^2*cosh(x)^2 + b^2)*sin
h(x)^6 + 10*b^2*cosh(x)^4 + 4*(63*b^2*cosh(x)^5 + 70*b^2*cosh(x)^3 + 15*b^2
*cosh(x))*sinh(x)^5 + 10*(21*b^2*cosh(x)^6 + 35*b^2*cosh(x)^4 + 15*b^2*cosh
(x)^2 + b^2)*sinh(x)^4 + 5*b^2*cosh(x)^2 + 40*(3*b^2*cosh(x)^7 + 7*b^2*cosh
(x)^5 + 5*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x)^3 + 5*(9*b^2*cosh(x)^8 + 28*
b^2*cosh(x)^6 + 30*b^2*cosh(x)^4 + 12*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2
+ 10*(b^2*cosh(x)^9 + 4*b^2*cosh(x)^7 + 6*b^2*cosh(x)^5 + 4*b^2*cosh(x)^3 +
```

```
 b^2*cosh(x))*sinh(x))*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 +
a*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*a*cosh(x)^2 + b)*sinh(x)^2 + sqrt(2)*(co
sh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a)*sqrt((a*cosh(x)^2 + a*
sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(a*co
sh(x)^3 + b*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^
2)) + 4*sqrt(2)*((2*a^2 + 10*a*b - 15*b^2)*cosh(x)^8 + 8*(2*a^2 + 10*a*b -
15*b^2)*cosh(x)*sinh(x)^7 + (2*a^2 + 10*a*b - 15*b^2)*sinh(x)^8 + 4*(2*a^2
+ 9*a*b - 5*b^2)*cosh(x)^6 + 4*(7*(2*a^2 + 10*a*b - 15*b^2)*cosh(x)^2 + 2*a
^2 + 9*a*b - 5*b^2)*sinh(x)^6 + 8*(7*(2*a^2 + 10*a*b - 15*b^2)*cosh(x)^3 +
3*(2*a^2 + 9*a*b - 5*b^2)*cosh(x))*sinh(x)^5 + 2*(6*a^2 + 26*a*b - 29*b^2)*
cosh(x)^4 + 2*(35*(2*a^2 + 10*a*b - 15*b^2)*cosh(x)^4 + 30*(2*a^2 + 9*a*b -
 5*b^2)*cosh(x)^2 + 6*a^2 + 26*a*b - 29*b^2)*sinh(x)^4 + 8*(7*(2*a^2 + 10*a
*b - 15*b^2)*cosh(x)^5 + 10*(2*a^2 + 9*a*b - 5*b^2)*cosh(x)^3 + (6*a^2 + 26
*a*b - 29*b^2)*cosh(x))*sinh(x)^3 + 4*(2*a^2 + 9*a*b - 5*b^2)*cosh(x)^2 + 4
*(7*(2*a^2 + 10*a*b - 15*b^2)*cosh(x)^6 + 15*(2*a^2 + 9*a*b - 5*b^2)*cosh(x
)^4 + 3*(6*a^2 + 26*a*b - 29*b^2)*cosh(x)^2 + 2*a^2 + 9*a*b - 5*b^2)*sinh(x
)^2 + 2*a^2 + 10*a*b - 15*b^2 + 8*((2*a^2 + 10*a*b - 15*b^2)*cosh(x)^7 + 3*
(2*a^2 + 9*a*b - 5*b^2)*cosh(x)^5 + (6*a^2 + 26*a*b - 29*b^2)*cosh(x)^3 + (
2*a^2 + 9*a*b - 5*b^2)*cosh(x))*sinh(x))*sqrt((a*cosh(x)^2 + a*sinh(x)^2 +
a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(b^2*cosh(x)^10 + 10
*b^2*cosh(x)*sinh(x)^9 + b^2*sinh(x)^10 + 5*b^2*cosh(x)^8 + 5*(9*b^2*cosh(x
)^2 + b^2)*sinh(x)^8 + 10*b^2*cosh(x)^6 + 40*(3*b^2*cosh(x)^3 + b^2*cosh(x)
)*sinh(x)^7 + 10*(21*b^2*cosh(x)^4 + 14*b^2*cosh(x)^2 + b^2)*sinh(x)^6 + 10
*b^2*cosh(x)^4 + 4*(63*b^2*cosh(x)^5 + 70*b^2*cosh(x)^3 + 15*b^2*cosh(x))*s
inh(x)^5 + 10*(21*b^2*cosh(x)^6 + 35*b^2*cosh(x)^4 + 15*b^2*cosh(x)^2 + b^2
)*sinh(x)^4 + 5*b^2*cosh(x)^2 + 40*(3*b^2*cosh(x)^7 + 7*b^2*cosh(x)^5 + 5*b
^2*cosh(x)^3 + b^2*cosh(x))*sinh(x)^3 + 5*(9*b^2*cosh(x)^8 + 28*b^2*cosh(x)
^6 + 30*b^2*cosh(x)^4 + 12*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 10*(b^2*c
osh(x)^9 + 4*b^2*cosh(x)^7 + 6*b^2*cosh(x)^5 + 4*b^2*cosh(x)^3 + b^2*cosh(x
))*sinh(x)), -1/30*(15*(b^2*cosh(x)^10 + 10*b^2*cosh(x)*sinh(x)^9 + b^2*sin
h(x)^10 + 5*b^2*cosh(x)^8 + 5*(9*b^2*cosh(x)^2 + b^2)*sinh(x)^8 + 10*b^2*co
sh(x)^6 + 40*(3*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x)^7 + 10*(21*b^2*cosh(x)
^4 + 14*b^2*cosh(x)^2 + b^2)*sinh(x)^6 + 10*b^2*cosh(x)^4 + 4*(63*b^2*cosh(
x)^5 + 70*b^2*cosh(x)^3 + 15*b^2*cosh(x))*sinh(x)^5 + 10*(21*b^2*cosh(x)^6
+ 35*b^2*cosh(x)^4 + 15*b^2*cosh(x)^2 + b^2)*sinh(x)^4 + 5*b^2*cosh(x)^2 +
40*(3*b^2*cosh(x)^7 + 7*b^2*cosh(x)^5 + 5*b^2*cosh(x)^3 + b^2*cosh(x))*sinh
(x)^3 + 5*(9*b^2*cosh(x)^8 + 28*b^2*cosh(x)^6 + 30*b^2*cosh(x)^4 + 12*b^2*c
osh(x)^2 + b^2)*sinh(x)^2 + b^2 + 10*(b^2*cosh(x)^9 + 4*b^2*cosh(x)^7 + 6*b
^2*cosh(x)^5 + 4*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(-a)*arctan(sqrt
(2)*((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a)
*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)
*sinh(x) + sinh(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(
x)^3 + (a^2 + a*b)*sinh(x)^4 + (2*a^2 + 3*a*b)*cosh(x)^2 + (6*(a^2 + a*b)*c
osh(x)^2 + 2*a^2 + 3*a*b)*sinh(x)^2 + a^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 + (2
*a^2 + 3*a*b)*cosh(x))*sinh(x))) + 15*(b^2*cosh(x)^10 + 10*b^2*cosh(x)*sinh
```

```
(x)^9 + b^2*sinh(x)^10 + 5*b^2*cosh(x)^8 + 5*(9*b^2*cosh(x)^2 + b^2)*sinh(x
)^8 + 10*b^2*cosh(x)^6 + 40*(3*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x)^7 + 10*
(21*b^2*cosh(x)^4 + 14*b^2*cosh(x)^2 + b^2)*sinh(x)^6 + 10*b^2*cosh(x)^4 +
4*(63*b^2*cosh(x)^5 + 70*b^2*cosh(x)^3 + 15*b^2*cosh(x))*sinh(x)^5 + 10*(21
*b^2*cosh(x)^6 + 35*b^2*cosh(x)^4 + 15*b^2*cosh(x)^2 + b^2)*sinh(x)^4 + 5*b
^2*cosh(x)^2 + 40*(3*b^2*cosh(x)^7 + 7*b^2*cosh(x)^5 + 5*b^2*cosh(x)^3 + b^
2*cosh(x))*sinh(x)^3 + 5*(9*b^2*cosh(x)^8 + 28*b^2*cosh(x)^6 + 30*b^2*cosh(
x)^4 + 12*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 10*(b^2*cosh(x)^9 + 4*b^2*
cosh(x)^7 + 6*b^2*cosh(x)^5 + 4*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(
-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a)
*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x)
+ sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2
*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a
 + 2*b)*cosh(x))*sinh(x) + a)) - 2*sqrt(2)*((2*a^2 + 10*a*b - 15*b^2)*cosh(
x)^8 + 8*(2*a^2 + 10*a*b - 15*b^2)*cosh(x)*sinh(x)^7 + (2*a^2 + 10*a*b - 15
*b^2)*sinh(x)^8 + 4*(2*a^2 + 9*a*b - 5*b^2)*cosh(x)^6 + 4*(7*(2*a^2 + 10*a*
b - 15*b^2)*cosh(x)^2 + 2*a^2 + 9*a*b - 5*b^2)*sinh(x)^6 + 8*(7*(2*a^2 + 10
*a*b - 15*b^2)*cosh(x)^3 + 3*(2*a^2 + 9*a*b - 5*b^2)*cosh(x))*sinh(x)^5 + 2
*(6*a^2 + 26*a*b - 29*b^2)*cosh(x)^4 + 2*(35*(2*a^2 + 10*a*b - 15*b^2)*cosh
(x)^4 + 30*(2*a^2 + 9*a*b - 5*b^2)*cosh(x)^2 + 6*a^2 + 26*a*b - 29*b^2)*sin
h(x)^4 + 8*(7*(2*a^2 + 10*a*b - 15*b^2)*cosh(x)^5 + 10*(2*a^2 + 9*a*b - 5*b
^2)*cosh(x)^3 + (6*a^2 + 26*a*b - 29*b^2)*cosh(x))*sinh(x)^3 + 4*(2*a^2 + 9
*a*b - 5*b^2)*cosh(x)^2 + 4*(7*(2*a^2 + 10*a*b - 15*b^2)*cosh(x)^6 + 15*(2*
a^2 + 9*a*b - 5*b^2)*cosh(x)^4 + 3*(6*a^2 + 26*a*b - 29*b^2)*cosh(x)^2 + 2*
a^2 + 9*a*b - 5*b^2)*sinh(x)^2 + 2*a^2 + 10*a*b - 15*b^2 + 8*((2*a^2 + 10*a
*b - 15*b^2)*cosh(x)^7 + 3*(2*a^2 + 9*a*b - 5*b^2)*cosh(x)^5 + (6*a^2 + 26*
a*b - 29*b^2)*cosh(x)^3 + (2*a^2 + 9*a*b - 5*b^2)*cosh(x))*sinh(x))*sqrt((a
*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x
)^2)))/(b^2*cosh(x)^10 + 10*b^2*cosh(x)*sinh(x)^9 + b^2*sinh(x)^10 + 5*b^2*
cosh(x)^8 + 5*(9*b^2*cosh(x)^2 + b^2)*sinh(x)^8 + 10*b^2*cosh(x)^6 + 40*(3*
b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x)^7 + 10*(21*b^2*cosh(x)^4 + 14*b^2*cosh
(x)^2 + b^2)*sinh(x)^6 + 10*b^2*cosh(x)^4 + 4*(63*b^2*cosh(x)^5 + 70*b^2*co
sh(x)^3 + 15*b^2*cosh(x))*sinh(x)^5 + 10*(21*b^2*cosh(x)^6 + 35*b^2*cosh(x)
^4 + 15*b^2*cosh(x)^2 + b^2)*sinh(x)^4 + 5*b^2*cosh(x)^2 + 40*(3*b^2*cosh(x
)^7 + 7*b^2*cosh(x)^5 + 5*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x)^3 + 5*(9*b^2
*cosh(x)^8 + 28*b^2*cosh(x)^6 + 30*b^2*cosh(x)^4 + 12*b^2*cosh(x)^2 + b^2)*
sinh(x)^2 + b^2 + 10*(b^2*cosh(x)^9 + 4*b^2*cosh(x)^7 + 6*b^2*cosh(x)^5 + 4
*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))]
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(1/2)*tanh(x)^5,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 0.47Error: Bad Argument Typ
e
```

maple [F] time = 0.45, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\mathrm{sech}(x)^2}\,\left(\tanh^5(x)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(x)^2)^(1/2)*tanh(x)^5,x)
[Out] int((a+b*sech(x)^2)^(1/2)*tanh(x)^5,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\,\mathrm{sech}(x)^2+a}\,\tanh(x)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(1/2)*tanh(x)^5,x, algorithm="maxima")
[Out] integrate(sqrt(b*sech(x)^2 + a)*tanh(x)^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tanh(x)^5\,\sqrt{a+\frac{b}{\cosh(x)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^5*(a + b/cosh(x)^2)^(1/2),x)
[Out] int(tanh(x)^5*(a + b/cosh(x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\,\mathrm{sech}^2(x)}\,\tanh^5(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)**2)**(1/2)*tanh(x)**5,x)
[Out] Integral(sqrt(a + b*sech(x)**2)*tanh(x)**5, x)
```

3.177 $\int \sqrt{a+b\,\mathrm{sech}^2(x)}\,\tanh^4(x)\,dx$

Optimal. Leaf size=125

$$-\frac{\left(a^2+6ab-3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{8b^{3/2}}+\frac{(a-3b)\tanh(x)\sqrt{a-b\tanh^2(x)+b}}{8b}+\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+\ldots}}\right.$$

```
[Out] -1/8*(a^2+6*a*b-3*b^2)*arctan(b^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))/b^(3
/2)+arctanh(a^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))*a^(1/2)+1/8*(a-3*b)*(a
+b-b*tanh(x)^2)^(1/2)*tanh(x)/b-1/4*(a+b-b*tanh(x)^2)^(1/2)*tanh(x)^3
```

Rubi [A] time = 0.31, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {4141, 1975, 478, 582, 523, 217, 203, 377, 206}

$$-\frac{\left(a^2+6ab-3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{8b^{3/2}}-\frac{1}{4}\tanh^3(x)\sqrt{a-b\tanh^2(x)+b}+\frac{(a-3b)\tanh(x)\sqrt{a-b\tanh^2(x)+b}}{8b}\ldots$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sech[x]^2]*Tanh[x]^4,x]
```

```
[Out] -((a^2 + 6*a*b - 3*b^2)*ArcTan[(Sqrt[b]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]
)/(8*b^(3/2)) + Sqrt[a]*ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]
] + ((a - 3*b)*Tanh[x]*Sqrt[a + b - b*Tanh[x]^2])/(8*b) - (Tanh[x]^3*Sqrt[a
 + b - b*Tanh[x]^2])/4
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*(m + n*(p + q) + 1)), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \sqrt{a+b\text{sech}^2(x)}\,\tanh^4(x)\,dx = \text{Subst}\left(\int \frac{x^4\sqrt{a+b\left(1-x^2\right)}}{1-x^2}\,dx, x, \tanh(x)\right)$$

$$= \text{Subst}\left(\int \frac{x^4\sqrt{a+b-bx^2}}{1-x^2}\,dx, x, \tanh(x)\right)$$

$$= -\frac{1}{4}\tanh^3(x)\sqrt{a+b-b\tanh^2(x)} + \frac{1}{4}\text{Subst}\left(\int \frac{x^2\left(3(a+b)+(a-3b)x^2\right)}{\left(1-x^2\right)\sqrt{a+b-bx^2}}\,dx,\right.$$

$$= \frac{(a-3b)\tanh(x)\sqrt{a+b-b\tanh^2(x)}}{8b} - \frac{1}{4}\tanh^3(x)\sqrt{a+b-b\tanh^2(x)} - \frac{\text{Sub}}{}$$

$$= \frac{(a-3b)\tanh(x)\sqrt{a+b-b\tanh^2(x)}}{8b} - \frac{1}{4}\tanh^3(x)\sqrt{a+b-b\tanh^2(x)} + a\,\text{Su}$$

$$= \frac{(a-3b)\tanh(x)\sqrt{a+b-b\tanh^2(x)}}{8b} - \frac{1}{4}\tanh^3(x)\sqrt{a+b-b\tanh^2(x)} + a\,\text{Su}$$

$$= -\frac{\left(a^2+6ab-3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\,\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{8b^{3/2}} + \sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right.$$

Mathematica [A] time = 0.47, size = 192, normalized size = 1.54

$$-\frac{\cosh(x)\sqrt{a+b\text{sech}^2(x)}\left(\sqrt{2}\left(a^2+6ab-3b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}\sinh(x)}{\sqrt{a\cosh(2x)+a+2b}}\right) - 8\sqrt{2}\sqrt{a}\,b^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sinh(x)}{\sqrt{a\cosh(2x)+a+2b}}\right) \right.}{8b^{3/2}\sqrt{a\cosh(2x)+}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sech[x]^2]*Tanh[x]^4,x]
```

```
[Out] -1/8*(Cosh[x]*Sqrt[a + b*Sech[x]^2]*(Sqrt[2]*(a^2 + 6*a*b - 3*b^2)*ArcTan[(
Sqrt[2]*Sqrt[b]*Sinh[x])/Sqrt[a + 2*b + a*Cosh[2*x]]] - 8*Sqrt[2]*Sqrt[a]*b
^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[a]*Sinh[x])/Sqrt[a + 2*b + a*Cosh[2*x]]] - (a
- 5*b)*Sqrt[b]*Sqrt[a + 2*b + a*Cosh[2*x]]*Sech[x]*Tanh[x] - 2*b^(3/2)*Sqrt
[a + 2*b + a*Cosh[2*x]]*Sech[x]^3*Tanh[x]))/(b^(3/2)*Sqrt[a + 2*b + a*Cosh[
2*x]])
```

fricas [B] time = 1.22, size = 8852, normalized size = 70.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(1/2)*tanh(x)^4,x, algorithm="fricas")
```

```
[Out] [1/16*(4*(b^2*cosh(x)^8 + 8*b^2*cosh(x)*sinh(x)^7 + b^2*sinh(x)^8 + 4*b^2*c
osh(x)^6 + 4*(7*b^2*cosh(x)^2 + b^2)*sinh(x)^6 + 6*b^2*cosh(x)^4 + 8*(7*b^2
*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^5 + 2*(35*b^2*cosh(x)^4 + 30*b^2*cosh(x
)^2 + 3*b^2)*sinh(x)^4 + 4*b^2*cosh(x)^2 + 8*(7*b^2*cosh(x)^5 + 10*b^2*cosh
(x)^3 + 3*b^2*cosh(x))*sinh(x)^3 + 4*(7*b^2*cosh(x)^6 + 15*b^2*cosh(x)^4 +
9*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 8*(b^2*cosh(x)^7 + 3*b^2*cosh(x)^5
 + 3*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(a)*log((a*b^2*cosh(x)^8 + 8
*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*
(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh(x)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a
*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*
a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 - 30*(a*b^2 - b^3)*cosh(x)^2)*sin
h(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a*b^2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2*
b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14
*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*cosh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*
a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh
(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)
*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 + 4*a*b)*
cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 - 4*a*b)*sinh(x)^2 -
 a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 + 4*a*b)*cosh(x))*sinh(x
))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x
)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*cosh(x)^7 - 3*(a*b^2 - b^3)*cosh(x)^5
+ (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3 + (a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(
cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sin
h(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + ((a^2
 + 6*a*b - 3*b^2)*cosh(x)^8 + 8*(a^2 + 6*a*b - 3*b^2)*cosh(x)*sinh(x)^7 + (
a^2 + 6*a*b - 3*b^2)*sinh(x)^8 + 4*(a^2 + 6*a*b - 3*b^2)*cosh(x)^6 + 4*(7*(
a^2 + 6*a*b - 3*b^2)*cosh(x)^2 + a^2 + 6*a*b - 3*b^2)*sinh(x)^6 + 8*(7*(a^2
 + 6*a*b - 3*b^2)*cosh(x)^3 + 3*(a^2 + 6*a*b - 3*b^2)*cosh(x))*sinh(x)^5 +
6*(a^2 + 6*a*b - 3*b^2)*cosh(x)^4 + 2*(35*(a^2 + 6*a*b - 3*b^2)*cosh(x)^4 +
```

```
30*(a^2 + 6*a*b - 3*b^2)*cosh(x)^2 + 3*a^2 + 18*a*b - 9*b^2)*sinh(x)^4 + 8
*(7*(a^2 + 6*a*b - 3*b^2)*cosh(x)^5 + 10*(a^2 + 6*a*b - 3*b^2)*cosh(x)^3 +
3*(a^2 + 6*a*b - 3*b^2)*cosh(x))*sinh(x)^3 + 4*(a^2 + 6*a*b - 3*b^2)*cosh(x
)^2 + 4*(7*(a^2 + 6*a*b - 3*b^2)*cosh(x)^6 + 15*(a^2 + 6*a*b - 3*b^2)*cosh(
x)^4 + 9*(a^2 + 6*a*b - 3*b^2)*cosh(x)^2 + a^2 + 6*a*b - 3*b^2)*sinh(x)^2 +
 a^2 + 6*a*b - 3*b^2 + 8*((a^2 + 6*a*b - 3*b^2)*cosh(x)^7 + 3*(a^2 + 6*a*b
- 3*b^2)*cosh(x)^5 + 3*(a^2 + 6*a*b - 3*b^2)*cosh(x)^3 + (a^2 + 6*a*b - 3*b
^2)*cosh(x))*sinh(x))*sqrt(-b)*log(-((a - b)*cosh(x)^4 + 4*(a - b)*cosh(x)*
sinh(x)^3 + (a - b)*sinh(x)^4 + 2*(a + 3*b)*cosh(x)^2 + 2*(3*(a - b)*cosh(x
)^2 + a + 3*b)*sinh(x)^2 + 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(
x)^2 - 1)*sqrt(-b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 -
2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a - b)*cosh(x)^3 + (a + 3*b)*cosh(x))
*sinh(x) + a - b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(
x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)) +
 4*(b^2*cosh(x)^8 + 8*b^2*cosh(x)*sinh(x)^7 + b^2*sinh(x)^8 + 4*b^2*cosh(x)
^6 + 4*(7*b^2*cosh(x)^2 + b^2)*sinh(x)^6 + 6*b^2*cosh(x)^4 + 8*(7*b^2*cosh(
x)^3 + 3*b^2*cosh(x))*sinh(x)^5 + 2*(35*b^2*cosh(x)^4 + 30*b^2*cosh(x)^2 +
3*b^2)*sinh(x)^4 + 4*b^2*cosh(x)^2 + 8*(7*b^2*cosh(x)^5 + 10*b^2*cosh(x)^3
+ 3*b^2*cosh(x))*sinh(x)^3 + 4*(7*b^2*cosh(x)^6 + 15*b^2*cosh(x)^4 + 9*b^2*
cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 8*(b^2*cosh(x)^7 + 3*b^2*cosh(x)^5 + 3*b
^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x
)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + b)
*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a
)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x)
 + sinh(x)^2)) + 4*(a*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a)/(cosh(x)^2
+ 2*cosh(x)*sinh(x) + sinh(x)^2)) + 2*sqrt(2)*((a*b - 5*b^2)*cosh(x)^6 + 6*
(a*b - 5*b^2)*cosh(x)*sinh(x)^5 + (a*b - 5*b^2)*sinh(x)^6 + (a*b + 3*b^2)*c
osh(x)^4 + (15*(a*b - 5*b^2)*cosh(x)^2 + a*b + 3*b^2)*sinh(x)^4 + 4*(5*(a*b
 - 5*b^2)*cosh(x)^3 + (a*b + 3*b^2)*cosh(x))*sinh(x)^3 - (a*b + 3*b^2)*cosh
(x)^2 + (15*(a*b - 5*b^2)*cosh(x)^4 + 6*(a*b + 3*b^2)*cosh(x)^2 - a*b - 3*b
^2)*sinh(x)^2 - a*b + 5*b^2 + 2*(3*(a*b - 5*b^2)*cosh(x)^5 + 2*(a*b + 3*b^2
)*cosh(x)^3 - (a*b + 3*b^2)*cosh(x))*sinh(x))*sqrt((a*cosh(x)^2 + a*sinh(x)
^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(b^2*cosh(x)^8
+ 8*b^2*cosh(x)*sinh(x)^7 + b^2*sinh(x)^8 + 4*b^2*cosh(x)^6 + 4*(7*b^2*cosh
(x)^2 + b^2)*sinh(x)^6 + 6*b^2*cosh(x)^4 + 8*(7*b^2*cosh(x)^3 + 3*b^2*cosh(
x))*sinh(x)^5 + 2*(35*b^2*cosh(x)^4 + 30*b^2*cosh(x)^2 + 3*b^2)*sinh(x)^4 +
 4*b^2*cosh(x)^2 + 8*(7*b^2*cosh(x)^5 + 10*b^2*cosh(x)^3 + 3*b^2*cosh(x))*s
inh(x)^3 + 4*(7*b^2*cosh(x)^6 + 15*b^2*cosh(x)^4 + 9*b^2*cosh(x)^2 + b^2)*s
inh(x)^2 + b^2 + 8*(b^2*cosh(x)^7 + 3*b^2*cosh(x)^5 + 3*b^2*cosh(x)^3 + b^2
*cosh(x))*sinh(x)), -1/8*(((a^2 + 6*a*b - 3*b^2)*cosh(x)^8 + 8*(a^2 + 6*a*b
 - 3*b^2)*cosh(x)*sinh(x)^7 + (a^2 + 6*a*b - 3*b^2)*sinh(x)^8 + 4*(a^2 + 6*
a*b - 3*b^2)*cosh(x)^6 + 4*(7*(a^2 + 6*a*b - 3*b^2)*cosh(x)^2 + a^2 + 6*a*b
 - 3*b^2)*sinh(x)^6 + 8*(7*(a^2 + 6*a*b - 3*b^2)*cosh(x)^3 + 3*(a^2 + 6*a*b
 - 3*b^2)*cosh(x))*sinh(x)^5 + 6*(a^2 + 6*a*b - 3*b^2)*cosh(x)^4 + 2*(35*(a
^2 + 6*a*b - 3*b^2)*cosh(x)^4 + 30*(a^2 + 6*a*b - 3*b^2)*cosh(x)^2 + 3*a^2
```

```
+ 18*a*b - 9*b^2)*sinh(x)^4 + 8*(7*(a^2 + 6*a*b - 3*b^2)*cosh(x)^5 + 10*(a^
2 + 6*a*b - 3*b^2)*cosh(x)^3 + 3*(a^2 + 6*a*b - 3*b^2)*cosh(x))*sinh(x)^3 +
 4*(a^2 + 6*a*b - 3*b^2)*cosh(x)^2 + 4*(7*(a^2 + 6*a*b - 3*b^2)*cosh(x)^6 +
 15*(a^2 + 6*a*b - 3*b^2)*cosh(x)^4 + 9*(a^2 + 6*a*b - 3*b^2)*cosh(x)^2 + a
^2 + 6*a*b - 3*b^2)*sinh(x)^2 + a^2 + 6*a*b - 3*b^2 + 8*((a^2 + 6*a*b - 3*b
^2)*cosh(x)^7 + 3*(a^2 + 6*a*b - 3*b^2)*cosh(x)^5 + 3*(a^2 + 6*a*b - 3*b^2)
*cosh(x)^3 + (a^2 + 6*a*b - 3*b^2)*cosh(x))*sinh(x))*sqrt(b)*arctan(sqrt(2)
*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(b)*sqrt((a*cosh(x)^2
+ a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*co
sh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(
3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*si
nh(x) + a)) - 2*(b^2*cosh(x)^8 + 8*b^2*cosh(x)*sinh(x)^7 + b^2*sinh(x)^8 +
4*b^2*cosh(x)^6 + 4*(7*b^2*cosh(x)^2 + b^2)*sinh(x)^6 + 6*b^2*cosh(x)^4 + 8
*(7*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^5 + 2*(35*b^2*cosh(x)^4 + 30*b^2
*cosh(x)^2 + 3*b^2)*sinh(x)^4 + 4*b^2*cosh(x)^2 + 8*(7*b^2*cosh(x)^5 + 10*b
^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^3 + 4*(7*b^2*cosh(x)^6 + 15*b^2*cosh(
x)^4 + 9*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 8*(b^2*cosh(x)^7 + 3*b^2*co
sh(x)^5 + 3*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(a)*log((a*b^2*cosh(x
)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^8 - 2*(a*b^2 - b^3)*cosh(x)
^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh(x)^6 + 4*(14*a*b^2*cosh(x)^3
 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^4
 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 - 30*(a*b^2 - b^3)*cosh(x)
^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a*b^2 - b^3)*cosh(x)^3 + (a^3 +
 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(a^3 + 3*a^2*b)*cosh(x)^2
+ 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*cosh(x)^4 + a^3 + 3*a^2*b + 3*(a
^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b
^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2
 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 +
4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 - 4*a*b)*sinh
(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 + 4*a*b)*cosh(x))
*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2
*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*cosh(x)^7 - 3*(a*b^2 - b^3)*cos
h(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3 + (a^3 + 3*a^2*b)*cosh(x))*sin
h(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x
)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6))
- 2*(b^2*cosh(x)^8 + 8*b^2*cosh(x)*sinh(x)^7 + b^2*sinh(x)^8 + 4*b^2*cosh(x
)^6 + 4*(7*b^2*cosh(x)^2 + b^2)*sinh(x)^6 + 6*b^2*cosh(x)^4 + 8*(7*b^2*cosh
(x)^3 + 3*b^2*cosh(x))*sinh(x)^5 + 2*(35*b^2*cosh(x)^4 + 30*b^2*cosh(x)^2 +
 3*b^2)*sinh(x)^4 + 4*b^2*cosh(x)^2 + 8*(7*b^2*cosh(x)^5 + 10*b^2*cosh(x)^3
 + 3*b^2*cosh(x))*sinh(x)^3 + 4*(7*b^2*cosh(x)^6 + 15*b^2*cosh(x)^4 + 9*b^2
*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 8*(b^2*cosh(x)^7 + 3*b^2*cosh(x)^5 + 3*
b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(
x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + b
)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(
a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x
```

```
) + sinh(x)^2)) + 4*(a*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a)/(cosh(x)^2
 + 2*cosh(x)*sinh(x) + sinh(x)^2)) - sqrt(2)*((a*b - 5*b^2)*cosh(x)^6 + 6*(
a*b - 5*b^2)*cosh(x)*sinh(x)^5 + (a*b - 5*b^2)*sinh(x)^6 + (a*b + 3*b^2)*co
sh(x)^4 + (15*(a*b - 5*b^2)*cosh(x)^2 + a*b + 3*b^2)*sinh(x)^4 + 4*(5*(a*b
- 5*b^2)*cosh(x)^3 + (a*b + 3*b^2)*cosh(x))*sinh(x)^3 - (a*b + 3*b^2)*cosh(
x)^2 + (15*(a*b - 5*b^2)*cosh(x)^4 + 6*(a*b + 3*b^2)*cosh(x)^2 - a*b - 3*b^
2)*sinh(x)^2 - a*b + 5*b^2 + 2*(3*(a*b - 5*b^2)*cosh(x)^5 + 2*(a*b + 3*b^2)
*cosh(x)^3 - (a*b + 3*b^2)*cosh(x))*sinh(x))*sqrt((a*cosh(x)^2 + a*sinh(x)^
2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(b^2*cosh(x)^8 +
 8*b^2*cosh(x)*sinh(x)^7 + b^2*sinh(x)^8 + 4*b^2*cosh(x)^6 + 4*(7*b^2*cosh(
x)^2 + b^2)*sinh(x)^6 + 6*b^2*cosh(x)^4 + 8*(7*b^2*cosh(x)^3 + 3*b^2*cosh(x
))*sinh(x)^5 + 2*(35*b^2*cosh(x)^4 + 30*b^2*cosh(x)^2 + 3*b^2)*sinh(x)^4 +
4*b^2*cosh(x)^2 + 8*(7*b^2*cosh(x)^5 + 10*b^2*cosh(x)^3 + 3*b^2*cosh(x))*si
nh(x)^3 + 4*(7*b^2*cosh(x)^6 + 15*b^2*cosh(x)^4 + 9*b^2*cosh(x)^2 + b^2)*si
nh(x)^2 + b^2 + 8*(b^2*cosh(x)^7 + 3*b^2*cosh(x)^5 + 3*b^2*cosh(x)^3 + b^2*
cosh(x))*sinh(x)), -1/16*(8*(b^2*cosh(x)^8 + 8*b^2*cosh(x)*sinh(x)^7 + b^2*
sinh(x)^8 + 4*b^2*cosh(x)^6 + 4*(7*b^2*cosh(x)^2 + b^2)*sinh(x)^6 + 6*b^2*c
osh(x)^4 + 8*(7*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^5 + 2*(35*b^2*cosh(x
)^4 + 30*b^2*cosh(x)^2 + 3*b^2)*sinh(x)^4 + 4*b^2*cosh(x)^2 + 8*(7*b^2*cosh
(x)^5 + 10*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^3 + 4*(7*b^2*cosh(x)^6 +
15*b^2*cosh(x)^4 + 9*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 8*(b^2*cosh(x)^
7 + 3*b^2*cosh(x)^5 + 3*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(-a)*arct
an(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + a)*sqrt(-a)*s
qrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) +
sinh(x)^2))/(a*b*cosh(x)^4 + 4*a*b*cosh(x)*sinh(x)^3 + a*b*sinh(x)^4 - (a^2
 + 3*a*b)*cosh(x)^2 + (6*a*b*cosh(x)^2 - a^2 - 3*a*b)*sinh(x)^2 - a^2 + 2*(
2*a*b*cosh(x)^3 - (a^2 + 3*a*b)*cosh(x))*sinh(x))) + 8*(b^2*cosh(x)^8 + 8*b
^2*cosh(x)*sinh(x)^7 + b^2*sinh(x)^8 + 4*b^2*cosh(x)^6 + 4*(7*b^2*cosh(x)^2
 + b^2)*sinh(x)^6 + 6*b^2*cosh(x)^4 + 8*(7*b^2*cosh(x)^3 + 3*b^2*cosh(x))*s
inh(x)^5 + 2*(35*b^2*cosh(x)^4 + 30*b^2*cosh(x)^2 + 3*b^2)*sinh(x)^4 + 4*b^
2*cosh(x)^2 + 8*(7*b^2*cosh(x)^5 + 10*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x
)^3 + 4*(7*b^2*cosh(x)^6 + 15*b^2*cosh(x)^4 + 9*b^2*cosh(x)^2 + b^2)*sinh(x
)^2 + b^2 + 8*(b^2*cosh(x)^7 + 3*b^2*cosh(x)^5 + 3*b^2*cosh(x)^3 + b^2*cosh
(x))*sinh(x))*sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh
(x)^2 + 1)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 -
 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*s
inh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 +
4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) - ((a^2 + 6*a*b - 3*b^2)*
cosh(x)^8 + 8*(a^2 + 6*a*b - 3*b^2)*cosh(x)*sinh(x)^7 + (a^2 + 6*a*b - 3*b^
2)*sinh(x)^8 + 4*(a^2 + 6*a*b - 3*b^2)*cosh(x)^6 + 4*(7*(a^2 + 6*a*b - 3*b^
2)*cosh(x)^2 + a^2 + 6*a*b - 3*b^2)*sinh(x)^6 + 8*(7*(a^2 + 6*a*b - 3*b^2)*
cosh(x)^3 + 3*(a^2 + 6*a*b - 3*b^2)*cosh(x))*sinh(x)^5 + 6*(a^2 + 6*a*b - 3
*b^2)*cosh(x)^4 + 2*(35*(a^2 + 6*a*b - 3*b^2)*cosh(x)^4 + 30*(a^2 + 6*a*b -
 3*b^2)*cosh(x)^2 + 3*a^2 + 18*a*b - 9*b^2)*sinh(x)^4 + 8*(7*(a^2 + 6*a*b -
 3*b^2)*cosh(x)^5 + 10*(a^2 + 6*a*b - 3*b^2)*cosh(x)^3 + 3*(a^2 + 6*a*b - 3
```

```
*b^2)*cosh(x))*sinh(x)^3 + 4*(a^2 + 6*a*b - 3*b^2)*cosh(x)^2 + 4*(7*(a^2 +
6*a*b - 3*b^2)*cosh(x)^6 + 15*(a^2 + 6*a*b - 3*b^2)*cosh(x)^4 + 9*(a^2 + 6*
a*b - 3*b^2)*cosh(x)^2 + a^2 + 6*a*b - 3*b^2)*sinh(x)^2 + a^2 + 6*a*b - 3*b
^2 + 8*((a^2 + 6*a*b - 3*b^2)*cosh(x)^7 + 3*(a^2 + 6*a*b - 3*b^2)*cosh(x)^5
 + 3*(a^2 + 6*a*b - 3*b^2)*cosh(x)^3 + (a^2 + 6*a*b - 3*b^2)*cosh(x))*sinh(
x))*sqrt(-b)*log(-((a - b)*cosh(x)^4 + 4*(a - b)*cosh(x)*sinh(x)^3 + (a - b
)*sinh(x)^4 + 2*(a + 3*b)*cosh(x)^2 + 2*(3*(a - b)*cosh(x)^2 + a + 3*b)*sin
h(x)^2 + 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-b)
*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x)
+ sinh(x)^2)) + 4*((a - b)*cosh(x)^3 + (a + 3*b)*cosh(x))*sinh(x) + a - b)/
(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^
2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)) - 2*sqrt(2)*((a*b -
 5*b^2)*cosh(x)^6 + 6*(a*b - 5*b^2)*cosh(x)*sinh(x)^5 + (a*b - 5*b^2)*sinh(
x)^6 + (a*b + 3*b^2)*cosh(x)^4 + (15*(a*b - 5*b^2)*cosh(x)^2 + a*b + 3*b^2)
*sinh(x)^4 + 4*(5*(a*b - 5*b^2)*cosh(x)^3 + (a*b + 3*b^2)*cosh(x))*sinh(x)^
3 - (a*b + 3*b^2)*cosh(x)^2 + (15*(a*b - 5*b^2)*cosh(x)^4 + 6*(a*b + 3*b^2)
*cosh(x)^2 - a*b - 3*b^2)*sinh(x)^2 - a*b + 5*b^2 + 2*(3*(a*b - 5*b^2)*cosh
(x)^5 + 2*(a*b + 3*b^2)*cosh(x)^3 - (a*b + 3*b^2)*cosh(x))*sinh(x))*sqrt((a
*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x
)^2)))/(b^2*cosh(x)^8 + 8*b^2*cosh(x)*sinh(x)^7 + b^2*sinh(x)^8 + 4*b^2*cos
h(x)^6 + 4*(7*b^2*cosh(x)^2 + b^2)*sinh(x)^6 + 6*b^2*cosh(x)^4 + 8*(7*b^2*c
osh(x)^3 + 3*b^2*cosh(x))*sinh(x)^5 + 2*(35*b^2*cosh(x)^4 + 30*b^2*cosh(x)^
2 + 3*b^2)*sinh(x)^4 + 4*b^2*cosh(x)^2 + 8*(7*b^2*cosh(x)^5 + 10*b^2*cosh(x
)^3 + 3*b^2*cosh(x))*sinh(x)^3 + 4*(7*b^2*cosh(x)^6 + 15*b^2*cosh(x)^4 + 9*
b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 8*(b^2*cosh(x)^7 + 3*b^2*cosh(x)^5 +
 3*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x)), -1/8*(4*(b^2*cosh(x)^8 + 8*b^2*co
sh(x)*sinh(x)^7 + b^2*sinh(x)^8 + 4*b^2*cosh(x)^6 + 4*(7*b^2*cosh(x)^2 + b^
2)*sinh(x)^6 + 6*b^2*cosh(x)^4 + 8*(7*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x
)^5 + 2*(35*b^2*cosh(x)^4 + 30*b^2*cosh(x)^2 + 3*b^2)*sinh(x)^4 + 4*b^2*cos
h(x)^2 + 8*(7*b^2*cosh(x)^5 + 10*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^3 +
 4*(7*b^2*cosh(x)^6 + 15*b^2*cosh(x)^4 + 9*b^2*cosh(x)^2 + b^2)*sinh(x)^2 +
 b^2 + 8*(b^2*cosh(x)^7 + 3*b^2*cosh(x)^5 + 3*b^2*cosh(x)^3 + b^2*cosh(x))*
sinh(x))*sqrt(-a)*arctan(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sin
h(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2
- 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*b*cosh(x)^4 + 4*a*b*cosh(x)*sinh(x)^3
+ a*b*sinh(x)^4 - (a^2 + 3*a*b)*cosh(x)^2 + (6*a*b*cosh(x)^2 - a^2 - 3*a*b)
*sinh(x)^2 - a^2 + 2*(2*a*b*cosh(x)^3 - (a^2 + 3*a*b)*cosh(x))*sinh(x))) +
4*(b^2*cosh(x)^8 + 8*b^2*cosh(x)*sinh(x)^7 + b^2*sinh(x)^8 + 4*b^2*cosh(x)^
6 + 4*(7*b^2*cosh(x)^2 + b^2)*sinh(x)^6 + 6*b^2*cosh(x)^4 + 8*(7*b^2*cosh(x
)^3 + 3*b^2*cosh(x))*sinh(x)^5 + 2*(35*b^2*cosh(x)^4 + 30*b^2*cosh(x)^2 + 3
*b^2)*sinh(x)^4 + 4*b^2*cosh(x)^2 + 8*(7*b^2*cosh(x)^5 + 10*b^2*cosh(x)^3 +
 3*b^2*cosh(x))*sinh(x)^3 + 4*(7*b^2*cosh(x)^6 + 15*b^2*cosh(x)^4 + 9*b^2*c
osh(x)^2 + b^2)*sinh(x)^2 + b^2 + 8*(b^2*cosh(x)^7 + 3*b^2*cosh(x)^5 + 3*b^
2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*
cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 +
```

```
 a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*c
osh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 +
 a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) + (
(a^2 + 6*a*b - 3*b^2)*cosh(x)^8 + 8*(a^2 + 6*a*b - 3*b^2)*cosh(x)*sinh(x)^7
 + (a^2 + 6*a*b - 3*b^2)*sinh(x)^8 + 4*(a^2 + 6*a*b - 3*b^2)*cosh(x)^6 + 4*
(7*(a^2 + 6*a*b - 3*b^2)*cosh(x)^2 + a^2 + 6*a*b - 3*b^2)*sinh(x)^6 + 8*(7*
(a^2 + 6*a*b - 3*b^2)*cosh(x)^3 + 3*(a^2 + 6*a*b - 3*b^2)*cosh(x))*sinh(x)^
5 + 6*(a^2 + 6*a*b - 3*b^2)*cosh(x)^4 + 2*(35*(a^2 + 6*a*b - 3*b^2)*cosh(x)
^4 + 30*(a^2 + 6*a*b - 3*b^2)*cosh(x)^2 + 3*a^2 + 18*a*b - 9*b^2)*sinh(x)^4
 + 8*(7*(a^2 + 6*a*b - 3*b^2)*cosh(x)^5 + 10*(a^2 + 6*a*b - 3*b^2)*cosh(x)^
3 + 3*(a^2 + 6*a*b - 3*b^2)*cosh(x))*sinh(x)^3 + 4*(a^2 + 6*a*b - 3*b^2)*co
sh(x)^2 + 4*(7*(a^2 + 6*a*b - 3*b^2)*cosh(x)^6 + 15*(a^2 + 6*a*b - 3*b^2)*c
osh(x)^4 + 9*(a^2 + 6*a*b - 3*b^2)*cosh(x)^2 + a^2 + 6*a*b - 3*b^2)*sinh(x)
^2 + a^2 + 6*a*b - 3*b^2 + 8*((a^2 + 6*a*b - 3*b^2)*cosh(x)^7 + 3*(a^2 + 6*
a*b - 3*b^2)*cosh(x)^5 + 3*(a^2 + 6*a*b - 3*b^2)*cosh(x)^3 + (a^2 + 6*a*b -
 3*b^2)*cosh(x))*sinh(x))*sqrt(b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sin
h(x) + sinh(x)^2 - 1)*sqrt(b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(c
osh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(
x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*si
nh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) - sqrt(2)*((a*b
 - 5*b^2)*cosh(x)^6 + 6*(a*b - 5*b^2)*cosh(x)*sinh(x)^5 + (a*b - 5*b^2)*sin
h(x)^6 + (a*b + 3*b^2)*cosh(x)^4 + (15*(a*b - 5*b^2)*cosh(x)^2 + a*b + 3*b^
2)*sinh(x)^4 + 4*(5*(a*b - 5*b^2)*cosh(x)^3 + (a*b + 3*b^2)*cosh(x))*sinh(x
)^3 - (a*b + 3*b^2)*cosh(x)^2 + (15*(a*b - 5*b^2)*cosh(x)^4 + 6*(a*b + 3*b^
2)*cosh(x)^2 - a*b - 3*b^2)*sinh(x)^2 - a*b + 5*b^2 + 2*(3*(a*b - 5*b^2)*co
sh(x)^5 + 2*(a*b + 3*b^2)*cosh(x)^3 - (a*b + 3*b^2)*cosh(x))*sinh(x))*sqrt(
(a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh
(x)^2)))/(b^2*cosh(x)^8 + 8*b^2*cosh(x)*sinh(x)^7 + b^2*sinh(x)^8 + 4*b^2*c
osh(x)^6 + 4*(7*b^2*cosh(x)^2 + b^2)*sinh(x)^6 + 6*b^2*cosh(x)^4 + 8*(7*b^2
*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^5 + 2*(35*b^2*cosh(x)^4 + 30*b^2*cosh(x
)^2 + 3*b^2)*sinh(x)^4 + 4*b^2*cosh(x)^2 + 8*(7*b^2*cosh(x)^5 + 10*b^2*cosh
(x)^3 + 3*b^2*cosh(x))*sinh(x)^3 + 4*(7*b^2*cosh(x)^6 + 15*b^2*cosh(x)^4 +
9*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 8*(b^2*cosh(x)^7 + 3*b^2*cosh(x)^5
 + 3*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(1/2)*tanh(x)^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\text{sech}(x)^2}\,\left(\tanh^4(x)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(x)^2)^(1/2)*tanh(x)^4,x)
[Out] int((a+b*sech(x)^2)^(1/2)*tanh(x)^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\,\text{sech}(x)^2 + a}\,\tanh(x)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(1/2)*tanh(x)^4,x, algorithm="maxima")
[Out] integrate(sqrt(b*sech(x)^2 + a)*tanh(x)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tanh(x)^4\sqrt{a + \frac{b}{\cosh(x)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4*(a + b/cosh(x)^2)^(1/2),x)
[Out] int(tanh(x)^4*(a + b/cosh(x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\,\text{sech}^2(x)}\,\tanh^4(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)**2)**(1/2)*tanh(x)**4,x)
[Out] Integral(sqrt(a + b*sech(x)**2)*tanh(x)**4, x)
```

3.178 $\int \sqrt{a+b\,\mathbf{sech}^2(x)}\,\tanh^3(x)\,dx$

Optimal. Leaf size=59

$$\frac{\left(a+b\,\text{sech}^2(x)\right)^{3/2}}{3b}-\sqrt{a+b\,\text{sech}^2(x)}+\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\,\text{sech}^2(x)}}{\sqrt{a}}\right)$$

```
[Out] 1/3*(a+b*sech(x)^2)^(3/2)/b+arctanh((a+b*sech(x)^2)^(1/2)/a^(1/2))*a^(1/2)-
(a+b*sech(x)^2)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {4139, 446, 80, 50, 63, 208}

$$\frac{\left(a+b\,\text{sech}^2(x)\right)^{3/2}}{3b}-\sqrt{a+b\,\text{sech}^2(x)}+\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\,\text{sech}^2(x)}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sech[x]^2]*Tanh[x]^3,x]
```

```
[Out] Sqrt[a]*ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]] - Sqrt[a + b*Sech[x]^2] + (a
 + b*Sech[x]^2)^(3/2)/(3*b)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \sqrt{a+b\text{sech}^2(x)}\,\tanh^3(x)\,dx = \text{Subst}\left(\int \frac{(-1+x^2)\sqrt{a+bx^2}}{x}\,dx, x, \text{sech}(x)\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{(-1+x)\sqrt{a+bx}}{x}\,dx, x, \text{sech}^2(x)\right)$$
$$= \frac{\left(a+b\text{sech}^2(x)\right)^{3/2}}{3b} - \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \text{sech}^2(x)\right)$$
$$= -\sqrt{a+b\text{sech}^2(x)} + \frac{\left(a+b\text{sech}^2(x)\right)^{3/2}}{3b} - \frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \text{sech}^2\right.$$
$$= -\sqrt{a+b\text{sech}^2(x)} + \frac{\left(a+b\text{sech}^2(x)\right)^{3/2}}{3b} - \frac{a\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{sech}}\right.}{b}$$
$$= \sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right) - \sqrt{a+b\text{sech}^2(x)} + \frac{\left(a+b\text{sech}^2(x)\right)^{3/2}}{3b}$$

Mathematica [A] time = 0.40, size = 90, normalized size = 1.53

$$\frac{1}{3}\cosh(x)\sqrt{a+b\text{sech}^2(x)}\left(\left(\frac{a}{b}-3\right)\text{sech}(x) + \frac{3\sqrt{2}\sqrt{a}\,\log\left(\sqrt{a\cosh(2x)+a+2b}+\sqrt{2}\sqrt{a}\,\cosh(x)\right)}{\sqrt{a\cosh(2x)+a+2b}} + \text{sech}^3(x\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sech[x]^2]*Tanh[x]^3,x]
```

```
[Out] (Cosh[x]*Sqrt[a + b*Sech[x]^2]*((3*Sqrt[2]*Sqrt[a]*Log[Sqrt[2]*Sqrt[a]*Cosh
[x] + Sqrt[a + 2*b + a*Cosh[2*x]]])/Sqrt[a + 2*b + a*Cosh[2*x]] + (-3 + a/b
)*Sech[x] + Sech[x]^3))/3
```

fricas [B] time = 0.73, size = 2394, normalized size = 40.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(1/2)*tanh(x)^3,x, algorithm="fricas")
```

```
[Out] [1/12*(3*(b*cosh(x)^6 + 6*b*cosh(x)*sinh(x)^5 + b*sinh(x)^6 + 3*b*cosh(x)^4
 + 3*(5*b*cosh(x)^2 + b)*sinh(x)^4 + 4*(5*b*cosh(x)^3 + 3*b*cosh(x))*sinh(x
```

```
)^3 + 3*b*cosh(x)^2 + 3*(5*b*cosh(x)^4 + 6*b*cosh(x)^2 + b)*sinh(x)^2 + 6*(
b*cosh(x)^5 + 2*b*cosh(x)^3 + b*cosh(x))*sinh(x) + b)*sqrt(a)*log(((a^3 + 2
*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^7 + (
a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh
(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*(a^3 + 2*a^2*b + a*b^2)*cos
h(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + 3*(2*a^3 + 5*
a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*co
sh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 6*a^3 + 14*a^2*b + 9*a*b^
2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3
+ 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)
^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(2*a^3 + 3*a
^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 + 15*(2*a^3 + 5*a
^2*b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3 + 3*a^2*b + 3*(6*a^3 + 14*a^2*b + 9
*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*((a^2 + 2*a*b + b^2)*cosh(x)^6 + 6*(
a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2 + 2*a*b + b^2)*sinh(x)^6 + 3*(a
^2 + 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 + 2*
a*b + b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 + 2*a*b
+ b^2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4*a*b)*cosh(x)^2 + (15*(a^2 + 2*a*b +
b^2)*cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*cosh(x)^2 + 3*a^2 + 4*a*b)*sinh(x)^
2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 6*(a^2 + 2*a*b + b^2)*cosh(x
)^3 + (3*a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(
x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 +
2*a^2*b + a*b^2)*cosh(x)^7 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^5
+ (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^3 + (2*a^3 + 3*a^2*b)*cosh(x))*sinh(
x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^
3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) +
3*(b*cosh(x)^6 + 6*b*cosh(x)*sinh(x)^5 + b*sinh(x)^6 + 3*b*cosh(x)^4 + 3*(5
*b*cosh(x)^2 + b)*sinh(x)^4 + 4*(5*b*cosh(x)^3 + 3*b*cosh(x))*sinh(x)^3 + 3
*b*cosh(x)^2 + 3*(5*b*cosh(x)^4 + 6*b*cosh(x)^2 + b)*sinh(x)^2 + 6*(b*cosh(
x)^5 + 2*b*cosh(x)^3 + b*cosh(x))*sinh(x) + b)*sqrt(a)*log(-(a*cosh(x)^4 +
4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*a*cosh(x)^2 + b)
*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a
)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x)
 + sinh(x)^2)) + 4*(a*cosh(x)^3 + b*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*co
sh(x)*sinh(x) + sinh(x)^2)) + 4*sqrt(2)*((a - 3*b)*cosh(x)^4 + 4*(a - 3*b)*
cosh(x)*sinh(x)^3 + (a - 3*b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a - 3
*b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a - 3*b)*cosh(x)^3 + (a - b)*cosh(x)
)*sinh(x) + a - 3*b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2
- 2*cosh(x)*sinh(x) + sinh(x)^2)))/(b*cosh(x)^6 + 6*b*cosh(x)*sinh(x)^5 + b
*sinh(x)^6 + 3*b*cosh(x)^4 + 3*(5*b*cosh(x)^2 + b)*sinh(x)^4 + 4*(5*b*cosh(
x)^3 + 3*b*cosh(x))*sinh(x)^3 + 3*b*cosh(x)^2 + 3*(5*b*cosh(x)^4 + 6*b*cosh
(x)^2 + b)*sinh(x)^2 + 6*(b*cosh(x)^5 + 2*b*cosh(x)^3 + b*cosh(x))*sinh(x)
+ b), -1/6*(3*(b*cosh(x)^6 + 6*b*cosh(x)*sinh(x)^5 + b*sinh(x)^6 + 3*b*cosh
(x)^4 + 3*(5*b*cosh(x)^2 + b)*sinh(x)^4 + 4*(5*b*cosh(x)^3 + 3*b*cosh(x))*s
inh(x)^3 + 3*b*cosh(x)^2 + 3*(5*b*cosh(x)^4 + 6*b*cosh(x)^2 + b)*sinh(x)^2
```

```
+ 6*(b*cosh(x)^5 + 2*b*cosh(x)^3 + b*cosh(x))*sinh(x) + b)*sqrt(-a)*arctan(
sqrt(2)*((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2
+ a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cos
h(x)*sinh(x) + sinh(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*s
inh(x)^3 + (a^2 + a*b)*sinh(x)^4 + (2*a^2 + 3*a*b)*cosh(x)^2 + (6*(a^2 + a*
b)*cosh(x)^2 + 2*a^2 + 3*a*b)*sinh(x)^2 + a^2 + 2*(2*(a^2 + a*b)*cosh(x)^3
+ (2*a^2 + 3*a*b)*cosh(x))*sinh(x))) + 3*(b*cosh(x)^6 + 6*b*cosh(x)*sinh(x)
^5 + b*sinh(x)^6 + 3*b*cosh(x)^4 + 3*(5*b*cosh(x)^2 + b)*sinh(x)^4 + 4*(5*b
*cosh(x)^3 + 3*b*cosh(x))*sinh(x)^3 + 3*b*cosh(x)^2 + 3*(5*b*cosh(x)^4 + 6*
b*cosh(x)^2 + b)*sinh(x)^2 + 6*(b*cosh(x)^5 + 2*b*cosh(x)^3 + b*cosh(x))*si
nh(x) + b)*sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)
^2 - 1)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*
cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh
(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(
a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) - 2*sqrt(2)*((a - 3*b)*cosh(
x)^4 + 4*(a - 3*b)*cosh(x)*sinh(x)^3 + (a - 3*b)*sinh(x)^4 + 2*(a - b)*cosh
(x)^2 + 2*(3*(a - 3*b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a - 3*b)*cosh(x)^
3 + (a - b)*cosh(x))*sinh(x) + a - 3*b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a
 + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(b*cosh(x)^6 + 6*b*co
sh(x)*sinh(x)^5 + b*sinh(x)^6 + 3*b*cosh(x)^4 + 3*(5*b*cosh(x)^2 + b)*sinh(
x)^4 + 4*(5*b*cosh(x)^3 + 3*b*cosh(x))*sinh(x)^3 + 3*b*cosh(x)^2 + 3*(5*b*c
osh(x)^4 + 6*b*cosh(x)^2 + b)*sinh(x)^2 + 6*(b*cosh(x)^5 + 2*b*cosh(x)^3 +
b*cosh(x))*sinh(x) + b)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(1/2)*tanh(x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\mathrm{sech}(x)^{2}}\,\left(\tanh^{3}(x)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(x)^2)^(1/2)*tanh(x)^3,x)
```

```
[Out] int((a+b*sech(x)^2)^(1/2)*tanh(x)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\,\mathrm{sech}(x)^{2}+a}\,\tanh(x)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(1/2)*tanh(x)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sech(x)^2 + a)*tanh(x)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \tanh(x)^3 \sqrt{a + \frac{b}{\cosh(x)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3*(a + b/cosh(x)^2)^(1/2),x)
```

```
[Out] int(tanh(x)^3*(a + b/cosh(x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\operatorname{sech}^2(x)}\,\tanh^3(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)**2)**(1/2)*tanh(x)**3,x)
```

```
[Out] Integral(sqrt(a + b*sech(x)**2)*tanh(x)**3, x)
```

3.179 $\int \sqrt{a+b\,\mathbf{sech}^2(x)}\,\tanh^2(x)\,dx$

Optimal. Leaf size=87

$$-\frac{1}{2}\tanh(x)\sqrt{a-b\tanh^2(x)+b}+\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)-\frac{(a-b)\tan^{-1}\left(\frac{\sqrt{b}\,\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{2\sqrt{b}}$$

```
[Out] arctanh(a^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))*a^(1/2)-1/2*(a-b)*arctan(b
^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))/b^(1/2)-1/2*(a+b-b*tanh(x)^2)^(1/2)
*tanh(x)
```

Rubi [A] time = 0.22, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {4141, 1975, 478, 523, 217, 203, 377, 206}

$$\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)-\frac{1}{2}\tanh(x)\sqrt{a-b\tanh^2(x)+b}-\frac{(a-b)\tan^{-1}\left(\frac{\sqrt{b}\,\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sech[x]^2]*Tanh[x]^2,x]
```

```
[Out] -((a - b)*ArcTan[(Sqrt[b]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]])/(2*Sqrt[b])
+ Sqrt[a]*ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]] - (Tanh[x]*S
qrt[a + b - b*Tanh[x]^2])/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*(m + n*(p + q) + 1)), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \sqrt{a+b\text{sech}^2(x)}\,\tanh^2(x)\,dx = \text{Subst}\left(\int \frac{x^2\sqrt{a+b\left(1-x^2\right)}}{1-x^2}\,dx, x, \tanh(x)\right)$$
$$= \text{Subst}\left(\int \frac{x^2\sqrt{a+b-bx^2}}{1-x^2}\,dx, x, \tanh(x)\right)$$
$$= -\frac{1}{2}\tanh(x)\sqrt{a+b-b\tanh^2(x)} + \frac{1}{2}\text{Subst}\left(\int \frac{a+b+(a-b)x^2}{\left(1-x^2\right)\sqrt{a+b-bx^2}}\,dx, x, \text{ta}$$
$$= -\frac{1}{2}\tanh(x)\sqrt{a+b-b\tanh^2(x)} + a\,\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)\sqrt{a+b-bx^2}}\,dx, x, \text{tan}$$
$$= -\frac{1}{2}\tanh(x)\sqrt{a+b-b\tanh^2(x)} + a\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b-b\tanh^2}}\right.$$
$$= -\frac{(a-b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{2\sqrt{b}} + \sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right) - \frac{1}{2}\text{tanh}$$

Mathematica [A] time = 0.29, size = 150, normalized size = 1.72

$$-\frac{\cosh(x)\sqrt{a+b\text{sech}^2(x)}\left(\sqrt{2}(a-b)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}\sinh(x)}{\sqrt{a\cosh(2x)+a+2b}}\right) - 2\sqrt{2}\sqrt{a}\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sinh(x)}{\sqrt{a\cosh(2x)+a+2b}}\right) + \sqrt{b}\,\text{tanh}\right.}{2\sqrt{b}\sqrt{a\cosh(2x)+a+2b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sech[x]^2]*Tanh[x]^2,x]
```

```
[Out] -1/2*(Cosh[x]*Sqrt[a + b*Sech[x]^2]*(Sqrt[2]*(a - b)*ArcTan[(Sqrt[2]*Sqrt[b
]*Sinh[x])/Sqrt[a + 2*b + a*Cosh[2*x]]] - 2*Sqrt[2]*Sqrt[a]*Sqrt[b]*ArcTanh
[(Sqrt[2]*Sqrt[a]*Sinh[x])/Sqrt[a + 2*b + a*Cosh[2*x]]] + Sqrt[b]*Sqrt[a +
2*b + a*Cosh[2*x]]*Sech[x]*Tanh[x]))/(Sqrt[b]*Sqrt[a + 2*b + a*Cosh[2*x]])
```

fricas [B] time = 0.89, size = 4316, normalized size = 49.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(1/2)*tanh(x)^2,x, algorithm="fricas")

[Out] [1/4*((b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*b*cosh(x)^2 +
2*(3*b*cosh(x)^2 + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + b*cosh(x))*sinh(x) + b)*
sqrt(a)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^8
- 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh(x)^
6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3 + 4*a
^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 -
 30*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a*b^2
- b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(
a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*cosh(x)
^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqr
t(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(
x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cos
h(x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh
(x)^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3
 - (a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2
+ a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*cosh(x
)^7 - 3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3 + (a^
3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x
)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)
*sinh(x)^5 + sinh(x)^6)) + ((a - b)*cosh(x)^4 + 4*(a - b)*cosh(x)*sinh(x)^3
 + (a - b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a - b)*cosh(x)^2 + a - b
)*sinh(x)^2 + 4*((a - b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a - b)*sqrt
(-b)*log(-((a - b)*cosh(x)^4 + 4*(a - b)*cosh(x)*sinh(x)^3 + (a - b)*sinh(x
)^4 + 2*(a + 3*b)*cosh(x)^2 + 2*(3*(a - b)*cosh(x)^2 + a + 3*b)*sinh(x)^2 +
 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-b)*sqrt((a
*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x
)^2)) + 4*((a - b)*cosh(x)^3 + (a + 3*b)*cosh(x))*sinh(x) + a - b)/(cosh(x)
^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*co
sh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)) + (b*cosh(x)^4 + 4*b*cosh(x
)*sinh(x)^3 + b*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*b*cosh(x)^2 + b)*sinh(x)^2
 + 4*(b*cosh(x)^3 + b*cosh(x))*sinh(x) + b)*sqrt(a)*log(-(a*cosh(x)^4 + 4*a
*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 +
 a + b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)
*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*
sinh(x) + sinh(x)^2)) + 4*(a*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a)/(cos
h(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) - 2*sqrt(2)*(b*cosh(x)^2 + 2*b*cos
h(x)*sinh(x) + b*sinh(x)^2 - b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/
(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(b*cosh(x)^4 + 4*b*cosh(x)*si
nh(x)^3 + b*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*b*cosh(x)^2 + b)*sinh(x)^2 + 4
*(b*cosh(x)^3 + b*cosh(x))*sinh(x) + b), -1/4*(2*((a - b)*cosh(x)^4 + 4*(a
- b)*cosh(x)*sinh(x)^3 + (a - b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a
- b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a - b)*cosh(x)^3 + (a - b)*cosh(x))
*sinh(x) + a - b)*sqrt(b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + s
```

```
inh(x)^2 - 1)*sqrt(b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2
 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a
*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2
+ 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) - (b*cosh(x)^4 + 4*b*co
sh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*b*cosh(x)^2 + b)*sinh(
x)^2 + 4*(b*cosh(x)^3 + b*cosh(x))*sinh(x) + b)*sqrt(a)*log((a*b^2*cosh(x)^
8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6
 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh(x)^6 + 4*(14*a*b^2*cosh(x)^3 -
 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^4 +
 (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 - 30*(a*b^2 - b^3)*cosh(x)^2
)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a*b^2 - b^3)*cosh(x)^3 + (a^3 + 4
*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(a^3 + 3*a^2*b)*cosh(x)^2 +
2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*cosh(x)^4 + a^3 + 3*a^2*b + 3*(a^3
 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2
*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 -
 b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 + 4*
a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 - 4*a*b)*sinh(x
)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 + 4*a*b)*cosh(x))*s
inh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*c
osh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*cosh(x)^7 - 3*(a*b^2 - b^3)*cosh(
x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3 + (a^3 + 3*a^2*b)*cosh(x))*sinh(
x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^
3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) -
(b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*b
*cosh(x)^2 + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + b*cosh(x))*sinh(x) + b)*sqrt(a
)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + b)*cosh(
x)^2 + 2*(3*a*cosh(x)^2 + a + b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)
*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b
)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(a*cosh(x)^3 + (a + b)*c
osh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) + 2*sqrt(
2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - b)*sqrt((a*cosh(x)^2
+ a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(b*c
osh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*b*cos
h(x)^2 + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + b*cosh(x))*sinh(x) + b), -1/4*(2*(
b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*b*
cosh(x)^2 + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + b*cosh(x))*sinh(x) + b)*sqrt(-a
)*arctan(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + a)*sqrt
(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh
(x) + sinh(x)^2))/(a*b*cosh(x)^4 + 4*a*b*cosh(x)*sinh(x)^3 + a*b*sinh(x)^4
- (a^2 + 3*a*b)*cosh(x)^2 + (6*a*b*cosh(x)^2 - a^2 - 3*a*b)*sinh(x)^2 - a^2
 + 2*(2*a*b*cosh(x)^3 - (a^2 + 3*a*b)*cosh(x))*sinh(x))) + 2*(b*cosh(x)^4 +
 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*b*cosh(x)^2 + b
)*sinh(x)^2 + 4*(b*cosh(x)^3 + b*cosh(x))*sinh(x) + b)*sqrt(-a)*arctan(sqrt
(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a)*sqrt((a*cosh(x
)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(
```

```
a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 +
 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x)
)*sinh(x) + a)) - ((a - b)*cosh(x)^4 + 4*(a - b)*cosh(x)*sinh(x)^3 + (a - b
)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a - b)*cosh(x)^2 + a - b)*sinh(x)
^2 + 4*((a - b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a - b)*sqrt(-b)*log(
-((a - b)*cosh(x)^4 + 4*(a - b)*cosh(x)*sinh(x)^3 + (a - b)*sinh(x)^4 + 2*(
a + 3*b)*cosh(x)^2 + 2*(3*(a - b)*cosh(x)^2 + a + 3*b)*sinh(x)^2 + 2*sqrt(2
)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-b)*sqrt((a*cosh(x)^
2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4
*((a - b)*cosh(x)^3 + (a + 3*b)*cosh(x))*sinh(x) + a - b)/(cosh(x)^4 + 4*co
sh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 +
 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)) + 2*sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(
x)*sinh(x) + b*sinh(x)^2 - b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(c
osh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(b*cosh(x)^4 + 4*b*cosh(x)*sinh
(x)^3 + b*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*b*cosh(x)^2 + b)*sinh(x)^2 + 4*(
b*cosh(x)^3 + b*cosh(x))*sinh(x) + b), -1/2*((b*cosh(x)^4 + 4*b*cosh(x)*sin
h(x)^3 + b*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*b*cosh(x)^2 + b)*sinh(x)^2 + 4*
(b*cosh(x)^3 + b*cosh(x))*sinh(x) + b)*sqrt(-a)*arctan(sqrt(2)*(b*cosh(x)^2
 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*si
nh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*b*cosh(x
)^4 + 4*a*b*cosh(x)*sinh(x)^3 + a*b*sinh(x)^4 - (a^2 + 3*a*b)*cosh(x)^2 + (
6*a*b*cosh(x)^2 - a^2 - 3*a*b)*sinh(x)^2 - a^2 + 2*(2*a*b*cosh(x)^3 - (a^2
+ 3*a*b)*cosh(x))*sinh(x))) + (b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh
(x)^4 + 2*b*cosh(x)^2 + 2*(3*b*cosh(x)^2 + b)*sinh(x)^2 + 4*(b*cosh(x)^3 +
b*cosh(x))*sinh(x) + b)*sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh
(x) + sinh(x)^2 + 1)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(c
osh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(
x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*si
nh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) + ((a - b)*cosh
(x)^4 + 4*(a - b)*cosh(x)*sinh(x)^3 + (a - b)*sinh(x)^4 + 2*(a - b)*cosh(x)
^2 + 2*(3*(a - b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a - b)*cosh(x)^3 + (a
- b)*cosh(x))*sinh(x) + a - b)*sqrt(b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x
)*sinh(x) + sinh(x)^2 - 1)*sqrt(b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*
b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*
sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*
b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) + sqrt(2)*
(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - b)*sqrt((a*cosh(x)^2 + a
*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(b*cosh
(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*b*cosh(x
)^2 + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + b*cosh(x))*sinh(x) + b)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(1/2)*tanh(x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\mathrm{sech}(x)^2}\,\left(\tanh^2(x)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(x)^2)^(1/2)*tanh(x)^2,x)
```

```
[Out] int((a+b*sech(x)^2)^(1/2)*tanh(x)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\,\mathrm{sech}(x)^2+a}\,\tanh(x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(1/2)*tanh(x)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sech(x)^2 + a)*tanh(x)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tanh(x)^2\sqrt{a+\frac{b}{\cosh(x)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2*(a + b/cosh(x)^2)^(1/2),x)
```

```
[Out] int(tanh(x)^2*(a + b/cosh(x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\,\mathrm{sech}^2\,(x)}\,\tanh^2\,(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)**2)**(1/2)*tanh(x)**2,x)
```

```
[Out] Integral(sqrt(a + b*sech(x)**2)*tanh(x)**2, x)
```

3.180 $\int \sqrt{a+b\,\mathrm{sech}^2(x)}\,\tanh(x)\,dx$

Optimal. Leaf size=40

$$\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}^2(x)}}{\sqrt{a}}\right)-\sqrt{a+b\,\mathrm{sech}^2(x)}$$

```
[Out] arctanh((a+b*sech(x)^2)^(1/2)/a^(1/2))*a^(1/2)-(a+b*sech(x)^2)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4139, 266, 50, 63, 208}

$$\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}^2(x)}}{\sqrt{a}}\right)-\sqrt{a+b\,\mathrm{sech}^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sech[x]^2]*Tanh[x],x]
```

```
[Out] Sqrt[a]*ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]] - Sqrt[a + b*Sech[x]^2]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\begin{aligned}
\int \sqrt{a+b\operatorname{sech}^2(x)}\,\tanh(x)\,dx &= -\operatorname{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x}\,dx, x, \operatorname{sech}(x)\right)\\
&= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \operatorname{sech}^2(x)\right)\right)\\
&= -\sqrt{a+b\operatorname{sech}^2(x)} - \frac{1}{2}a\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \operatorname{sech}^2(x)\right)\\
&= -\sqrt{a+b\operatorname{sech}^2(x)} - \frac{a\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{sech}^2(x)}\right)}{b}\\
&= \sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right) - \sqrt{a+b\operatorname{sech}^2(x)}
\end{aligned}$$

Mathematica [B] time = 0.27, size = 90, normalized size = 2.25

$$-\frac{\sqrt{a+b\operatorname{sech}^2(x)}\left(-\sqrt{2}\sqrt{a}\sqrt{b}\cosh(x)\sqrt{\frac{a\cosh(2x)+a+2b}{b}}\sinh^{-1}\left(\frac{\sqrt{a}\cosh(x)}{\sqrt{b}}\right)+a\cosh(2x)+a+2b\right)}{a\cosh(2x)+a+2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sech[x]^2]*Tanh[x],x]
```

```
[Out] -(((a + 2*b + a*Cosh[2*x] - Sqrt[2]*Sqrt[a]*Sqrt[b]*ArcSinh[(Sqrt[a]*Cosh[x
])/Sqrt[b]]*Cosh[x]*Sqrt[(a + 2*b + a*Cosh[2*x])/b])*Sqrt[a + b*Sech[x]^2])
/(a + 2*b + a*Cosh[2*x]))
```

fricas [B] time = 0.49, size = 1608, normalized size = 40.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(1/2)*tanh(x),x, algorithm="fricas")
```

```
[Out] [1/4*((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*log(((a^3 + 2
*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^7 + (
a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh
(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*(a^3 + 2*a^2*b + a*b^2)*cos
h(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + 3*(2*a^3 + 5*
a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*co
sh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 6*a^3 + 14*a^2*b + 9*a*b^
2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3
+ 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)
^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(2*a^3 + 3*a
^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 + 15*(2*a^3 + 5*a
^2*b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3 + 3*a^2*b + 3*(6*a^3 + 14*a^2*b + 9
*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*((a^2 + 2*a*b + b^2)*cosh(x)^6 + 6*(
a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2 + 2*a*b + b^2)*sinh(x)^6 + 3*(a
^2 + 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 + 2*
a*b + b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 + 2*a*b
+ b^2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4*a*b)*cosh(x)^2 + (15*(a^2 + 2*a*b +
b^2)*cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*cosh(x)^2 + 3*a^2 + 4*a*b)*sinh(x)^
2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 6*(a^2 + 2*a*b + b^2)*cosh(x
)^3 + (3*a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(
x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 +
2*a^2*b + a*b^2)*cosh(x)^7 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^5
+ (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^3 + (2*a^3 + 3*a^2*b)*cosh(x))*sinh(
x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^
3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) +
(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*log(-(a*cosh(x)^4 +
 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*a*cosh(x)^2 + b
)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(
a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x
) + sinh(x)^2)) + 4*(a*cosh(x)^3 + b*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*c
```

```
osh(x)*sinh(x) + sinh(x)^2)) - 4*sqrt(2)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 +
a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(cosh(x)^2 + 2*cosh(
x)*sinh(x) + sinh(x)^2 + 1), -1/2*((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)
^2 + 1)*sqrt(-a)*arctan(sqrt(2)*((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh
(x) + (a + b)*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a +
 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a^2 + a*b)*cosh(x)^4 +
 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 + (2*a^2 + 3*a*b)*
cosh(x)^2 + (6*(a^2 + a*b)*cosh(x)^2 + 2*a^2 + 3*a*b)*sinh(x)^2 + a^2 + 2*(
2*(a^2 + a*b)*cosh(x)^3 + (2*a^2 + 3*a*b)*cosh(x))*sinh(x))) + (cosh(x)^2 +
 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*
cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 +
 a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*c
osh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 +
 a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) + 2
*sqrt(2)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*
sinh(x) + sinh(x)^2)))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)]
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(1/2)*tanh(x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [**A**] time = 0.07, size = 43, normalized size = 1.08

$$-\sqrt{a+b\operatorname{sech}(x)^{2}}+\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\operatorname{sech}(x)^{2}}}{\operatorname{sech}(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(x)^2)^(1/2)*tanh(x),x)
```

```
[Out] -(a+b*sech(x)^2)^(1/2)+a^(1/2)*ln((2*a+2*a^(1/2)*(a+b*sech(x)^2)^(1/2))/sec
h(x))
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\operatorname{sech}(x)^{2}+a}\,\tanh(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(1/2)*tanh(x),x, algorithm="maxima")
[Out] integrate(sqrt(b*sech(x)^2 + a)*tanh(x), x)
```

mupad [B] time = 1.71, size = 32, normalized size = 0.80

$$\sqrt{a}\ \mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{\cosh(x)^2}}}{\sqrt{a}}\right)-\sqrt{a+\frac{b}{\cosh(x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)*(a + b/cosh(x)^2)^(1/2),x)
[Out] a^(1/2)*atanh((a + b/cosh(x)^2)^(1/2)/a^(1/2)) - (a + b/cosh(x)^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a+b\operatorname{sech}^2(x)}\ \tanh(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)**2)**(1/2)*tanh(x),x)
[Out] Integral(sqrt(a + b*sech(x)**2)*tanh(x), x)
```

3.181 $\int \sqrt{a+b\,\mathbf{sech}^2(x)}\,dx$

Optimal. Leaf size=59

$$\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)+\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)$$

```
[Out] arctanh(a^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))*a^(1/2)+arctan(b^(1/2)*tan
h(x)/(a+b-b*tanh(x)^2)^(1/2))*b^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4128, 402, 217, 203, 377, 206}

$$\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)+\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]] + Sqrt[a]*ArcTa
nh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+b\text{sech}^2(x)}\,dx &= \text{Subst}\left(\int \frac{\sqrt{a+b-bx^2}}{1-x^2}\,dx, x, \tanh(x)\right)\\ &= a\,\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)\sqrt{a+b-bx^2}}\,dx, x, \tanh(x)\right) + b\,\text{Subst}\left(\int \frac{1}{\sqrt{a+b-bx^2}}\,dx, x, \text{ta}\right.\\ &= a\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right) + b\,\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{\text{ta}}{\sqrt{a+b-}}\right.\\ &= \sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right) + \sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)\end{aligned}$$

Mathematica [B] time = 0.74, size = 134, normalized size = 2.27

$$\frac{\sqrt{2}\cosh(x)\sqrt{a+b\text{sech}^2(x)}\left(\sqrt{a}\sqrt{a+b}\sinh^{-1}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{a+b}}\right)\sqrt{\frac{a\cosh(2x)+a+2b}{a+b}} + \sqrt{b}\sqrt{a\cosh(2x)+a+2b}\tan^{-1}\right.}{a\cosh(2x)+a+2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] (Sqrt[2]*Cosh[x]*(Sqrt[b]*ArcTan[(Sqrt[2]*Sqrt[b]*Sinh[x])/Sqrt[a + 2*b + a
*Cosh[2*x]]]*Sqrt[a + 2*b + a*Cosh[2*x]] + Sqrt[a]*Sqrt[a + b]*ArcSinh[(Sqr
t[a]*Sinh[x])/Sqrt[a + b]]*Sqrt[(a + 2*b + a*Cosh[2*x])/(a + b)])*Sqrt[a +
b*Sech[x]^2])/(a + 2*b + a*Cosh[2*x])
```

fricas [B] time = 0.58, size = 2949, normalized size = 49.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(a)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(
x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sin
h(x)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3
+ 4*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*
b^2 - 30*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a
*b^2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3
+ 2*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*co
sh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2
+ sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*
cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^
2*cosh(x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2
*cosh(x)^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh
(x)^3 - (a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(
x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*c
osh(x)^7 - 3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3
+ (a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*c
osh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*co
sh(x)*sinh(x)^5 + sinh(x)^6)) + 1/2*sqrt(-b)*log(-((a - b)*cosh(x)^4 + 4*(a
 - b)*cosh(x)*sinh(x)^3 + (a - b)*sinh(x)^4 + 2*(a + 3*b)*cosh(x)^2 + 2*(3*
(a - b)*cosh(x)^2 + a + 3*b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*s
inh(x) + sinh(x)^2 - 1)*sqrt(-b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)
/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a - b)*cosh(x)^3 + (a +
 3*b)*cosh(x))*sinh(x) + a - b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^
4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*s
inh(x) + 1)) + 1/4*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*si
nh(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + b)*sinh(x)^2 + sqrt(
2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*sqrt((a*cosh(x)^
2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4
*(a*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x
) + sinh(x)^2)), sqrt(b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + si
nh(x)^2 - 1)*sqrt(b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2
```

```
- 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*
sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 +
 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) + 1/4*sqrt(a)*log((a*b^2
*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^8 - 2*(a*b^2 - b^3)*
cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh(x)^6 + 4*(14*a*b^2*co
sh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*co
sh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 - 30*(a*b^2 - b^3)*
cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a*b^2 - b^3)*cosh(x)^3 +
 (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(a^3 + 3*a^2*b)*cos
h(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*cosh(x)^4 + a^3 + 3*a^2*b
 + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^
6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*co
sh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 -
(a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 - 4*a*
b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 + 4*a*b)*c
osh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x
)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*cosh(x)^7 - 3*(a*b^2 - b
^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3 + (a^3 + 3*a^2*b)*cosh(
x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20
*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(
x)^6)) + 1/4*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^
4 + 2*(a + b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + b)*sinh(x)^2 + sqrt(2)*(co
sh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*sqrt((a*cosh(x)^2 + a*
sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(a*co
sh(x)^3 + (a + b)*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + si
nh(x)^2)), -1/2*sqrt(-a)*arctan(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x)
+ b*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cos
h(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*b*cosh(x)^4 + 4*a*b*cosh(x)*sin
h(x)^3 + a*b*sinh(x)^4 - (a^2 + 3*a*b)*cosh(x)^2 + (6*a*b*cosh(x)^2 - a^2 -
 3*a*b)*sinh(x)^2 - a^2 + 2*(2*a*b*cosh(x)^3 - (a^2 + 3*a*b)*cosh(x))*sinh(
x))) - 1/2*sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)
^2 + 1)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*
cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh
(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(
a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) + 1/2*sqrt(-b)*log(-((a - b)
*cosh(x)^4 + 4*(a - b)*cosh(x)*sinh(x)^3 + (a - b)*sinh(x)^4 + 2*(a + 3*b)*
cosh(x)^2 + 2*(3*(a - b)*cosh(x)^2 + a + 3*b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x
)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-b)*sqrt((a*cosh(x)^2 + a*sin
h(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a - b)
*cosh(x)^3 + (a + 3*b)*cosh(x))*sinh(x) + a - b)/(cosh(x)^4 + 4*cosh(x)*sin
h(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(
x)^3 + cosh(x))*sinh(x) + 1)), -1/2*sqrt(-a)*arctan(sqrt(2)*(b*cosh(x)^2 +
2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(
x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*b*cosh(x)^4
 + 4*a*b*cosh(x)*sinh(x)^3 + a*b*sinh(x)^4 - (a^2 + 3*a*b)*cosh(x)^2 + (6*a
```

```
*b*cosh(x)^2 - a^2 - 3*a*b)*sinh(x)^2 - a^2 + 2*(2*a*b*cosh(x)^3 - (a^2 + 3
*a*b)*cosh(x))*sinh(x))) - 1/2*sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(
x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a +
2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x
)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a +
2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) + sqrt(b
)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(b)*sq
rt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + s
inh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)
*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a +
2*b)*cosh(x))*sinh(x) + a))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, replacing 0 by ` u`, a substitution
 variable should perhaps be purged.Warning, replacing 0 by ` u`, a substitu
tion variable should perhaps be purged.Warning, replacing 0 by ` u`, a subs
titution variable should perhaps be purged.Warning, replacing 0 by ` u`, a
substitution variable should perhaps be purged.Warning, replacing 0 by ` u`
, a substitution variable should perhaps be purged.Warning, replacing 0 by
` u`, a substitution variable should perhaps be purged.Warning, replacing 0
 by ` u`, a substitution variable should perhaps be purged.Warning, replaci
ng 0 by ` u`, a substitution variable should perhaps be purged.Warning, rep
lacing 0 by ` u`, a substitution variable should perhaps be purged.Warning,
 integration of abs or sign assumes constant sign by intervals (correct if
the argument is real):Check [abs(t_nostep)]Warning, need to choose a branch
 for the root of a polynomial with parameters. This might be wrong.Non regu
lar value [0] was discarded and replaced randomly by 0=[-10]Warning, need t
o choose a branch for the root of a polynomial with parameters. This might
be wrong.Non regular value [0] was discarded and replaced randomly by 0=[-3
9]Warning, need to choose a branch for the root of a polynomial with parame
ters. This might be wrong.Non regular value [0] was discarded and replaced
randomly by 0=[35]Warning, need to choose a branch for the root of a polyno
mial with parameters. This might be wrong.Non regular value [0] was discard
ed and replaced randomly by 0=[90]Warning, need to choose a branch for the
root of a polynomial with parameters. This might be wrong.Non regular value
 [0] was discarded and replaced randomly by 0=[-98]Warning, need to choose
a branch for the root of a polynomial with parameters. This might be wrong.
```

```
Non regular value [0] was discarded and replaced randomly by 0=[-5]Evaluati
on time: 0.57index.cc index_m operator + Error: Bad Argument Value
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\operatorname{sech}(x)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(x)^2)^(1/2),x)
```

```
[Out] int((a+b*sech(x)^2)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\operatorname{sech}(x)^{2}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sech(x)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{a+\frac{b}{\cosh(x)^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cosh(x)^2)^(1/2),x)
```

```
[Out] int((a + b/cosh(x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\operatorname{sech}^{2}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sech(x)**2), x)
```

3.182 $\int \coth(x)\sqrt{a+b\,\mathrm{sech}^2(x)}\,dx$

Optimal. Leaf size=56

$$\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}^2(x)}}{\sqrt{a}}\right)-\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}^2(x)}}{\sqrt{a+b}}\right)$$

```
[Out] arctanh((a+b*sech(x)^2)^(1/2)/a^(1/2))*a^(1/2)-arctanh((a+b*sech(x)^2)^(1/2
)/(a+b)^(1/2))*(a+b)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4139, 446, 83, 63, 208}

$$\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}^2(x)}}{\sqrt{a}}\right)-\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}^2(x)}}{\sqrt{a+b}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]*Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] Sqrt[a]*ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]] - Sqrt[a + b]*ArcTanh[Sqrt[a
 + b*Sech[x]^2]/Sqrt[a + b]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 83

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x)
, x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\begin{aligned}
\int \coth(x)\sqrt{a+b\operatorname{sech}^2(x)}\,dx &= \text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x\left(-1+x^2\right)}\,dx, x, \operatorname{sech}(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx}}{(-1+x)x}\,dx, x, \operatorname{sech}^2(x)\right)\\
&= -\left(\frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \operatorname{sech}^2(x)\right)\right)+\frac{1}{2}(a+b)\,\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a}}\right.\\
&= -\frac{a\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{sech}^2(x)}\right)}{b}+\frac{(a+b)\,\text{Subst}\left(\int \frac{1}{-1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x,\right.}{b}\\
&= \sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)-\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a+b}}\right)
\end{aligned}$$

Mathematica [A] time = 0.13, size = 111, normalized size = 1.98

$$\frac{\sqrt{2}\cosh(x)\sqrt{a+b\operatorname{sech}^2(x)}\left(\sqrt{a}\log\left(\sqrt{a\cosh(2x)+a+2b}+\sqrt{2}\sqrt{a}\cosh(x)\right)-\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{2a+2b}\cosh(}{\sqrt{a\cosh(2x)+a}}\right.\right.}{\sqrt{a\cosh(2x)+a+2b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]*Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] (Sqrt[2]*Cosh[x]*(-(Sqrt[a + b]*ArcTanh[(Sqrt[2*a + 2*b]*Cosh[x])/Sqrt[a +
2*b + a*Cosh[2*x]]]) + Sqrt[a]*Log[Sqrt[2]*Sqrt[a]*Cosh[x] + Sqrt[a + 2*b +
 a*Cosh[2*x]]])*Sqrt[a + b*Sech[x]^2])/Sqrt[a + 2*b + a*Cosh[2*x]]
```

fricas [B] time = 0.62, size = 3597, normalized size = 64.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*sech(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(a)*log(((a^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*
b^2)*cosh(x)*sinh(x)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*a
^2*b + 4*a*b^2 + b^3)*cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*(
a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)
*cosh(x)^3 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + (6*a^
3 + 14*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 +
 6*a^3 + 14*a^2*b + 9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^
2)*sinh(x)^4 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a^2*
b + 4*a*b^2 + b^3)*cosh(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*sinh(x
)^3 + a^3 + 2*(2*a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*c
osh(x)^6 + 15*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3 + 3*a^2*b
 + 3*(6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*((a^2 + 2*
a*b + b^2)*cosh(x)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2 + 2*a
*b + b^2)*sinh(x)^6 + 3*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b +
 b^2)*cosh(x)^2 + a^2 + 2*a*b + b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*c
osh(x)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4*a*b)*cosh(
x)^2 + (15*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*cosh(x)^2
 + 3*a^2 + 4*a*b)*sinh(x)^2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 6*
(a^2 + 2*a*b + b^2)*cosh(x)^3 + (3*a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*s
qrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) +
sinh(x)^2)) + 4*(2*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^7 + 3*(2*a^3 + 5*a^2*b +
 4*a*b^2 + b^3)*cosh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^3 + (2*a^3
 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)
^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*
sinh(x)^5 + sinh(x)^6)) + 1/2*sqrt(a + b)*log(((2*a + b)*cosh(x)^4 + 4*(2*a
 + b)*cosh(x)*sinh(x)^3 + (2*a + b)*sinh(x)^4 + 2*(2*a + 3*b)*cosh(x)^2 + 2
*(3*(2*a + b)*cosh(x)^2 + 2*a + 3*b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*c
osh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2
 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((2*a + b)*cos
h(x)^3 + (2*a + 3*b)*cosh(x))*sinh(x) + 2*a + b)/(cosh(x)^4 + 4*cosh(x)*sin
```

```
h(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(
x)^3 - cosh(x))*sinh(x) + 1)) + 1/4*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)
*sinh(x)^3 + a*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*a*cosh(x)^2 + b)*sinh(x)^2
+ sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a)*sqrt((a*c
osh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^
2)) + 4*(a*cosh(x)^3 + b*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(
x) + sinh(x)^2)), sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x
) + sinh(x)^2 + 1)*sqrt(-a - b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/
(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sin
h(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*
sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) + 1/4*sqrt(a)
*log(((a^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*cosh(x)
*sinh(x)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*a^2*b + 4*a*b
^2 + b^3)*cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*(a^3 + 2*a^2*
b + a*b^2)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 +
 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + (6*a^3 + 14*a^2*b
 + 9*a*b^2)*cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 6*a^3 + 14*
a^2*b + 9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^4
 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a^2*b + 4*a*b^2
+ b^3)*cosh(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 +
2*(2*a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 + 1
5*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3 + 3*a^2*b + 3*(6*a^3
+ 14*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*((a^2 + 2*a*b + b^2)*c
osh(x)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2 + 2*a*b + b^2)*si
nh(x)^6 + 3*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(x
)^2 + a^2 + 2*a*b + b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(x)^3 + 3
*(a^2 + 2*a*b + b^2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4*a*b)*cosh(x)^2 + (15*(
a^2 + 2*a*b + b^2)*cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*cosh(x)^2 + 3*a^2 + 4
*a*b)*sinh(x)^2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 6*(a^2 + 2*a*b
 + b^2)*cosh(x)^3 + (3*a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(
x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))
+ 4*(2*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^7 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b
^3)*cosh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^3 + (2*a^3 + 3*a^2*b)*
cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2
 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 +
sinh(x)^6)) + 1/4*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sin
h(x)^4 + 2*b*cosh(x)^2 + 2*(3*a*cosh(x)^2 + b)*sinh(x)^2 + sqrt(2)*(cosh(x)
^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(
x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(a*cosh(x)
^3 + b*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)),
-1/2*sqrt(-a)*arctan(sqrt(2)*((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x)
 + (a + b)*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*
b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*
(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 + (2*a^2 + 3*a*b)*cos
h(x)^2 + (6*(a^2 + a*b)*cosh(x)^2 + 2*a^2 + 3*a*b)*sinh(x)^2 + a^2 + 2*(2*(
```

```
a^2 + a*b)*cosh(x)^3 + (2*a^2 + 3*a*b)*cosh(x))*sinh(x))) - 1/2*sqrt(-a)*ar
ctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a)*sqrt(
(a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh
(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*co
sh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b
)*cosh(x))*sinh(x) + a)) + 1/2*sqrt(a + b)*log(((2*a + b)*cosh(x)^4 + 4*(2*
a + b)*cosh(x)*sinh(x)^3 + (2*a + b)*sinh(x)^4 + 2*(2*a + 3*b)*cosh(x)^2 +
2*(3*(2*a + b)*cosh(x)^2 + 2*a + 3*b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*
cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*sqrt((a*cosh(x)^2 + a*sinh(x)^
2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((2*a + b)*co
sh(x)^3 + (2*a + 3*b)*cosh(x))*sinh(x) + 2*a + b)/(cosh(x)^4 + 4*cosh(x)*si
nh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh
(x)^3 - cosh(x))*sinh(x) + 1)), -1/2*sqrt(-a)*arctan(sqrt(2)*((a + b)*cosh(
x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*
cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)
^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)
*sinh(x)^4 + (2*a^2 + 3*a*b)*cosh(x)^2 + (6*(a^2 + a*b)*cosh(x)^2 + 2*a^2 +
 3*a*b)*sinh(x)^2 + a^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 + (2*a^2 + 3*a*b)*cosh
(x))*sinh(x))) - 1/2*sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x)
 + sinh(x)^2 - 1)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh
(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^
3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(
x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) + sqrt(-a - b)*arc
tan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a - b)*sq
rt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + s
inh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)
*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a +
2*b)*cosh(x))*sinh(x) + a))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, replacing 0 by ` u`, a substitution
 variable should perhaps be purged.Warning, replacing 0 by ` u`, a substitu
tion variable should perhaps be purged.Warning, replacing 0 by ` u`, a subs
titution variable should perhaps be purged.Warning, replacing 0 by ` u`, a
substitution variable should perhaps be purged.Warning, replacing 0 by ` u`
, a substitution variable should perhaps be purged.Warning, replacing 0 by
` u`, a substitution variable should perhaps be purged.Warning, replacing 0
 by ` u`, a substitution variable should perhaps be purged.Warning, replaci
```

```
ng 0 by ` u`, a substitution variable should perhaps be purged.Warning, rep
lacing 0 by ` u`, a substitution variable should perhaps be purged.Warning,
 integration of abs or sign assumes constant sign by intervals (correct if
the argument is real):Check [abs(t_nostep)]Warning, need to choose a branch
 for the root of a polynomial with parameters. This might be wrong.Non regu
lar value [0] was discarded and replaced randomly by 0=[-59]Warning, need t
o choose a branch for the root of a polynomial with parameters. This might
be wrong.Non regular value [0] was discarded and replaced randomly by 0=[-9
2]Warning, need to choose a branch for the root of a polynomial with parame
ters. This might be wrong.Non regular value [0] was discarded and replaced
randomly by 0=[80]Warning, need to choose a branch for the root of a polyno
mial with parameters. This might be wrong.Non regular value [0] was discard
ed and replaced randomly by 0=[84]Warning, need to choose a branch for the
root of a polynomial with parameters. This might be wrong.Non regular value
 [0] was discarded and replaced randomly by 0=[89]Warning, need to choose a
 branch for the root of a polynomial with parameters. This might be wrong.N
on regular value [0] was discarded and replaced randomly by 0=[-4]Precision
 problem choosing root in common_EXT, current precision 14Warning, need to
choose a branch for the root of a polynomial with parameters. This might be
 wrong.Non regular value [0] was discarded and replaced randomly by 0=[16]W
arning, need to choose a branch for the root of a polynomial with parameter
s. This might be wrong.Non regular value [0] was discarded and replaced ran
domly by 0=[-86]Precision problem choosing root in common_EXT, current prec
ision 14Evaluation time: 0.62index.cc index_m operator + Error: Bad Argumen
t Value
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \coth(x)\sqrt{a+b\,\mathrm{sech}(x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)*(a+b*sech(x)^2)^(1/2),x)
```

```
[Out] int(coth(x)*(a+b*sech(x)^2)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\,\mathrm{sech}(x)^2+a}\,\coth(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*sech(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sech(x)^2 + a)*coth(x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \coth(x)\sqrt{a+\frac{b}{\cosh(x)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)*(a + b/cosh(x)^2)^(1/2),x)
```

```
[Out] int(coth(x)*(a + b/cosh(x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\,\mathrm{sech}^2\,(x)}\,\coth\,(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*sech(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sech(x)**2)*coth(x), x)
```

3.183 $\int \coth^2(x)\sqrt{a+b\mathbf{sech}^2(x)}\,dx$

Optimal. Leaf size=48

$$\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)-\coth(x)\sqrt{a-b\tanh^2(x)+b}$$

```
[Out] arctanh(a^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))*a^(1/2)-coth(x)*(a+b-b*tan
h(x)^2)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {4141, 1975, 475, 12, 377, 206}

$$\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)-\coth(x)\sqrt{a-b\tanh^2(x)+b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2*Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] Sqrt[a]*ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]] - Coth[x]*Sqrt
[a + b - b*Tanh[x]^2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \coth^2(x)\sqrt{a+b\text{sech}^2(x)}\,dx = \text{Subst}\left(\int \frac{\sqrt{a+b\left(1-x^2\right)}}{x^2\left(1-x^2\right)}\,dx, x, \tanh(x)\right)$$

$$= \text{Subst}\left(\int \frac{\sqrt{a+b-bx^2}}{x^2\left(1-x^2\right)}\,dx, x, \tanh(x)\right)$$

$$= -\coth(x)\sqrt{a+b-b\tanh^2(x)} + \text{Subst}\left(\int \frac{a}{\left(1-x^2\right)\sqrt{a+b-bx^2}}\,dx, x, \text{tanh}$$

$$= -\coth(x)\sqrt{a+b-b\tanh^2(x)} + a\,\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)\sqrt{a+b-bx^2}}\,dx, x, \text{tan}$$

$$= -\coth(x)\sqrt{a+b-b\tanh^2(x)} + a\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b-b\tanh^2(}}$$

$$= \sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right) - \coth(x)\sqrt{a+b-b\tanh^2(x)}$$

Mathematica [A] time = 0.52, size = 75, normalized size = 1.56

$$\sqrt{a+b\text{sech}^2(x)}\left(\frac{\sqrt{2}\sqrt{a}\cosh(x)\sinh^{-1}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{a+b}}\right)}{\sqrt{a+b}\sqrt{\frac{a\cosh(2x)+a+2b}{a+b}}} - \coth(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2*Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] ((Sqrt[2]*Sqrt[a]*ArcSinh[(Sqrt[a]*Sinh[x])/Sqrt[a + b]]*Cosh[x])/(Sqrt[a +
 b]*Sqrt[(a + 2*b + a*Cosh[2*x])/(a + b)]) - Coth[x])*Sqrt[a + b*Sech[x]^2]
```

fricas [B] time = 0.52, size = 1303, normalized size = 27.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2*(a+b*sech(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a)*log((a*b^2*co
sh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^8 - 2*(a*b^2 - b^3)*cos
```

```
h(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh(x)^6 + 4*(14*a*b^2*cosh(
x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(
x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 - 30*(a*b^2 - b^3)*cos
h(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a*b^2 - b^3)*cosh(x)^3 + (a
^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(a^3 + 3*a^2*b)*cosh(x
)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*cosh(x)^4 + a^3 + 3*a^2*b +
3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 +
 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(
x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^
2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 - 4*a*b)*
sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 + 4*a*b)*cosh
(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2
 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*cosh(x)^7 - 3*(a*b^2 - b^3)
*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3 + (a^3 + 3*a^2*b)*cosh(x))
*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*co
sh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^
6)) + (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a)*log(-(a*cosh(
x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*a*c
osh(x)^2 + a + b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh
(x)^2 + 1)*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 -
2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(a*cosh(x)^3 + (a + b)*cosh(x))*sinh(x)
 + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) - 4*sqrt(2)*sqrt((a*cosh
(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))
)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1), -1/2*((cosh(x)^2 + 2*cos
h(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a)*arctan(sqrt(2)*(b*cosh(x)^2 + 2*b*co
sh(x)*sinh(x) + b*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 +
 a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*b*cosh(x)^4 + 4*a
*b*cosh(x)*sinh(x)^3 + a*b*sinh(x)^4 - (a^2 + 3*a*b)*cosh(x)^2 + (6*a*b*cos
h(x)^2 - a^2 - 3*a*b)*sinh(x)^2 - a^2 + 2*(2*a*b*cosh(x)^3 - (a^2 + 3*a*b)*
cosh(x))*sinh(x))) + (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-
a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a)*
sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) +
 sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*
b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a
+ 2*b)*cosh(x))*sinh(x) + a)) + 2*sqrt(2)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 +
 a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(cosh(x)^2 + 2*cosh
(x)*sinh(x) + sinh(x)^2 - 1)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2*(a+b*sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [F] time = 0.39, size = 0, normalized size = 0.00

$$\int \left(\coth^2(x)\right)\sqrt{a+b\mathrm{sech}(x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2*(a+b*sech(x)^2)^(1/2),x)
[Out] int(coth(x)^2*(a+b*sech(x)^2)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\,\mathrm{sech}(x)^2+a}\,\coth(x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2*(a+b*sech(x)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sech(x)^2 + a)*coth(x)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \coth(x)^2\sqrt{a+\frac{b}{\cosh(x)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2*(a + b/cosh(x)^2)^(1/2),x)
[Out] int(coth(x)^2*(a + b/cosh(x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\,\mathrm{sech}^2(x)}\,\coth^2(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2*(a+b*sech(x)**2)**(1/2),x)
[Out] Integral(sqrt(a + b*sech(x)**2)*coth(x)**2, x)
```

3.184 $\int \coth^3(x)\sqrt{a+b\,\mathbf{sech}^2(x)}\,dx$

Optimal. Leaf size=83

$$-\frac{1}{2}\coth^2(x)\sqrt{a+b\text{sech}^2(x)}+\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)-\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a+b}}\right)}{2\sqrt{a+b}}$$

```
[Out] arctanh((a+b*sech(x)^2)^(1/2)/a^(1/2))*a^(1/2)-1/2*(2*a+b)*arctanh((a+b*sec
h(x)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(1/2)-1/2*coth(x)^2*(a+b*sech(x)^2)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {4139, 446, 99, 156, 63, 208}

$$-\frac{1}{2}\coth^2(x)\sqrt{a+b\text{sech}^2(x)}+\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)-\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a+b}}\right)}{2\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^3*Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] Sqrt[a]*ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]] - ((2*a + b)*ArcTanh[Sqrt[a
+ b*Sech[x]^2]/Sqrt[a + b]])/(2*Sqrt[a + b]) - (Coth[x]^2*Sqrt[a + b*Sech[x
]^2])/2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
```

```
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \coth^3(x)\sqrt{a+b\text{sech}^2(x)}\,dx = -\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x\left(-1+x^2\right)^2}\,dx,x,\text{sech}(x)\right)$$
$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx}}{(-1+x)^2x}\,dx,x,\text{sech}^2(x)\right)\right)$$
$$= -\frac{1}{2}\coth^2(x)\sqrt{a+b\text{sech}^2(x)} - \frac{1}{2}\text{Subst}\left(\int \frac{-a-\frac{bx}{2}}{(-1+x)x\sqrt{a+bx}}\,dx,x,\text{sech}^2(x)\right)$$
$$= -\frac{1}{2}\coth^2(x)\sqrt{a+b\text{sech}^2(x)} - \frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,\text{sech}^2(x)\right) - \frac{1}{4}(-$$
$$= -\frac{1}{2}\coth^2(x)\sqrt{a+b\text{sech}^2(x)} - \frac{a\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+b\text{sech}^2(x)}\right)}{b} - \frac{(-}{}$$
$$= \sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right) - \frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a+b}}\right)}{2\sqrt{a+b}} - \frac{1}{2}\coth^2(x)\sqrt{}$$

Mathematica [A] time = 0.59, size = 156, normalized size = 1.88

$$-\frac{\sqrt{a+b\text{sech}^2(x)}\left(\sqrt{2}(2a+b)\cosh(x)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a+b}\cosh(x)}{\sqrt{a\cosh(2x)+a+2b}}\right)+\sqrt{a+b}\left(\coth^2(x)\sqrt{a\cosh(2x)+a+2b}-2\sqrt{}\right.\right.}{2\sqrt{a+b}\sqrt{a\cosh(2x)+a+2b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^3*Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] -1/2*((Sqrt[2]*(2*a + b)*ArcTanh[(Sqrt[2]*Sqrt[a + b]*Cosh[x])/Sqrt[a + 2*b
 + a*Cosh[2*x]]]*Cosh[x] + Sqrt[a + b]*(Sqrt[a + 2*b + a*Cosh[2*x]]*Coth[x]
^2 - 2*Sqrt[2]*Sqrt[a]*Cosh[x]*Log[Sqrt[2]*Sqrt[a]*Cosh[x] + Sqrt[a + 2*b +
 a*Cosh[2*x]]]))*Sqrt[a + b*Sech[x]^2])/(Sqrt[a + b]*Sqrt[a + 2*b + a*Cosh[
2*x]])
```

fricas [B] time = 0.63, size = 5247, normalized size = 63.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3*(a+b*sech(x)^2)^(1/2),x, algorithm="fricas")

[Out] [1/4*(((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4
- 2*(a + b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 - a - b)*sinh(x)^2 + 4*((a +
 b)*cosh(x)^3 - (a + b)*cosh(x))*sinh(x) + a + b)*sqrt(a)*log(((a^3 + 2*a^2
*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^7 + (a^3
+ 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^
6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)
^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + 3*(2*a^3 + 5*a^2*
b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x
)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 6*a^3 + 14*a^2*b + 9*a*b^2 +
30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + 2*
a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^3 +
 (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(2*a^3 + 3*a^2*b
)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 + 15*(2*a^3 + 5*a^2*b
 + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3 + 3*a^2*b + 3*(6*a^3 + 14*a^2*b + 9*a*b
^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*((a^2 + 2*a*b + b^2)*cosh(x)^6 + 6*(a^2
+ 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2 + 2*a*b + b^2)*sinh(x)^6 + 3*(a^2 +
 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 + 2*a*b
+ b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 + 2*a*b + b^
2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4*a*b)*cosh(x)^2 + (15*(a^2 + 2*a*b + b^2)
*cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*cosh(x)^2 + 3*a^2 + 4*a*b)*sinh(x)^2 +
a^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 6*(a^2 + 2*a*b + b^2)*cosh(x)^3
+ (3*a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2
 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + 2*a^
2*b + a*b^2)*cosh(x)^7 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^5 + (6
*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^3 + (2*a^3 + 3*a^2*b)*cosh(x))*sinh(x))/
(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*si
nh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + ((2*
a + b)*cosh(x)^4 + 4*(2*a + b)*cosh(x)*sinh(x)^3 + (2*a + b)*sinh(x)^4 - 2*
(2*a + b)*cosh(x)^2 + 2*(3*(2*a + b)*cosh(x)^2 - 2*a - b)*sinh(x)^2 + 4*((2
*a + b)*cosh(x)^3 - (2*a + b)*cosh(x))*sinh(x) + 2*a + b)*sqrt(a + b)*log((
(2*a + b)*cosh(x)^4 + 4*(2*a + b)*cosh(x)*sinh(x)^3 + (2*a + b)*sinh(x)^4 +
 2*(2*a + 3*b)*cosh(x)^2 + 2*(3*(2*a + b)*cosh(x)^2 + 2*a + 3*b)*sinh(x)^2
- 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*sqr
t((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + si
nh(x)^2)) + 4*((2*a + b)*cosh(x)^3 + (2*a + 3*b)*cosh(x))*sinh(x) + 2*a + b
)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x
)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)) + ((a + b)*cosh(x
)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 - 2*(a + b)*cosh(x)^2
 + 2*(3*(a + b)*cosh(x)^2 - a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 - (a +
b)*cosh(x))*sinh(x) + a + b)*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x
)^3 + a*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*a*cosh(x)^2 + b)*sinh(x)^2 + sqrt(
2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a)*sqrt((a*cosh(x)^
2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4
```

```
*(a*cosh(x)^3 + b*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + si
nh(x)^2)) - 2*sqrt(2)*((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a +
 b)*sinh(x)^2 + a + b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^
2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)
*sinh(x)^3 + (a + b)*sinh(x)^4 - 2*(a + b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)
^2 - a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 - (a + b)*cosh(x))*sinh(x) + a
 + b), 1/4*(2*((2*a + b)*cosh(x)^4 + 4*(2*a + b)*cosh(x)*sinh(x)^3 + (2*a +
 b)*sinh(x)^4 - 2*(2*a + b)*cosh(x)^2 + 2*(3*(2*a + b)*cosh(x)^2 - 2*a - b)
*sinh(x)^2 + 4*((2*a + b)*cosh(x)^3 - (2*a + b)*cosh(x))*sinh(x) + 2*a + b)
*sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1
)*sqrt(-a - b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*co
sh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x
)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*
cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) + ((a + b)*cosh(x)^4 + 4*(a +
b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 - 2*(a + b)*cosh(x)^2 + 2*(3*(a +
b)*cosh(x)^2 - a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 - (a + b)*cosh(x))*s
inh(x) + a + b)*sqrt(a)*log(((a^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2
*a^2*b + a*b^2)*cosh(x)*sinh(x)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(
2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 +
 b^3 + 14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2
*b + a*b^2)*cosh(x)^3 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x
)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*
cosh(x)^4 + 6*a^3 + 14*a^2*b + 9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^
3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a
^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh
(x))*sinh(x)^3 + a^3 + 2*(2*a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b
 + a*b^2)*cosh(x)^6 + 15*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^
3 + 3*a^2*b + 3*(6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)
*((a^2 + 2*a*b + b^2)*cosh(x)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 +
 (a^2 + 2*a*b + b^2)*sinh(x)^6 + 3*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^
2 + 2*a*b + b^2)*cosh(x)^2 + a^2 + 2*a*b + b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a
*b + b^2)*cosh(x)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4
*a*b)*cosh(x)^2 + (15*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 18*(a^2 + 2*a*b + b^2
)*cosh(x)^2 + 3*a^2 + 4*a*b)*sinh(x)^2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*cos
h(x)^5 + 6*(a^2 + 2*a*b + b^2)*cosh(x)^3 + (3*a^2 + 4*a*b)*cosh(x))*sinh(x)
)*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)
*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^7 + 3*(2*a^3
+ 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)
^3 + (2*a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) +
 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 +
 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + ((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)
*sinh(x)^3 + (a + b)*sinh(x)^4 - 2*(a + b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)
^2 - a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 - (a + b)*cosh(x))*sinh(x) + a
 + b)*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*b
*cosh(x)^2 + 2*(3*a*cosh(x)^2 + b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(
```

```
x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2
*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(a*cosh(x)^3 + b*cosh(
x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) - 2*sqrt(2)*(
(a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a + b)*
sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) +
 sinh(x)^2)))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*si
nh(x)^4 - 2*(a + b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 - a - b)*sinh(x)^2 +
 4*((a + b)*cosh(x)^3 - (a + b)*cosh(x))*sinh(x) + a + b), -1/4*(2*((a + b)
*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 - 2*(a + b)*co
sh(x)^2 + 2*(3*(a + b)*cosh(x)^2 - a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3
- (a + b)*cosh(x))*sinh(x) + a + b)*sqrt(-a)*arctan(sqrt(2)*((a + b)*cosh(x
)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*c
osh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^
2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*
sinh(x)^4 + (2*a^2 + 3*a*b)*cosh(x)^2 + (6*(a^2 + a*b)*cosh(x)^2 + 2*a^2 +
3*a*b)*sinh(x)^2 + a^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 + (2*a^2 + 3*a*b)*cosh(
x))*sinh(x))) + 2*((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b
)*sinh(x)^4 - 2*(a + b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 - a - b)*sinh(x)
^2 + 4*((a + b)*cosh(x)^3 - (a + b)*cosh(x))*sinh(x) + a + b)*sqrt(-a)*arct
an(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a)*sqrt((a
*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x
)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh
(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*
cosh(x))*sinh(x) + a)) - ((2*a + b)*cosh(x)^4 + 4*(2*a + b)*cosh(x)*sinh(x)
^3 + (2*a + b)*sinh(x)^4 - 2*(2*a + b)*cosh(x)^2 + 2*(3*(2*a + b)*cosh(x)^2
 - 2*a - b)*sinh(x)^2 + 4*((2*a + b)*cosh(x)^3 - (2*a + b)*cosh(x))*sinh(x)
 + 2*a + b)*sqrt(a + b)*log(((2*a + b)*cosh(x)^4 + 4*(2*a + b)*cosh(x)*sinh
(x)^3 + (2*a + b)*sinh(x)^4 + 2*(2*a + 3*b)*cosh(x)^2 + 2*(3*(2*a + b)*cosh
(x)^2 + 2*a + 3*b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + s
inh(x)^2 + 1)*sqrt(a + b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(
x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((2*a + b)*cosh(x)^3 + (2*a + 3*
b)*cosh(x))*sinh(x) + 2*a + b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4
 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*si
nh(x) + 1)) + 2*sqrt(2)*((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a
 + b)*sinh(x)^2 + a + b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x
)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(
x)*sinh(x)^3 + (a + b)*sinh(x)^4 - 2*(a + b)*cosh(x)^2 + 2*(3*(a + b)*cosh(
x)^2 - a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 - (a + b)*cosh(x))*sinh(x) +
 a + b), -1/2*(((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*s
inh(x)^4 - 2*(a + b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 - a - b)*sinh(x)^2
+ 4*((a + b)*cosh(x)^3 - (a + b)*cosh(x))*sinh(x) + a + b)*sqrt(-a)*arctan(
sqrt(2)*((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2
+ a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cos
h(x)*sinh(x) + sinh(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*s
inh(x)^3 + (a^2 + a*b)*sinh(x)^4 + (2*a^2 + 3*a*b)*cosh(x)^2 + (6*(a^2 + a*
```

```
b)*cosh(x)^2 + 2*a^2 + 3*a*b)*sinh(x)^2 + a^2 + 2*(2*(a^2 + a*b)*cosh(x)^3
+ (2*a^2 + 3*a*b)*cosh(x))*sinh(x))) + ((a + b)*cosh(x)^4 + 4*(a + b)*cosh(
x)*sinh(x)^3 + (a + b)*sinh(x)^4 - 2*(a + b)*cosh(x)^2 + 2*(3*(a + b)*cosh(
x)^2 - a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 - (a + b)*cosh(x))*sinh(x) +
 a + b)*sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2
- 1)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cos
h(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)
^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*c
osh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) - ((2*a + b)*cosh(x)^4 + 4*(2*a
 + b)*cosh(x)*sinh(x)^3 + (2*a + b)*sinh(x)^4 - 2*(2*a + b)*cosh(x)^2 + 2*(
3*(2*a + b)*cosh(x)^2 - 2*a - b)*sinh(x)^2 + 4*((2*a + b)*cosh(x)^3 - (2*a
+ b)*cosh(x))*sinh(x) + 2*a + b)*sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2 + 2
*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a - b)*sqrt((a*cosh(x)^2 + a*sinh(x
)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 +
4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x
)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)
) + sqrt(2)*((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x
)^2 + a + b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh
(x)*sinh(x) + sinh(x)^2)))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3
 + (a + b)*sinh(x)^4 - 2*(a + b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 - a - b
)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 - (a + b)*cosh(x))*sinh(x) + a + b)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3*(a+b*sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [F] time = 0.41, size = 0, normalized size = 0.00

$$\int \left(\coth^{3}(x)\right)\sqrt{a+b\,\mathrm{sech}(x)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3*(a+b*sech(x)^2)^(1/2),x)
```

```
[Out] int(coth(x)^3*(a+b*sech(x)^2)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\,\mathrm{sech}(x)^{2}+a}\,\coth(x)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3*(a+b*sech(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sech(x)^2 + a)*coth(x)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \coth(x)^3 \sqrt{a + \frac{b}{\cosh(x)^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3*(a + b/cosh(x)^2)^(1/2),x)
```

```
[Out] int(coth(x)^3*(a + b/cosh(x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\operatorname{sech}^2(x)} \coth^3(x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**3*(a+b*sech(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sech(x)**2)*coth(x)**3, x)
```

3.185 $\int \coth^4(x)\sqrt{a+b\mathbf{sech}^2(x)}\,dx$

Optimal. Leaf size=84

$$\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)-\frac{1}{3}\coth^3(x)\sqrt{a-b\tanh^2(x)+b}-\frac{(3a+2b)\coth(x)\sqrt{a-b\tanh^2(x)+b}}{3(a+b)}$$

```
[Out] arctanh(a^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))*a^(1/2)-1/3*(3*a+2*b)*coth
(x)*(a+b-b*tanh(x)^2)^(1/2)/(a+b)-1/3*coth(x)^3*(a+b-b*tanh(x)^2)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {4141, 1975, 475, 583, 12, 377, 206}

$$\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)-\frac{1}{3}\coth^3(x)\sqrt{a-b\tanh^2(x)+b}-\frac{(3a+2b)\coth(x)\sqrt{a-b\tanh^2(x)+b}}{3(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^4*Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] Sqrt[a]*ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]] - ((3*a + 2*b)
*Coth[x]*Sqrt[a + b - b*Tanh[x]^2])/(3*(a + b)) - (Coth[x]^3*Sqrt[a + b - b
*Tanh[x]^2])/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \coth^4(x)\sqrt{a+b\mathrm{sech}^2(x)}\,dx = \mathrm{Subst}\left(\int \frac{\sqrt{a+b\left(1-x^2\right)}}{x^4\left(1-x^2\right)}\,dx, x, \tanh(x)\right)$$

$$= \mathrm{Subst}\left(\int \frac{\sqrt{a+b-bx^2}}{x^4\left(1-x^2\right)}\,dx, x, \tanh(x)\right)$$

$$= -\frac{1}{3}\coth^3(x)\sqrt{a+b-b\tanh^2(x)} + \frac{1}{3}\mathrm{Subst}\left(\int \frac{3a+2b-2bx^2}{x^2\left(1-x^2\right)\sqrt{a+b-bx^2}}\,dx, x\right.$$

$$= -\frac{(3a+2b)\coth(x)\sqrt{a+b-b\tanh^2(x)}}{3(a+b)} - \frac{1}{3}\coth^3(x)\sqrt{a+b-b\tanh^2(x)} - \text{Su}$$

$$= -\frac{(3a+2b)\coth(x)\sqrt{a+b-b\tanh^2(x)}}{3(a+b)} - \frac{1}{3}\coth^3(x)\sqrt{a+b-b\tanh^2(x)} + a$$

$$= -\frac{(3a+2b)\coth(x)\sqrt{a+b-b\tanh^2(x)}}{3(a+b)} - \frac{1}{3}\coth^3(x)\sqrt{a+b-b\tanh^2(x)} + a$$

$$= \sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right) - \frac{(3a+2b)\coth(x)\sqrt{a+b-b\tanh^2(x)}}{3(a+b)} -$$

Mathematica [A] time = 0.62, size = 149, normalized size = 1.77

$$\frac{\sqrt{2}\cosh(x)\sqrt{a\sinh^2(x)+a+b}\sqrt{a+b\mathrm{sech}^2(x)}\left(3\sqrt{a}(a+b)\sinh^{-1}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{a+b}}\right)-\sqrt{a+b}\,\mathrm{csch}(x)\sqrt{\frac{a\sinh^2(x)+a+b}{a+b}}\right.}{3(a+b)^{3/2}\sqrt{\frac{a\sinh^2(x)+a+b}{a+b}}\sqrt{a\cosh(2x)+a+2b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^4*Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] (Sqrt[2]*Cosh[x]*Sqrt[a + b*Sech[x]^2]*Sqrt[a + b + a*Sinh[x]^2]*(3*Sqrt[a]
*(a + b)*ArcSinh[(Sqrt[a]*Sinh[x])/Sqrt[a + b]] - Sqrt[a + b]*Csch[x]*(4*a
+ 3*b + (a + b)*Csch[x]^2)*Sqrt[(a + b + a*Sinh[x]^2)/(a + b)]))/(3*(a + b)
^(3/2)*Sqrt[a + 2*b + a*Cosh[2*x]]*Sqrt[(a + b + a*Sinh[x]^2)/(a + b)])
```

fricas [B] time = 0.62, size = 2341, normalized size = 27.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4*(a+b*sech(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*((a + b)*cosh(x)^6 + 6*(a + b)*cosh(x)*sinh(x)^5 + (a + b)*sinh(x)
^6 - 3*(a + b)*cosh(x)^4 + 3*(5*(a + b)*cosh(x)^2 - a - b)*sinh(x)^4 + 4*(5
*(a + b)*cosh(x)^3 - 3*(a + b)*cosh(x))*sinh(x)^3 + 3*(a + b)*cosh(x)^2 + 3
*(5*(a + b)*cosh(x)^4 - 6*(a + b)*cosh(x)^2 + a + b)*sinh(x)^2 + 6*((a + b)
*cosh(x)^5 - 2*(a + b)*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) - a - b)*sqrt(a
)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^8 - 2*(a
*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh(x)^6 + 4*
(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3 + 4*a^2*b +
 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 - 30*(a
*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a*b^2 - b^3)
*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(a^3 +
3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*cosh(x)^4 + a
^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(
b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 +
 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*
sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2
- a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^
2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a +
2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*cosh(x)^7 -
3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3 + (a^3 + 3*
a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*si
nh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(
x)^5 + sinh(x)^6)) + 3*((a + b)*cosh(x)^6 + 6*(a + b)*cosh(x)*sinh(x)^5 + (
a + b)*sinh(x)^6 - 3*(a + b)*cosh(x)^4 + 3*(5*(a + b)*cosh(x)^2 - a - b)*si
nh(x)^4 + 4*(5*(a + b)*cosh(x)^3 - 3*(a + b)*cosh(x))*sinh(x)^3 + 3*(a + b)
*cosh(x)^2 + 3*(5*(a + b)*cosh(x)^4 - 6*(a + b)*cosh(x)^2 + a + b)*sinh(x)^
2 + 6*((a + b)*cosh(x)^5 - 2*(a + b)*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) -
 a - b)*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2
*(a + b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + b)*sinh(x)^2 + sqrt(2)*(cosh(x)
^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(
x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(a*cosh(x)
^3 + (a + b)*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)
^2)) - 4*sqrt(2)*((4*a + 3*b)*cosh(x)^4 + 4*(4*a + 3*b)*cosh(x)*sinh(x)^3 +
 (4*a + 3*b)*sinh(x)^4 - 2*(2*a + b)*cosh(x)^2 + 2*(3*(4*a + 3*b)*cosh(x)^2
 - 2*a - b)*sinh(x)^2 + 4*((4*a + 3*b)*cosh(x)^3 - (2*a + b)*cosh(x))*sinh(
x) + 4*a + 3*b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*c
osh(x)*sinh(x) + sinh(x)^2)))/((a + b)*cosh(x)^6 + 6*(a + b)*cosh(x)*sinh(x
)^5 + (a + b)*sinh(x)^6 - 3*(a + b)*cosh(x)^4 + 3*(5*(a + b)*cosh(x)^2 - a
- b)*sinh(x)^4 + 4*(5*(a + b)*cosh(x)^3 - 3*(a + b)*cosh(x))*sinh(x)^3 + 3*
(a + b)*cosh(x)^2 + 3*(5*(a + b)*cosh(x)^4 - 6*(a + b)*cosh(x)^2 + a + b)*s
inh(x)^2 + 6*((a + b)*cosh(x)^5 - 2*(a + b)*cosh(x)^3 + (a + b)*cosh(x))*si
```

```
nh(x) - a - b), -1/6*(3*((a + b)*cosh(x)^6 + 6*(a + b)*cosh(x)*sinh(x)^5 +
(a + b)*sinh(x)^6 - 3*(a + b)*cosh(x)^4 + 3*(5*(a + b)*cosh(x)^2 - a - b)*s
inh(x)^4 + 4*(5*(a + b)*cosh(x)^3 - 3*(a + b)*cosh(x))*sinh(x)^3 + 3*(a + b
)*cosh(x)^2 + 3*(5*(a + b)*cosh(x)^4 - 6*(a + b)*cosh(x)^2 + a + b)*sinh(x)
^2 + 6*((a + b)*cosh(x)^5 - 2*(a + b)*cosh(x)^3 + (a + b)*cosh(x))*sinh(x)
- a - b)*sqrt(-a)*arctan(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sin
h(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2
- 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*b*cosh(x)^4 + 4*a*b*cosh(x)*sinh(x)^3
+ a*b*sinh(x)^4 - (a^2 + 3*a*b)*cosh(x)^2 + (6*a*b*cosh(x)^2 - a^2 - 3*a*b)
*sinh(x)^2 - a^2 + 2*(2*a*b*cosh(x)^3 - (a^2 + 3*a*b)*cosh(x))*sinh(x))) +
3*((a + b)*cosh(x)^6 + 6*(a + b)*cosh(x)*sinh(x)^5 + (a + b)*sinh(x)^6 - 3*
(a + b)*cosh(x)^4 + 3*(5*(a + b)*cosh(x)^2 - a - b)*sinh(x)^4 + 4*(5*(a + b
)*cosh(x)^3 - 3*(a + b)*cosh(x))*sinh(x)^3 + 3*(a + b)*cosh(x)^2 + 3*(5*(a
+ b)*cosh(x)^4 - 6*(a + b)*cosh(x)^2 + a + b)*sinh(x)^2 + 6*((a + b)*cosh(x
)^5 - 2*(a + b)*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) - a - b)*sqrt(-a)*arct
an(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a)*sqrt((a
*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x
)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh
(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*
cosh(x))*sinh(x) + a)) + 2*sqrt(2)*((4*a + 3*b)*cosh(x)^4 + 4*(4*a + 3*b)*c
osh(x)*sinh(x)^3 + (4*a + 3*b)*sinh(x)^4 - 2*(2*a + b)*cosh(x)^2 + 2*(3*(4*
a + 3*b)*cosh(x)^2 - 2*a - b)*sinh(x)^2 + 4*((4*a + 3*b)*cosh(x)^3 - (2*a +
 b)*cosh(x))*sinh(x) + 4*a + 3*b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b
)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a + b)*cosh(x)^6 + 6*(a +
 b)*cosh(x)*sinh(x)^5 + (a + b)*sinh(x)^6 - 3*(a + b)*cosh(x)^4 + 3*(5*(a +
 b)*cosh(x)^2 - a - b)*sinh(x)^4 + 4*(5*(a + b)*cosh(x)^3 - 3*(a + b)*cosh(
x))*sinh(x)^3 + 3*(a + b)*cosh(x)^2 + 3*(5*(a + b)*cosh(x)^4 - 6*(a + b)*co
sh(x)^2 + a + b)*sinh(x)^2 + 6*((a + b)*cosh(x)^5 - 2*(a + b)*cosh(x)^3 + (
a + b)*cosh(x))*sinh(x) - a - b)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4*(a+b*sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [F] time = 0.41, size = 0, normalized size = 0.00

$$\int \left(\coth^{4}(x)\right) \sqrt{a+b\,\mathrm{sech}(x)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4*(a+b*sech(x)^2)^(1/2),x)
```

```
[Out] int(coth(x)^4*(a+b*sech(x)^2)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\,\mathrm{sech}(x)^2 + a}\,\coth(x)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4*(a+b*sech(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sech(x)^2 + a)*coth(x)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \coth(x)^4\,\sqrt{a + \frac{b}{\cosh(x)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4*(a + b/cosh(x)^2)^(1/2),x)
```

```
[Out] int(coth(x)^4*(a + b/cosh(x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\,\mathrm{sech}^2(x)}\,\coth^4(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**4*(a+b*sech(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sech(x)**2)*coth(x)**4, x)
```

3.186 $\int \coth^5(x)\sqrt{a+b\text{sech}^2(x)}\,dx$

Optimal. Leaf size=125

$$-\frac{\left(8a^2+12ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a+b}}\right)}{8(a+b)^{3/2}}-\frac{1}{4}\coth^4(x)\sqrt{a+b\text{sech}^2(x)}-\frac{(4a+3b)\coth^2(x)\sqrt{a+b\text{sech}^2(x)}}{8(a+b)}+\sqrt{a}$$

```
[Out] -1/8*(8*a^2+12*a*b+3*b^2)*arctanh((a+b*sech(x)^2)^(1/2)/(a+b)^(1/2))/(a+b)^
(3/2)+arctanh((a+b*sech(x)^2)^(1/2)/a^(1/2))*a^(1/2)-1/8*(4*a+3*b)*coth(x)^
2*(a+b*sech(x)^2)^(1/2)/(a+b)-1/4*coth(x)^4*(a+b*sech(x)^2)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {4139, 446, 99, 151, 156, 63, 208}

$$-\frac{\left(8a^2+12ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a+b}}\right)}{8(a+b)^{3/2}}-\frac{1}{4}\coth^4(x)\sqrt{a+b\text{sech}^2(x)}-\frac{(4a+3b)\coth^2(x)\sqrt{a+b\text{sech}^2(x)}}{8(a+b)}+\sqrt{a}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^5*Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] Sqrt[a]*ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]] - ((8*a^2 + 12*a*b + 3*b^2)*
ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a + b]])/(8*(a + b)^(3/2)) - ((4*a + 3*b
)*Coth[x]^2*Sqrt[a + b*Sech[x]^2])/(8*(a + b)) - (Coth[x]^4*Sqrt[a + b*Sech
[x]^2])/4
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
```

```
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \coth^5(x)\sqrt{a+b\text{sech}^2(x)}\,dx = \text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x\left(-1+x^2\right)^3}\,dx, x, \text{sech}(x)\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx}}{(-1+x)^3x}\,dx, x, \text{sech}^2(x)\right)$$

$$= -\frac{1}{4}\coth^4(x)\sqrt{a+b\text{sech}^2(x)} + \frac{1}{4}\text{Subst}\left(\int \frac{-2a-\frac{3bx}{2}}{(-1+x)^2x\sqrt{a+bx}}\,dx, x, \text{sech}^2(x)\right.$$

$$= -\frac{(4a+3b)\coth^2(x)\sqrt{a+b\text{sech}^2(x)}}{8(a+b)} - \frac{1}{4}\coth^4(x)\sqrt{a+b\text{sech}^2(x)} - \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{(4a+3b)\coth^2(x)\sqrt{a+b\text{sech}^2(x)}}{8(a+b)} - \frac{1}{4}\coth^4(x)\sqrt{a+b\text{sech}^2(x)} - \frac{1}{2}a\,\text{Subst}$$

$$= -\frac{(4a+3b)\coth^2(x)\sqrt{a+b\text{sech}^2(x)}}{8(a+b)} - \frac{1}{4}\coth^4(x)\sqrt{a+b\text{sech}^2(x)} - \frac{a\,\text{Subst}\left(\right.}{}$$

$$= \sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right) - \frac{\left(8a^2+12ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a+b}}\right)}{8(a+b)^{3/2}} - \frac{(4}{}$$

Mathematica [A] time = 1.02, size = 191, normalized size = 1.53

$$-\frac{\cosh(x)\sqrt{a+b\text{sech}^2(x)}\left(\sqrt{2}\left(8a^2+12ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a+b}\cosh(x)}{\sqrt{a\cosh(2x)+a+2b}}\right)+\sqrt{a+b}\left(\frac{1}{2}\coth(x)\text{csch}^3(x)\sqrt{a\cos}\right.\right.}{8(a+b)^{3/2}\sqrt{a\text{co}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^5*Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] -1/8*(Cosh[x]*(Sqrt[2]*(8*a^2 + 12*a*b + 3*b^2)*ArcTanh[(Sqrt[2]*Sqrt[a + b
]*Cosh[x])/Sqrt[a + 2*b + a*Cosh[2*x]]] + Sqrt[a + b]*((Sqrt[a + 2*b + a*Co
sh[2*x]]*(-2*a - b + (6*a + 5*b)*Cosh[2*x])*Coth[x]*Csch[x]^3)/2 - 8*Sqrt[2
]*Sqrt[a]*(a + b)*Log[Sqrt[2]*Sqrt[a]*Cosh[x] + Sqrt[a + 2*b + a*Cosh[2*x]]
]))*Sqrt[a + b*Sech[x]^2])/((a + b)^(3/2)*Sqrt[a + 2*b + a*Cosh[2*x]])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5*(a+b*sech(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5*(a+b*sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [F] time = 0.42, size = 0, normalized size = 0.00

$$\int \left(\coth^{5}(x)\right) \sqrt{a + b\operatorname{sech}(x)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^5*(a+b*sech(x)^2)^(1/2),x)
```

```
[Out] int(coth(x)^5*(a+b*sech(x)^2)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\operatorname{sech}(x)^{2} + a}\, \coth(x)^{5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5*(a+b*sech(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sech(x)^2 + a)*coth(x)^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\mathrm{coth}\left(x\right)}^5\,\sqrt{a+\frac{b}{{\mathrm{cosh}\left(x\right)}^2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^5*(a + b/cosh(x)^2)^(1/2),x)
```

```
[Out] int(coth(x)^5*(a + b/cosh(x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\,\text{sech}^2(x)}\,\coth^5(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**5*(a+b*sech(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sech(x)**2)*coth(x)**5, x)
```

3.187 $\int \left(a+b\,\mathbf{sech}^2(x)\right)^{3/2}\tanh^3(x)\,dx$

Optimal. Leaf size=76

$$a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)+\frac{\left(a+b\text{sech}^2(x)\right)^{5/2}}{5b}-\frac{1}{3}\left(a+b\text{sech}^2(x)\right)^{3/2}-a\sqrt{a+b\text{sech}^2(x)}$$

```
[Out] a^(3/2)*arctanh((a+b*sech(x)^2)^(1/2)/a^(1/2))-1/3*(a+b*sech(x)^2)^(3/2)+1/
5*(a+b*sech(x)^2)^(5/2)/b-a*(a+b*sech(x)^2)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {4139, 446, 80, 50, 63, 208}

$$a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)+\frac{\left(a+b\text{sech}^2(x)\right)^{5/2}}{5b}-\frac{1}{3}\left(a+b\text{sech}^2(x)\right)^{3/2}-a\sqrt{a+b\text{sech}^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[x]^2)^(3/2)*Tanh[x]^3,x]
```

```
[Out] a^(3/2)*ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]] - a*Sqrt[a + b*Sech[x]^2] -
(a + b*Sech[x]^2)^(3/2)/3 + (a + b*Sech[x]^2)^(5/2)/(5*b)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \left(a+b\text{sech}^2(x)\right)^{3/2}\tanh^3(x)\,dx = \text{Subst}\left(\int \frac{\left(-1+x^2\right)\left(a+bx^2\right)^{3/2}}{x}\,dx, x, \text{sech}(x)\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{(-1+x)(a+bx)^{3/2}}{x}\,dx, x, \text{sech}^2(x)\right)$$
$$= \frac{\left(a+b\text{sech}^2(x)\right)^{5/2}}{5b} - \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx, x, \text{sech}^2(x)\right)$$
$$= -\frac{1}{3}\left(a+b\text{sech}^2(x)\right)^{3/2} + \frac{\left(a+b\text{sech}^2(x)\right)^{5/2}}{5b} - \frac{1}{2}a\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x,\right.$$
$$= -a\sqrt{a+b\text{sech}^2(x)} - \frac{1}{3}\left(a+b\text{sech}^2(x)\right)^{3/2} + \frac{\left(a+b\text{sech}^2(x)\right)^{5/2}}{5b} - \frac{1}{2}a^2\,\text{Sub}$$
$$= -a\sqrt{a+b\text{sech}^2(x)} - \frac{1}{3}\left(a+b\text{sech}^2(x)\right)^{3/2} + \frac{\left(a+b\text{sech}^2(x)\right)^{5/2}}{5b} - \frac{a^2\,\text{Subst}}{}$$
$$= a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right) - a\sqrt{a+b\text{sech}^2(x)} - \frac{1}{3}\left(a+b\text{sech}^2(x)\right)^{3/2}$$

Mathematica [A] time = 0.97, size = 129, normalized size = 1.70

$$\cosh^3(x)\left(a+b\text{sech}^2(x)\right)^{3/2}\left(\frac{2\sqrt{2}\,a^{3/2}\log\left(\sqrt{a\cosh(2x)+a+2b}+\sqrt{2}\sqrt{a}\cosh(x)\right)}{(a\cosh(2x)+a+2b)^{3/2}} + \frac{2\left(b(6a-5b)\text{sech}^3(x)+\right.}{15b(a}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[x]^2)^(3/2)*Tanh[x]^3,x]
```

```
[Out] Cosh[x]^3*(a + b*Sech[x]^2)^(3/2)*((2*Sqrt[2]*a^(3/2)*Log[Sqrt[2]*Sqrt[a]*C
osh[x] + Sqrt[a + 2*b + a*Cosh[2*x]]])/(a + 2*b + a*Cosh[2*x])^(3/2) + (2*(
a*(3*a - 20*b)*Sech[x] + (6*a - 5*b)*b*Sech[x]^3 + 3*b^2*Sech[x]^5))/(15*b*
(a + 2*b + a*Cosh[2*x])))
```

fricas [B] time = 2.15, size = 4226, normalized size = 55.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(3/2)*tanh(x)^3,x, algorithm="fricas")

[Out] [1/60*(15*(a*b*cosh(x)^10 + 10*a*b*cosh(x)*sinh(x)^9 + a*b*sinh(x)^10 + 5*a
*b*cosh(x)^8 + 5*(9*a*b*cosh(x)^2 + a*b)*sinh(x)^8 + 10*a*b*cosh(x)^6 + 40*
(3*a*b*cosh(x)^3 + a*b*cosh(x))*sinh(x)^7 + 10*(21*a*b*cosh(x)^4 + 14*a*b*c
osh(x)^2 + a*b)*sinh(x)^6 + 10*a*b*cosh(x)^4 + 4*(63*a*b*cosh(x)^5 + 70*a*b
*cosh(x)^3 + 15*a*b*cosh(x))*sinh(x)^5 + 10*(21*a*b*cosh(x)^6 + 35*a*b*cosh
(x)^4 + 15*a*b*cosh(x)^2 + a*b)*sinh(x)^4 + 5*a*b*cosh(x)^2 + 40*(3*a*b*cos
h(x)^7 + 7*a*b*cosh(x)^5 + 5*a*b*cosh(x)^3 + a*b*cosh(x))*sinh(x)^3 + 5*(9*
a*b*cosh(x)^8 + 28*a*b*cosh(x)^6 + 30*a*b*cosh(x)^4 + 12*a*b*cosh(x)^2 + a*
b)*sinh(x)^2 + a*b + 10*(a*b*cosh(x)^9 + 4*a*b*cosh(x)^7 + 6*a*b*cosh(x)^5
+ 4*a*b*cosh(x)^3 + a*b*cosh(x))*sinh(x))*sqrt(a)*log(((a^3 + 2*a^2*b + a*b
^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^7 + (a^3 + 2*a^2*
b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^6 + 2*(2
*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh
(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + 3*(2*a^3 + 5*a^2*b + 4*a*
b^2 + b^3)*cosh(x))*sinh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^4 + (7
0*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 6*a^3 + 14*a^2*b + 9*a*b^2 + 30*(2*a^
3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + 2*a^2*b +
a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^3 + (6*a^3
+ 14*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(2*a^3 + 3*a^2*b)*cosh(x
)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 + 15*(2*a^3 + 5*a^2*b + 4*a*b
^2 + b^3)*cosh(x)^4 + 2*a^3 + 3*a^2*b + 3*(6*a^3 + 14*a^2*b + 9*a*b^2)*cosh
(x)^2)*sinh(x)^2 + sqrt(2)*((a^2 + 2*a*b + b^2)*cosh(x)^6 + 6*(a^2 + 2*a*b
+ b^2)*cosh(x)*sinh(x)^5 + (a^2 + 2*a*b + b^2)*sinh(x)^6 + 3*(a^2 + 2*a*b +
 b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 + 2*a*b + b^2)*s
inh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(
x))*sinh(x)^3 + (3*a^2 + 4*a*b)*cosh(x)^2 + (15*(a^2 + 2*a*b + b^2)*cosh(x)
^4 + 18*(a^2 + 2*a*b + b^2)*cosh(x)^2 + 3*a^2 + 4*a*b)*sinh(x)^2 + a^2 + 2*
(3*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 6*(a^2 + 2*a*b + b^2)*cosh(x)^3 + (3*a^2
 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2
*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + 2*a^2*b + a*
b^2)*cosh(x)^7 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^5 + (6*a^3 + 1
4*a^2*b + 9*a*b^2)*cosh(x)^3 + (2*a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)
^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3
+ 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + 15*(a*b*cosh
(x)^10 + 10*a*b*cosh(x)*sinh(x)^9 + a*b*sinh(x)^10 + 5*a*b*cosh(x)^8 + 5*(9
*a*b*cosh(x)^2 + a*b)*sinh(x)^8 + 10*a*b*cosh(x)^6 + 40*(3*a*b*cosh(x)^3 +
a*b*cosh(x))*sinh(x)^7 + 10*(21*a*b*cosh(x)^4 + 14*a*b*cosh(x)^2 + a*b)*sin
h(x)^6 + 10*a*b*cosh(x)^4 + 4*(63*a*b*cosh(x)^5 + 70*a*b*cosh(x)^3 + 15*a*b
*cosh(x))*sinh(x)^5 + 10*(21*a*b*cosh(x)^6 + 35*a*b*cosh(x)^4 + 15*a*b*cosh
(x)^2 + a*b)*sinh(x)^4 + 5*a*b*cosh(x)^2 + 40*(3*a*b*cosh(x)^7 + 7*a*b*cosh
(x)^5 + 5*a*b*cosh(x)^3 + a*b*cosh(x))*sinh(x)^3 + 5*(9*a*b*cosh(x)^8 + 28*
a*b*cosh(x)^6 + 30*a*b*cosh(x)^4 + 12*a*b*cosh(x)^2 + a*b)*sinh(x)^2 + a*b
+ 10*(a*b*cosh(x)^9 + 4*a*b*cosh(x)^7 + 6*a*b*cosh(x)^5 + 4*a*b*cosh(x)^3 +
```

a*b*cosh(x))*sinh(x))*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*a*cosh(x)^2 + b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(a*cosh(x)^3 + b*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*sqrt(2)*((3*a^2 - 20*a*b)*cosh(x)^8 + 8*(3*a^2 - 20*a*b)*cosh(x)*sinh(x)^7 + (3*a^2 - 20*a*b)*sinh(x)^8 + 4*(3*a^2 - 14*a*b - 5*b^2)*cosh(x)^6 + 4*(7*(3*a^2 - 20*a*b)*cosh(x)^2 + 3*a^2 - 14*a*b - 5*b^2)*sinh(x)^6 + 8*(7*(3*a^2 - 20*a*b)*cosh(x)^3 + 3*(3*a^2 - 14*a*b - 5*b^2)*cosh(x))*sinh(x)^5 + 2*(9*a^2 - 36*a*b + 4*b^2)*cosh(x)^4 + 2*(35*(3*a^2 - 20*a*b)*cosh(x)^4 + 30*(3*a^2 - 14*a*b - 5*b^2)*cosh(x)^2 + 9*a^2 - 36*a*b + 4*b^2)*sinh(x)^4 + 8*(7*(3*a^2 - 20*a*b)*cosh(x)^5 + 10*(3*a^2 - 14*a*b - 5*b^2)*cosh(x)^3 + (9*a^2 - 36*a*b + 4*b^2)*cosh(x))*sinh(x)^3 + 4*(3*a^2 - 14*a*b - 5*b^2)*cosh(x)^2 + 4*(7*(3*a^2 - 20*a*b)*cosh(x)^6 + 15*(3*a^2 - 14*a*b - 5*b^2)*cosh(x)^4 + 3*(9*a^2 - 36*a*b + 4*b^2)*cosh(x)^2 + 3*a^2 - 14*a*b - 5*b^2)*sinh(x)^2 + 3*a^2 - 20*a*b + 8*((3*a^2 - 20*a*b)*cosh(x)^7 + 3*(3*a^2 - 14*a*b - 5*b^2)*cosh(x)^5 + (9*a^2 - 36*a*b + 4*b^2)*cosh(x)^3 + (3*a^2 - 14*a*b - 5*b^2)*cosh(x))*sinh(x))*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(b*cosh(x)^10 + 10*b*cosh(x)*sinh(x)^9 + b*sinh(x)^10 + 5*b*cosh(x)^8 + 5*(9*b*cosh(x)^2 + b)*sinh(x)^8 + 40*(3*b*cosh(x)^3 + b*cosh(x))*sinh(x)^7 + 10*b*cosh(x)^6 + 10*(21*b*cosh(x)^4 + 14*b*cosh(x)^2 + b)*sinh(x)^6 + 4*(63*b*cosh(x)^5 + 70*b*cosh(x)^3 + 15*b*cosh(x))*sinh(x)^5 + 10*b*cosh(x)^4 + 10*(21*b*cosh(x)^6 + 35*b*cosh(x)^4 + 15*b*cosh(x)^2 + b)*sinh(x)^4 + 40*(3*b*cosh(x)^7 + 7*b*cosh(x)^5 + 5*b*cosh(x)^3 + b*cosh(x))*sinh(x)^3 + 5*b*cosh(x)^2 + 5*(9*b*cosh(x)^8 + 28*b*cosh(x)^6 + 30*b*cosh(x)^4 + 12*b*cosh(x)^2 + b)*sinh(x)^2 + 10*(b*cosh(x)^9 + 4*b*cosh(x)^7 + 6*b*cosh(x)^5 + 4*b*cosh(x)^3 + b*cosh(x))*sinh(x) + b), -1/30*(15*(a*b*cosh(x)^10 + 10*a*b*cosh(x)*sinh(x)^9 + a*b*sinh(x)^10 + 5*a*b*cosh(x)^8 + 5*(9*a*b*cosh(x)^2 + a*b)*sinh(x)^8 + 10*a*b*cosh(x)^6 + 40*(3*a*b*cosh(x)^3 + a*b*cosh(x))*sinh(x)^7 + 10*(21*a*b*cosh(x)^4 + 14*a*b*cosh(x)^2 + a*b)*sinh(x)^6 + 10*a*b*cosh(x)^4 + 4*(63*a*b*cosh(x)^5 + 70*a*b*cosh(x)^3 + 15*a*b*cosh(x))*sinh(x)^5 + 10*(21*a*b*cosh(x)^6 + 35*a*b*cosh(x)^4 + 15*a*b*cosh(x)^2 + a*b)*sinh(x)^4 + 5*a*b*cosh(x)^2 + 40*(3*a*b*cosh(x)^7 + 7*a*b*cosh(x)^5 + 5*a*b*cosh(x)^3 + a*b*cosh(x))*sinh(x)^3 + 5*(9*a*b*cosh(x)^8 + 28*a*b*cosh(x)^6 + 30*a*b*cosh(x)^4 + 12*a*b*cosh(x)^2 + a*b)*sinh(x)^2 + a*b + 10*(a*b*cosh(x)^9 + 4*a*b*cosh(x)^7 + 6*a*b*cosh(x)^5 + 4*a*b*cosh(x)^3 + a*b*cosh(x))*sinh(x))*sqrt(-a)*arctan(sqrt(2)*((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 + (2*a^2 + 3*a*b)*cosh(x)^2 + (6*(a^2 + a*b)*cosh(x)^2 + 2*a^2 + 3*a*b)*sinh(x)^2 + a^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 + (2*a^2 + 3*a*b)*cosh(x))*sinh(x))) + 15*(a*b*cosh(x)^10 + 10*a*b*cosh(x)*sinh(x)^9 + a*b*sinh(x)^10 + 5*a*b*cosh(x)^8 + 5*(9*a*b*cosh(x)^2 + a*b)*sinh(x)^8 + 10*a*b*cosh(x)^6 + 40*(3*a*b*cosh(x)^3 + a*b*cosh(x))*sinh(x)^7 + 10*(21*a*b

```
*cosh(x)^4 + 14*a*b*cosh(x)^2 + a*b)*sinh(x)^6 + 10*a*b*cosh(x)^4 + 4*(63*a
*b*cosh(x)^5 + 70*a*b*cosh(x)^3 + 15*a*b*cosh(x))*sinh(x)^5 + 10*(21*a*b*co
sh(x)^6 + 35*a*b*cosh(x)^4 + 15*a*b*cosh(x)^2 + a*b)*sinh(x)^4 + 5*a*b*cosh
(x)^2 + 40*(3*a*b*cosh(x)^7 + 7*a*b*cosh(x)^5 + 5*a*b*cosh(x)^3 + a*b*cosh(
x))*sinh(x)^3 + 5*(9*a*b*cosh(x)^8 + 28*a*b*cosh(x)^6 + 30*a*b*cosh(x)^4 +
12*a*b*cosh(x)^2 + a*b)*sinh(x)^2 + a*b + 10*(a*b*cosh(x)^9 + 4*a*b*cosh(x)
^7 + 6*a*b*cosh(x)^5 + 4*a*b*cosh(x)^3 + a*b*cosh(x))*sinh(x))*sqrt(-a)*arc
tan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a)*sqrt((
a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(
x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cos
h(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)
*cosh(x))*sinh(x) + a)) - 2*sqrt(2)*((3*a^2 - 20*a*b)*cosh(x)^8 + 8*(3*a^2
- 20*a*b)*cosh(x)*sinh(x)^7 + (3*a^2 - 20*a*b)*sinh(x)^8 + 4*(3*a^2 - 14*a*
b - 5*b^2)*cosh(x)^6 + 4*(7*(3*a^2 - 20*a*b)*cosh(x)^2 + 3*a^2 - 14*a*b - 5
*b^2)*sinh(x)^6 + 8*(7*(3*a^2 - 20*a*b)*cosh(x)^3 + 3*(3*a^2 - 14*a*b - 5*b
^2)*cosh(x))*sinh(x)^5 + 2*(9*a^2 - 36*a*b + 4*b^2)*cosh(x)^4 + 2*(35*(3*a^
2 - 20*a*b)*cosh(x)^4 + 30*(3*a^2 - 14*a*b - 5*b^2)*cosh(x)^2 + 9*a^2 - 36*
a*b + 4*b^2)*sinh(x)^4 + 8*(7*(3*a^2 - 20*a*b)*cosh(x)^5 + 10*(3*a^2 - 14*a
*b - 5*b^2)*cosh(x)^3 + (9*a^2 - 36*a*b + 4*b^2)*cosh(x))*sinh(x)^3 + 4*(3*
a^2 - 14*a*b - 5*b^2)*cosh(x)^2 + 4*(7*(3*a^2 - 20*a*b)*cosh(x)^6 + 15*(3*a
^2 - 14*a*b - 5*b^2)*cosh(x)^4 + 3*(9*a^2 - 36*a*b + 4*b^2)*cosh(x)^2 + 3*a
^2 - 14*a*b - 5*b^2)*sinh(x)^2 + 3*a^2 - 20*a*b + 8*((3*a^2 - 20*a*b)*cosh(
x)^7 + 3*(3*a^2 - 14*a*b - 5*b^2)*cosh(x)^5 + (9*a^2 - 36*a*b + 4*b^2)*cosh
(x)^3 + (3*a^2 - 14*a*b - 5*b^2)*cosh(x))*sinh(x))*sqrt((a*cosh(x)^2 + a*si
nh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(b*cosh(x)
^10 + 10*b*cosh(x)*sinh(x)^9 + b*sinh(x)^10 + 5*b*cosh(x)^8 + 5*(9*b*cosh(x
)^2 + b)*sinh(x)^8 + 40*(3*b*cosh(x)^3 + b*cosh(x))*sinh(x)^7 + 10*b*cosh(x
)^6 + 10*(21*b*cosh(x)^4 + 14*b*cosh(x)^2 + b)*sinh(x)^6 + 4*(63*b*cosh(x)^
5 + 70*b*cosh(x)^3 + 15*b*cosh(x))*sinh(x)^5 + 10*b*cosh(x)^4 + 10*(21*b*co
sh(x)^6 + 35*b*cosh(x)^4 + 15*b*cosh(x)^2 + b)*sinh(x)^4 + 40*(3*b*cosh(x)^
7 + 7*b*cosh(x)^5 + 5*b*cosh(x)^3 + b*cosh(x))*sinh(x)^3 + 5*b*cosh(x)^2 +
5*(9*b*cosh(x)^8 + 28*b*cosh(x)^6 + 30*b*cosh(x)^4 + 12*b*cosh(x)^2 + b)*si
nh(x)^2 + 10*(b*cosh(x)^9 + 4*b*cosh(x)^7 + 6*b*cosh(x)^5 + 4*b*cosh(x)^3 +
 b*cosh(x))*sinh(x) + b)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(3/2)*tanh(x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 0.77Error: Bad Argument Typ
e
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \left(a+b\,\mathrm{sech}(x)^{2}\right)^{\frac{3}{2}}\left(\tanh^{3}(x)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(x)^2)^(3/2)*tanh(x)^3,x)
[Out] int((a+b*sech(x)^2)^(3/2)*tanh(x)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\,\mathrm{sech}(x)^{2}+a\right)^{\frac{3}{2}}\tanh(x)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(3/2)*tanh(x)^3,x, algorithm="maxima")
[Out] integrate((b*sech(x)^2 + a)^(3/2)*tanh(x)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tanh(x)^{3}\left(a+\frac{b}{\cosh(x)^{2}}\right)^{3/2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3*(a + b/cosh(x)^2)^(3/2),x)
[Out] int(tanh(x)^3*(a + b/cosh(x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\,\mathrm{sech}^{2}(x)\right)^{\frac{3}{2}}\tanh^{3}(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)**2)**(3/2)*tanh(x)**3,x)
[Out] Integral((a + b*sech(x)**2)**(3/2)*tanh(x)**3, x)
```

3.188 $\int \left(a+b\text{sech}^2(x)\right)^{3/2} \tanh^2(x)\,dx$

Optimal. Leaf size=125

$$a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)-\frac{\left(3a^2-6ab-b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{8\sqrt{b}}-\frac{1}{8}(5a+b)\tanh(x)\sqrt{a-b\tanh^2(x)+$$

[Out] a^(3/2)*arctanh(a^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))-1/8*(3*a^2-6*a*b-b^2)*arctan(b^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))/b^(1/2)-1/8*(5*a+b)*(a+b-b*tanh(x)^2)^(1/2)*tanh(x)+1/4*b*(a+b-b*tanh(x)^2)^(1/2)*tanh(x)^3

Rubi [A] time = 0.36, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {4141, 1975, 477, 582, 523, 217, 203, 377, 206}

$$-\frac{\left(3a^2-6ab-b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{8\sqrt{b}}+a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)+\frac{1}{4}b\tanh^3(x)\sqrt{a-b\tanh^2(x)+b}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sech[x]^2)^(3/2)*Tanh[x]^2,x]

[Out] -((3*a^2 - 6*a*b - b^2)*ArcTan[(Sqrt[b]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]])/(8*Sqrt[b]) + a^(3/2)*ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]] - ((5*a + b)*Tanh[x]*Sqrt[a + b - b*Tanh[x]^2])/8 + (b*Tanh[x]^3*Sqrt[a + b - b*Tanh[x]^2])/4

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 477

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)
^(q - 1))/(b*e*(m + n*(p + q) + 1)), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \left(a+b\text{sech}^2(x)\right)^{3/2}\tanh^2(x)\,dx = \text{Subst}\left(\int \frac{x^2\left(a+b\left(1-x^2\right)\right)^{3/2}}{1-x^2}\,dx, x, \tanh(x)\right)$$

$$= \text{Subst}\left(\int \frac{x^2\left(a+b-bx^2\right)^{3/2}}{1-x^2}\,dx, x, \tanh(x)\right)$$

$$= \frac{1}{4}b\tanh^3(x)\sqrt{a+b-b\tanh^2(x)} - \frac{1}{4}\text{Subst}\left(\int \frac{x^2\left(-(a+b)(4a+b)+b(5a\right.}{\left(1-x^2\right)\sqrt{a+b-bx^2}}\right.$$

$$= -\frac{1}{8}(5a+b)\tanh(x)\sqrt{a+b-b\tanh^2(x)} + \frac{1}{4}b\tanh^3(x)\sqrt{a+b-b\tanh^2(x)}$$

$$= -\frac{1}{8}(5a+b)\tanh(x)\sqrt{a+b-b\tanh^2(x)} + \frac{1}{4}b\tanh^3(x)\sqrt{a+b-b\tanh^2(x)}$$

$$= -\frac{1}{8}(5a+b)\tanh(x)\sqrt{a+b-b\tanh^2(x)} + \frac{1}{4}b\tanh^3(x)\sqrt{a+b-b\tanh^2(x)}$$

$$= -\frac{\left(3a^2-6ab-b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{8\sqrt{b}} + a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x}}\right.$$

Mathematica [A] time = 0.85, size = 197, normalized size = 1.58

$$-\frac{\cosh^3(x)\left(a+b\text{sech}^2(x)\right)^{3/2}\left(-8\sqrt{2}\,a^{3/2}\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sinh(x)}{\sqrt{a\cosh(2x)+a+2b}}\right)+\sqrt{2}\left(3a^2-6ab-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}\sinh(x}{\sqrt{a\cosh(2x)+a+}}\right.\right.}{4\sqrt{b}\left(a\cosh(2x)+\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[x]^2)^(3/2)*Tanh[x]^2,x]
```

```
[Out] -1/4*(Cosh[x]^3*(a + b*Sech[x]^2)^(3/2)*(Sqrt[2]*(3*a^2 - 6*a*b - b^2)*ArcT
an[(Sqrt[2]*Sqrt[b]*Sinh[x])/Sqrt[a + 2*b + a*Cosh[2*x]]] - 8*Sqrt[2]*a^(3/
2)*Sqrt[b]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sinh[x])/Sqrt[a + 2*b + a*Cosh[2*x]]] +
 (5*a - b)*Sqrt[b]*Sqrt[a + 2*b + a*Cosh[2*x]]*Sech[x]*Tanh[x] + 2*b^(3/2)*
Sqrt[a + 2*b + a*Cosh[2*x]]*Sech[x]^3*Tanh[x]))/(Sqrt[b]*(a + 2*b + a*Cosh[
2*x])^(3/2))
```

fricas [B] time = 1.15, size = 8582, normalized size = 68.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(3/2)*tanh(x)^2,x, algorithm="fricas")
```

```
[Out] [1/16*(4*(a*b*cosh(x)^8 + 8*a*b*cosh(x)*sinh(x)^7 + a*b*sinh(x)^8 + 4*a*b*c
osh(x)^6 + 4*(7*a*b*cosh(x)^2 + a*b)*sinh(x)^6 + 6*a*b*cosh(x)^4 + 8*(7*a*b
*cosh(x)^3 + 3*a*b*cosh(x))*sinh(x)^5 + 2*(35*a*b*cosh(x)^4 + 30*a*b*cosh(x
)^2 + 3*a*b)*sinh(x)^4 + 4*a*b*cosh(x)^2 + 8*(7*a*b*cosh(x)^5 + 10*a*b*cosh
(x)^3 + 3*a*b*cosh(x))*sinh(x)^3 + 4*(7*a*b*cosh(x)^6 + 15*a*b*cosh(x)^4 +
9*a*b*cosh(x)^2 + a*b)*sinh(x)^2 + a*b + 8*(a*b*cosh(x)^7 + 3*a*b*cosh(x)^5
 + 3*a*b*cosh(x)^3 + a*b*cosh(x))*sinh(x))*sqrt(a)*log((a*b^2*cosh(x)^8 + 8
*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*
(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh(x)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a
*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*
a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 - 30*(a*b^2 - b^3)*cosh(x)^2)*sin
h(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a*b^2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2*
b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14
*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*cosh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*
a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh
(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)
*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 + 4*a*b)*
cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 - 4*a*b)*sinh(x)^2 -
 a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 + 4*a*b)*cosh(x))*sinh(x
))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x
)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*cosh(x)^7 - 3*(a*b^2 - b^3)*cosh(x)^5
+ (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3 + (a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(
cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sin
h(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + ((3*a
^2 - 6*a*b - b^2)*cosh(x)^8 + 8*(3*a^2 - 6*a*b - b^2)*cosh(x)*sinh(x)^7 + (
3*a^2 - 6*a*b - b^2)*sinh(x)^8 + 4*(3*a^2 - 6*a*b - b^2)*cosh(x)^6 + 4*(7*(
3*a^2 - 6*a*b - b^2)*cosh(x)^2 + 3*a^2 - 6*a*b - b^2)*sinh(x)^6 + 8*(7*(3*a
^2 - 6*a*b - b^2)*cosh(x)^3 + 3*(3*a^2 - 6*a*b - b^2)*cosh(x))*sinh(x)^5 +
6*(3*a^2 - 6*a*b - b^2)*cosh(x)^4 + 2*(35*(3*a^2 - 6*a*b - b^2)*cosh(x)^4 +
```

30*(3*a^2 - 6*a*b - b^2)*cosh(x)^2 + 9*a^2 - 18*a*b - 3*b^2)*sinh(x)^4 + 8
(7(3*a^2 - 6*a*b - b^2)*cosh(x)^5 + 10*(3*a^2 - 6*a*b - b^2)*cosh(x)^3 +
3*(3*a^2 - 6*a*b - b^2)*cosh(x))*sinh(x)^3 + 4*(3*a^2 - 6*a*b - b^2)*cosh(x
)^2 + 4*(7*(3*a^2 - 6*a*b - b^2)*cosh(x)^6 + 15*(3*a^2 - 6*a*b - b^2)*cosh(
x)^4 + 9*(3*a^2 - 6*a*b - b^2)*cosh(x)^2 + 3*a^2 - 6*a*b - b^2)*sinh(x)^2 +
3*a^2 - 6*a*b - b^2 + 8*((3*a^2 - 6*a*b - b^2)*cosh(x)^7 + 3*(3*a^2 - 6*a*
b - b^2)*cosh(x)^5 + 3*(3*a^2 - 6*a*b - b^2)*cosh(x)^3 + (3*a^2 - 6*a*b - b
^2)*cosh(x))*sinh(x))*sqrt(-b)*log(-((a - b)*cosh(x)^4 + 4*(a - b)*cosh(x)*
sinh(x)^3 + (a - b)*sinh(x)^4 + 2*(a + 3*b)*cosh(x)^2 + 2*(3*(a - b)*cosh(x
)^2 + a + 3*b)*sinh(x)^2 + 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(
x)^2 - 1)*sqrt(-b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 -
2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a - b)*cosh(x)^3 + (a + 3*b)*cosh(x))
*sinh(x) + a - b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(
x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)) +
4*(a*b*cosh(x)^8 + 8*a*b*cosh(x)*sinh(x)^7 + a*b*sinh(x)^8 + 4*a*b*cosh(x)
^6 + 4*(7*a*b*cosh(x)^2 + a*b)*sinh(x)^6 + 6*a*b*cosh(x)^4 + 8*(7*a*b*cosh(
x)^3 + 3*a*b*cosh(x))*sinh(x)^5 + 2*(35*a*b*cosh(x)^4 + 30*a*b*cosh(x)^2 +
3*a*b)*sinh(x)^4 + 4*a*b*cosh(x)^2 + 8*(7*a*b*cosh(x)^5 + 10*a*b*cosh(x)^3
+ 3*a*b*cosh(x))*sinh(x)^3 + 4*(7*a*b*cosh(x)^6 + 15*a*b*cosh(x)^4 + 9*a*b*
cosh(x)^2 + a*b)*sinh(x)^2 + a*b + 8*(a*b*cosh(x)^7 + 3*a*b*cosh(x)^5 + 3*a
*b*cosh(x)^3 + a*b*cosh(x))*sinh(x))*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x
)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + b)
sinh(x)^2 + sqrt(2)(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a
)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x)
+ sinh(x)^2)) + 4*(a*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a)/(cosh(x)^2
+ 2*cosh(x)*sinh(x) + sinh(x)^2)) - 2*sqrt(2)*((5*a*b - b^2)*cosh(x)^6 + 6*
(5*a*b - b^2)*cosh(x)*sinh(x)^5 + (5*a*b - b^2)*sinh(x)^6 + (5*a*b + 7*b^2)
cosh(x)^4 + (15(5*a*b - b^2)*cosh(x)^2 + 5*a*b + 7*b^2)*sinh(x)^4 + 4*(5*
(5*a*b - b^2)*cosh(x)^3 + (5*a*b + 7*b^2)*cosh(x))*sinh(x)^3 - (5*a*b + 7*b
^2)*cosh(x)^2 + (15*(5*a*b - b^2)*cosh(x)^4 + 6*(5*a*b + 7*b^2)*cosh(x)^2 -
5*a*b - 7*b^2)*sinh(x)^2 - 5*a*b + b^2 + 2*(3*(5*a*b - b^2)*cosh(x)^5 + 2*
(5*a*b + 7*b^2)*cosh(x)^3 - (5*a*b + 7*b^2)*cosh(x))*sinh(x))*sqrt((a*cosh(
x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))
/(b*cosh(x)^8 + 8*b*cosh(x)*sinh(x)^7 + b*sinh(x)^8 + 4*b*cosh(x)^6 + 4*(7*
b*cosh(x)^2 + b)*sinh(x)^6 + 8*(7*b*cosh(x)^3 + 3*b*cosh(x))*sinh(x)^5 + 6*
b*cosh(x)^4 + 2*(35*b*cosh(x)^4 + 30*b*cosh(x)^2 + 3*b)*sinh(x)^4 + 8*(7*b*
cosh(x)^5 + 10*b*cosh(x)^3 + 3*b*cosh(x))*sinh(x)^3 + 4*b*cosh(x)^2 + 4*(7*
b*cosh(x)^6 + 15*b*cosh(x)^4 + 9*b*cosh(x)^2 + b)*sinh(x)^2 + 8*(b*cosh(x)^
7 + 3*b*cosh(x)^5 + 3*b*cosh(x)^3 + b*cosh(x))*sinh(x) + b), -1/8*(((3*a^2
- 6*a*b - b^2)*cosh(x)^8 + 8*(3*a^2 - 6*a*b - b^2)*cosh(x)*sinh(x)^7 + (3*a
^2 - 6*a*b - b^2)*sinh(x)^8 + 4*(3*a^2 - 6*a*b - b^2)*cosh(x)^6 + 4*(7*(3*a
^2 - 6*a*b - b^2)*cosh(x)^2 + 3*a^2 - 6*a*b - b^2)*sinh(x)^6 + 8*(7*(3*a^2
- 6*a*b - b^2)*cosh(x)^3 + 3*(3*a^2 - 6*a*b - b^2)*cosh(x))*sinh(x)^5 + 6*(
3*a^2 - 6*a*b - b^2)*cosh(x)^4 + 2*(35*(3*a^2 - 6*a*b - b^2)*cosh(x)^4 + 30
*(3*a^2 - 6*a*b - b^2)*cosh(x)^2 + 9*a^2 - 18*a*b - 3*b^2)*sinh(x)^4 + 8*(7

```
*(3*a^2 - 6*a*b - b^2)*cosh(x)^5 + 10*(3*a^2 - 6*a*b - b^2)*cosh(x)^3 + 3*(
3*a^2 - 6*a*b - b^2)*cosh(x))*sinh(x)^3 + 4*(3*a^2 - 6*a*b - b^2)*cosh(x)^2
 + 4*(7*(3*a^2 - 6*a*b - b^2)*cosh(x)^6 + 15*(3*a^2 - 6*a*b - b^2)*cosh(x)^
4 + 9*(3*a^2 - 6*a*b - b^2)*cosh(x)^2 + 3*a^2 - 6*a*b - b^2)*sinh(x)^2 + 3*
a^2 - 6*a*b - b^2 + 8*((3*a^2 - 6*a*b - b^2)*cosh(x)^7 + 3*(3*a^2 - 6*a*b -
 b^2)*cosh(x)^5 + 3*(3*a^2 - 6*a*b - b^2)*cosh(x)^3 + (3*a^2 - 6*a*b - b^2)
*cosh(x))*sinh(x))*sqrt(b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) +
sinh(x)^2 - 1)*sqrt(b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^
2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 +
a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2
 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) - 2*(a*b*cosh(x)^8 + 8
*a*b*cosh(x)*sinh(x)^7 + a*b*sinh(x)^8 + 4*a*b*cosh(x)^6 + 4*(7*a*b*cosh(x)
^2 + a*b)*sinh(x)^6 + 6*a*b*cosh(x)^4 + 8*(7*a*b*cosh(x)^3 + 3*a*b*cosh(x))
*sinh(x)^5 + 2*(35*a*b*cosh(x)^4 + 30*a*b*cosh(x)^2 + 3*a*b)*sinh(x)^4 + 4*
a*b*cosh(x)^2 + 8*(7*a*b*cosh(x)^5 + 10*a*b*cosh(x)^3 + 3*a*b*cosh(x))*sinh
(x)^3 + 4*(7*a*b*cosh(x)^6 + 15*a*b*cosh(x)^4 + 9*a*b*cosh(x)^2 + a*b)*sinh
(x)^2 + a*b + 8*(a*b*cosh(x)^7 + 3*a*b*cosh(x)^5 + 3*a*b*cosh(x)^3 + a*b*co
sh(x))*sinh(x))*sqrt(a)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 +
a*b^2*sinh(x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2
 + b^3)*sinh(x)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x
)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a
^2*b + 9*a*b^2 - 30*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x
)^5 - 10*(a*b^2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(
x)^3 + a^3 + 2*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^
2 - b^3)*cosh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)
*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)
^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(
x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)
^4 - 18*b^2*cosh(x)^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 -
 6*b^2*cosh(x)^3 - (a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^
2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4
*(2*a*b^2*cosh(x)^7 - 3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)
*cosh(x)^3 + (a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sin
h(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(
x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) - 2*(a*b*cosh(x)^8 + 8*a*b*cosh(x)
*sinh(x)^7 + a*b*sinh(x)^8 + 4*a*b*cosh(x)^6 + 4*(7*a*b*cosh(x)^2 + a*b)*si
nh(x)^6 + 6*a*b*cosh(x)^4 + 8*(7*a*b*cosh(x)^3 + 3*a*b*cosh(x))*sinh(x)^5 +
 2*(35*a*b*cosh(x)^4 + 30*a*b*cosh(x)^2 + 3*a*b)*sinh(x)^4 + 4*a*b*cosh(x)^
2 + 8*(7*a*b*cosh(x)^5 + 10*a*b*cosh(x)^3 + 3*a*b*cosh(x))*sinh(x)^3 + 4*(7
*a*b*cosh(x)^6 + 15*a*b*cosh(x)^4 + 9*a*b*cosh(x)^2 + a*b)*sinh(x)^2 + a*b
+ 8*(a*b*cosh(x)^7 + 3*a*b*cosh(x)^5 + 3*a*b*cosh(x)^3 + a*b*cosh(x))*sinh(
x))*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a
+ b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 +
 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2
 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(a*cosh(x)^3 +
```

```
(a + b)*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2))
 + sqrt(2)*((5*a*b - b^2)*cosh(x)^6 + 6*(5*a*b - b^2)*cosh(x)*sinh(x)^5 + (
5*a*b - b^2)*sinh(x)^6 + (5*a*b + 7*b^2)*cosh(x)^4 + (15*(5*a*b - b^2)*cosh
(x)^2 + 5*a*b + 7*b^2)*sinh(x)^4 + 4*(5*(5*a*b - b^2)*cosh(x)^3 + (5*a*b +
7*b^2)*cosh(x))*sinh(x)^3 - (5*a*b + 7*b^2)*cosh(x)^2 + (15*(5*a*b - b^2)*c
osh(x)^4 + 6*(5*a*b + 7*b^2)*cosh(x)^2 - 5*a*b - 7*b^2)*sinh(x)^2 - 5*a*b +
 b^2 + 2*(3*(5*a*b - b^2)*cosh(x)^5 + 2*(5*a*b + 7*b^2)*cosh(x)^3 - (5*a*b
+ 7*b^2)*cosh(x))*sinh(x))*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh
(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(b*cosh(x)^8 + 8*b*cosh(x)*sinh(x)
^7 + b*sinh(x)^8 + 4*b*cosh(x)^6 + 4*(7*b*cosh(x)^2 + b)*sinh(x)^6 + 8*(7*b
*cosh(x)^3 + 3*b*cosh(x))*sinh(x)^5 + 6*b*cosh(x)^4 + 2*(35*b*cosh(x)^4 + 3
0*b*cosh(x)^2 + 3*b)*sinh(x)^4 + 8*(7*b*cosh(x)^5 + 10*b*cosh(x)^3 + 3*b*co
sh(x))*sinh(x)^3 + 4*b*cosh(x)^2 + 4*(7*b*cosh(x)^6 + 15*b*cosh(x)^4 + 9*b*
cosh(x)^2 + b)*sinh(x)^2 + 8*(b*cosh(x)^7 + 3*b*cosh(x)^5 + 3*b*cosh(x)^3 +
 b*cosh(x))*sinh(x) + b), -1/16*(8*(a*b*cosh(x)^8 + 8*a*b*cosh(x)*sinh(x)^7
 + a*b*sinh(x)^8 + 4*a*b*cosh(x)^6 + 4*(7*a*b*cosh(x)^2 + a*b)*sinh(x)^6 +
6*a*b*cosh(x)^4 + 8*(7*a*b*cosh(x)^3 + 3*a*b*cosh(x))*sinh(x)^5 + 2*(35*a*b
*cosh(x)^4 + 30*a*b*cosh(x)^2 + 3*a*b)*sinh(x)^4 + 4*a*b*cosh(x)^2 + 8*(7*a
*b*cosh(x)^5 + 10*a*b*cosh(x)^3 + 3*a*b*cosh(x))*sinh(x)^3 + 4*(7*a*b*cosh(
x)^6 + 15*a*b*cosh(x)^4 + 9*a*b*cosh(x)^2 + a*b)*sinh(x)^2 + a*b + 8*(a*b*c
osh(x)^7 + 3*a*b*cosh(x)^5 + 3*a*b*cosh(x)^3 + a*b*cosh(x))*sinh(x))*sqrt(-
a)*arctan(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + a)*sqr
t(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sin
h(x) + sinh(x)^2))/(a*b*cosh(x)^4 + 4*a*b*cosh(x)*sinh(x)^3 + a*b*sinh(x)^4
 - (a^2 + 3*a*b)*cosh(x)^2 + (6*a*b*cosh(x)^2 - a^2 - 3*a*b)*sinh(x)^2 - a^
2 + 2*(2*a*b*cosh(x)^3 - (a^2 + 3*a*b)*cosh(x))*sinh(x))) + 8*(a*b*cosh(x)^
8 + 8*a*b*cosh(x)*sinh(x)^7 + a*b*sinh(x)^8 + 4*a*b*cosh(x)^6 + 4*(7*a*b*co
sh(x)^2 + a*b)*sinh(x)^6 + 6*a*b*cosh(x)^4 + 8*(7*a*b*cosh(x)^3 + 3*a*b*cos
h(x))*sinh(x)^5 + 2*(35*a*b*cosh(x)^4 + 30*a*b*cosh(x)^2 + 3*a*b)*sinh(x)^4
 + 4*a*b*cosh(x)^2 + 8*(7*a*b*cosh(x)^5 + 10*a*b*cosh(x)^3 + 3*a*b*cosh(x))
*sinh(x)^3 + 4*(7*a*b*cosh(x)^6 + 15*a*b*cosh(x)^4 + 9*a*b*cosh(x)^2 + a*b)
*sinh(x)^2 + a*b + 8*(a*b*cosh(x)^7 + 3*a*b*cosh(x)^5 + 3*a*b*cosh(x)^3 + a
*b*cosh(x))*sinh(x))*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cosh(x)^2 + a
*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(
x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a)) - ((3*a^2 - 6*a*b - b^2)*cos
h(x)^8 + 8*(3*a^2 - 6*a*b - b^2)*cosh(x)*sinh(x)^7 + (3*a^2 - 6*a*b - b^2)*
sinh(x)^8 + 4*(3*a^2 - 6*a*b - b^2)*cosh(x)^6 + 4*(7*(3*a^2 - 6*a*b - b^2)*
cosh(x)^2 + 3*a^2 - 6*a*b - b^2)*sinh(x)^6 + 8*(7*(3*a^2 - 6*a*b - b^2)*cos
h(x)^3 + 3*(3*a^2 - 6*a*b - b^2)*cosh(x))*sinh(x)^5 + 6*(3*a^2 - 6*a*b - b^
2)*cosh(x)^4 + 2*(35*(3*a^2 - 6*a*b - b^2)*cosh(x)^4 + 30*(3*a^2 - 6*a*b -
b^2)*cosh(x)^2 + 9*a^2 - 18*a*b - 3*b^2)*sinh(x)^4 + 8*(7*(3*a^2 - 6*a*b -
b^2)*cosh(x)^5 + 10*(3*a^2 - 6*a*b - b^2)*cosh(x)^3 + 3*(3*a^2 - 6*a*b - b^
2)*cosh(x))*sinh(x)^3 + 4*(3*a^2 - 6*a*b - b^2)*cosh(x)^2 + 4*(7*(3*a^2 - 6
*a*b - b^2)*cosh(x)^6 + 15*(3*a^2 - 6*a*b - b^2)*cosh(x)^4 + 9*(3*a^2 - 6*a
*b - b^2)*cosh(x)^2 + 3*a^2 - 6*a*b - b^2)*sinh(x)^2 + 3*a^2 - 6*a*b - b^2
```

```
+ 8*((3*a^2 - 6*a*b - b^2)*cosh(x)^7 + 3*(3*a^2 - 6*a*b - b^2)*cosh(x)^5 +
3*(3*a^2 - 6*a*b - b^2)*cosh(x)^3 + (3*a^2 - 6*a*b - b^2)*cosh(x))*sinh(x))
*sqrt(-b)*log(-((a - b)*cosh(x)^4 + 4*(a - b)*cosh(x)*sinh(x)^3 + (a - b)*s
inh(x)^4 + 2*(a + 3*b)*cosh(x)^2 + 2*(3*(a - b)*cosh(x)^2 + a + 3*b)*sinh(x
)^2 + 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-b)*sq
rt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + s
inh(x)^2)) + 4*((a - b)*cosh(x)^3 + (a + 3*b)*cosh(x))*sinh(x) + a - b)/(co
sh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 +
 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)) + 2*sqrt(2)*((5*a*b -
b^2)*cosh(x)^6 + 6*(5*a*b - b^2)*cosh(x)*sinh(x)^5 + (5*a*b - b^2)*sinh(x)^
6 + (5*a*b + 7*b^2)*cosh(x)^4 + (15*(5*a*b - b^2)*cosh(x)^2 + 5*a*b + 7*b^2
)*sinh(x)^4 + 4*(5*(5*a*b - b^2)*cosh(x)^3 + (5*a*b + 7*b^2)*cosh(x))*sinh(
x)^3 - (5*a*b + 7*b^2)*cosh(x)^2 + (15*(5*a*b - b^2)*cosh(x)^4 + 6*(5*a*b +
 7*b^2)*cosh(x)^2 - 5*a*b - 7*b^2)*sinh(x)^2 - 5*a*b + b^2 + 2*(3*(5*a*b -
b^2)*cosh(x)^5 + 2*(5*a*b + 7*b^2)*cosh(x)^3 - (5*a*b + 7*b^2)*cosh(x))*sin
h(x))*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sin
h(x) + sinh(x)^2)))/(b*cosh(x)^8 + 8*b*cosh(x)*sinh(x)^7 + b*sinh(x)^8 + 4*
b*cosh(x)^6 + 4*(7*b*cosh(x)^2 + b)*sinh(x)^6 + 8*(7*b*cosh(x)^3 + 3*b*cosh
(x))*sinh(x)^5 + 6*b*cosh(x)^4 + 2*(35*b*cosh(x)^4 + 30*b*cosh(x)^2 + 3*b)*
sinh(x)^4 + 8*(7*b*cosh(x)^5 + 10*b*cosh(x)^3 + 3*b*cosh(x))*sinh(x)^3 + 4*
b*cosh(x)^2 + 4*(7*b*cosh(x)^6 + 15*b*cosh(x)^4 + 9*b*cosh(x)^2 + b)*sinh(x
)^2 + 8*(b*cosh(x)^7 + 3*b*cosh(x)^5 + 3*b*cosh(x)^3 + b*cosh(x))*sinh(x) +
 b), -1/8*(4*(a*b*cosh(x)^8 + 8*a*b*cosh(x)*sinh(x)^7 + a*b*sinh(x)^8 + 4*a
*b*cosh(x)^6 + 4*(7*a*b*cosh(x)^2 + a*b)*sinh(x)^6 + 6*a*b*cosh(x)^4 + 8*(7
*a*b*cosh(x)^3 + 3*a*b*cosh(x))*sinh(x)^5 + 2*(35*a*b*cosh(x)^4 + 30*a*b*co
sh(x)^2 + 3*a*b)*sinh(x)^4 + 4*a*b*cosh(x)^2 + 8*(7*a*b*cosh(x)^5 + 10*a*b*
cosh(x)^3 + 3*a*b*cosh(x))*sinh(x)^3 + 4*(7*a*b*cosh(x)^6 + 15*a*b*cosh(x)^
4 + 9*a*b*cosh(x)^2 + a*b)*sinh(x)^2 + a*b + 8*(a*b*cosh(x)^7 + 3*a*b*cosh(
x)^5 + 3*a*b*cosh(x)^3 + a*b*cosh(x))*sinh(x))*sqrt(-a)*arctan(sqrt(2)*(b*c
osh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^
2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*
b*cosh(x)^4 + 4*a*b*cosh(x)*sinh(x)^3 + a*b*sinh(x)^4 - (a^2 + 3*a*b)*cosh(
x)^2 + (6*a*b*cosh(x)^2 - a^2 - 3*a*b)*sinh(x)^2 - a^2 + 2*(2*a*b*cosh(x)^3
 - (a^2 + 3*a*b)*cosh(x))*sinh(x))) + ((3*a^2 - 6*a*b - b^2)*cosh(x)^8 + 8*
(3*a^2 - 6*a*b - b^2)*cosh(x)*sinh(x)^7 + (3*a^2 - 6*a*b - b^2)*sinh(x)^8 +
 4*(3*a^2 - 6*a*b - b^2)*cosh(x)^6 + 4*(7*(3*a^2 - 6*a*b - b^2)*cosh(x)^2 +
 3*a^2 - 6*a*b - b^2)*sinh(x)^6 + 8*(7*(3*a^2 - 6*a*b - b^2)*cosh(x)^3 + 3*
(3*a^2 - 6*a*b - b^2)*cosh(x))*sinh(x)^5 + 6*(3*a^2 - 6*a*b - b^2)*cosh(x)^
4 + 2*(35*(3*a^2 - 6*a*b - b^2)*cosh(x)^4 + 30*(3*a^2 - 6*a*b - b^2)*cosh(x
)^2 + 9*a^2 - 18*a*b - 3*b^2)*sinh(x)^4 + 8*(7*(3*a^2 - 6*a*b - b^2)*cosh(x
)^5 + 10*(3*a^2 - 6*a*b - b^2)*cosh(x)^3 + 3*(3*a^2 - 6*a*b - b^2)*cosh(x))
*sinh(x)^3 + 4*(3*a^2 - 6*a*b - b^2)*cosh(x)^2 + 4*(7*(3*a^2 - 6*a*b - b^2)
*cosh(x)^6 + 15*(3*a^2 - 6*a*b - b^2)*cosh(x)^4 + 9*(3*a^2 - 6*a*b - b^2)*c
osh(x)^2 + 3*a^2 - 6*a*b - b^2)*sinh(x)^2 + 3*a^2 - 6*a*b - b^2 + 8*((3*a^2
 - 6*a*b - b^2)*cosh(x)^7 + 3*(3*a^2 - 6*a*b - b^2)*cosh(x)^5 + 3*(3*a^2 -
```

```
6*a*b - b^2)*cosh(x)^3 + (3*a^2 - 6*a*b - b^2)*cosh(x))*sinh(x))*sqrt(b)*ar
ctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(b)*sqrt((
a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(
x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cos
h(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)
*cosh(x))*sinh(x) + a)) + 4*(a*b*cosh(x)^8 + 8*a*b*cosh(x)*sinh(x)^7 + a*b*
sinh(x)^8 + 4*a*b*cosh(x)^6 + 4*(7*a*b*cosh(x)^2 + a*b)*sinh(x)^6 + 6*a*b*c
osh(x)^4 + 8*(7*a*b*cosh(x)^3 + 3*a*b*cosh(x))*sinh(x)^5 + 2*(35*a*b*cosh(x
)^4 + 30*a*b*cosh(x)^2 + 3*a*b)*sinh(x)^4 + 4*a*b*cosh(x)^2 + 8*(7*a*b*cosh
(x)^5 + 10*a*b*cosh(x)^3 + 3*a*b*cosh(x))*sinh(x)^3 + 4*(7*a*b*cosh(x)^6 +
15*a*b*cosh(x)^4 + 9*a*b*cosh(x)^2 + a*b)*sinh(x)^2 + a*b + 8*(a*b*cosh(x)^
7 + 3*a*b*cosh(x)^5 + 3*a*b*cosh(x)^3 + a*b*cosh(x))*sinh(x))*sqrt(-a)*arct
an(sqrt(2)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 -
 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sin
h(x)^2 + a)) + sqrt(2)*((5*a*b - b^2)*cosh(x)^6 + 6*(5*a*b - b^2)*cosh(x)*s
inh(x)^5 + (5*a*b - b^2)*sinh(x)^6 + (5*a*b + 7*b^2)*cosh(x)^4 + (15*(5*a*b
 - b^2)*cosh(x)^2 + 5*a*b + 7*b^2)*sinh(x)^4 + 4*(5*(5*a*b - b^2)*cosh(x)^3
 + (5*a*b + 7*b^2)*cosh(x))*sinh(x)^3 - (5*a*b + 7*b^2)*cosh(x)^2 + (15*(5*
a*b - b^2)*cosh(x)^4 + 6*(5*a*b + 7*b^2)*cosh(x)^2 - 5*a*b - 7*b^2)*sinh(x)
^2 - 5*a*b + b^2 + 2*(3*(5*a*b - b^2)*cosh(x)^5 + 2*(5*a*b + 7*b^2)*cosh(x)
^3 - (5*a*b + 7*b^2)*cosh(x))*sinh(x))*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a
+ 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(b*cosh(x)^8 + 8*b*cos
h(x)*sinh(x)^7 + b*sinh(x)^8 + 4*b*cosh(x)^6 + 4*(7*b*cosh(x)^2 + b)*sinh(x
)^6 + 8*(7*b*cosh(x)^3 + 3*b*cosh(x))*sinh(x)^5 + 6*b*cosh(x)^4 + 2*(35*b*c
osh(x)^4 + 30*b*cosh(x)^2 + 3*b)*sinh(x)^4 + 8*(7*b*cosh(x)^5 + 10*b*cosh(x
)^3 + 3*b*cosh(x))*sinh(x)^3 + 4*b*cosh(x)^2 + 4*(7*b*cosh(x)^6 + 15*b*cosh
(x)^4 + 9*b*cosh(x)^2 + b)*sinh(x)^2 + 8*(b*cosh(x)^7 + 3*b*cosh(x)^5 + 3*b
*cosh(x)^3 + b*cosh(x))*sinh(x) + b)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(3/2)*tanh(x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 0.46Error: Bad Argument Typ
e
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \left(a + b\mathrm{sech}(x)^2\right)^{\frac{3}{2}} \left(\tanh^2(x)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(x)^2)^(3/2)*tanh(x)^2,x)
[Out] int((a+b*sech(x)^2)^(3/2)*tanh(x)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\,\mathrm{sech}(x)^2 + a\right)^{\frac{3}{2}} \tanh(x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(3/2)*tanh(x)^2,x, algorithm="maxima")
[Out] integrate((b*sech(x)^2 + a)^(3/2)*tanh(x)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tanh(x)^2 \left(a + \frac{b}{\cosh(x)^2}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2*(a + b/cosh(x)^2)^(3/2),x)
[Out] int(tanh(x)^2*(a + b/cosh(x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\,\mathrm{sech}^2\left(x\right)\right)^{\frac{3}{2}} \tanh^2\left(x\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)**2)**(3/2)*tanh(x)**2,x)
[Out] Integral((a + b*sech(x)**2)**(3/2)*tanh(x)**2, x)
```

3.189 $\int \left(a+b\,\mathbf{sech}^2(x)\right)^{3/2} \tanh(x)\,dx$

Optimal. Leaf size=57

$$a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)-a\sqrt{a+b\operatorname{sech}^2(x)}-\frac{1}{3}\left(a+b\operatorname{sech}^2(x)\right)^{3/2}$$

```
[Out] a^(3/2)*arctanh((a+b*sech(x)^2)^(1/2)/a^(1/2))-1/3*(a+b*sech(x)^2)^(3/2)-a*
(a+b*sech(x)^2)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4139, 266, 50, 63, 208}

$$a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)-a\sqrt{a+b\operatorname{sech}^2(x)}-\frac{1}{3}\left(a+b\operatorname{sech}^2(x)\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[x]^2)^(3/2)*Tanh[x],x]
```

```
[Out] a^(3/2)*ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]] - a*Sqrt[a + b*Sech[x]^2] -
(a + b*Sech[x]^2)^(3/2)/3
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \left(a+b\,\text{sech}^2(x)\right)^{3/2}\tanh(x)\,dx = -\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{x}\,dx, x, \text{sech}(x)\right)$$

$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx, x, \text{sech}^2(x)\right)\right)$$

$$= -\frac{1}{3}\left(a+b\,\text{sech}^2(x)\right)^{3/2} - \frac{1}{2}a\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \text{sech}^2(x)\right)$$

$$= -a\sqrt{a+b\,\text{sech}^2(x)} - \frac{1}{3}\left(a+b\,\text{sech}^2(x)\right)^{3/2} - \frac{1}{2}a^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x,$$

$$= -a\sqrt{a+b\,\text{sech}^2(x)} - \frac{1}{3}\left(a+b\,\text{sech}^2(x)\right)^{3/2} - \frac{a^2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\right.}{b}$$

$$= a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\,\text{sech}^2(x)}}{\sqrt{a}}\right) - a\sqrt{a+b\,\text{sech}^2(x)} - \frac{1}{3}\left(a+b\,\text{sech}^2(x)\right)^{3/2}$$

Mathematica [C] time = 0.12, size = 65, normalized size = 1.14

$$-\frac{2b\left(a+b\,\mathrm{sech}^2(x)\right)^{3/2}\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};-\frac{a\cosh^2(x)}{b}\right)}{3\sqrt{\frac{a\cosh^2(x)}{b}+1}\,(a\cosh(2x)+a+2b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[x]^2)^(3/2)*Tanh[x],x]
```

```
[Out] (-2*b*Hypergeometric2F1[-3/2, -3/2, -1/2, -((a*Cosh[x]^2)/b)]*(a + b*Sech[x
]^2)^(3/2))/(3*Sqrt[1 + (a*Cosh[x]^2)/b]*(a + 2*b + a*Cosh[2*x]))
```

fricas [B] time = 0.84, size = 2312, normalized size = 40.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(3/2)*tanh(x),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(a*cosh(x)^6 + 6*a*cosh(x)*sinh(x)^5 + a*sinh(x)^6 + 3*a*cosh(x)^4
 + 3*(5*a*cosh(x)^2 + a)*sinh(x)^4 + 4*(5*a*cosh(x)^3 + 3*a*cosh(x))*sinh(x
)^3 + 3*a*cosh(x)^2 + 3*(5*a*cosh(x)^4 + 6*a*cosh(x)^2 + a)*sinh(x)^2 + 6*(
a*cosh(x)^5 + 2*a*cosh(x)^3 + a*cosh(x))*sinh(x) + a)*sqrt(a)*log(((a^3 + 2
*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^7 + (
a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh
(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*(a^3 + 2*a^2*b + a*b^2)*cos
h(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + 3*(2*a^3 + 5*
a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*co
sh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 6*a^3 + 14*a^2*b + 9*a*b^
2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3
+ 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)
^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(2*a^3 + 3*a
^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 + 15*(2*a^3 + 5*a
^2*b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3 + 3*a^2*b + 3*(6*a^3 + 14*a^2*b + 9
*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*((a^2 + 2*a*b + b^2)*cosh(x)^6 + 6*(
a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2 + 2*a*b + b^2)*sinh(x)^6 + 3*(a
^2 + 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 + 2*
a*b + b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 + 2*a*b
+ b^2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4*a*b)*cosh(x)^2 + (15*(a^2 + 2*a*b +
b^2)*cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*cosh(x)^2 + 3*a^2 + 4*a*b)*sinh(x)^
2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 6*(a^2 + 2*a*b + b^2)*cosh(x
)^3 + (3*a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(
x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 +
```

```
2*a^2*b + a*b^2)*cosh(x)^7 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^5
+ (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^3 + (2*a^3 + 3*a^2*b)*cosh(x))*sinh(
x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^
3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) +
3*(a*cosh(x)^6 + 6*a*cosh(x)*sinh(x)^5 + a*sinh(x)^6 + 3*a*cosh(x)^4 + 3*(5
*a*cosh(x)^2 + a)*sinh(x)^4 + 4*(5*a*cosh(x)^3 + 3*a*cosh(x))*sinh(x)^3 + 3
*a*cosh(x)^2 + 3*(5*a*cosh(x)^4 + 6*a*cosh(x)^2 + a)*sinh(x)^2 + 6*(a*cosh(
x)^5 + 2*a*cosh(x)^3 + a*cosh(x))*sinh(x) + a)*sqrt(a)*log(-(a*cosh(x)^4 +
4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*a*cosh(x)^2 + b)
*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a
)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x)
 + sinh(x)^2)) + 4*(a*cosh(x)^3 + b*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*co
sh(x)*sinh(x) + sinh(x)^2)) - 16*sqrt(2)*(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)
^3 + a*sinh(x)^4 + (2*a + b)*cosh(x)^2 + (6*a*cosh(x)^2 + 2*a + b)*sinh(x)^
2 + 2*(2*a*cosh(x)^3 + (2*a + b)*cosh(x))*sinh(x) + a)*sqrt((a*cosh(x)^2 +
a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(cosh(
x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + 3*(5*cosh(x)^2 + 1)*sinh(x)^4 + 3*
cosh(x)^4 + 4*(5*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 3*(5*cosh(x)^4 + 6*cosh
(x)^2 + 1)*sinh(x)^2 + 3*cosh(x)^2 + 6*(cosh(x)^5 + 2*cosh(x)^3 + cosh(x))*
sinh(x) + 1), -1/6*(3*(a*cosh(x)^6 + 6*a*cosh(x)*sinh(x)^5 + a*sinh(x)^6 +
3*a*cosh(x)^4 + 3*(5*a*cosh(x)^2 + a)*sinh(x)^4 + 4*(5*a*cosh(x)^3 + 3*a*co
sh(x))*sinh(x)^3 + 3*a*cosh(x)^2 + 3*(5*a*cosh(x)^4 + 6*a*cosh(x)^2 + a)*si
nh(x)^2 + 6*(a*cosh(x)^5 + 2*a*cosh(x)^3 + a*cosh(x))*sinh(x) + a)*sqrt(-a)
*arctan(sqrt(2)*((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*si
nh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2
 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*c
osh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 + (2*a^2 + 3*a*b)*cosh(x)^2 + (6*(
a^2 + a*b)*cosh(x)^2 + 2*a^2 + 3*a*b)*sinh(x)^2 + a^2 + 2*(2*(a^2 + a*b)*co
sh(x)^3 + (2*a^2 + 3*a*b)*cosh(x))*sinh(x))) + 3*(a*cosh(x)^6 + 6*a*cosh(x)
*sinh(x)^5 + a*sinh(x)^6 + 3*a*cosh(x)^4 + 3*(5*a*cosh(x)^2 + a)*sinh(x)^4
+ 4*(5*a*cosh(x)^3 + 3*a*cosh(x))*sinh(x)^3 + 3*a*cosh(x)^2 + 3*(5*a*cosh(x
)^4 + 6*a*cosh(x)^2 + a)*sinh(x)^2 + 6*(a*cosh(x)^5 + 2*a*cosh(x)^3 + a*cos
h(x))*sinh(x) + a)*sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) +
 sinh(x)^2 - 1)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x
)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3
+ a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)
^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) + 8*sqrt(2)*(a*cosh(
x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + (2*a + b)*cosh(x)^2 + (6*a*cos
h(x)^2 + 2*a + b)*sinh(x)^2 + 2*(2*a*cosh(x)^3 + (2*a + b)*cosh(x))*sinh(x)
 + a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sin
h(x) + sinh(x)^2)))/(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + 3*(5*cos
h(x)^2 + 1)*sinh(x)^4 + 3*cosh(x)^4 + 4*(5*cosh(x)^3 + 3*cosh(x))*sinh(x)^3
 + 3*(5*cosh(x)^4 + 6*cosh(x)^2 + 1)*sinh(x)^2 + 3*cosh(x)^2 + 6*(cosh(x)^5
 + 2*cosh(x)^3 + cosh(x))*sinh(x) + 1)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(3/2)*tanh(x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [A] time = 0.05, size = 56, normalized size = 0.98

$$-\frac{\left(a+b\mathrm{sech}(x)^{2}\right)^{\frac{3}{2}}}{3}+\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\mathrm{sech}(x)^{2}}}{\mathrm{sech}(x)}\right)a^{\frac{3}{2}}-a\sqrt{a+b\mathrm{sech}(x)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(x)^2)^(3/2)*tanh(x),x)
```

```
[Out] -1/3*(a+b*sech(x)^2)^(3/2)+ln((2*a+2*a^(1/2)*(a+b*sech(x)^2)^(1/2))/sech(x)
)*a^(3/2)-a*(a+b*sech(x)^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\,\mathrm{sech}(x)^{2}+a\right)^{\frac{3}{2}}\tanh(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(3/2)*tanh(x),x, algorithm="maxima")
```

```
[Out] integrate((b*sech(x)^2 + a)^(3/2)*tanh(x), x)
```

mupad [B] time = 3.36, size = 45, normalized size = 0.79

$$a^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{\cosh(x)^{2}}}}{\sqrt{a}}\right)-\frac{\left(a+\frac{b}{\cosh(x)^{2}}\right)^{3/2}}{3}-a\sqrt{a+\frac{b}{\cosh(x)^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)*(a + b/cosh(x)^2)^(3/2),x)
```

```
[Out] a^(3/2)*atanh((a + b/cosh(x)^2)^(1/2)/a^(1/2)) - (a + b/cosh(x)^2)^(3/2)/3
- a*(a + b/cosh(x)^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\operatorname{sech}^2(x)\right)^{\frac{3}{2}} \tanh(x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)**2)**(3/2)*tanh(x),x)
```

```
[Out] Integral((a + b*sech(x)**2)**(3/2)*tanh(x), x)
```

3.190 $\int \left(a + b\,\mathrm{sech}^2(x)\right)^{3/2} dx$

Optimal. Leaf size=88

$$a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)+\frac{1}{2}b\tanh(x)\sqrt{a-b\tanh^2(x)+b}+\frac{1}{2}\sqrt{b}\,(3a+b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)$$

```
[Out] a^(3/2)*arctanh(a^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))+1/2*(3*a+b)*arctan
(b^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))*b^(1/2)+1/2*b*(a+b-b*tanh(x)^2)^(
1/2)*tanh(x)
```

Rubi [A] time = 0.09, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {4128, 416, 523, 217, 203, 377, 206}

$$a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)+\frac{1}{2}b\tanh(x)\sqrt{a-b\tanh^2(x)+b}+\frac{1}{2}\sqrt{b}\,(3a+b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[x]^2)^(3/2),x]
```

```
[Out] (Sqrt[b]*(3*a + b)*ArcTan[(Sqrt[b]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]])/2 +
 a^(3/2)*ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]] + (b*Tanh[x]*
Sqrt[a + b - b*Tanh[x]^2])/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \left(a+b\text{sech}^2(x)\right)^{3/2}\,dx = \text{Subst}\left(\int \frac{\left(a+b-bx^2\right)^{3/2}}{1-x^2}\,dx, x, \tanh(x)\right)$$

$$= \frac{1}{2}b\tanh(x)\sqrt{a+b-b\tanh^2(x)} - \frac{1}{2}\text{Subst}\left(\int \frac{-(a+b)(2a+b)+b(3a+b)x^2}{\left(1-x^2\right)\sqrt{a+b-bx^2}}\,dx, x, \text{t}\right.$$

$$= \frac{1}{2}b\tanh(x)\sqrt{a+b-b\tanh^2(x)} + a^2\,\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)\sqrt{a+b-bx^2}}\,dx, x, \tanh(x)\right)$$

$$= \frac{1}{2}b\tanh(x)\sqrt{a+b-b\tanh^2(x)} + a^2\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)$$

$$= \frac{1}{2}\sqrt{b}\,(3a+b)\tan^{-1}\left(\frac{\sqrt{b}\,\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right) + a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right) + \frac{1}{2}$$

Mathematica [A] time = 0.27, size = 152, normalized size = 1.73

$$\frac{\text{sech}(x)\left(a\cosh^2(x)+b\right)\sqrt{a+b\text{sech}^2(x)}\left(2\sqrt{2}\,a^{3/2}\cosh^2(x)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sinh(x)}{\sqrt{a\cosh(2x)+a+2b}}\right)+b\sinh(x)\sqrt{a\cosh(2x)+}\right.}{(a\cosh(2x)+a+2b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[x]^2)^(3/2),x]
```

```
[Out] ((b + a*Cosh[x]^2)*Sech[x]*Sqrt[a + b*Sech[x]^2]*(Sqrt[2]*Sqrt[b]*(3*a + b)
*ArcTan[(Sqrt[2]*Sqrt[b]*Sinh[x])/Sqrt[a + 2*b + a*Cosh[2*x]]]*Cosh[x]^2 +
2*Sqrt[2]*a^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[a]*Sinh[x])/Sqrt[a + 2*b + a*Cosh[2
*x]]]*Cosh[x]^2 + b*Sqrt[a + 2*b + a*Cosh[2*x]]*Sinh[x]))/(a + 2*b + a*Cosh
[2*x])^(3/2)
```

fricas [B] time = 1.85, size = 4140, normalized size = 47.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*a*cosh(x)^2 +
2*(3*a*cosh(x)^2 + a)*sinh(x)^2 + 4*(a*cosh(x)^3 + a*cosh(x))*sinh(x) + a)*
```

```
sqrt(a)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^8
- 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh(x)^
6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3 + 4*a
^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 -
 30*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a*b^2
- b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(
a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*cosh(x)
^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqr
t(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(
x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cos
h(x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh
(x)^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3
 - (a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2
+ a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*cosh(x
)^7 - 3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3 + (a^
3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x
)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)
*sinh(x)^5 + sinh(x)^6)) + ((3*a + b)*cosh(x)^4 + 4*(3*a + b)*cosh(x)*sinh(
x)^3 + (3*a + b)*sinh(x)^4 + 2*(3*a + b)*cosh(x)^2 + 2*(3*(3*a + b)*cosh(x)
^2 + 3*a + b)*sinh(x)^2 + 4*((3*a + b)*cosh(x)^3 + (3*a + b)*cosh(x))*sinh(
x) + 3*a + b)*sqrt(-b)*log(-((a - b)*cosh(x)^4 + 4*(a - b)*cosh(x)*sinh(x)^
3 + (a - b)*sinh(x)^4 + 2*(a + 3*b)*cosh(x)^2 + 2*(3*(a - b)*cosh(x)^2 + a
+ 3*b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1
)*sqrt(-b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x
)*sinh(x) + sinh(x)^2)) + 4*((a - b)*cosh(x)^3 + (a + 3*b)*cosh(x))*sinh(x)
 + a - b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1
)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)) + (a*cosh
(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*a*cosh(x)^2 + 2*(3*a*cosh(x
)^2 + a)*sinh(x)^2 + 4*(a*cosh(x)^3 + a*cosh(x))*sinh(x) + a)*sqrt(a)*log(-
(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + b)*cosh(x)^2 +
2*(3*a*cosh(x)^2 + a + b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x
) + sinh(x)^2 + 1)*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh
(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(a*cosh(x)^3 + (a + b)*cosh(x))
*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) + 2*sqrt(2)*(b*c
osh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - b)*sqrt((a*cosh(x)^2 + a*sin
h(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(cosh(x)^4
+ 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(
x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1), 1/4*(2*((3*a + b)*cosh(x)^4 +
4*(3*a + b)*cosh(x)*sinh(x)^3 + (3*a + b)*sinh(x)^4 + 2*(3*a + b)*cosh(x)^2
 + 2*(3*(3*a + b)*cosh(x)^2 + 3*a + b)*sinh(x)^2 + 4*((3*a + b)*cosh(x)^3 +
 (3*a + b)*cosh(x))*sinh(x) + 3*a + b)*sqrt(b)*arctan(sqrt(2)*(cosh(x)^2 +
2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2
+ a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*
cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2
+ a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) +
```

```
(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*a*cosh(x)^2 + 2*(3*a
*cosh(x)^2 + a)*sinh(x)^2 + 4*(a*cosh(x)^3 + a*cosh(x))*sinh(x) + a)*sqrt(a
)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^8 - 2*(a
*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh(x)^6 + 4*
(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3 + 4*a^2*b +
 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 - 30*(a
*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a*b^2 - b^3)
*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(a^3 +
3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*cosh(x)^4 + a
^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(
b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 +
 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*
sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2
- a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^
2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a +
2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*cosh(x)^7 -
3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3 + (a^3 + 3*
a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*si
nh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(
x)^5 + sinh(x)^6)) + (a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2
*a*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a)*sinh(x)^2 + 4*(a*cosh(x)^3 + a*cosh(x)
)*sinh(x) + a)*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x
)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + b)*sinh(x)^2 + sqrt(2)*(
cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*sqrt((a*cosh(x)^2 +
a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(a*
cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) +
sinh(x)^2)) + 2*sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 -
b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x
) + sinh(x)^2)))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x
)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1), -1
/4*(2*(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*a*cosh(x)^2 +
2*(3*a*cosh(x)^2 + a)*sinh(x)^2 + 4*(a*cosh(x)^3 + a*cosh(x))*sinh(x) + a)*
sqrt(-a)*arctan(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 +
a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(
x)*sinh(x) + sinh(x)^2))/(a*b*cosh(x)^4 + 4*a*b*cosh(x)*sinh(x)^3 + a*b*sin
h(x)^4 - (a^2 + 3*a*b)*cosh(x)^2 + (6*a*b*cosh(x)^2 - a^2 - 3*a*b)*sinh(x)^
2 - a^2 + 2*(2*a*b*cosh(x)^3 - (a^2 + 3*a*b)*cosh(x))*sinh(x))) + 2*(a*cosh
(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*a*cosh(x)^2 + 2*(3*a*cosh(x
)^2 + a)*sinh(x)^2 + 4*(a*cosh(x)^3 + a*cosh(x))*sinh(x) + a)*sqrt(-a)*arct
an(sqrt(2)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 -
 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sin
h(x)^2 + a)) - ((3*a + b)*cosh(x)^4 + 4*(3*a + b)*cosh(x)*sinh(x)^3 + (3*a
+ b)*sinh(x)^4 + 2*(3*a + b)*cosh(x)^2 + 2*(3*(3*a + b)*cosh(x)^2 + 3*a + b
)*sinh(x)^2 + 4*((3*a + b)*cosh(x)^3 + (3*a + b)*cosh(x))*sinh(x) + 3*a + b
)*sqrt(-b)*log(-((a - b)*cosh(x)^4 + 4*(a - b)*cosh(x)*sinh(x)^3 + (a - b)*
```

```
sinh(x)^4 + 2*(a + 3*b)*cosh(x)^2 + 2*(3*(a - b)*cosh(x)^2 + a + 3*b)*sinh(
x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-b)*s
qrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) +
sinh(x)^2)) + 4*((a - b)*cosh(x)^3 + (a + 3*b)*cosh(x))*sinh(x) + a - b)/(c
osh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2
+ 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)) - 2*sqrt(2)*(b*cosh(x
)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - b)*sqrt((a*cosh(x)^2 + a*sinh(x)^
2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(cosh(x)^4 + 4*c
osh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2
+ 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1), -1/2*((a*cosh(x)^4 + 4*a*cosh(x)*si
nh(x)^3 + a*sinh(x)^4 + 2*a*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a)*sinh(x)^2 + 4
*(a*cosh(x)^3 + a*cosh(x))*sinh(x) + a)*sqrt(-a)*arctan(sqrt(2)*(b*cosh(x)^
2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*s
inh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*b*cosh(
x)^4 + 4*a*b*cosh(x)*sinh(x)^3 + a*b*sinh(x)^4 - (a^2 + 3*a*b)*cosh(x)^2 +
(6*a*b*cosh(x)^2 - a^2 - 3*a*b)*sinh(x)^2 - a^2 + 2*(2*a*b*cosh(x)^3 - (a^2
 + 3*a*b)*cosh(x))*sinh(x))) - ((3*a + b)*cosh(x)^4 + 4*(3*a + b)*cosh(x)*s
inh(x)^3 + (3*a + b)*sinh(x)^4 + 2*(3*a + b)*cosh(x)^2 + 2*(3*(3*a + b)*cos
h(x)^2 + 3*a + b)*sinh(x)^2 + 4*((3*a + b)*cosh(x)^3 + (3*a + b)*cosh(x))*s
inh(x) + 3*a + b)*sqrt(b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + s
inh(x)^2 - 1)*sqrt(b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2
 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a
*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2
+ 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) + (a*cosh(x)^4 + 4*a*co
sh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*a*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a)*sinh(
x)^2 + 4*(a*cosh(x)^3 + a*cosh(x))*sinh(x) + a)*sqrt(-a)*arctan(sqrt(2)*sqr
t(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sin
h(x) + sinh(x)^2))/(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a)) -
 sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - b)*sqrt((a*cosh
(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))
)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x
)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \left(a + b\mathrm{sech}(x)^2\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(x)^2)^(3/2),x)
[Out] int((a+b*sech(x)^2)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\,\mathrm{sech}(x)^2 + a\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(3/2),x, algorithm="maxima")
[Out] integrate((b*sech(x)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a + \frac{b}{\cosh(x)^2}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cosh(x)^2)^(3/2),x)
[Out] int((a + b/cosh(x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\,\mathrm{sech}^2\,(x)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)**2)**(3/2),x)
[Out] Integral((a + b*sech(x)**2)**(3/2), x)
```

3.191 $\int \coth(x)\left(a+b\,\mathbf{sech}^2(x)\right)^{3/2}\,dx$

Optimal. Leaf size=70

$$a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)+b\sqrt{a+b\operatorname{sech}^2(x)}-(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a+b}}\right)$$

```
[Out] a^(3/2)*arctanh((a+b*sech(x)^2)^(1/2)/a^(1/2))-(a+b)^(3/2)*arctanh((a+b*sec
h(x)^2)^(1/2)/(a+b)^(1/2))+b*(a+b*sech(x)^2)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4139, 446, 84, 156, 63, 208}

$$a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)+b\sqrt{a+b\operatorname{sech}^2(x)}-(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a+b}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]*(a + b*Sech[x]^2)^(3/2),x]
```

```
[Out] a^(3/2)*ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]] - (a + b)^(3/2)*ArcTanh[Sqrt
[a + b*Sech[x]^2]/Sqrt[a + b]] + b*Sqrt[a + b*Sech[x]^2]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p - 1))/(b*d*(p - 1)), x] + Dist[1/(b*d), I
nt[((b*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*(e + f*x)^(p - 2))/
((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
```

```
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \coth(x)\left(a+b\text{sech}^2(x)\right)^{3/2}\,dx = \text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{x\left(-1+x^2\right)}\,dx, x, \text{sech}(x)\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{(-1+x)x}\,dx, x, \text{sech}^2(x)\right)$$
$$= b\sqrt{a+b\text{sech}^2(x)} + \frac{1}{2}\text{Subst}\left(\int \frac{a^2+b(2a+b)x}{(-1+x)x\sqrt{a+bx}}\,dx, x, \text{sech}^2(x)\right)$$
$$= b\sqrt{a+b\text{sech}^2(x)} - \frac{1}{2}a^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \text{sech}^2(x)\right) + \frac{1}{2}(a+b)^2\,\text{S}$$
$$= b\sqrt{a+b\text{sech}^2(x)} - \frac{a^2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{sech}^2(x)}\right)}{b} + \frac{(a+b)^2\,\text{S}}{}$$
$$= a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right) - (a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a+b}}\right) + b\sqrt{}$$

Mathematica [B] time = 0.52, size = 159, normalized size = 2.27

$$-\frac{2\left(a\cosh^2(x)+b\right)\sqrt{a+b\text{sech}^2(x)}\left(\sqrt{2}(a+b)^2\cosh(x)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a+b}\cosh(x)}{\sqrt{a\cosh(2x)+a+2b}}\right)-\sqrt{a+b}\left(\sqrt{2}a^{3/2}\cosh(x)\text{l}\right.\right.}{\sqrt{a+b}\,(a\cosh(2x)+a+2b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]*(a + b*Sech[x]^2)^(3/2),x]
```

```
[Out] (-2*(b + a*Cosh[x]^2)*(Sqrt[2]*(a + b)^2*ArcTanh[(Sqrt[2]*Sqrt[a + b]*Cosh[
x])/Sqrt[a + 2*b + a*Cosh[2*x]]]*Cosh[x] - Sqrt[a + b]*(b*Sqrt[a + 2*b + a*
Cosh[2*x]] + Sqrt[2]*a^(3/2)*Cosh[x]*Log[Sqrt[2]*Sqrt[a]*Cosh[x] + Sqrt[a +
 2*b + a*Cosh[2*x]]]))*Sqrt[a + b*Sech[x]^2])/(Sqrt[a + b]*(a + 2*b + a*Cos
h[2*x])^(3/2))
```

fricas [B] time = 1.31, size = 4123, normalized size = 58.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*sech(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a)*sqrt(a)*log(((a
^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)
^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3
)*cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*(a^3 + 2*a^2*b + a*b^
2)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + 3*(2*a^
3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b
^2)*cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 6*a^3 + 14*a^2*b +
9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14
*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*c
osh(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(2*a^3
 + 3*a^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 + 15*(2*a^3
 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3 + 3*a^2*b + 3*(6*a^3 + 14*a^2
*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*((a^2 + 2*a*b + b^2)*cosh(x)^6
 + 6*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2 + 2*a*b + b^2)*sinh(x)^6
+ 3*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^
2 + 2*a*b + b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 +
2*a*b + b^2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4*a*b)*cosh(x)^2 + (15*(a^2 + 2*
a*b + b^2)*cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*cosh(x)^2 + 3*a^2 + 4*a*b)*si
nh(x)^2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 6*(a^2 + 2*a*b + b^2)*
cosh(x)^3 + (3*a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a
*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(
a^3 + 2*a^2*b + a*b^2)*cosh(x)^7 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh
(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^3 + (2*a^3 + 3*a^2*b)*cosh(x))
*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*co
sh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^
6)) + 2*((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2
+ a + b)*sqrt(a + b)*log(((2*a + b)*cosh(x)^4 + 4*(2*a + b)*cosh(x)*sinh(x)
^3 + (2*a + b)*sinh(x)^4 + 2*(2*a + 3*b)*cosh(x)^2 + 2*(3*(2*a + b)*cosh(x)
^2 + 2*a + 3*b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh
(x)^2 + 1)*sqrt(a + b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^
2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((2*a + b)*cosh(x)^3 + (2*a + 3*b)*
cosh(x))*sinh(x) + 2*a + b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 +
2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(
x) + 1)) + (a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a)*sqrt(a)*lo
g(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(
3*a*cosh(x)^2 + b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sin
h(x)^2 - 1)*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 -
 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(a*cosh(x)^3 + b*cosh(x))*sinh(x) + a)
/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*sqrt(2)*b*sqrt((a*cosh(x)
^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(
cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1), 1/4*(4*((a + b)*cosh(x)^2 +
 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a + b)*sqrt(-a - b)*arctan
(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a - b)*sqrt(
(a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh
(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*co
```

```
sh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b
)*cosh(x))*sinh(x) + a)) + (a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2
 + a)*sqrt(a)*log(((a^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a
*b^2)*cosh(x)*sinh(x)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*
a^2*b + 4*a*b^2 + b^3)*cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*
(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2
)*cosh(x)^3 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + (6*a
^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4
+ 6*a^3 + 14*a^2*b + 9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)
^2)*sinh(x)^4 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a^2
*b + 4*a*b^2 + b^3)*cosh(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*sinh(
x)^3 + a^3 + 2*(2*a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*
cosh(x)^6 + 15*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3 + 3*a^2*
b + 3*(6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*((a^2 + 2
*a*b + b^2)*cosh(x)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2 + 2*
a*b + b^2)*sinh(x)^6 + 3*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b
+ b^2)*cosh(x)^2 + a^2 + 2*a*b + b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*
cosh(x)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4*a*b)*cosh
(x)^2 + (15*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*cosh(x)^
2 + 3*a^2 + 4*a*b)*sinh(x)^2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 6
*(a^2 + 2*a*b + b^2)*cosh(x)^3 + (3*a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*
sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) +
 sinh(x)^2)) + 4*(2*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^7 + 3*(2*a^3 + 5*a^2*b
+ 4*a*b^2 + b^3)*cosh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^3 + (2*a^
3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x
)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)
*sinh(x)^5 + sinh(x)^6)) + (a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2
 + a)*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*b
*cosh(x)^2 + 2*(3*a*cosh(x)^2 + b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(
x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2
*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(a*cosh(x)^3 + b*cosh(
x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*sqrt(2)*b
*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x)
+ sinh(x)^2)))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1), -1/2*((a*co
sh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a)*sqrt(-a)*arctan(sqrt(2)*((
a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a)*sqrt(
-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(
x) + sinh(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 +
 (a^2 + a*b)*sinh(x)^4 + (2*a^2 + 3*a*b)*cosh(x)^2 + (6*(a^2 + a*b)*cosh(x)
^2 + 2*a^2 + 3*a*b)*sinh(x)^2 + a^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 + (2*a^2 +
 3*a*b)*cosh(x))*sinh(x))) + (a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)
^2 + a)*sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2
- 1)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cos
h(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)
^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*c
```

```
osh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) - ((a + b)*cosh(x)^2 + 2*(a + b
)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a + b)*sqrt(a + b)*log(((2*a + b)*c
osh(x)^4 + 4*(2*a + b)*cosh(x)*sinh(x)^3 + (2*a + b)*sinh(x)^4 + 2*(2*a + 3
*b)*cosh(x)^2 + 2*(3*(2*a + b)*cosh(x)^2 + 2*a + 3*b)*sinh(x)^2 - 2*sqrt(2)
*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*sqrt((a*cosh(x
)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) +
 4*((2*a + b)*cosh(x)^3 + (2*a + 3*b)*cosh(x))*sinh(x) + 2*a + b)/(cosh(x)^
4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cos
h(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)) - 2*sqrt(2)*b*sqrt((a*cosh(x
)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/
(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1), -1/2*((a*cosh(x)^2 + 2*a*c
osh(x)*sinh(x) + a*sinh(x)^2 + a)*sqrt(-a)*arctan(sqrt(2)*((a + b)*cosh(x)^
2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*cos
h(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)
)/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*si
nh(x)^4 + (2*a^2 + 3*a*b)*cosh(x)^2 + (6*(a^2 + a*b)*cosh(x)^2 + 2*a^2 + 3*
a*b)*sinh(x)^2 + a^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 + (2*a^2 + 3*a*b)*cosh(x)
)*sinh(x))) + (a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a)*sqrt(-a
)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a)*s
qrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) +
sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b
)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a +
 2*b)*cosh(x))*sinh(x) + a)) - 2*((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sin
h(x) + (a + b)*sinh(x)^2 + a + b)*sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2 +
2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a - b)*sqrt((a*cosh(x)^2 + a*sinh(
x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 +
 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(
x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a
)) - 2*sqrt(2)*b*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*
cosh(x)*sinh(x) + sinh(x)^2)))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 +
 1)]
```

giac [B] time = 0.47, size = 134, normalized size = 1.91

$$-\frac{4\left(\left(\sqrt{a}e^{(2x)}-\sqrt{ae^{(4x)}+2ae^{(2x)}+4be^{(2x)}+a}\right)b^2-\sqrt{a}b^2\right)}{\left(\sqrt{a}e^{(2x)}-\sqrt{ae^{(4x)}+2ae^{(2x)}+4be^{(2x)}+a}\right)^2+2\left(\sqrt{a}e^{(2x)}-\sqrt{ae^{(4x)}+2ae^{(2x)}+4be^{(2x)}+a}\right)\sqrt{a}+a+4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*sech(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -4*((sqrt(a)*e^(2*x) - sqrt(a*e^(4*x) + 2*a*e^(2*x) + 4*b*e^(2*x) + a))*b^2
 - sqrt(a)*b^2)/((sqrt(a)*e^(2*x) - sqrt(a*e^(4*x) + 2*a*e^(2*x) + 4*b*e^(2
```

```
*x) + a))^2 + 2*(sqrt(a)*e^(2*x) - sqrt(a*e^(4*x) + 2*a*e^(2*x) + 4*b*e^(2*
x) + a))*sqrt(a) + a + 4*b)
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \coth(x)\left(a+b\operatorname{sech}(x)^{2}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)*(a+b*sech(x)^2)^(3/2),x)
[Out] int(coth(x)*(a+b*sech(x)^2)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\operatorname{sech}(x)^{2}+a\right)^{\frac{3}{2}}\coth(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*sech(x)^2)^(3/2),x, algorithm="maxima")
[Out] integrate((b*sech(x)^2 + a)^(3/2)*coth(x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \coth(x)\left(a+\frac{b}{\cosh(x)^{2}}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)*(a + b/cosh(x)^2)^(3/2),x)
[Out] int(coth(x)*(a + b/cosh(x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\operatorname{sech}^{2}(x)\right)^{\frac{3}{2}}\coth(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*sech(x)**2)**(3/2),x)
[Out] Integral((a + b*sech(x)**2)**(3/2)*coth(x), x)
```

3.192 $\int \coth^2(x) \left(a + b\mathbf{sech}^2(x)\right)^{3/2} dx$

Optimal. Leaf size=81

$$a^{3/2} \tanh^{-1}\left(\frac{\sqrt{a} \tanh(x)}{\sqrt{a - b\tanh^2(x) + b}}\right) - b^{3/2} \tan^{-1}\left(\frac{\sqrt{b} \tanh(x)}{\sqrt{a - b\tanh^2(x) + b}}\right) - (a+b)\coth(x)\sqrt{a - b\tanh^2(x) + b}$$

```
[Out] -b^(3/2)*arctan(b^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))+a^(3/2)*arctanh(a^
(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))-(a+b)*coth(x)*(a+b-b*tanh(x)^2)^(1/2
)
```

Rubi [A] time = 0.23, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {4141, 1975, 474, 523, 217, 203, 377, 206}

$$a^{3/2} \tanh^{-1}\left(\frac{\sqrt{a} \tanh(x)}{\sqrt{a - b\tanh^2(x) + b}}\right) - b^{3/2} \tan^{-1}\left(\frac{\sqrt{b} \tanh(x)}{\sqrt{a - b\tanh^2(x) + b}}\right) - (a+b)\coth(x)\sqrt{a - b\tanh^2(x) + b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2*(a + b*Sech[x]^2)^(3/2),x]
```

```
[Out] -(b^(3/2)*ArcTan[(Sqrt[b]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]) + a^(3/2)*Ar
cTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]] - (a + b)*Coth[x]*Sqrt[a
 + b - b*Tanh[x]^2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 474

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
(q - 1))/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \coth^2(x)\left(a+b\mathrm{sech}^2(x)\right)^{3/2}\,dx = \mathrm{Subst}\left(\int \frac{\left(a+b\left(1-x^2\right)\right)^{3/2}}{x^2\left(1-x^2\right)}\,dx, x, \tanh(x)\right)$$

$$= \mathrm{Subst}\left(\int \frac{\left(a+b-bx^2\right)^{3/2}}{x^2\left(1-x^2\right)}\,dx, x, \tanh(x)\right)$$

$$= -(a+b)\coth(x)\sqrt{a+b-b\tanh^2(x)} + \mathrm{Subst}\left(\int \frac{a^2-b^2+b^2x^2}{\left(1-x^2\right)\sqrt{a+b-bx^2}}\,dx,$$

$$= -(a+b)\coth(x)\sqrt{a+b-b\tanh^2(x)} + a^2\,\mathrm{Subst}\left(\int \frac{1}{\left(1-x^2\right)\sqrt{a+b-bx^2}}\right.$$

$$= -(a+b)\coth(x)\sqrt{a+b-b\tanh^2(x)} + a^2\,\mathrm{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\tan}{\sqrt{a+b-}}\right.$$

$$= -b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right) + a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right) - ($$

Mathematica [**A**] time = 0.35, size = 144, normalized size = 1.78

$$-\frac{2\left(a\cosh^2(x)+b\right)\sqrt{a+b\mathrm{sech}^2(x)}\left(-\sqrt{2}\,a^{3/2}\cosh(x)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sinh(x)}{\sqrt{a\cosh(2x)+a+2b}}\right)+\sqrt{2}\,b^{3/2}\cosh(x)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{}}{\sqrt{a\cosh}}\right.\right.}{(a\cosh(2x)+a+2b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2*(a + b*Sech[x]^2)^(3/2),x]
```

```
[Out] (-2*(b + a*Cosh[x]^2)*(Sqrt[2]*b^(3/2)*ArcTan[(Sqrt[2]*Sqrt[b]*Sinh[x])/Sqr
t[a + 2*b + a*Cosh[2*x]]]*Cosh[x] - Sqrt[2]*a^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[a
]*Sinh[x])/Sqrt[a + 2*b + a*Cosh[2*x]]]*Cosh[x] + (a + b)*Sqrt[a + 2*b + a*
Cosh[2*x]]*Coth[x])*Sqrt[a + b*Sech[x]^2])/(a + 2*b + a*Cosh[2*x])^(3/2)
```

fricas [**B**] time = 4.04, size = 3349, normalized size = 41.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2*(a+b*sech(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 - a)*sqrt(a)*log((a*
b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^8 - 2*(a*b^2 - b^
3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh(x)^6 + 4*(14*a*b^2
*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)
*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 - 30*(a*b^2 - b^
3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a*b^2 - b^3)*cosh(x)^
3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(a^3 + 3*a^2*b)*
cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*cosh(x)^4 + a^3 + 3*a^
2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(
x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2
*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3
 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 - 4
*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 + 4*a*b
)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cos
h(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*cosh(x)^7 - 3*(a*b^2
- b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3 + (a^3 + 3*a^2*b)*co
sh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 +
 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + si
nh(x)^6)) + 2*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - b)*sqrt(-b
)*log(-((a - b)*cosh(x)^4 + 4*(a - b)*cosh(x)*sinh(x)^3 + (a - b)*sinh(x)^4
 + 2*(a + 3*b)*cosh(x)^2 + 2*(3*(a - b)*cosh(x)^2 + a + 3*b)*sinh(x)^2 + 2*
sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-b)*sqrt((a*co
sh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2
)) + 4*((a - b)*cosh(x)^3 + (a + 3*b)*cosh(x))*sinh(x) + a - b)/(cosh(x)^4
+ 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(
x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)) + (a*cosh(x)^2 + 2*a*cosh(x)*s
inh(x) + a*sinh(x)^2 - a)*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3
 + a*sinh(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + b)*sinh(x)^2
+ sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*sqrt((a*c
osh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^
2)) + 4*(a*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)
*sinh(x) + sinh(x)^2)) - 4*sqrt(2)*(a + b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2
+ a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(cosh(x)^2 + 2*cos
h(x)*sinh(x) + sinh(x)^2 - 1), -1/4*(4*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) +
 b*sinh(x)^2 - b)*sqrt(b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + s
inh(x)^2 - 1)*sqrt(b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2
 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a
*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2
+ 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) - (a*cosh(x)^2 + 2*a*co
sh(x)*sinh(x) + a*sinh(x)^2 - a)*sqrt(a)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cos
h(x)*sinh(x)^7 + a*b^2*sinh(x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*
cosh(x)^2 - a*b^2 + b^3)*sinh(x)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3
)*cosh(x))*sinh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh
(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 - 30*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4
*(14*a*b^2*cosh(x)^5 - 10*(a*b^2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^
```

2)*cosh(x))*sinh(x)^3 + a^3 + 2*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cos
h(x)^6 - 15*(a*b^2 - b^3)*cosh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*
a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x
)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4
 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2
+ (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(
3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)
*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x)
+ sinh(x)^2)) + 4*(2*a*b^2*cosh(x)^7 - 3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4
*a^2*b + 9*a*b^2)*cosh(x)^3 + (a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6
+ 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 1
5*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) - (a*cosh(x)^2 +
2*a*cosh(x)*sinh(x) + a*sinh(x)^2 - a)*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh
(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a +
b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt
(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(
x) + sinh(x)^2)) + 4*(a*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a)/(cosh(x)^
2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*sqrt(2)*(a + b)*sqrt((a*cosh(x)^2 +
 a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(cosh
(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1), -1/2*((a*cosh(x)^2 + 2*a*cosh(x
)*sinh(x) + a*sinh(x)^2 - a)*sqrt(-a)*arctan(sqrt(2)*(b*cosh(x)^2 + 2*b*cos
h(x)*sinh(x) + b*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 +
a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*b*cosh(x)^4 + 4*a*
b*cosh(x)*sinh(x)^3 + a*b*sinh(x)^4 - (a^2 + 3*a*b)*cosh(x)^2 + (6*a*b*cosh
(x)^2 - a^2 - 3*a*b)*sinh(x)^2 - a^2 + 2*(2*a*b*cosh(x)^3 - (a^2 + 3*a*b)*c
osh(x))*sinh(x))) + (a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 - a)*s
qrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/
(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^2 + 2*a*cosh(x)*sin
h(x) + a*sinh(x)^2 + a)) - (b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2
 - b)*sqrt(-b)*log(-((a - b)*cosh(x)^4 + 4*(a - b)*cosh(x)*sinh(x)^3 + (a -
 b)*sinh(x)^4 + 2*(a + 3*b)*cosh(x)^2 + 2*(3*(a - b)*cosh(x)^2 + a + 3*b)*s
inh(x)^2 + 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-
b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x
) + sinh(x)^2)) + 4*((a - b)*cosh(x)^3 + (a + 3*b)*cosh(x))*sinh(x) + a - b
)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x
)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)) + 2*sqrt(2)*(a +
b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x
) + sinh(x)^2)))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1), -1/2*((a*
cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 - a)*sqrt(-a)*arctan(sqrt(2)*
(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh
(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))
/(a*b*cosh(x)^4 + 4*a*b*cosh(x)*sinh(x)^3 + a*b*sinh(x)^4 - (a^2 + 3*a*b)*c
osh(x)^2 + (6*a*b*cosh(x)^2 - a^2 - 3*a*b)*sinh(x)^2 - a^2 + 2*(2*a*b*cosh(
x)^3 - (a^2 + 3*a*b)*cosh(x))*sinh(x))) + 2*(b*cosh(x)^2 + 2*b*cosh(x)*sinh
(x) + b*sinh(x)^2 - b)*sqrt(b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x

```
) + sinh(x)^2 - 1)*sqrt(b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh
(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^
3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(
x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) + (a*cosh(x)^2 + 2
*a*cosh(x)*sinh(x) + a*sinh(x)^2 - a)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt
((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sin
h(x)^2))/(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a)) + 2*sqrt(2)
*(a + b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*
sinh(x) + sinh(x)^2)))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)]
```

giac [B] time = 0.52, size = 222, normalized size = 2.74

$$\frac{4\left(\left(\sqrt{a}\,e^{(2\,x)} - \sqrt{ae^{(4\,x)} + 2\,ae^{(2\,x)} + 4\,be^{(2\,x)} + a}\right)a^2 + 2\left(\sqrt{a}\,e^{(2\,x)} - \sqrt{ae^{(4\,x)} + 2\,ae^{(2\,x)} + 4\,be^{(2\,x)} + a}\right)ab + \left(\sqrt{a}\,e^{(2\right.\right.}{\left(\sqrt{a}\,e^{(2\,x)} - \sqrt{ae^{(4\,x)} + 2\,ae^{(2\,x)} + 4\,be^{(2\,x)} + a}\right)^2 - 2\left(\sqrt{a}\,e^{(2\,x)} - \sqrt{ae^{(4\,x)} + 2\,ae^{(2\,x)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2*(a+b*sech(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 4*((sqrt(a)*e^(2*x) - sqrt(a*e^(4*x) + 2*a*e^(2*x) + 4*b*e^(2*x) + a))*a^2
+ 2*(sqrt(a)*e^(2*x) - sqrt(a*e^(4*x) + 2*a*e^(2*x) + 4*b*e^(2*x) + a))*a*b
 + (sqrt(a)*e^(2*x) - sqrt(a*e^(4*x) + 2*a*e^(2*x) + 4*b*e^(2*x) + a))*b^2
+ a^(5/2) + 2*a^(3/2)*b + sqrt(a)*b^2)/((sqrt(a)*e^(2*x) - sqrt(a*e^(4*x) +
 2*a*e^(2*x) + 4*b*e^(2*x) + a))^2 - 2*(sqrt(a)*e^(2*x) - sqrt(a*e^(4*x) +
2*a*e^(2*x) + 4*b*e^(2*x) + a))*sqrt(a) - 3*a - 4*b)
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \left(\coth^2(x)\right)\left(a + b\mathrm{sech}(x)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2*(a+b*sech(x)^2)^(3/2),x)
```

```
[Out] int(coth(x)^2*(a+b*sech(x)^2)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\,\mathrm{sech}(x)^2 + a\right)^{\frac{3}{2}}\coth(x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2*(a+b*sech(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sech(x)^2 + a)^(3/2)*coth(x)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \coth(x)^2 \left(a + \frac{b}{\cosh(x)^2}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2*(a + b/cosh(x)^2)^(3/2),x)
[Out] int(coth(x)^2*(a + b/cosh(x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2*(a+b*sech(x)**2)**(3/2),x)
[Out] Timed out
```

3.193 $\int \left(a+b\,\mathrm{sech}^2(c+dx)\right)^{5/2}\,dx$

Optimal. Leaf size=170

$$\frac{a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\tanh(c+dx)}{\sqrt{a-b\tanh^2(c+dx)+b}}\right)}{d}+\frac{\sqrt{b}\left(15a^2+10ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a-b\tanh^2(c+dx)+b}}\right)}{8d}+\frac{b\tanh(c+dx)\left(a-b\tanh\right.}{4d}$$

```
[Out] a^(5/2)*arctanh(a^(1/2)*tanh(d*x+c)/(a+b-b*tanh(d*x+c)^2)^(1/2))/d+1/8*(15*
a^2+10*a*b+3*b^2)*arctan(b^(1/2)*tanh(d*x+c)/(a+b-b*tanh(d*x+c)^2)^(1/2))*b
^(1/2)/d+1/8*b*(7*a+3*b)*(a+b-b*tanh(d*x+c)^2)^(1/2)*tanh(d*x+c)/d+1/4*b*ta
nh(d*x+c)*(a+b-b*tanh(d*x+c)^2)^(3/2)/d
```

Rubi [A] time = 0.19, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4128, 416, 528, 523, 217, 203, 377, 206}

$$\frac{\sqrt{b}\left(15a^2+10ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a-b\tanh^2(c+dx)+b}}\right)}{8d}+\frac{a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\tanh(c+dx)}{\sqrt{a-b\tanh^2(c+dx)+b}}\right)}{d}+\frac{b\tanh(c+dx)\left(a-b\tanh\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^(5/2),x]
```

```
[Out] (Sqrt[b]*(15*a^2 + 10*a*b + 3*b^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a +
b - b*Tanh[c + d*x]^2]])/(8*d) + (a^(5/2)*ArcTanh[(Sqrt[a]*Tanh[c + d*x])/S
qrt[a + b - b*Tanh[c + d*x]^2]])/d + (b*(7*a + 3*b)*Tanh[c + d*x]*Sqrt[a +
b - b*Tanh[c + d*x]^2])/(8*d) + (b*Tanh[c + d*x]*(a + b - b*Tanh[c + d*x]^2
)^(3/2))/(4*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \left(a+b\operatorname{sech}^2(c+dx)\right)^{5/2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b-bx^2\right)^{5/2}}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b\tanh(c+dx)\left(a+b-b\tanh^2(c+dx)\right)^{3/2}}{4d} - \frac{\text{Subst}\left(\int \frac{\sqrt{a+b-bx^2}\left((a+b)(b-4(a+b))+\right.}{1-x^2}\right.}{4d}$$

$$= \frac{b(7a+3b)\tanh(c+dx)\sqrt{a+b-b\tanh^2(c+dx)}}{8d} + \frac{b\tanh(c+dx)\left(a+b-b\right.}{4d}$$

$$= \frac{b(7a+3b)\tanh(c+dx)\sqrt{a+b-b\tanh^2(c+dx)}}{8d} + \frac{b\tanh(c+dx)\left(a+b-b\right.}{4d}$$

$$= \frac{b(7a+3b)\tanh(c+dx)\sqrt{a+b-b\tanh^2(c+dx)}}{8d} + \frac{b\tanh(c+dx)\left(a+b-b\right.}{4d}$$

$$= \frac{\sqrt{b}\left(15a^2+10ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b-b\tanh^2(c+dx)}}\right)}{8d} + \frac{a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+d}{\sqrt{a+b-b\tanh^2(c}}\right.}{d}$$

Mathematica [A] time = 9.28, size = 280, normalized size = 1.65

$$\frac{\cosh^5(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^{5/2}\left(8a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a\sinh^2(c+dx)+a+b}}\right)+\sqrt{b}\left(15a^2+10ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}}{\sqrt{a\sin}}\right.\right.}{\sqrt{2}\,d(a\cosh(2c+2dx)+a+2b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^(5/2),x]
```

```
[Out] ((Sqrt[b]*(15*a^2 + 10*a*b + 3*b^2)*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a +
 b + a*Sinh[c + d*x]^2]] + 8*a^(5/2)*ArcTanh[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a
 + b + a*Sinh[c + d*x]^2]])*Cosh[c + d*x]^5*(a + b*Sech[c + d*x]^2)^(5/2))/
(Sqrt[2]*d*(a + 2*b + a*Cosh[2*c + 2*d*x])^(5/2)) + (Cosh[c + d*x]^5*(a + b
*Sech[c + d*x]^2)^(5/2)*((b^2*Sech[c]*Sech[c + d*x]^4*Sinh[d*x])/d + (3*Sec
h[c]*Sech[c + d*x]^2*(3*a*b*Sinh[d*x] + b^2*Sinh[d*x]))/(2*d) + (3*b*(3*a +
 b)*Sech[c + d*x]*Tanh[c])/(2*d) + (b^2*Sech[c + d*x]^3*Tanh[c])/d))/(a + 2
*b + a*Cosh[2*c + 2*d*x])^2
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 1.13Error: Bad Argument Typ
e
```

maple [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \left(a + b\mathrm{sech}\left(dx + c\right)^{2}\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)^(5/2),x)
```

```
[Out] int((a+b*sech(d*x+c)^2)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\,\mathrm{sech}\left(dx + c\right)^{2} + a\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sech(d*x + c)^2 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+\frac{b}{{\mathrm{cosh}\left(c+d\,x\right)}^2}\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cosh(c + d*x)^2)^(5/2),x)
```

```
[Out] int((a + b/cosh(c + d*x)^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \operatorname{sech}^{2}{\left(c + dx \right)}\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)**(5/2),x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**(5/2), x)
```

3.194 $\int \frac{\tanh^5(x)}{\sqrt{a+b\operatorname{sech}^2(x)}}\,dx$

Optimal. Leaf size=66

$$-\frac{\left(a+b\operatorname{sech}^2(x)\right)^{3/2}}{3b^2}+\frac{(a+2b)\sqrt{a+b\operatorname{sech}^2(x)}}{b^2}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] -1/3*(a+b*sech(x)^2)^(3/2)/b^2+arctanh((a+b*sech(x)^2)^(1/2)/a^(1/2))/a^(1/
2)+(a+2*b)*(a+b*sech(x)^2)^(1/2)/b^2
```

Rubi [A] time = 0.13, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {4139, 446, 88, 63, 208}

$$-\frac{\left(a+b\operatorname{sech}^2(x)\right)^{3/2}}{3b^2}+\frac{(a+2b)\sqrt{a+b\operatorname{sech}^2(x)}}{b^2}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^5/Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]]/Sqrt[a] + ((a + 2*b)*Sqrt[a + b*Sech
[x]^2])/b^2 - (a + b*Sech[x]^2)^(3/2)/(3*b^2)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \frac{\tanh^5(x)}{\sqrt{a+b\,\mathrm{sech}^2(x)}}\,dx = -\mathrm{Subst}\left(\int \frac{\left(-1+x^2\right)^2}{x\sqrt{a+bx^2}}\,dx, x, \mathrm{sech}(x)\right)$$

$$= -\left(\frac{1}{2}\mathrm{Subst}\left(\int \frac{(-1+x)^2}{x\sqrt{a+bx}}\,dx, x, \mathrm{sech}^2(x)\right)\right)$$

$$= -\left(\frac{1}{2}\mathrm{Subst}\left(\int \left(\frac{-a-2b}{b\sqrt{a+bx}}+\frac{1}{x\sqrt{a+bx}}+\frac{\sqrt{a+bx}}{b}\right)dx, x, \mathrm{sech}^2(x)\right)\right)$$

$$= \frac{(a+2b)\sqrt{a+b\,\mathrm{sech}^2(x)}}{b^2}-\frac{\left(a+b\,\mathrm{sech}^2(x)\right)^{3/2}}{3b^2}-\frac{1}{2}\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \mathrm{sech}^2(x$$

$$= \frac{(a+2b)\sqrt{a+b\,\mathrm{sech}^2(x)}}{b^2}-\frac{\left(a+b\,\mathrm{sech}^2(x)\right)^{3/2}}{3b^2}-\frac{\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\,\mathrm{sech}^2(}{b}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{(a+2b)\sqrt{a+b\,\mathrm{sech}^2(x)}}{b^2}-\frac{\left(a+b\,\mathrm{sech}^2(x)\right)^{3/2}}{3b^2}$$

Mathematica [A] time = 0.52, size = 109, normalized size = 1.65

$$\frac{\operatorname{sech}(x)\left(\frac{\operatorname{sech}(x)(a\cosh(2x)+a+2b)\left(2a-b\operatorname{sech}^2(x)+6b\right)}{3b^2}+\frac{\sqrt{2}\sqrt{a\cosh(2x)+a+2b}\log\left(\sqrt{a\cosh(2x)+a+2b}+\sqrt{2}\sqrt{a}\cosh(x)\right)}{\sqrt{a}}\right)}{2\sqrt{a+b\operatorname{sech}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^5/Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] (Sech[x]*((Sqrt[2]*Sqrt[a + 2*b + a*Cosh[2*x]]*Log[Sqrt[2]*Sqrt[a]*Cosh[x]
+ Sqrt[a + 2*b + a*Cosh[2*x]]])/Sqrt[a] + ((a + 2*b + a*Cosh[2*x])*Sech[x]*
(2*a + 6*b - b*Sech[x]^2))/(3*b^2)))/(2*Sqrt[a + b*Sech[x]^2])
```

fricas [B] time = 0.61, size = 2678, normalized size = 40.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+b*sech(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 + 3*b^2*c
osh(x)^4 + 3*(5*b^2*cosh(x)^2 + b^2)*sinh(x)^4 + 3*b^2*cosh(x)^2 + 4*(5*b^2
*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^3 + 3*(5*b^2*cosh(x)^4 + 6*b^2*cosh(x)^
2 + b^2)*sinh(x)^2 + b^2 + 6*(b^2*cosh(x)^5 + 2*b^2*cosh(x)^3 + b^2*cosh(x)
)*sinh(x))*sqrt(a)*log(((a^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*
b + a*b^2)*cosh(x)*sinh(x)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3
 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3
+ 14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b +
a*b^2)*cosh(x)^3 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 +
 (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(
x)^4 + 6*a^3 + 14*a^2*b + 9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*co
sh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3 +
5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*
sinh(x)^3 + a^3 + 2*(2*a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*
b^2)*cosh(x)^6 + 15*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3 + 3
*a^2*b + 3*(6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*((a^
2 + 2*a*b + b^2)*cosh(x)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2
 + 2*a*b + b^2)*sinh(x)^6 + 3*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2
*a*b + b^2)*cosh(x)^2 + a^2 + 2*a*b + b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b +
b^2)*cosh(x)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4*a*b)
*cosh(x)^2 + (15*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*cos
h(x)^2 + 3*a^2 + 4*a*b)*sinh(x)^2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^
5 + 6*(a^2 + 2*a*b + b^2)*cosh(x)^3 + (3*a^2 + 4*a*b)*cosh(x))*sinh(x))*sqr
```

t(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh
(x) + sinh(x)^2)) + 4*(2*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^7 + 3*(2*a^3 + 5*a
^2*b + 4*a*b^2 + b^3)*cosh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^3 +
(2*a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*c
osh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*co
sh(x)*sinh(x)^5 + sinh(x)^6)) + 3*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5
+ b^2*sinh(x)^6 + 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 + b^2)*sinh(x)^4 + 3
*b^2*cosh(x)^2 + 4*(5*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^3 + 3*(5*b^2*c
osh(x)^4 + 6*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 6*(b^2*cosh(x)^5 + 2*b^
2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)
*sinh(x)^3 + a*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*a*cosh(x)^2 + b)*sinh(x)^2
+ sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a)*sqrt((a*c
osh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^
2)) + 4*(a*cosh(x)^3 + b*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(
x) + sinh(x)^2)) + 8*sqrt(2)*((a^2 + 3*a*b)*cosh(x)^4 + 4*(a^2 + 3*a*b)*cos
h(x)*sinh(x)^3 + (a^2 + 3*a*b)*sinh(x)^4 + 2*(a^2 + 2*a*b)*cosh(x)^2 + 2*(3
*(a^2 + 3*a*b)*cosh(x)^2 + a^2 + 2*a*b)*sinh(x)^2 + a^2 + 3*a*b + 4*((a^2 +
3*a*b)*cosh(x)^3 + (a^2 + 2*a*b)*cosh(x))*sinh(x))*sqrt((a*cosh(x)^2 + a*s
inh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a*b^2*co
sh(x)^6 + 6*a*b^2*cosh(x)*sinh(x)^5 + a*b^2*sinh(x)^6 + 3*a*b^2*cosh(x)^4 +
3*a*b^2*cosh(x)^2 + 3*(5*a*b^2*cosh(x)^2 + a*b^2)*sinh(x)^4 + 4*(5*a*b^2*c
osh(x)^3 + 3*a*b^2*cosh(x))*sinh(x)^3 + a*b^2 + 3*(5*a*b^2*cosh(x)^4 + 6*a*
b^2*cosh(x)^2 + a*b^2)*sinh(x)^2 + 6*(a*b^2*cosh(x)^5 + 2*a*b^2*cosh(x)^3 +
a*b^2*cosh(x))*sinh(x)), -1/6*(3*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5
+ b^2*sinh(x)^6 + 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 + b^2)*sinh(x)^4 + 3
*b^2*cosh(x)^2 + 4*(5*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^3 + 3*(5*b^2*c
osh(x)^4 + 6*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 6*(b^2*cosh(x)^5 + 2*b^
2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(-a)*arctan(sqrt(2)*((a + b)*cosh(x
)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*c
osh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^
2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*
sinh(x)^4 + (2*a^2 + 3*a*b)*cosh(x)^2 + (6*(a^2 + a*b)*cosh(x)^2 + 2*a^2 +
3*a*b)*sinh(x)^2 + a^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 + (2*a^2 + 3*a*b)*cosh(
x))*sinh(x))) + 3*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6
+ 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 + b^2)*sinh(x)^4 + 3*b^2*cosh(x)^2 +
4*(5*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^3 + 3*(5*b^2*cosh(x)^4 + 6*b^2
*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 6*(b^2*cosh(x)^5 + 2*b^2*cosh(x)^3 + b^
2*cosh(x))*sinh(x))*sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x)
+ sinh(x)^2 - 1)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(
x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3
+ a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x
)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) - 4*sqrt(2)*((a^2 +
3*a*b)*cosh(x)^4 + 4*(a^2 + 3*a*b)*cosh(x)*sinh(x)^3 + (a^2 + 3*a*b)*sinh(
x)^4 + 2*(a^2 + 2*a*b)*cosh(x)^2 + 2*(3*(a^2 + 3*a*b)*cosh(x)^2 + a^2 + 2*a
*b)*sinh(x)^2 + a^2 + 3*a*b + 4*((a^2 + 3*a*b)*cosh(x)^3 + (a^2 + 2*a*b)*co

```
sh(x))*sinh(x))*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*c
osh(x)*sinh(x) + sinh(x)^2)))/(a*b^2*cosh(x)^6 + 6*a*b^2*cosh(x)*sinh(x)^5
+ a*b^2*sinh(x)^6 + 3*a*b^2*cosh(x)^4 + 3*a*b^2*cosh(x)^2 + 3*(5*a*b^2*cosh
(x)^2 + a*b^2)*sinh(x)^4 + 4*(5*a*b^2*cosh(x)^3 + 3*a*b^2*cosh(x))*sinh(x)^
3 + a*b^2 + 3*(5*a*b^2*cosh(x)^4 + 6*a*b^2*cosh(x)^2 + a*b^2)*sinh(x)^2 + 6
*(a*b^2*cosh(x)^5 + 2*a*b^2*cosh(x)^3 + a*b^2*cosh(x))*sinh(x))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+b*sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [F] time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{\tanh^5(x)}{\sqrt{a+b\mathrm{sech}(x)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^5/(a+b*sech(x)^2)^(1/2),x)
```

```
[Out] int(tanh(x)^5/(a+b*sech(x)^2)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)^5}{\sqrt{b\,\mathrm{sech}(x)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+b*sech(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)^5/sqrt(b*sech(x)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\tanh(x)^5}{\sqrt{a+\frac{b}{\cosh(x)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^5/(a + b/cosh(x)^2)^(1/2),x)
```

```
[Out] int(tanh(x)^5/(a + b/cosh(x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^5(x)}{\sqrt{a + b\,\mathrm{sech}^2(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**5/(a+b*sech(x)**2)**(1/2),x)
```

```
[Out] Integral(tanh(x)**5/sqrt(a + b*sech(x)**2), x)
```

3.195 $\int \frac{\tanh^4(x)}{\sqrt{a+b\,\mathrm{sech}^2(x)}}\,dx$

Optimal. Leaf size=90

$$-\frac{(a+3b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{2b^{3/2}}+\frac{\tanh(x)\sqrt{a-b\tanh^2(x)+b}}{2b}+\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{\sqrt{a}}$$

```
[Out] -1/2*(a+3*b)*arctan(b^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))/b^(3/2)+arctan
h(a^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))/a^(1/2)+1/2*(a+b-b*tanh(x)^2)^(1
/2)*tanh(x)/b
```

Rubi [A] time = 0.23, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {4141, 1975, 479, 523, 217, 203, 377, 206}

$$-\frac{(a+3b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{2b^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{\sqrt{a}}+\frac{\tanh(x)\sqrt{a-b\tanh^2(x)+b}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^4/Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] -((a + 3*b)*ArcTan[(Sqrt[b]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]])/(2*b^(3/2)
) + ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/Sqrt[a] + (Tanh[x]
*Sqrt[a + b - b*Tanh[x]^2])/(2*b)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh^4(x)}{\sqrt{a+b\text{sech}^2(x)}}\,dx &= \text{Subst}\left(\int \frac{x^4}{\left(1-x^2\right)\sqrt{a+b\left(1-x^2\right)}}\,dx, x, \tanh(x)\right) \\
&= \text{Subst}\left(\int \frac{x^4}{\left(1-x^2\right)\sqrt{a+b-bx^2}}\,dx, x, \tanh(x)\right) \\
&= \frac{\tanh(x)\sqrt{a+b-b\tanh^2(x)}}{2b} - \frac{\text{Subst}\left(\int \frac{a+b+(-a-3b)x^2}{(1-x^2)\sqrt{a+b-bx^2}}\,dx, x, \tanh(x)\right)}{2b} \\
&= \frac{\tanh(x)\sqrt{a+b-b\tanh^2(x)}}{2b} - \frac{(a+3b)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+b-bx^2}}\,dx, x, \tanh(x)\right)}{2b} + \text{Subst}\left(\int \right. \\
&= \frac{\tanh(x)\sqrt{a+b-b\tanh^2(x)}}{2b} - \frac{(a+3b)\,\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{2b} + \text{Subst}\left(\right. \\
&= -\frac{(a+3b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{2b^{3/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{\sqrt{a}} + \frac{\tanh(x)\sqrt{a+b-b\tanh}}{2b}
\end{aligned}
$$

Mathematica [A] time = 0.53, size = 169, normalized size = 1.88

$$
\frac{\text{sech}(x)\left(2\sqrt{2}\,b^{3/2}\sqrt{a\cosh(2x)+a+2b}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{a}\,\sinh(x)}{\sqrt{a\cosh(2x)+a+2b}}\right)+\sqrt{a}\,\left(\sqrt{b}\,\tanh(x)\text{sech}(x)(a\cosh(2x)+a+2b\right.\right.}{4\sqrt{a}\,b^{3/2}\sqrt{a+b\text{sech}^2(x)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^4/Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] (Sech[x]*(2*Sqrt[2]*b^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[a]*Sinh[x])/Sqrt[a + 2*b
+ a*Cosh[2*x]]]*Sqrt[a + 2*b + a*Cosh[2*x]] + Sqrt[a]*(-(Sqrt[2]*(a + 3*b)*
ArcTan[(Sqrt[2]*Sqrt[b]*Sinh[x])/Sqrt[a + 2*b + a*Cosh[2*x]]]*Sqrt[a + 2*b
+ a*Cosh[2*x]]) + Sqrt[b]*(a + 2*b + a*Cosh[2*x])*Sech[x]*Tanh[x])))/(4*Sqr
t[a]*b^(3/2)*Sqrt[a + b*Sech[x]^2])
```

fricas [B] time = 0.66, size = 4569, normalized size = 50.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*sech(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((b^2*cosh(x)^4 + 4*b^2*cosh(x)*sinh(x)^3 + b^2*sinh(x)^4 + 2*b^2*cosh
(x)^2 + 2*(3*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 4*(b^2*cosh(x)^3 + b^2*
cosh(x))*sinh(x))*sqrt(a)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7
+ a*b^2*sinh(x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b
^2 + b^3)*sinh(x)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh
(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4
*a^2*b + 9*a*b^2 - 30*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh
(x)^5 - 10*(a*b^2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sin
h(x)^3 + a^3 + 2*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*
b^2 - b^3)*cosh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^
2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(
x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cos
h(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(
x)^4 - 18*b^2*cosh(x)^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5
 - 6*b^2*cosh(x)^3 - (a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x
)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) +
 4*(2*a*b^2*cosh(x)^7 - 3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^
2)*cosh(x)^3 + (a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*s
inh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sin
h(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) - ((a^2 + 3*a*b)*cosh(x)^4 + 4*(
a^2 + 3*a*b)*cosh(x)*sinh(x)^3 + (a^2 + 3*a*b)*sinh(x)^4 + 2*(a^2 + 3*a*b)*
cosh(x)^2 + 2*(3*(a^2 + 3*a*b)*cosh(x)^2 + a^2 + 3*a*b)*sinh(x)^2 + a^2 + 3
*a*b + 4*((a^2 + 3*a*b)*cosh(x)^3 + (a^2 + 3*a*b)*cosh(x))*sinh(x))*sqrt(-b
)*log(-((a - b)*cosh(x)^4 + 4*(a - b)*cosh(x)*sinh(x)^3 + (a - b)*sinh(x)^4
 + 2*(a + 3*b)*cosh(x)^2 + 2*(3*(a - b)*cosh(x)^2 + a + 3*b)*sinh(x)^2 - 2*
sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-b)*sqrt((a*co
sh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2
)) + 4*((a - b)*cosh(x)^3 + (a + 3*b)*cosh(x))*sinh(x) + a - b)/(cosh(x)^4
+ 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(
x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)) + (b^2*cosh(x)^4 + 4*b^2*cosh(
x)*sinh(x)^3 + b^2*sinh(x)^4 + 2*b^2*cosh(x)^2 + 2*(3*b^2*cosh(x)^2 + b^2)*
sinh(x)^2 + b^2 + 4*(b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(a)*log(-(a*
cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(
3*a*cosh(x)^2 + a + b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) +
 sinh(x)^2 + 1)*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)
^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(a*cosh(x)^3 + (a + b)*cosh(x))*si
nh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) + 2*sqrt(2)*(a*b*co
sh(x)^2 + 2*a*b*cosh(x)*sinh(x) + a*b*sinh(x)^2 - a*b)*sqrt((a*cosh(x)^2 +
a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a*b^2
*cosh(x)^4 + 4*a*b^2*cosh(x)*sinh(x)^3 + a*b^2*sinh(x)^4 + 2*a*b^2*cosh(x)^
2 + a*b^2 + 2*(3*a*b^2*cosh(x)^2 + a*b^2)*sinh(x)^2 + 4*(a*b^2*cosh(x)^3 +
```

```
a*b^2*cosh(x))*sinh(x)), -1/4*(2*((a^2 + 3*a*b)*cosh(x)^4 + 4*(a^2 + 3*a*b)
*cosh(x)*sinh(x)^3 + (a^2 + 3*a*b)*sinh(x)^4 + 2*(a^2 + 3*a*b)*cosh(x)^2 +
2*(3*(a^2 + 3*a*b)*cosh(x)^2 + a^2 + 3*a*b)*sinh(x)^2 + a^2 + 3*a*b + 4*((a
^2 + 3*a*b)*cosh(x)^3 + (a^2 + 3*a*b)*cosh(x))*sinh(x))*sqrt(b)*arctan(sqrt
(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(b)*sqrt((a*cosh(x)
^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a
*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 +
2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))
*sinh(x) + a)) - (b^2*cosh(x)^4 + 4*b^2*cosh(x)*sinh(x)^3 + b^2*sinh(x)^4 +
 2*b^2*cosh(x)^2 + 2*(3*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 4*(b^2*cosh(
x)^3 + b^2*cosh(x))*sinh(x))*sqrt(a)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)
*sinh(x)^7 + a*b^2*sinh(x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh
(x)^2 - a*b^2 + b^3)*sinh(x)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*co
sh(x))*sinh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^
4 + a^3 + 4*a^2*b + 9*a*b^2 - 30*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14
*a*b^2*cosh(x)^5 - 10*(a*b^2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*c
osh(x))*sinh(x)^3 + a^3 + 2*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)
^6 - 15*(a*b^2 - b^3)*cosh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^
2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5
+ b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4
*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (1
5*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^
2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqr
t((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + si
nh(x)^2)) + 4*(2*a*b^2*cosh(x)^7 - 3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2
*b + 9*a*b^2)*cosh(x)^3 + (a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*
cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*co
sh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) - (b^2*cosh(x)^4 + 4*
b^2*cosh(x)*sinh(x)^3 + b^2*sinh(x)^4 + 2*b^2*cosh(x)^2 + 2*(3*b^2*cosh(x)^
2 + b^2)*sinh(x)^2 + b^2 + 4*(b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(a)
*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + b)*cosh(x
)^2 + 2*(3*a*cosh(x)^2 + a + b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*
sinh(x) + sinh(x)^2 + 1)*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)
/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(a*cosh(x)^3 + (a + b)*co
sh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) - 2*sqrt(2
)*(a*b*cosh(x)^2 + 2*a*b*cosh(x)*sinh(x) + a*b*sinh(x)^2 - a*b)*sqrt((a*cos
h(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)
))/(a*b^2*cosh(x)^4 + 4*a*b^2*cosh(x)*sinh(x)^3 + a*b^2*sinh(x)^4 + 2*a*b^2
*cosh(x)^2 + a*b^2 + 2*(3*a*b^2*cosh(x)^2 + a*b^2)*sinh(x)^2 + 4*(a*b^2*cos
h(x)^3 + a*b^2*cosh(x))*sinh(x)), -1/4*(2*(b^2*cosh(x)^4 + 4*b^2*cosh(x)*si
nh(x)^3 + b^2*sinh(x)^4 + 2*b^2*cosh(x)^2 + 2*(3*b^2*cosh(x)^2 + b^2)*sinh(
x)^2 + b^2 + 4*(b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(-a)*arctan(sqrt(
2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*c
osh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^
2))/(a*b*cosh(x)^4 + 4*a*b*cosh(x)*sinh(x)^3 + a*b*sinh(x)^4 - (a^2 + 3*a*b
```

)*cosh(x)^2 + (6*a*b*cosh(x)^2 - a^2 - 3*a*b)*sinh(x)^2 - a^2 + 2*(2*a*b*co
sh(x)^3 - (a^2 + 3*a*b)*cosh(x))*sinh(x))) + 2*(b^2*cosh(x)^4 + 4*b^2*cosh(
x)*sinh(x)^3 + b^2*sinh(x)^4 + 2*b^2*cosh(x)^2 + 2*(3*b^2*cosh(x)^2 + b^2)*
sinh(x)^2 + b^2 + 4*(b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(-a)*arctan(
sqrt(2)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*
cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x
)^2 + a)) + ((a^2 + 3*a*b)*cosh(x)^4 + 4*(a^2 + 3*a*b)*cosh(x)*sinh(x)^3 +
(a^2 + 3*a*b)*sinh(x)^4 + 2*(a^2 + 3*a*b)*cosh(x)^2 + 2*(3*(a^2 + 3*a*b)*co
sh(x)^2 + a^2 + 3*a*b)*sinh(x)^2 + a^2 + 3*a*b + 4*((a^2 + 3*a*b)*cosh(x)^3
 + (a^2 + 3*a*b)*cosh(x))*sinh(x))*sqrt(-b)*log(-((a - b)*cosh(x)^4 + 4*(a
- b)*cosh(x)*sinh(x)^3 + (a - b)*sinh(x)^4 + 2*(a + 3*b)*cosh(x)^2 + 2*(3*(
a - b)*cosh(x)^2 + a + 3*b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*si
nh(x) + sinh(x)^2 - 1)*sqrt(-b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/
(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a - b)*cosh(x)^3 + (a +
3*b)*cosh(x))*sinh(x) + a - b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4
 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*si
nh(x) + 1)) - 2*sqrt(2)*(a*b*cosh(x)^2 + 2*a*b*cosh(x)*sinh(x) + a*b*sinh(x
)^2 - a*b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x
)*sinh(x) + sinh(x)^2)))/(a*b^2*cosh(x)^4 + 4*a*b^2*cosh(x)*sinh(x)^3 + a*b
^2*sinh(x)^4 + 2*a*b^2*cosh(x)^2 + a*b^2 + 2*(3*a*b^2*cosh(x)^2 + a*b^2)*si
nh(x)^2 + 4*(a*b^2*cosh(x)^3 + a*b^2*cosh(x))*sinh(x)), -1/2*((b^2*cosh(x)^
4 + 4*b^2*cosh(x)*sinh(x)^3 + b^2*sinh(x)^4 + 2*b^2*cosh(x)^2 + 2*(3*b^2*co
sh(x)^2 + b^2)*sinh(x)^2 + b^2 + 4*(b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*s
qrt(-a)*arctan(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + a
)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x
)*sinh(x) + sinh(x)^2))/(a*b*cosh(x)^4 + 4*a*b*cosh(x)*sinh(x)^3 + a*b*sinh
(x)^4 - (a^2 + 3*a*b)*cosh(x)^2 + (6*a*b*cosh(x)^2 - a^2 - 3*a*b)*sinh(x)^2
 - a^2 + 2*(2*a*b*cosh(x)^3 - (a^2 + 3*a*b)*cosh(x))*sinh(x))) + ((a^2 + 3*
a*b)*cosh(x)^4 + 4*(a^2 + 3*a*b)*cosh(x)*sinh(x)^3 + (a^2 + 3*a*b)*sinh(x)^
4 + 2*(a^2 + 3*a*b)*cosh(x)^2 + 2*(3*(a^2 + 3*a*b)*cosh(x)^2 + a^2 + 3*a*b)
*sinh(x)^2 + a^2 + 3*a*b + 4*((a^2 + 3*a*b)*cosh(x)^3 + (a^2 + 3*a*b)*cosh(
x))*sinh(x))*sqrt(b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x
)^2 - 1)*sqrt(b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*
cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh
(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(
a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) + (b^2*cosh(x)^4 + 4*b^2*cos
h(x)*sinh(x)^3 + b^2*sinh(x)^4 + 2*b^2*cosh(x)^2 + 2*(3*b^2*cosh(x)^2 + b^2
)*sinh(x)^2 + b^2 + 4*(b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(-a)*arcta
n(sqrt(2)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 -
2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh
(x)^2 + a)) - sqrt(2)*(a*b*cosh(x)^2 + 2*a*b*cosh(x)*sinh(x) + a*b*sinh(x)^
2 - a*b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*
sinh(x) + sinh(x)^2)))/(a*b^2*cosh(x)^4 + 4*a*b^2*cosh(x)*sinh(x)^3 + a*b^2
*sinh(x)^4 + 2*a*b^2*cosh(x)^2 + a*b^2 + 2*(3*a*b^2*cosh(x)^2 + a*b^2)*sinh
(x)^2 + 4*(a*b^2*cosh(x)^3 + a*b^2*cosh(x))*sinh(x))]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{\tanh^{4}(x)}{\sqrt{a+b\,\mathrm{sech}(x)^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(a+b*sech(x)^2)^(1/2),x)
```

```
[Out] int(tanh(x)^4/(a+b*sech(x)^2)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)^{4}}{\sqrt{b\,\mathrm{sech}(x)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*sech(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)^4/sqrt(b*sech(x)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tanh(x)^{4}}{\sqrt{a+\frac{b}{\cosh(x)^{2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(a + b/cosh(x)^2)^(1/2),x)
```

```
[Out] int(tanh(x)^4/(a + b/cosh(x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^4(x)}{\sqrt{a + b\,\mathrm{sech}^2(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**4/(a+b*sech(x)**2)**(1/2),x)
```

```
[Out] Integral(tanh(x)**4/sqrt(a + b*sech(x)**2), x)
```

3.196 $\int \frac{\tanh^3(x)}{\sqrt{a+b\,\mathbf{sech}^2(x)}}\,dx$

Optimal. Leaf size=42

$$\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{b}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] arctanh((a+b*sech(x)^2)^(1/2)/a^(1/2))/a^(1/2)+(a+b*sech(x)^2)^(1/2)/b
```

Rubi [A] time = 0.09, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {4139, 446, 80, 63, 208}

$$\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{b}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^3/Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]]/Sqrt[a] + Sqrt[a + b*Sech[x]^2]/b
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^3(x)}{\sqrt{a+b\operatorname{sech}^2(x)}}\,dx &= \operatorname{Subst}\left(\int \frac{-1+x^2}{x\sqrt{a+bx^2}}\,dx, x, \operatorname{sech}(x)\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{-1+x}{x\sqrt{a+bx}}\,dx, x, \operatorname{sech}^2(x)\right)\\ &= \frac{\sqrt{a+b\operatorname{sech}^2(x)}}{b} - \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \operatorname{sech}^2(x)\right)\\ &= \frac{\sqrt{a+b\operatorname{sech}^2(x)}}{b} - \frac{\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{sech}^2(x)}\right)}{b}\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}} + \frac{\sqrt{a+b\operatorname{sech}^2(x)}}{b}\end{aligned}$$

Mathematica [B] time = 0.22, size = 105, normalized size = 2.50

$$\frac{\operatorname{sech}^2(x)(a\cosh(2x)+a+2b)}{2b\sqrt{a+b\operatorname{sech}^2(x)}}+\frac{\operatorname{sech}(x)\sqrt{a\cosh(2x)+a+2b}\,\log\left(\sqrt{a\cosh(2x)+a+2b}+\sqrt{2}\sqrt{a}\cosh(x)\right)}{\sqrt{2}\sqrt{a}\sqrt{a+b\operatorname{sech}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^3/Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] (Sqrt[a + 2*b + a*Cosh[2*x]]*Log[Sqrt[2]*Sqrt[a]*Cosh[x] + Sqrt[a + 2*b + a
*Cosh[2*x]]]*Sech[x])/(Sqrt[2]*Sqrt[a]*Sqrt[a + b*Sech[x]^2]) + ((a + 2*b +
 a*Cosh[2*x])*Sech[x]^2)/(2*b*Sqrt[a + b*Sech[x]^2])
```

fricas [B] time = 0.52, size = 1650, normalized size = 39.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*sech(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + b)*sqrt(a)*log(((a
^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)
^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3
)*cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*(a^3 + 2*a^2*b + a*b^
2)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + 3*(2*a^
3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b
^2)*cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 6*a^3 + 14*a^2*b +
9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14
*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*c
osh(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(2*a^3
 + 3*a^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 + 15*(2*a^3
 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3 + 3*a^2*b + 3*(6*a^3 + 14*a^2
*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*((a^2 + 2*a*b + b^2)*cosh(x)^6
 + 6*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2 + 2*a*b + b^2)*sinh(x)^6
+ 3*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^
2 + 2*a*b + b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 +
2*a*b + b^2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4*a*b)*cosh(x)^2 + (15*(a^2 + 2*
a*b + b^2)*cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*cosh(x)^2 + 3*a^2 + 4*a*b)*si
nh(x)^2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 6*(a^2 + 2*a*b + b^2)*
cosh(x)^3 + (3*a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a
*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(
a^3 + 2*a^2*b + a*b^2)*cosh(x)^7 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh
(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^3 + (2*a^3 + 3*a^2*b)*cosh(x))
*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*co
sh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^
6)) + (b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + b)*sqrt(a)*log(-(a
*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*a*c
osh(x)^2 + b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^
2 - 1)*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*co
sh(x)*sinh(x) + sinh(x)^2)) + 4*(a*cosh(x)^3 + b*cosh(x))*sinh(x) + a)/(cos
```

```
h(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*sqrt(2)*a*sqrt((a*cosh(x)^2 +
a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a*b*c
osh(x)^2 + 2*a*b*cosh(x)*sinh(x) + a*b*sinh(x)^2 + a*b), -1/2*((b*cosh(x)^2
 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + b)*sqrt(-a)*arctan(sqrt(2)*((a + b)*
cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a)*sqrt(-a)*sqr
t((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + si
nh(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 +
 a*b)*sinh(x)^4 + (2*a^2 + 3*a*b)*cosh(x)^2 + (6*(a^2 + a*b)*cosh(x)^2 + 2*
a^2 + 3*a*b)*sinh(x)^2 + a^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 + (2*a^2 + 3*a*b)
*cosh(x))*sinh(x))) + (b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + b)
*sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sq
rt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*si
nh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*
(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^
3 + (a + 2*b)*cosh(x))*sinh(x) + a)) - 2*sqrt(2)*a*sqrt((a*cosh(x)^2 + a*si
nh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a*b*cosh(
x)^2 + 2*a*b*cosh(x)*sinh(x) + a*b*sinh(x)^2 + a*b)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{\tanh^{3}(x)}{\sqrt{a+b\,\mathrm{sech}(x)^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(a+b*sech(x)^2)^(1/2),x)
```

```
[Out] int(tanh(x)^3/(a+b*sech(x)^2)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)^{3}}{\sqrt{b\,\mathrm{sech}(x)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*sech(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)^3/sqrt(b*sech(x)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\tanh(x)^3}{\sqrt{a + \frac{b}{\cosh(x)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(a + b/cosh(x)^2)^(1/2),x)
```

```
[Out] int(tanh(x)^3/(a + b/cosh(x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^3(x)}{\sqrt{a + b\,\mathrm{sech}^2(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**3/(a+b*sech(x)**2)**(1/2),x)
```

```
[Out] Integral(tanh(x)**3/sqrt(a + b*sech(x)**2), x)
```

3.197 $\int \frac{\tanh^2(x)}{\sqrt{a+b\operatorname{sech}^2(x)}}\,dx$

Optimal. Leaf size=60

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{\sqrt{a}}-\frac{\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{\sqrt{b}}$$

```
[Out] arctanh(a^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))/a^(1/2)-arctan(b^(1/2)*tan
h(x)/(a+b-b*tanh(x)^2)^(1/2))/b^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {4141, 1975, 483, 217, 203, 377, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{\sqrt{a}}-\frac{\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^2/Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] -(ArcTan[(Sqrt[b]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/Sqrt[b]) + ArcTanh[(S
qrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/Sqrt[a]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 483

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[(a*e^n)/b, Int[((e*x)^(m - n)*(c + d*x^n)^q)/(a + b*x^n), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\tanh^2(x)}{\sqrt{a+b\text{sech}^2(x)}}\,dx = \text{Subst}\left(\int \frac{x^2}{(1-x^2)\sqrt{a+b(1-x^2)}}\,dx, x, \tanh(x)\right)$$
$$= \text{Subst}\left(\int \frac{x^2}{(1-x^2)\sqrt{a+b-bx^2}}\,dx, x, \tanh(x)\right)$$
$$= -\text{Subst}\left(\int \frac{1}{\sqrt{a+b-bx^2}}\,dx, x, \tanh(x)\right) + \text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+b-bx^2}}\,dx, x, \text{ta}\right.$$
$$= \text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right) - \text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{\tanh}{\sqrt{a+b-b}}\right.$$
$$= -\frac{\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{\sqrt{b}} + \frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{\sqrt{a}}$$

Mathematica [A] time = 0.18, size = 107, normalized size = 1.78

$$\frac{\text{sech}(x)\sqrt{a\cosh(2x)+a+2b}\left(\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sinh(x)}{\sqrt{a\cosh(2x)+a+2b}}\right)}{\sqrt{a}} - \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}\sinh(x)}{\sqrt{a\cosh(2x)+a+2b}}\right)}{\sqrt{b}}\right)}{\sqrt{2}\sqrt{a+b\text{sech}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^2/Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] ((-(ArcTan[(Sqrt[2]*Sqrt[b]*Sinh[x])/Sqrt[a + 2*b + a*Cosh[2*x]]]/Sqrt[b])
+ ArcTanh[(Sqrt[2]*Sqrt[a]*Sinh[x])/Sqrt[a + 2*b + a*Cosh[2*x]]]/Sqrt[a])*S
qrt[a + 2*b + a*Cosh[2*x]]*Sech[x])/(Sqrt[2]*Sqrt[a + b*Sech[x]^2])
```

fricas [B] time = 0.56, size = 2856, normalized size = 47.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*sech(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a)*b*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*si
nh(x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*
```

```
sinh(x)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a
^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9
*a*b^2 - 30*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10
*(a*b^2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a
^3 + 2*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)
*cosh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)
^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b
^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3
*b^2*cosh(x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*
b^2*cosh(x)^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*c
osh(x)^3 - (a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*si
nh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^
2*cosh(x)^7 - 3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)
^3 + (a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 1
5*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6
*cosh(x)*sinh(x)^5 + sinh(x)^6)) - 2*a*sqrt(-b)*log(-((a - b)*cosh(x)^4 + 4
*(a - b)*cosh(x)*sinh(x)^3 + (a - b)*sinh(x)^4 + 2*(a + 3*b)*cosh(x)^2 + 2*
(3*(a - b)*cosh(x)^2 + a + 3*b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x
)*sinh(x) + sinh(x)^2 - 1)*sqrt(-b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2
*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a - b)*cosh(x)^3 + (
a + 3*b)*cosh(x))*sinh(x) + a - b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(
x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x)
)*sinh(x) + 1)) + sqrt(a)*b*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*s
inh(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + b)*sinh(x)^2 + sqrt
(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*sqrt((a*cosh(x)
^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) +
4*(a*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(
x) + sinh(x)^2)))/(a*b), -1/4*(4*a*sqrt(b)*arctan(sqrt(2)*(cosh(x)^2 + 2*co
sh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a
+ 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh
(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a
+ 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) - sqrt
(a)*b*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^8 -
2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh(x)^6
+ 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3 + 4*a^2
*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 - 3
0*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a*b^2 -
b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(a^
3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*cosh(x)^4
 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(
2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)
^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(
x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x
)^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 -
 (a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 +
```

```
a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*cosh(x)^
7 - 3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3 + (a^3
+ 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^
4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*s
inh(x)^5 + sinh(x)^6)) - sqrt(a)*b*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^
3 + a*sinh(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + b)*sinh(x)^2
 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*sqrt((a*
cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)
^2)) + 4*(a*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x
)*sinh(x) + sinh(x)^2)))/(a*b), -1/2*(sqrt(-a)*b*arctan(sqrt(2)*(b*cosh(x)^
2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*s
inh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*b*cosh(
x)^4 + 4*a*b*cosh(x)*sinh(x)^3 + a*b*sinh(x)^4 - (a^2 + 3*a*b)*cosh(x)^2 +
(6*a*b*cosh(x)^2 - a^2 - 3*a*b)*sinh(x)^2 - a^2 + 2*(2*a*b*cosh(x)^3 - (a^2
 + 3*a*b)*cosh(x))*sinh(x))) + sqrt(-a)*b*arctan(sqrt(2)*sqrt(-a)*sqrt((a*c
osh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^
2))/(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a)) + a*sqrt(-b)*log
(-((a - b)*cosh(x)^4 + 4*(a - b)*cosh(x)*sinh(x)^3 + (a - b)*sinh(x)^4 + 2*
(a + 3*b)*cosh(x)^2 + 2*(3*(a - b)*cosh(x)^2 + a + 3*b)*sinh(x)^2 - 2*sqrt(
2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-b)*sqrt((a*cosh(x)
^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) +
4*((a - b)*cosh(x)^3 + (a + 3*b)*cosh(x))*sinh(x) + a - b)/(cosh(x)^4 + 4*c
osh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2
+ 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)))/(a*b), -1/2*(sqrt(-a)*b*arctan(sqr
t(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + a)*sqrt(-a)*sqrt((a
*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x
)^2))/(a*b*cosh(x)^4 + 4*a*b*cosh(x)*sinh(x)^3 + a*b*sinh(x)^4 - (a^2 + 3*a
*b)*cosh(x)^2 + (6*a*b*cosh(x)^2 - a^2 - 3*a*b)*sinh(x)^2 - a^2 + 2*(2*a*b*
cosh(x)^3 - (a^2 + 3*a*b)*cosh(x))*sinh(x))) + 2*a*sqrt(b)*arctan(sqrt(2)*(
cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(b)*sqrt((a*cosh(x)^2 +
a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh
(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*
a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh
(x) + a)) + sqrt(-a)*b*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x
)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^2 +
2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a)))/(a*b)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
```

```
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [F] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{\tanh^2(x)}{\sqrt{a+b\operatorname{sech}(x)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(a+b*sech(x)^2)^(1/2),x)
[Out] int(tanh(x)^2/(a+b*sech(x)^2)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)^2}{\sqrt{b\operatorname{sech}(x)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*sech(x)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(tanh(x)^2/sqrt(b*sech(x)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\tanh(x)^2}{\sqrt{a+\frac{b}{\cosh(x)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(a + b/cosh(x)^2)^(1/2),x)
[Out] int(tanh(x)^2/(a + b/cosh(x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^2(x)}{\sqrt{a+b\operatorname{sech}^2(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**2/(a+b*sech(x)**2)**(1/2),x)
[Out] Integral(tanh(x)**2/sqrt(a + b*sech(x)**2), x)
```

3.198 $\int \frac{\tanh(x)}{\sqrt{a+b\operatorname{sech}^2(x)}}\,dx$

Optimal. Leaf size=25

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] arctanh((a+b*sech(x)^2)^(1/2)/a^(1/2))/a^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {4139, 266, 63, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]/Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]]/Sqrt[a]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh(x)}{\sqrt{a+b\operatorname{sech}^2(x)}}\,dx &= -\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx^2}}\,dx, x, \operatorname{sech}(x)\right)\\ &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \operatorname{sech}^2(x)\right)\right)\\ &= -\frac{\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{sech}^2(x)}\right)}{b}\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}\end{aligned}$$

Mathematica [B] time = 0.08, size = 70, normalized size = 2.80

$$\frac{\operatorname{sech}(x)\sqrt{a\cosh(2x)+a+2b}\,\log\left(\sqrt{a\cosh(2x)+a+2b}+\sqrt{2}\sqrt{a}\cosh(x)\right)}{\sqrt{2}\sqrt{a}\sqrt{a+b\operatorname{sech}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] (Sqrt[a + 2*b + a*Cosh[2*x]]*Log[Sqrt[2]*Sqrt[a]*Cosh[x] + Sqrt[a + 2*b + a
*Cosh[2*x]]]*Sech[x])/(Sqrt[2]*Sqrt[a]*Sqrt[a + b*Sech[x]^2])
```

fricas [B] time = 0.46, size = 1430, normalized size = 57.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sech(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a)*log(((a^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a
*b^2)*cosh(x)*sinh(x)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*
a^2*b + 4*a*b^2 + b^3)*cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*
(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2
)*cosh(x)^3 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + (6*a
^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4
+ 6*a^3 + 14*a^2*b + 9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)
^2)*sinh(x)^4 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a^2
*b + 4*a*b^2 + b^3)*cosh(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*sinh(
x)^3 + a^3 + 2*(2*a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*
cosh(x)^6 + 15*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3 + 3*a^2*
b + 3*(6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*((a^2 + 2
*a*b + b^2)*cosh(x)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2 + 2*
a*b + b^2)*sinh(x)^6 + 3*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b
+ b^2)*cosh(x)^2 + a^2 + 2*a*b + b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*
cosh(x)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4*a*b)*cosh
(x)^2 + (15*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*cosh(x)^
2 + 3*a^2 + 4*a*b)*sinh(x)^2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 6
*(a^2 + 2*a*b + b^2)*cosh(x)^3 + (3*a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*
sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) +
 sinh(x)^2)) + 4*(2*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^7 + 3*(2*a^3 + 5*a^2*b
+ 4*a*b^2 + b^3)*cosh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^3 + (2*a^
3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x
)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)
*sinh(x)^5 + sinh(x)^6)) + sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^
3 + a*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*a*cosh(x)^2 + b)*sinh(x)^2 + sqrt(2)
*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a)*sqrt((a*cosh(x)^2
+ a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(
a*cosh(x)^3 + b*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh
(x)^2)))/a, -1/2*(sqrt(-a)*arctan(sqrt(2)*((a + b)*cosh(x)^2 + 2*(a + b)*co
sh(x)*sinh(x) + (a + b)*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(
x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a^2 + a*b)*c
osh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 + (2*a^2
 + 3*a*b)*cosh(x)^2 + (6*(a^2 + a*b)*cosh(x)^2 + 2*a^2 + 3*a*b)*sinh(x)^2 +
 a^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 + (2*a^2 + 3*a*b)*cosh(x))*sinh(x))) + sq
rt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(
-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(
x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a
+ 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 +
 (a + 2*b)*cosh(x))*sinh(x) + a)))/a]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [A] time = 0.12, size = 30, normalized size = 1.20

$$\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\,\mathrm{sech}(x)^2}}{\mathrm{sech}(x)}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a+b*sech(x)^2)^(1/2),x)
```

```
[Out] 1/a^(1/2)*ln((2*a+2*a^(1/2)*(a+b*sech(x)^2)^(1/2))/sech(x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)}{\sqrt{b\,\mathrm{sech}(x)^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sech(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)/sqrt(b*sech(x)^2 + a), x)
```

mupad [B] time = 1.67, size = 19, normalized size = 0.76

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{\cosh(x)^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a + b/cosh(x)^2)^(1/2),x)
```

```
[Out] atanh((a + b/cosh(x)^2)^(1/2)/a^(1/2))/a^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh{\left(x \right)}}{\sqrt{a + b\,\mathrm{sech}^{2}{\left(x \right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sech(x)**2)**(1/2),x)
[Out] Integral(tanh(x)/sqrt(a + b*sech(x)**2), x)
```

3.199 $\int \frac{1}{\sqrt{a+b\,\mathbf{sech}^2(x)}}\,dx$

Optimal. Leaf size=29

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{\sqrt{a}}$$

[Out] arctanh(a^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))/a^(1/2)

Rubi [A] time = 0.03, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4128, 377, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[a + b*Sech[x]^2],x]

[Out] ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/Sqrt[a]

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\operatorname{sech}^2(x)}}\,dx = \text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+b-bx^2}}\,dx, x, \tanh(x)\right)$$
$$= \text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{\sqrt{a}}$$

Mathematica [B] time = 0.04, size = 62, normalized size = 2.14

$$\frac{\operatorname{sech}(x)\sqrt{a\cosh(2x)+a+2b}\,\tanh^{-1}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{a\sinh^2(x)+a+b}}\right)}{\sqrt{2}\sqrt{a}\sqrt{a+b\operatorname{sech}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] (ArcTanh[(Sqrt[a]*Sinh[x])/Sqrt[a + b + a*Sinh[x]^2]]*Sqrt[a + 2*b + a*Cosh
[2*x]]*Sech[x])/(Sqrt[2]*Sqrt[a]*Sqrt[a + b*Sech[x]^2])
```

fricas [B] time = 0.45, size = 1059, normalized size = 36.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh
(x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*si
nh(x)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3
 + 4*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a
*b^2 - 30*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(
a*b^2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3
 + 2*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*c
osh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2
 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2
```

```
*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b
^2*cosh(x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^
2*cosh(x)^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cos
h(x)^3 - (a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh
(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*
cosh(x)^7 - 3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3
 + (a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*
cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*c
osh(x)*sinh(x)^5 + sinh(x)^6)) + sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*si
nh(x)^3 + a*sinh(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + b)*sin
h(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*sq
rt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + s
inh(x)^2)) + 4*(a*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*
cosh(x)*sinh(x) + sinh(x)^2)))/a, -1/2*(sqrt(-a)*arctan(sqrt(2)*(b*cosh(x)^
2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*s
inh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*b*cosh(
x)^4 + 4*a*b*cosh(x)*sinh(x)^3 + a*b*sinh(x)^4 - (a^2 + 3*a*b)*cosh(x)^2 +
(6*a*b*cosh(x)^2 - a^2 - 3*a*b)*sinh(x)^2 - a^2 + 2*(2*a*b*cosh(x)^3 - (a^2
 + 3*a*b)*cosh(x))*sinh(x))) + sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos
h(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)
)/(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a)))/a]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [F] time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\,\mathrm{sech}(x)^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sech(x)^2)^(1/2),x)
```

```
[Out] int(1/(a+b*sech(x)^2)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\,\mathrm{sech}\left(x\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(x)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(b*sech(x)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{a + \frac{b}{\cosh(x)^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cosh(x)^2)^(1/2),x)
[Out] int(1/(a + b/cosh(x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b\,\mathrm{sech}^2\,(x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(x)**2)**(1/2),x)
[Out] Integral(1/sqrt(a + b*sech(x)**2), x)
```

3.200 $\int \frac{\coth(x)}{\sqrt{a+b\,\mathrm{sech}^2(x)}}\,dx$

Optimal. Leaf size=56

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}$$

```
[Out] arctanh((a+b*sech(x)^2)^(1/2)/a^(1/2))/a^(1/2)-arctanh((a+b*sech(x)^2)^(1/2
)/(a+b)^(1/2))/(a+b)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4139, 446, 86, 63, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]/Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]]/Sqrt[a] - ArcTanh[Sqrt[a + b*Sech[x]
^2]/Sqrt[a + b]]/Sqrt[a + b]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] && !IntegerQ[p]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth(x)}{\sqrt{a+b\operatorname{sech}^2(x)}}\,dx &= \text{Subst}\left(\int \frac{1}{x\left(-1+x^2\right)\sqrt{a+bx^2}}\,dx, x, \operatorname{sech}(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(-1+x)x\sqrt{a+bx}}\,dx, x, \operatorname{sech}^2(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a+bx}}\,dx, x, \operatorname{sech}^2(x)\right)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \operatorname{sech}^2(x)\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{-1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{sech}^2(x)}\right)}{b}-\frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{sech}^2(x)}\right)}{b}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}
\end{aligned}$$

Mathematica [B] time = 0.26, size = 124, normalized size = 2.21

$$\frac{\operatorname{sech}(x)\sqrt{a\cosh(2x)+a+2b}\left(\sqrt{a+b}\log\left(\sqrt{a\cosh(2x)+a+2b}+\sqrt{2}\sqrt{a}\cosh(x)\right)-\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a+b}\cosh}{\sqrt{a\cosh(2x)+a}}\right.\right.}{\sqrt{2}\sqrt{a}\sqrt{a+b}\sqrt{a+b\operatorname{sech}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] (Sqrt[a + 2*b + a*Cosh[2*x]]*(-(Sqrt[a]*ArcTanh[(Sqrt[2]*Sqrt[a + b]*Cosh[x
])/Sqrt[a + 2*b + a*Cosh[2*x]]]) + Sqrt[a + b]*Log[Sqrt[2]*Sqrt[a]*Cosh[x]
+ Sqrt[a + 2*b + a*Cosh[2*x]]])*Sech[x])/(Sqrt[2]*Sqrt[a]*Sqrt[a + b]*Sqrt[
a + b*Sech[x]^2])
```

fricas [B] time = 0.57, size = 3663, normalized size = 65.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sech(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a + b)*sqrt(a)*log(((a^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a
^2*b + a*b^2)*cosh(x)*sinh(x)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*
a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b
^3 + 14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b
 + a*b^2)*cosh(x)^3 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^
5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*co
sh(x)^4 + 6*a^3 + 14*a^2*b + 9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)
*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3
 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x
))*sinh(x)^3 + a^3 + 2*(2*a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b +
 a*b^2)*cosh(x)^6 + 15*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3
+ 3*a^2*b + 3*(6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(
(a^2 + 2*a*b + b^2)*cosh(x)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (
a^2 + 2*a*b + b^2)*sinh(x)^6 + 3*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^2
+ 2*a*b + b^2)*cosh(x)^2 + a^2 + 2*a*b + b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b
 + b^2)*cosh(x)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4*a
*b)*cosh(x)^2 + (15*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*
cosh(x)^2 + 3*a^2 + 4*a*b)*sinh(x)^2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(
x)^5 + 6*(a^2 + 2*a*b + b^2)*cosh(x)^3 + (3*a^2 + 4*a*b)*cosh(x))*sinh(x))*
sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*s
inh(x) + sinh(x)^2)) + 4*(2*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^7 + 3*(2*a^3 +
5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^3
 + (2*a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 1
```

```
5*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6
*cosh(x)*sinh(x)^5 + sinh(x)^6)) + 2*sqrt(a + b)*a*log(((2*a + b)*cosh(x)^4
 + 4*(2*a + b)*cosh(x)*sinh(x)^3 + (2*a + b)*sinh(x)^4 + 2*(2*a + 3*b)*cosh
(x)^2 + 2*(3*(2*a + b)*cosh(x)^2 + 2*a + 3*b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x
)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*sqrt((a*cosh(x)^2 + a*
sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((2*a
 + b)*cosh(x)^3 + (2*a + 3*b)*cosh(x))*sinh(x) + 2*a + b)/(cosh(x)^4 + 4*co
sh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 +
 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)) + (a + b)*sqrt(a)*log(-(a*cosh(x)^4
+ 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*a*cosh(x)^2 +
b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt
(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(
x) + sinh(x)^2)) + 4*(a*cosh(x)^3 + b*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*
cosh(x)*sinh(x) + sinh(x)^2)))/(a^2 + a*b), 1/4*(4*a*sqrt(-a - b)*arctan(sq
rt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a - b)*sqrt((a*
cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)
^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(
x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*c
osh(x))*sinh(x) + a)) + (a + b)*sqrt(a)*log(((a^3 + 2*a^2*b + a*b^2)*cosh(x
)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^7 + (a^3 + 2*a^2*b + a*b^2)
*sinh(x)^8 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^6 + 2*(2*a^3 + 5*a
^2*b + 4*a*b^2 + b^3 + 14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^6 + 4*
(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)
*cosh(x))*sinh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*(a^3 + 2
*a^2*b + a*b^2)*cosh(x)^4 + 6*a^3 + 14*a^2*b + 9*a*b^2 + 30*(2*a^3 + 5*a^2*
b + 4*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cos
h(x)^5 + 10*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^3 + (6*a^3 + 14*a^2*b
 + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(2*a^3 + 3*a^2*b)*cosh(x)^2 + 2*(1
4*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 + 15*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*
cosh(x)^4 + 2*a^3 + 3*a^2*b + 3*(6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^2)*sin
h(x)^2 + sqrt(2)*((a^2 + 2*a*b + b^2)*cosh(x)^6 + 6*(a^2 + 2*a*b + b^2)*cos
h(x)*sinh(x)^5 + (a^2 + 2*a*b + b^2)*sinh(x)^6 + 3*(a^2 + 2*a*b + b^2)*cosh
(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 + 2*a*b + b^2)*sinh(x)^4 +
 4*(5*(a^2 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(x))*sinh(x
)^3 + (3*a^2 + 4*a*b)*cosh(x)^2 + (15*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 18*(a
^2 + 2*a*b + b^2)*cosh(x)^2 + 3*a^2 + 4*a*b)*sinh(x)^2 + a^2 + 2*(3*(a^2 +
2*a*b + b^2)*cosh(x)^5 + 6*(a^2 + 2*a*b + b^2)*cosh(x)^3 + (3*a^2 + 4*a*b)*
cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(
x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + 2*a^2*b + a*b^2)*cosh(
x)^7 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^5 + (6*a^3 + 14*a^2*b +
9*a*b^2)*cosh(x)^3 + (2*a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cos
h(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(
x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + (a + b)*sqrt(a)*log(-(
a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*a*
cosh(x)^2 + b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)
```

```
^2 - 1)*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*c
osh(x)*sinh(x) + sinh(x)^2)) + 4*(a*cosh(x)^3 + b*cosh(x))*sinh(x) + a)/(co
sh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a^2 + a*b), -1/2*(sqrt(-a)*(a +
 b)*arctan(sqrt(2)*((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)
*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x
)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b
)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 + (2*a^2 + 3*a*b)*cosh(x)^2 + (
6*(a^2 + a*b)*cosh(x)^2 + 2*a^2 + 3*a*b)*sinh(x)^2 + a^2 + 2*(2*(a^2 + a*b)
*cosh(x)^3 + (2*a^2 + 3*a*b)*cosh(x))*sinh(x))) + sqrt(-a)*(a + b)*arctan(s
qrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a)*sqrt((a*cos
h(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)
)/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^
2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh
(x))*sinh(x) + a)) - sqrt(a + b)*a*log(((2*a + b)*cosh(x)^4 + 4*(2*a + b)*c
osh(x)*sinh(x)^3 + (2*a + b)*sinh(x)^4 + 2*(2*a + 3*b)*cosh(x)^2 + 2*(3*(2*
a + b)*cosh(x)^2 + 2*a + 3*b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*
sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a +
2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((2*a + b)*cosh(x)^3
+ (2*a + 3*b)*cosh(x))*sinh(x) + 2*a + b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3
+ sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 -
cosh(x))*sinh(x) + 1)))/(a^2 + a*b), -1/2*(sqrt(-a)*(a + b)*arctan(sqrt(2)*
((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a)*sqr
t(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sin
h(x) + sinh(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3
 + (a^2 + a*b)*sinh(x)^4 + (2*a^2 + 3*a*b)*cosh(x)^2 + (6*(a^2 + a*b)*cosh(
x)^2 + 2*a^2 + 3*a*b)*sinh(x)^2 + a^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 + (2*a^2
 + 3*a*b)*cosh(x))*sinh(x))) + sqrt(-a)*(a + b)*arctan(sqrt(2)*(cosh(x)^2 +
 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^
2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*
a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^
2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a))
- 2*a*sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^
2 + 1)*sqrt(-a - b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 -
 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*s
inh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 +
4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)))/(a^2 + a*b)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
```

```
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{\coth(x)}{\sqrt{a + b\mathrm{sech}(x)^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a+b*sech(x)^2)^(1/2),x)
[Out] int(coth(x)/(a+b*sech(x)^2)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(x)}{\sqrt{b\,\mathrm{sech}(x)^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sech(x)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(coth(x)/sqrt(b*sech(x)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\coth(x)}{\sqrt{a + \frac{b}{\cosh(x)^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a + b/cosh(x)^2)^(1/2),x)
[Out] int(coth(x)/(a + b/cosh(x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth{\left(x \right)}}{\sqrt{a + b\,\mathrm{sech}^{2}{\left(x \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sech(x)**2)**(1/2),x)
[Out] Integral(coth(x)/sqrt(a + b*sech(x)**2), x)
```

3.201 $\int \frac{\coth^2(x)}{\sqrt{a+b\,\mathrm{sech}^2(x)}}\,dx$

Optimal. Leaf size=53

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{\sqrt{a}}-\frac{\coth(x)\sqrt{a-b\tanh^2(x)+b}}{a+b}$$

```
[Out] arctanh(a^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))/a^(1/2)-coth(x)*(a+b-b*tan
h(x)^2)^(1/2)/(a+b)
```

Rubi [A] time = 0.19, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {4141, 1975, 480, 12, 377, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{\sqrt{a}}-\frac{\coth(x)\sqrt{a-b\tanh^2(x)+b}}{a+b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2/Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/Sqrt[a] - (Coth[x]*Sqr
t[a + b - b*Tanh[x]^2])/(a + b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\coth^2(x)}{\sqrt{a+b\text{sech}^2(x)}}\,dx = \text{Subst}\left(\int \frac{1}{x^2(1-x^2)\sqrt{a+b(1-x^2)}}\,dx, x, \tanh(x)\right)$$
$$= \text{Subst}\left(\int \frac{1}{x^2(1-x^2)\sqrt{a+b-bx^2}}\,dx, x, \tanh(x)\right)$$
$$= -\frac{\coth(x)\sqrt{a+b-b\tanh^2(x)}}{a+b} + \frac{\text{Subst}\left(\int \frac{a+b}{(1-x^2)\sqrt{a+b-bx^2}}\,dx, x, \tanh(x)\right)}{a+b}$$
$$= -\frac{\coth(x)\sqrt{a+b-b\tanh^2(x)}}{a+b} + \text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+b-bx^2}}\,dx, x, \tanh(x)\right)$$
$$= -\frac{\coth(x)\sqrt{a+b-b\tanh^2(x)}}{a+b} + \text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{\sqrt{a}} - \frac{\coth(x)\sqrt{a+b-b\tanh^2(x)}}{a+b}$$

Mathematica [A] time = 0.11, size = 94, normalized size = 1.77

$$\frac{\text{sech}(x)\sqrt{a\cosh(2x)+a+2b}\left((a+b)\tanh^{-1}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{a\sinh^2(x)+a+b}}\right)-\sqrt{a}\,\text{csch}(x)\sqrt{a\sinh^2(x)+a+b}\right)}{\sqrt{2}\sqrt{a}(a+b)\sqrt{a+b\text{sech}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2/Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] (Sqrt[a + 2*b + a*Cosh[2*x]]*Sech[x]*((a + b)*ArcTanh[(Sqrt[a]*Sinh[x])/Sqr
t[a + b + a*Sinh[x]^2]] - Sqrt[a]*Csch[x]*Sqrt[a + b + a*Sinh[x]^2]))/(Sqrt
[2]*Sqrt[a]*(a + b)*Sqrt[a + b*Sech[x]^2])
```

fricas [B] time = 0.51, size = 1365, normalized size = 25.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*sech(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 -
a - b)*sqrt(a)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sin
h(x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*s
inh(x)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^
3 + 4*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*
a*b^2 - 30*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*
(a*b^2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^
3 + 2*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*
cosh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^
2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^
2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*
b^2*cosh(x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b
^2*cosh(x)^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*co
sh(x)^3 - (a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sin
h(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2
*cosh(x)^7 - 3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^
3 + (a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15
*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*
cosh(x)*sinh(x)^5 + sinh(x)^6)) + ((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*si
nh(x) + (a + b)*sinh(x)^2 - a - b)*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*
sinh(x)^3 + a*sinh(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + b)*s
inh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*
sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) +
 sinh(x)^2)) + 4*(a*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a)/(cosh(x)^2 +
2*cosh(x)*sinh(x) + sinh(x)^2)) - 4*sqrt(2)*a*sqrt((a*cosh(x)^2 + a*sinh(x)
^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^2 + a*b)*co
sh(x)^2 + 2*(a^2 + a*b)*cosh(x)*sinh(x) + (a^2 + a*b)*sinh(x)^2 - a^2 - a*b
), -1/2*(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2
 - a - b)*sqrt(-a)*arctan(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*si
nh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2
 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*b*cosh(x)^4 + 4*a*b*cosh(x)*sinh(x)^3
 + a*b*sinh(x)^4 - (a^2 + 3*a*b)*cosh(x)^2 + (6*a*b*cosh(x)^2 - a^2 - 3*a*b
)*sinh(x)^2 - a^2 + 2*(2*a*b*cosh(x)^3 - (a^2 + 3*a*b)*cosh(x))*sinh(x))) +
 ((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - a - b
)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*
b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^2 + 2*a*cosh(x)*
sinh(x) + a*sinh(x)^2 + a)) + 2*sqrt(2)*a*sqrt((a*cosh(x)^2 + a*sinh(x)^2 +
 a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^2 + a*b)*cosh(x
)^2 + 2*(a^2 + a*b)*cosh(x)*sinh(x) + (a^2 + a*b)*sinh(x)^2 - a^2 - a*b)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [F] time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(x)}{\sqrt{a + b\mathrm{sech}(x)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(a+b*sech(x)^2)^(1/2),x)
```

```
[Out] int(coth(x)^2/(a+b*sech(x)^2)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(x)^2}{\sqrt{b\,\mathrm{sech}(x)^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*sech(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(x)^2/sqrt(b*sech(x)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\coth(x)^2}{\sqrt{a + \frac{b}{\cosh(x)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(a + b/cosh(x)^2)^(1/2),x)
```

```
[Out] int(coth(x)^2/(a + b/cosh(x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(x)}{\sqrt{a + b\,\mathrm{sech}^2(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2/(a+b*sech(x)**2)**(1/2),x)
```

```
[Out] Integral(coth(x)**2/sqrt(a + b*sech(x)**2), x)
```

3.202 $\int \frac{\coth^3(x)}{\sqrt{a+b\mathbf{sech}^2(x)}} dx$

Optimal. Leaf size=90

$$-\frac{\coth^2(x)\sqrt{a+b\text{sech}^2(x)}}{2(a+b)}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a+b}}\right)}{2(a+b)^{3/2}}$$

```
[Out] -1/2*(2*a+3*b)*arctanh((a+b*sech(x)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(3/2)+arcta
nh((a+b*sech(x)^2)^(1/2)/a^(1/2))/a^(1/2)-1/2*coth(x)^2*(a+b*sech(x)^2)^(1/
2)/(a+b)
```

Rubi [A] time = 0.16, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {4139, 446, 103, 156, 63, 208}

$$-\frac{\coth^2(x)\sqrt{a+b\text{sech}^2(x)}}{2(a+b)}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a+b}}\right)}{2(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^3/Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]]/Sqrt[a] - ((2*a + 3*b)*ArcTanh[Sqrt[
a + b*Sech[x]^2]/Sqrt[a + b]])/(2*(a + b)^(3/2)) - (Coth[x]^2*Sqrt[a + b*Se
ch[x]^2])/(2*(a + b))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
```

```
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \frac{\coth^3(x)}{\sqrt{a+b\text{sech}^2(x)}}\,dx = -\text{Subst}\left(\int \frac{1}{x\left(-1+x^2\right)^2\sqrt{a+bx^2}}\,dx, x, \text{sech}(x)\right)$$

$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{(-1+x)^2x\sqrt{a+bx}}\,dx, x, \text{sech}^2(x)\right)\right)$$

$$= -\frac{\coth^2(x)\sqrt{a+b\text{sech}^2(x)}}{2(a+b)} + \frac{\text{Subst}\left(\int \frac{a+b+\frac{bx}{2}}{(-1+x)x\sqrt{a+bx}}\,dx, x, \text{sech}^2(x)\right)}{2(a+b)}$$

$$= -\frac{\coth^2(x)\sqrt{a+b\text{sech}^2(x)}}{2(a+b)} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \text{sech}^2(x)\right) + \frac{(2a+3b)\,\text{Subs}}{}$$

$$= -\frac{\coth^2(x)\sqrt{a+b\text{sech}^2(x)}}{2(a+b)} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{sech}^2(x)}\right)}{b} + \frac{(2a+3b)\,\text{Subs}}{}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a+b}}\right)}{2(a+b)^{3/2}} - \frac{\coth^2(x)\sqrt{a+b\text{sech}^2(x)}}{2(a+b)}$$

Mathematica [A] time = 0.85, size = 159, normalized size = 1.77

$$\frac{\frac{\sqrt{2}\,\text{sech}(x)\sqrt{a\cosh(2x)+a+2b}\left(2(a+b)^{3/2}\log\left(\sqrt{a\cosh(2x)+a+2b}+\sqrt{2}\sqrt{a}\cosh(x)\right)-\sqrt{a}(2a+3b)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a+b}\cosh(x)}{\sqrt{a\cosh(2x)+a+2b}}\right)\right)}{\sqrt{a}\sqrt{a+b}} - \text{csch}^2(x)(a\cosh}{4(a+b)\sqrt{a+b\text{sech}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^3/Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] (-((a + 2*b + a*Cosh[2*x])*Csch[x]^2) + (Sqrt[2]*Sqrt[a + 2*b + a*Cosh[2*x]
]*(-(Sqrt[a]*(2*a + 3*b)*ArcTanh[(Sqrt[2]*Sqrt[a + b]*Cosh[x])/Sqrt[a + 2*b
 + a*Cosh[2*x]]]) + 2*(a + b)^(3/2)*Log[Sqrt[2]*Sqrt[a]*Cosh[x] + Sqrt[a +
2*b + a*Cosh[2*x]]])*Sech[x])/(Sqrt[a]*Sqrt[a + b]))/(4*(a + b)*Sqrt[a + b*
Sech[x]^2])
```

fricas [B] time = 0.73, size = 6475, normalized size = 71.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^3/(a+b*sech(x)^2)^(1/2),x, algorithm="fricas")

[Out] [1/4*(((a^2 + 2*a*b + b^2)*cosh(x)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^3 + (a^2 + 2*a*b + b^2)*sinh(x)^4 - 2*(a^2 + 2*a*b + b^2)*cosh(x)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^2 - a^2 - 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(x)^3 - (a^2 + 2*a*b + b^2)*cosh(x))*sinh(x))*sqrt(a)*log(((a^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 6*a^3 + 14*a^2*b + 9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(2*a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 + 15*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3 + 3*a^2*b + 3*(6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*((a^2 + 2*a*b + b^2)*cosh(x)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2 + 2*a*b + b^2)*sinh(x)^6 + 3*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 + 2*a*b + b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4*a*b)*cosh(x)^2 + (15*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*cosh(x)^2 + 3*a^2 + 4*a*b)*sinh(x)^2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 6*(a^2 + 2*a*b + b^2)*cosh(x)^3 + (3*a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^7 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^3 + (2*a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + ((2*a^2 + 3*a*b)*cosh(x)^4 + 4*(2*a^2 + 3*a*b)*cosh(x)*sinh(x)^3 + (2*a^2 + 3*a*b)*sinh(x)^4 - 2*(2*a^2 + 3*a*b)*cosh(x)^2 + 2*(3*(2*a^2 + 3*a*b)*cosh(x)^2 - 2*a^2 - 3*a*b)*sinh(x)^2 + 2*a^2 + 3*a*b + 4*((2*a^2 + 3*a*b)*cosh(x)^3 - (2*a^2 + 3*a*b)*cosh(x))*sinh(x))*sqrt(a + b)*log(((2*a + b)*cosh(x)^4 + 4*(2*a + b)*cosh(x)*sinh(x)^3 + (2*a + b)*sinh(x)^4 + 2*(2*a + 3*b)*cosh(x)^2 + 2*(3*(2*a + b)*cosh(x)^2 + 2*a + 3*b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((2*a + b)*cosh(x)^3 + (2*a + 3*b)*cosh(x))*sinh(x) + 2*a + b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)) + ((a^2 + 2*a*b + b^2)*cosh(x)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^3 + (a^2 + 2*a*b + b^2)*sinh(x)^4 - 2*(a^2 + 2*a*b + b^2)*cosh(x)^2 + 2*(3*(a^2 +

```
2*a*b + b^2)*cosh(x)^2 - a^2 - 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 +
 4*((a^2 + 2*a*b + b^2)*cosh(x)^3 - (a^2 + 2*a*b + b^2)*cosh(x))*sinh(x))*s
qrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*b*cosh(x
)^2 + 2*(3*a*cosh(x)^2 + b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh
(x) + sinh(x)^2 - 1)*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(co
sh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(a*cosh(x)^3 + b*cosh(x))*sin
h(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) - 2*sqrt(2)*((a^2 +
a*b)*cosh(x)^2 + 2*(a^2 + a*b)*cosh(x)*sinh(x) + (a^2 + a*b)*sinh(x)^2 + a^
2 + a*b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*
sinh(x) + sinh(x)^2)))/((a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 4*(a^3 + 2*a^2*
b + a*b^2)*cosh(x)*sinh(x)^3 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^4 + a^3 + 2*
a^2*b + a*b^2 - 2*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2 - 2*(a^3 + 2*a^2*b + a*
b^2 - 3*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 + 2*a^2*b +
a*b^2)*cosh(x)^3 - (a^3 + 2*a^2*b + a*b^2)*cosh(x))*sinh(x)), 1/4*(2*((2*a^
2 + 3*a*b)*cosh(x)^4 + 4*(2*a^2 + 3*a*b)*cosh(x)*sinh(x)^3 + (2*a^2 + 3*a*b
)*sinh(x)^4 - 2*(2*a^2 + 3*a*b)*cosh(x)^2 + 2*(3*(2*a^2 + 3*a*b)*cosh(x)^2
- 2*a^2 - 3*a*b)*sinh(x)^2 + 2*a^2 + 3*a*b + 4*((2*a^2 + 3*a*b)*cosh(x)^3 -
 (2*a^2 + 3*a*b)*cosh(x))*sinh(x))*sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2 +
 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a - b)*sqrt((a*cosh(x)^2 + a*sinh
(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4
+ 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh
(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) +
a)) + ((a^2 + 2*a*b + b^2)*cosh(x)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x
)^3 + (a^2 + 2*a*b + b^2)*sinh(x)^4 - 2*(a^2 + 2*a*b + b^2)*cosh(x)^2 + 2*(
3*(a^2 + 2*a*b + b^2)*cosh(x)^2 - a^2 - 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*
b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(x)^3 - (a^2 + 2*a*b + b^2)*cosh(x))*s
inh(x))*sqrt(a)*log(((a^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b +
 a*b^2)*cosh(x)*sinh(x)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 +
5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 1
4*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b
^2)*cosh(x)^3 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + (6
*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^
4 + 6*a^3 + 14*a^2*b + 9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(
x)^2)*sinh(x)^4 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a
^2*b + 4*a*b^2 + b^3)*cosh(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*sin
h(x)^3 + a^3 + 2*(2*a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2
)*cosh(x)^6 + 15*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3 + 3*a^
2*b + 3*(6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*((a^2 +
 2*a*b + b^2)*cosh(x)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2 +
2*a*b + b^2)*sinh(x)^6 + 3*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*
b + b^2)*cosh(x)^2 + a^2 + 2*a*b + b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2
)*cosh(x)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4*a*b)*co
sh(x)^2 + (15*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*cosh(x
)^2 + 3*a^2 + 4*a*b)*sinh(x)^2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^5 +
 6*(a^2 + 2*a*b + b^2)*cosh(x)^3 + (3*a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a
```

```
)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x)
 + sinh(x)^2)) + 4*(2*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^7 + 3*(2*a^3 + 5*a^2*
b + 4*a*b^2 + b^3)*cosh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^3 + (2*
a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh
(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(
x)*sinh(x)^5 + sinh(x)^6)) + ((a^2 + 2*a*b + b^2)*cosh(x)^4 + 4*(a^2 + 2*a*
b + b^2)*cosh(x)*sinh(x)^3 + (a^2 + 2*a*b + b^2)*sinh(x)^4 - 2*(a^2 + 2*a*b
 + b^2)*cosh(x)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^2 - a^2 - 2*a*b - b^2)
*sinh(x)^2 + a^2 + 2*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(x)^3 - (a^2 +
2*a*b + b^2)*cosh(x))*sinh(x))*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh
(x)^3 + a*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*a*cosh(x)^2 + b)*sinh(x)^2 + sqr
t(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a)*sqrt((a*cosh(x
)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) +
 4*(a*cosh(x)^3 + b*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) +
sinh(x)^2)) - 2*sqrt(2)*((a^2 + a*b)*cosh(x)^2 + 2*(a^2 + a*b)*cosh(x)*sinh
(x) + (a^2 + a*b)*sinh(x)^2 + a^2 + a*b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 +
a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^3 + 2*a^2*b + a*
b^2)*cosh(x)^4 + 4*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^3 + (a^3 + 2*a^2
*b + a*b^2)*sinh(x)^4 + a^3 + 2*a^2*b + a*b^2 - 2*(a^3 + 2*a^2*b + a*b^2)*c
osh(x)^2 - 2*(a^3 + 2*a^2*b + a*b^2 - 3*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*
sinh(x)^2 + 4*((a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 - (a^3 + 2*a^2*b + a*b^2)*
cosh(x))*sinh(x)), -1/4*(2*((a^2 + 2*a*b + b^2)*cosh(x)^4 + 4*(a^2 + 2*a*b
+ b^2)*cosh(x)*sinh(x)^3 + (a^2 + 2*a*b + b^2)*sinh(x)^4 - 2*(a^2 + 2*a*b +
 b^2)*cosh(x)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^2 - a^2 - 2*a*b - b^2)*s
inh(x)^2 + a^2 + 2*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(x)^3 - (a^2 + 2*
a*b + b^2)*cosh(x))*sinh(x))*sqrt(-a)*arctan(sqrt(2)*((a + b)*cosh(x)^2 + 2
*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^
2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a
^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)
^4 + (2*a^2 + 3*a*b)*cosh(x)^2 + (6*(a^2 + a*b)*cosh(x)^2 + 2*a^2 + 3*a*b)*
sinh(x)^2 + a^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 + (2*a^2 + 3*a*b)*cosh(x))*sin
h(x))) + 2*((a^2 + 2*a*b + b^2)*cosh(x)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(x)*s
inh(x)^3 + (a^2 + 2*a*b + b^2)*sinh(x)^4 - 2*(a^2 + 2*a*b + b^2)*cosh(x)^2
+ 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^2 - a^2 - 2*a*b - b^2)*sinh(x)^2 + a^2 +
 2*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(x)^3 - (a^2 + 2*a*b + b^2)*cosh(
x))*sinh(x))*sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(
x)^2 - 1)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 -
2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*si
nh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4
*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) - ((2*a^2 + 3*a*b)*cosh(x)
^4 + 4*(2*a^2 + 3*a*b)*cosh(x)*sinh(x)^3 + (2*a^2 + 3*a*b)*sinh(x)^4 - 2*(2
*a^2 + 3*a*b)*cosh(x)^2 + 2*(3*(2*a^2 + 3*a*b)*cosh(x)^2 - 2*a^2 - 3*a*b)*s
inh(x)^2 + 2*a^2 + 3*a*b + 4*((2*a^2 + 3*a*b)*cosh(x)^3 - (2*a^2 + 3*a*b)*c
osh(x))*sinh(x))*sqrt(a + b)*log(((2*a + b)*cosh(x)^4 + 4*(2*a + b)*cosh(x)
*sinh(x)^3 + (2*a + b)*sinh(x)^4 + 2*(2*a + 3*b)*cosh(x)^2 + 2*(3*(2*a + b)
```

```
*cosh(x)^2 + 2*a + 3*b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x
) + sinh(x)^2 + 1)*sqrt(a + b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(
cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((2*a + b)*cosh(x)^3 + (2*a
 + 3*b)*cosh(x))*sinh(x) + 2*a + b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh
(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x
))*sinh(x) + 1)) + 2*sqrt(2)*((a^2 + a*b)*cosh(x)^2 + 2*(a^2 + a*b)*cosh(x)
*sinh(x) + (a^2 + a*b)*sinh(x)^2 + a^2 + a*b)*sqrt((a*cosh(x)^2 + a*sinh(x)
^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^3 + 2*a^2*b
 + a*b^2)*cosh(x)^4 + 4*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^3 + (a^3 +
2*a^2*b + a*b^2)*sinh(x)^4 + a^3 + 2*a^2*b + a*b^2 - 2*(a^3 + 2*a^2*b + a*b
^2)*cosh(x)^2 - 2*(a^3 + 2*a^2*b + a*b^2 - 3*(a^3 + 2*a^2*b + a*b^2)*cosh(x
)^2)*sinh(x)^2 + 4*((a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 - (a^3 + 2*a^2*b + a*
b^2)*cosh(x))*sinh(x)), -1/2*(((a^2 + 2*a*b + b^2)*cosh(x)^4 + 4*(a^2 + 2*a
*b + b^2)*cosh(x)*sinh(x)^3 + (a^2 + 2*a*b + b^2)*sinh(x)^4 - 2*(a^2 + 2*a*
b + b^2)*cosh(x)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^2 - a^2 - 2*a*b - b^2
)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(x)^3 - (a^2 +
 2*a*b + b^2)*cosh(x))*sinh(x))*sqrt(-a)*arctan(sqrt(2)*((a + b)*cosh(x)^2
+ 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(
x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/
((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh
(x)^4 + (2*a^2 + 3*a*b)*cosh(x)^2 + (6*(a^2 + a*b)*cosh(x)^2 + 2*a^2 + 3*a*
b)*sinh(x)^2 + a^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 + (2*a^2 + 3*a*b)*cosh(x))*
sinh(x))) + ((a^2 + 2*a*b + b^2)*cosh(x)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(x)*
sinh(x)^3 + (a^2 + 2*a*b + b^2)*sinh(x)^4 - 2*(a^2 + 2*a*b + b^2)*cosh(x)^2
 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^2 - a^2 - 2*a*b - b^2)*sinh(x)^2 + a^2
+ 2*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(x)^3 - (a^2 + 2*a*b + b^2)*cosh
(x))*sinh(x))*sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh
(x)^2 - 1)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 -
 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*s
inh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 +
4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) - ((2*a^2 + 3*a*b)*cosh(x
)^4 + 4*(2*a^2 + 3*a*b)*cosh(x)*sinh(x)^3 + (2*a^2 + 3*a*b)*sinh(x)^4 - 2*(
2*a^2 + 3*a*b)*cosh(x)^2 + 2*(3*(2*a^2 + 3*a*b)*cosh(x)^2 - 2*a^2 - 3*a*b)*
sinh(x)^2 + 2*a^2 + 3*a*b + 4*((2*a^2 + 3*a*b)*cosh(x)^3 - (2*a^2 + 3*a*b)*
cosh(x))*sinh(x))*sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x
) + sinh(x)^2 + 1)*sqrt(-a - b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/
(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sin
h(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*
sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) + sqrt(2)*((a
^2 + a*b)*cosh(x)^2 + 2*(a^2 + a*b)*cosh(x)*sinh(x) + (a^2 + a*b)*sinh(x)^2
 + a^2 + a*b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cos
h(x)*sinh(x) + sinh(x)^2)))/((a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 4*(a^3 + 2
*a^2*b + a*b^2)*cosh(x)*sinh(x)^3 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^4 + a^3
 + 2*a^2*b + a*b^2 - 2*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2 - 2*(a^3 + 2*a^2*b
 + a*b^2 - 3*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 + 2*a^2
```

```
*b + a*b^2)*cosh(x)^3 - (a^3 + 2*a^2*b + a*b^2)*cosh(x))*sinh(x))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [F] time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{\coth^3(x)}{\sqrt{a + b\mathrm{sech}(x)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(a+b*sech(x)^2)^(1/2),x)
```

```
[Out] int(coth(x)^3/(a+b*sech(x)^2)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(x)^3}{\sqrt{b\,\mathrm{sech}(x)^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*sech(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(x)^3/sqrt(b*sech(x)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\coth(x)^3}{\sqrt{a + \frac{b}{\cosh(x)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(a + b/cosh(x)^2)^(1/2),x)
```

```
[Out] int(coth(x)^3/(a + b/cosh(x)^2)^(1/2), x)
```

<u>sympy</u> [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^3(x)}{\sqrt{a+b\,\mathrm{sech}^2(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**3/(a+b*sech(x)**2)**(1/2),x)
```

```
[Out] Integral(coth(x)**3/sqrt(a + b*sech(x)**2), x)
```

3.203 $\int \frac{\tanh^5(x)}{\left(a+b\,\mathrm{sech}^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=68

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}^2(x)}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{(a+b)^2}{ab^2\sqrt{a+b\,\mathrm{sech}^2(x)}}-\frac{\sqrt{a+b\,\mathrm{sech}^2(x)}}{b^2}$$

```
[Out] arctanh((a+b*sech(x)^2)^(1/2)/a^(1/2))/a^(3/2)-(a+b)^2/a/b^2/(a+b*sech(x)^2
)^(1/2)-(a+b*sech(x)^2)^(1/2)/b^2
```

Rubi [A] time = 0.15, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {4139, 446, 87, 63, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}^2(x)}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{(a+b)^2}{ab^2\sqrt{a+b\,\mathrm{sech}^2(x)}}-\frac{\sqrt{a+b\,\mathrm{sech}^2(x)}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^5/(a + b*Sech[x]^2)^(3/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]]/a^(3/2) - (a + b)^2/(a*b^2*Sqrt[a +
b*Sech[x]^2]) - Sqrt[a + b*Sech[x]^2]/b^2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 87

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], ((c + d
*x)^n*(e + f*x)^IntegerPart[p])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh^5(x)}{\left(a+b\,\mathrm{sech}^2(x)\right)^{3/2}}\,dx &= -\mathrm{Subst}\left(\int \frac{\left(-1+x^2\right)^2}{x\left(a+bx^2\right)^{3/2}}\,dx, x, \mathrm{sech}(x)\right)\\
&= -\left(\frac{1}{2}\mathrm{Subst}\left(\int \frac{(-1+x)^2}{x(a+bx)^{3/2}}\,dx, x, \mathrm{sech}^2(x)\right)\right)\\
&= -\left(\frac{1}{2}\mathrm{Subst}\left(\int \left(-\frac{(a+b)^2}{ab(a+bx)^{3/2}}+\frac{1}{b\sqrt{a+bx}}+\frac{1}{ax\sqrt{a+bx}}\right)dx, x, \mathrm{sech}^2(x)\right)\right)\\
&= -\frac{(a+b)^2}{ab^2\sqrt{a+b\,\mathrm{sech}^2(x)}}-\frac{\sqrt{a+b\,\mathrm{sech}^2(x)}}{b^2}-\frac{\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \mathrm{sech}^2(x)\right)}{2a}\\
&= -\frac{(a+b)^2}{ab^2\sqrt{a+b\,\mathrm{sech}^2(x)}}-\frac{\sqrt{a+b\,\mathrm{sech}^2(x)}}{b^2}-\frac{\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\,\mathrm{sech}^2(x)}\right)}{ab}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}^2(x)}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{(a+b)^2}{ab^2\sqrt{a+b\,\mathrm{sech}^2(x)}}-\frac{\sqrt{a+b\,\mathrm{sech}^2(x)}}{b^2}
\end{aligned}
$$

Mathematica [A] time = 0.53, size = 129, normalized size = 1.90

$$
\frac{\mathrm{sech}^3(x)\left(\frac{\sqrt{2}\,(a\cosh(2x)+a+2b)^{3/2}\log\left(\sqrt{a\cosh(2x)+a+2b}+\sqrt{2}\,\sqrt{a}\,\cosh(x)\right)}{a^{3/2}}-\frac{\mathrm{sech}(x)(a\cosh(2x)+a+2b)\left(\left(2a^2+2ab+b^2\right)\cosh(2x)+2a^2+4ab+\right.}{ab^2}\right.}{4\left(a+b\,\mathrm{sech}^2(x)\right)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^5/(a + b*Sech[x]^2)^(3/2),x]
```

```
[Out] (Sech[x]^3*((Sqrt[2]*(a + 2*b + a*Cosh[2*x])^(3/2)*Log[Sqrt[2]*Sqrt[a]*Cosh
[x] + Sqrt[a + 2*b + a*Cosh[2*x]]])/a^(3/2) - ((a + 2*b + a*Cosh[2*x])*(2*a
^2 + 4*a*b + b^2 + (2*a^2 + 2*a*b + b^2)*Cosh[2*x])*Sech[x])/(a*b^2)))/(4*(
a + b*Sech[x]^2)^(3/2))
```

fricas [B] time = 0.68, size = 3360, normalized size = 49.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+b*sech(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a*b^2*cosh(x)^6 + 6*a*b^2*cosh(x)*sinh(x)^5 + a*b^2*sinh(x)^6 + (3*a
*b^2 + 4*b^3)*cosh(x)^4 + (15*a*b^2*cosh(x)^2 + 3*a*b^2 + 4*b^3)*sinh(x)^4
+ 4*(5*a*b^2*cosh(x)^3 + (3*a*b^2 + 4*b^3)*cosh(x))*sinh(x)^3 + a*b^2 + (3*
a*b^2 + 4*b^3)*cosh(x)^2 + (15*a*b^2*cosh(x)^4 + 3*a*b^2 + 4*b^3 + 6*(3*a*b
^2 + 4*b^3)*cosh(x)^2)*sinh(x)^2 + 2*(3*a*b^2*cosh(x)^5 + 2*(3*a*b^2 + 4*b^
3)*cosh(x)^3 + (3*a*b^2 + 4*b^3)*cosh(x))*sinh(x))*sqrt(a)*log(((a^3 + 2*a^
2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^7 + (a^3
 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)
^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*(a^3 + 2*a^2*b + a*b^2)*cosh(x
)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + 3*(2*a^3 + 5*a^2
*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(
x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 6*a^3 + 14*a^2*b + 9*a*b^2 +
 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + 2
*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^3
+ (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(2*a^3 + 3*a^2*
b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 + 15*(2*a^3 + 5*a^2*
b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3 + 3*a^2*b + 3*(6*a^3 + 14*a^2*b + 9*a*
b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*((a^2 + 2*a*b + b^2)*cosh(x)^6 + 6*(a^2
 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2 + 2*a*b + b^2)*sinh(x)^6 + 3*(a^2
+ 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 + 2*a*b
 + b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 + 2*a*b + b
^2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4*a*b)*cosh(x)^2 + (15*(a^2 + 2*a*b + b^2
)*cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*cosh(x)^2 + 3*a^2 + 4*a*b)*sinh(x)^2 +
 a^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 6*(a^2 + 2*a*b + b^2)*cosh(x)^3
 + (3*a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^
2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + 2*a
^2*b + a*b^2)*cosh(x)^7 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^5 + (
6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^3 + (2*a^3 + 3*a^2*b)*cosh(x))*sinh(x))
/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*s
inh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + (a*
b^2*cosh(x)^6 + 6*a*b^2*cosh(x)*sinh(x)^5 + a*b^2*sinh(x)^6 + (3*a*b^2 + 4*
b^3)*cosh(x)^4 + (15*a*b^2*cosh(x)^2 + 3*a*b^2 + 4*b^3)*sinh(x)^4 + 4*(5*a*
b^2*cosh(x)^3 + (3*a*b^2 + 4*b^3)*cosh(x))*sinh(x)^3 + a*b^2 + (3*a*b^2 + 4
*b^3)*cosh(x)^2 + (15*a*b^2*cosh(x)^4 + 3*a*b^2 + 4*b^3 + 6*(3*a*b^2 + 4*b^
3)*cosh(x)^2)*sinh(x)^2 + 2*(3*a*b^2*cosh(x)^5 + 2*(3*a*b^2 + 4*b^3)*cosh(x
)^3 + (3*a*b^2 + 4*b^3)*cosh(x))*sinh(x))*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*c
osh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*a*cosh(x)^2 + b)*sinh
(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a)*sqr
t((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + si
nh(x)^2)) + 4*(a*cosh(x)^3 + b*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)
*sinh(x) + sinh(x)^2)) - 4*sqrt(2)*((2*a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 4
*(2*a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^3 + (2*a^3 + 2*a^2*b + a*b^2)*si
nh(x)^4 + 2*a^3 + 2*a^2*b + a*b^2 + 2*(2*a^3 + 4*a^2*b + a*b^2)*cosh(x)^2 +
```

```
 2*(2*a^3 + 4*a^2*b + a*b^2 + 3*(2*a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x
)^2 + 4*((2*a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + (2*a^3 + 4*a^2*b + a*b^2)*co
sh(x))*sinh(x))*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*c
osh(x)*sinh(x) + sinh(x)^2)))/(a^3*b^2*cosh(x)^6 + 6*a^3*b^2*cosh(x)*sinh(x
)^5 + a^3*b^2*sinh(x)^6 + a^3*b^2 + (3*a^3*b^2 + 4*a^2*b^3)*cosh(x)^4 + (15
*a^3*b^2*cosh(x)^2 + 3*a^3*b^2 + 4*a^2*b^3)*sinh(x)^4 + 4*(5*a^3*b^2*cosh(x
)^3 + (3*a^3*b^2 + 4*a^2*b^3)*cosh(x))*sinh(x)^3 + (3*a^3*b^2 + 4*a^2*b^3)*
cosh(x)^2 + (15*a^3*b^2*cosh(x)^4 + 3*a^3*b^2 + 4*a^2*b^3 + 6*(3*a^3*b^2 +
4*a^2*b^3)*cosh(x)^2)*sinh(x)^2 + 2*(3*a^3*b^2*cosh(x)^5 + 2*(3*a^3*b^2 + 4
*a^2*b^3)*cosh(x)^3 + (3*a^3*b^2 + 4*a^2*b^3)*cosh(x))*sinh(x)), -1/2*((a*b
^2*cosh(x)^6 + 6*a*b^2*cosh(x)*sinh(x)^5 + a*b^2*sinh(x)^6 + (3*a*b^2 + 4*b
^3)*cosh(x)^4 + (15*a*b^2*cosh(x)^2 + 3*a*b^2 + 4*b^3)*sinh(x)^4 + 4*(5*a*b
^2*cosh(x)^3 + (3*a*b^2 + 4*b^3)*cosh(x))*sinh(x)^3 + a*b^2 + (3*a*b^2 + 4*
b^3)*cosh(x)^2 + (15*a*b^2*cosh(x)^4 + 3*a*b^2 + 4*b^3 + 6*(3*a*b^2 + 4*b^3
)*cosh(x)^2)*sinh(x)^2 + 2*(3*a*b^2*cosh(x)^5 + 2*(3*a*b^2 + 4*b^3)*cosh(x)
^3 + (3*a*b^2 + 4*b^3)*cosh(x))*sinh(x))*sqrt(-a)*arctan(sqrt(2)*((a + b)*c
osh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a)*sqrt(-a)*sqrt
((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sin
h(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 +
a*b)*sinh(x)^4 + (2*a^2 + 3*a*b)*cosh(x)^2 + (6*(a^2 + a*b)*cosh(x)^2 + 2*a
^2 + 3*a*b)*sinh(x)^2 + a^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 + (2*a^2 + 3*a*b)*
cosh(x))*sinh(x))) + (a*b^2*cosh(x)^6 + 6*a*b^2*cosh(x)*sinh(x)^5 + a*b^2*s
inh(x)^6 + (3*a*b^2 + 4*b^3)*cosh(x)^4 + (15*a*b^2*cosh(x)^2 + 3*a*b^2 + 4*
b^3)*sinh(x)^4 + 4*(5*a*b^2*cosh(x)^3 + (3*a*b^2 + 4*b^3)*cosh(x))*sinh(x)^
3 + a*b^2 + (3*a*b^2 + 4*b^3)*cosh(x)^2 + (15*a*b^2*cosh(x)^4 + 3*a*b^2 + 4
*b^3 + 6*(3*a*b^2 + 4*b^3)*cosh(x)^2)*sinh(x)^2 + 2*(3*a*b^2*cosh(x)^5 + 2*
(3*a*b^2 + 4*b^3)*cosh(x)^3 + (3*a*b^2 + 4*b^3)*cosh(x))*sinh(x))*sqrt(-a)*
arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a)*sqr
t((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + si
nh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*
cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2
*b)*cosh(x))*sinh(x) + a)) + 2*sqrt(2)*((2*a^3 + 2*a^2*b + a*b^2)*cosh(x)^4
 + 4*(2*a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^3 + (2*a^3 + 2*a^2*b + a*b^2
)*sinh(x)^4 + 2*a^3 + 2*a^2*b + a*b^2 + 2*(2*a^3 + 4*a^2*b + a*b^2)*cosh(x)
^2 + 2*(2*a^3 + 4*a^2*b + a*b^2 + 3*(2*a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*si
nh(x)^2 + 4*((2*a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + (2*a^3 + 4*a^2*b + a*b^2
)*cosh(x))*sinh(x))*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 -
 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a^3*b^2*cosh(x)^6 + 6*a^3*b^2*cosh(x)*si
nh(x)^5 + a^3*b^2*sinh(x)^6 + a^3*b^2 + (3*a^3*b^2 + 4*a^2*b^3)*cosh(x)^4 +
 (15*a^3*b^2*cosh(x)^2 + 3*a^3*b^2 + 4*a^2*b^3)*sinh(x)^4 + 4*(5*a^3*b^2*co
sh(x)^3 + (3*a^3*b^2 + 4*a^2*b^3)*cosh(x))*sinh(x)^3 + (3*a^3*b^2 + 4*a^2*b
^3)*cosh(x)^2 + (15*a^3*b^2*cosh(x)^4 + 3*a^3*b^2 + 4*a^2*b^3 + 6*(3*a^3*b^
2 + 4*a^2*b^3)*cosh(x)^2)*sinh(x)^2 + 2*(3*a^3*b^2*cosh(x)^5 + 2*(3*a^3*b^2
 + 4*a^2*b^3)*cosh(x)^3 + (3*a^3*b^2 + 4*a^2*b^3)*cosh(x))*sinh(x))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+b*sech(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{\tanh^{5}(x)}{\left(a+b\,\mathrm{sech}(x)^{2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^5/(a+b*sech(x)^2)^(3/2),x)
```

```
[Out] int(tanh(x)^5/(a+b*sech(x)^2)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)^{5}}{\left(b\,\mathrm{sech}(x)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+b*sech(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)^5/(b*sech(x)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tanh(x)^{5}}{\left(a+\frac{b}{\cosh(x)^{2}}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^5/(a + b/cosh(x)^2)^(3/2),x)
```

```
[Out] int(tanh(x)^5/(a + b/cosh(x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^5(x)}{\left(a+b\,\mathrm{sech}^2(x)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**5/(a+b*sech(x)**2)**(3/2),x)
```

```
[Out] Integral(tanh(x)**5/(a + b*sech(x)**2)**(3/2), x)
```

3.204 $\int \frac{\tanh^4(x)}{\left(a+b\,\mathbf{sech}^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=86

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{a^{3/2}}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{b^{3/2}}-\frac{(a+b)\tanh(x)}{ab\sqrt{a-b\tanh^2(x)+b}}$$

```
[Out] arctan(b^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))/b^(3/2)+arctanh(a^(1/2)*tan
h(x)/(a+b-b*tanh(x)^2)^(1/2))/a^(3/2)-(a+b)*tanh(x)/a/b/(a+b-b*tanh(x)^2)^(
1/2)
```

Rubi [A] time = 0.25, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {4141, 1975, 470, 523, 217, 203, 377, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{a^{3/2}}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{b^{3/2}}-\frac{(a+b)\tanh(x)}{ab\sqrt{a-b\tanh^2(x)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^4/(a + b*Sech[x]^2)^(3/2),x]
```

```
[Out] ArcTan[(Sqrt[b]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/b^(3/2) + ArcTanh[(Sqrt
[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/a^(3/2) - ((a + b)*Tanh[x])/(a*b*Sq
rt[a + b - b*Tanh[x]^2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\tanh^4(x)}{\left(a+b\operatorname{sech}^2(x)\right)^{3/2}}\,dx = \text{Subst}\left(\int \frac{x^4}{\left(1-x^2\right)\left(a+b\left(1-x^2\right)\right)^{3/2}}\,dx, x, \tanh(x)\right)$$

$$= \text{Subst}\left(\int \frac{x^4}{\left(1-x^2\right)\left(a+b-bx^2\right)^{3/2}}\,dx, x, \tanh(x)\right)$$

$$= -\frac{(a+b)\tanh(x)}{ab\sqrt{a+b-b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{a+b-ax^2}{(1-x^2)\sqrt{a+b-bx^2}}\,dx, x, \tanh(x)\right)}{ab}$$

$$= -\frac{(a+b)\tanh(x)}{ab\sqrt{a+b-b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+b-bx^2}}\,dx, x, \tanh(x)\right)}{a} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+b-}}\right.}{}$$

$$= -\frac{(a+b)\tanh(x)}{ab\sqrt{a+b-b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{a} + \frac{\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx\right.}{}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{b^{3/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{a^{3/2}} - \frac{(a+b)\tanh(x)}{ab\sqrt{a+b-b\tanh^2(x)}}$$

Mathematica [A] time = 0.41, size = 169, normalized size = 1.97

$$-\frac{\operatorname{sech}^3(x)(a\cosh(2x)+a+2b)\left(\sqrt{a}\left(2\sqrt{b}(a+b)\sinh(x)-\sqrt{2}a\sqrt{a\cosh(2x)+a+2b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}\sinh(x)}{\sqrt{a\cosh(2x)+a+2b}}\right)\right)\right.}{4a^{3/2}b^{3/2}\left(a+b\operatorname{sech}^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^4/(a + b*Sech[x]^2)^(3/2),x]
```

```
[Out] -1/4*((a + 2*b + a*Cosh[2*x])*Sech[x]^3*(-(Sqrt[2]*b^(3/2)*ArcTanh[(Sqrt[2]
*Sqrt[a]*Sinh[x])/Sqrt[a + 2*b + a*Cosh[2*x]]]*Sqrt[a + 2*b + a*Cosh[2*x]])
 + Sqrt[a]*(-(Sqrt[2]*a*ArcTan[(Sqrt[2]*Sqrt[b]*Sinh[x])/Sqrt[a + 2*b + a*C
osh[2*x]]]*Sqrt[a + 2*b + a*Cosh[2*x]]) + 2*Sqrt[b]*(a + b)*Sinh[x])))/(a^(
3/2)*b^(3/2)*(a + b*Sech[x]^2)^(3/2))
```

fricas [B] time = 0.70, size = 5170, normalized size = 60.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*sech(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a*b^2*cosh(x)^4 + 4*a*b^2*cosh(x)*sinh(x)^3 + a*b^2*sinh(x)^4 + a*b^
2 + 2*(a*b^2 + 2*b^3)*cosh(x)^2 + 2*(3*a*b^2*cosh(x)^2 + a*b^2 + 2*b^3)*sin
h(x)^2 + 4*(a*b^2*cosh(x)^3 + (a*b^2 + 2*b^3)*cosh(x))*sinh(x))*sqrt(a)*log
((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^8 - 2*(a*b^2
- b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh(x)^6 + 4*(14*a
*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3 + 4*a^2*b + 9*a*
b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 - 30*(a*b^2
- b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a*b^2 - b^3)*cosh
(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(a^3 + 3*a^2
*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*cosh(x)^4 + a^3 +
3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*c
osh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5
*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(
x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2
 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 + 4
*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/
(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*cosh(x)^7 - 3*(a*
b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3 + (a^3 + 3*a^2*b
)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)
^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5
+ sinh(x)^6)) - 2*(a^3*cosh(x)^4 + 4*a^3*cosh(x)*sinh(x)^3 + a^3*sinh(x)^4
+ a^3 + 2*(a^3 + 2*a^2*b)*cosh(x)^2 + 2*(3*a^3*cosh(x)^2 + a^3 + 2*a^2*b)*s
inh(x)^2 + 4*(a^3*cosh(x)^3 + (a^3 + 2*a^2*b)*cosh(x))*sinh(x))*sqrt(-b)*lo
g(-((a - b)*cosh(x)^4 + 4*(a - b)*cosh(x)*sinh(x)^3 + (a - b)*sinh(x)^4 + 2
*(a + 3*b)*cosh(x)^2 + 2*(3*(a - b)*cosh(x)^2 + a + 3*b)*sinh(x)^2 + 2*sqrt
(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-b)*sqrt((a*cosh(x
)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) +
 4*((a - b)*cosh(x)^3 + (a + 3*b)*cosh(x))*sinh(x) + a - b)/(cosh(x)^4 + 4*
cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2
 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)) + (a*b^2*cosh(x)^4 + 4*a*b^2*cosh(
x)*sinh(x)^3 + a*b^2*sinh(x)^4 + a*b^2 + 2*(a*b^2 + 2*b^3)*cosh(x)^2 + 2*(3
*a*b^2*cosh(x)^2 + a*b^2 + 2*b^3)*sinh(x)^2 + 4*(a*b^2*cosh(x)^3 + (a*b^2 +
 2*b^3)*cosh(x))*sinh(x))*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3
 + a*sinh(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + b)*sinh(x)^2
+ sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*sqrt((a*c
osh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^
2)) + 4*(a*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)
*sinh(x) + sinh(x)^2)) + 4*sqrt(2)*(a^2*b + a*b^2 - (a^2*b + a*b^2)*cosh(x)
^2 - 2*(a^2*b + a*b^2)*cosh(x)*sinh(x) - (a^2*b + a*b^2)*sinh(x)^2)*sqrt((a
*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x
)^2)))/(a^3*b^2*cosh(x)^4 + 4*a^3*b^2*cosh(x)*sinh(x)^3 + a^3*b^2*sinh(x)^4
```

```
 + a^3*b^2 + 2*(a^3*b^2 + 2*a^2*b^3)*cosh(x)^2 + 2*(3*a^3*b^2*cosh(x)^2 + a
^3*b^2 + 2*a^2*b^3)*sinh(x)^2 + 4*(a^3*b^2*cosh(x)^3 + (a^3*b^2 + 2*a^2*b^3
)*cosh(x))*sinh(x)), 1/4*(4*(a^3*cosh(x)^4 + 4*a^3*cosh(x)*sinh(x)^3 + a^3*
sinh(x)^4 + a^3 + 2*(a^3 + 2*a^2*b)*cosh(x)^2 + 2*(3*a^3*cosh(x)^2 + a^3 +
2*a^2*b)*sinh(x)^2 + 4*(a^3*cosh(x)^3 + (a^3 + 2*a^2*b)*cosh(x))*sinh(x))*s
qrt(b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(
b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x
) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a +
 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 +
(a + 2*b)*cosh(x))*sinh(x) + a)) + (a*b^2*cosh(x)^4 + 4*a*b^2*cosh(x)*sinh(
x)^3 + a*b^2*sinh(x)^4 + a*b^2 + 2*(a*b^2 + 2*b^3)*cosh(x)^2 + 2*(3*a*b^2*c
osh(x)^2 + a*b^2 + 2*b^3)*sinh(x)^2 + 4*(a*b^2*cosh(x)^3 + (a*b^2 + 2*b^3)*
cosh(x))*sinh(x))*sqrt(a)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7
+ a*b^2*sinh(x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b
^2 + b^3)*sinh(x)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh
(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4
*a^2*b + 9*a*b^2 - 30*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh
(x)^5 - 10*(a*b^2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sin
h(x)^3 + a^3 + 2*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*
b^2 - b^3)*cosh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^
2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(
x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cos
h(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(
x)^4 - 18*b^2*cosh(x)^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5
 - 6*b^2*cosh(x)^3 - (a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x
)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) +
 4*(2*a*b^2*cosh(x)^7 - 3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^
2)*cosh(x)^3 + (a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*s
inh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sin
h(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + (a*b^2*cosh(x)^4 + 4*a*b^2*cos
h(x)*sinh(x)^3 + a*b^2*sinh(x)^4 + a*b^2 + 2*(a*b^2 + 2*b^3)*cosh(x)^2 + 2*
(3*a*b^2*cosh(x)^2 + a*b^2 + 2*b^3)*sinh(x)^2 + 4*(a*b^2*cosh(x)^3 + (a*b^2
 + 2*b^3)*cosh(x))*sinh(x))*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)
^3 + a*sinh(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + b)*sinh(x)^
2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*sqrt((a
*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x
)^2)) + 4*(a*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(
x)*sinh(x) + sinh(x)^2)) + 4*sqrt(2)*(a^2*b + a*b^2 - (a^2*b + a*b^2)*cosh(
x)^2 - 2*(a^2*b + a*b^2)*cosh(x)*sinh(x) - (a^2*b + a*b^2)*sinh(x)^2)*sqrt(
(a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh
(x)^2)))/(a^3*b^2*cosh(x)^4 + 4*a^3*b^2*cosh(x)*sinh(x)^3 + a^3*b^2*sinh(x)
^4 + a^3*b^2 + 2*(a^3*b^2 + 2*a^2*b^3)*cosh(x)^2 + 2*(3*a^3*b^2*cosh(x)^2 +
 a^3*b^2 + 2*a^2*b^3)*sinh(x)^2 + 4*(a^3*b^2*cosh(x)^3 + (a^3*b^2 + 2*a^2*b
^3)*cosh(x))*sinh(x)), -1/2*((a*b^2*cosh(x)^4 + 4*a*b^2*cosh(x)*sinh(x)^3 +
 a*b^2*sinh(x)^4 + a*b^2 + 2*(a*b^2 + 2*b^3)*cosh(x)^2 + 2*(3*a*b^2*cosh(x)
```

```
^2 + a*b^2 + 2*b^3)*sinh(x)^2 + 4*(a*b^2*cosh(x)^3 + (a*b^2 + 2*b^3)*cosh(x
))*sinh(x))*sqrt(-a)*arctan(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*
sinh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)
^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*b*cosh(x)^4 + 4*a*b*cosh(x)*sinh(x)
^3 + a*b*sinh(x)^4 - (a^2 + 3*a*b)*cosh(x)^2 + (6*a*b*cosh(x)^2 - a^2 - 3*a
*b)*sinh(x)^2 - a^2 + 2*(2*a*b*cosh(x)^3 - (a^2 + 3*a*b)*cosh(x))*sinh(x)))
 + (a*b^2*cosh(x)^4 + 4*a*b^2*cosh(x)*sinh(x)^3 + a*b^2*sinh(x)^4 + a*b^2 +
 2*(a*b^2 + 2*b^3)*cosh(x)^2 + 2*(3*a*b^2*cosh(x)^2 + a*b^2 + 2*b^3)*sinh(x
)^2 + 4*(a*b^2*cosh(x)^3 + (a*b^2 + 2*b^3)*cosh(x))*sinh(x))*sqrt(-a)*arcta
n(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a)*sqrt((a*
cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)
^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(
x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*c
osh(x))*sinh(x) + a)) + (a^3*cosh(x)^4 + 4*a^3*cosh(x)*sinh(x)^3 + a^3*sinh
(x)^4 + a^3 + 2*(a^3 + 2*a^2*b)*cosh(x)^2 + 2*(3*a^3*cosh(x)^2 + a^3 + 2*a^
2*b)*sinh(x)^2 + 4*(a^3*cosh(x)^3 + (a^3 + 2*a^2*b)*cosh(x))*sinh(x))*sqrt(
-b)*log(-((a - b)*cosh(x)^4 + 4*(a - b)*cosh(x)*sinh(x)^3 + (a - b)*sinh(x)
^4 + 2*(a + 3*b)*cosh(x)^2 + 2*(3*(a - b)*cosh(x)^2 + a + 3*b)*sinh(x)^2 +
2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-b)*sqrt((a*
cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)
^2)) + 4*((a - b)*cosh(x)^3 + (a + 3*b)*cosh(x))*sinh(x) + a - b)/(cosh(x)^
4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cos
h(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)) - 2*sqrt(2)*(a^2*b + a*b^2 -
 (a^2*b + a*b^2)*cosh(x)^2 - 2*(a^2*b + a*b^2)*cosh(x)*sinh(x) - (a^2*b + a
*b^2)*sinh(x)^2)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*
cosh(x)*sinh(x) + sinh(x)^2)))/(a^3*b^2*cosh(x)^4 + 4*a^3*b^2*cosh(x)*sinh(
x)^3 + a^3*b^2*sinh(x)^4 + a^3*b^2 + 2*(a^3*b^2 + 2*a^2*b^3)*cosh(x)^2 + 2*
(3*a^3*b^2*cosh(x)^2 + a^3*b^2 + 2*a^2*b^3)*sinh(x)^2 + 4*(a^3*b^2*cosh(x)^
3 + (a^3*b^2 + 2*a^2*b^3)*cosh(x))*sinh(x)), -1/2*((a*b^2*cosh(x)^4 + 4*a*b
^2*cosh(x)*sinh(x)^3 + a*b^2*sinh(x)^4 + a*b^2 + 2*(a*b^2 + 2*b^3)*cosh(x)^
2 + 2*(3*a*b^2*cosh(x)^2 + a*b^2 + 2*b^3)*sinh(x)^2 + 4*(a*b^2*cosh(x)^3 +
(a*b^2 + 2*b^3)*cosh(x))*sinh(x))*sqrt(-a)*arctan(sqrt(2)*(b*cosh(x)^2 + 2*
b*cosh(x)*sinh(x) + b*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)
^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*b*cosh(x)^4 +
 4*a*b*cosh(x)*sinh(x)^3 + a*b*sinh(x)^4 - (a^2 + 3*a*b)*cosh(x)^2 + (6*a*b
*cosh(x)^2 - a^2 - 3*a*b)*sinh(x)^2 - a^2 + 2*(2*a*b*cosh(x)^3 - (a^2 + 3*a
*b)*cosh(x))*sinh(x))) + (a*b^2*cosh(x)^4 + 4*a*b^2*cosh(x)*sinh(x)^3 + a*b
^2*sinh(x)^4 + a*b^2 + 2*(a*b^2 + 2*b^3)*cosh(x)^2 + 2*(3*a*b^2*cosh(x)^2 +
 a*b^2 + 2*b^3)*sinh(x)^2 + 4*(a*b^2*cosh(x)^3 + (a*b^2 + 2*b^3)*cosh(x))*s
inh(x))*sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2
+ 1)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cos
h(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)
^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*c
osh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) - 2*(a^3*cosh(x)^4 + 4*a^3*cosh
(x)*sinh(x)^3 + a^3*sinh(x)^4 + a^3 + 2*(a^3 + 2*a^2*b)*cosh(x)^2 + 2*(3*a^
```

```
3*cosh(x)^2 + a^3 + 2*a^2*b)*sinh(x)^2 + 4*(a^3*cosh(x)^3 + (a^3 + 2*a^2*b)
*cosh(x))*sinh(x))*sqrt(b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) +
sinh(x)^2 - 1)*sqrt(b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^
2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 +
a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2
 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) - 2*sqrt(2)*(a^2*b + a
*b^2 - (a^2*b + a*b^2)*cosh(x)^2 - 2*(a^2*b + a*b^2)*cosh(x)*sinh(x) - (a^2
*b + a*b^2)*sinh(x)^2)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^
2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a^3*b^2*cosh(x)^4 + 4*a^3*b^2*cosh(x)
*sinh(x)^3 + a^3*b^2*sinh(x)^4 + a^3*b^2 + 2*(a^3*b^2 + 2*a^2*b^3)*cosh(x)^
2 + 2*(3*a^3*b^2*cosh(x)^2 + a^3*b^2 + 2*a^2*b^3)*sinh(x)^2 + 4*(a^3*b^2*co
sh(x)^3 + (a^3*b^2 + 2*a^2*b^3)*cosh(x))*sinh(x))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*sech(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{\tanh^4(x)}{\left(a + b\operatorname{sech}(x)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(a+b*sech(x)^2)^(3/2),x)
```

```
[Out] int(tanh(x)^4/(a+b*sech(x)^2)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)^4}{\left(b\operatorname{sech}(x)^2 + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*sech(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)^4/(b*sech(x)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tanh(x)^4}{\left(a+\frac{b}{\cosh(x)^2}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(a + b/cosh(x)^2)^(3/2),x)
[Out] int(tanh(x)^4/(a + b/cosh(x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^4(x)}{\left(a+b\operatorname{sech}^2(x)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**4/(a+b*sech(x)**2)**(3/2),x)
[Out] Integral(tanh(x)**4/(a + b*sech(x)**2)**(3/2), x)
```

3.205 $\int \frac{\tanh^3(x)}{\left(a+b\mathbf{sech}^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=49

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{a+b}{ab\sqrt{a+b\text{sech}^2(x)}}$$

```
[Out] arctanh((a+b*sech(x)^2)^(1/2)/a^(1/2))/a^(3/2)+(-a-b)/a/b/(a+b*sech(x)^2)^(
1/2)
```

Rubi [A] time = 0.11, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {4139, 446, 78, 63, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{a+b}{ab\sqrt{a+b\text{sech}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^3/(a + b*Sech[x]^2)^(3/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]]/a^(3/2) - (a + b)/(a*b*Sqrt[a + b*Se
ch[x]^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
```

```
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \frac{\tanh^3(x)}{\left(a+b\text{sech}^2(x)\right)^{3/2}}\,dx = \text{Subst}\left(\int \frac{-1+x^2}{x\left(a+bx^2\right)^{3/2}}\,dx, x, \text{sech}(x)\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{-1+x}{x(a+bx)^{3/2}}\,dx, x, \text{sech}^2(x)\right)$$

$$= -\frac{a+b}{ab\sqrt{a+b\text{sech}^2(x)}} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \text{sech}^2(x)\right)}{2a}$$

$$= -\frac{a+b}{ab\sqrt{a+b\text{sech}^2(x)}} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{sech}^2(x)}\right)}{ab}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{a+b}{ab\sqrt{a+b\text{sech}^2(x)}}$$

Mathematica [B] time = 0.23, size = 103, normalized size = 2.10

$$\frac{\text{sech}^3(x)\left(\sqrt{2}\,(a\cosh(2x)+a+2b)^{3/2}\log\left(\sqrt{a\cosh(2x)+a+2b}+\sqrt{2}\sqrt{a}\cosh(x)\right)-\frac{2\sqrt{a}\,(a+b)\cosh(x)(a\cosh(2x)+a+}{b}\right.}{4a^{3/2}\left(a+b\text{sech}^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^3/(a + b*Sech[x]^2)^(3/2),x]
```

```
[Out] (((-2*Sqrt[a]*(a + b)*Cosh[x]*(a + 2*b + a*Cosh[2*x]))/b + Sqrt[2]*(a + 2*b
 + a*Cosh[2*x])^(3/2)*Log[Sqrt[2]*Sqrt[a]*Cosh[x] + Sqrt[a + 2*b + a*Cosh[2
*x]]])*Sech[x]^3)/(4*a^(3/2)*(a + b*Sech[x]^2)^(3/2))
```

fricas [B] time = 0.54, size = 2194, normalized size = 44.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*sech(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a*b*cosh(x)^4 + 4*a*b*cosh(x)*sinh(x)^3 + a*b*sinh(x)^4 + 2*(a*b + 2
*b^2)*cosh(x)^2 + 2*(3*a*b*cosh(x)^2 + a*b + 2*b^2)*sinh(x)^2 + a*b + 4*(a*
```

```
b*cosh(x)^3 + (a*b + 2*b^2)*cosh(x))*sinh(x))*sqrt(a)*log(((a^3 + 2*a^2*b +
 a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^7 + (a^3 + 2*
a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^6 +
2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*
sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + 3*(2*a^3 + 5*a^2*b +
4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^4
+ (70*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 6*a^3 + 14*a^2*b + 9*a*b^2 + 30*(
2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + 2*a^2*
b + a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^3 + (6*
a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(2*a^3 + 3*a^2*b)*co
sh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 + 15*(2*a^3 + 5*a^2*b + 4
*a*b^2 + b^3)*cosh(x)^4 + 2*a^3 + 3*a^2*b + 3*(6*a^3 + 14*a^2*b + 9*a*b^2)*
cosh(x)^2)*sinh(x)^2 + sqrt(2)*((a^2 + 2*a*b + b^2)*cosh(x)^6 + 6*(a^2 + 2*
a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2 + 2*a*b + b^2)*sinh(x)^6 + 3*(a^2 + 2*a
*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 + 2*a*b + b^
2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 + 2*a*b + b^2)*c
osh(x))*sinh(x)^3 + (3*a^2 + 4*a*b)*cosh(x)^2 + (15*(a^2 + 2*a*b + b^2)*cos
h(x)^4 + 18*(a^2 + 2*a*b + b^2)*cosh(x)^2 + 3*a^2 + 4*a*b)*sinh(x)^2 + a^2
+ 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 6*(a^2 + 2*a*b + b^2)*cosh(x)^3 + (3
*a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a
 + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + 2*a^2*b
+ a*b^2)*cosh(x)^7 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^5 + (6*a^3
 + 14*a^2*b + 9*a*b^2)*cosh(x)^3 + (2*a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cos
h(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x
)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + (a*b*cos
h(x)^4 + 4*a*b*cosh(x)*sinh(x)^3 + a*b*sinh(x)^4 + 2*(a*b + 2*b^2)*cosh(x)^
2 + 2*(3*a*b*cosh(x)^2 + a*b + 2*b^2)*sinh(x)^2 + a*b + 4*(a*b*cosh(x)^3 +
(a*b + 2*b^2)*cosh(x))*sinh(x))*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sin
h(x)^3 + a*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*a*cosh(x)^2 + b)*sinh(x)^2 + sq
rt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a)*sqrt((a*cosh(
x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))
+ 4*(a*cosh(x)^3 + b*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) +
 sinh(x)^2)) - 4*sqrt(2)*((a^2 + a*b)*cosh(x)^2 + 2*(a^2 + a*b)*cosh(x)*sin
h(x) + (a^2 + a*b)*sinh(x)^2 + a^2 + a*b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 +
 a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a^3*b*cosh(x)^4 +
4*a^3*b*cosh(x)*sinh(x)^3 + a^3*b*sinh(x)^4 + a^3*b + 2*(a^3*b + 2*a^2*b^2)
*cosh(x)^2 + 2*(3*a^3*b*cosh(x)^2 + a^3*b + 2*a^2*b^2)*sinh(x)^2 + 4*(a^3*b
*cosh(x)^3 + (a^3*b + 2*a^2*b^2)*cosh(x))*sinh(x)), -1/2*((a*b*cosh(x)^4 +
4*a*b*cosh(x)*sinh(x)^3 + a*b*sinh(x)^4 + 2*(a*b + 2*b^2)*cosh(x)^2 + 2*(3*
a*b*cosh(x)^2 + a*b + 2*b^2)*sinh(x)^2 + a*b + 4*(a*b*cosh(x)^3 + (a*b + 2*
b^2)*cosh(x))*sinh(x))*sqrt(-a)*arctan(sqrt(2)*((a + b)*cosh(x)^2 + 2*(a +
b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*
sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a^2 + a
*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 + (
2*a^2 + 3*a*b)*cosh(x)^2 + (6*(a^2 + a*b)*cosh(x)^2 + 2*a^2 + 3*a*b)*sinh(x
```

```
)^2 + a^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 + (2*a^2 + 3*a*b)*cosh(x))*sinh(x)))
 + (a*b*cosh(x)^4 + 4*a*b*cosh(x)*sinh(x)^3 + a*b*sinh(x)^4 + 2*(a*b + 2*b^
2)*cosh(x)^2 + 2*(3*a*b*cosh(x)^2 + a*b + 2*b^2)*sinh(x)^2 + a*b + 4*(a*b*c
osh(x)^3 + (a*b + 2*b^2)*cosh(x))*sinh(x))*sqrt(-a)*arctan(sqrt(2)*(cosh(x)
^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh
(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4
+ 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh
(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) +
a)) + 2*sqrt(2)*((a^2 + a*b)*cosh(x)^2 + 2*(a^2 + a*b)*cosh(x)*sinh(x) + (a
^2 + a*b)*sinh(x)^2 + a^2 + a*b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)
/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a^3*b*cosh(x)^4 + 4*a^3*b*c
osh(x)*sinh(x)^3 + a^3*b*sinh(x)^4 + a^3*b + 2*(a^3*b + 2*a^2*b^2)*cosh(x)^
2 + 2*(3*a^3*b*cosh(x)^2 + a^3*b + 2*a^2*b^2)*sinh(x)^2 + 4*(a^3*b*cosh(x)^
3 + (a^3*b + 2*a^2*b^2)*cosh(x))*sinh(x))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*sech(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, replacing 0 by ` u`, a substitution
 variable should perhaps be purged.Warning, replacing 0 by ` u`, a substitu
tion variable should perhaps be purged.Warning, replacing 0 by ` u`, a subs
titution variable should perhaps be purged.Error: Bad Argument Type
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{\tanh^3(x)}{\left(a+b\,\mathrm{sech}(x)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(a+b*sech(x)^2)^(3/2),x)
```

```
[Out] int(tanh(x)^3/(a+b*sech(x)^2)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)^3}{\left(b\,\mathrm{sech}(x)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*sech(x)^2)^(3/2),x, algorithm="maxima")
[Out] integrate(tanh(x)^3/(b*sech(x)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\tanh(x)^3}{\left(a + \frac{b}{\cosh(x)^2}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(a + b/cosh(x)^2)^(3/2),x)
[Out] int(tanh(x)^3/(a + b/cosh(x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^3(x)}{\left(a + b\,\mathrm{sech}^2(x)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**3/(a+b*sech(x)**2)**(3/2),x)
[Out] Integral(tanh(x)**3/(a + b*sech(x)**2)**(3/2), x)
```

3.206 $\int \frac{\tanh^2(x)}{\left(a+b\,\mathbf{sech}^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=51

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{a^{3/2}}-\frac{\tanh(x)}{a\sqrt{a-b\tanh^2(x)+b}}$$

```
[Out] arctanh(a^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))/a^(3/2)-tanh(x)/a/(a+b-b*t
anh(x)^2)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {4141, 1975, 471, 377, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{a^{3/2}}-\frac{\tanh(x)}{a\sqrt{a-b\tanh^2(x)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^2/(a + b*Sech[x]^2)^(3/2),x]
```

```
[Out] ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/a^(3/2) - Tanh[x]/(a*S
qrt[a + b - b*Tanh[x]^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
```

```
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\tanh^2(x)}{\left(a+b\text{sech}^2(x)\right)^{3/2}}\,dx = \text{Subst}\left(\int \frac{x^2}{\left(1-x^2\right)\left(a+b\left(1-x^2\right)\right)^{3/2}}\,dx, x, \tanh(x)\right)$$

$$= \text{Subst}\left(\int \frac{x^2}{\left(1-x^2\right)\left(a+b-bx^2\right)^{3/2}}\,dx, x, \tanh(x)\right)$$

$$= -\frac{\tanh(x)}{a\sqrt{a+b-b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)\sqrt{a+b-bx^2}}\,dx, x, \tanh(x)\right)}{a}$$

$$= -\frac{\tanh(x)}{a\sqrt{a+b-b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{a}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{a^{3/2}} - \frac{\tanh(x)}{a\sqrt{a+b-b\tanh^2(x)}}$$

Mathematica [B] time = 0.76, size = 128, normalized size = 2.51

$$\frac{\text{sech}^2(x)(a\cosh(2x)+a+2b)\left(\text{sech}(x)\sinh^{-1}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{a+b}}\right)(a\cosh(2x)+a+2b)-2\sqrt{a}\sqrt{a+b}\tanh(x)\sqrt{\frac{a\sinh^2(x}{a+}}\right.}{4a^{3/2}\sqrt{a+b}\sqrt{\frac{a\sinh^2(x)+a+b}{a+b}}\left(a+b\text{sech}^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^2/(a + b*Sech[x]^2)^(3/2),x]
```

```
[Out] ((a + 2*b + a*Cosh[2*x])*Sech[x]^2*(ArcSinh[(Sqrt[a]*Sinh[x])/Sqrt[a + b]]*
(a + 2*b + a*Cosh[2*x])*Sech[x] - 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(a + b + a*Sin
h[x]^2)/(a + b)]*Tanh[x]))/(4*a^(3/2)*Sqrt[a + b]*(a + b*Sech[x]^2)^(3/2)*S
qrt[(a + b + a*Sinh[x]^2)/(a + b)])
```

fricas [B] time = 0.54, size = 1733, normalized size = 33.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*sech(x)^2)^(3/2),x, algorithm="fricas")

[Out] [1/4*((a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh
(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*
cosh(x))*sinh(x) + a)*sqrt(a)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x
)^7 + a*b^2*sinh(x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 -
 a*b^2 + b^3)*sinh(x)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*
sinh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3
 + 4*a^2*b + 9*a*b^2 - 30*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*
cosh(x)^5 - 10*(a*b^2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))
*sinh(x)^3 + a^3 + 2*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15
*(a*b^2 - b^3)*cosh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh
(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*s
inh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2
*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*c
osh(x)^4 - 18*b^2*cosh(x)^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(
x)^5 - 6*b^2*cosh(x)^3 - (a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*co
sh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2
)) + 4*(2*a*b^2*cosh(x)^7 - 3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*
a*b^2)*cosh(x)^3 + (a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)
^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2
*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + (a*cosh(x)^4 + 4*a*cosh(x)
*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2
*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)*sqrt(a)*lo
g(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + b)*cosh(x)^2
 + 2*(3*a*cosh(x)^2 + a + b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sin
h(x) + sinh(x)^2 + 1)*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(c
osh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(a*cosh(x)^3 + (a + b)*cosh(
x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) - 4*sqrt(2)*(
a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 - a)*sqrt((a*cosh(x)^2 + a*
sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a^3*cos
h(x)^4 + 4*a^3*cosh(x)*sinh(x)^3 + a^3*sinh(x)^4 + a^3 + 2*(a^3 + 2*a^2*b)*
cosh(x)^2 + 2*(3*a^3*cosh(x)^2 + a^3 + 2*a^2*b)*sinh(x)^2 + 4*(a^3*cosh(x)^
3 + (a^3 + 2*a^2*b)*cosh(x))*sinh(x)), -1/2*((a*cosh(x)^4 + 4*a*cosh(x)*sin
h(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*
sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)*sqrt(-a)*arcta
n(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + a)*sqrt(-a)*sq
rt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + s
inh(x)^2))/(a*b*cosh(x)^4 + 4*a*b*cosh(x)*sinh(x)^3 + a*b*sinh(x)^4 - (a^2
+ 3*a*b)*cosh(x)^2 + (6*a*b*cosh(x)^2 - a^2 - 3*a*b)*sinh(x)^2 - a^2 + 2*(2
*a*b*cosh(x)^3 - (a^2 + 3*a*b)*cosh(x))*sinh(x))) + (a*cosh(x)^4 + 4*a*cosh
(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a
+ 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)*sqrt(-a
)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a)*s
qrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) +
```

```
sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b
)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a +
 2*b)*cosh(x))*sinh(x) + a)) + 2*sqrt(2)*(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x)
 + a*sinh(x)^2 - a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 -
 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a^3*cosh(x)^4 + 4*a^3*cosh(x)*sinh(x)^3
+ a^3*sinh(x)^4 + a^3 + 2*(a^3 + 2*a^2*b)*cosh(x)^2 + 2*(3*a^3*cosh(x)^2 +
a^3 + 2*a^2*b)*sinh(x)^2 + 4*(a^3*cosh(x)^3 + (a^3 + 2*a^2*b)*cosh(x))*sinh
(x))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*sech(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, replacing 0 by ` u`, a substitution
 variable should perhaps be purged.Warning, replacing 0 by ` u`, a substitu
tion variable should perhaps be purged.Warning, replacing 0 by ` u`, a subs
titution variable should perhaps be purged.Error: Bad Argument Type
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{\tanh^2(x)}{\left(a + b\operatorname{sech}(x)^2\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(a+b*sech(x)^2)^(3/2),x)
```

```
[Out] int(tanh(x)^2/(a+b*sech(x)^2)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)^2}{\left(b\operatorname{sech}(x)^2 + a\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*sech(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)^2/(b*sech(x)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\tanh(x)^2}{\left(a+\frac{b}{\cosh(x)^2}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(a + b/cosh(x)^2)^(3/2),x)
```

```
[Out] int(tanh(x)^2/(a + b/cosh(x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^2(x)}{\left(a+b\operatorname{sech}^2(x)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**2/(a+b*sech(x)**2)**(3/2),x)
```

```
[Out] Integral(tanh(x)**2/(a + b*sech(x)**2)**(3/2), x)
```

3.207 $\int \frac{\tanh(x)}{\left(a+b\mathbf{sech}^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=43

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{a\sqrt{a+b\operatorname{sech}^2(x)}}$$

```
[Out] arctanh((a+b*sech(x)^2)^(1/2)/a^(1/2))/a^(3/2)-1/a/(a+b*sech(x)^2)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4139, 266, 51, 63, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{a\sqrt{a+b\operatorname{sech}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]/(a + b*Sech[x]^2)^(3/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]]/a^(3/2) - 1/(a*Sqrt[a + b*Sech[x]^2]
)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh(x)}{\left(a+b\operatorname{sech}^2(x)\right)^{3/2}}\,dx &= -\operatorname{Subst}\left(\int \frac{1}{x\left(a+bx^2\right)^{3/2}}\,dx, x, \operatorname{sech}(x)\right)\\
&= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, \operatorname{sech}^2(x)\right)\right)\\
&= -\frac{1}{a\sqrt{a+b\operatorname{sech}^2(x)}} - \frac{\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \operatorname{sech}^2(x)\right)}{2a}\\
&= -\frac{1}{a\sqrt{a+b\operatorname{sech}^2(x)}} - \frac{\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{sech}^2(x)}\right)}{ab}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{1}{a\sqrt{a+b\operatorname{sech}^2(x)}}
\end{aligned}
$$

Mathematica [B] time = 0.40, size = 98, normalized size = 2.28

$$-\frac{\operatorname{sech}^3(x)(a\cosh(2x)+a+2b)\left(2\sqrt{a}\cosh(x)-\sqrt{2}\sqrt{a\cosh(2x)+a+2b}\log\left(\sqrt{a\cosh(2x)+a+2b}+\sqrt{2}\sqrt{a}\right.\right.}{4a^{3/2}\left(a+b\operatorname{sech}^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/(a + b*Sech[x]^2)^(3/2),x]
```

```
[Out] -1/4*((a + 2*b + a*Cosh[2*x])*(2*Sqrt[a]*Cosh[x] - Sqrt[2]*Sqrt[a + 2*b + a
*Cosh[2*x]]*Log[Sqrt[2]*Sqrt[a]*Cosh[x] + Sqrt[a + 2*b + a*Cosh[2*x]]])*Sec
h[x]^3)/(a^(3/2)*(a + b*Sech[x]^2)^(3/2))
```

fricas [B] time = 0.52, size = 2034, normalized size = 47.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sech(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh
(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*
cosh(x))*sinh(x) + a)*sqrt(a)*log(((a^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a
^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8
 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a
*b^2 + b^3 + 14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 +
 2*a^2*b + a*b^2)*cosh(x)^3 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x))*
sinh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*(a^3 + 2*a^2*b + a
*b^2)*cosh(x)^4 + 6*a^3 + 14*a^2*b + 9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^
2 + b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 1
0*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b^2
)*cosh(x))*sinh(x)^3 + a^3 + 2*(2*a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2
*a^2*b + a*b^2)*cosh(x)^6 + 15*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^4
+ 2*a^3 + 3*a^2*b + 3*(6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + s
qrt(2)*((a^2 + 2*a*b + b^2)*cosh(x)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(
x)^5 + (a^2 + 2*a*b + b^2)*sinh(x)^6 + 3*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 3*
(5*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 + 2*a*b + b^2)*sinh(x)^4 + 4*(5*(a^2
 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(x))*sinh(x)^3 + (3*a
^2 + 4*a*b)*cosh(x)^2 + (15*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 18*(a^2 + 2*a*b
 + b^2)*cosh(x)^2 + 3*a^2 + 4*a*b)*sinh(x)^2 + a^2 + 2*(3*(a^2 + 2*a*b + b^
2)*cosh(x)^5 + 6*(a^2 + 2*a*b + b^2)*cosh(x)^3 + (3*a^2 + 4*a*b)*cosh(x))*s
inh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*c
osh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^7 + 3*(
2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*c
```

```
osh(x)^3 + (2*a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sin
h(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(
x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + (a*cosh(x)^4 + 4*a*cosh(x)*sinh(
x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*si
nh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)*sqrt(a)*log(-(a*
cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*a*co
sh(x)^2 + b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2
 - 1)*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cos
h(x)*sinh(x) + sinh(x)^2)) + 4*(a*cosh(x)^3 + b*cosh(x))*sinh(x) + a)/(cosh
(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) - 4*sqrt(2)*(a*cosh(x)^2 + 2*a*cosh
(x)*sinh(x) + a*sinh(x)^2 + a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(
cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a^3*cosh(x)^4 + 4*a^3*cosh(x)
*sinh(x)^3 + a^3*sinh(x)^4 + a^3 + 2*(a^3 + 2*a^2*b)*cosh(x)^2 + 2*(3*a^3*c
osh(x)^2 + a^3 + 2*a^2*b)*sinh(x)^2 + 4*(a^3*cosh(x)^3 + (a^3 + 2*a^2*b)*co
sh(x))*sinh(x)), -1/2*((a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 +
 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(
x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)*sqrt(-a)*arctan(sqrt(2)*((a + b)*cos
h(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a)*sqrt(-a)*sqrt((
a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(
x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*
b)*sinh(x)^4 + (2*a^2 + 3*a*b)*cosh(x)^2 + (6*(a^2 + a*b)*cosh(x)^2 + 2*a^2
 + 3*a*b)*sinh(x)^2 + a^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 + (2*a^2 + 3*a*b)*co
sh(x))*sinh(x))) + (a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(
a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3
 + (a + 2*b)*cosh(x))*sinh(x) + a)*sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*c
osh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 +
a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*co
sh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 +
a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) + 2*
sqrt(2)*(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a)*sqrt((a*cosh(
x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))
/(a^3*cosh(x)^4 + 4*a^3*cosh(x)*sinh(x)^3 + a^3*sinh(x)^4 + a^3 + 2*(a^3 +
2*a^2*b)*cosh(x)^2 + 2*(3*a^3*cosh(x)^2 + a^3 + 2*a^2*b)*sinh(x)^2 + 4*(a^3
*cosh(x)^3 + (a^3 + 2*a^2*b)*cosh(x))*sinh(x))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sech(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, replacing 0 by ` u`, a substitution
 variable should perhaps be purged.Warning, replacing 0 by ` u`, a substitu
```

```
tion variable should perhaps be purged.Warning, replacing 0 by ` u`, a subs
titution variable should perhaps be purged.Error: Bad Argument Type
```

maple [A] time = 0.10, size = 46, normalized size = 1.07

$$-\frac{1}{a\sqrt{a+b\operatorname{sech}(x)^2}}+\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\operatorname{sech}(x)^2}}{\operatorname{sech}(x)}\right)}{a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a+b*sech(x)^2)^(3/2),x)
```

```
[Out] -1/a/(a+b*sech(x)^2)^(1/2)+1/a^(3/2)*ln((2*a+2*a^(1/2)*(a+b*sech(x)^2)^(1/2
))/sech(x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)}{\left(b\operatorname{sech}(x)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sech(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)/(b*sech(x)^2 + a)^(3/2), x)
```

mupad [B] time = 1.74, size = 35, normalized size = 0.81

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{\cosh(x)^2}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{a\sqrt{a+\frac{b}{\cosh(x)^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a + b/cosh(x)^2)^(3/2),x)
```

```
[Out] atanh((a + b/cosh(x)^2)^(1/2)/a^(1/2))/a^(3/2) - 1/(a*(a + b/cosh(x)^2)^(1/
2))
```

sympy [A] time = 5.93, size = 44, normalized size = 1.02

$$-\frac{1}{a\sqrt{a+b\operatorname{sech}^2(x)}}-\frac{\operatorname{atan}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{-a}}\right)}{a\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sech(x)**2)**(3/2),x)
```

```
[Out] -1/(a*sqrt(a + b*sech(x)**2)) - atan(sqrt(a + b*sech(x)**2)/sqrt(-a))/(a*sq
rt(-a))
```

3.208 $\int \frac{1}{\left(a+b\,\mathbf{sech}^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=57

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\,\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{a^{3/2}}-\frac{b\tanh(x)}{a(a+b)\sqrt{a-b\tanh^2(x)+b}}$$

```
[Out] arctanh(a^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))/a^(3/2)-b*tanh(x)/a/(a+b)/
(a+b-b*tanh(x)^2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4128, 382, 377, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\,\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{a^{3/2}}-\frac{b\tanh(x)}{a(a+b)\sqrt{a-b\tanh^2(x)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[x]^2)^(-3/2),x]
```

```
[Out] ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/a^(3/2) - (b*Tanh[x])/
(a*(a + b)*Sqrt[a + b - b*Tanh[x]^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
```

```
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+b\operatorname{sech}^2(x)\right)^{3/2}}\,dx &= \text{Subst}\left(\int \frac{1}{\left(1-x^2\right)\left(a+b-bx^2\right)^{3/2}}\,dx, x, \tanh(x)\right)\\ &= -\frac{b\tanh(x)}{a(a+b)\sqrt{a+b-b\tanh^2(x)}}+\frac{\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)\sqrt{a+b-bx^2}}\,dx, x, \tanh(x)\right)}{a}\\ &= -\frac{b\tanh(x)}{a(a+b)\sqrt{a+b-b\tanh^2(x)}}+\frac{\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{a}\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{a^{3/2}}-\frac{b\tanh(x)}{a(a+b)\sqrt{a+b-b\tanh^2(x)}}\end{aligned}$$

Mathematica [A] time = 0.84, size = 107, normalized size = 1.88

$$\frac{\operatorname{sech}^3(x)(a\cosh(2x)+a+2b)\left((a+b)^{3/2}\sinh^{-1}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{a+b}}\right)\sqrt{\frac{a\cosh(2x)+a+2b}{a+b}}-\sqrt{2}\sqrt{a}\,b\sinh(x)\right)}{2\sqrt{2}\,a^{3/2}(a+b)\left(a+b\operatorname{sech}^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[x]^2)^(-3/2),x]
```

```
[Out] ((a + 2*b + a*Cosh[2*x])*Sech[x]^3*((a + b)^(3/2)*ArcSinh[(Sqrt[a]*Sinh[x])
/Sqrt[a + b]]*Sqrt[(a + 2*b + a*Cosh[2*x])/(a + b)] - Sqrt[2]*Sqrt[a]*b*Sin
h[x]))/(2*Sqrt[2]*a^(3/2)*(a + b)*(a + b*Sech[x]^2)^(3/2))
```

fricas [B] time = 0.55, size = 2095, normalized size = 36.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b
)*sinh(x)^4 + 2*(a^2 + 3*a*b + 2*b^2)*cosh(x)^2 + 2*(3*(a^2 + a*b)*cosh(x)^
2 + a^2 + 3*a*b + 2*b^2)*sinh(x)^2 + a^2 + a*b + 4*((a^2 + a*b)*cosh(x)^3 +
 (a^2 + 3*a*b + 2*b^2)*cosh(x))*sinh(x))*sqrt(a)*log((a*b^2*cosh(x)^8 + 8*a
*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(1
4*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh(x)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b
^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*
b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 - 30*(a*b^2 - b^3)*cosh(x)^2)*sinh(
x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a*b^2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2*b
+ 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a
*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*cosh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^
2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x
)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*s
inh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 + 4*a*b)*co
sh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 - 4*a*b)*sinh(x)^2 - a
^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 + 4*a*b)*cosh(x))*sinh(x))
*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*
sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*cosh(x)^7 - 3*(a*b^2 - b^3)*cosh(x)^5 +
(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3 + (a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(co
sh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(
x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + ((a^2 +
 a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 +
 2*(a^2 + 3*a*b + 2*b^2)*cosh(x)^2 + 2*(3*(a^2 + a*b)*cosh(x)^2 + a^2 + 3*a
*b + 2*b^2)*sinh(x)^2 + a^2 + a*b + 4*((a^2 + a*b)*cosh(x)^3 + (a^2 + 3*a*b
 + 2*b^2)*cosh(x))*sinh(x))*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)
^3 + a*sinh(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + b)*sinh(x)^
2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*sqrt((a
*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x
)^2)) + 4*(a*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(
x)*sinh(x) + sinh(x)^2)) - 4*sqrt(2)*(a*b*cosh(x)^2 + 2*a*b*cosh(x)*sinh(x)
 + a*b*sinh(x)^2 - a*b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)
^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^4 + a^3*b)*cosh(x)^4 + 4*(a^4 + a
^3*b)*cosh(x)*sinh(x)^3 + (a^4 + a^3*b)*sinh(x)^4 + a^4 + a^3*b + 2*(a^4 +
3*a^3*b + 2*a^2*b^2)*cosh(x)^2 + 2*(a^4 + 3*a^3*b + 2*a^2*b^2 + 3*(a^4 + a^
```

```
3*b)*cosh(x)^2)*sinh(x)^2 + 4*((a^4 + a^3*b)*cosh(x)^3 + (a^4 + 3*a^3*b + 2
*a^2*b^2)*cosh(x))*sinh(x)), -1/2*(((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*c
osh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 + 2*(a^2 + 3*a*b + 2*b^2)*cosh(x)^
2 + 2*(3*(a^2 + a*b)*cosh(x)^2 + a^2 + 3*a*b + 2*b^2)*sinh(x)^2 + a^2 + a*b
 + 4*((a^2 + a*b)*cosh(x)^3 + (a^2 + 3*a*b + 2*b^2)*cosh(x))*sinh(x))*sqrt(
-a)*arctan(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + a)*sq
rt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*si
nh(x) + sinh(x)^2))/(a*b*cosh(x)^4 + 4*a*b*cosh(x)*sinh(x)^3 + a*b*sinh(x)^
4 - (a^2 + 3*a*b)*cosh(x)^2 + (6*a*b*cosh(x)^2 - a^2 - 3*a*b)*sinh(x)^2 - a
^2 + 2*(2*a*b*cosh(x)^3 - (a^2 + 3*a*b)*cosh(x))*sinh(x))) + ((a^2 + a*b)*c
osh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 + 2*(a^2
 + 3*a*b + 2*b^2)*cosh(x)^2 + 2*(3*(a^2 + a*b)*cosh(x)^2 + a^2 + 3*a*b + 2*
b^2)*sinh(x)^2 + a^2 + a*b + 4*((a^2 + a*b)*cosh(x)^3 + (a^2 + 3*a*b + 2*b^
2)*cosh(x))*sinh(x))*sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x)
 + sinh(x)^2 + 1)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh
(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^
3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(
x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) + 2*sqrt(2)*(a*b*c
osh(x)^2 + 2*a*b*cosh(x)*sinh(x) + a*b*sinh(x)^2 - a*b)*sqrt((a*cosh(x)^2 +
 a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^4
 + a^3*b)*cosh(x)^4 + 4*(a^4 + a^3*b)*cosh(x)*sinh(x)^3 + (a^4 + a^3*b)*sin
h(x)^4 + a^4 + a^3*b + 2*(a^4 + 3*a^3*b + 2*a^2*b^2)*cosh(x)^2 + 2*(a^4 + 3
*a^3*b + 2*a^2*b^2 + 3*(a^4 + a^3*b)*cosh(x)^2)*sinh(x)^2 + 4*((a^4 + a^3*b
)*cosh(x)^3 + (a^4 + 3*a^3*b + 2*a^2*b^2)*cosh(x))*sinh(x))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, replacing 0 by ` u`, a substitution
 variable should perhaps be purged.Warning, replacing 0 by ` u`, a substitu
tion variable should perhaps be purged.Warning, replacing 0 by ` u`, a subs
titution variable should perhaps be purged.Error: Bad Argument Type
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b\,\mathrm{sech}(x)^2\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sech(x)^2)^(3/2),x)
```

```
[Out] int(1/(a+b*sech(x)^2)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,\mathrm{sech}(x)^2 + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sech(x)^2 + a)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(a + \frac{b}{\cosh(x)^2}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cosh(x)^2)^(3/2),x)
```

```
[Out] int(1/(a + b/cosh(x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b\,\mathrm{sech}^2\,(x)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(x)**2)**(3/2),x)
```

```
[Out] Integral((a + b*sech(x)**2)**(-3/2), x)
```

3.209 $\int \frac{\coth(x)}{\left(a+b\,\mathrm{sech}^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=79

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}^2(x)}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{b}{a(a+b)\sqrt{a+b\,\mathrm{sech}^2(x)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}$$

```
[Out] arctanh((a+b*sech(x)^2)^(1/2)/a^(1/2))/a^(3/2)-arctanh((a+b*sech(x)^2)^(1/2
)/(a+b)^(1/2))/(a+b)^(3/2)-b/a/(a+b)/(a+b*sech(x)^2)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4139, 446, 85, 156, 63, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}^2(x)}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{b}{a(a+b)\sqrt{a+b\,\mathrm{sech}^2(x)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]/(a + b*Sech[x]^2)^(3/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]]/a^(3/2) - ArcTanh[Sqrt[a + b*Sech[x]
^2]/Sqrt[a + b]]/(a + b)^(3/2) - b/(a*(a + b)*Sqrt[a + b*Sech[x]^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p + 1))/((p + 1)*(b*e - a*f)*(d*e - c*f)),
x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[((b*d*e - b*c*f - a*d*f - b*d*f*
x)*(e + f*x)^(p + 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && LtQ[p, -1]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \frac{\coth(x)}{\left(a+b\text{sech}^2(x)\right)^{3/2}}\,dx = \text{Subst}\left(\int \frac{1}{x\left(-1+x^2\right)\left(a+bx^2\right)^{3/2}}\,dx, x, \text{sech}(x)\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(-1+x)x(a+bx)^{3/2}}\,dx, x, \text{sech}^2(x)\right)$$
$$= -\frac{b}{a(a+b)\sqrt{a+b\text{sech}^2(x)}} + \frac{\text{Subst}\left(\int \frac{a+b-bx}{(-1+x)x\sqrt{a+bx}}\,dx, x, \text{sech}^2(x)\right)}{2a(a+b)}$$
$$= -\frac{b}{a(a+b)\sqrt{a+b\text{sech}^2(x)}} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \text{sech}^2(x)\right)}{2a} + \frac{\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a+}}\right.}{2(a}$$
$$= -\frac{b}{a(a+b)\sqrt{a+b\text{sech}^2(x)}} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{sech}^2(x)}\right)}{ab} + \frac{\text{Subst}\left(\int -\right.}{-1}$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}} - \frac{b}{a(a+b)\sqrt{a+b\text{sech}^2(x)}}$$

Mathematica [A] time = 0.69, size = 155, normalized size = 1.96

$$\frac{\text{sech}^2(x)\left(\frac{\sqrt{2}\,\text{sech}(x)(a\cosh(2x)+a+2b)^{3/2}\left((a+b)^{3/2}\log\left(\sqrt{a\cosh(2x)+a+2b}+\sqrt{2}\sqrt{a}\cosh(x)\right)-a^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a+b}\cosh(x)}{\sqrt{a\cosh(2x)+a+2b}}\right)\right)}{\sqrt{a}\sqrt{a+b}} - 2b(a\cosh\right.}{4a(a+b)\left(a+b\text{sech}^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/(a + b*Sech[x]^2)^(3/2),x]
```

```
[Out] (Sech[x]^2*(-2*b*(a + 2*b + a*Cosh[2*x]) + (Sqrt[2]*(a + 2*b + a*Cosh[2*x])
^(3/2)*(-(a^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[a + b]*Cosh[x])/Sqrt[a + 2*b + a*Co
sh[2*x]]]) + (a + b)^(3/2)*Log[Sqrt[2]*Sqrt[a]*Cosh[x] + Sqrt[a + 2*b + a*C
osh[2*x]]])*Sech[x])/(Sqrt[a]*Sqrt[a + b])))/(4*a*(a + b)*(a + b*Sech[x]^2)
^(3/2))
```

fricas [B] time = 0.74, size = 6939, normalized size = 87.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sech(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 4*(a^3 + 2*a^2*b + a*b^2)*cosh(x
)*sinh(x)^3 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^4 + a^3 + 2*a^2*b + a*b^2 + 2
*(a^3 + 4*a^2*b + 5*a*b^2 + 2*b^3)*cosh(x)^2 + 2*(a^3 + 4*a^2*b + 5*a*b^2 +
 2*b^3 + 3*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 + 2*a^2*b
 + a*b^2)*cosh(x)^3 + (a^3 + 4*a^2*b + 5*a*b^2 + 2*b^3)*cosh(x))*sinh(x))*s
qrt(a)*log(((a^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*c
osh(x)*sinh(x)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*a^2*b +
 4*a*b^2 + b^3)*cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*(a^3 +
2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(
x)^3 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + (6*a^3 + 14
*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 6*a^3
 + 14*a^2*b + 9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^2)*sin
h(x)^4 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a^2*b + 4*
a*b^2 + b^3)*cosh(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 +
a^3 + 2*(2*a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)
^6 + 15*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3 + 3*a^2*b + 3*(
6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*((a^2 + 2*a*b +
b^2)*cosh(x)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2 + 2*a*b + b
^2)*sinh(x)^6 + 3*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)*
cosh(x)^2 + a^2 + 2*a*b + b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(x)
^3 + 3*(a^2 + 2*a*b + b^2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4*a*b)*cosh(x)^2 +
 (15*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*cosh(x)^2 + 3*a
^2 + 4*a*b)*sinh(x)^2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 6*(a^2 +
 2*a*b + b^2)*cosh(x)^3 + (3*a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a
*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x
)^2)) + 4*(2*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^7 + 3*(2*a^3 + 5*a^2*b + 4*a*b
^2 + b^3)*cosh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^3 + (2*a^3 + 3*a
^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sin
h(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x
)^5 + sinh(x)^6)) + 2*(a^3*cosh(x)^4 + 4*a^3*cosh(x)*sinh(x)^3 + a^3*sinh(x
)^4 + a^3 + 2*(a^3 + 2*a^2*b)*cosh(x)^2 + 2*(3*a^3*cosh(x)^2 + a^3 + 2*a^2*
b)*sinh(x)^2 + 4*(a^3*cosh(x)^3 + (a^3 + 2*a^2*b)*cosh(x))*sinh(x))*sqrt(a
+ b)*log(((2*a + b)*cosh(x)^4 + 4*(2*a + b)*cosh(x)*sinh(x)^3 + (2*a + b)*s
inh(x)^4 + 2*(2*a + 3*b)*cosh(x)^2 + 2*(3*(2*a + b)*cosh(x)^2 + 2*a + 3*b)*
sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(
a + b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*si
nh(x) + sinh(x)^2)) + 4*((2*a + b)*cosh(x)^3 + (2*a + 3*b)*cosh(x))*sinh(x)
 + 2*a + b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 -
 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)) + ((a^3
 + 2*a^2*b + a*b^2)*cosh(x)^4 + 4*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^3
 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^4 + a^3 + 2*a^2*b + a*b^2 + 2*(a^3 + 4*a
```

```
^2*b + 5*a*b^2 + 2*b^3)*cosh(x)^2 + 2*(a^3 + 4*a^2*b + 5*a*b^2 + 2*b^3 + 3*
(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 + 2*a^2*b + a*b^2)*c
osh(x)^3 + (a^3 + 4*a^2*b + 5*a*b^2 + 2*b^3)*cosh(x))*sinh(x))*sqrt(a)*log(
-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*
a*cosh(x)^2 + b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(
x)^2 - 1)*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2
*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(a*cosh(x)^3 + b*cosh(x))*sinh(x) + a)/(
cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) - 4*sqrt(2)*(a^2*b + a*b^2 + (a
^2*b + a*b^2)*cosh(x)^2 + 2*(a^2*b + a*b^2)*cosh(x)*sinh(x) + (a^2*b + a*b^
2)*sinh(x)^2)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cos
h(x)*sinh(x) + sinh(x)^2)))/(a^5 + 2*a^4*b + a^3*b^2 + (a^5 + 2*a^4*b + a^3
*b^2)*cosh(x)^4 + 4*(a^5 + 2*a^4*b + a^3*b^2)*cosh(x)*sinh(x)^3 + (a^5 + 2*
a^4*b + a^3*b^2)*sinh(x)^4 + 2*(a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3)*cosh
(x)^2 + 2*(a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3 + 3*(a^5 + 2*a^4*b + a^3*b
^2)*cosh(x)^2)*sinh(x)^2 + 4*((a^5 + 2*a^4*b + a^3*b^2)*cosh(x)^3 + (a^5 +
4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3)*cosh(x))*sinh(x)), 1/4*(4*(a^3*cosh(x)^4 +
 4*a^3*cosh(x)*sinh(x)^3 + a^3*sinh(x)^4 + a^3 + 2*(a^3 + 2*a^2*b)*cosh(x)^
2 + 2*(3*a^3*cosh(x)^2 + a^3 + 2*a^2*b)*sinh(x)^2 + 4*(a^3*cosh(x)^3 + (a^3
 + 2*a^2*b)*cosh(x))*sinh(x))*sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2 + 2*co
sh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a - b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2
 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a
*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2
 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) +
 ((a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 4*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sin
h(x)^3 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^4 + a^3 + 2*a^2*b + a*b^2 + 2*(a^3
 + 4*a^2*b + 5*a*b^2 + 2*b^3)*cosh(x)^2 + 2*(a^3 + 4*a^2*b + 5*a*b^2 + 2*b^
3 + 3*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 + 2*a^2*b + a*
b^2)*cosh(x)^3 + (a^3 + 4*a^2*b + 5*a*b^2 + 2*b^3)*cosh(x))*sinh(x))*sqrt(a
)*log(((a^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*cosh(x
)*sinh(x)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*a^2*b + 4*a*
b^2 + b^3)*cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*(a^3 + 2*a^2
*b + a*b^2)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3
+ 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + (6*a^3 + 14*a^2*
b + 9*a*b^2)*cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 6*a^3 + 14
*a^2*b + 9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^
4 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a^2*b + 4*a*b^2
 + b^3)*cosh(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 +
 2*(2*a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 +
15*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3 + 3*a^2*b + 3*(6*a^3
 + 14*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*((a^2 + 2*a*b + b^2)*
cosh(x)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2 + 2*a*b + b^2)*s
inh(x)^6 + 3*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(
x)^2 + a^2 + 2*a*b + b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(x)^3 +
3*(a^2 + 2*a*b + b^2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4*a*b)*cosh(x)^2 + (15*
(a^2 + 2*a*b + b^2)*cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*cosh(x)^2 + 3*a^2 +
```

```
4*a*b)*sinh(x)^2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 6*(a^2 + 2*a*
b + b^2)*cosh(x)^3 + (3*a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh
(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))
 + 4*(2*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^7 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 +
b^3)*cosh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^3 + (2*a^3 + 3*a^2*b)
*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^
2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 +
 sinh(x)^6)) + ((a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 4*(a^3 + 2*a^2*b + a*b^
2)*cosh(x)*sinh(x)^3 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^4 + a^3 + 2*a^2*b +
a*b^2 + 2*(a^3 + 4*a^2*b + 5*a*b^2 + 2*b^3)*cosh(x)^2 + 2*(a^3 + 4*a^2*b +
5*a*b^2 + 2*b^3 + 3*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^2 + 4*((a^3
+ 2*a^2*b + a*b^2)*cosh(x)^3 + (a^3 + 4*a^2*b + 5*a*b^2 + 2*b^3)*cosh(x))*s
inh(x))*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2
*b*cosh(x)^2 + 2*(3*a*cosh(x)^2 + b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cos
h(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a +
 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(a*cosh(x)^3 + b*cos
h(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) - 4*sqrt(2)
*(a^2*b + a*b^2 + (a^2*b + a*b^2)*cosh(x)^2 + 2*(a^2*b + a*b^2)*cosh(x)*sin
h(x) + (a^2*b + a*b^2)*sinh(x)^2)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b
)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a^5 + 2*a^4*b + a^3*b^2 +
(a^5 + 2*a^4*b + a^3*b^2)*cosh(x)^4 + 4*(a^5 + 2*a^4*b + a^3*b^2)*cosh(x)*s
inh(x)^3 + (a^5 + 2*a^4*b + a^3*b^2)*sinh(x)^4 + 2*(a^5 + 4*a^4*b + 5*a^3*b
^2 + 2*a^2*b^3)*cosh(x)^2 + 2*(a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3 + 3*(a
^5 + 2*a^4*b + a^3*b^2)*cosh(x)^2)*sinh(x)^2 + 4*((a^5 + 2*a^4*b + a^3*b^2)
*cosh(x)^3 + (a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3)*cosh(x))*sinh(x)), -1/
2*(((a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 4*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*s
inh(x)^3 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^4 + a^3 + 2*a^2*b + a*b^2 + 2*(a
^3 + 4*a^2*b + 5*a*b^2 + 2*b^3)*cosh(x)^2 + 2*(a^3 + 4*a^2*b + 5*a*b^2 + 2*
b^3 + 3*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 + 2*a^2*b +
a*b^2)*cosh(x)^3 + (a^3 + 4*a^2*b + 5*a*b^2 + 2*b^3)*cosh(x))*sinh(x))*sqrt
(-a)*arctan(sqrt(2)*((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b
)*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(
x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*
b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 + (2*a^2 + 3*a*b)*cosh(x)^2 +
(6*(a^2 + a*b)*cosh(x)^2 + 2*a^2 + 3*a*b)*sinh(x)^2 + a^2 + 2*(2*(a^2 + a*b
)*cosh(x)^3 + (2*a^2 + 3*a*b)*cosh(x))*sinh(x))) + ((a^3 + 2*a^2*b + a*b^2)
*cosh(x)^4 + 4*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^3 + (a^3 + 2*a^2*b +
 a*b^2)*sinh(x)^4 + a^3 + 2*a^2*b + a*b^2 + 2*(a^3 + 4*a^2*b + 5*a*b^2 + 2*
b^3)*cosh(x)^2 + 2*(a^3 + 4*a^2*b + 5*a*b^2 + 2*b^3 + 3*(a^3 + 2*a^2*b + a*
b^2)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + (a^3 + 4
*a^2*b + 5*a*b^2 + 2*b^3)*cosh(x))*sinh(x))*sqrt(-a)*arctan(sqrt(2)*(cosh(x
)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sin
h(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4
 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cos
h(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) +
```

```
 a)) - (a^3*cosh(x)^4 + 4*a^3*cosh(x)*sinh(x)^3 + a^3*sinh(x)^4 + a^3 + 2*(
a^3 + 2*a^2*b)*cosh(x)^2 + 2*(3*a^3*cosh(x)^2 + a^3 + 2*a^2*b)*sinh(x)^2 +
4*(a^3*cosh(x)^3 + (a^3 + 2*a^2*b)*cosh(x))*sinh(x))*sqrt(a + b)*log(((2*a
+ b)*cosh(x)^4 + 4*(2*a + b)*cosh(x)*sinh(x)^3 + (2*a + b)*sinh(x)^4 + 2*(2
*a + 3*b)*cosh(x)^2 + 2*(3*(2*a + b)*cosh(x)^2 + 2*a + 3*b)*sinh(x)^2 - 2*s
qrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*sqrt((a*
cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)
^2)) + 4*((2*a + b)*cosh(x)^3 + (2*a + 3*b)*cosh(x))*sinh(x) + 2*a + b)/(co
sh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 -
 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)) + 2*sqrt(2)*(a^2*b + a
*b^2 + (a^2*b + a*b^2)*cosh(x)^2 + 2*(a^2*b + a*b^2)*cosh(x)*sinh(x) + (a^2
*b + a*b^2)*sinh(x)^2)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^
2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a^5 + 2*a^4*b + a^3*b^2 + (a^5 + 2*a^
4*b + a^3*b^2)*cosh(x)^4 + 4*(a^5 + 2*a^4*b + a^3*b^2)*cosh(x)*sinh(x)^3 +
(a^5 + 2*a^4*b + a^3*b^2)*sinh(x)^4 + 2*(a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*
b^3)*cosh(x)^2 + 2*(a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3 + 3*(a^5 + 2*a^4*
b + a^3*b^2)*cosh(x)^2)*sinh(x)^2 + 4*((a^5 + 2*a^4*b + a^3*b^2)*cosh(x)^3
+ (a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3)*cosh(x))*sinh(x)), -1/2*(((a^3 +
2*a^2*b + a*b^2)*cosh(x)^4 + 4*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^3 +
(a^3 + 2*a^2*b + a*b^2)*sinh(x)^4 + a^3 + 2*a^2*b + a*b^2 + 2*(a^3 + 4*a^2*
b + 5*a*b^2 + 2*b^3)*cosh(x)^2 + 2*(a^3 + 4*a^2*b + 5*a*b^2 + 2*b^3 + 3*(a^
3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 + 2*a^2*b + a*b^2)*cosh
(x)^3 + (a^3 + 4*a^2*b + 5*a*b^2 + 2*b^3)*cosh(x))*sinh(x))*sqrt(-a)*arctan
(sqrt(2)*((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2
 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*co
sh(x)*sinh(x) + sinh(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*
sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 + (2*a^2 + 3*a*b)*cosh(x)^2 + (6*(a^2 + a
*b)*cosh(x)^2 + 2*a^2 + 3*a*b)*sinh(x)^2 + a^2 + 2*(2*(a^2 + a*b)*cosh(x)^3
 + (2*a^2 + 3*a*b)*cosh(x))*sinh(x))) + ((a^3 + 2*a^2*b + a*b^2)*cosh(x)^4
+ 4*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^3 + (a^3 + 2*a^2*b + a*b^2)*sin
h(x)^4 + a^3 + 2*a^2*b + a*b^2 + 2*(a^3 + 4*a^2*b + 5*a*b^2 + 2*b^3)*cosh(x
)^2 + 2*(a^3 + 4*a^2*b + 5*a*b^2 + 2*b^3 + 3*(a^3 + 2*a^2*b + a*b^2)*cosh(x
)^2)*sinh(x)^2 + 4*((a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + (a^3 + 4*a^2*b + 5*
a*b^2 + 2*b^3)*cosh(x))*sinh(x))*sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cos
h(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a
+ 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh
(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a
+ 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) - 2*(a
^3*cosh(x)^4 + 4*a^3*cosh(x)*sinh(x)^3 + a^3*sinh(x)^4 + a^3 + 2*(a^3 + 2*a
^2*b)*cosh(x)^2 + 2*(3*a^3*cosh(x)^2 + a^3 + 2*a^2*b)*sinh(x)^2 + 4*(a^3*co
sh(x)^3 + (a^3 + 2*a^2*b)*cosh(x))*sinh(x))*sqrt(-a - b)*arctan(sqrt(2)*(co
sh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a - b)*sqrt((a*cosh(x)^2
 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*c
osh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*
(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*s
```

```
inh(x) + a)) + 2*sqrt(2)*(a^2*b + a*b^2 + (a^2*b + a*b^2)*cosh(x)^2 + 2*(a^
2*b + a*b^2)*cosh(x)*sinh(x) + (a^2*b + a*b^2)*sinh(x)^2)*sqrt((a*cosh(x)^2
 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a^
5 + 2*a^4*b + a^3*b^2 + (a^5 + 2*a^4*b + a^3*b^2)*cosh(x)^4 + 4*(a^5 + 2*a^
4*b + a^3*b^2)*cosh(x)*sinh(x)^3 + (a^5 + 2*a^4*b + a^3*b^2)*sinh(x)^4 + 2*
(a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3)*cosh(x)^2 + 2*(a^5 + 4*a^4*b + 5*a^
3*b^2 + 2*a^2*b^3 + 3*(a^5 + 2*a^4*b + a^3*b^2)*cosh(x)^2)*sinh(x)^2 + 4*((
a^5 + 2*a^4*b + a^3*b^2)*cosh(x)^3 + (a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3
)*cosh(x))*sinh(x))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sech(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [F] time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{\coth(x)}{\left(a+b\,\mathrm{sech}(x)^{2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a+b*sech(x)^2)^(3/2),x)
```

```
[Out] int(coth(x)/(a+b*sech(x)^2)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(x)}{\left(b\,\mathrm{sech}(x)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sech(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(x)/(b*sech(x)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\coth(x)}{\left(a+\frac{b}{\cosh(x)^{2}}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a + b/cosh(x)^2)^(3/2),x)
[Out] int(coth(x)/(a + b/cosh(x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(x)}{\left(a + b\,\mathrm{sech}^2(x)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sech(x)**2)**(3/2),x)
[Out] Integral(coth(x)/(a + b*sech(x)**2)**(3/2), x)
```

3.210 $\int \frac{\coth^2(x)}{\left(a+b\,\mathrm{sech}^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=88

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{a^{3/2}}-\frac{(a-b)\coth(x)\sqrt{a-b\tanh^2(x)+b}}{a(a+b)^2}-\frac{b\coth(x)}{a(a+b)\sqrt{a-b\tanh^2(x)+b}}$$

```
[Out] arctanh(a^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))/a^(3/2)-b*coth(x)/a/(a+b)/
(a+b-b*tanh(x)^2)^(1/2)-(a-b)*coth(x)*(a+b-b*tanh(x)^2)^(1/2)/a/(a+b)^2
```

Rubi [A] time = 0.27, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {4141, 1975, 472, 583, 12, 377, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{a^{3/2}}-\frac{(a-b)\coth(x)\sqrt{a-b\tanh^2(x)+b}}{a(a+b)^2}-\frac{b\coth(x)}{a(a+b)\sqrt{a-b\tanh^2(x)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2/(a + b*Sech[x]^2)^(3/2),x]
```

```
[Out] ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/a^(3/2) - (b*Coth[x])/
(a*(a + b)*Sqrt[a + b - b*Tanh[x]^2]) - ((a - b)*Coth[x]*Sqrt[a + b - b*Tan
h[x]^2])/(a*(a + b)^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\coth^2(x)}{\left(a+b\operatorname{sech}^2(x)\right)^{3/2}}\,dx = \text{Subst}\left(\int \frac{1}{x^2\left(1-x^2\right)\left(a+b\left(1-x^2\right)\right)^{3/2}}\,dx, x, \tanh(x)\right)$$

$$= \text{Subst}\left(\int \frac{1}{x^2\left(1-x^2\right)\left(a+b-bx^2\right)^{3/2}}\,dx, x, \tanh(x)\right)$$

$$= -\frac{b\coth(x)}{a(a+b)\sqrt{a+b-b\tanh^2(x)}} - \frac{\text{Subst}\left(\int \frac{-a+b-2bx^2}{x^2\left(1-x^2\right)\sqrt{a+b-bx^2}}\,dx, x, \tanh(x)\right)}{a(a+b)}$$

$$= -\frac{b\coth(x)}{a(a+b)\sqrt{a+b-b\tanh^2(x)}} - \frac{(a-b)\coth(x)\sqrt{a+b-b\tanh^2(x)}}{a(a+b)^2} + \frac{\text{Subst}\left(\int \frac{}{(1-}\right.}{}$$

$$= -\frac{b\coth(x)}{a(a+b)\sqrt{a+b-b\tanh^2(x)}} - \frac{(a-b)\coth(x)\sqrt{a+b-b\tanh^2(x)}}{a(a+b)^2} + \frac{\text{Subst}\left(\int \frac{}{(1-}\right.}{}$$

$$= -\frac{b\coth(x)}{a(a+b)\sqrt{a+b-b\tanh^2(x)}} - \frac{(a-b)\coth(x)\sqrt{a+b-b\tanh^2(x)}}{a(a+b)^2} + \frac{\text{Subst}\left(\int \frac{1}{1-a}\right.}{}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{a^{3/2}} - \frac{b\coth(x)}{a(a+b)\sqrt{a+b-b\tanh^2(x)}} - \frac{(a-b)\coth(x)\sqrt{a+b-b}}{a(a+b)^2}$$

<u>Mathematica</u> [A] time = 0.46, size = 120, normalized size = 1.36

$$\frac{\operatorname{sech}^3(x)\left(\frac{\sqrt{2}\,(a\cosh(2x)+a+2b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{a}\sinh(x)}{\sqrt{a\cosh(2x)+a+2b}}\right)}{a^{3/2}} - \frac{(a\cosh(2x)+a+2b)\left(a\operatorname{csch}(x)(a\cosh(2x)+a+2b)+2b^2\sinh(x)\right)}{a(a+b)^2}\right)}{4\left(a+b\operatorname{sech}^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2/(a + b*Sech[x]^2)^(3/2),x]
```

```
[Out] (Sech[x]^3*((Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sinh[x])/Sqrt[a + 2*b + a*Cos
h[2*x]]]*(a + 2*b + a*Cosh[2*x])^(3/2))/a^(3/2) - ((a + 2*b + a*Cosh[2*x])*
(a*(a + 2*b + a*Cosh[2*x])*Csch[x] + 2*b^2*Sinh[x]))/(a*(a + b)^2)))/(4*(a
+ b*Sech[x]^2)^(3/2))
```

fricas [B] time = 0.67, size = 3941, normalized size = 44.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*sech(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 + 6*(a^3 + 2*a^2*b + a*b^2)*cosh(x
)*sinh(x)^5 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^6 + (a^3 + 6*a^2*b + 9*a*b^2
+ 4*b^3)*cosh(x)^4 + (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3 + 15*(a^3 + 2*a^2*b +
 a*b^2)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + (a^
3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*cosh(x))*sinh(x)^3 - a^3 - 2*a^2*b - a*b^2 -
 (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*cosh(x)^2 + (15*(a^3 + 2*a^2*b + a*b^2)*
cosh(x)^4 - a^3 - 6*a^2*b - 9*a*b^2 - 4*b^3 + 6*(a^3 + 6*a^2*b + 9*a*b^2 +
4*b^3)*cosh(x)^2)*sinh(x)^2 + 2*(3*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 2*(a
^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*cosh(x)^3 - (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^
3)*cosh(x))*sinh(x))*sqrt(a)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)
^7 + a*b^2*sinh(x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 -
a*b^2 + b^3)*sinh(x)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*s
inh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3
+ 4*a^2*b + 9*a*b^2 - 30*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*c
osh(x)^5 - 10*(a*b^2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*
sinh(x)^3 + a^3 + 2*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*
(a*b^2 - b^3)*cosh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(
x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*si
nh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*
cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*co
sh(x)^4 - 18*b^2*cosh(x)^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x
)^5 - 6*b^2*cosh(x)^3 - (a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cos
h(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)
) + 4*(2*a*b^2*cosh(x)^7 - 3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a
*b^2)*cosh(x)^3 + (a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^
5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*
sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + ((a^3 + 2*a^2*b + a*b^2)*co
sh(x)^6 + 6*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^5 + (a^3 + 2*a^2*b + a*
b^2)*sinh(x)^6 + (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*cosh(x)^4 + (a^3 + 6*a^2
*b + 9*a*b^2 + 4*b^3 + 15*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^4 + 4*
(5*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*co
sh(x))*sinh(x)^3 - a^3 - 2*a^2*b - a*b^2 - (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3
)*cosh(x)^2 + (15*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 - a^3 - 6*a^2*b - 9*a*b
```

```
^2 - 4*b^3 + 6*(a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*cosh(x)^2)*sinh(x)^2 + 2*(
3*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 2*(a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*c
osh(x)^3 - (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*cosh(x))*sinh(x))*sqrt(a)*log(
-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + b)*cosh(x)^2 +
 2*(3*a*cosh(x)^2 + a + b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(
x) + sinh(x)^2 + 1)*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cos
h(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(a*cosh(x)^3 + (a + b)*cosh(x)
)*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) - 4*sqrt(2)*((a
^3 + a*b^2)*cosh(x)^4 + 4*(a^3 + a*b^2)*cosh(x)*sinh(x)^3 + (a^3 + a*b^2)*s
inh(x)^4 + a^3 + a*b^2 + 2*(a^3 + 2*a^2*b - a*b^2)*cosh(x)^2 + 2*(a^3 + 2*a
^2*b - a*b^2 + 3*(a^3 + a*b^2)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 + a*b^2)*cosh
(x)^3 + (a^3 + 2*a^2*b - a*b^2)*cosh(x))*sinh(x))*sqrt((a*cosh(x)^2 + a*sin
h(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^5 + 2*a
^4*b + a^3*b^2)*cosh(x)^6 + 6*(a^5 + 2*a^4*b + a^3*b^2)*cosh(x)*sinh(x)^5 +
 (a^5 + 2*a^4*b + a^3*b^2)*sinh(x)^6 - a^5 - 2*a^4*b - a^3*b^2 + (a^5 + 6*a
^4*b + 9*a^3*b^2 + 4*a^2*b^3)*cosh(x)^4 + (a^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^
2*b^3 + 15*(a^5 + 2*a^4*b + a^3*b^2)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^5 + 2*a
^4*b + a^3*b^2)*cosh(x)^3 + (a^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*cosh(x)
)*sinh(x)^3 - (a^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*cosh(x)^2 - (a^5 + 6*
a^4*b + 9*a^3*b^2 + 4*a^2*b^3 - 15*(a^5 + 2*a^4*b + a^3*b^2)*cosh(x)^4 - 6*
(a^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*cosh(x)^2)*sinh(x)^2 + 2*(3*(a^5 +
2*a^4*b + a^3*b^2)*cosh(x)^5 + 2*(a^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*co
sh(x)^3 - (a^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*cosh(x))*sinh(x)), -1/2*(
((a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 + 6*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh
(x)^5 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^6 + (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^
3)*cosh(x)^4 + (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3 + 15*(a^3 + 2*a^2*b + a*b^2
)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + (a^3 + 6*
a^2*b + 9*a*b^2 + 4*b^3)*cosh(x))*sinh(x)^3 - a^3 - 2*a^2*b - a*b^2 - (a^3
+ 6*a^2*b + 9*a*b^2 + 4*b^3)*cosh(x)^2 + (15*(a^3 + 2*a^2*b + a*b^2)*cosh(x
)^4 - a^3 - 6*a^2*b - 9*a*b^2 - 4*b^3 + 6*(a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)
*cosh(x)^2)*sinh(x)^2 + 2*(3*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 2*(a^3 + 6
*a^2*b + 9*a*b^2 + 4*b^3)*cosh(x)^3 - (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*cos
h(x))*sinh(x))*sqrt(-a)*arctan(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) +
 b*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh
(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*b*cosh(x)^4 + 4*a*b*cosh(x)*sinh
(x)^3 + a*b*sinh(x)^4 - (a^2 + 3*a*b)*cosh(x)^2 + (6*a*b*cosh(x)^2 - a^2 -
3*a*b)*sinh(x)^2 - a^2 + 2*(2*a*b*cosh(x)^3 - (a^2 + 3*a*b)*cosh(x))*sinh(x
))) + ((a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 + 6*(a^3 + 2*a^2*b + a*b^2)*cosh(x
)*sinh(x)^5 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^6 + (a^3 + 6*a^2*b + 9*a*b^2
+ 4*b^3)*cosh(x)^4 + (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3 + 15*(a^3 + 2*a^2*b +
 a*b^2)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + (a^
3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*cosh(x))*sinh(x)^3 - a^3 - 2*a^2*b - a*b^2 -
 (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*cosh(x)^2 + (15*(a^3 + 2*a^2*b + a*b^2)*
cosh(x)^4 - a^3 - 6*a^2*b - 9*a*b^2 - 4*b^3 + 6*(a^3 + 6*a^2*b + 9*a*b^2 +
4*b^3)*cosh(x)^2)*sinh(x)^2 + 2*(3*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 2*(a
```

```
^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*cosh(x)^3 - (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^
3)*cosh(x))*sinh(x))*sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x)
 + sinh(x)^2 + 1)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh
(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^
3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(
x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) + 2*sqrt(2)*((a^3
+ a*b^2)*cosh(x)^4 + 4*(a^3 + a*b^2)*cosh(x)*sinh(x)^3 + (a^3 + a*b^2)*sinh
(x)^4 + a^3 + a*b^2 + 2*(a^3 + 2*a^2*b - a*b^2)*cosh(x)^2 + 2*(a^3 + 2*a^2*
b - a*b^2 + 3*(a^3 + a*b^2)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 + a*b^2)*cosh(x)
^3 + (a^3 + 2*a^2*b - a*b^2)*cosh(x))*sinh(x))*sqrt((a*cosh(x)^2 + a*sinh(x
)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^5 + 2*a^4*
b + a^3*b^2)*cosh(x)^6 + 6*(a^5 + 2*a^4*b + a^3*b^2)*cosh(x)*sinh(x)^5 + (a
^5 + 2*a^4*b + a^3*b^2)*sinh(x)^6 - a^5 - 2*a^4*b - a^3*b^2 + (a^5 + 6*a^4*
b + 9*a^3*b^2 + 4*a^2*b^3)*cosh(x)^4 + (a^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^2*b
^3 + 15*(a^5 + 2*a^4*b + a^3*b^2)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^5 + 2*a^4*
b + a^3*b^2)*cosh(x)^3 + (a^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*cosh(x))*s
inh(x)^3 - (a^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*cosh(x)^2 - (a^5 + 6*a^4
*b + 9*a^3*b^2 + 4*a^2*b^3 - 15*(a^5 + 2*a^4*b + a^3*b^2)*cosh(x)^4 - 6*(a^
5 + 6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*cosh(x)^2)*sinh(x)^2 + 2*(3*(a^5 + 2*a
^4*b + a^3*b^2)*cosh(x)^5 + 2*(a^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*cosh(
x)^3 - (a^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*cosh(x))*sinh(x))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*sech(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(x)}{\left(a + b\operatorname{sech}(x)^2\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(a+b*sech(x)^2)^(3/2),x)
```

```
[Out] int(coth(x)^2/(a+b*sech(x)^2)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(x)^2}{\left(b\,\mathrm{sech}(x)^2 + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*sech(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(x)^2/(b*sech(x)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\coth(x)^2}{\left(a + \frac{b}{\cosh(x)^2}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(a + b/cosh(x)^2)^(3/2),x)
```

```
[Out] int(coth(x)^2/(a + b/cosh(x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(x)}{\left(a + b\,\mathrm{sech}^2(x)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2/(a+b*sech(x)**2)**(3/2),x)
```

```
[Out] Integral(coth(x)**2/(a + b*sech(x)**2)**(3/2), x)
```

3.211 $\int \frac{\tanh^6(x)}{\left(a+b\,\mathbf{sech}^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=118

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{a^{5/2}}-\frac{\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}-\frac{\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{b^{5/2}}-\frac{(a+b)\tanh^3(x)}{3ab\left(a-b\tanh^2(x)+b\right)^{3/2}}$$

```
[Out] -arctan(b^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))/b^(5/2)+arctanh(a^(1/2)*ta
nh(x)/(a+b-b*tanh(x)^2)^(1/2))/a^(5/2)-(1/a^2-1/b^2)*tanh(x)/(a+b-b*tanh(x)
^2)^(1/2)-1/3*(a+b)*tanh(x)^3/a/b/(a+b-b*tanh(x)^2)^(3/2)
```

Rubi [A] time = 0.34, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {4141, 1975, 470, 578, 523, 217, 203, 377, 206}

$$-\frac{\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{a^{5/2}}-\frac{\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{b^{5/2}}-\frac{(a+b)\tanh^3(x)}{3ab\left(a-b\tanh^2(x)+b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^6/(a + b*Sech[x]^2)^(5/2),x]
```

```
[Out] -(ArcTan[(Sqrt[b]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/b^(5/2)) + ArcTanh[(S
qrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/a^(5/2) - ((a + b)*Tanh[x]^3)/(3
*a*b*(a + b - b*Tanh[x]^2)^(3/2)) - ((a^(-2) - b^(-2))*Tanh[x])/Sqrt[a + b
- b*Tanh[x]^2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
```

```
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\tanh^6(x)}{\left(a+b\text{sech}^2(x)\right)^{5/2}}\,dx = \text{Subst}\left(\int \frac{x^6}{\left(1-x^2\right)\left(a+b\left(1-x^2\right)\right)^{5/2}}\,dx, x, \tanh(x)\right)$$

$$= \text{Subst}\left(\int \frac{x^6}{\left(1-x^2\right)\left(a+b-bx^2\right)^{5/2}}\,dx, x, \tanh(x)\right)$$

$$= -\frac{(a+b)\tanh^3(x)}{3ab\left(a+b-b\tanh^2(x)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{x^2\left(3(a+b)-3ax^2\right)}{\left(1-x^2\right)\left(a+b-bx^2\right)^{3/2}}\,dx, x, \tanh(x)\right)}{3ab}$$

$$= -\frac{(a+b)\tanh^3(x)}{3ab\left(a+b-b\tanh^2(x)\right)^{3/2}} - \frac{\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}} - \frac{\text{Subst}\left(\int \frac{3\left(a^2-b^2\right)-3a^2x^2}{\left(1-x^2\right)\sqrt{a+b-bx^2}}\,dx, x\right.}{3a^2b^2}$$

$$= -\frac{(a+b)\tanh^3(x)}{3ab\left(a+b-b\tanh^2(x)\right)^{3/2}} - \frac{\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)\sqrt{a+b-bx^2}}\,dx, x\right.}{a^2}$$

$$= -\frac{(a+b)\tanh^3(x)}{3ab\left(a+b-b\tanh^2(x)\right)^{3/2}} - \frac{\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\tanh}{\sqrt{a+b-b}}\right.}{a^2}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{b^{5/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{a^{5/2}} - \frac{(a+b)\tanh^3(x)}{3ab\left(a+b-b\tanh^2(x)\right)^{3/2}} -$$

Mathematica [A] time = 0.75, size = 178, normalized size = 1.51

$$\frac{\operatorname{sech}^5(x)\left(\frac{\sqrt{2}(a\cosh(2x)+a+2b)^{5/2}\left(b^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sinh(x)}{\sqrt{a\cosh(2x)+a+2b}}\right)-a^{5/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}\sinh(x)}{\sqrt{a\cosh(2x)+a+2b}}\right)\right)}{a^{5/2}b^{5/2}}+\frac{2(a+b)\sinh(x)\left(3a^2+a(3a-4b)\cosh(2x)+4ab\right.}{3a^2b^2}\right.}{8\left(a+b\operatorname{sech}^2(x)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^6/(a + b*Sech[x]^2)^(5/2),x]
```

```
[Out] (Sech[x]^5*((Sqrt[2]*(-(a^(5/2)*ArcTan[(Sqrt[2]*Sqrt[b]*Sinh[x])/Sqrt[a + 2
*b + a*Cosh[2*x]]]) + b^(5/2)*ArcTanh[(Sqrt[2]*Sqrt[a]*Sinh[x])/Sqrt[a + 2*
b + a*Cosh[2*x]]])*(a + 2*b + a*Cosh[2*x])^(5/2))/(a^(5/2)*b^(5/2)) + (2*(a
 + b)*(a + 2*b + a*Cosh[2*x])*(3*a^2 + 4*a*b - 6*b^2 + a*(3*a - 4*b)*Cosh[2
*x])*Sinh[x])/(3*a^2*b^2)))/(8*(a + b*Sech[x]^2)^(5/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^6/(a+b*sech(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^6/(a+b*sech(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{\tanh^6(x)}{\left(a+b\operatorname{sech}(x)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^6/(a+b*sech(x)^2)^(5/2),x)
```

```
[Out] int(tanh(x)^6/(a+b*sech(x)^2)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)^6}{\left(b\,\mathrm{sech}(x)^2 + a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^6/(a+b*sech(x)^2)^(5/2),x, algorithm="maxima")
[Out] integrate(tanh(x)^6/(b*sech(x)^2 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tanh(x)^6}{\left(a + \frac{b}{\cosh(x)^2}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^6/(a + b/cosh(x)^2)^(5/2),x)
[Out] int(tanh(x)^6/(a + b/cosh(x)^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^6(x)}{\left(a + b\,\mathrm{sech}^2(x)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**6/(a+b*sech(x)**2)**(5/2),x)
[Out] Integral(tanh(x)**6/(a + b*sech(x)**2)**(5/2), x)
```

3.212 $\int \frac{\tanh^5(x)}{\left(a+b\,\mathbf{sech}^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=76

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\mathrm{sech}^2(x)}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{\frac{1}{a^2}-\frac{1}{b^2}}{\sqrt{a+b\mathrm{sech}^2(x)}}-\frac{(a+b)^2}{3ab^2\left(a+b\mathrm{sech}^2(x)\right)^{3/2}}$$

```
[Out] arctanh((a+b*sech(x)^2)^(1/2)/a^(1/2))/a^(5/2)-1/3*(a+b)^2/a/b^2/(a+b*sech(
x)^2)^(3/2)+(-1/a^2+1/b^2)/(a+b*sech(x)^2)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {4139, 446, 87, 63, 208}

$$-\frac{\frac{1}{a^2}-\frac{1}{b^2}}{\sqrt{a+b\mathrm{sech}^2(x)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\mathrm{sech}^2(x)}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{(a+b)^2}{3ab^2\left(a+b\mathrm{sech}^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^5/(a + b*Sech[x]^2)^(5/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]]/a^(5/2) - (a + b)^2/(3*a*b^2*(a + b*
Sech[x]^2)^(3/2)) - (a^(-2) - b^(-2))/Sqrt[a + b*Sech[x]^2]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 87

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], ((c + d
*x)^n*(e + f*x)^IntegerPart[p])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \frac{\tanh^5(x)}{\left(a+b\text{sech}^2(x)\right)^{5/2}}\,dx = -\text{Subst}\left(\int \frac{\left(-1+x^2\right)^2}{x\left(a+bx^2\right)^{5/2}}\,dx, x, \text{sech}(x)\right)$$

$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{(-1+x)^2}{x(a+bx)^{5/2}}\,dx, x, \text{sech}^2(x)\right)\right)$$

$$= -\left(\frac{1}{2}\text{Subst}\left(\int \left(-\frac{(a+b)^2}{ab(a+bx)^{5/2}} + \frac{a^2-b^2}{a^2b(a+bx)^{3/2}} + \frac{1}{a^2x\sqrt{a+bx}}\right)dx, x, \text{sech}^2(x)\right)\right)$$

$$= -\frac{(a+b)^2}{3ab^2\left(a+b\text{sech}^2(x)\right)^{3/2}} - \frac{\frac{1}{a^2}-\frac{1}{b^2}}{\sqrt{a+b\text{sech}^2(x)}} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \text{sech}^2(x)\right)}{2a^2}$$

$$= -\frac{(a+b)^2}{3ab^2\left(a+b\text{sech}^2(x)\right)^{3/2}} - \frac{\frac{1}{a^2}-\frac{1}{b^2}}{\sqrt{a+b\text{sech}^2(x)}} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{sech}^2(x)}\right)}{a^2b}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)}{a^{5/2}} - \frac{(a+b)^2}{3ab^2\left(a+b\text{sech}^2(x)\right)^{3/2}} - \frac{\frac{1}{a^2}-\frac{1}{b^2}}{\sqrt{a+b\text{sech}^2(x)}}$$

Mathematica [A] time = 0.56, size = 126, normalized size = 1.66

$$\frac{\text{sech}^5(x)\left(\frac{\sqrt{2}(a\cosh(2x)+a+2b)^{5/2}\log\left(\sqrt{a\cosh(2x)+a+2b}+\sqrt{2}\sqrt{a}\cosh(x)\right)}{a^{5/2}} + \frac{4(a+b)\cosh(x)\left(a^2+a(a-2b)\cosh(2x)+ab-3b^2\right)(a\cosh(2x)+a+2b}{3a^2b^2}\right.}{8\left(a+b\text{sech}^2(x)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^5/(a + b*Sech[x]^2)^(5/2),x]
```

```
[Out] (((4*(a + b)*Cosh[x]*(a + 2*b + a*Cosh[2*x])*(a^2 + a*b - 3*b^2 + a*(a - 2*
b)*Cosh[2*x]))/(3*a^2*b^2) + (Sqrt[2]*(a + 2*b + a*Cosh[2*x])^(5/2)*Log[Sqr
t[2]*Sqrt[a]*Cosh[x] + Sqrt[a + 2*b + a*Cosh[2*x]]])/a^(5/2))*Sech[x]^5)/(8
*(a + b*Sech[x]^2)^(5/2))
```

fricas [B] time = 0.76, size = 5184, normalized size = 68.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+b*sech(x)^2)^(5/2),x, algorithm="fricas")

[Out] [1/12*(3*(a^2*b^2*cosh(x)^8 + 8*a^2*b^2*cosh(x)*sinh(x)^7 + a^2*b^2*sinh(x)
^8 + 4*(a^2*b^2 + 2*a*b^3)*cosh(x)^6 + 4*(7*a^2*b^2*cosh(x)^2 + a^2*b^2 + 2
*a*b^3)*sinh(x)^6 + 8*(7*a^2*b^2*cosh(x)^3 + 3*(a^2*b^2 + 2*a*b^3)*cosh(x))
*sinh(x)^5 + 2*(3*a^2*b^2 + 8*a*b^3 + 8*b^4)*cosh(x)^4 + 2*(35*a^2*b^2*cosh
(x)^4 + 3*a^2*b^2 + 8*a*b^3 + 8*b^4 + 30*(a^2*b^2 + 2*a*b^3)*cosh(x)^2)*sin
h(x)^4 + a^2*b^2 + 8*(7*a^2*b^2*cosh(x)^5 + 10*(a^2*b^2 + 2*a*b^3)*cosh(x)^
3 + (3*a^2*b^2 + 8*a*b^3 + 8*b^4)*cosh(x))*sinh(x)^3 + 4*(a^2*b^2 + 2*a*b^3
)*cosh(x)^2 + 4*(7*a^2*b^2*cosh(x)^6 + 15*(a^2*b^2 + 2*a*b^3)*cosh(x)^4 + a
^2*b^2 + 2*a*b^3 + 3*(3*a^2*b^2 + 8*a*b^3 + 8*b^4)*cosh(x)^2)*sinh(x)^2 + 8
*(a^2*b^2*cosh(x)^7 + 3*(a^2*b^2 + 2*a*b^3)*cosh(x)^5 + (3*a^2*b^2 + 8*a*b^
3 + 8*b^4)*cosh(x)^3 + (a^2*b^2 + 2*a*b^3)*cosh(x))*sinh(x))*sqrt(a)*log(((
a^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x
)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^
3)*cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*(a^3 + 2*a^2*b + a*b
^2)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + 3*(2*a
^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*
b^2)*cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 6*a^3 + 14*a^2*b +
 9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^4 + 4*(1
4*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*
cosh(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(2*a^
3 + 3*a^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 + 15*(2*a^
3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3 + 3*a^2*b + 3*(6*a^3 + 14*a^
2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*((a^2 + 2*a*b + b^2)*cosh(x)^
6 + 6*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2 + 2*a*b + b^2)*sinh(x)^6
 + 3*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a
^2 + 2*a*b + b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 +
 2*a*b + b^2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4*a*b)*cosh(x)^2 + (15*(a^2 + 2
*a*b + b^2)*cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*cosh(x)^2 + 3*a^2 + 4*a*b)*s
inh(x)^2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 6*(a^2 + 2*a*b + b^2)
*cosh(x)^3 + (3*a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 +
a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*
(a^3 + 2*a^2*b + a*b^2)*cosh(x)^7 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cos
h(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^3 + (2*a^3 + 3*a^2*b)*cosh(x)
)*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*c
osh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)
^6)) + 3*(a^2*b^2*cosh(x)^8 + 8*a^2*b^2*cosh(x)*sinh(x)^7 + a^2*b^2*sinh(x)
^8 + 4*(a^2*b^2 + 2*a*b^3)*cosh(x)^6 + 4*(7*a^2*b^2*cosh(x)^2 + a^2*b^2 + 2
*a*b^3)*sinh(x)^6 + 8*(7*a^2*b^2*cosh(x)^3 + 3*(a^2*b^2 + 2*a*b^3)*cosh(x))
*sinh(x)^5 + 2*(3*a^2*b^2 + 8*a*b^3 + 8*b^4)*cosh(x)^4 + 2*(35*a^2*b^2*cosh
(x)^4 + 3*a^2*b^2 + 8*a*b^3 + 8*b^4 + 30*(a^2*b^2 + 2*a*b^3)*cosh(x)^2)*sin
h(x)^4 + a^2*b^2 + 8*(7*a^2*b^2*cosh(x)^5 + 10*(a^2*b^2 + 2*a*b^3)*cosh(x)^
3 + (3*a^2*b^2 + 8*a*b^3 + 8*b^4)*cosh(x))*sinh(x)^3 + 4*(a^2*b^2 + 2*a*b^3
)*cosh(x)^2 + 4*(7*a^2*b^2*cosh(x)^6 + 15*(a^2*b^2 + 2*a*b^3)*cosh(x)^4 + a
```

^2*b^2 + 2*a*b^3 + 3*(3*a^2*b^2 + 8*a*b^3 + 8*b^4)*cosh(x)^2)*sinh(x)^2 + 8
*(a^2*b^2*cosh(x)^7 + 3*(a^2*b^2 + 2*a*b^3)*cosh(x)^5 + (3*a^2*b^2 + 8*a*b^
3 + 8*b^4)*cosh(x)^3 + (a^2*b^2 + 2*a*b^3)*cosh(x))*sinh(x))*sqrt(a)*log(-(
a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*a*
cosh(x)^2 + b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)
^2 - 1)*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*c
osh(x)*sinh(x) + sinh(x)^2)) + 4*(a*cosh(x)^3 + b*cosh(x))*sinh(x) + a)/(co
sh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) + 8*sqrt(2)*((a^4 - a^3*b - 2*a^2
*b^2)*cosh(x)^6 + 6*(a^4 - a^3*b - 2*a^2*b^2)*cosh(x)*sinh(x)^5 + (a^4 - a^
3*b - 2*a^2*b^2)*sinh(x)^6 + 3*(a^4 + a^3*b - 2*a^2*b^2 - 2*a*b^3)*cosh(x)^
4 + 3*(a^4 + a^3*b - 2*a^2*b^2 - 2*a*b^3 + 5*(a^4 - a^3*b - 2*a^2*b^2)*cosh
(x)^2)*sinh(x)^4 + a^4 - a^3*b - 2*a^2*b^2 + 4*(5*(a^4 - a^3*b - 2*a^2*b^2)
cosh(x)^3 + 3(a^4 + a^3*b - 2*a^2*b^2 - 2*a*b^3)*cosh(x))*sinh(x)^3 + 3*(
a^4 + a^3*b - 2*a^2*b^2 - 2*a*b^3)*cosh(x)^2 + 3*(5*(a^4 - a^3*b - 2*a^2*b^
2)*cosh(x)^4 + a^4 + a^3*b - 2*a^2*b^2 - 2*a*b^3 + 6*(a^4 + a^3*b - 2*a^2*b
^2 - 2*a*b^3)*cosh(x)^2)*sinh(x)^2 + 6*((a^4 - a^3*b - 2*a^2*b^2)*cosh(x)^5
 + 2*(a^4 + a^3*b - 2*a^2*b^2 - 2*a*b^3)*cosh(x)^3 + (a^4 + a^3*b - 2*a^2*b
^2 - 2*a*b^3)*cosh(x))*sinh(x))*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/
(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a^5*b^2*cosh(x)^8 + 8*a^5*b^
2*cosh(x)*sinh(x)^7 + a^5*b^2*sinh(x)^8 + a^5*b^2 + 4*(a^5*b^2 + 2*a^4*b^3)
cosh(x)^6 + 4(7*a^5*b^2*cosh(x)^2 + a^5*b^2 + 2*a^4*b^3)*sinh(x)^6 + 8*(7
*a^5*b^2*cosh(x)^3 + 3*(a^5*b^2 + 2*a^4*b^3)*cosh(x))*sinh(x)^5 + 2*(3*a^5*
b^2 + 8*a^4*b^3 + 8*a^3*b^4)*cosh(x)^4 + 2*(35*a^5*b^2*cosh(x)^4 + 3*a^5*b^
2 + 8*a^4*b^3 + 8*a^3*b^4 + 30*(a^5*b^2 + 2*a^4*b^3)*cosh(x)^2)*sinh(x)^4 +
 8*(7*a^5*b^2*cosh(x)^5 + 10*(a^5*b^2 + 2*a^4*b^3)*cosh(x)^3 + (3*a^5*b^2 +
 8*a^4*b^3 + 8*a^3*b^4)*cosh(x))*sinh(x)^3 + 4*(a^5*b^2 + 2*a^4*b^3)*cosh(x
)^2 + 4*(7*a^5*b^2*cosh(x)^6 + a^5*b^2 + 2*a^4*b^3 + 15*(a^5*b^2 + 2*a^4*b^
3)*cosh(x)^4 + 3*(3*a^5*b^2 + 8*a^4*b^3 + 8*a^3*b^4)*cosh(x)^2)*sinh(x)^2 +
 8*(a^5*b^2*cosh(x)^7 + 3*(a^5*b^2 + 2*a^4*b^3)*cosh(x)^5 + (3*a^5*b^2 + 8*
a^4*b^3 + 8*a^3*b^4)*cosh(x)^3 + (a^5*b^2 + 2*a^4*b^3)*cosh(x))*sinh(x)), -
1/6*(3*(a^2*b^2*cosh(x)^8 + 8*a^2*b^2*cosh(x)*sinh(x)^7 + a^2*b^2*sinh(x)^8
 + 4*(a^2*b^2 + 2*a*b^3)*cosh(x)^6 + 4*(7*a^2*b^2*cosh(x)^2 + a^2*b^2 + 2*a
*b^3)*sinh(x)^6 + 8*(7*a^2*b^2*cosh(x)^3 + 3*(a^2*b^2 + 2*a*b^3)*cosh(x))*s
inh(x)^5 + 2*(3*a^2*b^2 + 8*a*b^3 + 8*b^4)*cosh(x)^4 + 2*(35*a^2*b^2*cosh(x
)^4 + 3*a^2*b^2 + 8*a*b^3 + 8*b^4 + 30*(a^2*b^2 + 2*a*b^3)*cosh(x)^2)*sinh(
x)^4 + a^2*b^2 + 8*(7*a^2*b^2*cosh(x)^5 + 10*(a^2*b^2 + 2*a*b^3)*cosh(x)^3
+ (3*a^2*b^2 + 8*a*b^3 + 8*b^4)*cosh(x))*sinh(x)^3 + 4*(a^2*b^2 + 2*a*b^3)*
cosh(x)^2 + 4*(7*a^2*b^2*cosh(x)^6 + 15*(a^2*b^2 + 2*a*b^3)*cosh(x)^4 + a^2
*b^2 + 2*a*b^3 + 3*(3*a^2*b^2 + 8*a*b^3 + 8*b^4)*cosh(x)^2)*sinh(x)^2 + 8*(
a^2*b^2*cosh(x)^7 + 3*(a^2*b^2 + 2*a*b^3)*cosh(x)^5 + (3*a^2*b^2 + 8*a*b^3
+ 8*b^4)*cosh(x)^3 + (a^2*b^2 + 2*a*b^3)*cosh(x))*sinh(x))*sqrt(-a)*arctan(
sqrt(2)*((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2
+ a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cos
h(x)*sinh(x) + sinh(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*s
inh(x)^3 + (a^2 + a*b)*sinh(x)^4 + (2*a^2 + 3*a*b)*cosh(x)^2 + (6*(a^2 + a*

```
b)*cosh(x)^2 + 2*a^2 + 3*a*b)*sinh(x)^2 + a^2 + 2*(2*(a^2 + a*b)*cosh(x)^3
+ (2*a^2 + 3*a*b)*cosh(x))*sinh(x))) + 3*(a^2*b^2*cosh(x)^8 + 8*a^2*b^2*cos
h(x)*sinh(x)^7 + a^2*b^2*sinh(x)^8 + 4*(a^2*b^2 + 2*a*b^3)*cosh(x)^6 + 4*(7
*a^2*b^2*cosh(x)^2 + a^2*b^2 + 2*a*b^3)*sinh(x)^6 + 8*(7*a^2*b^2*cosh(x)^3
+ 3*(a^2*b^2 + 2*a*b^3)*cosh(x))*sinh(x)^5 + 2*(3*a^2*b^2 + 8*a*b^3 + 8*b^4
)*cosh(x)^4 + 2*(35*a^2*b^2*cosh(x)^4 + 3*a^2*b^2 + 8*a*b^3 + 8*b^4 + 30*(a
^2*b^2 + 2*a*b^3)*cosh(x)^2)*sinh(x)^4 + a^2*b^2 + 8*(7*a^2*b^2*cosh(x)^5 +
 10*(a^2*b^2 + 2*a*b^3)*cosh(x)^3 + (3*a^2*b^2 + 8*a*b^3 + 8*b^4)*cosh(x))*
sinh(x)^3 + 4*(a^2*b^2 + 2*a*b^3)*cosh(x)^2 + 4*(7*a^2*b^2*cosh(x)^6 + 15*(
a^2*b^2 + 2*a*b^3)*cosh(x)^4 + a^2*b^2 + 2*a*b^3 + 3*(3*a^2*b^2 + 8*a*b^3 +
 8*b^4)*cosh(x)^2)*sinh(x)^2 + 8*(a^2*b^2*cosh(x)^7 + 3*(a^2*b^2 + 2*a*b^3)
*cosh(x)^5 + (3*a^2*b^2 + 8*a*b^3 + 8*b^4)*cosh(x)^3 + (a^2*b^2 + 2*a*b^3)*
cosh(x))*sinh(x))*sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) +
sinh(x)^2 - 1)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)
^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 +
 a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^
2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) - 4*sqrt(2)*((a^4 - a
^3*b - 2*a^2*b^2)*cosh(x)^6 + 6*(a^4 - a^3*b - 2*a^2*b^2)*cosh(x)*sinh(x)^5
 + (a^4 - a^3*b - 2*a^2*b^2)*sinh(x)^6 + 3*(a^4 + a^3*b - 2*a^2*b^2 - 2*a*b
^3)*cosh(x)^4 + 3*(a^4 + a^3*b - 2*a^2*b^2 - 2*a*b^3 + 5*(a^4 - a^3*b - 2*a
^2*b^2)*cosh(x)^2)*sinh(x)^4 + a^4 - a^3*b - 2*a^2*b^2 + 4*(5*(a^4 - a^3*b
- 2*a^2*b^2)*cosh(x)^3 + 3*(a^4 + a^3*b - 2*a^2*b^2 - 2*a*b^3)*cosh(x))*sin
h(x)^3 + 3*(a^4 + a^3*b - 2*a^2*b^2 - 2*a*b^3)*cosh(x)^2 + 3*(5*(a^4 - a^3*
b - 2*a^2*b^2)*cosh(x)^4 + a^4 + a^3*b - 2*a^2*b^2 - 2*a*b^3 + 6*(a^4 + a^3
*b - 2*a^2*b^2 - 2*a*b^3)*cosh(x)^2)*sinh(x)^2 + 6*((a^4 - a^3*b - 2*a^2*b^
2)*cosh(x)^5 + 2*(a^4 + a^3*b - 2*a^2*b^2 - 2*a*b^3)*cosh(x)^3 + (a^4 + a^3
*b - 2*a^2*b^2 - 2*a*b^3)*cosh(x))*sinh(x))*sqrt((a*cosh(x)^2 + a*sinh(x)^2
 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a^5*b^2*cosh(x)^
8 + 8*a^5*b^2*cosh(x)*sinh(x)^7 + a^5*b^2*sinh(x)^8 + a^5*b^2 + 4*(a^5*b^2
+ 2*a^4*b^3)*cosh(x)^6 + 4*(7*a^5*b^2*cosh(x)^2 + a^5*b^2 + 2*a^4*b^3)*sinh
(x)^6 + 8*(7*a^5*b^2*cosh(x)^3 + 3*(a^5*b^2 + 2*a^4*b^3)*cosh(x))*sinh(x)^5
 + 2*(3*a^5*b^2 + 8*a^4*b^3 + 8*a^3*b^4)*cosh(x)^4 + 2*(35*a^5*b^2*cosh(x)^
4 + 3*a^5*b^2 + 8*a^4*b^3 + 8*a^3*b^4 + 30*(a^5*b^2 + 2*a^4*b^3)*cosh(x)^2)
*sinh(x)^4 + 8*(7*a^5*b^2*cosh(x)^5 + 10*(a^5*b^2 + 2*a^4*b^3)*cosh(x)^3 +
(3*a^5*b^2 + 8*a^4*b^3 + 8*a^3*b^4)*cosh(x))*sinh(x)^3 + 4*(a^5*b^2 + 2*a^4
*b^3)*cosh(x)^2 + 4*(7*a^5*b^2*cosh(x)^6 + a^5*b^2 + 2*a^4*b^3 + 15*(a^5*b^
2 + 2*a^4*b^3)*cosh(x)^4 + 3*(3*a^5*b^2 + 8*a^4*b^3 + 8*a^3*b^4)*cosh(x)^2)
*sinh(x)^2 + 8*(a^5*b^2*cosh(x)^7 + 3*(a^5*b^2 + 2*a^4*b^3)*cosh(x)^5 + (3*
a^5*b^2 + 8*a^4*b^3 + 8*a^3*b^4)*cosh(x)^3 + (a^5*b^2 + 2*a^4*b^3)*cosh(x))
*sinh(x))]
```

giac [F(-2)]   time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+b*sech(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, replacing 0 by ` u`, a substitution
 variable should perhaps be purged.Warning, replacing 0 by ` u`, a substitu
tion variable should perhaps be purged.Warning, replacing 0 by ` u`, a subs
titution variable should perhaps be purged.Error: Bad Argument Type
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{\tanh^5(x)}{\left(a + b\mathrm{sech}(x)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^5/(a+b*sech(x)^2)^(5/2),x)
```

```
[Out] int(tanh(x)^5/(a+b*sech(x)^2)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)^5}{\left(b\,\mathrm{sech}(x)^2 + a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+b*sech(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)^5/(b*sech(x)^2 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tanh(x)^5}{\left(a + \frac{b}{\cosh(x)^2}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^5/(a + b/cosh(x)^2)^(5/2),x)
```

```
[Out] int(tanh(x)^5/(a + b/cosh(x)^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^5(x)}{\left(a + b\,\mathrm{sech}^2(x)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**5/(a+b*sech(x)**2)**(5/2),x)
[Out] Integral(tanh(x)**5/(a + b*sech(x)**2)**(5/2), x)
```

3.213 $\int \frac{\tanh^4(x)}{\left(a+b\,\mathbf{sech}^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=90

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{a^{5/2}}+\frac{(a-3b)\tanh(x)}{3a^2b\sqrt{a-b\tanh^2(x)+b}}-\frac{(a+b)\tanh(x)}{3ab\left(a-b\tanh^2(x)+b\right)^{3/2}}$$

```
[Out] arctanh(a^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))/a^(5/2)+1/3*(a-3*b)*tanh(x
)/a^2/b/(a+b-b*tanh(x)^2)^(1/2)-1/3*(a+b)*tanh(x)/a/b/(a+b-b*tanh(x)^2)^(3/
2)
```

Rubi [A] time = 0.26, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {4141, 1975, 470, 527, 12, 377, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{a^{5/2}}+\frac{(a-3b)\tanh(x)}{3a^2b\sqrt{a-b\tanh^2(x)+b}}-\frac{(a+b)\tanh(x)}{3ab\left(a-b\tanh^2(x)+b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^4/(a + b*Sech[x]^2)^(5/2),x]
```

```
[Out] ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/a^(5/2) - ((a + b)*Tan
h[x])/(3*a*b*(a + b - b*Tanh[x]^2)^(3/2)) + ((a - 3*b)*Tanh[x])/(3*a^2*b*Sq
rt[a + b - b*Tanh[x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\tanh^4(x)}{\left(a+b\text{sech}^2(x)\right)^{5/2}}\,dx = \text{Subst}\left(\int \frac{x^4}{\left(1-x^2\right)\left(a+b\left(1-x^2\right)\right)^{5/2}}\,dx, x, \tanh(x)\right)$$

$$= \text{Subst}\left(\int \frac{x^4}{\left(1-x^2\right)\left(a+b-bx^2\right)^{5/2}}\,dx, x, \tanh(x)\right)$$

$$= -\frac{(a+b)\tanh(x)}{3ab\left(a+b-b\tanh^2(x)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{a+b+(-a+2b)x^2}{\left(1-x^2\right)\left(a+b-bx^2\right)^{3/2}}\,dx, x, \tanh(x)\right)}{3ab}$$

$$= -\frac{(a+b)\tanh(x)}{3ab\left(a+b-b\tanh^2(x)\right)^{3/2}} + \frac{(a-3b)\tanh(x)}{3a^2b\sqrt{a+b-b\tanh^2(x)}} - \frac{\text{Subst}\left(\int -\frac{3b(a+b)}{\left(1-x^2\right)\sqrt{a+b-bx^2}}\right.}{3a^2b(a+b}$$

$$= -\frac{(a+b)\tanh(x)}{3ab\left(a+b-b\tanh^2(x)\right)^{3/2}} + \frac{(a-3b)\tanh(x)}{3a^2b\sqrt{a+b-b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)\sqrt{a+b-bx^2}}\right.}{a^2}$$

$$= -\frac{(a+b)\tanh(x)}{3ab\left(a+b-b\tanh^2(x)\right)^{3/2}} + \frac{(a-3b)\tanh(x)}{3a^2b\sqrt{a+b-b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{}{\sqrt{a+}}\right.}{a^2}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\,\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{a^{5/2}} - \frac{(a+b)\tanh(x)}{3ab\left(a+b-b\tanh^2(x)\right)^{3/2}} + \frac{(a-3b)\tanh(x)}{3a^2b\sqrt{a+b-b\tanh^2(x)}}$$

Mathematica [B] time = 2.17, size = 290, normalized size = 3.22

$$\frac{\operatorname{sech}^4(x)\left(\frac{\sqrt{2}\operatorname{csch}(x)\operatorname{sech}(x)\left(-\frac{16\left(a\sinh^2(x)+a+b\right)\left(\frac{a\sinh^2(x)}{a+b}+1\right)\left(\frac{a^2(a+b)\sinh^4(x)}{\left(a\sinh^2(x)+a+b\right)^2}+\frac{3a(a+b)\sinh^2(x)}{a\sinh^2(x)+a+b}-\frac{3\sqrt{a}\sqrt{a+b}\sinh(x)\sinh^{-1}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{a+b}}\right)}{\sqrt{\frac{a\sinh^2(x)+a+b}{a+b}}}\right)}{a^3}+\frac{12\sinh^4(x)}{a+b}+\cdots\right)}{\left(a\sinh^2(x)+a+b\right)^{3/2}}\cdots\right)}{384\,(\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^4/(a + b*Sech[x]^2)^(5/2),x]
```

```
[Out] (Sech[x]^4*((Sqrt[2]*(a + 2*b + a*Cosh[2*x])^(5/2)*Csch[x]*Sech[x]*(Sinh[x]
^2/(a + b) + (12*Sinh[x]^4)/(a + b) + (2*Sinh[x]^2*(a + b + a*Sinh[x]^2))/(
a + b)^2 - (16*(a + b + a*Sinh[x]^2)*(1 + (a*Sinh[x]^2)/(a + b))*((a^2*(a +
 b)*Sinh[x]^4)/(a + b + a*Sinh[x]^2)^2 + (3*a*(a + b)*Sinh[x]^2)/(a + b + a
*Sinh[x]^2) - (3*Sqrt[a]*Sqrt[a + b]*ArcSinh[(Sqrt[a]*Sinh[x])/Sqrt[a + b]]
*Sinh[x])/Sqrt[(a + b + a*Sinh[x]^2)/(a + b)]))/a^3))/(a + b + a*Sinh[x]^2)
^(3/2) + (8*(a + 2*b + a*Cosh[2*x])*(2*a + 3*b + a*Cosh[2*x])*Tanh[x])/(a +
 b)^2 - (12*(a + 2*b + a*Cosh[2*x])*(b + (3*a + 2*b)*Cosh[2*x])*Tanh[x])/(a
 + b)^2))/(384*(a + b*Sech[x]^2)^(5/2))
```

fricas [B] time = 0.70, size = 3559, normalized size = 39.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*sech(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(a^2*cosh(x)^8 + 8*a^2*cosh(x)*sinh(x)^7 + a^2*sinh(x)^8 + 4*(a^2
+ 2*a*b)*cosh(x)^6 + 4*(7*a^2*cosh(x)^2 + a^2 + 2*a*b)*sinh(x)^6 + 8*(7*a^2
*cosh(x)^3 + 3*(a^2 + 2*a*b)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 8*a*b + 8*b^2)
*cosh(x)^4 + 2*(35*a^2*cosh(x)^4 + 30*(a^2 + 2*a*b)*cosh(x)^2 + 3*a^2 + 8*a
*b + 8*b^2)*sinh(x)^4 + 8*(7*a^2*cosh(x)^5 + 10*(a^2 + 2*a*b)*cosh(x)^3 + (
3*a^2 + 8*a*b + 8*b^2)*cosh(x))*sinh(x)^3 + 4*(a^2 + 2*a*b)*cosh(x)^2 + 4*(
```

```
7*a^2*cosh(x)^6 + 15*(a^2 + 2*a*b)*cosh(x)^4 + 3*(3*a^2 + 8*a*b + 8*b^2)*co
sh(x)^2 + a^2 + 2*a*b)*sinh(x)^2 + a^2 + 8*(a^2*cosh(x)^7 + 3*(a^2 + 2*a*b)
*cosh(x)^5 + (3*a^2 + 8*a*b + 8*b^2)*cosh(x)^3 + (a^2 + 2*a*b)*cosh(x))*sin
h(x))*sqrt(a)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh
(x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*si
nh(x)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3
 + 4*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a
*b^2 - 30*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(
a*b^2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3
 + 2*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*c
osh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2
 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2
*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b
^2*cosh(x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^
2*cosh(x)^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cos
h(x)^3 - (a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh
(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*
cosh(x)^7 - 3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3
 + (a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*
cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*c
osh(x)*sinh(x)^5 + sinh(x)^6)) + 3*(a^2*cosh(x)^8 + 8*a^2*cosh(x)*sinh(x)^7
 + a^2*sinh(x)^8 + 4*(a^2 + 2*a*b)*cosh(x)^6 + 4*(7*a^2*cosh(x)^2 + a^2 + 2
*a*b)*sinh(x)^6 + 8*(7*a^2*cosh(x)^3 + 3*(a^2 + 2*a*b)*cosh(x))*sinh(x)^5 +
 2*(3*a^2 + 8*a*b + 8*b^2)*cosh(x)^4 + 2*(35*a^2*cosh(x)^4 + 30*(a^2 + 2*a*
b)*cosh(x)^2 + 3*a^2 + 8*a*b + 8*b^2)*sinh(x)^4 + 8*(7*a^2*cosh(x)^5 + 10*(
a^2 + 2*a*b)*cosh(x)^3 + (3*a^2 + 8*a*b + 8*b^2)*cosh(x))*sinh(x)^3 + 4*(a^
2 + 2*a*b)*cosh(x)^2 + 4*(7*a^2*cosh(x)^6 + 15*(a^2 + 2*a*b)*cosh(x)^4 + 3*
(3*a^2 + 8*a*b + 8*b^2)*cosh(x)^2 + a^2 + 2*a*b)*sinh(x)^2 + a^2 + 8*(a^2*c
osh(x)^7 + 3*(a^2 + 2*a*b)*cosh(x)^5 + (3*a^2 + 8*a*b + 8*b^2)*cosh(x)^3 +
(a^2 + 2*a*b)*cosh(x))*sinh(x))*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sin
h(x)^3 + a*sinh(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + b)*sinh
(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*sqr
t((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + si
nh(x)^2)) + 4*(a*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*c
osh(x)*sinh(x) + sinh(x)^2)) - 16*sqrt(2)*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*si
nh(x)^5 + a^2*sinh(x)^6 + 3*a*b*cosh(x)^4 + 3*(5*a^2*cosh(x)^2 + a*b)*sinh(
x)^4 - 3*a*b*cosh(x)^2 + 4*(5*a^2*cosh(x)^3 + 3*a*b*cosh(x))*sinh(x)^3 + 3*
(5*a^2*cosh(x)^4 + 6*a*b*cosh(x)^2 - a*b)*sinh(x)^2 - a^2 + 6*(a^2*cosh(x)^
5 + 2*a*b*cosh(x)^3 - a*b*cosh(x))*sinh(x))*sqrt((a*cosh(x)^2 + a*sinh(x)^2
 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a^5*cosh(x)^8 +
8*a^5*cosh(x)*sinh(x)^7 + a^5*sinh(x)^8 + 4*(a^5 + 2*a^4*b)*cosh(x)^6 + 4*(
7*a^5*cosh(x)^2 + a^5 + 2*a^4*b)*sinh(x)^6 + 8*(7*a^5*cosh(x)^3 + 3*(a^5 +
2*a^4*b)*cosh(x))*sinh(x)^5 + a^5 + 2*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*cosh(x)
^4 + 2*(35*a^5*cosh(x)^4 + 3*a^5 + 8*a^4*b + 8*a^3*b^2 + 30*(a^5 + 2*a^4*b)
*cosh(x)^2)*sinh(x)^4 + 8*(7*a^5*cosh(x)^5 + 10*(a^5 + 2*a^4*b)*cosh(x)^3 +
```

```
 (3*a^5 + 8*a^4*b + 8*a^3*b^2)*cosh(x))*sinh(x)^3 + 4*(a^5 + 2*a^4*b)*cosh(
x)^2 + 4*(7*a^5*cosh(x)^6 + a^5 + 2*a^4*b + 15*(a^5 + 2*a^4*b)*cosh(x)^4 +
3*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*cosh(x)^2)*sinh(x)^2 + 8*(a^5*cosh(x)^7 + 3
*(a^5 + 2*a^4*b)*cosh(x)^5 + (3*a^5 + 8*a^4*b + 8*a^3*b^2)*cosh(x)^3 + (a^5
 + 2*a^4*b)*cosh(x))*sinh(x)), -1/6*(3*(a^2*cosh(x)^8 + 8*a^2*cosh(x)*sinh(
x)^7 + a^2*sinh(x)^8 + 4*(a^2 + 2*a*b)*cosh(x)^6 + 4*(7*a^2*cosh(x)^2 + a^2
 + 2*a*b)*sinh(x)^6 + 8*(7*a^2*cosh(x)^3 + 3*(a^2 + 2*a*b)*cosh(x))*sinh(x)
^5 + 2*(3*a^2 + 8*a*b + 8*b^2)*cosh(x)^4 + 2*(35*a^2*cosh(x)^4 + 30*(a^2 +
2*a*b)*cosh(x)^2 + 3*a^2 + 8*a*b + 8*b^2)*sinh(x)^4 + 8*(7*a^2*cosh(x)^5 +
10*(a^2 + 2*a*b)*cosh(x)^3 + (3*a^2 + 8*a*b + 8*b^2)*cosh(x))*sinh(x)^3 + 4
*(a^2 + 2*a*b)*cosh(x)^2 + 4*(7*a^2*cosh(x)^6 + 15*(a^2 + 2*a*b)*cosh(x)^4
+ 3*(3*a^2 + 8*a*b + 8*b^2)*cosh(x)^2 + a^2 + 2*a*b)*sinh(x)^2 + a^2 + 8*(a
^2*cosh(x)^7 + 3*(a^2 + 2*a*b)*cosh(x)^5 + (3*a^2 + 8*a*b + 8*b^2)*cosh(x)^
3 + (a^2 + 2*a*b)*cosh(x))*sinh(x))*sqrt(-a)*arctan(sqrt(2)*(b*cosh(x)^2 +
2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(
x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*b*cosh(x)^4
 + 4*a*b*cosh(x)*sinh(x)^3 + a*b*sinh(x)^4 - (a^2 + 3*a*b)*cosh(x)^2 + (6*a
*b*cosh(x)^2 - a^2 - 3*a*b)*sinh(x)^2 - a^2 + 2*(2*a*b*cosh(x)^3 - (a^2 + 3
*a*b)*cosh(x))*sinh(x))) + 3*(a^2*cosh(x)^8 + 8*a^2*cosh(x)*sinh(x)^7 + a^2
*sinh(x)^8 + 4*(a^2 + 2*a*b)*cosh(x)^6 + 4*(7*a^2*cosh(x)^2 + a^2 + 2*a*b)*
sinh(x)^6 + 8*(7*a^2*cosh(x)^3 + 3*(a^2 + 2*a*b)*cosh(x))*sinh(x)^5 + 2*(3*
a^2 + 8*a*b + 8*b^2)*cosh(x)^4 + 2*(35*a^2*cosh(x)^4 + 30*(a^2 + 2*a*b)*cos
h(x)^2 + 3*a^2 + 8*a*b + 8*b^2)*sinh(x)^4 + 8*(7*a^2*cosh(x)^5 + 10*(a^2 +
2*a*b)*cosh(x)^3 + (3*a^2 + 8*a*b + 8*b^2)*cosh(x))*sinh(x)^3 + 4*(a^2 + 2*
a*b)*cosh(x)^2 + 4*(7*a^2*cosh(x)^6 + 15*(a^2 + 2*a*b)*cosh(x)^4 + 3*(3*a^2
 + 8*a*b + 8*b^2)*cosh(x)^2 + a^2 + 2*a*b)*sinh(x)^2 + a^2 + 8*(a^2*cosh(x)
^7 + 3*(a^2 + 2*a*b)*cosh(x)^5 + (3*a^2 + 8*a*b + 8*b^2)*cosh(x)^3 + (a^2 +
 2*a*b)*cosh(x))*sinh(x))*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cosh(x)^
2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*
cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a)) + 8*sqrt(2)*(a^2*cosh(x
)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)^6 + 3*a*b*cosh(x)^4 + 3*(5*a^2*
cosh(x)^2 + a*b)*sinh(x)^4 - 3*a*b*cosh(x)^2 + 4*(5*a^2*cosh(x)^3 + 3*a*b*c
osh(x))*sinh(x)^3 + 3*(5*a^2*cosh(x)^4 + 6*a*b*cosh(x)^2 - a*b)*sinh(x)^2 -
 a^2 + 6*(a^2*cosh(x)^5 + 2*a*b*cosh(x)^3 - a*b*cosh(x))*sinh(x))*sqrt((a*c
osh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^
2)))/(a^5*cosh(x)^8 + 8*a^5*cosh(x)*sinh(x)^7 + a^5*sinh(x)^8 + 4*(a^5 + 2*
a^4*b)*cosh(x)^6 + 4*(7*a^5*cosh(x)^2 + a^5 + 2*a^4*b)*sinh(x)^6 + 8*(7*a^5
*cosh(x)^3 + 3*(a^5 + 2*a^4*b)*cosh(x))*sinh(x)^5 + a^5 + 2*(3*a^5 + 8*a^4*
b + 8*a^3*b^2)*cosh(x)^4 + 2*(35*a^5*cosh(x)^4 + 3*a^5 + 8*a^4*b + 8*a^3*b^
2 + 30*(a^5 + 2*a^4*b)*cosh(x)^2)*sinh(x)^4 + 8*(7*a^5*cosh(x)^5 + 10*(a^5
+ 2*a^4*b)*cosh(x)^3 + (3*a^5 + 8*a^4*b + 8*a^3*b^2)*cosh(x))*sinh(x)^3 + 4
*(a^5 + 2*a^4*b)*cosh(x)^2 + 4*(7*a^5*cosh(x)^6 + a^5 + 2*a^4*b + 15*(a^5 +
 2*a^4*b)*cosh(x)^4 + 3*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*cosh(x)^2)*sinh(x)^2
+ 8*(a^5*cosh(x)^7 + 3*(a^5 + 2*a^4*b)*cosh(x)^5 + (3*a^5 + 8*a^4*b + 8*a^3
*b^2)*cosh(x)^3 + (a^5 + 2*a^4*b)*cosh(x))*sinh(x))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*sech(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, replacing 0 by ` u`, a substitution
 variable should perhaps be purged.Warning, replacing 0 by ` u`, a substitu
tion variable should perhaps be purged.Warning, replacing 0 by ` u`, a subs
titution variable should perhaps be purged.Error: Bad Argument Type
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{\tanh^4(x)}{\left(a + b\operatorname{sech}(x)^2\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(a+b*sech(x)^2)^(5/2),x)
```

```
[Out] int(tanh(x)^4/(a+b*sech(x)^2)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)^4}{\left(b \operatorname{sech}(x)^2 + a\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*sech(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)^4/(b*sech(x)^2 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tanh(x)^4}{\left(a + \frac{b}{\cosh(x)^2}\right)^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(a + b/cosh(x)^2)^(5/2),x)
```

```
[Out] int(tanh(x)^4/(a + b/cosh(x)^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^4(x)}{\left(a + b\operatorname{sech}^2(x)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**4/(a+b*sech(x)**2)**(5/2),x)
```

```
[Out] Integral(tanh(x)**4/(a + b*sech(x)**2)**(5/2), x)
```

3.214 $\int \frac{\tanh^3(x)}{\left(a+b\mathbf{sech}^2(x)\right)^{5/2}} dx$

Optimal. Leaf size=68

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{1}{a^2\sqrt{a+b\text{sech}^2(x)}}-\frac{a+b}{3ab\left(a+b\text{sech}^2(x)\right)^{3/2}}$$

```
[Out] arctanh((a+b*sech(x)^2)^(1/2)/a^(1/2))/a^(5/2)+1/3*(-a-b)/a/b/(a+b*sech(x)^
2)^(3/2)-1/a^2/(a+b*sech(x)^2)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {4139, 446, 78, 51, 63, 208}

$$-\frac{1}{a^2\sqrt{a+b\text{sech}^2(x)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{a+b}{3ab\left(a+b\text{sech}^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^3/(a + b*Sech[x]^2)^(5/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]]/a^(5/2) - (a + b)/(3*a*b*(a + b*Sech
[x]^2)^(3/2)) - 1/(a^2*Sqrt[a + b*Sech[x]^2])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \frac{\tanh^3(x)}{\left(a+b\operatorname{sech}^2(x)\right)^{5/2}}\,dx = \operatorname{Subst}\left(\int \frac{-1+x^2}{x\left(a+bx^2\right)^{5/2}}\,dx, x, \operatorname{sech}(x)\right)$$

$$= \frac{1}{2}\operatorname{Subst}\left(\int \frac{-1+x}{x(a+bx)^{5/2}}\,dx, x, \operatorname{sech}^2(x)\right)$$

$$= -\frac{a+b}{3ab\left(a+b\operatorname{sech}^2(x)\right)^{3/2}} - \frac{\operatorname{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, \operatorname{sech}^2(x)\right)}{2a}$$

$$= -\frac{a+b}{3ab\left(a+b\operatorname{sech}^2(x)\right)^{3/2}} - \frac{1}{a^2\sqrt{a+b\operatorname{sech}^2(x)}} - \frac{\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \operatorname{sech}^2(x)\right)}{2a^2}$$

$$= -\frac{a+b}{3ab\left(a+b\operatorname{sech}^2(x)\right)^{3/2}} - \frac{1}{a^2\sqrt{a+b\operatorname{sech}^2(x)}} - \frac{\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{sech}^2(}\right.}{a^2b}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{a^{5/2}} - \frac{a+b}{3ab\left(a+b\operatorname{sech}^2(x)\right)^{3/2}} - \frac{1}{a^2\sqrt{a+b\operatorname{sech}^2(x)}}$$

Mathematica [A] time = 1.00, size = 124, normalized size = 1.82

$$-\frac{\operatorname{sech}^5(x)(a\cosh(2x)+a+2b)\left(a^{3/2}(a+4b)\cosh(3x)+3\sqrt{a}\,(a+2b)^2\cosh(x)-3\sqrt{2}\,b(a\cosh(2x)+a+2b)^{3/2}\right.}{24a^{5/2}b\left(a+b\operatorname{sech}^2(x)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^3/(a + b*Sech[x]^2)^(5/2),x]
```

```
[Out] -1/24*((a + 2*b + a*Cosh[2*x])*(3*Sqrt[a]*(a + 2*b)^2*Cosh[x] + a^(3/2)*(a
+ 4*b)*Cosh[3*x] - 3*Sqrt[2]*b*(a + 2*b + a*Cosh[2*x])^(3/2)*Log[Sqrt[2]*Sq
rt[a]*Cosh[x] + Sqrt[a + 2*b + a*Cosh[2*x]]])*Sech[x]^5)/(a^(5/2)*b*(a + b*
Sech[x]^2)^(5/2))
```

fricas [B] time = 0.73, size = 4644, normalized size = 68.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*sech(x)^2)^(5/2),x, algorithm="fricas")

[Out] [1/12*(3*(a^2*b*cosh(x)^8 + 8*a^2*b*cosh(x)*sinh(x)^7 + a^2*b*sinh(x)^8 + 4
*(a^2*b + 2*a*b^2)*cosh(x)^6 + 4*(7*a^2*b*cosh(x)^2 + a^2*b + 2*a*b^2)*sinh
(x)^6 + 8*(7*a^2*b*cosh(x)^3 + 3*(a^2*b + 2*a*b^2)*cosh(x))*sinh(x)^5 + 2*(
3*a^2*b + 8*a*b^2 + 8*b^3)*cosh(x)^4 + 2*(35*a^2*b*cosh(x)^4 + 3*a^2*b + 8*
a*b^2 + 8*b^3 + 30*(a^2*b + 2*a*b^2)*cosh(x)^2)*sinh(x)^4 + 8*(7*a^2*b*cosh
(x)^5 + 10*(a^2*b + 2*a*b^2)*cosh(x)^3 + (3*a^2*b + 8*a*b^2 + 8*b^3)*cosh(x
))*sinh(x)^3 + a^2*b + 4*(a^2*b + 2*a*b^2)*cosh(x)^2 + 4*(7*a^2*b*cosh(x)^6
 + 15*(a^2*b + 2*a*b^2)*cosh(x)^4 + a^2*b + 2*a*b^2 + 3*(3*a^2*b + 8*a*b^2
+ 8*b^3)*cosh(x)^2)*sinh(x)^2 + 8*(a^2*b*cosh(x)^7 + 3*(a^2*b + 2*a*b^2)*co
sh(x)^5 + (3*a^2*b + 8*a*b^2 + 8*b^3)*cosh(x)^3 + (a^2*b + 2*a*b^2)*cosh(x)
)*sinh(x))*sqrt(a)*log(((a^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*
b + a*b^2)*cosh(x)*sinh(x)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3
 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3
+ 14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b +
a*b^2)*cosh(x)^3 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 +
 (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(
x)^4 + 6*a^3 + 14*a^2*b + 9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*co
sh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3 +
5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*
sinh(x)^3 + a^3 + 2*(2*a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*
b^2)*cosh(x)^6 + 15*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3 + 3
*a^2*b + 3*(6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*((a^
2 + 2*a*b + b^2)*cosh(x)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2
 + 2*a*b + b^2)*sinh(x)^6 + 3*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2
*a*b + b^2)*cosh(x)^2 + a^2 + 2*a*b + b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b +
b^2)*cosh(x)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4*a*b)
*cosh(x)^2 + (15*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*cos
h(x)^2 + 3*a^2 + 4*a*b)*sinh(x)^2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^
5 + 6*(a^2 + 2*a*b + b^2)*cosh(x)^3 + (3*a^2 + 4*a*b)*cosh(x))*sinh(x))*sqr
t(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh
(x) + sinh(x)^2)) + 4*(2*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^7 + 3*(2*a^3 + 5*a
^2*b + 4*a*b^2 + b^3)*cosh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^3 +
(2*a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*c
osh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*co
sh(x)*sinh(x)^5 + sinh(x)^6)) + 3*(a^2*b*cosh(x)^8 + 8*a^2*b*cosh(x)*sinh(x
)^7 + a^2*b*sinh(x)^8 + 4*(a^2*b + 2*a*b^2)*cosh(x)^6 + 4*(7*a^2*b*cosh(x)^
2 + a^2*b + 2*a*b^2)*sinh(x)^6 + 8*(7*a^2*b*cosh(x)^3 + 3*(a^2*b + 2*a*b^2)
*cosh(x))*sinh(x)^5 + 2*(3*a^2*b + 8*a*b^2 + 8*b^3)*cosh(x)^4 + 2*(35*a^2*b
*cosh(x)^4 + 3*a^2*b + 8*a*b^2 + 8*b^3 + 30*(a^2*b + 2*a*b^2)*cosh(x)^2)*si
nh(x)^4 + 8*(7*a^2*b*cosh(x)^5 + 10*(a^2*b + 2*a*b^2)*cosh(x)^3 + (3*a^2*b
+ 8*a*b^2 + 8*b^3)*cosh(x))*sinh(x)^3 + a^2*b + 4*(a^2*b + 2*a*b^2)*cosh(x)
^2 + 4*(7*a^2*b*cosh(x)^6 + 15*(a^2*b + 2*a*b^2)*cosh(x)^4 + a^2*b + 2*a*b^
2 + 3*(3*a^2*b + 8*a*b^2 + 8*b^3)*cosh(x)^2)*sinh(x)^2 + 8*(a^2*b*cosh(x)^7
```

```
 + 3*(a^2*b + 2*a*b^2)*cosh(x)^5 + (3*a^2*b + 8*a*b^2 + 8*b^3)*cosh(x)^3 +
(a^2*b + 2*a*b^2)*cosh(x))*sinh(x))*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)
*sinh(x)^3 + a*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*a*cosh(x)^2 + b)*sinh(x)^2
+ sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a)*sqrt((a*c
osh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^
2)) + 4*(a*cosh(x)^3 + b*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(
x) + sinh(x)^2)) - 4*sqrt(2)*((a^3 + 4*a^2*b)*cosh(x)^6 + 6*(a^3 + 4*a^2*b)
*cosh(x)*sinh(x)^5 + (a^3 + 4*a^2*b)*sinh(x)^6 + 3*(a^3 + 4*a^2*b + 4*a*b^2
)*cosh(x)^4 + 3*(a^3 + 4*a^2*b + 4*a*b^2 + 5*(a^3 + 4*a^2*b)*cosh(x)^2)*sin
h(x)^4 + 4*(5*(a^3 + 4*a^2*b)*cosh(x)^3 + 3*(a^3 + 4*a^2*b + 4*a*b^2)*cosh(
x))*sinh(x)^3 + a^3 + 4*a^2*b + 3*(a^3 + 4*a^2*b + 4*a*b^2)*cosh(x)^2 + 3*(
5*(a^3 + 4*a^2*b)*cosh(x)^4 + a^3 + 4*a^2*b + 4*a*b^2 + 6*(a^3 + 4*a^2*b +
4*a*b^2)*cosh(x)^2)*sinh(x)^2 + 6*((a^3 + 4*a^2*b)*cosh(x)^5 + 2*(a^3 + 4*a
^2*b + 4*a*b^2)*cosh(x)^3 + (a^3 + 4*a^2*b + 4*a*b^2)*cosh(x))*sinh(x))*sqr
t((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + si
nh(x)^2)))/(a^5*b*cosh(x)^8 + 8*a^5*b*cosh(x)*sinh(x)^7 + a^5*b*sinh(x)^8 +
 4*(a^5*b + 2*a^4*b^2)*cosh(x)^6 + 4*(7*a^5*b*cosh(x)^2 + a^5*b + 2*a^4*b^2
)*sinh(x)^6 + a^5*b + 8*(7*a^5*b*cosh(x)^3 + 3*(a^5*b + 2*a^4*b^2)*cosh(x))
*sinh(x)^5 + 2*(3*a^5*b + 8*a^4*b^2 + 8*a^3*b^3)*cosh(x)^4 + 2*(35*a^5*b*co
sh(x)^4 + 3*a^5*b + 8*a^4*b^2 + 8*a^3*b^3 + 30*(a^5*b + 2*a^4*b^2)*cosh(x)^
2)*sinh(x)^4 + 8*(7*a^5*b*cosh(x)^5 + 10*(a^5*b + 2*a^4*b^2)*cosh(x)^3 + (3
*a^5*b + 8*a^4*b^2 + 8*a^3*b^3)*cosh(x))*sinh(x)^3 + 4*(a^5*b + 2*a^4*b^2)*
cosh(x)^2 + 4*(7*a^5*b*cosh(x)^6 + a^5*b + 2*a^4*b^2 + 15*(a^5*b + 2*a^4*b^
2)*cosh(x)^4 + 3*(3*a^5*b + 8*a^4*b^2 + 8*a^3*b^3)*cosh(x)^2)*sinh(x)^2 + 8
*(a^5*b*cosh(x)^7 + 3*(a^5*b + 2*a^4*b^2)*cosh(x)^5 + (3*a^5*b + 8*a^4*b^2
+ 8*a^3*b^3)*cosh(x)^3 + (a^5*b + 2*a^4*b^2)*cosh(x))*sinh(x)), -1/6*(3*(a^
2*b*cosh(x)^8 + 8*a^2*b*cosh(x)*sinh(x)^7 + a^2*b*sinh(x)^8 + 4*(a^2*b + 2*
a*b^2)*cosh(x)^6 + 4*(7*a^2*b*cosh(x)^2 + a^2*b + 2*a*b^2)*sinh(x)^6 + 8*(7
*a^2*b*cosh(x)^3 + 3*(a^2*b + 2*a*b^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2*b + 8*
a*b^2 + 8*b^3)*cosh(x)^4 + 2*(35*a^2*b*cosh(x)^4 + 3*a^2*b + 8*a*b^2 + 8*b^
3 + 30*(a^2*b + 2*a*b^2)*cosh(x)^2)*sinh(x)^4 + 8*(7*a^2*b*cosh(x)^5 + 10*(
a^2*b + 2*a*b^2)*cosh(x)^3 + (3*a^2*b + 8*a*b^2 + 8*b^3)*cosh(x))*sinh(x)^3
 + a^2*b + 4*(a^2*b + 2*a*b^2)*cosh(x)^2 + 4*(7*a^2*b*cosh(x)^6 + 15*(a^2*b
 + 2*a*b^2)*cosh(x)^4 + a^2*b + 2*a*b^2 + 3*(3*a^2*b + 8*a*b^2 + 8*b^3)*cos
h(x)^2)*sinh(x)^2 + 8*(a^2*b*cosh(x)^7 + 3*(a^2*b + 2*a*b^2)*cosh(x)^5 + (3
*a^2*b + 8*a*b^2 + 8*b^3)*cosh(x)^3 + (a^2*b + 2*a*b^2)*cosh(x))*sinh(x))*s
qrt(-a)*arctan(sqrt(2)*((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a
+ b)*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(co
sh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 +
 a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 + (2*a^2 + 3*a*b)*cosh(x)^2
 + (6*(a^2 + a*b)*cosh(x)^2 + 2*a^2 + 3*a*b)*sinh(x)^2 + a^2 + 2*(2*(a^2 +
a*b)*cosh(x)^3 + (2*a^2 + 3*a*b)*cosh(x))*sinh(x))) + 3*(a^2*b*cosh(x)^8 +
8*a^2*b*cosh(x)*sinh(x)^7 + a^2*b*sinh(x)^8 + 4*(a^2*b + 2*a*b^2)*cosh(x)^6
 + 4*(7*a^2*b*cosh(x)^2 + a^2*b + 2*a*b^2)*sinh(x)^6 + 8*(7*a^2*b*cosh(x)^3
 + 3*(a^2*b + 2*a*b^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2*b + 8*a*b^2 + 8*b^3)*c
```

```
osh(x)^4 + 2*(35*a^2*b*cosh(x)^4 + 3*a^2*b + 8*a*b^2 + 8*b^3 + 30*(a^2*b +
2*a*b^2)*cosh(x)^2)*sinh(x)^4 + 8*(7*a^2*b*cosh(x)^5 + 10*(a^2*b + 2*a*b^2)
*cosh(x)^3 + (3*a^2*b + 8*a*b^2 + 8*b^3)*cosh(x))*sinh(x)^3 + a^2*b + 4*(a^
2*b + 2*a*b^2)*cosh(x)^2 + 4*(7*a^2*b*cosh(x)^6 + 15*(a^2*b + 2*a*b^2)*cosh
(x)^4 + a^2*b + 2*a*b^2 + 3*(3*a^2*b + 8*a*b^2 + 8*b^3)*cosh(x)^2)*sinh(x)^
2 + 8*(a^2*b*cosh(x)^7 + 3*(a^2*b + 2*a*b^2)*cosh(x)^5 + (3*a^2*b + 8*a*b^2
 + 8*b^3)*cosh(x)^3 + (a^2*b + 2*a*b^2)*cosh(x))*sinh(x))*sqrt(-a)*arctan(s
qrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a)*sqrt((a*cos
h(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)
)/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^
2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh
(x))*sinh(x) + a)) + 2*sqrt(2)*((a^3 + 4*a^2*b)*cosh(x)^6 + 6*(a^3 + 4*a^2*
b)*cosh(x)*sinh(x)^5 + (a^3 + 4*a^2*b)*sinh(x)^6 + 3*(a^3 + 4*a^2*b + 4*a*b
^2)*cosh(x)^4 + 3*(a^3 + 4*a^2*b + 4*a*b^2 + 5*(a^3 + 4*a^2*b)*cosh(x)^2)*s
inh(x)^4 + 4*(5*(a^3 + 4*a^2*b)*cosh(x)^3 + 3*(a^3 + 4*a^2*b + 4*a*b^2)*cos
h(x))*sinh(x)^3 + a^3 + 4*a^2*b + 3*(a^3 + 4*a^2*b + 4*a*b^2)*cosh(x)^2 + 3
*(5*(a^3 + 4*a^2*b)*cosh(x)^4 + a^3 + 4*a^2*b + 4*a*b^2 + 6*(a^3 + 4*a^2*b
+ 4*a*b^2)*cosh(x)^2)*sinh(x)^2 + 6*((a^3 + 4*a^2*b)*cosh(x)^5 + 2*(a^3 + 4
*a^2*b + 4*a*b^2)*cosh(x)^3 + (a^3 + 4*a^2*b + 4*a*b^2)*cosh(x))*sinh(x))*s
qrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) +
sinh(x)^2)))/(a^5*b*cosh(x)^8 + 8*a^5*b*cosh(x)*sinh(x)^7 + a^5*b*sinh(x)^8
 + 4*(a^5*b + 2*a^4*b^2)*cosh(x)^6 + 4*(7*a^5*b*cosh(x)^2 + a^5*b + 2*a^4*b
^2)*sinh(x)^6 + a^5*b + 8*(7*a^5*b*cosh(x)^3 + 3*(a^5*b + 2*a^4*b^2)*cosh(x
))*sinh(x)^5 + 2*(3*a^5*b + 8*a^4*b^2 + 8*a^3*b^3)*cosh(x)^4 + 2*(35*a^5*b*
cosh(x)^4 + 3*a^5*b + 8*a^4*b^2 + 8*a^3*b^3 + 30*(a^5*b + 2*a^4*b^2)*cosh(x
)^2)*sinh(x)^4 + 8*(7*a^5*b*cosh(x)^5 + 10*(a^5*b + 2*a^4*b^2)*cosh(x)^3 +
(3*a^5*b + 8*a^4*b^2 + 8*a^3*b^3)*cosh(x))*sinh(x)^3 + 4*(a^5*b + 2*a^4*b^2
)*cosh(x)^2 + 4*(7*a^5*b*cosh(x)^6 + a^5*b + 2*a^4*b^2 + 15*(a^5*b + 2*a^4*
b^2)*cosh(x)^4 + 3*(3*a^5*b + 8*a^4*b^2 + 8*a^3*b^3)*cosh(x)^2)*sinh(x)^2 +
 8*(a^5*b*cosh(x)^7 + 3*(a^5*b + 2*a^4*b^2)*cosh(x)^5 + (3*a^5*b + 8*a^4*b^
2 + 8*a^3*b^3)*cosh(x)^3 + (a^5*b + 2*a^4*b^2)*cosh(x))*sinh(x))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*sech(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, replacing 0 by ` u`, a substitution
 variable should perhaps be purged.Warning, replacing 0 by ` u`, a substitu
tion variable should perhaps be purged.Warning, replacing 0 by ` u`, a subs
titution variable should perhaps be purged.Error: Bad Argument Type
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{\tanh^3(x)}{\left(a + b\operatorname{sech}(x)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(a+b*sech(x)^2)^(5/2),x)
[Out] int(tanh(x)^3/(a+b*sech(x)^2)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)^3}{\left(b\operatorname{sech}(x)^2 + a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*sech(x)^2)^(5/2),x, algorithm="maxima")
[Out] integrate(tanh(x)^3/(b*sech(x)^2 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tanh(x)^3}{\left(a + \frac{b}{\cosh(x)^2}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(a + b/cosh(x)^2)^(5/2),x)
[Out] int(tanh(x)^3/(a + b/cosh(x)^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^3(x)}{\left(a + b\operatorname{sech}^2(x)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**3/(a+b*sech(x)**2)**(5/2),x)
[Out] Integral(tanh(x)**3/(a + b*sech(x)**2)**(5/2), x)
```

3.215 $\int \frac{\tanh^2(x)}{\left(a+b\,\mathrm{sech}^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=88

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{a^{5/2}}-\frac{(2a+3b)\tanh(x)}{3a^2(a+b)\sqrt{a-b\tanh^2(x)+b}}-\frac{\tanh(x)}{3a\left(a-b\tanh^2(x)+b\right)^{3/2}}$$

```
[Out] arctanh(a^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))/a^(5/2)-1/3*(2*a+3*b)*tanh
(x)/a^2/(a+b)/(a+b-b*tanh(x)^2)^(1/2)-1/3*tanh(x)/a/(a+b-b*tanh(x)^2)^(3/2)
```

Rubi [A] time = 0.25, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {4141, 1975, 471, 527, 12, 377, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{a^{5/2}}-\frac{(2a+3b)\tanh(x)}{3a^2(a+b)\sqrt{a-b\tanh^2(x)+b}}-\frac{\tanh(x)}{3a\left(a-b\tanh^2(x)+b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^2/(a + b*Sech[x]^2)^(5/2),x]
```

```
[Out] ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/a^(5/2) - Tanh[x]/(3*a
*(a + b - b*Tanh[x]^2)^(3/2)) - ((2*a + 3*b)*Tanh[x])/(3*a^2*(a + b)*Sqrt[a
 + b - b*Tanh[x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\tanh^2(x)}{\left(a+b\text{sech}^2(x)\right)^{5/2}}\,dx = \text{Subst}\left(\int \frac{x^2}{\left(1-x^2\right)\left(a+b\left(1-x^2\right)\right)^{5/2}}\,dx, x, \tanh(x)\right)$$

$$= \text{Subst}\left(\int \frac{x^2}{\left(1-x^2\right)\left(a+b-bx^2\right)^{5/2}}\,dx, x, \tanh(x)\right)$$

$$= -\frac{\tanh(x)}{3a\left(a+b-b\tanh^2(x)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{1+2x^2}{\left(1-x^2\right)\left(a+b-bx^2\right)^{3/2}}\,dx, x, \tanh(x)\right)}{3a}$$

$$= -\frac{\tanh(x)}{3a\left(a+b-b\tanh^2(x)\right)^{3/2}} - \frac{(2a+3b)\tanh(x)}{3a^2(a+b)\sqrt{a+b-b\tanh^2(x)}} - \frac{\text{Subst}\left(\int -\frac{3(a+}{\left(1-x^2\right)\sqrt{a}}\right.}{3a^2(}$$

$$= -\frac{\tanh(x)}{3a\left(a+b-b\tanh^2(x)\right)^{3/2}} - \frac{(2a+3b)\tanh(x)}{3a^2(a+b)\sqrt{a+b-b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)\sqrt{a+}}\right.}{a}$$

$$= -\frac{\tanh(x)}{3a\left(a+b-b\tanh^2(x)\right)^{3/2}} - \frac{(2a+3b)\tanh(x)}{3a^2(a+b)\sqrt{a+b-b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx,\right.}{a^2}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{a^{5/2}} - \frac{\tanh(x)}{3a\left(a+b-b\tanh^2(x)\right)^{3/2}} - \frac{(2a+3b)\tanh(x)}{3a^2(a+b)\sqrt{a+b-b\tanh^2}}$$

Mathematica [B] time = 1.20, size = 290, normalized size = 3.30

$$\frac{\operatorname{sech}^4(x)\left(\frac{\sqrt{2}\operatorname{csch}(x)\operatorname{sech}(x)\left(-\frac{16\left(a\sinh^2(x)+a+b\right)\left(\frac{a\sinh^2(x)}{a+b}+1\right)\left(\frac{a^2(a+b)\sinh^4(x)}{\left(a\sinh^2(x)+a+b\right)^2}+\frac{3a(a+b)\sinh^2(x)}{a\sinh^2(x)+a+b}-\frac{3\sqrt{a}\sqrt{a+b}\sinh(x)\sinh^{-1}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{a+b}}\right)}{\sqrt{\frac{a\sinh^2(x)+a+b}{a+b}}}\right)}{a^3}+\frac{12\sinh^4(x)}{a+b}+\cdots\right)}{\left(a\sinh^2(x)+a+b\right)^{3/2}}\cdots\right)}{384\left(a\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^2/(a + b*Sech[x]^2)^(5/2),x]
```

```
[Out] (Sech[x]^4*((Sqrt[2]*(a + 2*b + a*Cosh[2*x])^(5/2)*Csch[x]*Sech[x]*(Sinh[x]
^2/(a + b) + (12*Sinh[x]^4)/(a + b) + (2*Sinh[x]^2*(a + b + a*Sinh[x]^2))/(
a + b)^2 - (16*(a + b + a*Sinh[x]^2)*(1 + (a*Sinh[x]^2)/(a + b))*((a^2*(a +
 b)*Sinh[x]^4)/(a + b + a*Sinh[x]^2)^2 + (3*a*(a + b)*Sinh[x]^2)/(a + b + a
*Sinh[x]^2) - (3*Sqrt[a]*Sqrt[a + b]*ArcSinh[(Sqrt[a]*Sinh[x])/Sqrt[a + b]]
*Sinh[x])/Sqrt[(a + b + a*Sinh[x]^2)/(a + b)]))/a^3))/(a + b + a*Sinh[x]^2)
^(3/2) - (8*(a + 2*b + a*Cosh[2*x])*(2*a + 3*b + a*Cosh[2*x])*Tanh[x])/(a +
 b)^2 + (4*(a + 2*b + a*Cosh[2*x])*(b + (3*a + 2*b)*Cosh[2*x])*Tanh[x])/(a
+ b)^2))/(384*(a + b*Sech[x]^2)^(5/2))
```

fricas [B] time = 0.77, size = 4989, normalized size = 56.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*sech(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^7 + (a^
3 + a^2*b)*sinh(x)^8 + 4*(a^3 + 3*a^2*b + 2*a*b^2)*cosh(x)^6 + 4*(a^3 + 3*a
^2*b + 2*a*b^2 + 7*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 + 8*(7*(a^3 + a^2*b)*
cosh(x)^3 + 3*(a^3 + 3*a^2*b + 2*a*b^2)*cosh(x))*sinh(x)^5 + 2*(3*a^3 + 11*
a^2*b + 16*a*b^2 + 8*b^3)*cosh(x)^4 + 2*(35*(a^3 + a^2*b)*cosh(x)^4 + 3*a^3
 + 11*a^2*b + 16*a*b^2 + 8*b^3 + 30*(a^3 + 3*a^2*b + 2*a*b^2)*cosh(x)^2)*si
```

```
nh(x)^4 + 8*(7*(a^3 + a^2*b)*cosh(x)^5 + 10*(a^3 + 3*a^2*b + 2*a*b^2)*cosh(
x)^3 + (3*a^3 + 11*a^2*b + 16*a*b^2 + 8*b^3)*cosh(x))*sinh(x)^3 + a^3 + a^2
*b + 4*(a^3 + 3*a^2*b + 2*a*b^2)*cosh(x)^2 + 4*(7*(a^3 + a^2*b)*cosh(x)^6 +
 15*(a^3 + 3*a^2*b + 2*a*b^2)*cosh(x)^4 + a^3 + 3*a^2*b + 2*a*b^2 + 3*(3*a^
3 + 11*a^2*b + 16*a*b^2 + 8*b^3)*cosh(x)^2)*sinh(x)^2 + 8*((a^3 + a^2*b)*co
sh(x)^7 + 3*(a^3 + 3*a^2*b + 2*a*b^2)*cosh(x)^5 + (3*a^3 + 11*a^2*b + 16*a*
b^2 + 8*b^3)*cosh(x)^3 + (a^3 + 3*a^2*b + 2*a*b^2)*cosh(x))*sinh(x))*sqrt(a
)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^8 - 2*(a
*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh(x)^6 + 4*
(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3 + 4*a^2*b +
 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 - 30*(a
*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a*b^2 - b^3)
*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(a^3 +
3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*cosh(x)^4 + a
^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(
b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 +
 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*
sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2
- a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^
2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a +
2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*cosh(x)^7 -
3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3 + (a^3 + 3*
a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*si
nh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(
x)^5 + sinh(x)^6)) + 3*((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*s
inh(x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 4*(a^3 + 3*a^2*b + 2*a*b^2)*cosh(x)^6
+ 4*(a^3 + 3*a^2*b + 2*a*b^2 + 7*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 + 8*(7*
(a^3 + a^2*b)*cosh(x)^3 + 3*(a^3 + 3*a^2*b + 2*a*b^2)*cosh(x))*sinh(x)^5 +
2*(3*a^3 + 11*a^2*b + 16*a*b^2 + 8*b^3)*cosh(x)^4 + 2*(35*(a^3 + a^2*b)*cos
h(x)^4 + 3*a^3 + 11*a^2*b + 16*a*b^2 + 8*b^3 + 30*(a^3 + 3*a^2*b + 2*a*b^2)
*cosh(x)^2)*sinh(x)^4 + 8*(7*(a^3 + a^2*b)*cosh(x)^5 + 10*(a^3 + 3*a^2*b +
2*a*b^2)*cosh(x)^3 + (3*a^3 + 11*a^2*b + 16*a*b^2 + 8*b^3)*cosh(x))*sinh(x)
^3 + a^3 + a^2*b + 4*(a^3 + 3*a^2*b + 2*a*b^2)*cosh(x)^2 + 4*(7*(a^3 + a^2*
b)*cosh(x)^6 + 15*(a^3 + 3*a^2*b + 2*a*b^2)*cosh(x)^4 + a^3 + 3*a^2*b + 2*a
*b^2 + 3*(3*a^3 + 11*a^2*b + 16*a*b^2 + 8*b^3)*cosh(x)^2)*sinh(x)^2 + 8*((a
^3 + a^2*b)*cosh(x)^7 + 3*(a^3 + 3*a^2*b + 2*a*b^2)*cosh(x)^5 + (3*a^3 + 11
*a^2*b + 16*a*b^2 + 8*b^3)*cosh(x)^3 + (a^3 + 3*a^2*b + 2*a*b^2)*cosh(x))*s
inh(x))*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2
*(a + b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + b)*sinh(x)^2 + sqrt(2)*(cosh(x)
^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(
x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(a*cosh(x)
^3 + (a + b)*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)
^2)) - 4*sqrt(2)*((3*a^3 + 4*a^2*b)*cosh(x)^6 + 6*(3*a^3 + 4*a^2*b)*cosh(x)
*sinh(x)^5 + (3*a^3 + 4*a^2*b)*sinh(x)^6 + 3*(a^3 + 4*a^2*b + 4*a*b^2)*cosh
(x)^4 + 3*(a^3 + 4*a^2*b + 4*a*b^2 + 5*(3*a^3 + 4*a^2*b)*cosh(x)^2)*sinh(x)
```

```
^4 + 4*(5*(3*a^3 + 4*a^2*b)*cosh(x)^3 + 3*(a^3 + 4*a^2*b + 4*a*b^2)*cosh(x)
)*sinh(x)^3 - 3*a^3 - 4*a^2*b - 3*(a^3 + 4*a^2*b + 4*a*b^2)*cosh(x)^2 + 3*(
5*(3*a^3 + 4*a^2*b)*cosh(x)^4 - a^3 - 4*a^2*b - 4*a*b^2 + 6*(a^3 + 4*a^2*b
+ 4*a*b^2)*cosh(x)^2)*sinh(x)^2 + 6*((3*a^3 + 4*a^2*b)*cosh(x)^5 + 2*(a^3 +
 4*a^2*b + 4*a*b^2)*cosh(x)^3 - (a^3 + 4*a^2*b + 4*a*b^2)*cosh(x))*sinh(x))
*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x)
+ sinh(x)^2)))/((a^6 + a^5*b)*cosh(x)^8 + 8*(a^6 + a^5*b)*cosh(x)*sinh(x)^7
 + (a^6 + a^5*b)*sinh(x)^8 + 4*(a^6 + 3*a^5*b + 2*a^4*b^2)*cosh(x)^6 + 4*(a
^6 + 3*a^5*b + 2*a^4*b^2 + 7*(a^6 + a^5*b)*cosh(x)^2)*sinh(x)^6 + a^6 + a^5
*b + 8*(7*(a^6 + a^5*b)*cosh(x)^3 + 3*(a^6 + 3*a^5*b + 2*a^4*b^2)*cosh(x))*
sinh(x)^5 + 2*(3*a^6 + 11*a^5*b + 16*a^4*b^2 + 8*a^3*b^3)*cosh(x)^4 + 2*(3*
a^6 + 11*a^5*b + 16*a^4*b^2 + 8*a^3*b^3 + 35*(a^6 + a^5*b)*cosh(x)^4 + 30*(
a^6 + 3*a^5*b + 2*a^4*b^2)*cosh(x)^2)*sinh(x)^4 + 8*(7*(a^6 + a^5*b)*cosh(x
)^5 + 10*(a^6 + 3*a^5*b + 2*a^4*b^2)*cosh(x)^3 + (3*a^6 + 11*a^5*b + 16*a^4
*b^2 + 8*a^3*b^3)*cosh(x))*sinh(x)^3 + 4*(a^6 + 3*a^5*b + 2*a^4*b^2)*cosh(x
)^2 + 4*(7*(a^6 + a^5*b)*cosh(x)^6 + a^6 + 3*a^5*b + 2*a^4*b^2 + 15*(a^6 +
3*a^5*b + 2*a^4*b^2)*cosh(x)^4 + 3*(3*a^6 + 11*a^5*b + 16*a^4*b^2 + 8*a^3*b
^3)*cosh(x)^2)*sinh(x)^2 + 8*((a^6 + a^5*b)*cosh(x)^7 + 3*(a^6 + 3*a^5*b +
2*a^4*b^2)*cosh(x)^5 + (3*a^6 + 11*a^5*b + 16*a^4*b^2 + 8*a^3*b^3)*cosh(x)^
3 + (a^6 + 3*a^5*b + 2*a^4*b^2)*cosh(x))*sinh(x)), -1/6*(3*((a^3 + a^2*b)*c
osh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 4*
(a^3 + 3*a^2*b + 2*a*b^2)*cosh(x)^6 + 4*(a^3 + 3*a^2*b + 2*a*b^2 + 7*(a^3 +
 a^2*b)*cosh(x)^2)*sinh(x)^6 + 8*(7*(a^3 + a^2*b)*cosh(x)^3 + 3*(a^3 + 3*a^
2*b + 2*a*b^2)*cosh(x))*sinh(x)^5 + 2*(3*a^3 + 11*a^2*b + 16*a*b^2 + 8*b^3)
*cosh(x)^4 + 2*(35*(a^3 + a^2*b)*cosh(x)^4 + 3*a^3 + 11*a^2*b + 16*a*b^2 +
8*b^3 + 30*(a^3 + 3*a^2*b + 2*a*b^2)*cosh(x)^2)*sinh(x)^4 + 8*(7*(a^3 + a^2
*b)*cosh(x)^5 + 10*(a^3 + 3*a^2*b + 2*a*b^2)*cosh(x)^3 + (3*a^3 + 11*a^2*b
+ 16*a*b^2 + 8*b^3)*cosh(x))*sinh(x)^3 + a^3 + a^2*b + 4*(a^3 + 3*a^2*b + 2
*a*b^2)*cosh(x)^2 + 4*(7*(a^3 + a^2*b)*cosh(x)^6 + 15*(a^3 + 3*a^2*b + 2*a*
b^2)*cosh(x)^4 + a^3 + 3*a^2*b + 2*a*b^2 + 3*(3*a^3 + 11*a^2*b + 16*a*b^2 +
 8*b^3)*cosh(x)^2)*sinh(x)^2 + 8*((a^3 + a^2*b)*cosh(x)^7 + 3*(a^3 + 3*a^2*
b + 2*a*b^2)*cosh(x)^5 + (3*a^3 + 11*a^2*b + 16*a*b^2 + 8*b^3)*cosh(x)^3 +
(a^3 + 3*a^2*b + 2*a*b^2)*cosh(x))*sinh(x))*sqrt(-a)*arctan(sqrt(2)*(b*cosh
(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 +
 a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*b*c
osh(x)^4 + 4*a*b*cosh(x)*sinh(x)^3 + a*b*sinh(x)^4 - (a^2 + 3*a*b)*cosh(x)^
2 + (6*a*b*cosh(x)^2 - a^2 - 3*a*b)*sinh(x)^2 - a^2 + 2*(2*a*b*cosh(x)^3 -
(a^2 + 3*a*b)*cosh(x))*sinh(x))) + 3*((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^
2*b)*cosh(x)*sinh(x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 4*(a^3 + 3*a^2*b + 2*a*b
^2)*cosh(x)^6 + 4*(a^3 + 3*a^2*b + 2*a*b^2 + 7*(a^3 + a^2*b)*cosh(x)^2)*sin
h(x)^6 + 8*(7*(a^3 + a^2*b)*cosh(x)^3 + 3*(a^3 + 3*a^2*b + 2*a*b^2)*cosh(x)
)*sinh(x)^5 + 2*(3*a^3 + 11*a^2*b + 16*a*b^2 + 8*b^3)*cosh(x)^4 + 2*(35*(a^
3 + a^2*b)*cosh(x)^4 + 3*a^3 + 11*a^2*b + 16*a*b^2 + 8*b^3 + 30*(a^3 + 3*a^
2*b + 2*a*b^2)*cosh(x)^2)*sinh(x)^4 + 8*(7*(a^3 + a^2*b)*cosh(x)^5 + 10*(a^
3 + 3*a^2*b + 2*a*b^2)*cosh(x)^3 + (3*a^3 + 11*a^2*b + 16*a*b^2 + 8*b^3)*co
```

```
sh(x))*sinh(x)^3 + a^3 + a^2*b + 4*(a^3 + 3*a^2*b + 2*a*b^2)*cosh(x)^2 + 4*
(7*(a^3 + a^2*b)*cosh(x)^6 + 15*(a^3 + 3*a^2*b + 2*a*b^2)*cosh(x)^4 + a^3 +
 3*a^2*b + 2*a*b^2 + 3*(3*a^3 + 11*a^2*b + 16*a*b^2 + 8*b^3)*cosh(x)^2)*sin
h(x)^2 + 8*((a^3 + a^2*b)*cosh(x)^7 + 3*(a^3 + 3*a^2*b + 2*a*b^2)*cosh(x)^5
 + (3*a^3 + 11*a^2*b + 16*a*b^2 + 8*b^3)*cosh(x)^3 + (a^3 + 3*a^2*b + 2*a*b
^2)*cosh(x))*sinh(x))*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cosh(x)^2 +
a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh
(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a)) + 2*sqrt(2)*((3*a^3 + 4*a^2
*b)*cosh(x)^6 + 6*(3*a^3 + 4*a^2*b)*cosh(x)*sinh(x)^5 + (3*a^3 + 4*a^2*b)*s
inh(x)^6 + 3*(a^3 + 4*a^2*b + 4*a*b^2)*cosh(x)^4 + 3*(a^3 + 4*a^2*b + 4*a*b
^2 + 5*(3*a^3 + 4*a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(5*(3*a^3 + 4*a^2*b)*cosh
(x)^3 + 3*(a^3 + 4*a^2*b + 4*a*b^2)*cosh(x))*sinh(x)^3 - 3*a^3 - 4*a^2*b -
3*(a^3 + 4*a^2*b + 4*a*b^2)*cosh(x)^2 + 3*(5*(3*a^3 + 4*a^2*b)*cosh(x)^4 -
a^3 - 4*a^2*b - 4*a*b^2 + 6*(a^3 + 4*a^2*b + 4*a*b^2)*cosh(x)^2)*sinh(x)^2
+ 6*((3*a^3 + 4*a^2*b)*cosh(x)^5 + 2*(a^3 + 4*a^2*b + 4*a*b^2)*cosh(x)^3 -
(a^3 + 4*a^2*b + 4*a*b^2)*cosh(x))*sinh(x))*sqrt((a*cosh(x)^2 + a*sinh(x)^2
 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^6 + a^5*b)*co
sh(x)^8 + 8*(a^6 + a^5*b)*cosh(x)*sinh(x)^7 + (a^6 + a^5*b)*sinh(x)^8 + 4*(
a^6 + 3*a^5*b + 2*a^4*b^2)*cosh(x)^6 + 4*(a^6 + 3*a^5*b + 2*a^4*b^2 + 7*(a^
6 + a^5*b)*cosh(x)^2)*sinh(x)^6 + a^6 + a^5*b + 8*(7*(a^6 + a^5*b)*cosh(x)^
3 + 3*(a^6 + 3*a^5*b + 2*a^4*b^2)*cosh(x))*sinh(x)^5 + 2*(3*a^6 + 11*a^5*b
+ 16*a^4*b^2 + 8*a^3*b^3)*cosh(x)^4 + 2*(3*a^6 + 11*a^5*b + 16*a^4*b^2 + 8*
a^3*b^3 + 35*(a^6 + a^5*b)*cosh(x)^4 + 30*(a^6 + 3*a^5*b + 2*a^4*b^2)*cosh(
x)^2)*sinh(x)^4 + 8*(7*(a^6 + a^5*b)*cosh(x)^5 + 10*(a^6 + 3*a^5*b + 2*a^4*
b^2)*cosh(x)^3 + (3*a^6 + 11*a^5*b + 16*a^4*b^2 + 8*a^3*b^3)*cosh(x))*sinh(
x)^3 + 4*(a^6 + 3*a^5*b + 2*a^4*b^2)*cosh(x)^2 + 4*(7*(a^6 + a^5*b)*cosh(x)
^6 + a^6 + 3*a^5*b + 2*a^4*b^2 + 15*(a^6 + 3*a^5*b + 2*a^4*b^2)*cosh(x)^4 +
 3*(3*a^6 + 11*a^5*b + 16*a^4*b^2 + 8*a^3*b^3)*cosh(x)^2)*sinh(x)^2 + 8*((a
^6 + a^5*b)*cosh(x)^7 + 3*(a^6 + 3*a^5*b + 2*a^4*b^2)*cosh(x)^5 + (3*a^6 +
11*a^5*b + 16*a^4*b^2 + 8*a^3*b^3)*cosh(x)^3 + (a^6 + 3*a^5*b + 2*a^4*b^2)*
cosh(x))*sinh(x))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*sech(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, replacing 0 by ` u`, a substitution
 variable should perhaps be purged.Warning, replacing 0 by ` u`, a substitu
tion variable should perhaps be purged.Warning, replacing 0 by ` u`, a subs
titution variable should perhaps be purged.Error: Bad Argument Type
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{\tanh^2(x)}{\left(a+b\operatorname{sech}(x)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(a+b*sech(x)^2)^(5/2),x)
[Out] int(tanh(x)^2/(a+b*sech(x)^2)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)^2}{\left(b\operatorname{sech}(x)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*sech(x)^2)^(5/2),x, algorithm="maxima")
[Out] integrate(tanh(x)^2/(b*sech(x)^2 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tanh(x)^2}{\left(a+\frac{b}{\cosh(x)^2}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(a + b/cosh(x)^2)^(5/2),x)
[Out] int(tanh(x)^2/(a + b/cosh(x)^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^2(x)}{\left(a+b\operatorname{sech}^2(x)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**2/(a+b*sech(x)**2)**(5/2),x)
[Out] Integral(tanh(x)**2/(a + b*sech(x)**2)**(5/2), x)
```

3.216 $\int \frac{\tanh(x)}{\left(a+b\mathbf{sech}^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=62

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{1}{a^2\sqrt{a+b\text{sech}^2(x)}}-\frac{1}{3a\left(a+b\text{sech}^2(x)\right)^{3/2}}$$

```
[Out] arctanh((a+b*sech(x)^2)^(1/2)/a^(1/2))/a^(5/2)-1/3/a/(a+b*sech(x)^2)^(3/2)-
1/a^2/(a+b*sech(x)^2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4139, 266, 51, 63, 208}

$$-\frac{1}{a^2\sqrt{a+b\text{sech}^2(x)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{1}{3a\left(a+b\text{sech}^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]/(a + b*Sech[x]^2)^(5/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]]/a^(5/2) - 1/(3*a*(a + b*Sech[x]^2)^(
3/2)) - 1/(a^2*Sqrt[a + b*Sech[x]^2])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \frac{\tanh(x)}{\left(a+b\text{sech}^2(x)\right)^{5/2}}\,dx = -\text{Subst}\left(\int \frac{1}{x\left(a+bx^2\right)^{5/2}}\,dx, x, \text{sech}(x)\right)$$
$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{x(a+bx)^{5/2}}\,dx, x, \text{sech}^2(x)\right)\right)$$
$$= -\frac{1}{3a\left(a+b\text{sech}^2(x)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, \text{sech}^2(x)\right)}{2a}$$
$$= -\frac{1}{3a\left(a+b\text{sech}^2(x)\right)^{3/2}} - \frac{1}{a^2\sqrt{a+b\text{sech}^2(x)}} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \text{sech}^2(x)\right)}{2a^2}$$
$$= -\frac{1}{3a\left(a+b\text{sech}^2(x)\right)^{3/2}} - \frac{1}{a^2\sqrt{a+b\text{sech}^2(x)}} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{sech}^2(}\right.}{a^2b}$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)}{a^{5/2}} - \frac{1}{3a\left(a+b\text{sech}^2(x)\right)^{3/2}} - \frac{1}{a^2\sqrt{a+b\text{sech}^2(x)}}$$

Mathematica [A] time = 0.72, size = 112, normalized size = 1.81

$$-\frac{\text{sech}^5(x)(a\cosh(2x)+a+2b)\left(4a^{3/2}\cosh(3x)+12\sqrt{a}\,(a+b)\cosh(x)-3\sqrt{2}\,(a\cosh(2x)+a+2b)^{3/2}\log\left(\sqrt{a}\right.\right.}{24a^{5/2}\left(a+b\text{sech}^2(x)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/(a + b*Sech[x]^2)^(5/2),x]
```

```
[Out] -1/24*((a + 2*b + a*Cosh[2*x])*(12*Sqrt[a]*(a + b)*Cosh[x] + 4*a^(3/2)*Cosh
[3*x] - 3*Sqrt[2]*(a + 2*b + a*Cosh[2*x])^(3/2)*Log[Sqrt[2]*Sqrt[a]*Cosh[x]
 + Sqrt[a + 2*b + a*Cosh[2*x]]])*Sech[x]^5)/(a^(5/2)*(a + b*Sech[x]^2)^(5/2
))
```

fricas [B] time = 0.70, size = 3994, normalized size = 64.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sech(x)^2)^(5/2),x, algorithm="fricas")

[Out] [1/12*(3*(a^2*cosh(x)^8 + 8*a^2*cosh(x)*sinh(x)^7 + a^2*sinh(x)^8 + 4*(a^2
+ 2*a*b)*cosh(x)^6 + 4*(7*a^2*cosh(x)^2 + a^2 + 2*a*b)*sinh(x)^6 + 8*(7*a^2
*cosh(x)^3 + 3*(a^2 + 2*a*b)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 8*a*b + 8*b^2)
*cosh(x)^4 + 2*(35*a^2*cosh(x)^4 + 30*(a^2 + 2*a*b)*cosh(x)^2 + 3*a^2 + 8*a
*b + 8*b^2)*sinh(x)^4 + 8*(7*a^2*cosh(x)^5 + 10*(a^2 + 2*a*b)*cosh(x)^3 + (
3*a^2 + 8*a*b + 8*b^2)*cosh(x))*sinh(x)^3 + 4*(a^2 + 2*a*b)*cosh(x)^2 + 4*(
7*a^2*cosh(x)^6 + 15*(a^2 + 2*a*b)*cosh(x)^4 + 3*(3*a^2 + 8*a*b + 8*b^2)*co
sh(x)^2 + a^2 + 2*a*b)*sinh(x)^2 + a^2 + 8*(a^2*cosh(x)^7 + 3*(a^2 + 2*a*b)
*cosh(x)^5 + (3*a^2 + 8*a*b + 8*b^2)*cosh(x)^3 + (a^2 + 2*a*b)*cosh(x))*sin
h(x))*sqrt(a)*log(((a^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a
*b^2)*cosh(x)*sinh(x)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*
a^2*b + 4*a*b^2 + b^3)*cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*
(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2
)*cosh(x)^3 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + (6*a
^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4
+ 6*a^3 + 14*a^2*b + 9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)
^2)*sinh(x)^4 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a^2
*b + 4*a*b^2 + b^3)*cosh(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*sinh(
x)^3 + a^3 + 2*(2*a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*
cosh(x)^6 + 15*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3 + 3*a^2*
b + 3*(6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*((a^2 + 2
*a*b + b^2)*cosh(x)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2 + 2*
a*b + b^2)*sinh(x)^6 + 3*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b
+ b^2)*cosh(x)^2 + a^2 + 2*a*b + b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*
cosh(x)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4*a*b)*cosh
(x)^2 + (15*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*cosh(x)^
2 + 3*a^2 + 4*a*b)*sinh(x)^2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 6
*(a^2 + 2*a*b + b^2)*cosh(x)^3 + (3*a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*
sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) +
 sinh(x)^2)) + 4*(2*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^7 + 3*(2*a^3 + 5*a^2*b
+ 4*a*b^2 + b^3)*cosh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^3 + (2*a^
3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x
)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)
*sinh(x)^5 + sinh(x)^6)) + 3*(a^2*cosh(x)^8 + 8*a^2*cosh(x)*sinh(x)^7 + a^2
*sinh(x)^8 + 4*(a^2 + 2*a*b)*cosh(x)^6 + 4*(7*a^2*cosh(x)^2 + a^2 + 2*a*b)*
sinh(x)^6 + 8*(7*a^2*cosh(x)^3 + 3*(a^2 + 2*a*b)*cosh(x))*sinh(x)^5 + 2*(3*
a^2 + 8*a*b + 8*b^2)*cosh(x)^4 + 2*(35*a^2*cosh(x)^4 + 30*(a^2 + 2*a*b)*cos
h(x)^2 + 3*a^2 + 8*a*b + 8*b^2)*sinh(x)^4 + 8*(7*a^2*cosh(x)^5 + 10*(a^2 +
2*a*b)*cosh(x)^3 + (3*a^2 + 8*a*b + 8*b^2)*cosh(x))*sinh(x)^3 + 4*(a^2 + 2*
a*b)*cosh(x)^2 + 4*(7*a^2*cosh(x)^6 + 15*(a^2 + 2*a*b)*cosh(x)^4 + 3*(3*a^2
 + 8*a*b + 8*b^2)*cosh(x)^2 + a^2 + 2*a*b)*sinh(x)^2 + a^2 + 8*(a^2*cosh(x)
^7 + 3*(a^2 + 2*a*b)*cosh(x)^5 + (3*a^2 + 8*a*b + 8*b^2)*cosh(x)^3 + (a^2 +
 2*a*b)*cosh(x))*sinh(x))*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3
```

```
 + a*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*a*cosh(x)^2 + b)*sinh(x)^2 + sqrt(2)*
(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a)*sqrt((a*cosh(x)^2 +
 a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(a
*cosh(x)^3 + b*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(
x)^2)) - 16*sqrt(2)*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)^
6 + 3*(a^2 + a*b)*cosh(x)^4 + 3*(5*a^2*cosh(x)^2 + a^2 + a*b)*sinh(x)^4 + 4
*(5*a^2*cosh(x)^3 + 3*(a^2 + a*b)*cosh(x))*sinh(x)^3 + 3*(a^2 + a*b)*cosh(x
)^2 + 3*(5*a^2*cosh(x)^4 + 6*(a^2 + a*b)*cosh(x)^2 + a^2 + a*b)*sinh(x)^2 +
 a^2 + 6*(a^2*cosh(x)^5 + 2*(a^2 + a*b)*cosh(x)^3 + (a^2 + a*b)*cosh(x))*si
nh(x))*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*si
nh(x) + sinh(x)^2)))/(a^5*cosh(x)^8 + 8*a^5*cosh(x)*sinh(x)^7 + a^5*sinh(x)
^8 + 4*(a^5 + 2*a^4*b)*cosh(x)^6 + 4*(7*a^5*cosh(x)^2 + a^5 + 2*a^4*b)*sinh
(x)^6 + 8*(7*a^5*cosh(x)^3 + 3*(a^5 + 2*a^4*b)*cosh(x))*sinh(x)^5 + a^5 + 2
*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*cosh(x)^4 + 2*(35*a^5*cosh(x)^4 + 3*a^5 + 8*
a^4*b + 8*a^3*b^2 + 30*(a^5 + 2*a^4*b)*cosh(x)^2)*sinh(x)^4 + 8*(7*a^5*cosh
(x)^5 + 10*(a^5 + 2*a^4*b)*cosh(x)^3 + (3*a^5 + 8*a^4*b + 8*a^3*b^2)*cosh(x
))*sinh(x)^3 + 4*(a^5 + 2*a^4*b)*cosh(x)^2 + 4*(7*a^5*cosh(x)^6 + a^5 + 2*a
^4*b + 15*(a^5 + 2*a^4*b)*cosh(x)^4 + 3*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*cosh(
x)^2)*sinh(x)^2 + 8*(a^5*cosh(x)^7 + 3*(a^5 + 2*a^4*b)*cosh(x)^5 + (3*a^5 +
 8*a^4*b + 8*a^3*b^2)*cosh(x)^3 + (a^5 + 2*a^4*b)*cosh(x))*sinh(x)), -1/6*(
3*(a^2*cosh(x)^8 + 8*a^2*cosh(x)*sinh(x)^7 + a^2*sinh(x)^8 + 4*(a^2 + 2*a*b
)*cosh(x)^6 + 4*(7*a^2*cosh(x)^2 + a^2 + 2*a*b)*sinh(x)^6 + 8*(7*a^2*cosh(x
)^3 + 3*(a^2 + 2*a*b)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 8*a*b + 8*b^2)*cosh(x
)^4 + 2*(35*a^2*cosh(x)^4 + 30*(a^2 + 2*a*b)*cosh(x)^2 + 3*a^2 + 8*a*b + 8*
b^2)*sinh(x)^4 + 8*(7*a^2*cosh(x)^5 + 10*(a^2 + 2*a*b)*cosh(x)^3 + (3*a^2 +
 8*a*b + 8*b^2)*cosh(x))*sinh(x)^3 + 4*(a^2 + 2*a*b)*cosh(x)^2 + 4*(7*a^2*c
osh(x)^6 + 15*(a^2 + 2*a*b)*cosh(x)^4 + 3*(3*a^2 + 8*a*b + 8*b^2)*cosh(x)^2
 + a^2 + 2*a*b)*sinh(x)^2 + a^2 + 8*(a^2*cosh(x)^7 + 3*(a^2 + 2*a*b)*cosh(x
)^5 + (3*a^2 + 8*a*b + 8*b^2)*cosh(x)^3 + (a^2 + 2*a*b)*cosh(x))*sinh(x))*s
qrt(-a)*arctan(sqrt(2)*((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a
+ b)*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(co
sh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 +
 a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 + (2*a^2 + 3*a*b)*cosh(x)^2
 + (6*(a^2 + a*b)*cosh(x)^2 + 2*a^2 + 3*a*b)*sinh(x)^2 + a^2 + 2*(2*(a^2 +
a*b)*cosh(x)^3 + (2*a^2 + 3*a*b)*cosh(x))*sinh(x))) + 3*(a^2*cosh(x)^8 + 8*
a^2*cosh(x)*sinh(x)^7 + a^2*sinh(x)^8 + 4*(a^2 + 2*a*b)*cosh(x)^6 + 4*(7*a^
2*cosh(x)^2 + a^2 + 2*a*b)*sinh(x)^6 + 8*(7*a^2*cosh(x)^3 + 3*(a^2 + 2*a*b)
*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 8*a*b + 8*b^2)*cosh(x)^4 + 2*(35*a^2*cosh(
x)^4 + 30*(a^2 + 2*a*b)*cosh(x)^2 + 3*a^2 + 8*a*b + 8*b^2)*sinh(x)^4 + 8*(7
*a^2*cosh(x)^5 + 10*(a^2 + 2*a*b)*cosh(x)^3 + (3*a^2 + 8*a*b + 8*b^2)*cosh(
x))*sinh(x)^3 + 4*(a^2 + 2*a*b)*cosh(x)^2 + 4*(7*a^2*cosh(x)^6 + 15*(a^2 +
2*a*b)*cosh(x)^4 + 3*(3*a^2 + 8*a*b + 8*b^2)*cosh(x)^2 + a^2 + 2*a*b)*sinh(
x)^2 + a^2 + 8*(a^2*cosh(x)^7 + 3*(a^2 + 2*a*b)*cosh(x)^5 + (3*a^2 + 8*a*b
+ 8*b^2)*cosh(x)^3 + (a^2 + 2*a*b)*cosh(x))*sinh(x))*sqrt(-a)*arctan(sqrt(2
)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a)*sqrt((a*cosh(x)^
```

```
2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*
cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2
*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*
sinh(x) + a)) + 8*sqrt(2)*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*si
nh(x)^6 + 3*(a^2 + a*b)*cosh(x)^4 + 3*(5*a^2*cosh(x)^2 + a^2 + a*b)*sinh(x)
^4 + 4*(5*a^2*cosh(x)^3 + 3*(a^2 + a*b)*cosh(x))*sinh(x)^3 + 3*(a^2 + a*b)*
cosh(x)^2 + 3*(5*a^2*cosh(x)^4 + 6*(a^2 + a*b)*cosh(x)^2 + a^2 + a*b)*sinh(
x)^2 + a^2 + 6*(a^2*cosh(x)^5 + 2*(a^2 + a*b)*cosh(x)^3 + (a^2 + a*b)*cosh(
x))*sinh(x))*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh
(x)*sinh(x) + sinh(x)^2)))/(a^5*cosh(x)^8 + 8*a^5*cosh(x)*sinh(x)^7 + a^5*s
inh(x)^8 + 4*(a^5 + 2*a^4*b)*cosh(x)^6 + 4*(7*a^5*cosh(x)^2 + a^5 + 2*a^4*b
)*sinh(x)^6 + 8*(7*a^5*cosh(x)^3 + 3*(a^5 + 2*a^4*b)*cosh(x))*sinh(x)^5 + a
^5 + 2*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*cosh(x)^4 + 2*(35*a^5*cosh(x)^4 + 3*a^
5 + 8*a^4*b + 8*a^3*b^2 + 30*(a^5 + 2*a^4*b)*cosh(x)^2)*sinh(x)^4 + 8*(7*a^
5*cosh(x)^5 + 10*(a^5 + 2*a^4*b)*cosh(x)^3 + (3*a^5 + 8*a^4*b + 8*a^3*b^2)*
cosh(x))*sinh(x)^3 + 4*(a^5 + 2*a^4*b)*cosh(x)^2 + 4*(7*a^5*cosh(x)^6 + a^5
 + 2*a^4*b + 15*(a^5 + 2*a^4*b)*cosh(x)^4 + 3*(3*a^5 + 8*a^4*b + 8*a^3*b^2)
*cosh(x)^2)*sinh(x)^2 + 8*(a^5*cosh(x)^7 + 3*(a^5 + 2*a^4*b)*cosh(x)^5 + (3
*a^5 + 8*a^4*b + 8*a^3*b^2)*cosh(x)^3 + (a^5 + 2*a^4*b)*cosh(x))*sinh(x))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sech(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, replacing 0 by ` u`, a substitution
 variable should perhaps be purged.Warning, replacing 0 by ` u`, a substitu
tion variable should perhaps be purged.Warning, replacing 0 by ` u`, a subs
titution variable should perhaps be purged.Error: Bad Argument Type
```

maple [A] time = 0.11, size = 61, normalized size = 0.98

$$-\frac{1}{3a\left(a+b\operatorname{sech}(x)^{2}\right)^{\frac{3}{2}}}-\frac{1}{a^{2}\sqrt{a+b\operatorname{sech}(x)^{2}}}+\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\operatorname{sech}(x)^{2}}}{\operatorname{sech}(x)}\right)}{a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a+b*sech(x)^2)^(5/2),x)
```

```
[Out] -1/3/a/(a+b*sech(x)^2)^(3/2)-1/a^2/(a+b*sech(x)^2)^(1/2)+1/a^(5/2)*ln((2*a+
2*a^(1/2)*(a+b*sech(x)^2)^(1/2))/sech(x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)}{\left(b\,\mathrm{sech}(x)^2 + a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sech(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)/(b*sech(x)^2 + a)^(5/2), x)
```

mupad [B] time = 3.07, size = 50, normalized size = 0.81

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{\cosh(x)^2}}}{\sqrt{a}}\right)}{a^{5/2}} - \frac{\frac{1}{3\,a} + \frac{a+\frac{b}{\cosh(x)^2}}{a^2}}{\left(a + \frac{b}{\cosh(x)^2}\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a + b/cosh(x)^2)^(5/2),x)
```

```
[Out] atanh((a + b/cosh(x)^2)^(1/2)/a^(1/2))/a^(5/2) - (1/(3*a) + (a + b/cosh(x)^
2)/a^2)/(a + b/cosh(x)^2)^(3/2)
```

sympy [A] time = 12.68, size = 65, normalized size = 1.05

$$-\frac{1}{3a\left(a + b\,\mathrm{sech}^2\,(x)\right)^{\frac{3}{2}}} - \frac{1}{a^2\sqrt{a + b\,\mathrm{sech}^2\,(x)}} - \frac{\operatorname{atan}\left(\frac{\sqrt{a+b\,\mathrm{sech}^2\,(x)}}{\sqrt{-a}}\right)}{a^2\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sech(x)**2)**(5/2),x)
```

```
[Out] -1/(3*a*(a + b*sech(x)**2)**(3/2)) - 1/(a**2*sqrt(a + b*sech(x)**2)) - atan
(sqrt(a + b*sech(x)**2)/sqrt(-a))/(a**2*sqrt(-a))
```

3.217 $\int \frac{1}{\left(a+b\,\mathbf{sech}^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=95

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\,\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{a^{5/2}}-\frac{b(5a+3b)\tanh(x)}{3a^2(a+b)^2\sqrt{a-b\tanh^2(x)+b}}-\frac{b\tanh(x)}{3a(a+b)\left(a-b\tanh^2(x)+b\right)^{3/2}}$$

```
[Out] arctanh(a^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))/a^(5/2)-1/3*b*(5*a+3*b)*ta
nh(x)/a^2/(a+b)^2/(a+b-b*tanh(x)^2)^(1/2)-1/3*b*tanh(x)/a/(a+b)/(a+b-b*tanh
(x)^2)^(3/2)
```

Rubi [A] time = 0.09, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4128, 414, 527, 12, 377, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\,\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{a^{5/2}}-\frac{b(5a+3b)\tanh(x)}{3a^2(a+b)^2\sqrt{a-b\tanh^2(x)+b}}-\frac{b\tanh(x)}{3a(a+b)\left(a-b\tanh^2(x)+b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[x]^2)^(-5/2),x]
```

```
[Out] ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/a^(5/2) - (b*Tanh[x])/
(3*a*(a + b)*(a + b - b*Tanh[x]^2)^(3/2)) - (b*(5*a + 3*b)*Tanh[x])/(3*a^2*
(a + b)^2*Sqrt[a + b - b*Tanh[x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+b\text{sech}^2(x)\right)^{5/2}}\,dx = \text{Subst}\left(\int \frac{1}{\left(1-x^2\right)\left(a+b-bx^2\right)^{5/2}}\,dx, x, \tanh(x)\right)$$

$$= -\frac{b\tanh(x)}{3a(a+b)\left(a+b-b\tanh^2(x)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{-3a-b-2bx^2}{\left(1-x^2\right)\left(a+b-bx^2\right)^{3/2}}\,dx, x, \tanh(x)\right)}{3a(a+b)}$$

$$= -\frac{b\tanh(x)}{3a(a+b)\left(a+b-b\tanh^2(x)\right)^{3/2}} - \frac{b(5a+3b)\tanh(x)}{3a^2(a+b)^2\sqrt{a+b-b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{\ }{(1-\ }\right.}{\ }$$

$$= -\frac{b\tanh(x)}{3a(a+b)\left(a+b-b\tanh^2(x)\right)^{3/2}} - \frac{b(5a+3b)\tanh(x)}{3a^2(a+b)^2\sqrt{a+b-b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{\ }{(1-\ }\right.}{\ }$$

$$= -\frac{b\tanh(x)}{3a(a+b)\left(a+b-b\tanh^2(x)\right)^{3/2}} - \frac{b(5a+3b)\tanh(x)}{3a^2(a+b)^2\sqrt{a+b-b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{1-a\ }\right.}{\ }$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{a^{5/2}} - \frac{b\tanh(x)}{3a(a+b)\left(a+b-b\tanh^2(x)\right)^{3/2}} - \frac{b(5a+3b)\tanh}{3a^2(a+b)^2\sqrt{a+b-b}}$$

Mathematica [A] time = 0.45, size = 130, normalized size = 1.37

$$\frac{\text{sech}^5(x)\left(\frac{\sqrt{2}(a\cosh(2x)+a+2b)^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sinh(x)}{\sqrt{a\cosh(2x)+a+2b}}\right)}{a^{5/2}} - \frac{4b\sinh(x)(a\cosh(2x)+a+2b)\left(3a^2+a(3a+2b)\cosh(2x)+7ab+3b^2\right)}{3a^2(a+b)^2}\right)}{8\left(a+b\text{sech}^2(x)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[x]^2)^(-5/2),x]
```

```
[Out] (Sech[x]^5*((Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sinh[x])/Sqrt[a + 2*b + a*Cos
h[2*x]]]*(a + 2*b + a*Cosh[2*x])^(5/2))/a^(5/2) - (4*b*(a + 2*b + a*Cosh[2*
x])*(3*a^2 + 7*a*b + 3*b^2 + a*(3*a + 2*b)*Cosh[2*x])*Sinh[x])/(3*a^2*(a +
b)^2)))/(8*(a + b*Sech[x]^2)^(5/2))
```

fricas [B] time = 0.80, size = 6299, normalized size = 66.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*((a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^8 + 8*(a^4 + 2*a^3*b + a^2*b^2)
*cosh(x)*sinh(x)^7 + (a^4 + 2*a^3*b + a^2*b^2)*sinh(x)^8 + 4*(a^4 + 4*a^3*b
 + 5*a^2*b^2 + 2*a*b^3)*cosh(x)^6 + 4*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3
+ 7*(a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^2)*sinh(x)^6 + 8*(7*(a^4 + 2*a^3*b +
a^2*b^2)*cosh(x)^3 + 3*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*cosh(x))*sinh(
x)^5 + 2*(3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4)*cosh(x)^4 + 2*(
35*(a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^4 + 3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24
*a*b^3 + 8*b^4 + 30*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*cosh(x)^2)*sinh(x
)^4 + a^4 + 2*a^3*b + a^2*b^2 + 8*(7*(a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^5 +
10*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*cosh(x)^3 + (3*a^4 + 14*a^3*b + 27
*a^2*b^2 + 24*a*b^3 + 8*b^4)*cosh(x))*sinh(x)^3 + 4*(a^4 + 4*a^3*b + 5*a^2*
b^2 + 2*a*b^3)*cosh(x)^2 + 4*(7*(a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^6 + 15*(a
^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*cosh(x)^4 + a^4 + 4*a^3*b + 5*a^2*b^2 +
 2*a*b^3 + 3*(3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4)*cosh(x)^2)*
sinh(x)^2 + 8*((a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^7 + 3*(a^4 + 4*a^3*b + 5*a
^2*b^2 + 2*a*b^3)*cosh(x)^5 + (3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8
*b^4)*cosh(x)^3 + (a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*cosh(x))*sinh(x))*s
qrt(a)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^8 -
 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh(x)^6
 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3 + 4*a^
2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 -
30*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a*b^2 -
 b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(a
^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*cosh(x)^
4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt
(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x
)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh
(x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(
x)^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3
- (a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 +
 a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*cosh(x)
^7 - 3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3 + (a^3
 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)
^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*
sinh(x)^5 + sinh(x)^6)) + 3*((a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^8 + 8*(a^4 +
 2*a^3*b + a^2*b^2)*cosh(x)*sinh(x)^7 + (a^4 + 2*a^3*b + a^2*b^2)*sinh(x)^8
 + 4*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*cosh(x)^6 + 4*(a^4 + 4*a^3*b + 5
```

```
*a^2*b^2 + 2*a*b^3 + 7*(a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^2)*sinh(x)^6 + 8*(
7*(a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^3 + 3*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*
b^3)*cosh(x))*sinh(x)^5 + 2*(3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b
^4)*cosh(x)^4 + 2*(35*(a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^4 + 3*a^4 + 14*a^3*
b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4 + 30*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3
)*cosh(x)^2)*sinh(x)^4 + a^4 + 2*a^3*b + a^2*b^2 + 8*(7*(a^4 + 2*a^3*b + a^
2*b^2)*cosh(x)^5 + 10*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*cosh(x)^3 + (3*
a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4)*cosh(x))*sinh(x)^3 + 4*(a^4
 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*cosh(x)^2 + 4*(7*(a^4 + 2*a^3*b + a^2*b^2
)*cosh(x)^6 + 15*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*cosh(x)^4 + a^4 + 4*
a^3*b + 5*a^2*b^2 + 2*a*b^3 + 3*(3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 +
 8*b^4)*cosh(x)^2)*sinh(x)^2 + 8*((a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^7 + 3*(
a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*cosh(x)^5 + (3*a^4 + 14*a^3*b + 27*a^2
*b^2 + 24*a*b^3 + 8*b^4)*cosh(x)^3 + (a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*
cosh(x))*sinh(x))*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sin
h(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + b)*sinh(x)^2 + sqrt(2
)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*sqrt((a*cosh(x)^2
 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*
(a*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x)
 + sinh(x)^2)) - 8*sqrt(2)*((3*a^3*b + 2*a^2*b^2)*cosh(x)^6 + 6*(3*a^3*b +
2*a^2*b^2)*cosh(x)*sinh(x)^5 + (3*a^3*b + 2*a^2*b^2)*sinh(x)^6 + 3*(a^3*b +
 4*a^2*b^2 + 2*a*b^3)*cosh(x)^4 + 3*(a^3*b + 4*a^2*b^2 + 2*a*b^3 + 5*(3*a^3
*b + 2*a^2*b^2)*cosh(x)^2)*sinh(x)^4 - 3*a^3*b - 2*a^2*b^2 + 4*(5*(3*a^3*b
+ 2*a^2*b^2)*cosh(x)^3 + 3*(a^3*b + 4*a^2*b^2 + 2*a*b^3)*cosh(x))*sinh(x)^3
 - 3*(a^3*b + 4*a^2*b^2 + 2*a*b^3)*cosh(x)^2 + 3*(5*(3*a^3*b + 2*a^2*b^2)*c
osh(x)^4 - a^3*b - 4*a^2*b^2 - 2*a*b^3 + 6*(a^3*b + 4*a^2*b^2 + 2*a*b^3)*co
sh(x)^2)*sinh(x)^2 + 6*((3*a^3*b + 2*a^2*b^2)*cosh(x)^5 + 2*(a^3*b + 4*a^2*
b^2 + 2*a*b^3)*cosh(x)^3 - (a^3*b + 4*a^2*b^2 + 2*a*b^3)*cosh(x))*sinh(x))*
sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) +
 sinh(x)^2)))/((a^7 + 2*a^6*b + a^5*b^2)*cosh(x)^8 + 8*(a^7 + 2*a^6*b + a^5
*b^2)*cosh(x)*sinh(x)^7 + (a^7 + 2*a^6*b + a^5*b^2)*sinh(x)^8 + a^7 + 2*a^6
*b + a^5*b^2 + 4*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*cosh(x)^6 + 4*(a^7
 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3 + 7*(a^7 + 2*a^6*b + a^5*b^2)*cosh(x)^2)
*sinh(x)^6 + 8*(7*(a^7 + 2*a^6*b + a^5*b^2)*cosh(x)^3 + 3*(a^7 + 4*a^6*b +
5*a^5*b^2 + 2*a^4*b^3)*cosh(x))*sinh(x)^5 + 2*(3*a^7 + 14*a^6*b + 27*a^5*b^
2 + 24*a^4*b^3 + 8*a^3*b^4)*cosh(x)^4 + 2*(3*a^7 + 14*a^6*b + 27*a^5*b^2 +
24*a^4*b^3 + 8*a^3*b^4 + 35*(a^7 + 2*a^6*b + a^5*b^2)*cosh(x)^4 + 30*(a^7 +
 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*cosh(x)^2)*sinh(x)^4 + 8*(7*(a^7 + 2*a^6*
b + a^5*b^2)*cosh(x)^5 + 10*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*cosh(x)
^3 + (3*a^7 + 14*a^6*b + 27*a^5*b^2 + 24*a^4*b^3 + 8*a^3*b^4)*cosh(x))*sinh
(x)^3 + 4*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*cosh(x)^2 + 4*(a^7 + 4*a^
6*b + 5*a^5*b^2 + 2*a^4*b^3 + 7*(a^7 + 2*a^6*b + a^5*b^2)*cosh(x)^6 + 15*(a
^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*cosh(x)^4 + 3*(3*a^7 + 14*a^6*b + 27*
a^5*b^2 + 24*a^4*b^3 + 8*a^3*b^4)*cosh(x)^2)*sinh(x)^2 + 8*((a^7 + 2*a^6*b
+ a^5*b^2)*cosh(x)^7 + 3*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*cosh(x)^5
```

```
+ (3*a^7 + 14*a^6*b + 27*a^5*b^2 + 24*a^4*b^3 + 8*a^3*b^4)*cosh(x)^3 + (a^7
 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*cosh(x))*sinh(x)), -1/6*(3*((a^4 + 2*a^
3*b + a^2*b^2)*cosh(x)^8 + 8*(a^4 + 2*a^3*b + a^2*b^2)*cosh(x)*sinh(x)^7 +
(a^4 + 2*a^3*b + a^2*b^2)*sinh(x)^8 + 4*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^
3)*cosh(x)^6 + 4*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3 + 7*(a^4 + 2*a^3*b +
a^2*b^2)*cosh(x)^2)*sinh(x)^6 + 8*(7*(a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^3 +
3*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*cosh(x))*sinh(x)^5 + 2*(3*a^4 + 14*
a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4)*cosh(x)^4 + 2*(35*(a^4 + 2*a^3*b + a
^2*b^2)*cosh(x)^4 + 3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4 + 30*(
a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*cosh(x)^2)*sinh(x)^4 + a^4 + 2*a^3*b +
 a^2*b^2 + 8*(7*(a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^5 + 10*(a^4 + 4*a^3*b + 5
*a^2*b^2 + 2*a*b^3)*cosh(x)^3 + (3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 +
 8*b^4)*cosh(x))*sinh(x)^3 + 4*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*cosh(x
)^2 + 4*(7*(a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^6 + 15*(a^4 + 4*a^3*b + 5*a^2*
b^2 + 2*a*b^3)*cosh(x)^4 + a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3 + 3*(3*a^4 +
 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4)*cosh(x)^2)*sinh(x)^2 + 8*((a^4 +
 2*a^3*b + a^2*b^2)*cosh(x)^7 + 3*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*cos
h(x)^5 + (3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4)*cosh(x)^3 + (a^
4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*cosh(x))*sinh(x))*sqrt(-a)*arctan(sqrt(2
)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*co
sh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2
))/(a*b*cosh(x)^4 + 4*a*b*cosh(x)*sinh(x)^3 + a*b*sinh(x)^4 - (a^2 + 3*a*b)
*cosh(x)^2 + (6*a*b*cosh(x)^2 - a^2 - 3*a*b)*sinh(x)^2 - a^2 + 2*(2*a*b*cos
h(x)^3 - (a^2 + 3*a*b)*cosh(x))*sinh(x))) + 3*((a^4 + 2*a^3*b + a^2*b^2)*co
sh(x)^8 + 8*(a^4 + 2*a^3*b + a^2*b^2)*cosh(x)*sinh(x)^7 + (a^4 + 2*a^3*b +
a^2*b^2)*sinh(x)^8 + 4*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*cosh(x)^6 + 4*
(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3 + 7*(a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^
2)*sinh(x)^6 + 8*(7*(a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^3 + 3*(a^4 + 4*a^3*b
+ 5*a^2*b^2 + 2*a*b^3)*cosh(x))*sinh(x)^5 + 2*(3*a^4 + 14*a^3*b + 27*a^2*b^
2 + 24*a*b^3 + 8*b^4)*cosh(x)^4 + 2*(35*(a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^4
 + 3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4 + 30*(a^4 + 4*a^3*b + 5
*a^2*b^2 + 2*a*b^3)*cosh(x)^2)*sinh(x)^4 + a^4 + 2*a^3*b + a^2*b^2 + 8*(7*(
a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^5 + 10*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^
3)*cosh(x)^3 + (3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4)*cosh(x))*
sinh(x)^3 + 4*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*cosh(x)^2 + 4*(7*(a^4 +
 2*a^3*b + a^2*b^2)*cosh(x)^6 + 15*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*co
sh(x)^4 + a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3 + 3*(3*a^4 + 14*a^3*b + 27*a^
2*b^2 + 24*a*b^3 + 8*b^4)*cosh(x)^2)*sinh(x)^2 + 8*((a^4 + 2*a^3*b + a^2*b^
2)*cosh(x)^7 + 3*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*cosh(x)^5 + (3*a^4 +
 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4)*cosh(x)^3 + (a^4 + 4*a^3*b + 5*a
^2*b^2 + 2*a*b^3)*cosh(x))*sinh(x))*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((
a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(
x)^2))/(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a)) + 4*sqrt(2)*(
(3*a^3*b + 2*a^2*b^2)*cosh(x)^6 + 6*(3*a^3*b + 2*a^2*b^2)*cosh(x)*sinh(x)^5
 + (3*a^3*b + 2*a^2*b^2)*sinh(x)^6 + 3*(a^3*b + 4*a^2*b^2 + 2*a*b^3)*cosh(x
```

```
)^4 + 3*(a^3*b + 4*a^2*b^2 + 2*a*b^3 + 5*(3*a^3*b + 2*a^2*b^2)*cosh(x)^2)*s
inh(x)^4 - 3*a^3*b - 2*a^2*b^2 + 4*(5*(3*a^3*b + 2*a^2*b^2)*cosh(x)^3 + 3*(
a^3*b + 4*a^2*b^2 + 2*a*b^3)*cosh(x))*sinh(x)^3 - 3*(a^3*b + 4*a^2*b^2 + 2*
a*b^3)*cosh(x)^2 + 3*(5*(3*a^3*b + 2*a^2*b^2)*cosh(x)^4 - a^3*b - 4*a^2*b^2
 - 2*a*b^3 + 6*(a^3*b + 4*a^2*b^2 + 2*a*b^3)*cosh(x)^2)*sinh(x)^2 + 6*((3*a
^3*b + 2*a^2*b^2)*cosh(x)^5 + 2*(a^3*b + 4*a^2*b^2 + 2*a*b^3)*cosh(x)^3 - (
a^3*b + 4*a^2*b^2 + 2*a*b^3)*cosh(x))*sinh(x))*sqrt((a*cosh(x)^2 + a*sinh(x
)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^7 + 2*a^6*
b + a^5*b^2)*cosh(x)^8 + 8*(a^7 + 2*a^6*b + a^5*b^2)*cosh(x)*sinh(x)^7 + (a
^7 + 2*a^6*b + a^5*b^2)*sinh(x)^8 + a^7 + 2*a^6*b + a^5*b^2 + 4*(a^7 + 4*a^
6*b + 5*a^5*b^2 + 2*a^4*b^3)*cosh(x)^6 + 4*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a
^4*b^3 + 7*(a^7 + 2*a^6*b + a^5*b^2)*cosh(x)^2)*sinh(x)^6 + 8*(7*(a^7 + 2*a
^6*b + a^5*b^2)*cosh(x)^3 + 3*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*cosh(
x))*sinh(x)^5 + 2*(3*a^7 + 14*a^6*b + 27*a^5*b^2 + 24*a^4*b^3 + 8*a^3*b^4)*
cosh(x)^4 + 2*(3*a^7 + 14*a^6*b + 27*a^5*b^2 + 24*a^4*b^3 + 8*a^3*b^4 + 35*
(a^7 + 2*a^6*b + a^5*b^2)*cosh(x)^4 + 30*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4
*b^3)*cosh(x)^2)*sinh(x)^4 + 8*(7*(a^7 + 2*a^6*b + a^5*b^2)*cosh(x)^5 + 10*
(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*cosh(x)^3 + (3*a^7 + 14*a^6*b + 27*
a^5*b^2 + 24*a^4*b^3 + 8*a^3*b^4)*cosh(x))*sinh(x)^3 + 4*(a^7 + 4*a^6*b + 5
*a^5*b^2 + 2*a^4*b^3)*cosh(x)^2 + 4*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3
+ 7*(a^7 + 2*a^6*b + a^5*b^2)*cosh(x)^6 + 15*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2
*a^4*b^3)*cosh(x)^4 + 3*(3*a^7 + 14*a^6*b + 27*a^5*b^2 + 24*a^4*b^3 + 8*a^3
*b^4)*cosh(x)^2)*sinh(x)^2 + 8*((a^7 + 2*a^6*b + a^5*b^2)*cosh(x)^7 + 3*(a^
7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*cosh(x)^5 + (3*a^7 + 14*a^6*b + 27*a^5
*b^2 + 24*a^4*b^3 + 8*a^3*b^4)*cosh(x)^3 + (a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a
^4*b^3)*cosh(x))*sinh(x))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, replacing 0 by ` u`, a substitution
 variable should perhaps be purged.Warning, replacing 0 by ` u`, a substitu
tion variable should perhaps be purged.Warning, replacing 0 by ` u`, a subs
titution variable should perhaps be purged.Error: Bad Argument Type
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\,\mathrm{sech}(x)^{2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sech(x)^2)^(5/2),x)
```

```
[Out] int(1/(a+b*sech(x)^2)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,\mathrm{sech}(x)^2 + a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sech(x)^2 + a)^(-5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a + \frac{b}{\cosh(x)^2}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cosh(x)^2)^(5/2),x)
```

```
[Out] int(1/(a + b/cosh(x)^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b\,\mathrm{sech}^2\,(x)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(x)**2)**(5/2),x)
```

```
[Out] Integral((a + b*sech(x)**2)**(-5/2), x)
```

3.218 $\int \frac{\coth(x)}{\left(a+b\,\mathbf{sech}^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=109

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{b(2a+b)}{a^2(a+b)^2\sqrt{a+b\text{sech}^2(x)}}-\frac{b}{3a(a+b)\left(a+b\text{sech}^2(x)\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}}$$

```
[Out] arctanh((a+b*sech(x)^2)^(1/2)/a^(1/2))/a^(5/2)-arctanh((a+b*sech(x)^2)^(1/2
)/(a+b)^(1/2))/(a+b)^(5/2)-1/3*b/a/(a+b)/(a+b*sech(x)^2)^(3/2)-b*(2*a+b)/a^
2/(a+b)^2/(a+b*sech(x)^2)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {4139, 446, 85, 152, 156, 63, 208}

$$-\frac{b(2a+b)}{a^2(a+b)^2\sqrt{a+b\text{sech}^2(x)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{b}{3a(a+b)\left(a+b\text{sech}^2(x)\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]/(a + b*Sech[x]^2)^(5/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]]/a^(5/2) - ArcTanh[Sqrt[a + b*Sech[x]
^2]/Sqrt[a + b]]/(a + b)^(5/2) - b/(3*a*(a + b)*(a + b*Sech[x]^2)^(3/2)) -
(b*(2*a + b))/(a^2*(a + b)^2*Sqrt[a + b*Sech[x]^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p + 1))/((p + 1)*(b*e - a*f)*(d*e - c*f)),
x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[((b*d*e - b*c*f - a*d*f - b*d*f*
x)*(e + f*x)^(p + 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e
```

```
, f}, x] && LtQ[p, -1]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \frac{\coth(x)}{\left(a+b\text{sech}^2(x)\right)^{5/2}}\,dx = \text{Subst}\left(\int \frac{1}{x\left(-1+x^2\right)\left(a+bx^2\right)^{5/2}}\,dx, x, \text{sech}(x)\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(-1+x)x(a+bx)^{5/2}}\,dx, x, \text{sech}^2(x)\right)$$

$$= -\frac{b}{3a(a+b)\left(a+b\text{sech}^2(x)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{a+b-bx}{(-1+x)x(a+bx)^{3/2}}\,dx, x, \text{sech}^2(x)\right)}{2a(a+b)}$$

$$= -\frac{b}{3a(a+b)\left(a+b\text{sech}^2(x)\right)^{3/2}} - \frac{b(2a+b)}{a^2(a+b)^2\sqrt{a+b\text{sech}^2(x)}} - \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}(a+b)^2+\frac{1}{2}b(2a}{(-1+x)x\sqrt{a+b}}\right.}{a^2(a+}$$

$$= -\frac{b}{3a(a+b)\left(a+b\text{sech}^2(x)\right)^{3/2}} - \frac{b(2a+b)}{a^2(a+b)^2\sqrt{a+b\text{sech}^2(x)}} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x,\right.}{2a^2}$$

$$= -\frac{b}{3a(a+b)\left(a+b\text{sech}^2(x)\right)^{3/2}} - \frac{b(2a+b)}{a^2(a+b)^2\sqrt{a+b\text{sech}^2(x)}} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x,\right.}{a^2b}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)}{a^{5/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}} - \frac{b}{3a(a+b)\left(a+b\text{sech}^2(x)\right)^{3/2}} - \frac{}{a^2(a}$$

Mathematica [B] time = 1.11, size = 242, normalized size = 2.22

$$\frac{\text{sech}^5(x)\left(-\frac{2b\cosh(x)\left(7a^2+a(7a+4b)\cosh(2x)+16ab+6b^2\right)(a\cosh(2x)+a+2b)}{3a^2(a+b)^2} - \frac{(a\cosh(2x)+a+2b)^{5/2}\left(\sqrt{a}\left(a^2-2ab-b^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a+b}\cosh(x)}{\sqrt{a\cosh(2x)+a+2b}}\right.\right.}{}\right.}{8\left(a+b\text{sech}^2(x)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/(a + b*Sech[x]^2)^(5/2),x]
```

```
[Out] (((-2*b*Cosh[x]*(a + 2*b + a*Cosh[2*x])*(7*a^2 + 16*a*b + 6*b^2 + a*(7*a +
4*b)*Cosh[2*x]))/(3*a^2*(a + b)^2) - ((a + 2*b + a*Cosh[2*x])^(5/2)*(Sqrt[a
```

```
]*(a^2 - 2*a*b - b^2)*ArcTanh[(Sqrt[2]*Sqrt[a + b]*Cosh[x])/Sqrt[a + 2*b +
a*Cosh[2*x]]] + (a + b)^2*(Sqrt[a]*ArcTanh[(Sqrt[2*a + 2*b]*Cosh[x])/Sqrt[a
 + 2*b + a*Cosh[2*x]]] - 2*Sqrt[a + b]*Log[Sqrt[2]*Sqrt[a]*Cosh[x] + Sqrt[a
 + 2*b + a*Cosh[2*x]]])))/(Sqrt[2]*a^(5/2)*(a + b)^(5/2)))*Sech[x]^5)/(8*(a
 + b*Sech[x]^2)^(5/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sech(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sech(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{\coth(x)}{\left(a + b\,\mathrm{sech}(x)^{2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a+b*sech(x)^2)^(5/2),x)
```

```
[Out] int(coth(x)/(a+b*sech(x)^2)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(x)}{\left(b\,\mathrm{sech}(x)^{2} + a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sech(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(x)/(b*sech(x)^2 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\coth(x)}{\left(a+\frac{b}{\cosh(x)^2}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a + b/cosh(x)^2)^(5/2),x)
[Out] int(coth(x)/(a + b/cosh(x)^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth{(x)}}{\left(a+b\operatorname{sech}^{2}{(x)}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sech(x)**2)**(5/2),x)
[Out] Integral(coth(x)/(a + b*sech(x)**2)**(5/2), x)
```

3.219 $\int \frac{\coth^2(x)}{\left(a+b\,\mathbf{sech}^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=133

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{a^{5/2}}-\frac{(a-3b)(3a+b)\coth(x)\sqrt{a-b\tanh^2(x)+b}}{3a^2(a+b)^3}-\frac{b(7a+3b)\coth(x)}{3a^2(a+b)^2\sqrt{a-b\tanh^2(x)+b}}-\frac{}{3a(a}$$

```
[Out] arctanh(a^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))/a^(5/2)-1/3*b*(7*a+3*b)*co
th(x)/a^2/(a+b)^2/(a+b-b*tanh(x)^2)^(1/2)-1/3*(a-3*b)*(3*a+b)*coth(x)*(a+b-
b*tanh(x)^2)^(1/2)/a^2/(a+b)^3-1/3*b*coth(x)/a/(a+b)/(a+b-b*tanh(x)^2)^(3/2
)
```

Rubi [A] time = 0.38, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {4141, 1975, 472, 579, 583, 12, 377, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{a^{5/2}}-\frac{(a-3b)(3a+b)\coth(x)\sqrt{a-b\tanh^2(x)+b}}{3a^2(a+b)^3}-\frac{b(7a+3b)\coth(x)}{3a^2(a+b)^2\sqrt{a-b\tanh^2(x)+b}}-\frac{}{3a(a}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2/(a + b*Sech[x]^2)^(5/2),x]
```

```
[Out] ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/a^(5/2) - (b*Coth[x])/
(3*a*(a + b)*(a + b - b*Tanh[x]^2)^(3/2)) - (b*(7*a + 3*b)*Coth[x])/(3*a^2*
(a + b)^2*Sqrt[a + b - b*Tanh[x]^2]) - ((a - 3*b)*(3*a + b)*Coth[x]*Sqrt[a
+ b - b*Tanh[x]^2])/(3*a^2*(a + b)^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\coth^2(x)}{\left(a+b\operatorname{sech}^2(x)\right)^{5/2}}\,dx = \text{Subst}\left(\int \frac{1}{x^2\left(1-x^2\right)\left(a+b\left(1-x^2\right)\right)^{5/2}}\,dx, x, \tanh(x)\right)$$

$$= \text{Subst}\left(\int \frac{1}{x^2\left(1-x^2\right)\left(a+b-bx^2\right)^{5/2}}\,dx, x, \tanh(x)\right)$$

$$= -\frac{b\coth(x)}{3a(a+b)\left(a+b-b\tanh^2(x)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{-3a+b-4bx^2}{x^2\left(1-x^2\right)\left(a+b-bx^2\right)^{3/2}}\,dx, x, \tanh(x)\right)}{3a(a+b)}$$

$$= -\frac{b\coth(x)}{3a(a+b)\left(a+b-b\tanh^2(x)\right)^{3/2}} - \frac{b(7a+3b)\coth(x)}{3a^2(a+b)^2\sqrt{a+b-b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{(a}{}\right.}{}$$

$$= -\frac{b\coth(x)}{3a(a+b)\left(a+b-b\tanh^2(x)\right)^{3/2}} - \frac{b(7a+3b)\coth(x)}{3a^2(a+b)^2\sqrt{a+b-b\tanh^2(x)}} - \frac{(a-3b)(3a}{}$$

$$= -\frac{b\coth(x)}{3a(a+b)\left(a+b-b\tanh^2(x)\right)^{3/2}} - \frac{b(7a+3b)\coth(x)}{3a^2(a+b)^2\sqrt{a+b-b\tanh^2(x)}} - \frac{(a-3b)(3a}{}$$

$$= -\frac{b\coth(x)}{3a(a+b)\left(a+b-b\tanh^2(x)\right)^{3/2}} - \frac{b(7a+3b)\coth(x)}{3a^2(a+b)^2\sqrt{a+b-b\tanh^2(x)}} - \frac{(a-3b)(3a}{}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{a^{5/2}} - \frac{b\coth(x)}{3a(a+b)\left(a+b-b\tanh^2(x)\right)^{3/2}} - \frac{b(7a+3b)\coth}{3a^2(a+b)^2\sqrt{a+b-}}$$

Mathematica [A] time = 0.83, size = 155, normalized size = 1.17

$$\frac{\operatorname{sech}^5(x)\left(\frac{\sqrt{2}\,(a\cosh(2x)+a+2b)^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{a}\,\sinh(x)}{\sqrt{a\cosh(2x)+a+2b}}\right)}{a^{5/2}}-\frac{(a\cosh(2x)+a+2b)\left(3a^2\operatorname{csch}(x)(a\cosh(2x)+a+2b)^2-4b^3(a+b)\sinh(x)+2b^2(9a+4b}{3a^2(a+b)^3}\right.}{8\left(a+b\operatorname{sech}^2(x)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2/(a + b*Sech[x]^2)^(5/2),x]
```

```
[Out] (Sech[x]^5*((Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sinh[x])/Sqrt[a + 2*b + a*Cos
h[2*x]]]*(a + 2*b + a*Cosh[2*x])^(5/2))/a^(5/2) - ((a + 2*b + a*Cosh[2*x])*
(3*a^2*(a + 2*b + a*Cosh[2*x])^2*Csch[x] - 4*b^3*(a + b)*Sinh[x] + 2*b^2*(9
*a + 4*b)*(a + 2*b + a*Cosh[2*x])*Sinh[x]))/(3*a^2*(a + b)^3)))/(8*(a + b*S
ech[x]^2)^(5/2))
```

fricas [B] time = 4.18, size = 11205, normalized size = 84.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*sech(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^10 + 10*(a^5 + 3*a^
4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)*sinh(x)^9 + (a^5 + 3*a^4*b + 3*a^3*b^2 +
 a^2*b^3)*sinh(x)^10 + (3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^
4)*cosh(x)^8 + (3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4 + 45*(
a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^2)*sinh(x)^8 + 8*(15*(a^5 + 3*
a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^3 + (3*a^5 + 17*a^4*b + 33*a^3*b^2 + 2
7*a^2*b^3 + 8*a*b^4)*cosh(x))*sinh(x)^7 + 2*(a^5 + 7*a^4*b + 23*a^3*b^2 + 3
7*a^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x)^6 + 2*(a^5 + 7*a^4*b + 23*a^3*b^2 + 3
7*a^2*b^3 + 28*a*b^4 + 8*b^5 + 105*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*co
sh(x)^4 + 14*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*cosh(x)
^2)*sinh(x)^6 + 4*(63*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^5 + 14*
(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*cosh(x)^3 + 3*(a^5 +
 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x))*sinh(x)^5 -
 a^5 - 3*a^4*b - 3*a^3*b^2 - a^2*b^3 - 2*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a
^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x)^4 + 2*(105*(a^5 + 3*a^4*b + 3*a^3*b^2 +
a^2*b^3)*cosh(x)^6 - a^5 - 7*a^4*b - 23*a^3*b^2 - 37*a^2*b^3 - 28*a*b^4 - 8
*b^5 + 35*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*cosh(x)^4
+ 15*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x)^2
)*sinh(x)^4 + 8*(15*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^7 + 7*(3*
a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*cosh(x)^5 + 5*(a^5 + 7*
```

a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x)^3 - (a^5 + 7*a^
4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x))*sinh(x)^3 - (3*a
^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*cosh(x)^2 + (45*(a^5 + 3
*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^8 + 28*(3*a^5 + 17*a^4*b + 33*a^3*b^2
 + 27*a^2*b^3 + 8*a*b^4)*cosh(x)^6 - 3*a^5 - 17*a^4*b - 33*a^3*b^2 - 27*a^2
*b^3 - 8*a*b^4 + 30*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8
*b^5)*cosh(x)^4 - 12*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 +
8*b^5)*cosh(x)^2)*sinh(x)^2 + 2*(5*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*co
sh(x)^9 + 4*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*cosh(x)^
7 + 6*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x)^
5 - 4*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x)^
3 - (3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*cosh(x))*sinh(x)
)*sqrt(a)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^
8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh(x
)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3 + 4
*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^2
 - 30*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a*b^
2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2
*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*cosh(
x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + s
qrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cos
h(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*c
osh(x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*co
sh(x)^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)
^3 - (a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^
2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*cosh
(x)^7 - 3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3 + (
a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh
(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(
x)*sinh(x)^5 + sinh(x)^6)) + 3*((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(
x)^10 + 10*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)*sinh(x)^9 + (a^5 +
 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*sinh(x)^10 + (3*a^5 + 17*a^4*b + 33*a^3*b^2
 + 27*a^2*b^3 + 8*a*b^4)*cosh(x)^8 + (3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^
2*b^3 + 8*a*b^4 + 45*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^2)*sinh(
x)^8 + 8*(15*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^3 + (3*a^5 + 17*
a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*cosh(x))*sinh(x)^7 + 2*(a^5 + 7*
a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x)^6 + 2*(a^5 + 7*
a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5 + 105*(a^5 + 3*a^4*b + 3
*a^3*b^2 + a^2*b^3)*cosh(x)^4 + 14*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*
b^3 + 8*a*b^4)*cosh(x)^2)*sinh(x)^6 + 4*(63*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^
2*b^3)*cosh(x)^5 + 14*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4
)*cosh(x)^3 + 3*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5
)*cosh(x))*sinh(x)^5 - a^5 - 3*a^4*b - 3*a^3*b^2 - a^2*b^3 - 2*(a^5 + 7*a^4
*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x)^4 + 2*(105*(a^5 +
3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^6 - a^5 - 7*a^4*b - 23*a^3*b^2 - 37*

```
a^2*b^3 - 28*a*b^4 - 8*b^5 + 35*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3
 + 8*a*b^4)*cosh(x)^4 + 15*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*
b^4 + 8*b^5)*cosh(x)^2)*sinh(x)^4 + 8*(15*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*
b^3)*cosh(x)^7 + 7*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*c
osh(x)^5 + 5*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*c
osh(x)^3 - (a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*cos
h(x))*sinh(x)^3 - (3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*co
sh(x)^2 + (45*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^8 + 28*(3*a^5 +
 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*cosh(x)^6 - 3*a^5 - 17*a^4*b
 - 33*a^3*b^2 - 27*a^2*b^3 - 8*a*b^4 + 30*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*
a^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x)^4 - 12*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37
*a^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x)^2)*sinh(x)^2 + 2*(5*(a^5 + 3*a^4*b + 3
*a^3*b^2 + a^2*b^3)*cosh(x)^9 + 4*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b
^3 + 8*a*b^4)*cosh(x)^7 + 6*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a
*b^4 + 8*b^5)*cosh(x)^5 - 4*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a
*b^4 + 8*b^5)*cosh(x)^3 - (3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a
*b^4)*cosh(x))*sinh(x))*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 +
 a*sinh(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + b)*sinh(x)^2 +
sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*sqrt((a*cos
h(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)
) + 4*(a*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*s
inh(x) + sinh(x)^2)) - 4*sqrt(2)*((3*a^5 + 9*a^3*b^2 + 4*a^2*b^3)*cosh(x)^8
 + 8*(3*a^5 + 9*a^3*b^2 + 4*a^2*b^3)*cosh(x)*sinh(x)^7 + (3*a^5 + 9*a^3*b^2
 + 4*a^2*b^3)*sinh(x)^8 + 4*(3*a^5 + 6*a^4*b + 8*a^2*b^3 + 3*a*b^4)*cosh(x)
^6 + 4*(3*a^5 + 6*a^4*b + 8*a^2*b^3 + 3*a*b^4 + 7*(3*a^5 + 9*a^3*b^2 + 4*a^
2*b^3)*cosh(x)^2)*sinh(x)^6 + 8*(7*(3*a^5 + 9*a^3*b^2 + 4*a^2*b^3)*cosh(x)^
3 + 3*(3*a^5 + 6*a^4*b + 8*a^2*b^3 + 3*a*b^4)*cosh(x))*sinh(x)^5 + 3*a^5 +
9*a^3*b^2 + 4*a^2*b^3 + 6*(3*a^5 + 8*a^4*b + 5*a^3*b^2 - 12*a^2*b^3 - 4*a*b
^4)*cosh(x)^4 + 2*(9*a^5 + 24*a^4*b + 15*a^3*b^2 - 36*a^2*b^3 - 12*a*b^4 +
35*(3*a^5 + 9*a^3*b^2 + 4*a^2*b^3)*cosh(x)^4 + 30*(3*a^5 + 6*a^4*b + 8*a^2*
b^3 + 3*a*b^4)*cosh(x)^2)*sinh(x)^4 + 8*(7*(3*a^5 + 9*a^3*b^2 + 4*a^2*b^3)*
cosh(x)^5 + 10*(3*a^5 + 6*a^4*b + 8*a^2*b^3 + 3*a*b^4)*cosh(x)^3 + 3*(3*a^5
 + 8*a^4*b + 5*a^3*b^2 - 12*a^2*b^3 - 4*a*b^4)*cosh(x))*sinh(x)^3 + 4*(3*a^
5 + 6*a^4*b + 8*a^2*b^3 + 3*a*b^4)*cosh(x)^2 + 4*(7*(3*a^5 + 9*a^3*b^2 + 4*
a^2*b^3)*cosh(x)^6 + 3*a^5 + 6*a^4*b + 8*a^2*b^3 + 3*a*b^4 + 15*(3*a^5 + 6*
a^4*b + 8*a^2*b^3 + 3*a*b^4)*cosh(x)^4 + 9*(3*a^5 + 8*a^4*b + 5*a^3*b^2 - 1
2*a^2*b^3 - 4*a*b^4)*cosh(x)^2)*sinh(x)^2 + 8*((3*a^5 + 9*a^3*b^2 + 4*a^2*b
^3)*cosh(x)^7 + 3*(3*a^5 + 6*a^4*b + 8*a^2*b^3 + 3*a*b^4)*cosh(x)^5 + 3*(3*
a^5 + 8*a^4*b + 5*a^3*b^2 - 12*a^2*b^3 - 4*a*b^4)*cosh(x)^3 + (3*a^5 + 6*a^
4*b + 8*a^2*b^3 + 3*a*b^4)*cosh(x))*sinh(x))*sqrt((a*cosh(x)^2 + a*sinh(x)^
2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^8 + 3*a^7*b
+ 3*a^6*b^2 + a^5*b^3)*cosh(x)^10 + 10*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3
)*cosh(x)*sinh(x)^9 + (a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*sinh(x)^10 + (3
*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*cosh(x)^8 + (3*a^8 +
 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4 + 45*(a^8 + 3*a^7*b + 3*a^6
```

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*b^2 + a^5*b^3)*cosh(x)^2)*sinh(x)^8 - a^8 - 3*a^7*b - 3*a^6*b^2 - a^5*b^3
+ 8*(15*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*cosh(x)^3 + (3*a^8 + 17*a^7*b
 + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*cosh(x))*sinh(x)^7 + 2*(a^8 + 7*a^7
*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*cosh(x)^6 + 2*(a^8 +
 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5 + 105*(a^8 + 3*
a^7*b + 3*a^6*b^2 + a^5*b^3)*cosh(x)^4 + 14*(3*a^8 + 17*a^7*b + 33*a^6*b^2
+ 27*a^5*b^3 + 8*a^4*b^4)*cosh(x)^2)*sinh(x)^6 + 4*(63*(a^8 + 3*a^7*b + 3*a
^6*b^2 + a^5*b^3)*cosh(x)^5 + 14*(3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^
3 + 8*a^4*b^4)*cosh(x)^3 + 3*(a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*
a^4*b^4 + 8*a^3*b^5)*cosh(x))*sinh(x)^5 - 2*(a^8 + 7*a^7*b + 23*a^6*b^2 + 3
7*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*cosh(x)^4 - 2*(a^8 + 7*a^7*b + 23*a^6*b
^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5 - 105*(a^8 + 3*a^7*b + 3*a^6*b^2 +
 a^5*b^3)*cosh(x)^6 - 35*(3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^
4*b^4)*cosh(x)^4 - 15*(a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4
 + 8*a^3*b^5)*cosh(x)^2)*sinh(x)^4 + 8*(15*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5
*b^3)*cosh(x)^7 + 7*(3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4
)*cosh(x)^5 + 5*(a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a
^3*b^5)*cosh(x)^3 - (a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 +
 8*a^3*b^5)*cosh(x))*sinh(x)^3 - (3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^
3 + 8*a^4*b^4)*cosh(x)^2 + (45*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*cosh(x
)^8 - 3*a^8 - 17*a^7*b - 33*a^6*b^2 - 27*a^5*b^3 - 8*a^4*b^4 + 28*(3*a^8 +
17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*cosh(x)^6 + 30*(a^8 + 7*a^7
*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*cosh(x)^4 - 12*(a^8
+ 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*cosh(x)^2)*si
nh(x)^2 + 2*(5*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*cosh(x)^9 + 4*(3*a^8 +
 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*cosh(x)^7 + 6*(a^8 + 7*a^7
*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*cosh(x)^5 - 4*(a^8 +
 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*cosh(x)^3 - (3
*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*cosh(x))*sinh(x)), -
1/6*(3*((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^10 + 10*(a^5 + 3*a^4*
b + 3*a^3*b^2 + a^2*b^3)*cosh(x)*sinh(x)^9 + (a^5 + 3*a^4*b + 3*a^3*b^2 + a
^2*b^3)*sinh(x)^10 + (3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)
*cosh(x)^8 + (3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4 + 45*(a^
5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^2)*sinh(x)^8 + 8*(15*(a^5 + 3*a^
4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^3 + (3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*
a^2*b^3 + 8*a*b^4)*cosh(x))*sinh(x)^7 + 2*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*
a^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x)^6 + 2*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*
a^2*b^3 + 28*a*b^4 + 8*b^5 + 105*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh
(x)^4 + 14*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*cosh(x)^2
)*sinh(x)^6 + 4*(63*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^5 + 14*(3
*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*cosh(x)^3 + 3*(a^5 + 7
*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x))*sinh(x)^5 - a
^5 - 3*a^4*b - 3*a^3*b^2 - a^2*b^3 - 2*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2
*b^3 + 28*a*b^4 + 8*b^5)*cosh(x)^4 + 2*(105*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^
2*b^3)*cosh(x)^6 - a^5 - 7*a^4*b - 23*a^3*b^2 - 37*a^2*b^3 - 28*a*b^4 - 8*b
```

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^5 + 35*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*cosh(x)^4 +
15*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x)^2)*
sinh(x)^4 + 8*(15*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^7 + 7*(3*a^
5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*cosh(x)^5 + 5*(a^5 + 7*a^
4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x)^3 - (a^5 + 7*a^4*
b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x))*sinh(x)^3 - (3*a^5
 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*cosh(x)^2 + (45*(a^5 + 3*a
^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^8 + 28*(3*a^5 + 17*a^4*b + 33*a^3*b^2 +
 27*a^2*b^3 + 8*a*b^4)*cosh(x)^6 - 3*a^5 - 17*a^4*b - 33*a^3*b^2 - 27*a^2*b
^3 - 8*a*b^4 + 30*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b
^5)*cosh(x)^4 - 12*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*
b^5)*cosh(x)^2)*sinh(x)^2 + 2*(5*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh
(x)^9 + 4*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*cosh(x)^7
+ 6*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x)^5
- 4*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x)^3
- (3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*cosh(x))*sinh(x))*
sqrt(-a)*arctan(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 +
a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(
x)*sinh(x) + sinh(x)^2))/(a*b*cosh(x)^4 + 4*a*b*cosh(x)*sinh(x)^3 + a*b*sin
h(x)^4 - (a^2 + 3*a*b)*cosh(x)^2 + (6*a*b*cosh(x)^2 - a^2 - 3*a*b)*sinh(x)^
2 - a^2 + 2*(2*a*b*cosh(x)^3 - (a^2 + 3*a*b)*cosh(x))*sinh(x))) + 3*((a^5 +
 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^10 + 10*(a^5 + 3*a^4*b + 3*a^3*b^2
+ a^2*b^3)*cosh(x)*sinh(x)^9 + (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*sinh(x
)^10 + (3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*cosh(x)^8 + (
3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4 + 45*(a^5 + 3*a^4*b +
3*a^3*b^2 + a^2*b^3)*cosh(x)^2)*sinh(x)^8 + 8*(15*(a^5 + 3*a^4*b + 3*a^3*b^
2 + a^2*b^3)*cosh(x)^3 + (3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*
b^4)*cosh(x))*sinh(x)^7 + 2*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a
*b^4 + 8*b^5)*cosh(x)^6 + 2*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a
*b^4 + 8*b^5 + 105*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^4 + 14*(3*
a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*cosh(x)^2)*sinh(x)^6 +
4*(63*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^5 + 14*(3*a^5 + 17*a^4*
b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*cosh(x)^3 + 3*(a^5 + 7*a^4*b + 23*a^
3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x))*sinh(x)^5 - a^5 - 3*a^4*b -
 3*a^3*b^2 - a^2*b^3 - 2*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^
4 + 8*b^5)*cosh(x)^4 + 2*(105*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)
^6 - a^5 - 7*a^4*b - 23*a^3*b^2 - 37*a^2*b^3 - 28*a*b^4 - 8*b^5 + 35*(3*a^5
 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*cosh(x)^4 + 15*(a^5 + 7*a^
4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x)^2)*sinh(x)^4 + 8*
(15*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^7 + 7*(3*a^5 + 17*a^4*b +
 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*cosh(x)^5 + 5*(a^5 + 7*a^4*b + 23*a^3*b
^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x)^3 - (a^5 + 7*a^4*b + 23*a^3*b^2
 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x))*sinh(x)^3 - (3*a^5 + 17*a^4*b +
33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*cosh(x)^2 + (45*(a^5 + 3*a^4*b + 3*a^3*b
^2 + a^2*b^3)*cosh(x)^8 + 28*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 +
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8*a*b^4)*cosh(x)^6 - 3*a^5 - 17*a^4*b - 33*a^3*b^2 - 27*a^2*b^3 - 8*a*b^4 +
 30*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x)^4
- 12*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x)^2
)*sinh(x)^2 + 2*(5*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^9 + 4*(3*a
^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*cosh(x)^7 + 6*(a^5 + 7*a
^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x)^5 - 4*(a^5 + 7*a
^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x)^3 - (3*a^5 + 17*
a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*cosh(x))*sinh(x))*sqrt(-a)*arcta
n(sqrt(2)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 -
2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh
(x)^2 + a)) + 2*sqrt(2)*((3*a^5 + 9*a^3*b^2 + 4*a^2*b^3)*cosh(x)^8 + 8*(3*a
^5 + 9*a^3*b^2 + 4*a^2*b^3)*cosh(x)*sinh(x)^7 + (3*a^5 + 9*a^3*b^2 + 4*a^2*
b^3)*sinh(x)^8 + 4*(3*a^5 + 6*a^4*b + 8*a^2*b^3 + 3*a*b^4)*cosh(x)^6 + 4*(3
*a^5 + 6*a^4*b + 8*a^2*b^3 + 3*a*b^4 + 7*(3*a^5 + 9*a^3*b^2 + 4*a^2*b^3)*co
sh(x)^2)*sinh(x)^6 + 8*(7*(3*a^5 + 9*a^3*b^2 + 4*a^2*b^3)*cosh(x)^3 + 3*(3*
a^5 + 6*a^4*b + 8*a^2*b^3 + 3*a*b^4)*cosh(x))*sinh(x)^5 + 3*a^5 + 9*a^3*b^2
 + 4*a^2*b^3 + 6*(3*a^5 + 8*a^4*b + 5*a^3*b^2 - 12*a^2*b^3 - 4*a*b^4)*cosh(
x)^4 + 2*(9*a^5 + 24*a^4*b + 15*a^3*b^2 - 36*a^2*b^3 - 12*a*b^4 + 35*(3*a^5
 + 9*a^3*b^2 + 4*a^2*b^3)*cosh(x)^4 + 30*(3*a^5 + 6*a^4*b + 8*a^2*b^3 + 3*a
*b^4)*cosh(x)^2)*sinh(x)^4 + 8*(7*(3*a^5 + 9*a^3*b^2 + 4*a^2*b^3)*cosh(x)^5
 + 10*(3*a^5 + 6*a^4*b + 8*a^2*b^3 + 3*a*b^4)*cosh(x)^3 + 3*(3*a^5 + 8*a^4*
b + 5*a^3*b^2 - 12*a^2*b^3 - 4*a*b^4)*cosh(x))*sinh(x)^3 + 4*(3*a^5 + 6*a^4
*b + 8*a^2*b^3 + 3*a*b^4)*cosh(x)^2 + 4*(7*(3*a^5 + 9*a^3*b^2 + 4*a^2*b^3)*
cosh(x)^6 + 3*a^5 + 6*a^4*b + 8*a^2*b^3 + 3*a*b^4 + 15*(3*a^5 + 6*a^4*b + 8
*a^2*b^3 + 3*a*b^4)*cosh(x)^4 + 9*(3*a^5 + 8*a^4*b + 5*a^3*b^2 - 12*a^2*b^3
 - 4*a*b^4)*cosh(x)^2)*sinh(x)^2 + 8*((3*a^5 + 9*a^3*b^2 + 4*a^2*b^3)*cosh(
x)^7 + 3*(3*a^5 + 6*a^4*b + 8*a^2*b^3 + 3*a*b^4)*cosh(x)^5 + 3*(3*a^5 + 8*a
^4*b + 5*a^3*b^2 - 12*a^2*b^3 - 4*a*b^4)*cosh(x)^3 + (3*a^5 + 6*a^4*b + 8*a
^2*b^3 + 3*a*b^4)*cosh(x))*sinh(x))*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2
*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^8 + 3*a^7*b + 3*a^6*b
^2 + a^5*b^3)*cosh(x)^10 + 10*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*cosh(x)
*sinh(x)^9 + (a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*sinh(x)^10 + (3*a^8 + 17
*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*cosh(x)^8 + (3*a^8 + 17*a^7*b
 + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4 + 45*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^
5*b^3)*cosh(x)^2)*sinh(x)^8 - a^8 - 3*a^7*b - 3*a^6*b^2 - a^5*b^3 + 8*(15*(
a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*cosh(x)^3 + (3*a^8 + 17*a^7*b + 33*a^6
*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*cosh(x))*sinh(x)^7 + 2*(a^8 + 7*a^7*b + 23*a
^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*cosh(x)^6 + 2*(a^8 + 7*a^7*b
+ 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5 + 105*(a^8 + 3*a^7*b + 3
*a^6*b^2 + a^5*b^3)*cosh(x)^4 + 14*(3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*
b^3 + 8*a^4*b^4)*cosh(x)^2)*sinh(x)^6 + 4*(63*(a^8 + 3*a^7*b + 3*a^6*b^2 +
a^5*b^3)*cosh(x)^5 + 14*(3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4
*b^4)*cosh(x)^3 + 3*(a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 +
 8*a^3*b^5)*cosh(x))*sinh(x)^5 - 2*(a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3
 + 28*a^4*b^4 + 8*a^3*b^5)*cosh(x)^4 - 2*(a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a
```

```
^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5 - 105*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)
*cosh(x)^6 - 35*(3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*co
sh(x)^4 - 15*(a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*
b^5)*cosh(x)^2)*sinh(x)^4 + 8*(15*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*cos
h(x)^7 + 7*(3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*cosh(x)
^5 + 5*(a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*c
osh(x)^3 - (a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^
5)*cosh(x))*sinh(x)^3 - (3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4
*b^4)*cosh(x)^2 + (45*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*cosh(x)^8 - 3*a
^8 - 17*a^7*b - 33*a^6*b^2 - 27*a^5*b^3 - 8*a^4*b^4 + 28*(3*a^8 + 17*a^7*b
+ 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*cosh(x)^6 + 30*(a^8 + 7*a^7*b + 23*a
^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*cosh(x)^4 - 12*(a^8 + 7*a^7*b
 + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*cosh(x)^2)*sinh(x)^2 +
 2*(5*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*cosh(x)^9 + 4*(3*a^8 + 17*a^7*b
 + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*cosh(x)^7 + 6*(a^8 + 7*a^7*b + 23*a
^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*cosh(x)^5 - 4*(a^8 + 7*a^7*b
+ 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*cosh(x)^3 - (3*a^8 + 17
*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*cosh(x))*sinh(x))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*sech(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 0.51Error: Bad Argument Typ
e
```

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{\coth^{2}(x)}{\left(a+b\,\mathrm{sech}(x)^{2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(a+b*sech(x)^2)^(5/2),x)
```

```
[Out] int(coth(x)^2/(a+b*sech(x)^2)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(x)^{2}}{\left(b\,\mathrm{sech}(x)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*sech(x)^2)^(5/2),x, algorithm="maxima")
[Out] integrate(coth(x)^2/(b*sech(x)^2 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\coth(x)^2}{\left(a+\frac{b}{\cosh(x)^2}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(a + b/cosh(x)^2)^(5/2),x)
[Out] int(coth(x)^2/(a + b/cosh(x)^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(x)}{\left(a+b\operatorname{sech}^2(x)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2/(a+b*sech(x)**2)**(5/2),x)
[Out] Integral(coth(x)**2/(a + b*sech(x)**2)**(5/2), x)
```

3.220 $\int \frac{1}{\left(a+b\,\mathbf{sech}^2(c+dx)\right)^{7/2}}\,dx$

Optimal. Leaf size=183

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a-b\tanh^2(c+dx)+b}}\right)}{a^{7/2}d}-\frac{b(9a+5b)\tanh(c+dx)}{15a^2d(a+b)^2\left(a-b\tanh^2(c+dx)+b\right)^{3/2}}-\frac{b\left(33a^2+40ab+15b^2\right)\tanh(c+dx)}{15a^3d(a+b)^3\sqrt{a-b\tanh^2(c+dx)+b}}-$$

```
[Out] arctanh(a^(1/2)*tanh(d*x+c)/(a+b-b*tanh(d*x+c)^2)^(1/2))/a^(7/2)/d-1/15*b*(
33*a^2+40*a*b+15*b^2)*tanh(d*x+c)/a^3/(a+b)^3/d/(a+b-b*tanh(d*x+c)^2)^(1/2)
-1/5*b*tanh(d*x+c)/a/(a+b)/d/(a+b-b*tanh(d*x+c)^2)^(5/2)-1/15*b*(9*a+5*b)*t
anh(d*x+c)/a^2/(a+b)^2/d/(a+b-b*tanh(d*x+c)^2)^(3/2)
```

Rubi [A] time = 0.19, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4128, 414, 527, 12, 377, 206}

$$-\frac{b\left(33a^2+40ab+15b^2\right)\tanh(c+dx)}{15a^3d(a+b)^3\sqrt{a-b\tanh^2(c+dx)+b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a-b\tanh^2(c+dx)+b}}\right)}{a^{7/2}d}-\frac{b(9a+5b)\tanh(c+dx)}{15a^2d(a+b)^2\left(a-b\tanh^2(c+dx)+b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^(-7/2),x]
```

```
[Out] ArcTanh[(Sqrt[a]*Tanh[c + d*x])/Sqrt[a + b - b*Tanh[c + d*x]^2]]/(a^(7/2)*d
) - (b*Tanh[c + d*x])/(5*a*(a + b)*d*(a + b - b*Tanh[c + d*x]^2)^(5/2)) - (
b*(9*a + 5*b)*Tanh[c + d*x])/(15*a^2*(a + b)^2*d*(a + b - b*Tanh[c + d*x]^2
)^(3/2)) - (b*(33*a^2 + 40*a*b + 15*b^2)*Tanh[c + d*x])/(15*a^3*(a + b)^3*d
*Sqrt[a + b - b*Tanh[c + d*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+b\operatorname{sech}^2(c+dx)\right)^{7/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)(a+b-bx^2)^{7/2}}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{b\tanh(c+dx)}{5a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^{5/2}} - \frac{\text{Subst}\left(\int \frac{-5a-b-4bx^2}{(1-x^2)(a+b-bx^2)^{5/2}}\,dx, x, \tanh\right.}{5a(a+b)d}$$

$$= -\frac{b\tanh(c+dx)}{5a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^{5/2}} - \frac{b(9a+5b)\tanh(c+dx)}{15a^2(a+b)^2d\left(a+b-b\tanh^2(c+dx\right.}$$

$$= -\frac{b\tanh(c+dx)}{5a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^{5/2}} - \frac{b(9a+5b)\tanh(c+dx)}{15a^2(a+b)^2d\left(a+b-b\tanh^2(c+dx\right.}$$

$$= -\frac{b\tanh(c+dx)}{5a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^{5/2}} - \frac{b(9a+5b)\tanh(c+dx)}{15a^2(a+b)^2d\left(a+b-b\tanh^2(c+dx\right.}$$

$$= -\frac{b\tanh(c+dx)}{5a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^{5/2}} - \frac{b(9a+5b)\tanh(c+dx)}{15a^2(a+b)^2d\left(a+b-b\tanh^2(c+dx\right.}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a+b-b\tanh^2(c+dx)}}\right)}{a^{7/2}d} - \frac{b\tanh(c+dx)}{5a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^{5/2}} - \frac{}{15a^2(a+}$$

Mathematica [A] time = 7.98, size = 330, normalized size = 1.80

$$\frac{\operatorname{sech}^7(c+dx)\left(\frac{15}{4}e^{-7(c+dx)}\left(a\left(e^{2(c+dx)}+1\right)^2+4be^{2(c+dx)}\right)^{7/2}\left(\tanh^{-1}\left(\frac{ae^{2(c+dx)}+a+2b}{\sqrt{a}\sqrt{a\left(e^{2(c+dx)}+1\right)^2+4be^{2(c+dx)}}}\right)-\tanh^{-1}\left(\frac{ae^{2(c}}{\sqrt{a}\sqrt{a(e}}\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^(-7/2),x]
```

```
[Out] (Sech[c + d*x]^7*((15*(4*b*E^(2*(c + d*x)) + a*(1 + E^(2*(c + d*x)))^2)^(7/
2)*(ArcTanh[(a + 2*b + a*E^(2*(c + d*x)))/(Sqrt[a]*Sqrt[4*b*E^(2*(c + d*x))
 + a*(1 + E^(2*(c + d*x)))^2])] - ArcTanh[(a + a*E^(2*(c + d*x)) + 2*b*E^(2
*(c + d*x)))/(Sqrt[a]*Sqrt[4*b*E^(2*(c + d*x)) + a*(1 + E^(2*(c + d*x)))^2]
)]))/(4*E^(7*(c + d*x))) - (4*Sqrt[a]*b*(a + 2*b + a*Cosh[2*(c + d*x)])*(13
5*a^4 + 480*a^3*b + 709*a^2*b^2 + 460*a*b^3 + 120*b^4 + 4*a*(45*a^3 + 135*a
^2*b + 117*a*b^2 + 35*b^3)*Cosh[2*(c + d*x)] + a^2*(45*a^2 + 60*a*b + 23*b^
2)*Cosh[4*(c + d*x)])*Sinh[c + d*x])/(a + b)^3))/(960*a^(7/2)*d*(a + b*Sech
[c + d*x]^2)^(7/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)^2)^(7/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)^2)^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 1.44Error: Bad Argument Typ
e
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\,\mathrm{sech}\left(dx+c\right)^{2}\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sech(d*x+c)^2)^(7/2),x)
```

```
[Out] int(1/(a+b*sech(d*x+c)^2)^(7/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,\mathrm{sech}\left(dx+c\right)^{2}+a\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)^2)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sech(d*x + c)^2 + a)^(-7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a + \frac{b}{\cosh(c+d\,x)^2}\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cosh(c + d*x)^2)^(7/2),x)
```

```
[Out] int(1/(a + b/cosh(c + d*x)^2)^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b\operatorname{sech}^{2}\left(c + dx\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)**2)**(7/2),x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**(-7/2), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
```

```
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
```

```
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
    as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
```

```
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:
```

```
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)
   ))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type
   (expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
   Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
```

```
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
    ']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
    sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
```

```
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.
   func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
   expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
   is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
   expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
   expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
   expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
   leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
   expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
```

```
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
        else: #result contains complex but optimal is not
            return "C"
    else: # result do not contain complex, this assumes optimal do not as
well
        if leaf_count_result <= 2*leaf_count_optimal:
            return "A"
        else:
            return "B"
else:
    return "C"
```